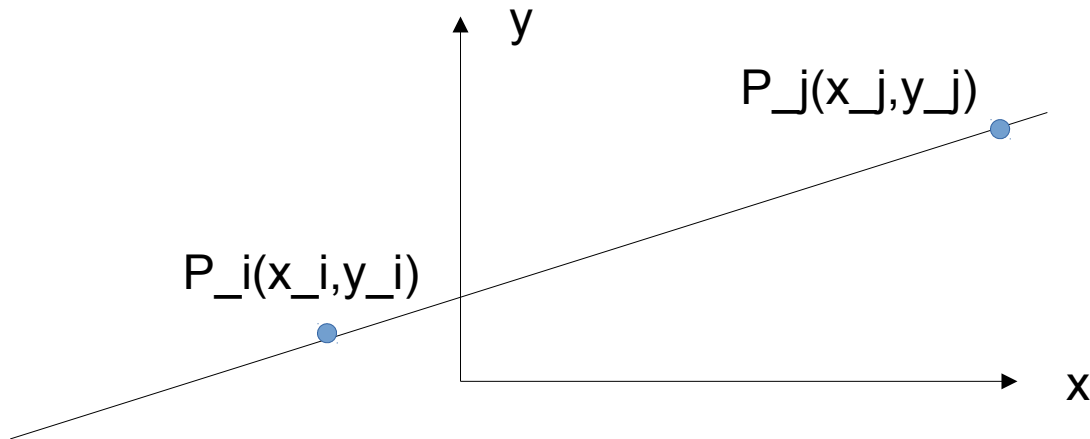


# Introduction to 2D Vector Graphics



Note:

(1) for  $\alpha = 0$  and  $1$  equation (2) returns  $P(x,y) = P_i(x_i, y_i)$ ,  $P(x,y) = P_j(x_j, y_j)$ ;

Based on the above vector form equation, generate screen savers and trees. (Reference: H. Li, IEEE Transactions on Education)

Direction vector  $d(x,y) = (dx, dy)$ , which is defined as

$$d(x,y) = P_i(x_i, y_i) - P_j(x_j, y_j) \quad \dots (1)$$

$$= (x_i - x_j, y_i - y_j) \quad \dots (1-1)$$

Now the vector form equation for the straight line:

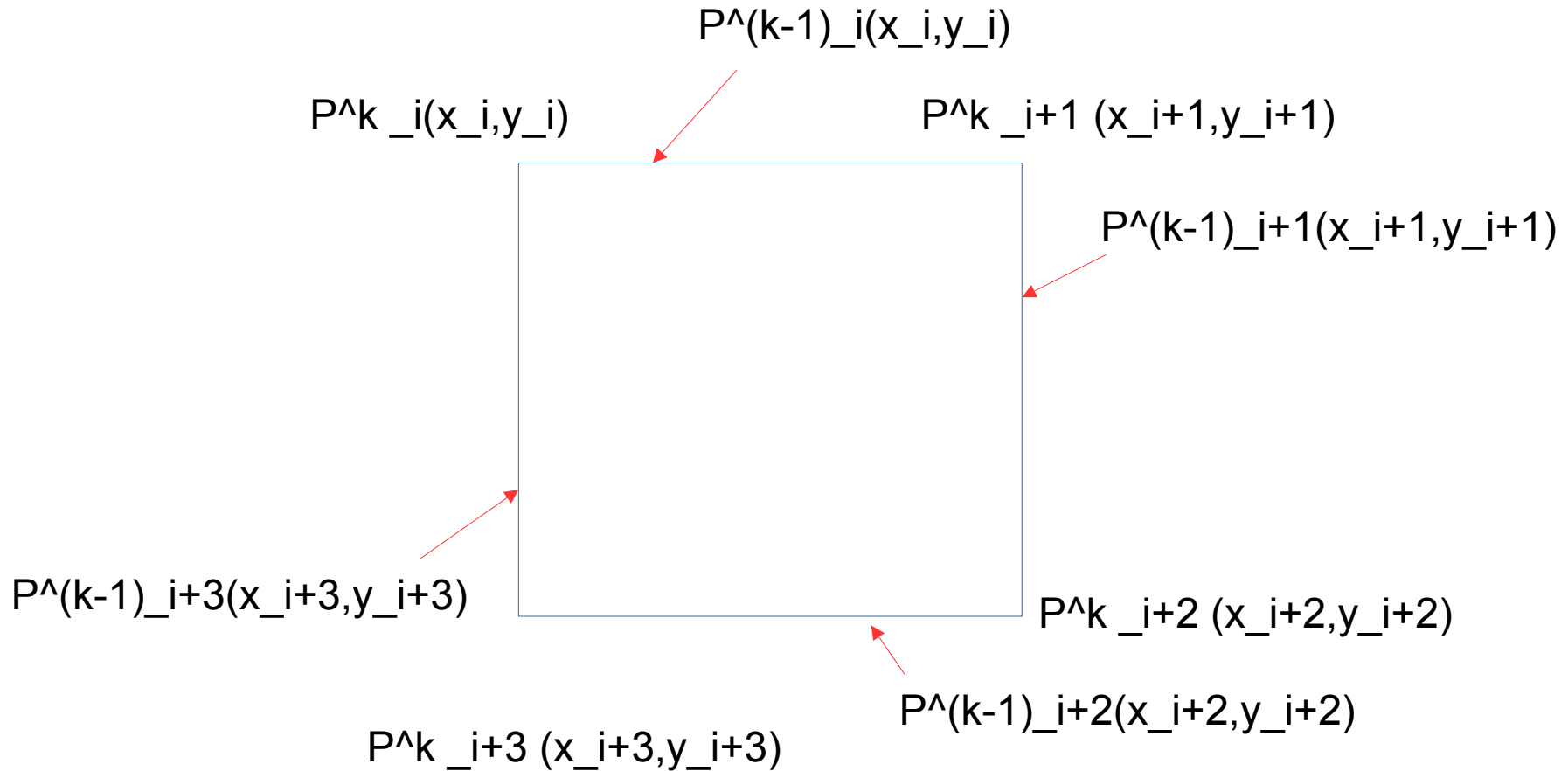
$$P(x,y) = P_i(x_i, y_i) + \alpha * (P_i(x_i, y_i) - P_j(x_j, y_j))$$

$$\dots (2)$$

Where  $0 \leq \alpha \leq 1$



# 2D Vector Graphics to Create Rotating Pattern



From equation (1), we can derive the following equation

$$P^k(x, y) = P^{(k-1)}_i(x_i, y_i) + \alpha * (P^{(k-1)}_i(x_i, y_i) - P^{(k-1)}_j(x_j, y_j)) \dots (3)$$

Choose  $\alpha = 0.8$

# 2D Rotating Pattern Technique

I have created an algorithm to summarize the rotating pattern technique:

Defining a polygon with a set of vertices  $\{(x_i, y_i) | i = 1, 2, \dots, k\}$ , one can use a vector formula to describe an object reduction and rotation as:

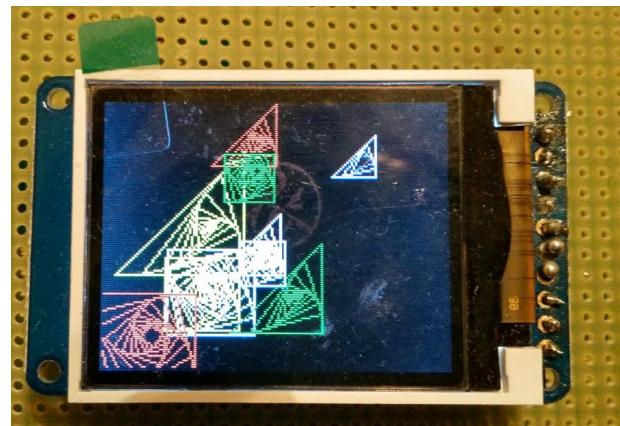
$$(x_i^{l+1}, y_i^{l+1}) = (x_i^l, y_i^l) + \mu(x_{i+1}^l - x_i^l, y_{i+1}^l - y_i^l) \quad (1)$$

