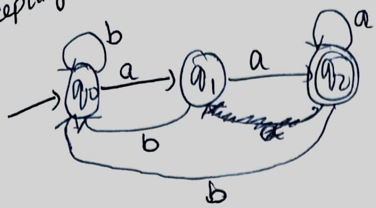
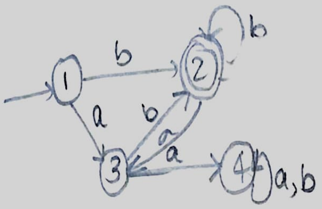
Introduction to language and Theory of Computation

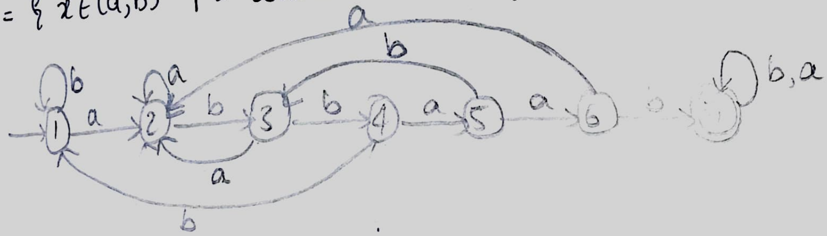
1) DFA accepting the language of strings ending in aa $L: \{a,b\}$



2) L = { Strings ending in b and not containing substring aa



3) $L = \{ x \in (a,b)^* \mid x$ contains substring abbaab $\}$

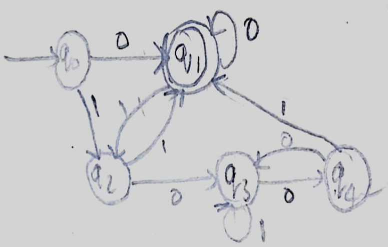


4) FA accepting binary representations of integers divisible by 3

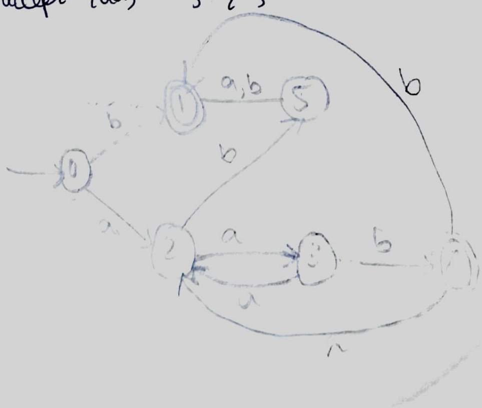
0, 11, 110, 1001, 1100, 1111, 10010, 10101, 11000, 11011, 11110

100001, 100100, 100111, 101010, 101101, 110000

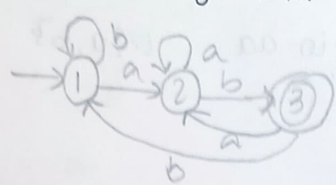
110111



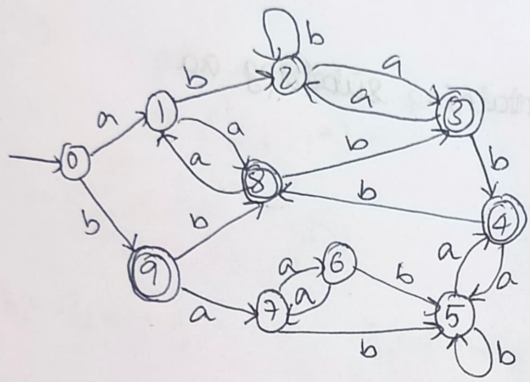
5) FA to accept $\{aa, aab\}^* \{b\}$ $\quad$ * : zero or more occurrence



6) FA accepting strings with a in nth symbol from end / n=2



7) Minimize DFA



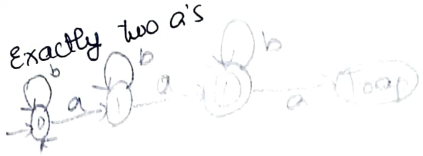
	0	1	2	3	4	5	6	7	8
1	✓								
2	✓								
3	X	X	X						
4	X	X	X						
5	✓			✓	✓				
6	✓	✓	✓	✓	✓	✓			
7	✓	✓	✓	✓	✓	✓			
8	X	X	X			X	X	X	
9	X	X	X	X	X	X	X	X	X

	a	b
[0,5]	[1,4]	[9,5]
[1,5]	[8,4]	[
[2,5]	[3,4]	[
[3,5]	[2,4]	
[4,5]	[5,4]	
[3,4]	[2,5]	[4,8]
[0,2]	[1,3]	[9,
[1,2]	[2,3]	[2,2]

