Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

ANS:

Mean(μ)=45 min

Standard deviation(σ)=8 min

Time commitment(X)=1 hour=60 min

But service manager begin the work after 10 min .

So,

X=60-10=50

In the given ,we have to find Z-score

$$\text{Z-score}=X-\mu/\sigma$$

Z-score = 50-45/8

=5/8

Z-score =0.625

Z-Score:

```
[7] #z-score mean=45,std=8,x=50
    z=(50-45)/8
```

```
z
```

```
0.625
```

The probability of that service manager cannot meet his commitment is 0.266

```
#find probability P(X>50)
p_value=1-stats.norm.cdf(0.625)
```

```
p_value
```

```
0.26598552904870054
```

B) Option is Correct

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.

ANS:

A) False.Because the probability for employees at the processing center are more between 38 and 44 than older than 44
Mean=38
Std=6

```
for_38=stats.norm.cdf(38,mean,std)
for_38
```

```
0.5
```

```
for_44=stats.norm.cdf(44,mean,std)
```

```
for_44
```

```
0.8413447460685429
```

```
between_38_and_44=for_44-for_38
print("probability of employee age between 38 and 44 is:",between_38_and_44)
```

```
probability of employee age between 38 and 44 is: 0.3413447460685429
```

```
more_than_44=1-stats.norm.cdf(44,mean,std)
```

```
print("probability of employee age more than 44 is :",more_than_44)
```

```
probability of employee age more than 44 is : 0.15865525393145707
```

```
true_or_false=(more_than_44 > between_38_and_44)
print("answer:",true_or_false)
```

```
answer: False
```

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees

ANS: True

```
[37] for_30=stats.norm.cdf(30,mean,std)
     for_30

     0.09121121972586788

[38] print ("Employees under 30 at the center would be expected to attract about:",np.round((for_30*400)),"employees")

     Employees under 30 at the center would be expected to attract about: 36.0 employees
```

3. If X_1~ $N(\mu, \sigma^2)$ and X_2 ~ $N(\mu, \sigma^2)$ are *iid*normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS:

According to central limit theorem , any large sum of independent identically distribution(iid) random variables is approximately normal.
The normal distribution is defined by two parameters, the mean and the variance and written as X~N(μ, σ^2)
Given
X_1~ $N(\mu, \sigma^2)$
X_2 ~ $N(\mu, \sigma^2)$
Where X_1 and X_2 are independent and identically distributed (iid) normal random variables.

<u>Difference between 2X_1 and $X_1 + X_2$:</u>

2 X_1= Scalinga random variable by a factor 2
X_1 +X_2 = Sum of two independent variables
<u>Distributions:</u>

2X_1:

When you scale a normal random variables by a constant , the mean is also Calculated by a constant but the variance is calculated by the square of that constant.

Mean: [2X_1]=2μ
Variance :Var(2X_1)=(2^2)σ^2= 4σ^2
Therefore,
2 X_1~N(2μ,4σ^2)

X_1 +X_2:

The sum of two independent variables is also normally distributed.
Mean: (X_1 +X_2)=μ+μ=2μ

Variance: Var(X_1+X_2)=$\sigma^2+\sigma^2=2\sigma^2$
Therefore,
X_1+X_2 ~N($2\mu,2\sigma^2$)

Discussion:

- **Both $2X_1$ and X_1+X_2 are normally distributed, but with different parameters**
- **$2X_1$ has greater spread compared to X_1+X_2 because its variance is 4 times that of X_1+X_2**
- **The mean of both distributions is the same 2μ. However,the spread around the mean is different due to the variance.**
- **In Summary, both random variables have the same expected value(mean),they differ in terms of variability with 2X,being more spread out compared to X_1+X_2.**

4. Let X ~ N(100, 20^2). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

   ANS: D is correct
   Mean=100,Std=20

```
print("The two values of a and b:",stats.norm.interval(0.99,100,20))
```

```
The two values of a and b: (48.48341392902199, 151.516586070978)
```

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1$ ~ N(5, 3^2) and $Profit_2$ ~ N(7, 4^2) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   ANS:
   Rupee range between Rs.99 million to 981 million Rupees, 95% of the time for the annual profit of the company
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   ANS:
   The 5^{th} percentile of profit for the company is Rs.170million.

C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:

The Division of 2 (profit2~N(7,42)) has a larger probability of making a loss in a given year.