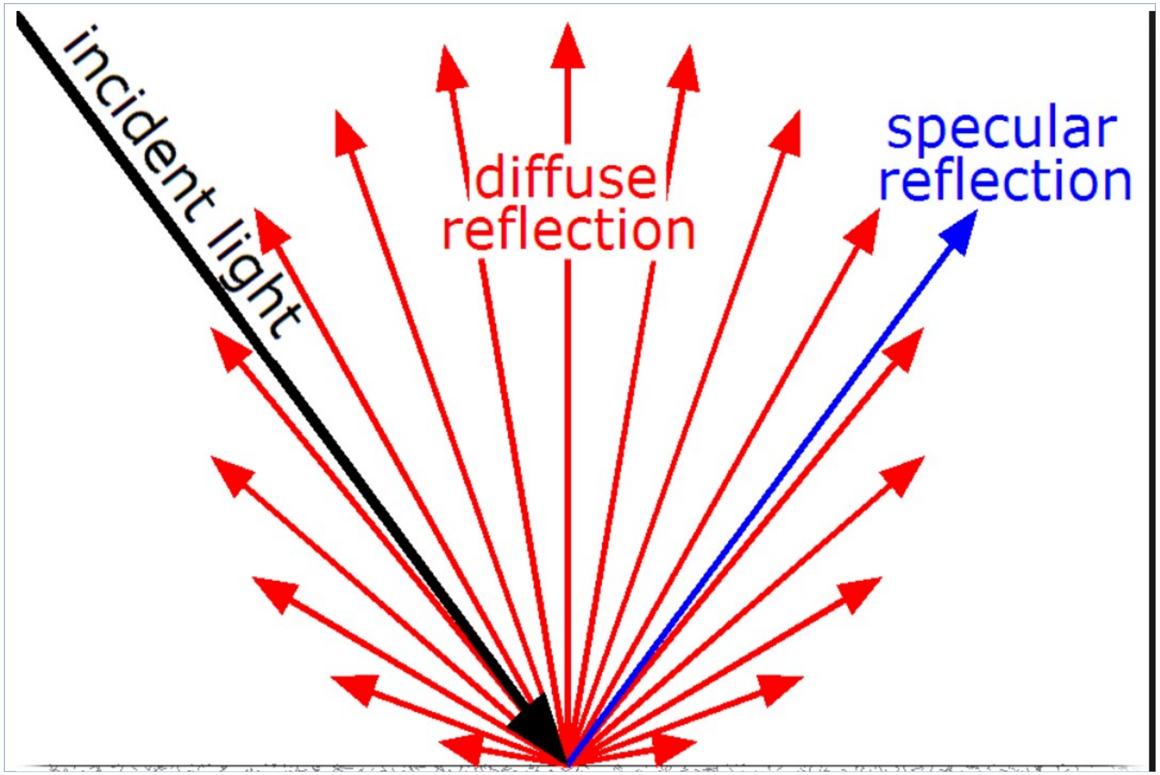
Diffuse Reflection



https://en.wikipedia.org/wiki/Diffuse_reflection

Diffuse Reflection: the reflection of light uniformly in all different directions, the surface of this reflection exhibits Lambert reflection, e.g., equal luminance when viewed from all directions.

Two Key Characteristics:

1. The surface with reflectivity as K_d = (k_r, k_g, k_b), e.g., diffuse coefficients;

2. The decay of incident light is inverse proportional to its distance from the source to the surface point. e.g., 1/(r*r), where r is bing the distance from the light source to the surface.
 Specular vs. diffuse reflection

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Diffuse Reflection Formulation

Light source I_s(x,y) consists of r, g, b 3 primitive colors as follows, but let's simplify it as white color, so r, g, b all equal and have the highest value (if in graphics, they are 255)

$$\boxed{\vec{I_s}(x,y,z) = \left(I_r(x,y,z), I_g(x,y,z), I_b(x,y,z)\right)} \quad \ldots (1)$$

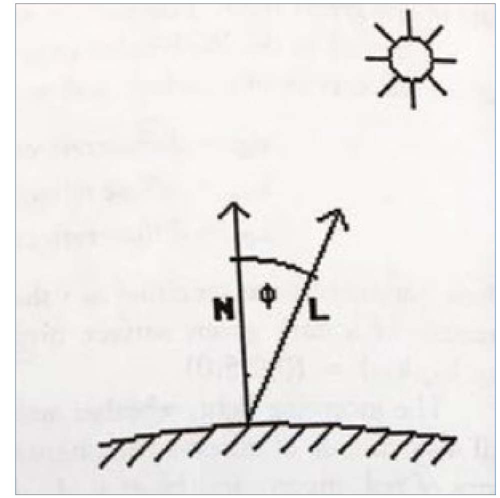
Object surface consists of reflectivity, e.g., coefficient of reflection

$$\boxed{\vec{K_d} = (K_r, K_g, K_b)} \quad \ldots (2)$$

r_d vector in Equation (1) is a ray equation, just like I_s(x,y,z) but has no r, g, b primitive color defined in it for the matter of simplicity.

Diffuse Reflection Equation

Let's consider white color of the point light source, then each primitive color r, g, b of the object surface I(x,y,z) can be computed as follows:



$$I_r = K_{d_r} \frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|} \frac{1}{\|\vec{r}\|_2^2} \quad \dots (1.1)$$

