

Crypto 14

Note: this material is not intended to replace the live lecture for students.

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14.1 Simultaneous Authentication of Equals (SAE)

NOTE 14.1.1

Wi-Fi point access, WPA3 uses SAE.

As in the previous protocols two users share a password $P = \textit{password}$ from which SAE is going to produce secure session keys.

SAE is peer-to-peer and Not Lock-step: no initiator or responder, nor client-server.

14.1.2 SAE parameters

a finite cyclic group Γ , e.g. FFC or ECC, with r elements.

an ordering function L taking two users and returning the "greater".

a random oracle equally probable, one-way function $H : \{0, 1\}^* \rightarrow \{0, 1\}^s$.

a KDF that stretch an arbitrary string *smallstr* to a string *bigstr* of a given *length* :

$$\textit{bigstr} = \text{KDF}(\textit{smallstr}, \textit{length})$$

a bijective function F from Γ to a set of numbers.

The protocol begins upon discovering of a peer.

Prior to sending any messages **Alice** and **Bob** select an element PWE , password element, from the group Γ .

14.1.3 PWE: SAE-ECC

```

i = 0
repeat
  i = i + 1
  if  $L(\text{Alice}, \text{Bob}) = \text{Alice}$  then
     $\text{pwdseed} = H(\text{Alice} \mid \text{Bob} \mid \text{password} \mid i)$ 
  else
     $\text{pwdseed} = H(\text{Bob} \mid \text{Alice} \mid \text{password} \mid i)$ 
  end if
   $x = (KDF(\text{pwdseed}, \text{len})) \bmod p$ 
  solve for  $y$  using the equation for the curve with  $x$ 
  if  $\text{pwdseed}$  is odd then
     $PWE = (x, -y)$ 
  else
     $PWE = (x, y)$ 
  end if
until  $PWE$  is on the curve
where  $p$  is the prime of the curve and  $\text{len}$  is the length of  $p$ .

```

14.1.4 PWE: SAE-FFC

```

if  $L(\text{Alice}, \text{Bob}) = \text{Alice}$  then
   $\text{pwdseed} = H(\text{Alice} \mid \text{Bob} \mid \text{password})$ 
else
   $\text{pwdseed} = H(\text{Bob} \mid \text{Alice} \mid \text{password})$ 
end if
 $\text{pwdvalue} = (KDF(\text{pwdseed}, \text{len})) \bmod p$ 
 $PWE = \text{pwdvalue}^{((p-1)/r)} \bmod p$ 
where  $p$  is the group prime,  $r$  is the order, and  $\text{len}$  is the
length of  $p$ .

```

14.1.5 SAE

Alice

generate a RND

 $rand_A, mask_A$ $scal_A = (rand_A + mask_A) \pmod r$ $elem_A = (mask_A \cdot PWE)^{-1}$ **Bob**

generate a RND

 $rand_B, mask_B$ $scal_B = (rand_B + mask_B) \pmod r$ $elem_B = (mask_B \cdot PWE)^{-1}$

$$\xrightarrow{scal_A, elem_A}$$

$$\xleftarrow{scal_B, elem_B}$$
 $K = rand_A \cdot ((scal_B \cdot PWE) \cdot elem_B)$ $k = F(K)$ $K = rand_B \cdot ((scal_A \cdot PWE) \cdot elem_A)$ $k = F(K)$ $tok_A = H(k || F(elem_A) || scal_A || F(elem_B) || scal_B)$ $\xrightarrow{tok_A}$

$$tok_B = H(k || F(elem_B) || scal_B || F(elem_A) || scal_A)$$

$$\xleftarrow{tok_B}$$

token verification

 $R = H(k || F(elem_A \cdot elem_B) || (scal_A + scal_B) \pmod r)$

token verification

 $R = idem$

14.2 Secure Remote Password (SRP)

SRP is a client-server secure password-based authentication and key-exchange protocol (PAKE).

NOTE 14.2.1

Authentication protocols are of two types:

plaintext-equivalent = requires the server to store a copy of P or something from which P is computationally feasible to obtain.

verifier-based = requires the server to store a V or something from which P is computationally infeasible to obtain. But from which P can be computationally verified.

SRP is verifier-based which reduce the damage that a Trojan can inflict.

The client, to be regarded as a human, as the P stored in his brain.

The server store a *verifier* V that allows him to check P yet it is computationally infeasible to get P from the verifier V .

14.2.1 Asymmetric key exchange (AKE)

Alice is going to be the client.

AKE: parameters and primitives

A Alice's password.

S Server's password.

a one-way function $P(x)$.

$Q(x, y), R(x, y)$ mixing functions.

