Data Mining

Week 1: Introduction, Association Rules

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Course Outline:

- Introduction: KDD Process
- Data Preprocessing
- Association Rule Mining
- Classification
- Clustering and Anomaly Detection
- Regression
- Case Studies

Data Mining

Introduction

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Why Data Mining?

- The Explosive Growth of Data: from terabytes to petabytes

    - Data collection and data availability

        - Automated data collection tools, database systems, Web, computerized society

    - Major sources of abundant data

        - Business: Web, e-commerce, transactions, stocks, …

        - Science: Remote sensing, bioinformatics, scientific simulation, …

        - Society and everyone: news, digital cameras, YouTube

- <u>We are drowning in data, but starving for knowledge!</u>

- "Necessity is the mother of invention"—Data mining—Automated analysis of massive data
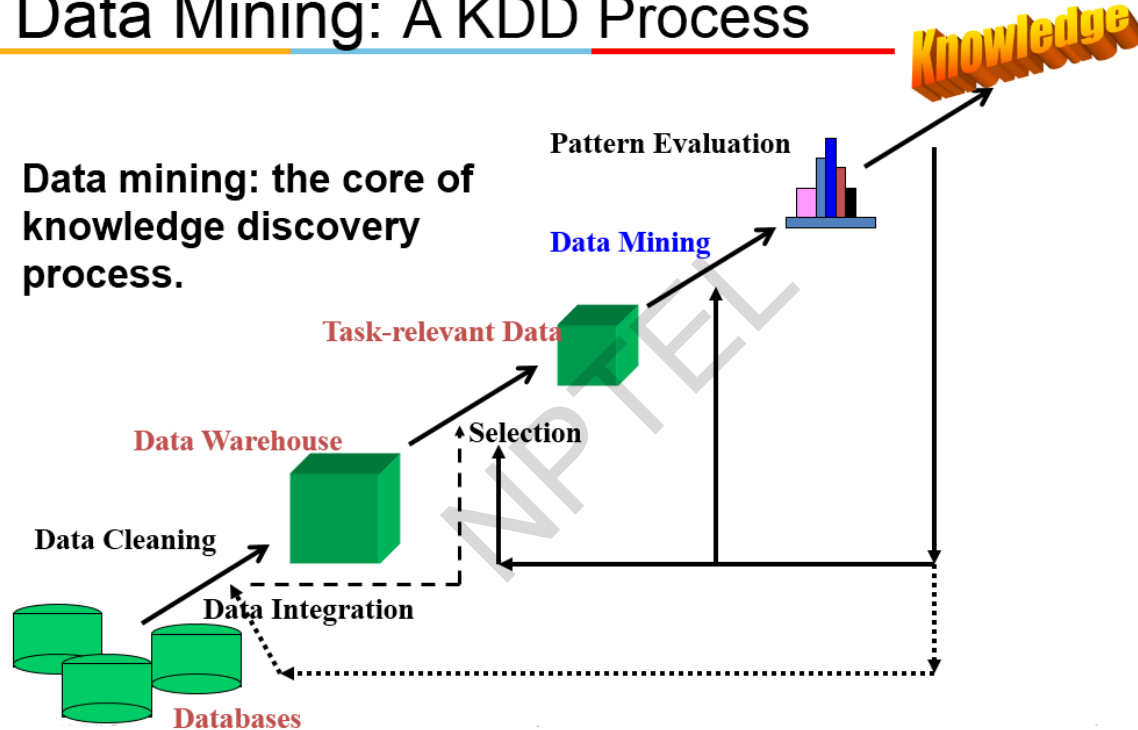
What Is Data Mining?

- Data mining (knowledge discovery from data)
  - Extraction of interesting (<u>non-trivial,</u> <u>implicit</u>, <u>previously unknown</u> and <u>potentially useful)</u> patterns or knowledge from huge amount of data

- Alternative names
  - Knowledge discovery (mining) in databases (KDD), knowledge extraction, data/pattern analysis, data archeology, data dredging, information harvesting, business intelligence, etc.