where

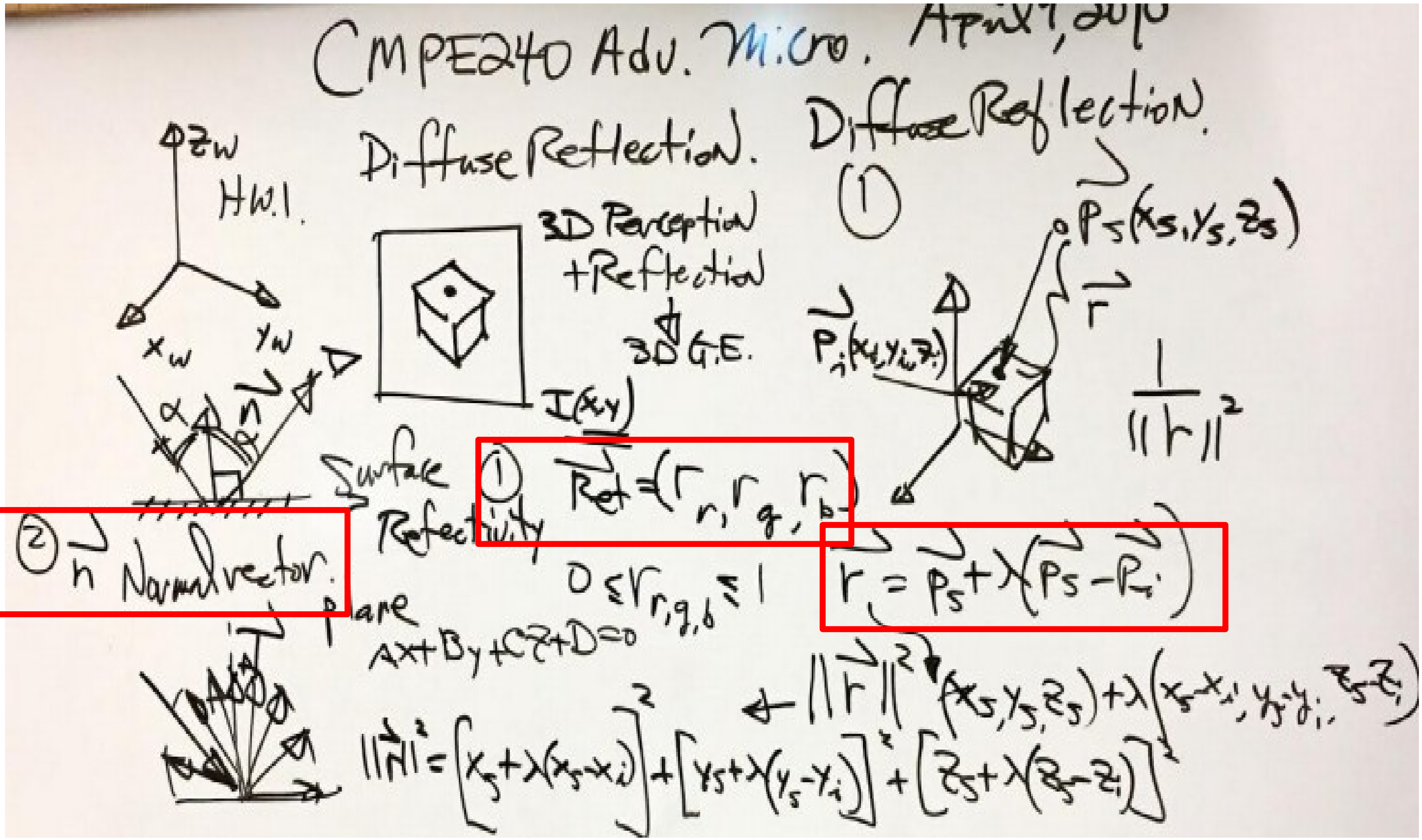
$$\|\vec{r}\|_2^2 = x_r^2 + y_r^2 + z_r^2$$

$$I_g = K_{d_g} \frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|} \cdot \frac{1}{\|\vec{r}\|_2^2} \quad \dots (1.2)$$

$$I_b = K_{d_b} \frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|} \cdot \frac{1}{\|\vec{r}\|_2^2} \quad \dots (1.3)$$

