Solution to reanalysis of a football kick using the Kalman Filter

Let's start with the solution:

$$\mathbf{x} = \mathbf{x}_b + \mathbf{BH}^T(\mathbf{R}+\mathbf{HBH}^T)^{-1}(\mathbf{y}-H[\mathbf{x}_b])$$

B and **R** are both diagonal matrices with time-invariant diagonal vectors **b** = $\{b, b\}$ and **r** = $\{r, r\}$

B = $\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$

R = $\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$

H is the Jacobi matrix of H: $\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}$. In this special case **H** = $\mathbf{H}^\mathbf{T}$

Thus, the matrix **R** + $\mathbf{HBH}^\mathbf{T}$ can be written as

$$\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}\begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} = \begin{bmatrix} a^2b+r & 0 \\ 0 & a^2b+r \end{bmatrix}$$

the inverse of which is $\begin{bmatrix} \frac{1}{a^2b+r} & 0 \\ 0 & \frac{1}{a^2b+r} \end{bmatrix}$

The entire solution thus is **x** = $\mathbf{x_b}$ + $\begin{bmatrix} \frac{ab}{a^2b+r} & 0 \\ 0 & \frac{-ab}{a^2b+r} \end{bmatrix}\begin{bmatrix} i-ax_b \\ j-(k-az_b) \end{bmatrix}$

For the implementation in Excel we can therefore treat the two dimensions separately:

$$x = x_b + \frac{ab}{a^2b+r}(i-ax_b)$$

$$z = z_b + \frac{-ab}{a^2b+r}(j-(k-az_b))$$