

## Solution to reanalysis of a football kick using the Kalman Filter

Let's start with the solution:

$$\mathbf{x} = \mathbf{x}_b + \mathbf{B}\mathbf{H}^T(\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}(\mathbf{y} - \mathbf{H}[\mathbf{x}_b])$$

$\mathbf{B}$  and  $\mathbf{R}$  are both diagonal matrices with time-invariant diagonal vectors  $\mathbf{b} = \{b, b\}$  and  $\mathbf{r} = \{r, r\}$

$$\mathbf{B} = \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$$

$\mathbf{H}$  is the Jacobi matrix of  $H$ :  $\begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}$ . In this special case  $\mathbf{H} = \mathbf{H}^T$

Thus, the matrix  $\mathbf{R} + \mathbf{H}\mathbf{B}\mathbf{H}^T$  can be written as

$$\begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} + \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} = \begin{bmatrix} a^2b + r & 0 \\ 0 & a^2b + r \end{bmatrix}$$

the inverse of which is  $\begin{bmatrix} \frac{1}{a^2b + r} & 0 \\ 0 & \frac{1}{a^2b + r} \end{bmatrix}$

The entire solution thus is  $\mathbf{x} = \mathbf{x}_b + \begin{bmatrix} \frac{ab}{a^2b + r} & 0 \\ 0 & \frac{-ab}{a^2b + r} \end{bmatrix} \begin{bmatrix} i - ax \\ j - (500 - az) \end{bmatrix}$

For the implementation in Excel we can therefore treat the two dimensions separately:

$$x = \frac{ab}{a^2b + r}(i - ax)$$

$$z = \frac{-ab}{a^2b + r}(j - (500 - az))$$