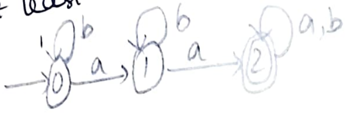
	a	b
[3,9]	=[2,7]	[4,8]
[4,9]	[5,7]	[8,8]
[8,9]	[1,7]	[3,8]
[3,8]	[2,1]	[4,3]
[4,8]	[5,1]	[8,3]
[0,7]	[1,6]	[9,5]
[1,7]	[8,6]	[2,5]
[2,7]	[3,6]	[2,5]
[3,7]	[2,6]	[4,5]
[4,7]	[5,6]	[8,5]
[5,7]	[4,6]	[5,5]
[6,7]	[7,6]	[5,5]
[0,6]	[1,7]	[9,5]
[1,6]	[8,7]	
[2,6]	[3,7]	
[3,6]	[2,7]	[4,5]
[4,6]	[5,7]	[8,5]
[5,6]	[4,7]	
[0,1]	[1,8]	[9,2]

L = {a,b}

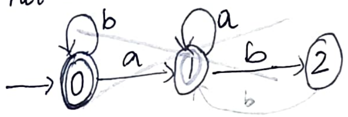
8) a) L = Exactly two a's

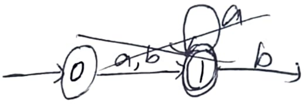


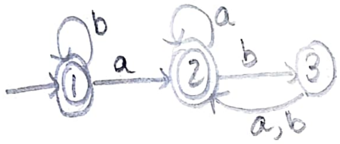
b) At least two a's



c) Do not end with ab

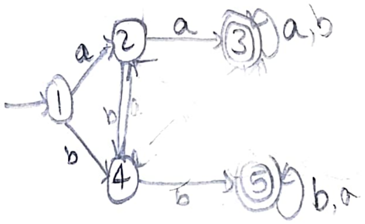






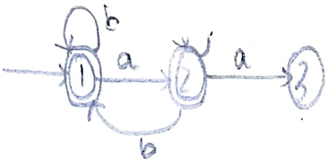
d) Begin or end with aa or bb

Doubt

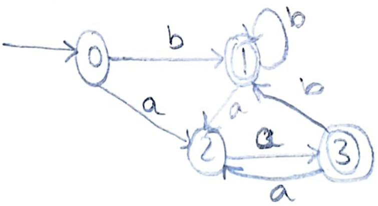


e) Not containing substring aa

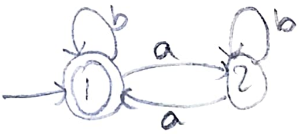
baba



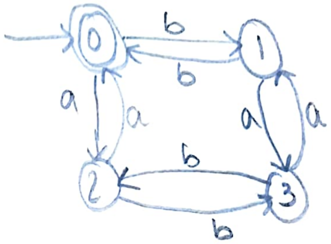
f) No. of a's is even



(or)



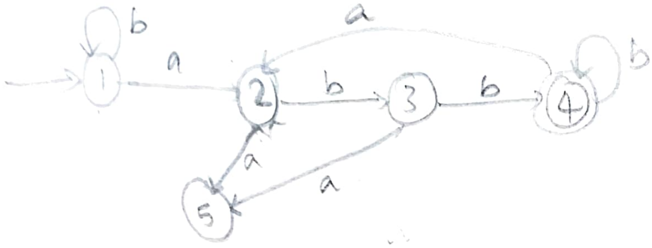
g) No. of a's and b's are even



h) no more than one occurrence of aa.

Doubt

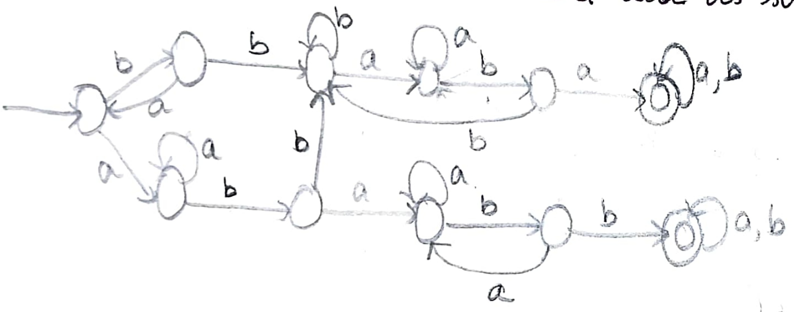
i) Strings in which every a is followed by bb.