- Watch out: Is everything "data mining"?
  - Simple search and query processing
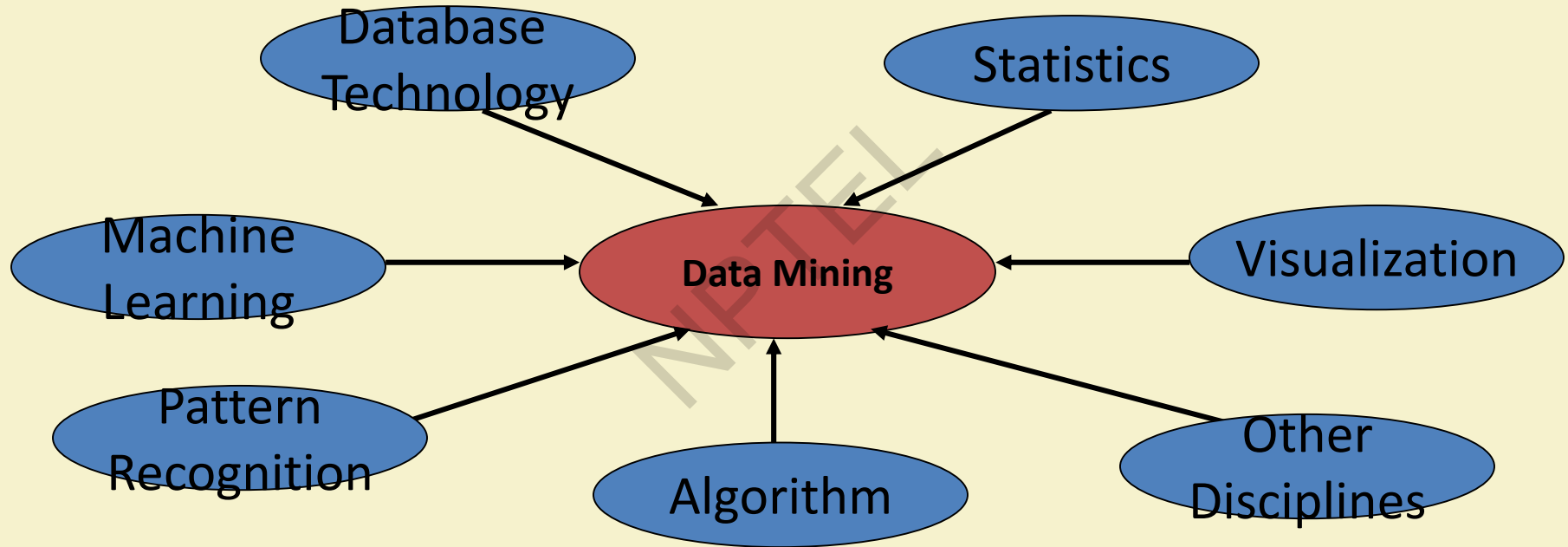 - (Deductive) expert systems

Data Mining: A KDD Process

Knowledge

Data mining: the core of knowledge discovery process.

Pattern Evaluation

Data Mining

Task-relevant Data

Data Warehouse

Selection

Data Cleaning

Data Integration

Databases

Data Mining: Confluence of Multiple Disciplines

Why Not Traditional Data Analysis?

- Tremendous amount of data
  - Algorithms must be highly scalable to handle such as tera-bytes of data
- High-dimensionality of data
  - Micro-array may have tens of thousands of dimensions
- High complexity of data
  - Data streams and sensor data
  - Time-series data, temporal data, sequence data
  - Structure data, graphs, social networks and multi-linked data
  - Heterogeneous databases and legacy databases
  - Spatial, spatiotemporal, multimedia, text and Web data

Data Mining: On What Kinds of Data?