where the subscript  $i$  is used to denote each vertex of a given object,  $i = 1, 2, \dots, k$ . If  $i = k$  and  $i + 1 > k$ , then  $i + 1 = 1$ . The superscript  $l$  is used to denote the level of iteration. The constant  $\mu$  is defined as  $0 \leq \mu \leq 1$ , which is used to define the rate of reduction. For example, if  $\mu = 0.5$  then each side of the object is reduced to half. This  $\mu$  is also related to the direction of the rotation. For  $\mu$  less than 0.5, the rotation is toward the current reference point  $(x_i^l, y_i^l)$ , otherwise the rotation is away from the current point. Equation (1) is, in fact, derived directly from a vector addition. Let us assume  $\bar{a} = (x_i^l, y_i^l)$ ,  $\bar{b} = (x_{i+1}^l - x_i^l, y_{i+1}^l - y_i^l)$ , and  $\bar{a}' = (x_i^{l+1}, y_i^{l+1})$ , then (1) can be written as  $\bar{a}' = \bar{a} + \mu(\bar{b} - \bar{a})$  for vertices  $i$  and  $i + 1$  with reduction at level  $l$ . Following

the argument above,  $\mu$  can also be expanded to the range greater than 1 to produce magnification.

Reference:

IEEE TRANSACTIONS ON EDUCATION, VOL. 35, NO. 1,  
Three-Dimensional Computer Graphics  
Using EGA or VGA Card By H. Li



# Computing 2D Rotation Example

# Computing 2D Rotation Example

Example:  $P_1(0, -10)$ ,  $P_2(0, 10)$ ,  
 $\lambda = 0.8$ , find the first left  
 rotation coordinate

Example: System y Dataset

Condition 1: Coordinate established @ the center of the display device.

2.  $\vec{P}_1(0, -10)$ ,  $\vec{P}_2(0, 10)$

Step 1:  $\vec{P}_1 = \vec{P}_0 + \lambda(\vec{P}_2 - \vec{P}_0) \mid \lambda = 0.8$

After  $(x'_i, y'_i, 1)^T = T_{Pos} R \cdot T_{pre} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \dots (?)$

Before  $(x_i, y_i, 1)^T$

Coordinate established @ the center of the display device.

$\vec{P}_0 = (0, 10) + (0, 16) = (0, 26)$

$= (0, 10) + 0.8(0, 20)$

$= (0, 10) + 0.8(0 - 0, 10 - (-10)) \dots (1)$

Step 2. Compute delta-x  
 and delta-y as shown below

$$\begin{aligned} x'_i &= x_i + \Delta x \\ 0 &= x_i + \Delta x \\ \Delta x &= -x_i \mid x_i = 0 = 0 \\ y'_i &= y_i + \Delta y \\ 0 &= y_i + \Delta y \\ \therefore \Delta y &= -y_i \mid y_i = 10 = -10 \end{aligned}$$

Step 2. Recall

After  $(x'_i, y'_i, 1)^T = T_{Pos} R \cdot T_{pre} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} \dots (?)$

Before  $(x_i, y_i, 1)^T$

$$\begin{aligned} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} &\rightarrow \begin{cases} x'_i = 0.866x_i - 0.5y_i + 5 \\ y'_i = 0.5x_i + 0.866y_i + 14 \end{cases} \\ \begin{pmatrix} 0.866 & -0.5 & 5 \\ 0.5 & 0.866 & 14 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ y_i \\ 1 \end{pmatrix} &\rightarrow \begin{cases} x'_i = 0.866x_i - 0.5y_i + 5 \\ y'_i = 0.5x_i + 0.866y_i + 14 \end{cases} \end{aligned}$$

$x'_i = 0.866x_i - 0.5y_i + 5 \mid x_i = 0, y_i = 26 \rightarrow 0 - 13 + 5 = -8$

$y'_i = 0.5x_i + 0.866y_i + 14 \mid x_i = 0, y_i = 26 \rightarrow 0 + 22.716 + 14 = 36.716$