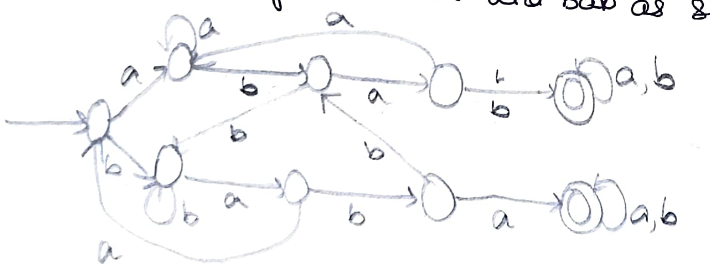
abbabbbb ✓

abababb ✗

j) Strings containing ~~no mo~~ both bb and aba as substring.



k) Strings containing both aba and bab as substring



abab

abaabab

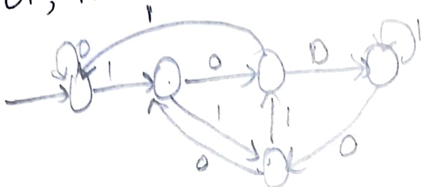
babbbaba

babaabaa

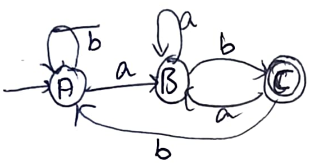
9) Strings accepted by binary represented integer divisible by 5.