- Database-oriented data sets and applications
  - Relational database, data warehouse, transactional database
- Advanced data sets and advanced applications
  - Data streams and sensor data
  - Time-series data, temporal data, sequence data (incl. bio-sequences)
  - Structure data, graphs, social networks and multi-linked data
  - Object-relational databases
  - Heterogeneous databases and legacy databases
  - Spatial data and spatiotemporal data
  - Multimedia database
  - Text databases
  - The World-Wide Web

Data Mining Functionalities

- Multidimensional concept description: Characterization and discrimination
  - Generalize, summarize, and contrast data characteristics, e.g., dry vs. wet regions
- Frequent patterns, association, correlation vs. causality
  - Tea → Sugar [0.5%, 75%] (Correlation or causality?)
- Classification and prediction
  - Construct models (functions) that describe and distinguish classes or concepts for future prediction
    - E.g., classify countries based on (climate), or classify cars based on (gas mileage)
  - Predict some unknown or missing numerical values

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Data Mining Functionalities

- Cluster analysis
  - Class label is unknown: Group data to form new classes, e.g., cluster houses to find distribution patterns
  - Maximizing intra-class similarity & minimizing interclass similarity
- Outlier analysis
  - Outlier: Data object that does not comply with the general behavior of the data
  - Noise or exception? Useful in fraud detection, rare events analysis
- Trend and evolution analysis
  - Trend and deviation: e.g., regression analysis
  - Sequential pattern mining: e.g., digital camera → large SD memory
  - Periodicity analysis
  - Similarity-based analysis
- Other pattern-directed or statistical analyses

Major Issues in Data Mining

- <u>Mining methodology</u>
  - Mining different kinds of knowledge from diverse data types, e.g., bio, stream, Web
  - Performance: efficiency, effectiveness, and scalability
  - Pattern evaluation: the interestingness problem
  - Incorporation of background knowledge
  - Handling noise and incomplete data
  - Parallel, distributed and incremental mining methods
  - Integration of the discovered knowledge with existing one: knowledge fusion
- <u>User interaction</u>
  - Data mining query languages and ad-hoc mining
  - Expression and visualization of data mining results
  - Interactive mining of knowledge at multiple levels of abstraction
- <u>Applications and social impacts</u>
  - Domain-specific data mining & invisible data mining
  - Protection of data security, integrity, and privacy

