

# 10-29-2018 3D Graphics Engine: Shade Computation

CMP210 Advanced Microprocessor Systems Oct 29, 2018. Harry Li

Today's Topics:

1<sup>st</sup> Homework (Due A Week from Today)

Design, Plot

(1) World Coordinate Systems

$X_w$ : Red;  $Y_w$ : Green;  $Z_w$ : Blue;  $E(200, 200, 200)$

OR  $E(100, 100, 100)$ ;  $D = 10 \sim 20$ ;

Each Axis with Length  $\approx 50$

Virtual Display Device

physical Device

(2) Display 3D Cube (with Same Setting)

Size of Each Side of the Cube  $50 \sim 100$ , Wireframe model.

Note: Sin/cos Computation use Example in the class (Ref: 2018F\_1114\_~; (3) Optional (Linear Decoration)

Submission on Line (CANVAS - as Exported project)

Example:

① Generate/Design 2D Pattern (Font)

from your 2 letters Initials  $\rightarrow$  "HL"   
 L Simpler One.

② Decorate  $S_2$ .

Redefine 2D  $\{P_i\}$  in 3D world Coordinate System.  $\{P'_i(x'_i, y'_i, z'_i)\}$

then, Reproject the Data  $\{P'_i(x'_i, y'_i, z'_i)\}$  onto one of the 3 planes

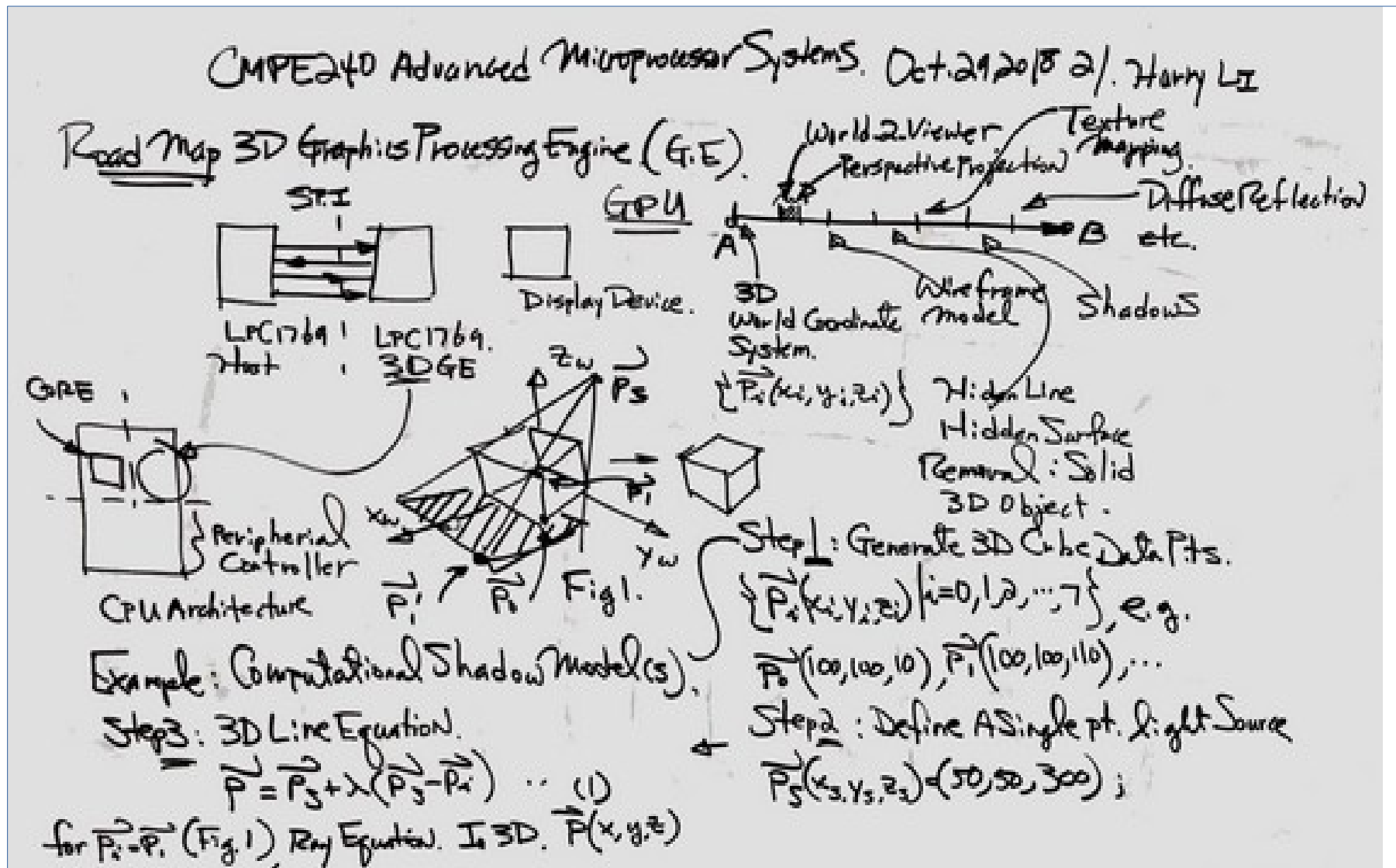
e.g.  $X_w-Y_w, Y_w-Z_w, Z_w-X_w$  (Right Hand System)