0, 101, 1010, 1111, 10100, 11001, 11110, 100011, 101000

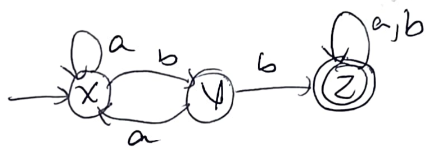
101101, 110010



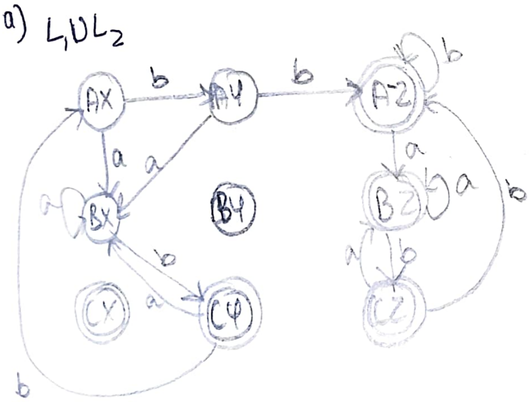
10) M_1 =



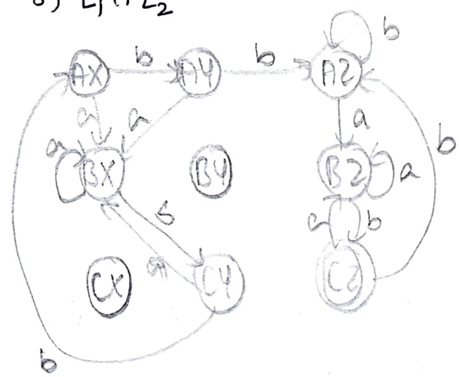
M_2 =



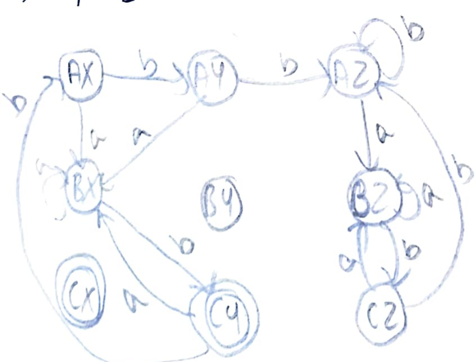
a) $L_1 \cup L_2$



b) $L_1 \cap L_2$

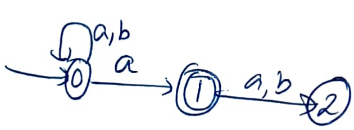


c) $L_1 - L_2$

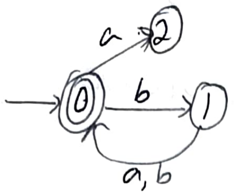


10) Draw DFA

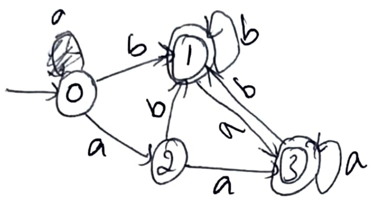
a) $\{a, b\}^* \{a\}$



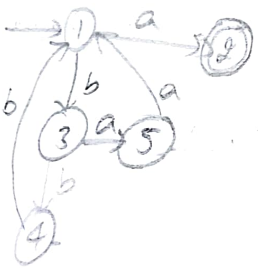
b) $\{bb, ba\}^*$



c) $\{a, b\}^* \{b, aa\} \{a, b\}^*$

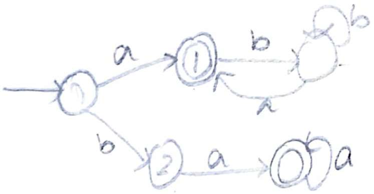


d) $\{bbb, baa\}^* \{a\}$

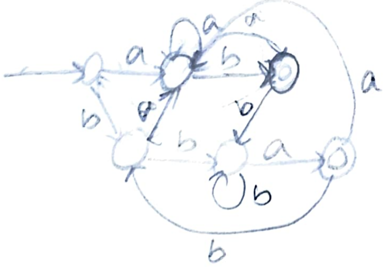


e) $\{a\} \cup \{b\}\{a\}^* \cup \{a\}\{b\}^*\{a\}$ Doubt ?

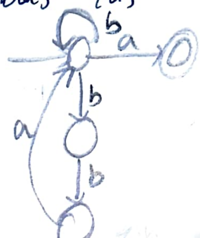
$\{a, ba^*, ab^*a\}$



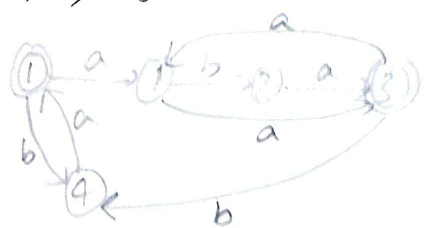
f) $\{a, b\}^* \{ab, bba\}$



g) $\{b, bba\}^* \{a\}$



h) $\{aba, aa\}^* \{ba\}^*$



11) In each case below, find a string of minimum length in $\{a,b\}^*$ not in language corresponding to given regular expression

a) $b^* (ab)^* a^*$

Acceptable strings = ϵ, b, a, ab, bab, aba, ...

~~Ans~~ String not in lang : ~~ba~~ aab

~~Bcoz b^* must followed by $(ab)^*$ and ba breaks the required order.~~

b) $(a^* + b^*)(a^* + b^*)(a^* + b^*)$

AS = aaa, bbb, abb, aba, baa, bba, ϵ ....

SN = abab

c) $a^* (baa^*)^* b^*$

AS = a, ϵ, b, baa, abaa, abb, ~~abb~~ ...

NS = ~~abba~~ bba

d) $b^* (a + ba)^* b^*$

AS = b, ϵ, bbb, a, ba,

NS = abba

12) Consider 2 regular expression $r = a^* + b^*$ $s = ab^* + ba^* + b^*a + (a^*b)^*$

AS of r = ϵ, aaa, a, b, bbb

AS of s = ϵ, a, b, ab, ba, bbba, aaabaaab, abb, baa ..

a) Find string corresponding to r but not in s.

aa, aaa, aaaa, bb, bbb

b) Find string corresponding to s but not to r.

ab, ba, bbba, aaab

c) Find string corresponding to both r and s

d) Find a string in $\{a,b\}^*$ corresponding to neither r nor s.

aba

13) Simplify

a) $r(r^*r + r^*) + r^*$

$r(r^*) + r^*$

$r^* + r^*$

r^*

b) $(r+\varepsilon)^*$

$(r+\varepsilon)^*$

r^*

c) $(r+s)^* rs (r+s)^* + r^*s^*$

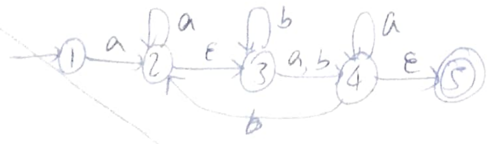
$(r+s)^*(\varepsilon . rs . \varepsilon) + r^*s^*$

$(r+s)^* rs + r^*s^*$

14) RE $(b+ab)^*(a+ab)^*$ describes set of all strings in $\{a,b\}^*$ not containing the substring <u>abbab</u> x <u>babb</u> for any x

