AMORO Lab: Kinematics and Dynamics of a Biglide

Master CORO-IMARO: Control and Robotics

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1 Introduction

The purpose of this lab is to derive the geometric, kinematic, and dynamic models of a Biglide mechanism and to compare them with the simulation results obtained from GAZEBO. This report describes the procedures undertaken to develop these models and examines the corresponding outcomes.

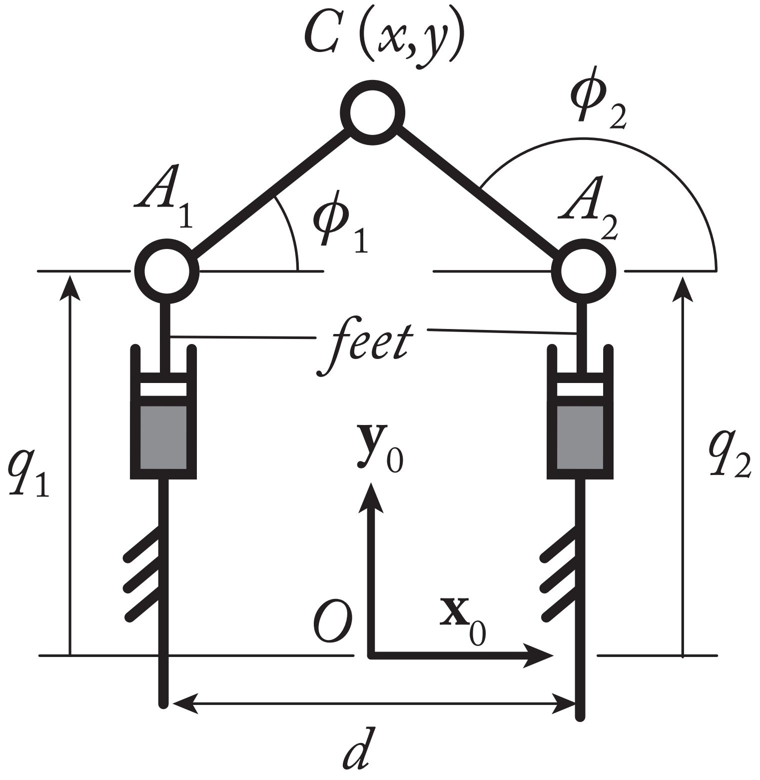

Figure 1: Kinematic model of the Biglide

2 Model

The kinematic architecture of the biglide mechanism is shown in Fig.1. For the GAZEBO model, the geometric parameters are:

- $d = 0.4$ m
- $l_{A1C} = 0.3606$ m
- $l_{A2C} = 0.3606$ m

The two prismatic joints are actuated.

The base dynamic parameters are:

- $m_p = 3$ kg the mass of the end-effector
- $m_f = 1$ kg the mass of each foot

All other dynamic parameters are neglected.

3 Geometric models

3.1 Direct geometric model

The coordinate definitions are:

$$\overrightarrow{OA_1} = \begin{bmatrix} -\frac{d}{2} \\ q_1 \end{bmatrix}, \quad \overrightarrow{OA_2} = \begin{bmatrix} \frac{d}{2} \\ q_2 \end{bmatrix}, \quad \overrightarrow{OC} = \begin{bmatrix} x \\ y \end{bmatrix}$$

The direct geometric model is given by:

$$\overrightarrow{OC} = \overrightarrow{OM} + \overrightarrow{MC}$$

Expanding the terms:

$$\overrightarrow{OC} = \overrightarrow{OA_2} + \overrightarrow{A_2M} + \overrightarrow{MC}$$

The midpoint M between A_1 and A_2 is defined as:

$$\overrightarrow{A_2M} = \frac{1}{2}(\overrightarrow{OA_1} - \overrightarrow{OA_2})$$

Thus,

$$\overrightarrow{OM} = \overrightarrow{OA_2} + \overrightarrow{A_2M} = \overrightarrow{OA_2} + \frac{1}{2}(\overrightarrow{OA_1} - \overrightarrow{OA_2})$$

$$\Rightarrow \overrightarrow{OM} = \frac{1}{2}(\overrightarrow{OA_1} + \overrightarrow{OA_2})$$

The vector $\overrightarrow{MC}$ is expressed as:

$$\overrightarrow{MC} = \gamma \frac{h}{a} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \overrightarrow{A_2M}$$

where $\gamma = \pm 1$ represents the two assembly modes.