After Before Ind. Func Ind. Func Ind. Func.

$\begin{cases} x'_i = x_i \checkmark \\ y'_i = y_i \checkmark \\ z'_i = C \end{cases} \quad \begin{matrix} S_1: \\ Y_w-Z_w \text{ plane} \end{matrix} \quad \begin{cases} x'_i = C \\ y'_i = x_i \\ z'_i = y_i \end{cases} \quad \begin{matrix} S_0: \\ Z_w-X_w \end{matrix} \quad \begin{cases} x'_i = y_i \\ y'_i = C \\ z'_i = x_i \end{cases}$

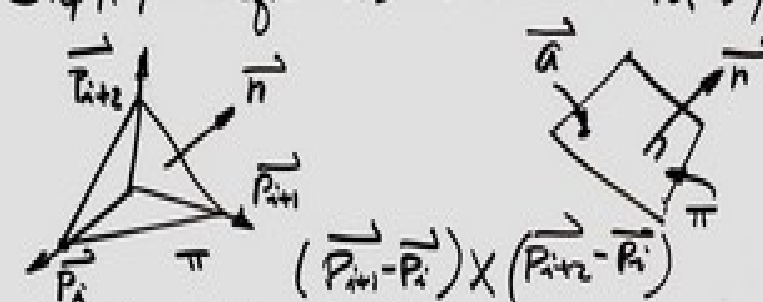
$C = 100$

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Step 4. plane Equation  $\Rightarrow$  Normal Vector  $\vec{n}(n_x, n_y, n_z)$



$$\vec{n} = (\vec{P}_{i+1} - \vec{P}_i) \times (\vec{P}_{i+2} - \vec{P}_i)$$

$$= \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_{i+1} - x_i & y_{i+1} - y_i & z_{i+1} - z_i \\ x_{i+2} - x_i & y_{i+2} - y_i & z_{i+2} - z_i \end{bmatrix} \dots (2)$$

$\vec{n} \cdot (\vec{a} - \vec{v}) = 0 \dots (3)$  where  $\vec{a} = (a_x, a_y, a_z)$   
Any Arbitrary known pt. On the plane.