RE $(a+b)^*(aa^*bb^*aa^* + bb^*aa^*bb^*)(a+b)^*$ describes the set of all string in $\{a,b\}^*$ containing both substring <u>aa</u> <u>bb</u>

15)



Say whether string accepts it

a) aba

$\hat{\delta}(1,a) = \{2\}$

$\hat{\delta}(1,ab) = \delta(2,b) = \hat{\delta}(3,b) = \{4,3\}$

$\hat{\delta}(1,aba) = \delta(3,a) \cup \delta(4,a) = \{3,4,5\}$

b) abab

$\hat{\delta}(1,a) = 2,3$

$\hat{\delta}(1,aba) = \{3,4,5\}$

$\hat{\delta}(1,abab) = \{3,4,2,5\}$

c) ~~aaabbb~~

c) aaabbb

$\hat{\delta}(1,a) = 2,3$

$\hat{\delta}(1,aa) = \delta(2,a) \cup \delta(3,a) = \{2,3,4\}$

$\hat{\delta}(1,aaa) = \delta(2,a) \cup \delta(3,a) \cup \delta(4,a) = \{2,3,4,5\}$

$\hat{\delta}(1,aaab) = \delta(2,b) \cup \delta(3,b) \cup \delta(4,b) \cup \delta(5,b) = \{3,4,2,5\}$

$\hat{\delta}(1,aaabb) = \delta(2,b)$ 〃 $= \{3,4,2,5\}$

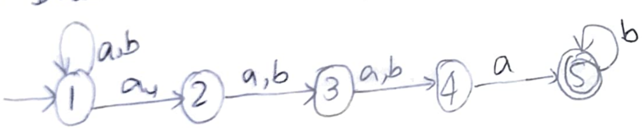
〃 $= \{2,3,4,5\}$

Accepted

16) Given Transition table of NFA with states 1-5 & input alphabet $\{a,b\}$. There are no ϵ transitions.

q	$\delta(q,a)$	$\delta(q,b)$
$\rightarrow 1$	1,2	1
2	3	3
3	4	4
4	5	$\emptyset$
$*5$	$\emptyset$	5

a) Draw transition diagram



b) Calculate $\delta^*(1,ab)$

$\delta^*(1,a) = \{1,2\}$

$\delta^*(1,ab) = \delta(1,b) \cup \delta(2,b) = \{1,3\}$ Not accepted

c) Calculate $\delta^*(1,abaab)$

$\delta^*(1,ab) = \{1,3\}$

$\delta^*(1,aba) = \delta(1,a) \cup \delta(3,a) = \{1,2,4\}$

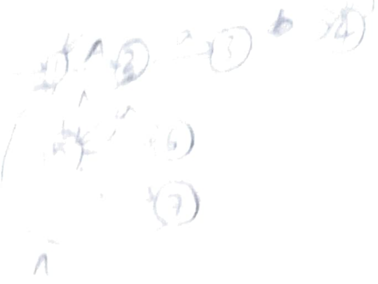
$\delta^*(1,abaa) = \delta(1,a) \cup \delta(2,a) \cup \delta(4,a) = \{1,2,3,5\}$

$\delta^*(1,abaab) = \delta(1,b) \cup \delta(2,b) \cup \delta(3,b) \cup \delta(5,b) = \{1,3,4,5\}$

Accepted

17)

q	$\delta(q,a)$	$\delta(q,b)$	$\delta(q,\Lambda)$
1	$\emptyset$	$\emptyset$	2
2	3	$\emptyset$	5
3	$\emptyset$	4	$\emptyset$
4	4	$\emptyset$	1
5	$\emptyset$	6, 7	$\emptyset$
6	5	$\emptyset$	$\emptyset$
7	$\emptyset$	$\emptyset$	1