Architecture: Typical Data Mining System
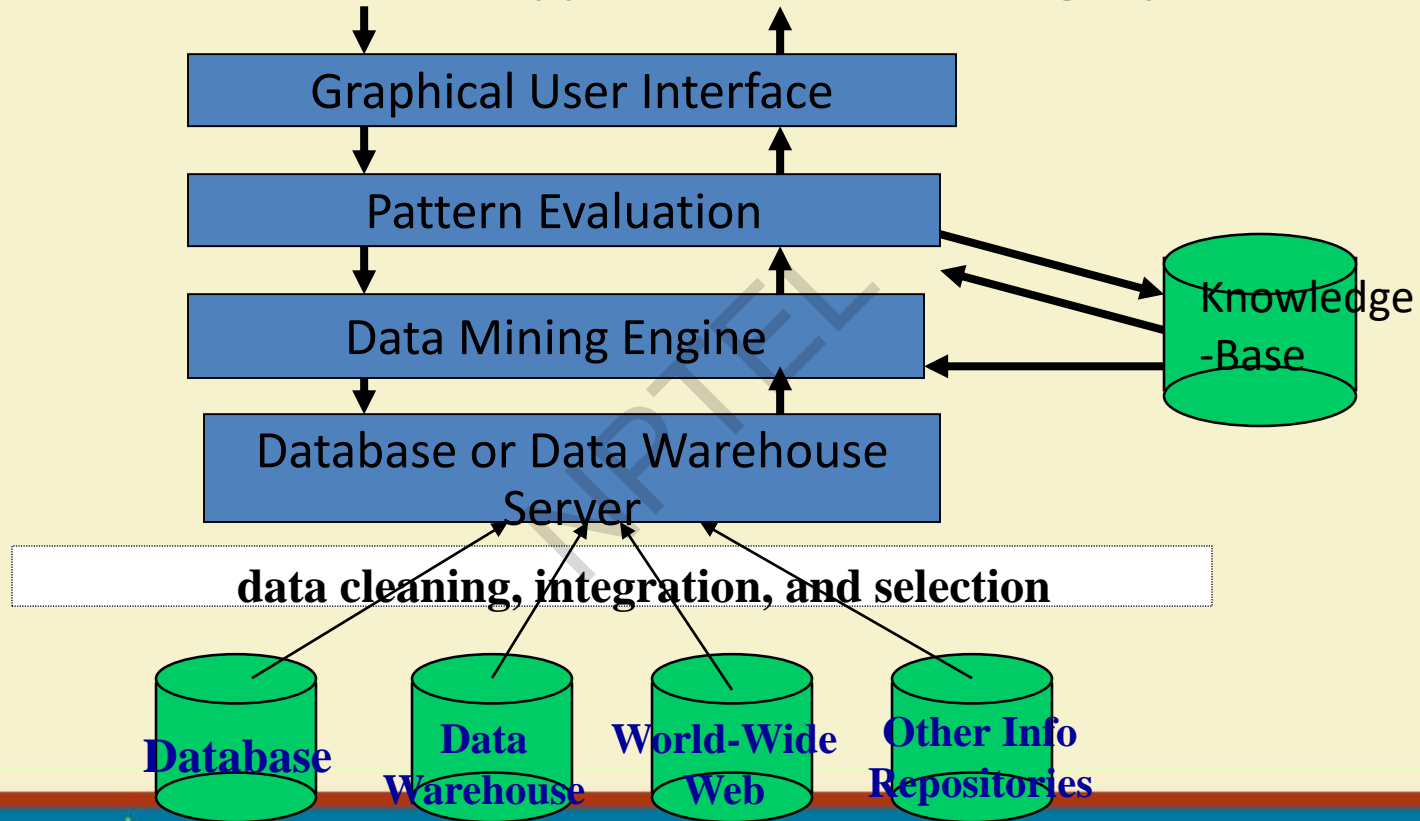
KDD Process: Summary

- Learning the application domain
  - relevant prior knowledge and goals of application
- Creating a target data set: data selection
- Data cleaning and preprocessing: (may take 60% of effort!)
- Data reduction and transformation
  - Find useful features, dimensionality/variable reduction, invariant representation
- Choosing functions of data mining
  - summarization, classification, regression, association, clustering
- Choosing the mining algorithm(s)
- Data mining: search for patterns of interest
- Pattern evaluation and knowledge presentation
  - visualization, transformation, removing redundant patterns, etc.
- Use of discovered knowledge

End of Introduction

Data Mining

Data Preprocessing

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What is Data?

- Collection of data objects and their attributes

- An attribute is a property or characteristic of an object
  - Examples: eye color of a person, temperature, etc.
  - Attribute is also known as variable, field, characteristic, or feature

- A collection of attributes describe an object
  - Object is also known as record, point, case, sample, entity, or instance

Attributes

Objects

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Types of Attributes

- There are different types of attributes
  - Nominal
    - Examples: ID numbers, eye color, zip codes
  - Ordinal
    - Examples: rankings (e.g., taste of potato chips on a scale from 1-10), grades, height in {tall, medium, short}
  - Interval
    - Examples: calendar dates, temperatures in Celsius or Fahrenheit.
  - Ratio
    - Examples: temperature in Kelvin, length, time, counts

Properties of Attribute Values

- The type of an attribute depends on which of the following properties it possesses:
  - Distinctness: $=\ \neq$
  - Order: $<\ >$
  - Addition: $+\ -$
  - Multiplication: $*\ /$

  - Nominal attribute: distinctness
  - Ordinal attribute: distinctness & order
  - Interval attribute: distinctness, order & addition
  - Ratio attribute: all 4 properties

