

Assignment 3

Audio signal decimation and interpolation

Downsampler:

Downsampling is compressing the signal in the time domain or reducing the length of the sequence in the time domain. In the frequency domain, the spectrum gets expanded. The input output relation in time domain is given by

$$y[n] = x[Mn] \text{ where } M \text{ is the downsampling factor}$$

Decimation:

The process of downsampling an input sequence $x[n]$, after passing through an Anti-aliasing filter is called Decimation. The Anti-aliasing filter acts as an LPF with cut off frequency

$$|f_c| \leq (\pi)/M,$$

where M =down sampling factor And Gain of the filter=1

The Anti-aliasing filter is used to avoid the aliasing of the spectral components, which may occur due to the decrease in the sampling rate after downsampling a signal. In Decimation, the sampling rate is reduced from f_s to f_s/M by discarding $(M - 1)$ samples for every M sample in the original sequence.

Upsampling:

Upsampling increases the sampling rate by padding the input signal with zeros between the original samples in the time domain. In the frequency domain, the spectrum gets compressed. The input output relation in time domain is given by

$$y[n] = [n/L] \text{ if } n \text{ is a multiple of } L = 0 \text{ otherwise}$$

Where L is the upsampling factor

Interpolation:

The process of upsampling an input sequence $x[n]$, and passing through an Anti-imaging filter called Interpolation. The Anti-imaging filter acts as an LPF with cut off frequency

$$|f_c| \leq (\pi)/L, \text{ where } L=\text{up sampling factor And Gain of the filter}=1$$

The Anti-imaging filter is used to avoid the unwanted replica of the spectral components due to the increase in the sampling rate after upsampling a signal. In Interpolation, the sampling rate

increases from f_s to $f_s \cdot L$ by padding $(L - 1)$ zero samples for every L sample in the original sequence.

Analysis and observations:

- Initially, the given audio signal is read using an audio read function by which we obtain the respective samples and the sampling frequency of the given audio signal.
- In figure 1, a plot of frequency vs time can be seen which tells about the amplitude of different frequencies of the audio signal being present at that particular time instant.

Decimation and Interpolation by a factor 2:

- The given audio signal is used as an input to the anti-aliasing filter, with the cut-off frequency $\omega_c = \pi/2, f_c = f_s/4 = f_s'/2, f_s' = f_s/2$ and then followed by a downsampler with a factor 2.
- reconstruction can be difficult if the sampling rate is chosen to be just above the Nyquist frequency. Reconstruction would be much easier for a higher sampling rate.
- As $M=2$, this means that every alternate sample is being discarded at the output of the decimated signal.
- When we hear the decimated audio signal, the loudness or amplitude of the output signal is decreased compared to that of the original signal
- Now the decimated output audio signal is given to the upsampler with $L=2$, which adds a sample zero between every alternate sample and then passed through an anti-imaging filter.
- The same is repeated for $L=M=4,8$.
- Now the obtained output signal from the interpolation is expected to be similar to that of the original signal, but the output audio is observed to be with decreased amplitude. This is due to the decimation and interpolation, which eliminates and adds samples respectively according to the factor M, L .
- As Downsampling a signal by a factor M is simply the process of discarding $M-1$ samples between a sample and the M th sample.
- Similarly Upsampling a signal by a factor of L is simply the process of inserting $L - 1$ zero in between each sample

- Although we have considered the downsampling and upsampling by the same factor, due to the deletion and addition of $M-1$ and $L-1$ samples, the audio signal at the output gets altered/distorted and hence the output would not have the exact amplitude i.e it has decreased amplitude version or the overall energy is said to be decreased
- This could also be explained by the spectrograms that are generated
- If we could observe the spectrums i.e figure 2 and 3, as we are passing through the low pass filters i.e anti aliasing and anti imaging filters with cut off frequency to be ω_c or f_c , the higher frequency components of the audio signal gets attenuated and therefore the spectrum has lower frequencies present in it.
- Another observation could be is, as the factor of decimation and interpolation i.e L ($L=M$) increases, more and more samples would be deleted during downsampling and more zero samples are added to the signal due to which the clarity of the output signal is greatly lost for higher factors of L when compared to that of the lower factors of L .
- As the L (or M as $L=M$ here) value increases, the cut off frequency for the low pass filters decreases hence more frequencies get attenuated,
- This can be visually seen from the figures of the output spectrums where the y-axis of the spectrum could be seen decreasing as L value increases
- In the spectrogram the frequencies with higher amplitude are seen in red colour while the frequencies that are less stronger are seen in orange and blue respectively.
- The command `specgram` computes short time fourier transform