This can be visualized as follows: the vector $\overrightarrow{A_2M}$ is divided by its magnitude a to obtain a unit vector in the direction of $\overrightarrow{A_2M}$. This unit vector is then rotated by 90° using the rotation matrix, the direction of rotation is determined by the γ and multiplied by the magnitude h. The resulting vector corresponds to $\overrightarrow{MC}$.

The parameters are defined as:

$$a = \|\overrightarrow{A_2M}\|, \quad h = \sqrt{l^2 - a^2}$$

since the distance between A_2 and C is l (i.e., $l = A_2C$).

3.2 Passive joints geometric model

The angles of the passive joints ϕ_1 and ϕ_2 are obtained from the Cartesian position of point $C(x, y)$ and the prismatic joint variables q_1 and q_2 as follows:

$$\phi_1 = \tan^{-1}\left(\frac{y - q_1}{x + \frac{d}{2}}\right)$$

$$\phi_2 = \tan^{-1}\left(\frac{y - q_2}{x - \frac{d}{2}}\right)$$

These equations define the orientation of each link with respect to the horizontal axis based on the geometry of the Biglide mechanism.

3.3 Inverse geometric model

From the geometric configuration, we can write:

$$(x + \tfrac{d}{2})^2 + (y - q_1)^2 = l^2$$

$$(x - \tfrac{d}{2})^2 + (y - q_2)^2 = l^2$$

Solving for the prismatic joint variables gives:

$$q_1 = y \pm \sqrt{l^2 - (x + \tfrac{d}{2})^2}$$

$$q_2 = y \pm \sqrt{l^2 - (x - \tfrac{d}{2})^2}$$

4 First order kinematic models

4.1 Forward and inverse kinematic model

The velocity of the end-effector point $\boldsymbol{\varepsilon}$ can be expressed for each leg as:

$$\boldsymbol{\varepsilon} = -\frac{d}{2}\,x_0 + q_1\,\mathbf{y}_0 + u_1\,\mathbf{l}_{A_1C} \quad \text{(Left leg)}$$

$$\boldsymbol{\varepsilon} = \frac{d}{2}\,\dot{x}_0 + \dot{q}_2\,\mathbf{y}_0 + \dot{u}_2\,\mathbf{l}_{A_2C} \quad \text{(Right leg)}$$

where:

$$\mathbf{u}_1 = \frac{\overrightarrow{A_1C}}{\|\overrightarrow{A_1C}\|} = \begin{bmatrix} \cos\phi_1 \\ \sin\phi_1 \end{bmatrix}, \quad \mathbf{u}_2 = \frac{\overrightarrow{A_2C}}{\|\overrightarrow{A_2C}\|} = \begin{bmatrix} \cos\phi_2 \\ \sin\phi_2 \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{u}_1, \mathbf{u}_2 = \text{unit vectors along } A_1C \text{ and } A_2C, \text{ respectively.}$$

The time derivative of $\boldsymbol{\varepsilon}$ gives:

$$\dot{\boldsymbol{\varepsilon}} = \dot{q}_1\,\mathbf{y}_0 + \dot{\phi}_1\,\mathbf{v}_1\mathbf{l}_{A_1C} \quad \text{(Left leg)}$$

$$\dot{\boldsymbol{\varepsilon}} = \dot{q}_2\,\mathbf{y}_0 + \dot{\phi}_2\,\mathbf{v}_2\mathbf{l}_{A_2C} \quad \text{(Right leg)}$$

where:

$$\dot{\mathbf{u}} = \dot{q}\,\mathbf{v}, \quad \dot{\mathbf{v}} = q\mathbf{u}$$

Here, $\mathbf{u}$ is the unit vector along the link, $\dot{\phi}$ is its angular velocity, and $\mathbf{v}$ is the unit vector rotated 90° counterclockwise with respect to $\mathbf{u}$.