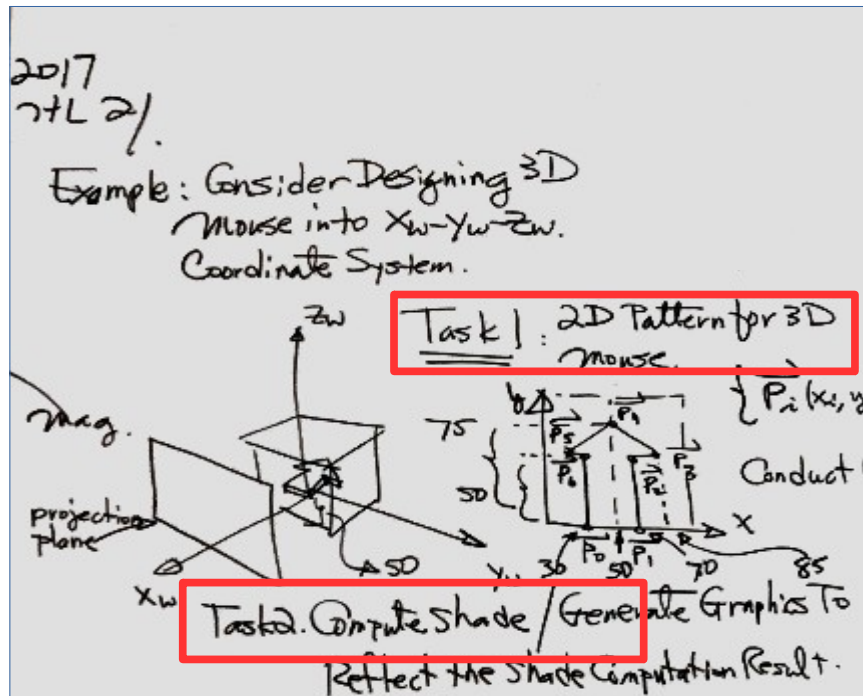
Step 5. Intersect pt.

$$\vec{P} = \vec{P}_s + \lambda (\vec{P}_s - \vec{P}_i) \dots (4)$$

$$\begin{cases} \vec{n} \cdot (\vec{a} - \vec{v}) = 0 \end{cases}$$

$$\vec{n} \cdot (\vec{a} - \vec{v}) \Big|_{\vec{v} = \vec{P}} = 0, \vec{n} \cdot (\vec{a} - \vec{P}) \Big|_{\vec{P} = \vec{P}_s + \lambda (\vec{P}_s - \vec{P}_i)} = 0$$