Attribute Type	Description	Examples	Operations
Nominal	The values of a nominal attribute are just different names, i.e., nominal attributes provide only enough information to distinguish one object from another. ($=$, $\neq$)	zip codes, employee ID numbers, eye color, sex: {*male, female*}	mode, entropy, contingency correlation, χ^2 test
Ordinal	The values of an ordinal attribute provide enough information to order objects ($<$ $>$).	hardness of minerals, {*good, better, best*}, grades, street numbers	median, percentiles, rank correlation, run tests, sign tests
Interval	For interval attributes, the differences between values are meaningful, i.e., a unit of measurement exists. ($+$, $-$)	calendar dates, temperature in Celsius or Fahrenheit	mean, standard deviation, Pearson's correlation, t and F tests
Ratio	For ratio variables, both differences and ratios are meaningful. ($*$, $/$)	temperature in Kelvin, monetary quantities, counts, age, mass, length, electrical current	geometric mean, harmonic mean, percent variation

Discrete and Continuous Attributes

- Discrete Attribute
  - Has only a finite or countably infinite set of values
  - Examples: zip codes, counts, or the set of words in a collection of documents
  - Often represented as integer variables.
  - Note: binary attributes are a special case of discrete attributes

- Continuous Attribute
  - Has real numbers as attribute values
  - Examples: temperature, height, or weight.
  - Practically, real values can only be measured and represented using a finite number of digits.
  - Continuous attributes are typically represented as floating-point variables.

Types of data sets

- **Record**
  - Data Matrix
  - Document Data
  - Transaction Data
- **Graph**
  - World Wide Web
  - Molecular Structures
- **Ordered**
  - Spatial Data
  - Temporal Data
  - Sequential Data
  - Genetic Sequence Data

Record Data

- Data that consists of a collection of records, each of which consists of a fixed set of attributes

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Data Matrix

- If data objects have the same fixed set of numeric attributes, then the data objects can be thought of as points in a multi-dimensional space, where each dimension represents a distinct attribute

- Such data set can be represented by an m by n matrix, where there are m rows, one for each object, and n columns, one for each attribute

Projection of x Load	Projection of y load	Distance	Load	Thickness
10.23	5.27	15.22	2.7	1.2
12.65	6.25	16.22	2.2	1.1

Text Data

- Each document becomes a `term' vector,
  - each term is a component (attribute) of the vector,
  - the value of each component is the number of times the corresponding term occurs in the document.

	team	coach	play	ball	score	game	win	lost	timeout	season
Document 1	3	0	5	0	2	6	0	2	0	2
Document 2	0	7	0	2	1	0	0	3	0	0
Document 3	0	1	0	0	1	2	2	0	3	0

Transaction Data

- A special type of record data, where

  – each record (transaction) involves a set of items.

  – For example, consider a grocery store. The set of products purchased by a customer during one shopping trip constitute a transaction, while the individual products that were purchased are the items.

TID	Items
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Graph Data

- Examples: Facebook graph and HTML Links
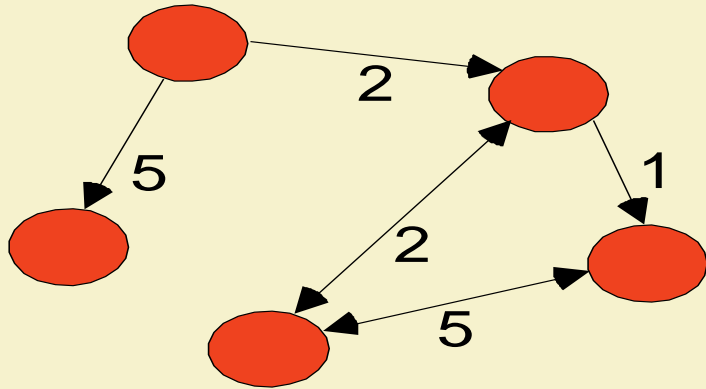
Ordered Data

- Genomic sequence data

GGTTCCGCCTTCAGCCCCGCGCC
CGCAGGGCCCGCCCCGCGCCGTC
GAGAAGGGCCCGCCTGGCGGGCG
GGGGGAGGCGGGGCCGCCCGAGC
CCAACCGAGTCCGACCAGGTGCC
CCCTCTGCTCGGCCTAGACCTGA
GCTCATTAGGCGGCAGCGGACAG
GCCAAGTAGAACACGCGAAGCGC
TGGGCTGCCTGCTGCGACCAGGG

Data Quality

- What kinds of data quality problems?

