Reanalysis of a football kick using the Kalman Filter

As in the video, we will use the model of the inclined throw (with a simple friction term) and images of a camera to obtain a reanalysis of the ball's trajectory.

For convenience we perform a number of simplifications. First, we assume that the state vector has only two elements, namely the ball's coordinates in x (horizontal) and z directions (vertical), hence $\mathbf{x} = \{x, z\}$. We treat the corresponding velocities u, (in x-direction) and w (in z-direction) as "diagnostic variables", which we derive after each model time step. Here are the equations of the model

$$x_t = x_{t-1} + u_{t-0.5}\,\Delta t - 0.5\,f\,sgn(u_{t-0.5})\,u_{t-0.5}^2\,\Delta t$$

$$z_t = z_{t-1} + w_{t-0.5}\,\Delta t - 0.5\,f\,sgn(w_{t-0.5})\,w_{t-0.5}^2\,\Delta t - 0.5\,g\,\Delta t^2$$

where f (= 0.03 s m^{-1}) is used to parametrise friction and g is gravity (= 9.80665 m s^{-2}). Here, t and t-1 refer to model time steps, t-0.5 indicates that wind refers to the mean of the time step. The diagnostic winds are based on the resulting wind speed from the previous time step (calculated from the analysis x, z) plus the effects of friction and gravity:

$$u_{t-0.5} = ((x_{t-1} - x_{t-2}) / \Delta t) - f\,sgn(u_{t-1.5})\,u_{t-1.5}^2\,\Delta t$$

$$w_{t-0.5} = ((z_{t-1} - z_{t-2}) / \Delta t) - f\,sgn(w_{t-1.5})\,w_{t-1.5}^2\,\Delta t - g\,\Delta t$$

Note that the starting conditions are such that the football (as in the video) is kicked from right to left, hence, u_0 is negative and the starting position x_0 is at some 23.4 m

As to the observations, we assume that these are photos are taken from one camera. For simplicity we assume that the projection plane of all photos aligns with the plane of the trajectory. However, the camera uses a pixel coordinate system (i, j) with origin in top left corner (positive to the right for i and downward for j). The H operators for the two pixel dimensions (i, j) are:

$$H(x) = a\,x$$

$$H(z) = 500 - a\,x$$

with a = 44.2 pixels m^{-1}

We further assume that the errors (standard deviations) of model and camera are 0.1 m and 3 pixels, respectively, and we assume that there is no correlation between x and z coordinates of the ball in $\mathbf{B}$. Note that this a massive oversimplification. In fact, data assimilation in essence draws from correlations of variables within the state vector, but this is less trivial to calculate. As a consequence of our assumptions, $\mathbf{B}$ and $\mathbf{R}$ are both diagonal matrices with time-invariant diagonal vectors $\mathbf{b}$ = {0.01 m^2, 0.01 m^2} and $\mathbf{r}$ = {9 pixels^2, 9 pixels^2}.

Observations of the camera for each time step (0.1 s) are given in the Excel Sheet. Before turning to the sheet, typing the model and solving the assimilation, we should have a look at the algebra.

Please simplify the solution:

$$\mathbf{x} = \mathbf{x}_b + \mathbf{B}\mathbf{H}^T(\mathbf{R}+\mathbf{H}\mathbf{B}\mathbf{H}^T)^{-1}\,(\mathbf{y}-H[\mathbf{x}_b])$$

for the assumptions made in this exercise, so that it can easily be implemented in Excel.

Then use the Excel file to solve the assimilation.