Reference: Computer Graphics, C. K. Pokorny, C. F. Gerald, pp. 514

Formulation Of Diffuse Reflection Equation



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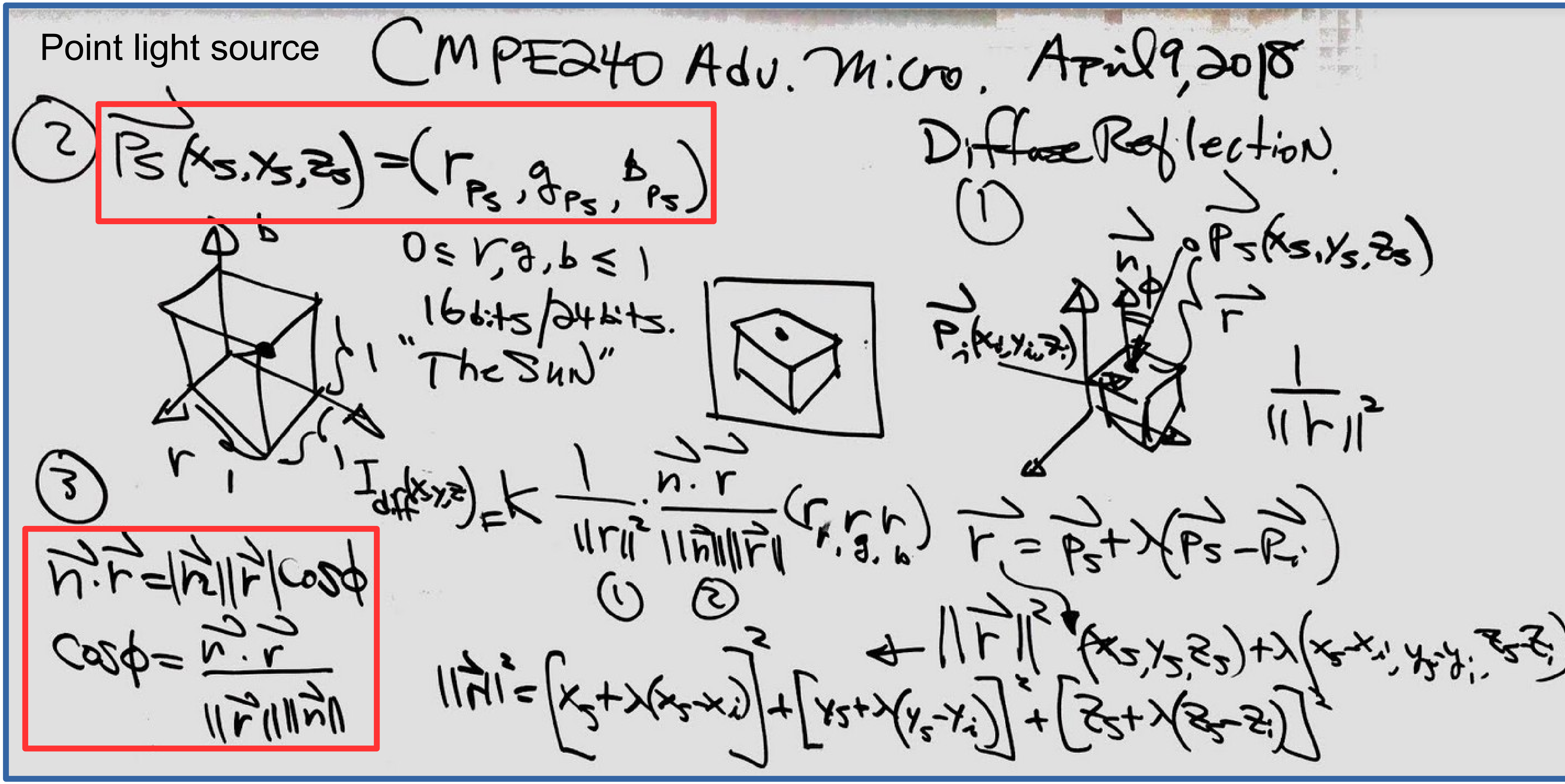
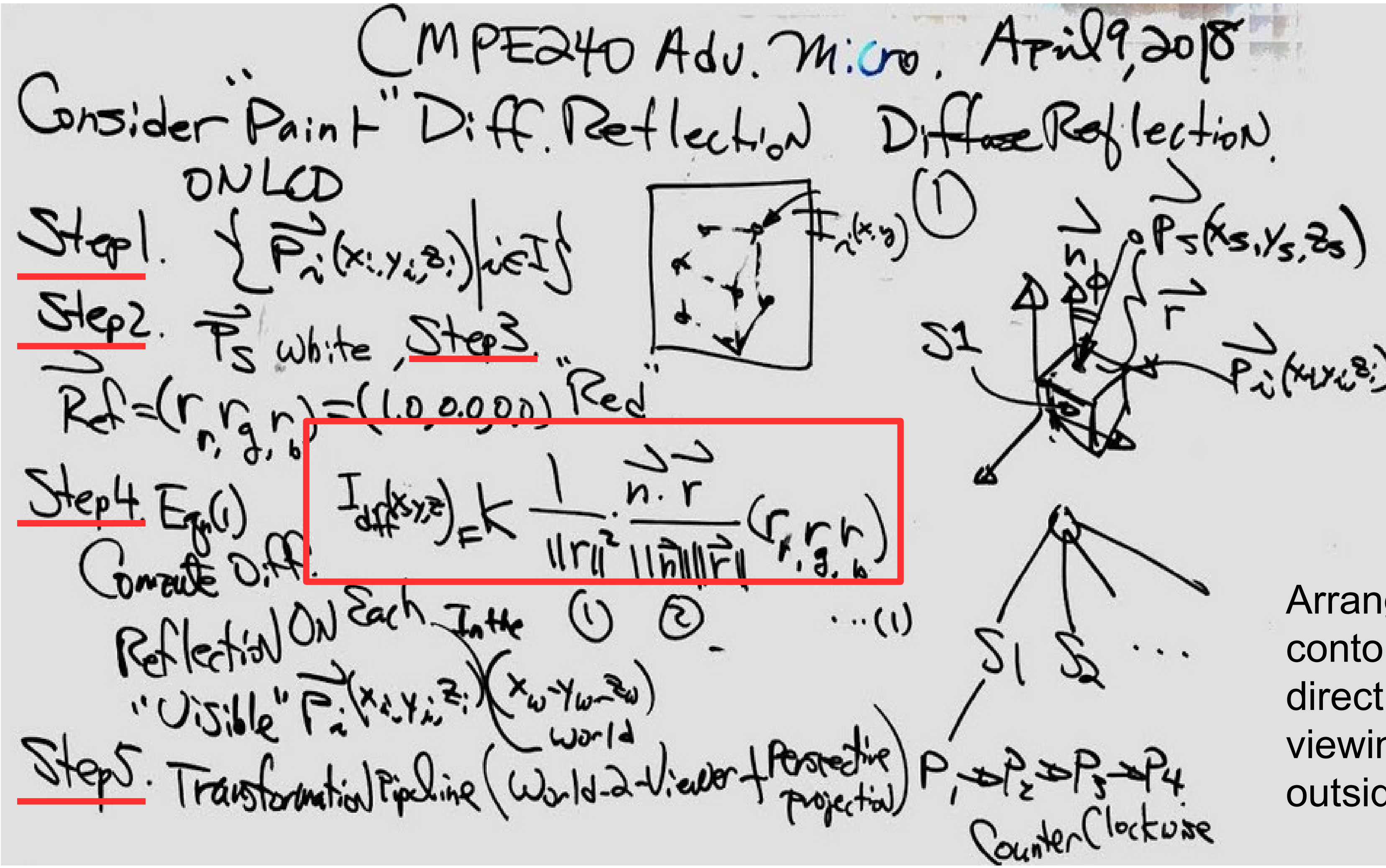
Point Light Source And Incident Angle