- How can we detect problems with the data?

- What can we do about these problems?

- Examples of data quality problems:
  - Noise and outliers
  - missing values
  - duplicate data

Noise

- Noise refers to modification of original values
  - Examples: distortion of a person's voice when talking on a poor phone and "snow" on television screen
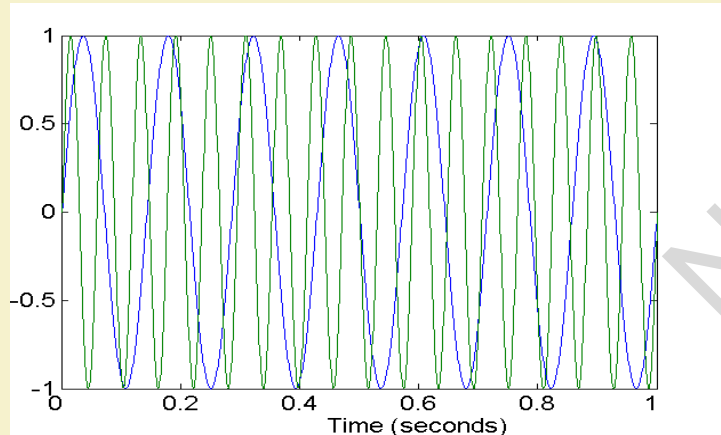
Two Sine Waves
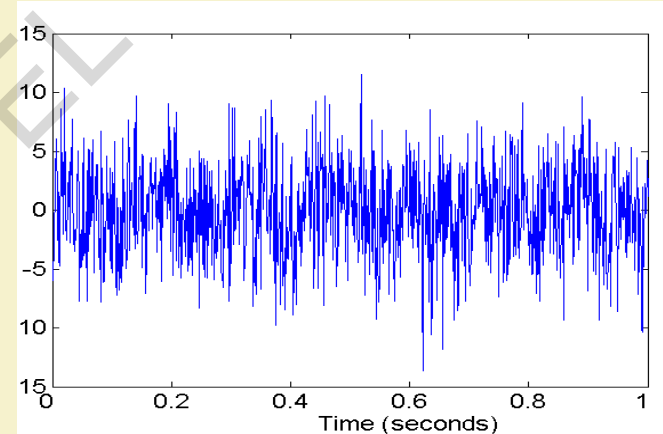
Two Sine Waves + Noise

Outliers

- Outliers are data objects with characteristics that are considerably different than most of the other data objects in the data set

Missing Values

- Reasons for missing values
  - Information is not collected
    (e.g., people decline to give their age and weight)
  - Attributes may not be applicable to all cases
    (e.g., annual income is not applicable to children)

- Handling missing values
  - Eliminate Data Objects
  - Estimate Missing Values
  - Ignore the Missing Value During Analysis
  - Replace with all possible values (weighted by their probabilities)

Duplicate Data

- Data set may include data objects that are duplicates, or almost duplicates of one another
  - Major issue when merging data from heterogenous sources

- Examples:
  - Same person with multiple email addresses

- Data cleaning
  - Process of dealing with duplicate data issues

Data Preprocessing

- Aggregation
- Sampling
- Dimensionality Reduction
- Feature subset selection
- Feature creation
- Discretization and Binarization
- Attribute Transformation

Aggregation

- Combining two or more attributes (or objects) into a single attribute (or object)
- Purpose
  - Data reduction
    - Reduce the number of attributes or objects
  - Change of scale
    - Cities aggregated into regions, states, countries, etc
  - More "stable" data
    - Aggregated data tends to have less variability

Sampling

- Sampling is the main technique employed for data selection.
  - It is often used for both the preliminary investigation of the data and the final data analysis.