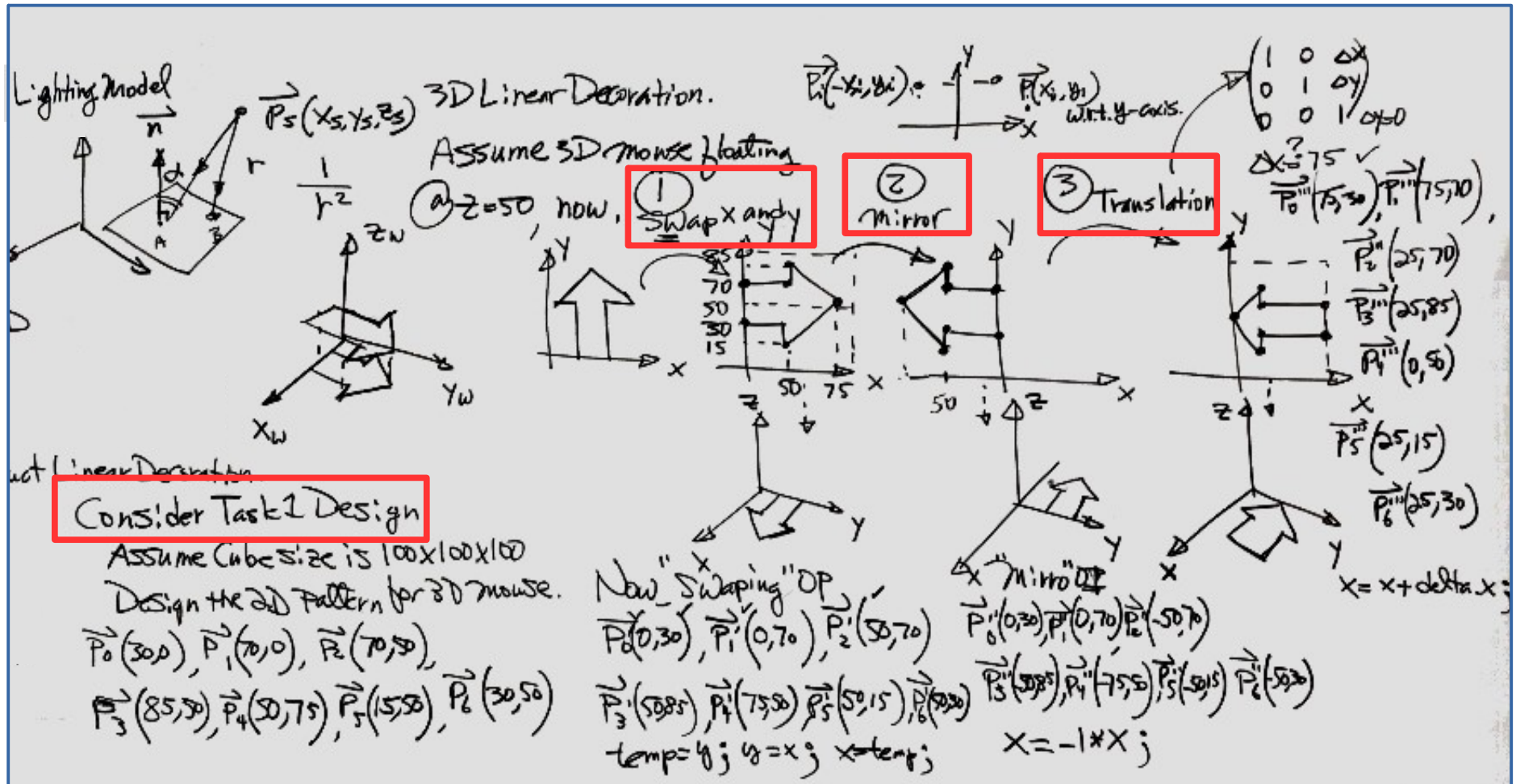
# 2017 Design of 3D Virtual Display



Three tasks:

1. 2D pattern for 3D mouse and perform 3D linear decoration algorithm
2. Compute shade
3. Hidden Line/Surface Removal

# Design 2D Cursor Pattern then 3D Decoration





# 3D Decoration

CMPE163 Introduction To  
Computer Graphics & AR HL.3/

Now, With Linear Decoration we  
can change  $\{\vec{P}_i(x_i, y_i) | i=0, 1, \dots, 6\}$   
to 3D mouse, by adding z-dimension,  
such that  $z_i = 50$ , Hence, we have

$$\left\{ \vec{P}_i(x_i, y_i, z_i) \mid i=0, 1, 2, \dots, 6 \right\}$$

$$z_i = 50$$

$$\vec{P}_0(75, 30, 50), \vec{P}_1(75, 70, 50), \vec{P}_2(25, 70, 50), \vec{P}_3(25, 85, 50)$$

$$\vec{P}_4(0, 50, 50), \vec{P}_5(25, 15, 50), \vec{P}_6(25, 30, 50)$$

Make the pattern with Thickness=5.

$$S1: \{ \vec{P}_i(x_i, y_i, z_i) \mid i=0, 1, 2, \dots, 6 \}$$

Then, (Layer Beneath  $S_1$ )

$$S2: \{ \vec{P}_i(x_i, y_i, z_i - 5) \mid i=0, 1, 2, \dots, 6 \}$$

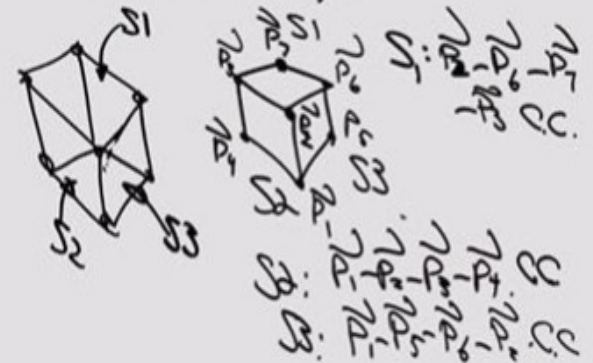
"Wire frame"  $\rightarrow$  Solid Object

Hidden Line / Surface Removal.