Find

a) $\Lambda(\{2,3\})$
$= \{2, 3, 5\}$

b) $\Lambda(\{1\})$
$\{2, 5, 1\}$

c) $\Lambda(\{3,4\})$
$\{3, 4, 1, 2, 5\}$

d) $\delta^*(1, ba)$ -

$\delta^*(1,b) = EC(\delta(1,b))$ -
$= EC(\emptyset)$
$= \emptyset$

$\delta^*(1, ba)$ = Dead state

e) $\delta^*(1, ab)$

$\delta^*(1,a) = EC(\delta(1,a))$
$= EC(\emptyset)$
$= \emptyset$

d) $\delta^*(1, ba)$ $\qquad$ $EC(1) = \{1,2,5\}$

$\delta^*(\delta(EC(1), b), a)$

$\delta^*(\delta(1,2,5, b), a)$

$\delta^*(\delta(6,7), a)$ $\qquad$ $E(6) = 6$
$E(7) = 7,1,2,5$

$\delta^*(\delta(7,1,2,5,6), a)$

$\{3\ 5\}$

on epsilon $= \{3, 5\}$

e) $\delta^*(1, ab)$ $\qquad$ $EC(1) = 1,2,5$

$\delta^*(1,a) = \delta^*(1,a)$
$= \delta(1,2,5, a)$
$= 3$

$E(3) = 3$

$\delta^*(1,ab) = \delta^*(3, b)$
$= \delta(3,b)$
$= 4$

on epsilon $\{4,1,2,5\}$

f) $\delta^*(1, ababa)$

$\delta^*(1, ab) = 4$ (prev ans)

$EC(4) = \{1,2,5,4\}$

$\delta^*(1, aba) = \delta^*(4, a)$
$= \delta(1,2,4,5, a)$
$= \{3, 4\}$

$\delta^*(1, abab) = \delta^*(\{3,4\}, b)$
$= \delta(1,2,3,4,5, b)$
$= 4, 6, 7$

$\delta^*(1, ababa) = \delta^*(4,6,7, a)$
$= \delta(1,2,4,5,6,7, a)$
[illegible]

Extra

1) ~~Draw~~ Give regular expression $\Sigma = \{0,1\}$

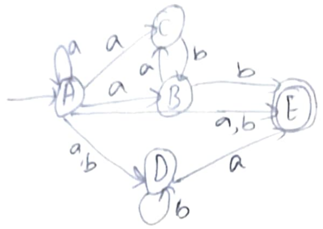
a) Set of all strings that do not end with 00

$(0+1)^*(1+0$ [illegible]

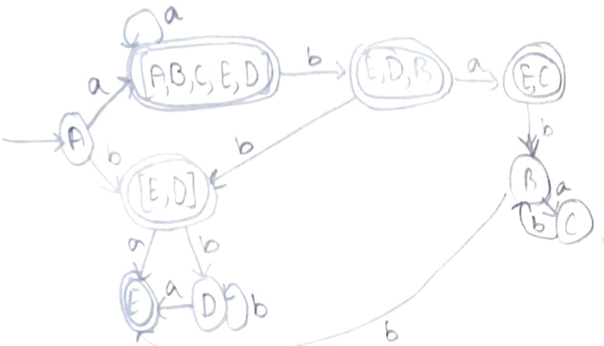
b) Set of all strings that do not contain substring 01

$0^* + 1^* 0^*$

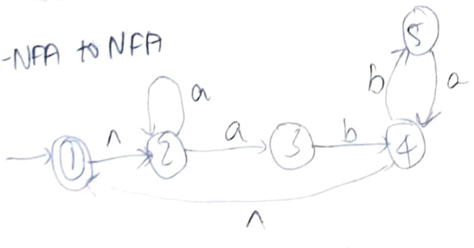
2) NFA given, using subset construction draw equivalent DFA



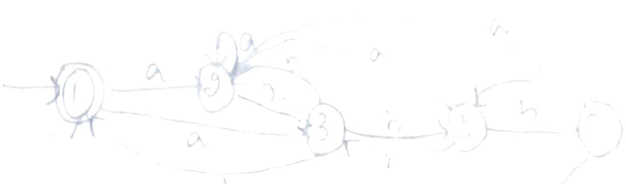
States	a	b
→ A	A, B, C, E, D	E, D
A, B, C, E, D	A, B, C, E, D	E, D, B
E, D	E	D
E, D, B	E, C	D, E
E	∅	∅
D	E	D
E, C	∅	B
B	C	E
C	∅	B



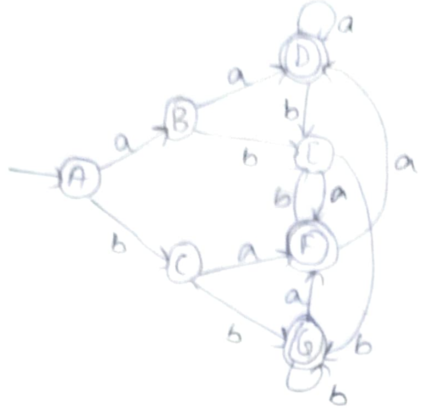
3) ε-NFA to NFA



	εC	a	$εC_a$	b	$εC_b$
→ 1	1, 2	2, 3	2, 3	∅	∅
2	2	2, 3	2, 3	∅	∅
3	3	∅	∅	4	4, 1, 2
4	4, 1, 2	2, 3	2, 3	5	5
* 5	5	4	4, 1, 2	∅	∅



