Topics: Normal distribution, Functions of Random Variables

- 1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
 - A. 0.3875
 - B. 0.2676
 - C. 0.5
 - D. 0.6987

ANS:

Mean(μ)=45 min

Standard deviation(σ)=8 min

Time commitment(X)=1 hour=60 min

But service manager begin the work after 10 min .

So,

X=60-10=50

In the given ,we have to find Z-score

Z-score=X-
$$\mu/\sigma$$

Z-score = 50-45/8

=5/8

Z-score =0.625

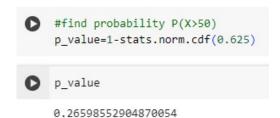
Z-Score:

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[7] #z-score mean=45,std=8,x=50
z=(50-45)/8

► z

→ 0.625
```

The probability of that service manager cannot meet his commitment is 0.266



B) Option is Correct

- 2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
 - A. More employees at the processing center are older than 44 than between 38 and 44.

ANS:

→ answer: False

A) False.Because the probability for employees at the processing center are more between 38 and 44 than older than 44

Mean=38 Std=6 for_38=stats.norm.cdf(38,mean,std) for_38 → 0.5 [] for_44=stats.norm.cdf(44,mean,std) [] for_44 0.8413447460685429 [] between_38_and_44=for_44-for_38 print("probability of employee age between 38 and 44 is:",between_38_and_44) probability of employee age between 38 and 44 is: 0.3413447460685429 [] more_than_44=1-stats.norm.cdf(44,mean,std) [] print("probability of employee age more than 44 is :",more_than_44) probability of employee age more than 44 is : 0.15865525393145707 true_or_false=(more_than_44 > between_38_and_44) print("answer:",true_or_false)

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees

ANS: True

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[37] for_30=stats.norm.cdf(30,mean,std)
    for_30
      0.09121121972586788

[38] print ("Employees under 30 at the center would be expected to attract about:",np.round((for_30*400)),"employees")
      Employees under 30 at the center would be expected to attract about: 36.0 employees
```

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid*normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS:

According to central limit theorem , any large sum of independent identically distribution(iid) random variables is approximately normal.

The normal distribution is defined by two parameters, the mean and the variance and written as $X^{\sim}N(\mu,\sigma^2)$

Given

 $X_1 \sim N(\mu, \sigma^2)$ $X_2 \sim N(\mu, \sigma^2)$

Where X_1 and X_2 are independent and identically distributed (iid) normal random variables.

Difference between $2X_1$ and $X_1 + X_2$:

```
2 X_1= Scalinga random variable by a factor 2 X_1+X_2 = Sum of two independent variables Distributions:
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2X₁:

When you scale a normal random variables by a constant, the mean is also Calculated by a constant but the variance is calculated by the square of that constant.

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Mean: [2X_1]=2\mu
Variance :Var(2X_1)=(2^2)\sigma^2=4\sigma^2
Therefore,
2X_1\sim N(2\mu,4\sigma^2)
```

X_1+X_2 :

The sum of two independent variables is also normally distributed.

Mean: $(X_1 + X_2) = \mu + \mu = 2\mu$

Variance: Var(X_1+X_2)= $\sigma^2+\sigma^2=2\sigma^2$ Therefore, $X_1+X_2 \sim N(2\mu,2\sigma^2)$

Discussion:

- Both 2X₁ and X₁+X₂ are normally distributed, but with different parameters
- 2X₁ has greater spread compared to X₁+X₂ because its variance is 4 times that
 of X₁+X₂
- The mean of both distributions is the same 2 μ . However,the spread around the mean is different due to the variance.
- In Summary, both random variables have the same expected value(mean), they
 differ in terms of variability with 2X, being more spread out compared to X₁+X₂.
- 4. Let $X \sim N(100, 20^2)$. Find two values, α and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
 - A. 90.5, 105.9
 - B. 80.2, 119.8
 - C. 22, 78
 - D. 48.5, 151.5
 - E. 90.1, 109.9

ANS: D is correct
Mean=100,Std=20

- print("The two values of a and b:", stats.norm.interval(0.99,100,20))
- The two values of a and b: (48.48341392902199, 151.516586070978)
- 5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45
 - A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS:

Rupee range between Rs.99 million to 981 million Rupees, 95% of the time for the annual profit of the company

B. Specify the 5th percentile of profit (in Rupees) for the company **ANS:**

The 5th percentile of profit for the company is Rs.170million.

C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:

The Division of 2 (profit2~N(7,42)) has a larger probability of making a loss in a given year.