Background

1. Define vertices of 3D Object(s) in Counter Clock Wise Direction.  
(When Viewing the Object from the outside)

Simple ~ Based ON  
Vector Cross Product  
Z-Buffer Algorithm.



# Single Point Light Source

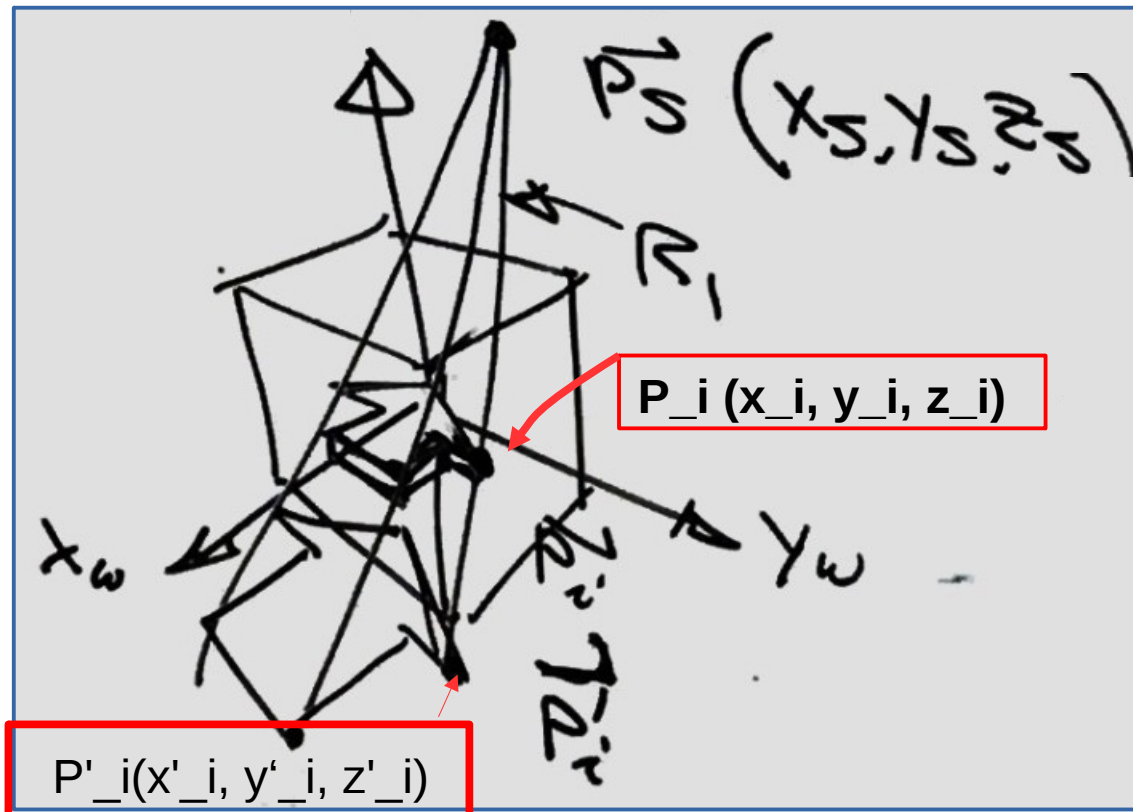
Give a single point light source  $P_s$ , and the 3D cursor as