From the velocity equations of the two legs:

$$\mathbf{u}_1^T\dot{\boldsymbol{\varepsilon}} = \dot{q}_1\,\mathbf{u}_1^T\mathbf{y}_0, \quad \mathbf{u}_2^T\dot{\boldsymbol{\varepsilon}} = \dot{q}_2\,\mathbf{u}_2^T\mathbf{y}_0$$

These can be written in matrix form as:

$$\begin{bmatrix} \mathbf{u}_1^T \\ \mathbf{u}_2^T \end{bmatrix} \dot{\boldsymbol{\varepsilon}} = \begin{bmatrix} \mathbf{u}_1^T\mathbf{y}_0 & 0 \\ 0 & \mathbf{u}_2^T\mathbf{y}_0 \end{bmatrix} \dot{\mathbf{q}}$$

or compactly:

$$A\,\dot{\boldsymbol{\varepsilon}} = B\,\dot{\mathbf{q}}$$

where

$$A = \begin{bmatrix} \mathbf{u}_1^T \\ \mathbf{u}_2^T \end{bmatrix}, \quad B = \begin{bmatrix} \mathbf{u}_1^T \mathbf{y}_0 & 0 \\ 0 & \mathbf{u}_2^T \mathbf{y}_0 \end{bmatrix}, \quad \dot{\mathbf{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Hence, the direct first order kinematic model is:

$$\dot{\boldsymbol{\varepsilon}} = A^{-1} B \, \dot{\mathbf{q}}$$

and the inverse first order kinematic model is:

$$\dot{\mathbf{q}} = B^{-1} A \, \dot{\boldsymbol{\varepsilon}}$$

4.2 Passive joints kinematic model

From the geometric relationships of each leg, the velocity of the end-effector $\dot{\boldsymbol{\varepsilon}}$ can be written as:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\phi}_1 \mathbf{v}_1 l_{A_1C} + \dot{q}_1 \, \mathbf{y}_0 \quad \text{(Left leg)}$$

$$\dot{\boldsymbol{\varepsilon}} = \dot{\phi}_2 \mathbf{v}_2 l_{A_2C} + \dot{q}_2 \, \mathbf{y}_0 \quad \text{(Right leg)}$$

Multiplying by $\mathbf{v}_1^T$ and $\mathbf{v}_2^T$ respectively gives:

$$\mathbf{v}_1^T \dot{\boldsymbol{\varepsilon}} = \dot{\phi}_1 l_{A_1C} + \dot{q}_1 \, \mathbf{v}_1^T \mathbf{y}_0$$

$$\mathbf{v}_2^T \dot{\boldsymbol{\varepsilon}} = \dot{\phi}_2 l_{A_2C} + \dot{q}_2 \, \mathbf{v}_2^T \mathbf{y}_0$$

or equivalently in matrix form:

$$\begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix} \dot{\boldsymbol{\varepsilon}} = \begin{bmatrix} \mathbf{v}_1^T \mathbf{y}_0 & 0 \\ 0 & \mathbf{v}_2^T \mathbf{y}_0 \end{bmatrix} \dot{\mathbf{q}} + l \dot{\boldsymbol{\phi}}$$

Defining:

$$A_p = \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix}, \quad B_p = \begin{bmatrix} \mathbf{v}_1^T \mathbf{y}_0 & 0 \\ 0 & \mathbf{v}_2^T \mathbf{y}_0 \end{bmatrix}$$

we can write:

$$A_p \, \dot{\boldsymbol{\varepsilon}} = B_p \, \dot{\mathbf{q}} + l \, \dot{\boldsymbol{\phi}}$$

Hence, the inverse first order kinematic model for the passive joints is:

$$\dot{\boldsymbol{\phi}} = l^{-1} (A_p \, \dot{\boldsymbol{\varepsilon}} - B_p \, \dot{\mathbf{q}})$$

Note: Here we need ϕ_1 and ϕ_2, which are already known from the inverse geometric model (IGM), and we compute their time derivatives using the above relation.