...mize DFA



	a	b
[D, G]	D, F	E, G
[F, G]	D, F	E, G
[D, F]	D, D	E, E
[B, E]	D, F	F, G
[C, E]	F, F	G,
[D, E]	D, F	E
[B, C]	D, F	E, G

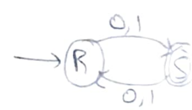
	A	B	C	D	E	F
B	X					
C	X	✓				
D	X	X	X			
E	X	✓		✓		
F	X		X		X	
G	X	X	X	✓	X	✓

E = C D = F

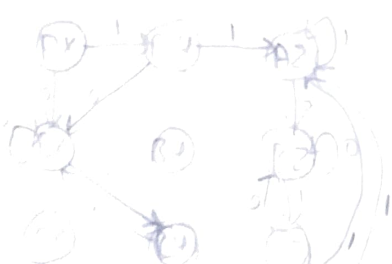
5) m1:

m2:

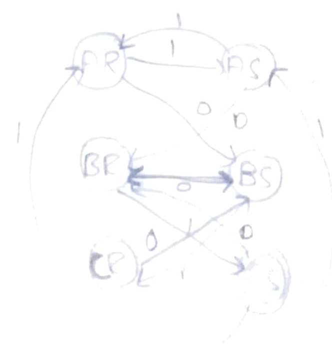
m_3:



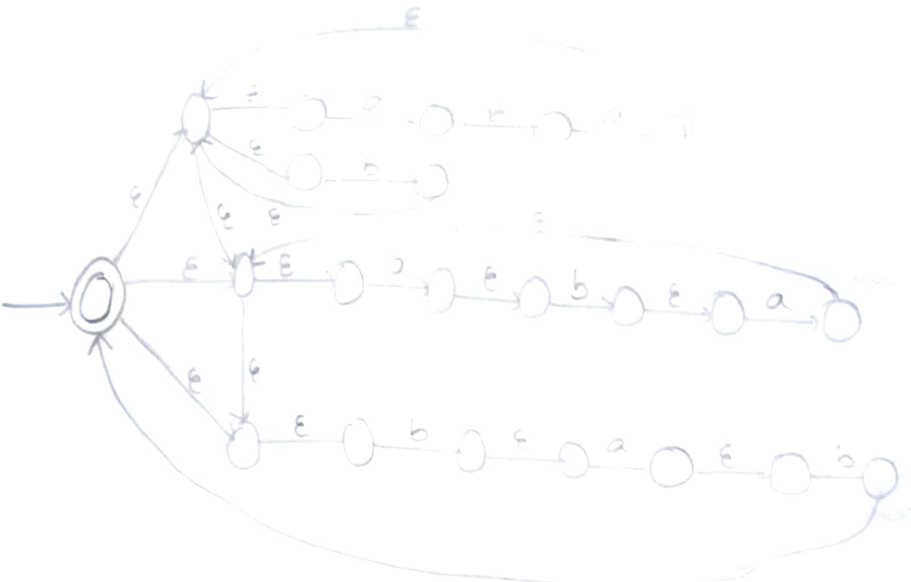
a) L1 − L2



b) L1 ∩ L3



6) NFA corresponding to $(aa+b)^*(aba)^*bab)^*$



7) Given RE Draw NFA accepting the corresponding lang,

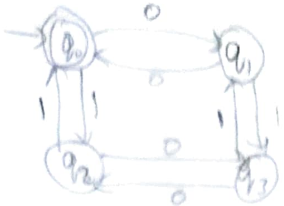
a) $(b+bba)^*a$

RE → ENFA → NFA

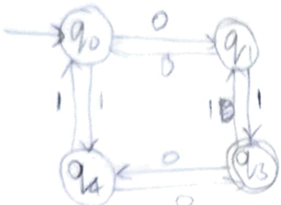
	ϵ	a	ϵn_a	b	ϵn_b
1	1, 2, 4, 7, 13, 14	15	15, 16	5, 8	2, 4, 5, 7, 8, 9, 13, 14
2	4, 7, 2, 13, 14	15	15, [illegible]	5, 8	[illegible]
4	4	$\emptyset$	$\emptyset$	5	2, 4, 5, [illegible]
5	5, 2, 4, 7, 13, 14	15	15, 16	5, 8	[illegible]
7	7	$\emptyset$	$\emptyset$	8	[illegible]
8	8, 9	$\emptyset$	$\emptyset$	$\emptyset$	[illegible]
9	9	$\emptyset$	$\emptyset$	[illegible]	[illegible]
10	[illegible]	12	[illegible]	$\emptyset$	[illegible]
11	11	[illegible]	[illegible]	[illegible]	[illegible]
12	2, 2, 4, [illegible]	[illegible]	[illegible]	5, 8	[illegible]
13	13, 14	[illegible]	[illegible]	[illegible]	[illegible]
14	14	[illegible]	15, [illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

8) Construct DFA

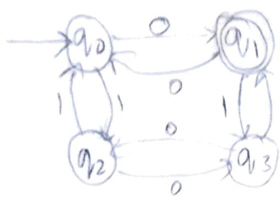
a) $L = \{w \mid w$ has both even no. of 0's & 1's$\}$



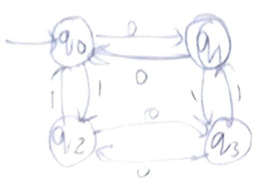
b) $L = \{w \mid w$ has both odd ~~even~~ no. of 0's & 1's$\}$



c) $L = \{w \mid w$ has both odd no. of 0s & even no. of 1's$\}$



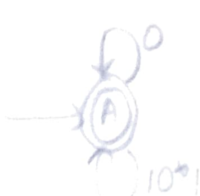
d) $L = \{w \mid w$ has both even no. of 0's & odd no. of 1's$\}$



9) DFA $\longrightarrow$ RE

State Elimination

a)

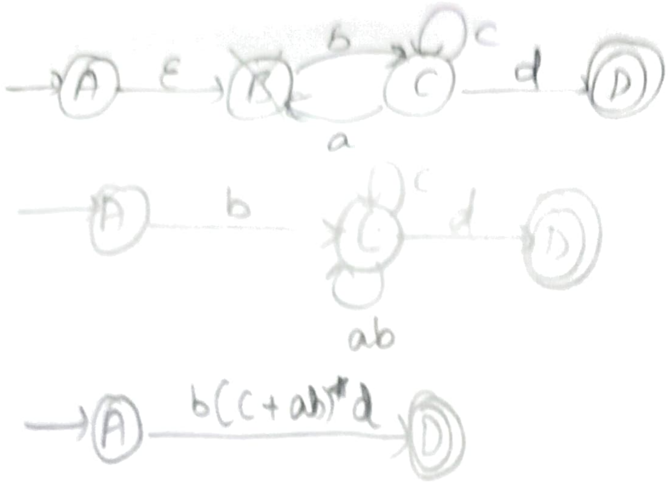


RE: $(0 + 10^*1)^*$

b)

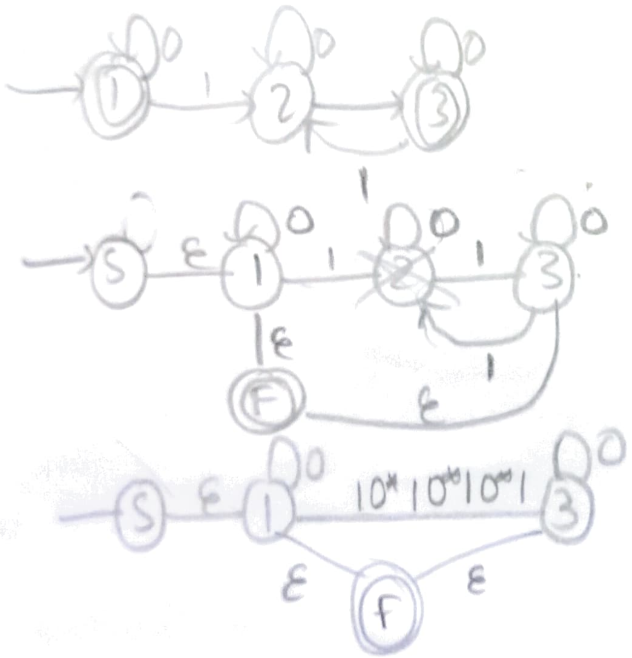
$(0 + 10^*$ [illegible] $)^*$

c)



RE = $b(c+ab)^*d$

d)



RE = $(0 + 10^*1)^*$