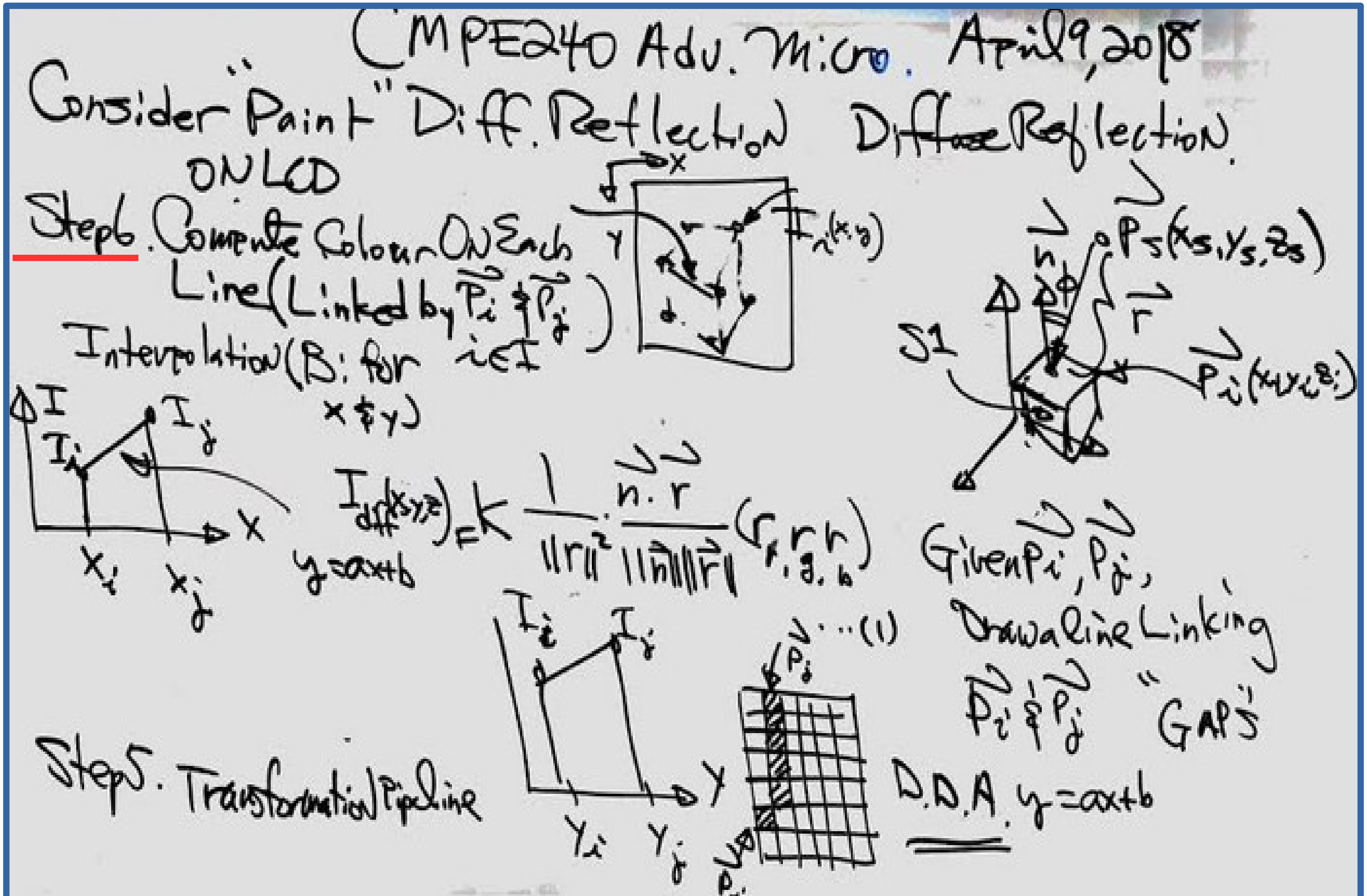
Angle of incident light

Step 1-5 For Diffuse Reflection Computation

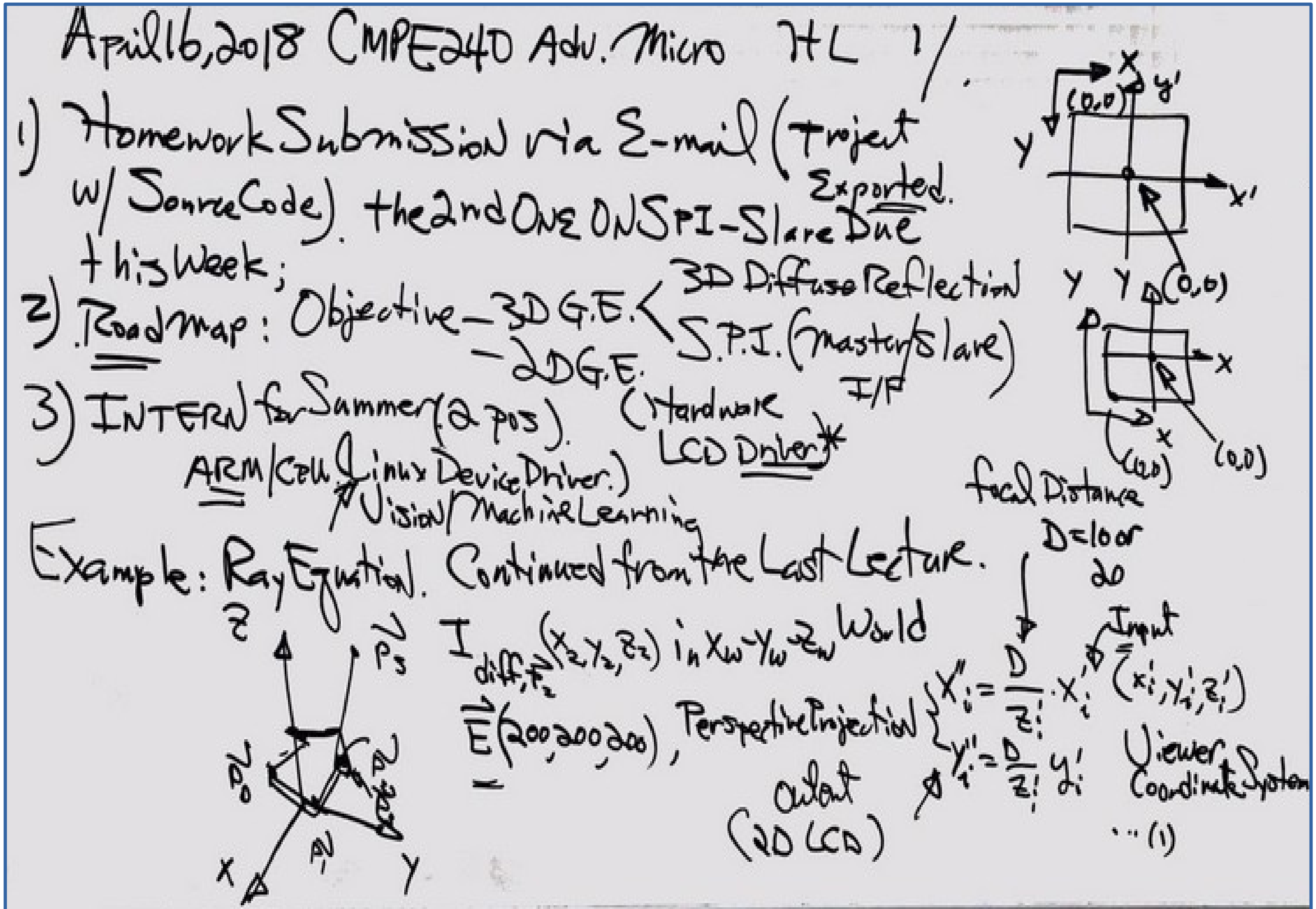


Arrange vertex in contour clock wise direction when viewing from outside

Step 6 For Diffuse Reflection Computation



Example On Diffuse Reflection Computation (1)



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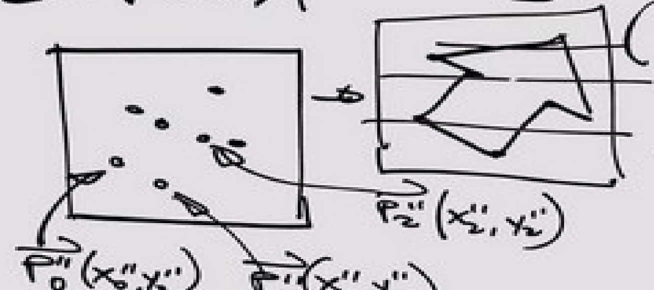
Example On Diffuse Reflection Computation (2)

$\{\vec{P_i}(x_i', y_i', z_i') \mid i=0,1,\dots,6\}$

Hence, from P.P. (Eqn-1), we have

Suppose D=10, find

$\{\vec{P_i}''(x_i'', y_i'') \mid i=0,1,\dots,6\}$



(Location + Color)

{ Interpolation to find Color
{ Find Boundaries?