5 Second order kinematic models

5.1 Forward and inverse kinematic model

From the first-order relations, the acceleration of the end-effector $\ddot{\boldsymbol{\varepsilon}}$ can be expressed for each leg as:

$$\ddot{\boldsymbol{\varepsilon}} = l_{A_1C}\,\ddot{\phi}_1\,\mathbf{v}_1 - l_{A_1C}\,\dot{\phi}_1^2\,\mathbf{u}_1 + \ddot{q}_1\,\mathbf{y}_0 \quad \text{(Left leg)}$$
$$\ddot{\boldsymbol{\varepsilon}} = l_{A_2C}\,\ddot{\phi}_2\,\mathbf{v}_2 - l_{A_2C}\,\dot{\phi}_2^2\,\mathbf{u}_2 + \ddot{q}_2\,\mathbf{y}_0 \quad \text{(Right leg)}$$

Rewriting in matrix form:

$$\begin{bmatrix}\mathbf{u}_1^T \\ \mathbf{u}_2^T\end{bmatrix}\ddot{\boldsymbol{\varepsilon}} = \begin{bmatrix}\mathbf{u}_1^T\mathbf{y}_0 & 0 \\ 0 & \mathbf{u}_2^T\mathbf{y}_0\end{bmatrix}\ddot{\mathbf{q}} - l\begin{bmatrix}\dot{\phi}_1^2 \\ \dot{\phi}_2^2\end{bmatrix}$$

Defining:

$$A = \begin{bmatrix}\mathbf{u}_1^T \\ \mathbf{u}_2^T\end{bmatrix}, \quad B = \begin{bmatrix}\mathbf{u}_1^T\mathbf{y}_0 & 0 \\ 0 & \mathbf{u}_2^T\mathbf{y}_0\end{bmatrix}, \quad \mathbf{d} = l\begin{bmatrix}\dot{\phi}_1^2 \\ \dot{\phi}_2^2\end{bmatrix}$$

The compact form of the second order model is:

$$A\,\ddot{\boldsymbol{\varepsilon}} = B\,\ddot{\mathbf{q}} - \mathbf{d}$$

Hence, the forward second order kinematic model is:

$$\ddot{\boldsymbol{\varepsilon}} = A^{-1}(B\,\ddot{\mathbf{q}} - \mathbf{d})$$

and the inverse second order kinematic model is:

$$\ddot{\mathbf{q}} = B^{-1}(A\,\ddot{\boldsymbol{\varepsilon}} + \mathbf{d})$$

These relations describe the mapping between actuator accelerations and the end-effector accelerations, including the effects of the passive joint angular velocities.

5.2 Passive joints kinematic model

To obtain the angular accelerations of the passive joints $\ddot{\phi}_1$ and $\ddot{\phi}_2$, we start from the relation:

$$\begin{bmatrix}\mathbf{v}_1^T \\ \mathbf{v}_2^T\end{bmatrix}\ddot{\boldsymbol{\varepsilon}} = l\begin{bmatrix}\ddot{\phi}_1 \\ \ddot{\phi}_2\end{bmatrix} + \begin{bmatrix}\mathbf{v}_1^T\mathbf{y}_0 & 0 \\ 0 & \mathbf{v}_2^T\mathbf{y}_0\end{bmatrix}\ddot{\mathbf{q}}$$

