Calculate Gini Index of Attribute "Outlook".

Outlook

	Y	N	T	P(Y)	P(N)	P(Attr)
Rainy	3	2	5	3/5	2/5	5/14
overcast	4	0	4	4/4	0/4	4/14
Sunny	2	3	5	2/5	3/5	5/14

$$\text{gini(Rainy)} = \text{gini}(3,2) = 1 - \left[\left(\frac{3}{5}\right)^2 + \left(\frac{2}{5}\right)^2\right] = 0.48$$

$$\text{gini(overcast)} = \text{gini}(4,0) = 1 - \left[\left(\frac{4}{4}\right)^2 + \left(\frac{0}{4}\right)^2\right] = 0$$

$$\text{gini(Sunny)} = \text{gini}(2,3) = 1 - \left[\left(\frac{2}{5}\right)^2 + \left(\frac{3}{5}\right)^2\right] = 0.48$$

$$\text{gini(Attribute.value)} = 1 - \sum_{i \in \text{class}} [P(i)]^2$$

$$\text{gini(Target.class, Attribute)} = \sum P(\text{Attribute, Target}) \cdot \text{gini(Attribute)}$$

$$\text{Gini}(\text{Target}, \text{outlook}) = \frac{5}{14}(0.48) + \frac{4}{14}(0) + \frac{5}{14}(0.48) = 0.343$$

Temperature

	Y	N	T	P(Y)	P(N)	P(A)
Hot	2	2	4	2/4	2/4	4/14
cold	3	1	4	3/4	1/4	4/14
mild	4	2	6	4/6	2/6	6/14

windy

$$\text{Gini(Hot)} = 1 - \left[\left(\frac{2}{4}\right)^2 + \left(\frac{2}{4}\right)^2\right] = 0.5$$

$$\text{Gini(cold)} = 1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right] = 0.375$$

$$\text{Gini(mild)} = 1 - \left[\left[\frac{4}{6}\right]^2 + \left[\frac{2}{6}\right]^2\right] = 0.444$$

$$\text{Gini(PlaySoft, Temp)} = \frac{4}{14} \times 0.5 + \frac{4}{14} \times 0.375 + \frac{6}{14} \times 0.444$$

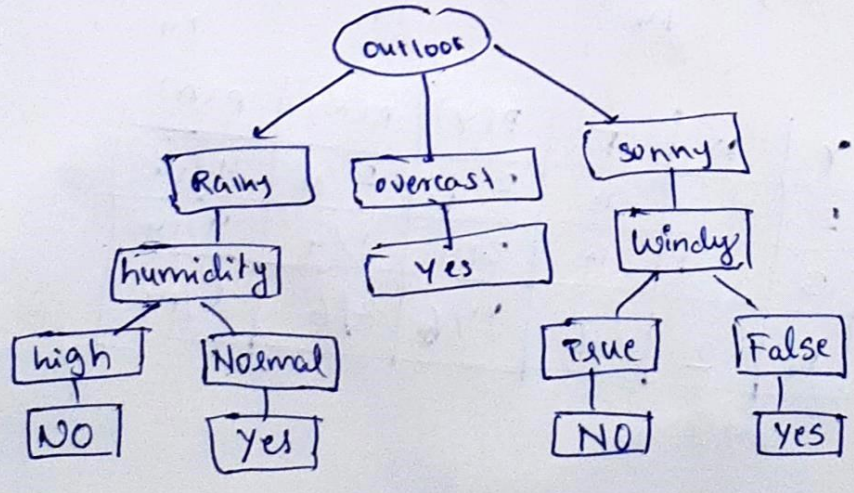
$$= 0.44403$$

	Y	N	P(Y)	P(N)	P(T)
Humidity					
High	3	4	3/7	4/7	7/14
Normal	6	1	6/7	1/7	7/14

$$\text{Gini(High)} = 1 - \left[\left(\frac{3}{7}\right)^2 + \left(\frac{4}{7}\right)^2\right] = 0.4898 \text{ ~~0.7959~~}$$

$$\text{Gini(Normal)} = 1 - \left[\left(\frac{6}{7}\right)^2 + \left(\frac{1}{7}\right)^2\right] = 0.2448$$

$$\text{Gini(PG, Humidity)} = \left(\frac{7}{14}\right) 0.4898 + \frac{7}{14} 0.2448 = 0.3673$$