ON Boundary(s)

"GAPs"

mult

Find Boundary: $y=ax+b$

$y_{k+1}=ax_{k+1}+b,\ x_{k+1}=x_k+1$

D.D.A. (Digital Differential Algorithm)

Example: Given $(x_0,y_0)=(1,1)$, $(x_1,y_1)=(3,5)$

D.D.A. to Draw a Line

$$\text{Slope} = a = \frac{y_1-y_0}{x_1-x_0} = \frac{5-1}{3-1} = 2$$

(Note, if $|a|<1$, then No GAP, $y_{k+1}=ax_{k+1}+b,\ x_{k+1}=x_k$)

Switch x and y.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1},$$

$$y = y_1 + \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

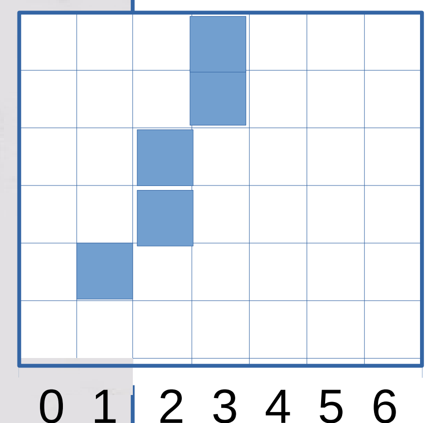
$$= \frac{y_2-y_1}{x_2-x_1}x + y_1 - \frac{y_2-y_1}{x_2-x_1}x_1$$

$y=ax+b$, $\frac{1}{a}y = x + \frac{b}{a}$ (1,1)

$\therefore x = \frac{1}{a}y - \frac{b}{a}$ ...(2)

$$x = \frac{1}{2}y + \frac{1}{2}$$

$$\begin{cases} y_{k+1}=y_k+1 \\ x_{k+1}=\frac{1}{2}y_{k+1}+\frac{1}{2} \end{cases}$$



Example On Diffuse Reflection Computation (3)

Algorithm: 1° Given (x_0, y_0) Starting pt, (x_{N-1}, y_{N-1}) Ending pt.

Find Slop a;

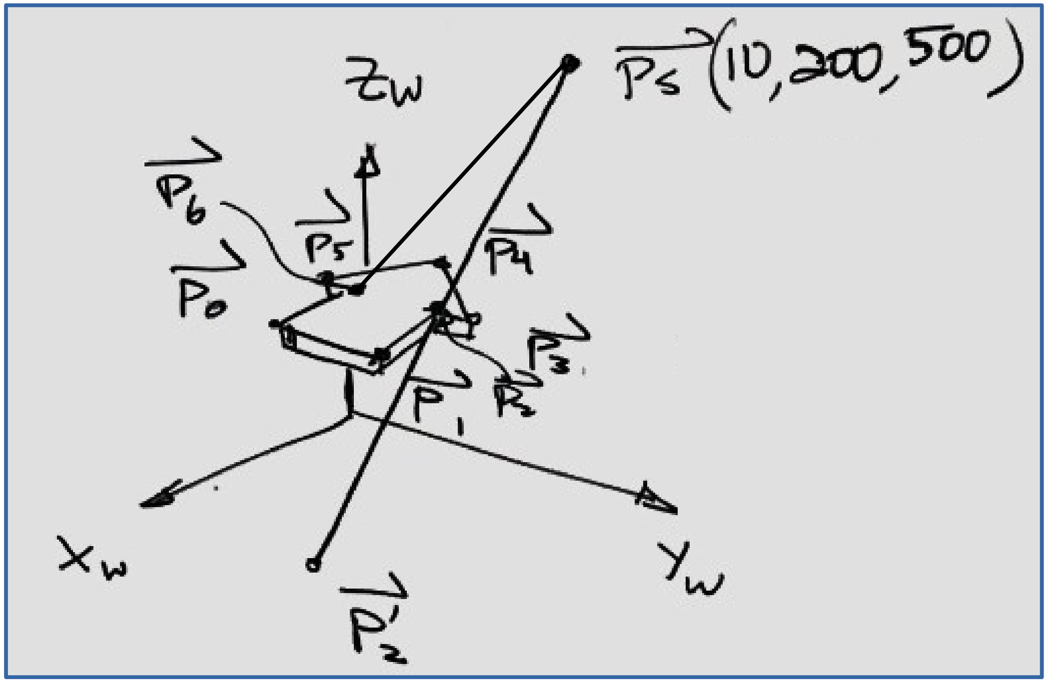
2° if $|a| < 1$, then $\begin{cases} x_{k+1} = x_k + 1 \\ y_{k+1} = a x_{k+1} + b \end{cases}$

o/w $x = \frac{1}{a} y - \frac{b}{a}$, then $\begin{cases} y_{k+1} = y_k + 1 \\ x_{k+1} = \frac{1}{a} y_{k+1} - \frac{b}{a} \end{cases}$

3° Finish all pts till Reaching (x_{N-1}, y_{N-1})

DDA Algorithm

Diffuse Reflection Example



From equation (1.1),

$$I_r = K_{d_r} \frac{\vec{n}\cdot\vec{r}}{\|\vec{n}\|\|\vec{r}\|} \frac{1}{\|\vec{r}\|_2^2} \quad \ldots (1.1)$$

First, find ray equation to, say, one of the vertex, P2(25, 70, 50).

Then find the distance from light source to P2.

Then use the given condition, find the color internsity at P2 location.

Repeat this process to find color intensity for all the vertex from P0 to P6.

Example: Suppose we have a single light source P_s(10,200, 500), now define its (r, g, b) color, so we have single color light source as I_s(r_s, g_s, b_s) = (1.0, 0.0, 0.0), Find the diffuse reflection on the 3D floating arrow by first find color intensity on each of the marked vertex, and then find the color of each pixel of the cursor.
Assume reflection coefficient Kd=(1.0, 0.0, 0.0)

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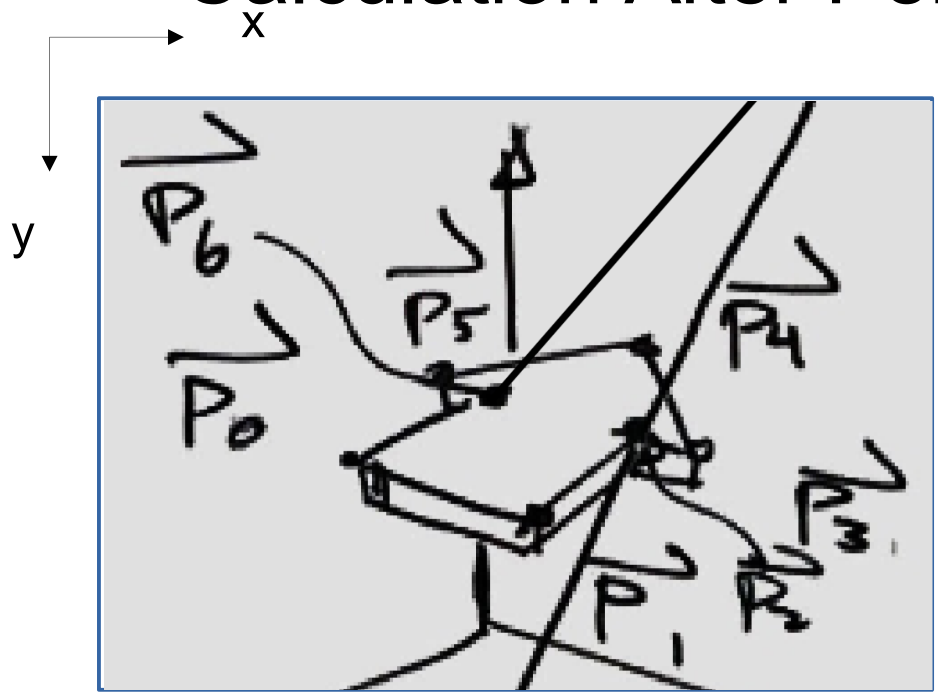
Calculation After Perspective Projection