$\{P_i \mid i = 0, 1, \dots, N-1\}$

Find the intersection points on  $X_w$ - $Y_w$  plane,

$\{P'_i \mid i = 0, 1, \dots, N-1\}$

e.g.,

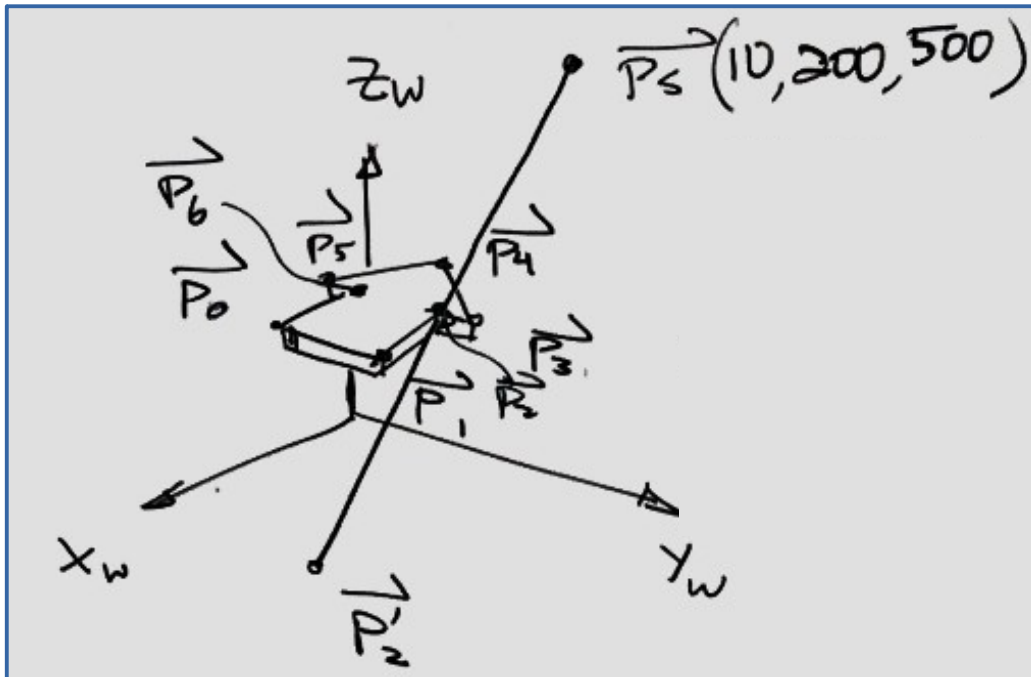


$P_i(x_i, y_i, z_i)$  from the 3D cursor, linked to single point light source

$P_s(x_s, y_s, z_s)$  and formed intersection point

$P'_i(x'_i, y'_i, z'_i)$

# Computing Shade From A Single Point Light Source (1)



$$\text{Ray Equation (Line ~)} \\ \vec{R}_z = \vec{P}_s + \lambda(\vec{P}_s - \vec{P}_2) \quad (1)$$

Plane equation below:

$$\vec{n} \cdot (\vec{r} - \vec{a}) = 0 \quad (2)$$

Where the normal vector of the  $X_w$ - $Y_w$  plane is

$$\vec{n} = (0, 0, 1)$$

And the known point vertex  $a$  is:

$$\vec{a} = (0, 0, 0)$$

$$\text{for } i=0, 1, 2, \dots, 6, \text{ we have generalized} \\ \vec{R}_i(x_i, y_i, z_i) = \vec{P}_s(x_s, y_s, z_s) + \lambda(x_s - x_i, y_s - y_i, z_s - z_i)$$

7 Ray equations for each vertex of the 3D cursor (1.1)



# Computing Shade From A Single Point Light Source (2)

Substitute the known condition into the plane equation, we have

$$(0,0,1) * (\vec{v} - \vec{a}) \Big|_{\vec{a}=\vec{0}} = 0$$

(3)

Note vector V is the common shared point (intersection) of the ray vector, so we have

$$\vec{n} * \vec{v} \Big|_{\vec{v}=\vec{r}_i} = 0$$

(4)

e.g.

$$\vec{n} * \vec{r}_i = 0$$

$$\vec{r}_i = \vec{p}_s + \lambda (\vec{p}_s - \vec{p}_i)$$

Hence,

$$\vec{n} * (\vec{p}_s + \lambda (\vec{p}_s - \vec{p}_i)) = 0$$

(5)

Or

$$\vec{n} * \vec{p}_s + \lambda \vec{n} * (\vec{p}_s - \vec{p}_i) = 0$$

(5.1)