		Y	N	T	P(Y)	P(N)	P(windy)
Windy	False	6	2	8	6/8	2/8	8/14
	True	3	3	6	3/6	3/6	6/14

$$\text{Gini(False)} = \text{Gini}(6,2) = 1 - \left(\left(\frac{6}{8}\right)^2 + \left(\frac{2}{8}\right)^2\right) = 0.375$$

$$\text{Gini(True)} = \text{Gini}(3,3) = 1 - \left(\left(\frac{3}{6}\right)^2 + \left(\frac{3}{6}\right)^2\right) = 0.5$$

$$\text{Gini(OH, windy)} = \frac{8}{14} \times 0.375 + \frac{6}{14} \times 0.5 = 0.429$$

→ Outlook sunny → Temparature (subtree)

		Y	N	T	P(Y)	P(N)	P(temp.)
Temparature	Mild	2	1	3	2/3	1/3	3/5
	Cool.	1	1	2	1/2	1/2	2/5

$$\text{Gini(Mild)} = \text{Gini}(2,1) = 1 - \left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right) = 0.44$$

$$\text{Gini(Cold)} = \text{Gini}(1,1) = 1 - \left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = 0.5$$

$$\text{Gini(pg, temp.)} = \frac{3}{5} \times 0.44 + \frac{2}{5} \times 0.5 = 0.464$$

→ Outlook sunny → Humidity (subtree)

		Y	N	T	P(Y)	P(N)	P(humidity)
Humidity	Normal	2	1	3	2/3	1/3	3/5
	High.	1	1	2	1/2	1/2	2/5

$$\text{Gini(Normal)} = \text{Gini}(2,1) = 1 - \left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2\right) = 0.44$$

$$\text{Gini(high)} = \text{Gini}(1,1) = 1 - \left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = 0.5$$

$$\text{Gini(pg, humidity)} = \frac{3}{5} \times 0.44 + \frac{2}{5} \times 0.5 = 0.464$$

→ outlook sunny → windy (subtree)

windy	Y	N	T	P(Y)	P(N)	P(windy)
True	0	2	2	0/2 = 0	2/2 = 1	2/5
False	3	0	3	3/3 = 1	0/3 = 0	3/5

Gini (True) = Gini (0,2) = $1 - (0^2 + 1^2) = 0$

Gini (False) = Gini (3,0) = $1 - (1^2 + 0^2) = 0$.

Gini (Pg, windy) = $\frac{2}{5} \times 0 + \frac{3}{5} \times 0 = 0$.

→ outlook rainy → Temperature (subtree)

Temperature	Y	N	T	P(Y)	P(N)	P(Temp.)
Hot	0	2	2	0	1	2/5
mild	1	1	2	1/2	1/2	2/5
cool	1	0	1	1	0	1/5

Gini (Hot) = Gini (0,2) = $1 - (0^2 + 1^2) = 0$

Gini (mild) = Gini (1,1) = $1 - \left(\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right) = 0.5$

Gini (cool) = Gini (1,0) = $1 - (1^2 + 0^2) = 0$

Gini (Pg, temp.) = $\frac{2}{5} \times 0 + \frac{2}{5} \times 0.5 + \frac{1}{5} \times 0 = 0.2$.

→ outlook rainy → Humidity (subtree)

Humidity	Y	N	T	P(Y)	P(N)	P(humidity)
High	0	3	3	0/3 = 0	3/3 = 1	3/5
Normal	2	0	2	2/2 = 1	0/2 = 0	2/5

Gini (high) = Gini (0, 3) = $1-(0^2+1^2) = 0$.

Gini (normal) = Gini(2,0) = $1-(1^2+0^2) = 0$

Gini (pg, humidity) = $\frac{3}{5}\times 0 + \frac{2}{5}\times 0 = 0$.

→ outlook rainy → windy (subtree)

windy	Y	N	T	P(Y)	P(N)	P(windy)
True	1	1	2	1/2	1/2	2/5
False	1	2	3	1/3	2/3	3/5

Gini(True) = Gini (1,1) = $1-\left(\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2\right) = 0.5$

Gini (False) = Gini (1,2) = $1-\left(\left(\frac{1}{3}\right)^2+\left(\frac{2}{3}\right)^2\right) = 0.45$

Gini (pg, windy) = $\frac{2}{5}\times 0.5 + \frac{3}{5}\times 0.45 = 0.47$.