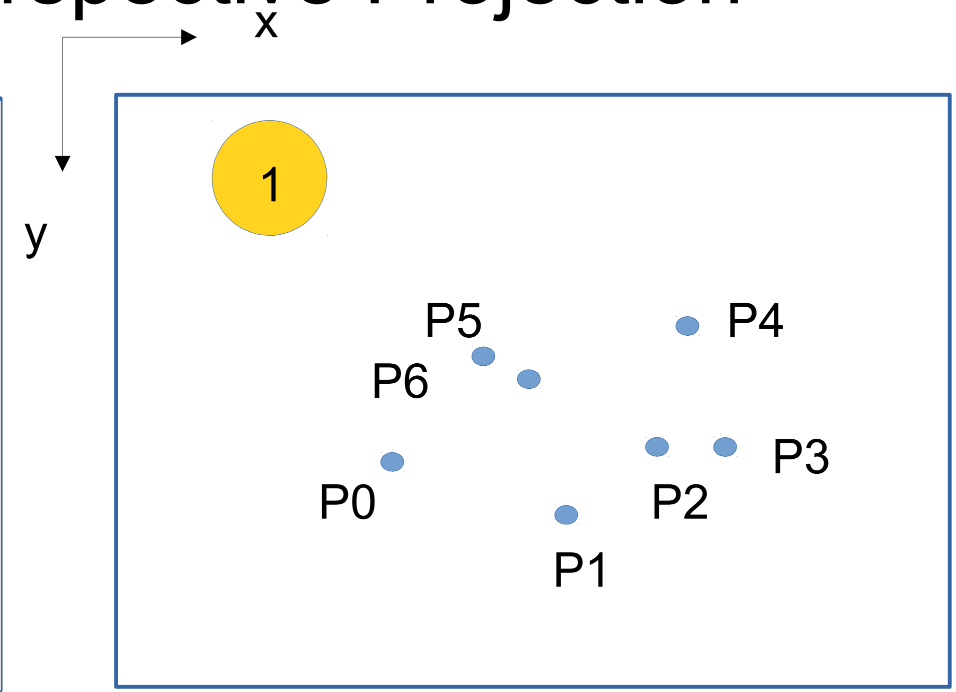
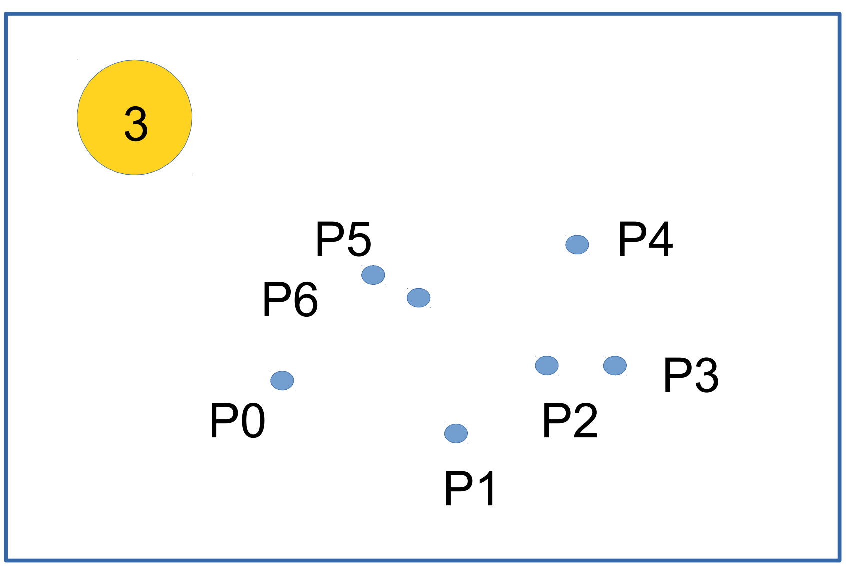
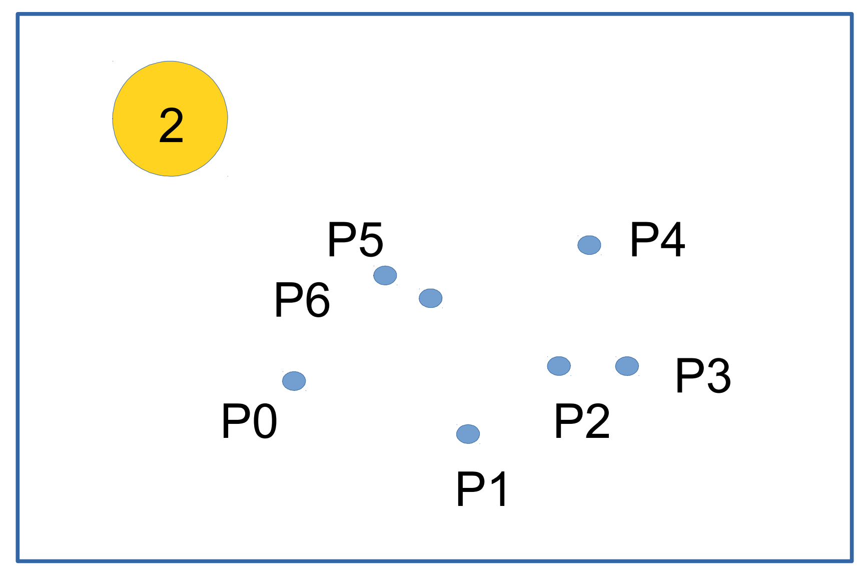


Image (graphics) plane after perspective projection.





