**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

To solve this problem, we can use the z-score formula to standardize the time it takes to service a transmission and then use the standard normal distribution table to find the probability.

The z-score is calculated as follows:

Z = (X – )/

*Where:*

*X is the time commitment (1 hour or 60 minutes)*

*is the standard deviation,*

*is the mean service time (45 minutes)*

*Calculate z score*

*Z = (60 – 45)/ 8 = 15/8*

*Now, find the probability using the z score:*

*P (Z < 1.875)*

*You can look up this value in standard normal distribution table or use a calculator or software that provides cumulative distribution function (CDF) for the standard normal distribution.*

*The correct answer is approximately:*

*P(Z < 1.875) 0.9693*

*Now, calculate the probability that the service manager cannot meet his commitment:*

*P (cannot meet commitment) = 1 – P(Z < 1.875)*

*Now, lets calculate this probability.*

*P(cannot meet commitment) 1 – 0.9693*

*P(cannot meet commitment) 0.0307*

*The answer is approximately 0.0307. Therefore, none of the provided options (A, B, C, D)*

*Exactly match the calculated probability. It’s possible that there’s slightly discrepancy in the options, or there might be a rounding difference. Please check the provided options and see if any of them are close to 0.0307.*

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

* A) . More employees at the processing center are older than 44 than between 38 and 44.

To determine this, we need to compare the areas under the normal distribution curve for the two intervals: (1) older than 44 and (2) between 38 and 44. The z-scores for these values can be calculated as follows:

= 44 -38/ 6 = 1

= 38-38/6 = 0

Using a standard normal distribution table or calculator, we can find that the area to the right of Z =1 is less than the area between Z = 0 and Z = 1. Therefore, Statement A is True.

B) A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

To assess this statement, we need to find the probability that a randomly selected employee is under the age of 30. The z-score for X = 30 is calculated as :

= 30 – 38/6 = -4/3

Using a standard normal distribution table or a calculator, we find the probability associated with Z = -4/3.

The probability is about 0.0912. If we multiply this probability by the total number of employees (400), we get :

Expected number of employees under 30 = 0.0912 \* 400 36.48

Since the number of employees must be a whole number, its reasonable to expect about 36 employees must be a whole number, its reasonable to expect about 36 employees to be under the age of 30. Therefore, Statement B is True.

In summary, both statements A and B are True based on the normal distribution properties and calculations.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: The Normal Distribution has its link with the Central Limit Theorem, which states that 'Any large sum of independent identically distribution random variables are approximately Normal then

(X1 + X2) and (2X1) tends to have Normal distribution only if X1 and X2 are i.i.d and n is large.

The Difference between 2X1 and (X1 + X2) is magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N( ) then ,

2X1 ~ N (, 4 2)

If X1 ~ N( ) and X2 ~ N( ) are iid normal random variables then (X1 + X2) (2 , 2 2)

Hence , 2X1 – (X1 + X2) ~ (2 - 2 , 4 2 + 2 2 ) The distribution remains the same for every sample subset of similar source , it tends to fall under Normal distribution and slight deviations in parameters.

The Normal Distribution has 2 parameters, the mean, , and the variance , 2. and 2 satisfy

* < < , 2 > 0. We write X ~ Normal ( ) or X ~ N( ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

To find two values, a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99, we need to use the properties of the normal distribution.

For normal distribution N( , ), the interval (*μ*−*z* \* *σ*, *μ* +*z* \* *σ*) contains a proportion P of the distribution, where z is the z-score associated with the cumulative probability 1+P/2. In other words:

P(*μ*−*z* \* *σ*<*X*<*μ* +*z* \* *σ* ) = p

In this case, we want the probability to be 0.99, so p – 0.99. We need to find the z-score associated with 1 + 0.99/2 = 0.995.

Using a standard normal distribution table or a calculator, we find the z-score for 0.995, which is approximately 2.576.

Now, we can calculate a and b:

a = *μ* −*z* \* *σ*

b = *μ* +*z* \* *σ*

Substitute the values:

a = 100 – 2.576 \*

b = 100 + 2.576 \*

Now, calculate a and b:

a 100 – 2.576 \* 20

a 100 – 51.52

a 48.48

b 100 + 2.576 \* 20

b 100 + 51.52

b 151.52

Therefore, the correct values for a and b are approximately 48.48 and 151.52. None of the provided options (A, B, C, D, E) exactly match these values. Its possible that there’s a slight discrepancy in the options, or there might be a rounding difference. Please check the provided options and see if any of them are close to these calculated values.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?
5. Rupee Range for 95% Probability:

For a normal distribution, a 95% confidence interval (CI) corresponds to approximately

1.96 standard deviation from the mean. We can use the z-score formula:

Z = (*X*−*μ*)/

Where Z is the z-score, X is the value, is the mean, is the standard deviation.

Lets calculate the range for the total profit of the company in Rupees:

For Profit1:

= (X – 5)/

For profit 2:

= (X – 7) /

Since the total profit is the sum of Profit1 and Profit2, the z-score for the total profit (Ztotal = Z1 + Z2

Now, we can use Ztotal to find the Rupee range.

1. 5th Percentile , we need to find the z-score (Z 5th) corresponding to the cumulative probability of 0.05 and then convert it to Rupees.
2. Probability of Making a Loss:

A division is making a loss when its profit is below zero. We can calculate the probability of making a loss for each division using the cumulative distribution function (CDF) of the normal distribution.

Calculations in Jupyter Python :

Attached .ipynb file