Defining:

$$A_p = \begin{bmatrix}\mathbf{v}_1^T \\ \mathbf{v}_2^T\end{bmatrix}, \quad B_p = \begin{bmatrix}\mathbf{v}_1^T\mathbf{y}_0 & 0 \\ 0 & \mathbf{v}_2^T\mathbf{y}_0\end{bmatrix}$$

the equation becomes:

$$A_p\,\ddot{\boldsymbol{\varepsilon}} = l\,\ddot{\boldsymbol{\phi}} + B_p\,\ddot{\mathbf{q}}$$

Hence, the inverse second order kinematic model for passive joints is:

$$\ddot{\boldsymbol{\phi}} = \frac{1}{l}(A_p\,\ddot{\boldsymbol{\varepsilon}} - B_p\,\ddot{\mathbf{q}})$$

This expression allows the computation of the passive joint angular accelerations once the actuator accelerations $\ddot{\mathbf{q}}$ and the Cartesian accelerations $\ddot{\boldsymbol{\varepsilon}}$ are known.

6 Dynamic model

For the Biglide mechanism, we consider the following dynamic parameters, as given in the lab statement:

- $m_p = 3$ kg: mass of the end-effector (moving platform),
- $m_f = 1$ kg: mass of each foot (actuated slider).

These parameters correspond to the configuration used in the AMORO Biglide simulation. No other inertial parameters are provided, and gravity is neglected because the Biglide operates on a horizontal plane. Therefore, the total potential energy is zero, and the Lagrangian reduces to the total kinetic energy of the system:

$$L = E - U = E, \qquad U = 0.$$

The total kinetic energy of the Biglide is composed of two parts:

1. The kinetic energy of the two actuated feet (tree structure),
2. The kinetic energy of the moving platform (end-effector).

For each prismatic actuator i ($i = 1, 2$), the foot translates linearly with velocity $\dot{q}_i$. Thus, its kinetic energy is:

$$E_i = \frac{1}{2} m_f \dot{q}_i^2.$$

Summing both actuator contributions gives:

$$E_{\text{legs}} = \frac{1}{2} m_f \dot{q}_1^2 + \frac{1}{2} m_f \dot{q}_2^2.$$

The moving platform (end-effector) of mass m_p moves with Cartesian velocity $\dot{\xi} = [\dot{x},\ \dot{y}]^T$. Its kinetic energy is:

$$E_{\text{plat}} = \frac{1}{2} m_p (\dot{x}^2 + \dot{y}^2) = \frac{1}{2} m_p \dot{\xi}^T \dot{\xi}.$$

Hence, the total kinetic energy of the Biglide system is:

$$E_{\text{tot}} = E_{\text{legs}} + E_{\text{plat}}.$$

The corresponding Lagrangian is:

$$L = \frac{1}{2} m_f (\dot{q}_1^2 + \dot{q}_2^2) + \frac{1}{2} m_p \dot{\xi}^T \dot{\xi}.$$

6.1 Kinematic Relationships

We denote the actuator coordinates by

$$q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \qquad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \qquad \ddot{q} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}.$$

The Cartesian position of the end-effector is

$$\xi = \begin{bmatrix} x \\ y \end{bmatrix}.$$

From the first-order kinematic model of the Biglide, we have

$$A(q)\,\dot{\xi} = B(q)\,\dot{q},$$

which gives

$$\dot{\xi} = J(q)\,\dot{q}, \qquad J(q) = A(q)^{-1}B(q).$$

Differentiating with respect to time gives the Cartesian acceleration:

$$\ddot{\xi} = J(q)\,\ddot{q} + \dot{J}(q,\dot{q})\,\dot{q}.$$

From the second-order kinematic model of the Biglide, we also have

$$A(q)\,\ddot{\xi} = B(q)\,\ddot{q} + d(q,\dot{q}),$$

where $d(q,\dot{q})$ contains the nonlinear velocity-squared terms (centripetal and Coriolis-like effects arising from the closed-chain constraints).

Solving for $\ddot{\xi}$:

$$\ddot{\xi} = A(q)^{-1}B(q)\,\ddot{q} + A(q)^{-1}d(q,\dot{q}) = J(q)\,\ddot{q} + A(q)^{-1}d(q,\dot{q}).$$

6.2 Lagrange Formulation

We now express the forces required at the actuators.