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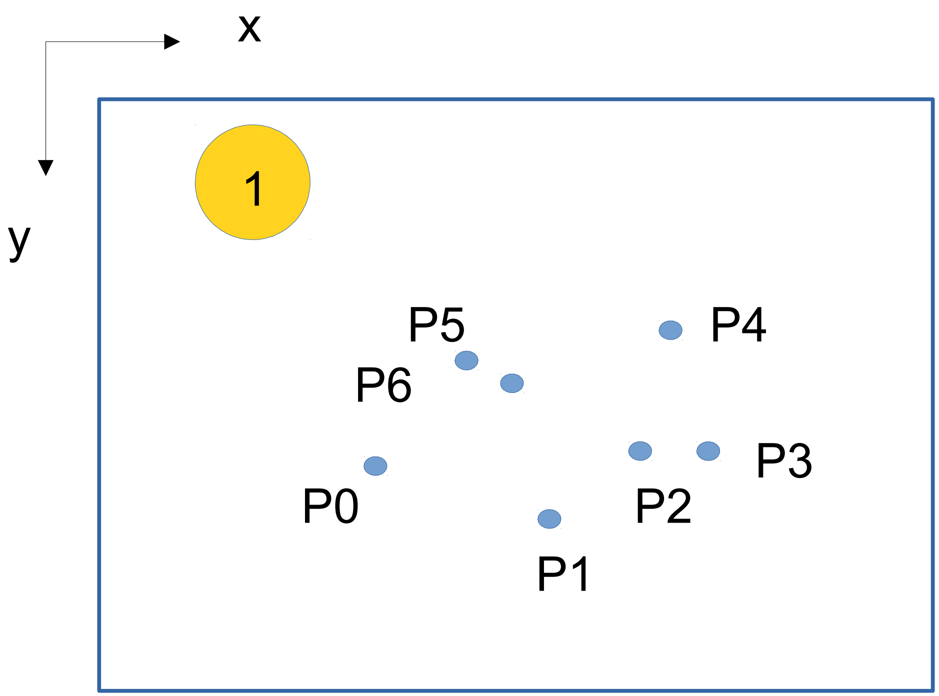
Color On The Line Segments (DDA)



Given P_i and P_(i+1), find line equation first;

Then, rasterization of the line by using DDA (Digital Differential Algorithm);

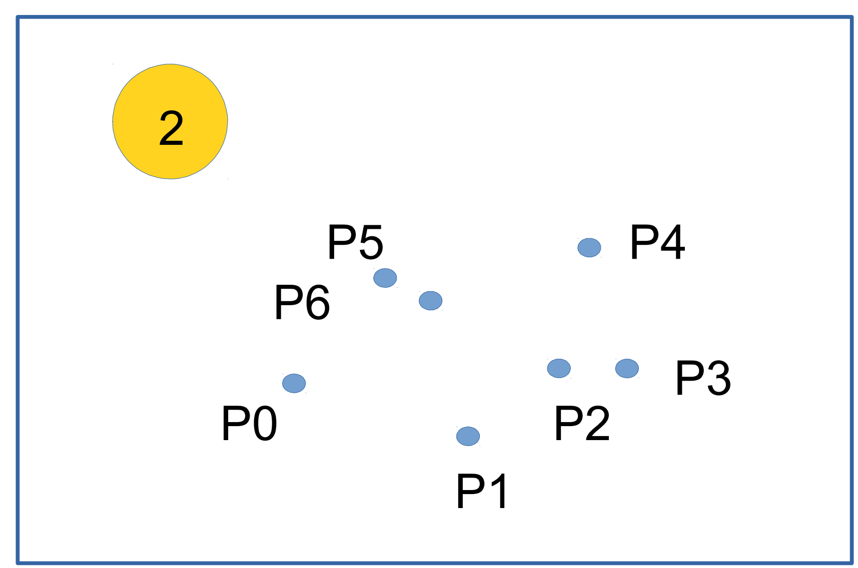
Then use bilinear interpolation to find the color on each point on the line. As a result you will have figure labeled as figure 2.

DDA Algorithm Example

The key :
Slop |a| should be less than or equal to 1,
Otherwise will have to swap x and y;

$y_{(k+1)} = a * x_{(k+1)} + b$ … (1)

$x_{(k+1)} = x_k;$ … (2)



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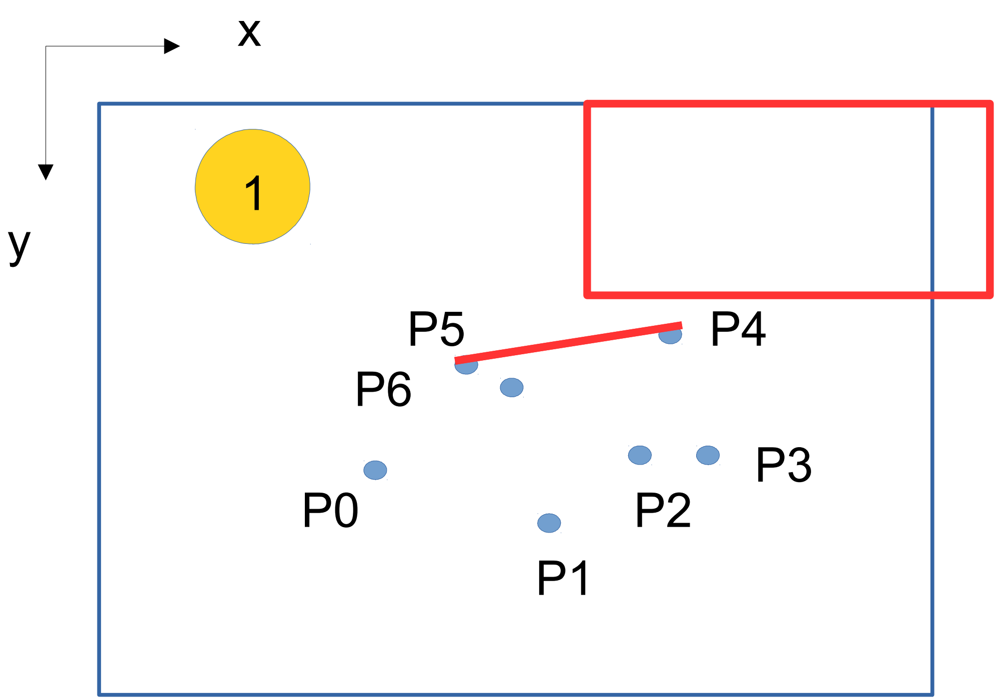
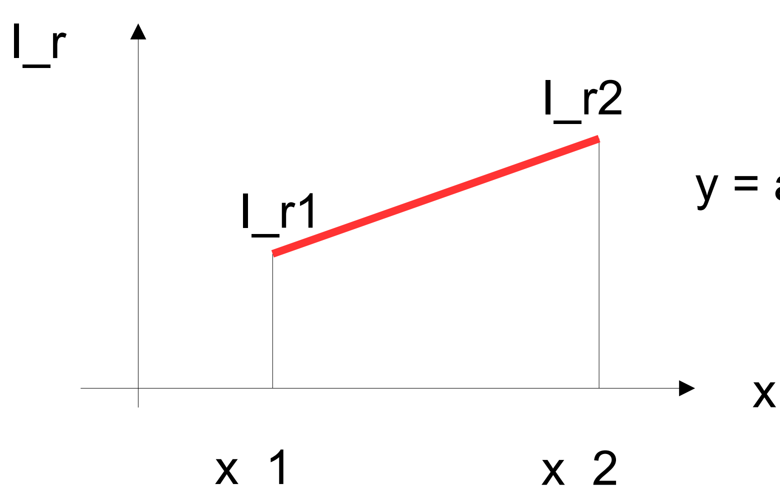
Color On The Line Segments (Interpolation)



Example: when rasterization is done, then find color for each point on the line by interpolation.