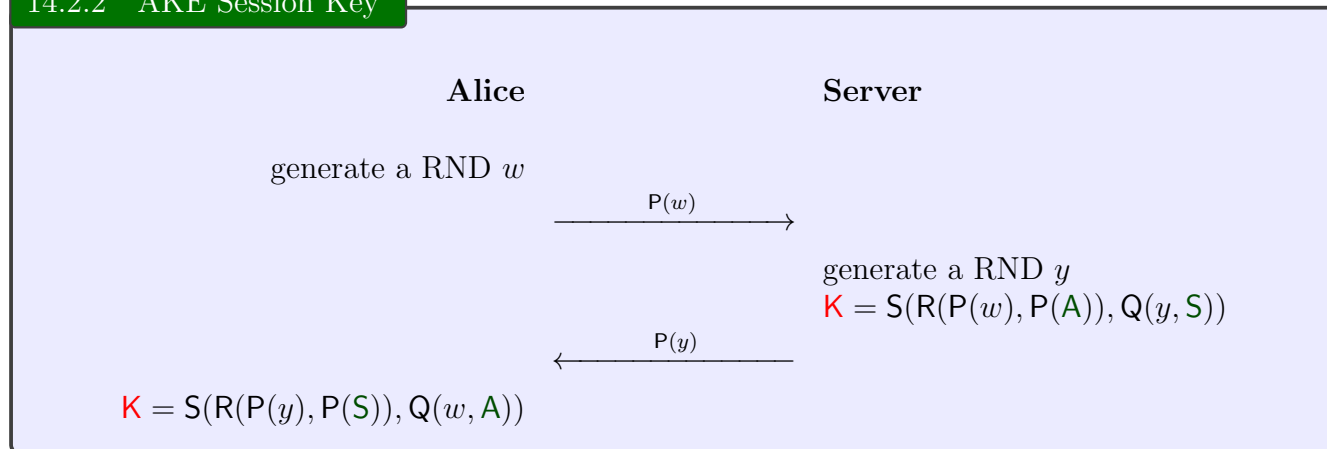
$S(x, y)$ the session key K generation function.

All this must satisfies for all w, y, S, A :

$$S(R(P(w), P(A)), Q(y, S)) = S(R(P(y), P(S)), Q(w, A))$$

At the set up **Alice** computes $P(A)$ and give it to the **Server**, and the **Server** computes $P(S)$ and gives it to **Alice**.

14.2.2 AKE Session Key



NOTE 14.2.3

To complete AKE authentication process **Alice** and **Server** can use a mutually agreeable method of key verification e.g. Challenge-Response see Crypto13 **Validation of Keys**.

Notice that AKE is [Zero-knowledge password](#).

Instead EKE protocol use prearranged shared secret. More precisely, both parties keep exactly the same secret P .

14.2.2 SRP

14.2.4 SRP specifications

Computations performed in the Galois finite field $\text{GF}(p)$, p a large prime number. That is to say, the arguments and values of P, Q, R, S are integers between 0 and $p - 1$.

Here P :

$$P(x) = g^x$$

where g is a generator of $\text{GF}(p)$.

Here the functions Q, R, S :

$$\begin{cases} Q(w, x) = w + u \cdot x \\ R(w, x) = w \cdot x^u \\ S(w, x) = w^x \end{cases}$$

where $u = u(w, x)$ is a function to be explained later.

At set-up **Alice** generate a random salt s , his long term password **A** and compute $x = H(s||\mathbf{A})$. Then she gives to the **Server** in a secure way s and $v = g^x$. The **Server** stores the verifier v and the salt s .

Exercise 14.2.5

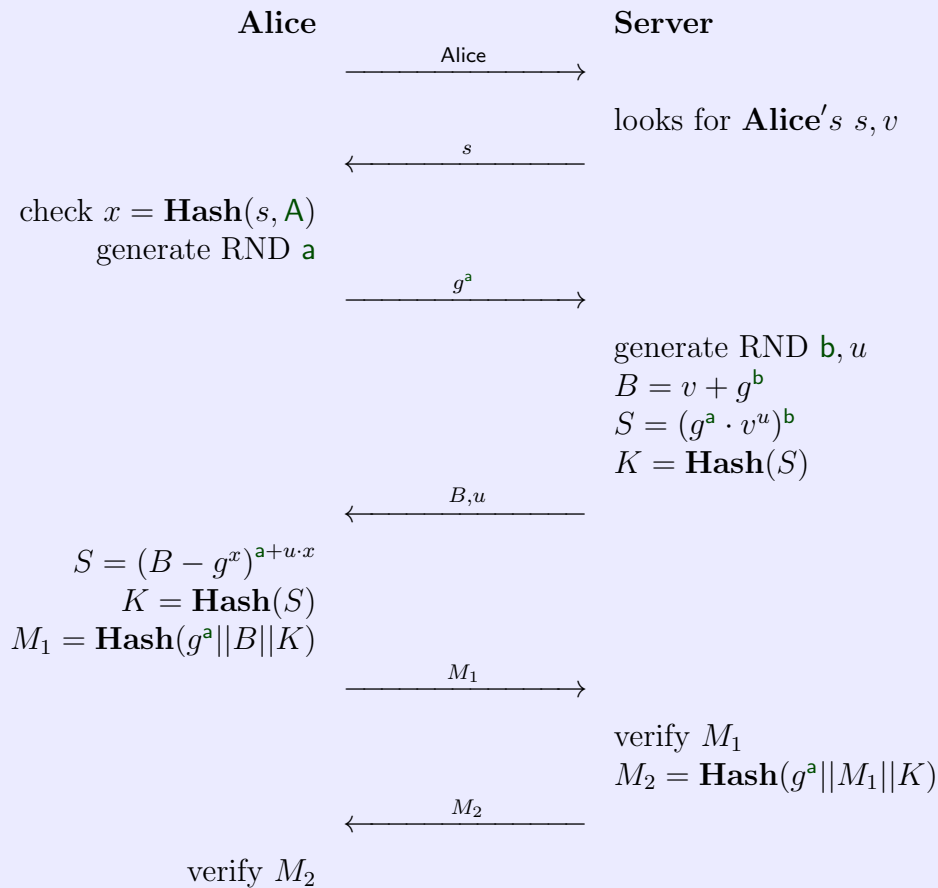
Verify that:

$$S(R(P(w), P(\mathbf{A})), Q(y, \mathbf{S})) = S(R(P(y), P(\mathbf{S})), Q(w, \mathbf{A}))$$

NOTE 14.2.6

SRP parameters domain: the prime number p , the generator g and also a **Hash** function H .

14.2.7 SRP protocol



Exercise 14.2.8

Explain:

- 1) Why just not make $B = g^b$, instead of the sum of two exponentials $B = v + g^b$, and simplify the protocol?
- 2) Partition attack: Why not make $B = v \oplus g^b$? Hint: $B = v + g^b \in \text{GF}(p)$ so $B \leq p$.
- 3) How an intruder can gain access to the **Server** knowing 1) **Alice's** verifier v and 2) how the **Server** generate u . Hint: the intruder send $g^a \cdot v^{-u}$ to **Server**. In particular, this shows why the **Server** reveals u **after** he receives **Alice's** g^a .

NOTE 14.2.9

SRP is not a verified-base Post-Quantum protocol. Indeed, this is so because the DLP instance $v = g^x$ which allows the server to get x by using a quantum computer.

14.3 J-PAKE

https://en.wikipedia.org/wiki/Password_Authenticated_Key_Exchange_by_Juggling

The Password Authenticated Key Exchange by Juggling (or J-PAKE) is a password-authenticated key agreement protocol, proposed by Feng Hao and Peter Ryan <https://eprint.iacr.org/2010/190.pdf>. This protocol allows two parties to establish private and authenticated communication solely based on their shared (low-entropy) password without requiring a Public Key Infrastructure. It provides mutual authentication to the key exchange, a feature that is lacking in the Diffie-Hellman key exchange protocol.

It is implemented in the WiFi protocols Thread and Open Thread : <https://openthread.io/>

14.4 Bibliography

Books I used to prepare this note:

- [Paar10] Paar, Christof, Pelzl, Jan, *Understanding Cryptography, A Textbook for Students and Practitioners*, Springer-Verlag, 2010.

Papers I used to prepare this note:

- [BeMe92] S. Bellovin and M. Merritt; *Encrypted Key Exchange: Password-based Protocols Secure Against Dictionary Attacks*, Proc of the Symposium on Security and Privacy, pages 72-84, IEEE, 1992 <https://www.cs.columbia.edu/~smb/papers/neke.pdf>
- [Ha08] Harkins, Dan; *Simultaneous Authentication of Equals: A Secure, Password-Based Key Exchange for Mesh Networks*, 2008 Second International Conference on Sensor Technologies and Applications. IEEE, DOI 10.1109/SENSORCOMM.2008.131 <https://ieeexplore.ieee.org/document/4622764>
- [Ha08] Harkins, Dan; *Simultaneous Authentication of Equals: A Secure, Password-Based Key Exchange for Mesh Networks*, 2008 Second International Conference on Sensor Technologies and Applications. IEEE, DOI 10.1109/SENSORCOMM.2008.131 <https://pdfs.semanticscholar.org/cac2/887626aa79bc2c30ff09330e0920cadb24db.pdf>
- [Wu97] Wu, Thomas; *The Secure Remote Password Protocol*, 1998 Internet Society Symposium on Network and Distributed System Security. <http://www.scs.stanford.edu/nyu/02sp/sched/srp.pdf>

and some interesting links:

<https://www.youtube.com/watch?v=iaH8UG2yMg4>

[https://en.wikipedia.org/wiki/Kerberos_\(protocol\)](https://en.wikipedia.org/wiki/Kerberos_(protocol))

https://en.wikipedia.org/wiki/Security_protocol_notation

https://en.wikipedia.org/wiki/Simultaneous_Authentication_of_Equals

<http://homes.di.unimi.it/visconti/PBKDF2.pdf>

https://en.wikipedia.org/wiki/Needham%E2%80%93Schroeder_protocol

http://www.cs.unc.edu/~fabian/course_papers/needham.pdf

<http://pages.cs.wisc.edu/~remzi/Courses/736/Spring2005/Papers/data-encryption-denning.pdf>

<http://srp.stanford.edu/>