Tree structure (actuated feet). The actuated feet of mass m_f move along q_1 and q_2. From the Lagrangian

$$L_{\text{legs}} = \frac{1}{2}m_f(\dot{q}_1^2 + \dot{q}_2^2),$$

the generalized effort associated to each actuator is:

$$\tau^{\text{legs}} = \begin{bmatrix} m_f & 0 \\ 0 & m_f \end{bmatrix}\ddot{q} \equiv M_{\text{legs}}\ddot{q}.$$

Platform (end-effector). The moving platform of mass m_p produces a Cartesian wrench

$$w_p = m_p\ddot{\xi}.$$

In a closed-chain parallel robot like the Biglide, this Cartesian wrench is transmitted to the actuators through the Jacobian transpose. Therefore, the total actuator forces are:

$$\tau = \tau^{\text{legs}} + J(q)^T w_p.$$

Substituting $\tau^{\text{legs}} = M_{\text{legs}}\ddot{q}$ and $w_p = m_p\ddot{\xi}$:

$$\tau = M_{\text{legs}}\ddot{q} + J(q)^T\big(m_p\ddot{\xi}\big).$$

Now replace $\ddot{\xi}$ by the expression from the second-order kinematics:

$$\ddot{\xi} = J(q)\,\ddot{q} + A(q)^{-1}d(q,\dot{q}).$$

We obtain:

$$\tau = M_{\text{legs}}\ddot{q} + J^T m_p\left(J\ddot{q} + A^{-1}d\right).$$

Rearranging terms:

$$\tau = \underbrace{\big(M_{\text{legs}} + m_p J(q)^T J(q)\big)}_{M(q)}\ddot{q} + \underbrace{m_p J(q)^T\left(A(q)^{-1}d(q,\dot{q})\right)}_{c(q,\dot{q})}.$$

6.3 Final Inverse Dynamic Model of the Biglide

We can now write the dynamic model of the Biglide in the compact standard form:

$$\boxed{\tau = M(q)\,\ddot{q} + c(q,\dot{q})}$$

where

$$M(q) = \begin{bmatrix} m_f & 0 \\ 0 & m_f \end{bmatrix} + m_p J(q)^T J(q),$$

and

$$c(q,\dot{q}) = m_p J(q)^T \left(A(q)^{-1} d(q,\dot{q}) \right).$$

Here:

- $q = [q_1,\ q_2]^T$ are the prismatic actuator coordinates,
- $A(q)$ and $B(q)$ are the kinematic matrices of the Biglide,
- $J(q) = A(q)^{-1}B(q)$ is the Jacobian,
- $d(q,\dot{q})$ is the nonlinear velocity-squared term obtained from the second-order kinematics,
- $M(q)$ is the joint-space inertia matrix,
- $c(q,\dot{q})$ is the Coriolis / centrifugal term,
- $m_f = 1$ kg and $m_p = 3$ kg are given in the lab statement.

Note: The sign of $c(q,\dot{q})$ may vary depending on the sign convention used in the matrices $A(q)$ and $B(q)$. In our implementation, a negative sign appears due to the direction convention of $A(q)$, which is consistent with the `biglide_models.py` program.

This final expression matches the implementation used in `biglide_models.py` for the inverse dynamic model, where:

$$M = m_f I + m_p J^T J, \qquad c = -\,m_p J^T (A^{-1} d).$$

The derived model provides the theoretical basis for the computed-torque control of the Biglide mechanism in simulation, ensuring that both the analytical formulation and the implemented model remain consistent.

7 Conclusion

In this lab, the geometric, kinematic, and dynamic models of the Biglide mechanism were derived and analyzed. The results obtained from the analytical models were compared with the simulations in GAZEBO, showing good consistency between theory and simulation.The lab provided a clear understanding of the Biglide's mechanism and confirmed the validity of the developed models which are further dicussed in the other report.