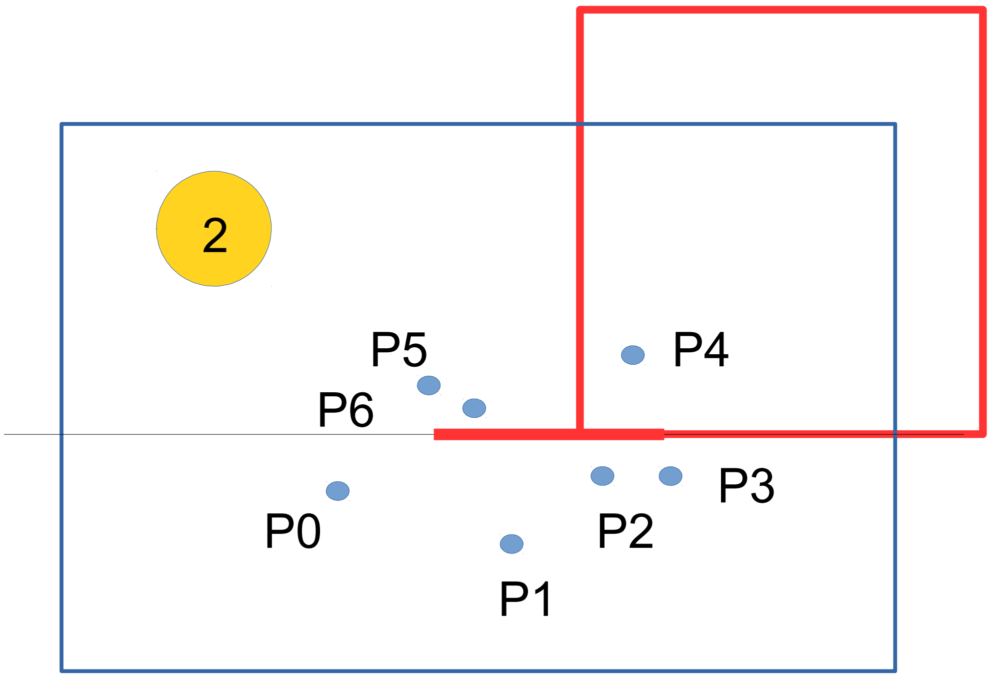
Example (Interpolation)

Step 1. Interpolation along x-direction;
Step 2. Interpolation along y-direction;
Where at each direction, we just use one dimensional interpolation.



$y = a * x + b;$

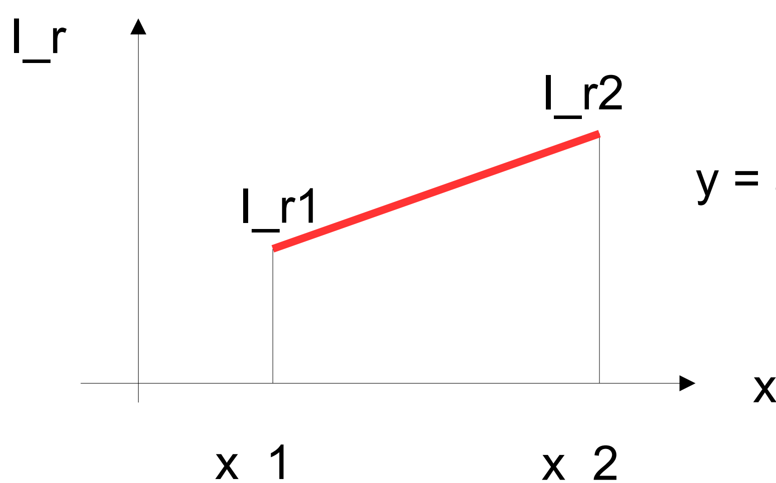
Color Inside the Line Boundary (Interpolation)



Example: when rasterization is done, then find color for each point on the line inside the boundary lines by interpolation.

Example (Interpolation)

Raster scan the image, use linear interpolation find the color between boundaries.



y = a * x + b;

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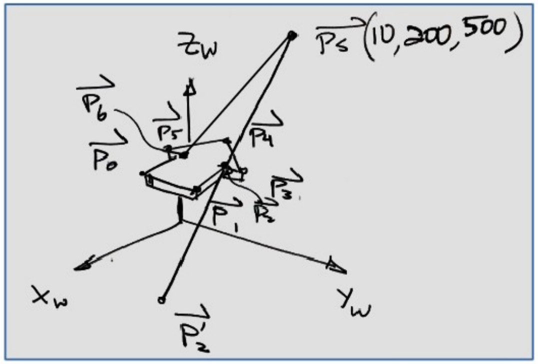
Diffuse Reflection Example



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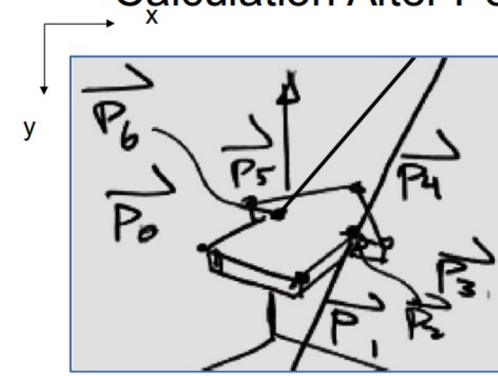

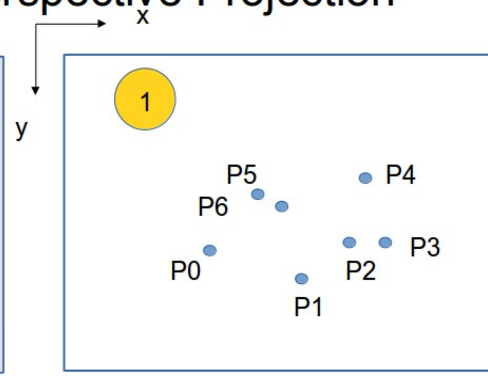

Image (graphics) plane after perspective projection.