Solve for lamda,

$$\lambda = - \frac{\vec{n} * \vec{p}_s}{\vec{n} * (\vec{p}_s - \vec{p}_i)}$$

(6)

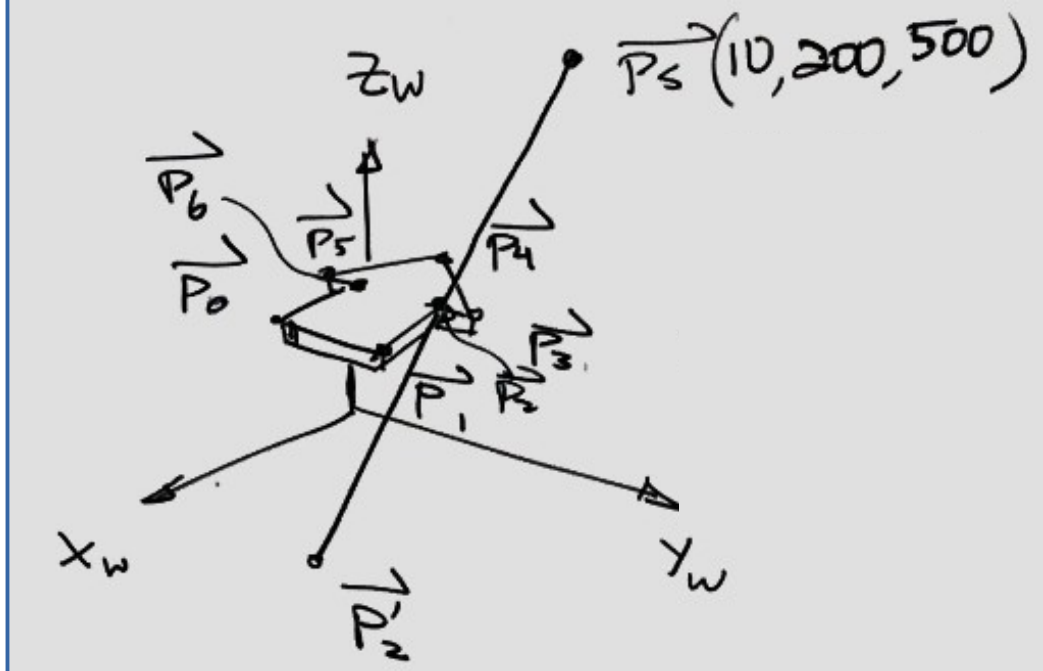
e.g.

$$\lambda = \frac{-(n_x, n_y, n_z) * (x_s, y_s, z_s)}{(n_x, n_y, n_z) * (x_s - x_i, y_s - y_i, z_s - z_i)}$$

(6.1)

## Computing Shade From A Single Point Light Source (3)

Example: Compute the Shade point  $\vec{P}_2'$   
Given  $\vec{P}_2(25, 70, 50)$ ,  $\vec{P}_S(10, 200, 500)$



Then substitute the lamda back to the ray equation to find the intersection point as follows

$$\begin{aligned}\vec{P}_2' &= \vec{P}_5 + \lambda(\vec{P}_5 - \vec{P}_2) \\ &= (10, 200, 500) - \frac{10}{9}(10-25, 200-70, 500-80) \\ &= \left(10 + \frac{150}{9}, 200 - \frac{1300}{9}, \underbrace{500 - \frac{4500}{9}}_0\right)\end{aligned}$$

From equation (6), compute lamda

$$\lambda = - \frac{\sum n_i \cdot P_i}{\sum n_i (P_i - P_2)} = - \frac{n_x \cdot x_5 + n_y \cdot y_5 + n_z \cdot z_5}{n_x(x_5 - x_2) + n_y(y_5 - y_2) + n_z(z_5 - z_2)}$$
$$= - \frac{0 + 0 + 1 \cdot z_5}{0 + 0 + 1 \cdot (z_5 - z_2)} = - \frac{z_5}{z_5 - z_2} = - \frac{500}{500 - 50} = - \frac{500}{450} = - \frac{10}{9}$$

The rest of the points can be computed similarly.