- Statisticians sample because obtaining the entire set of data of interest is too expensive or time consuming.

- Sampling is used in data mining because processing the entire set of data of interest is too expensive or time consuming.
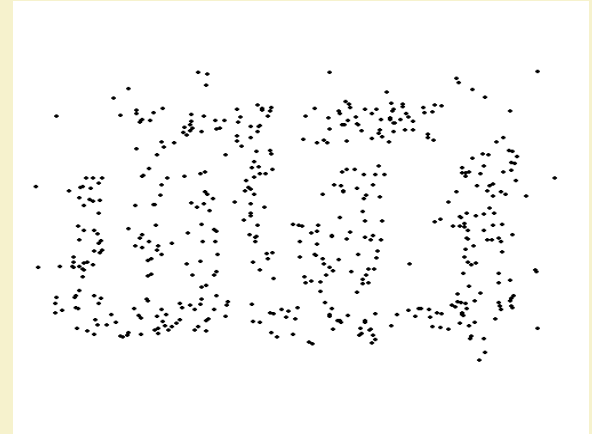
Sample Size
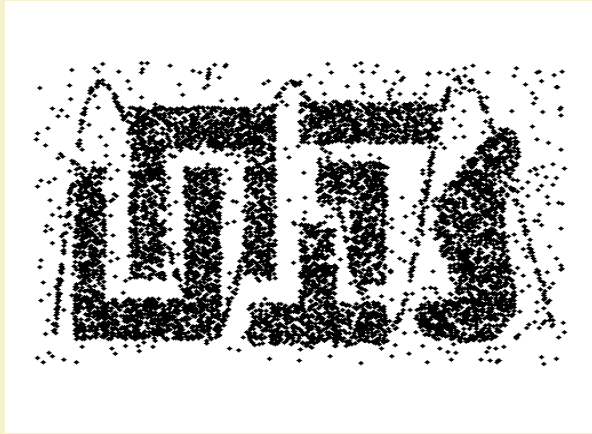
8000 points
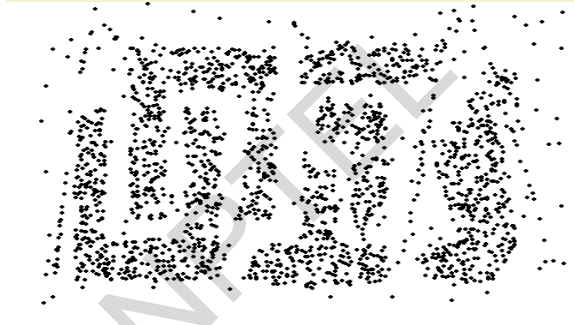
2000 Points

500 Points

Sampling …

- The key principle for effective sampling is the following:

  - using a sample will work almost as well as using the entire data sets, if the sample is representative

  - A sample is representative if it has approximately the same property (of interest) as the original set of data

Types of Sampling

- Simple Random Sampling
  - There is an equal probability of selecting any particular item

- Sampling without replacement
  - As each item is selected, it is removed from the population

- Sampling with replacement
  - Objects are not removed from the population as they are selected for the sample.
    - In sampling with replacement, the same object can be picked up more than once

- Stratified sampling
  - Split the data into several partitions; then draw random samples from each partition

Curse of Dimensionality

- When dimensionality increases, data becomes increasingly sparse in the space that it occupies

- Definitions of density and distance between points, which is critical for clustering and outlier detection, become less meaningful

Dimensionality Reduction

- Purpose:
  - Avoid curse of dimensionality
  - Reduce amount of time and memory required by data mining algorithms
  - Allow data to be more easily visualized
  - May help to eliminate irrelevant features or reduce noise

- Techniques
  - Principle Component Analysis
  - Singular Value Decomposition
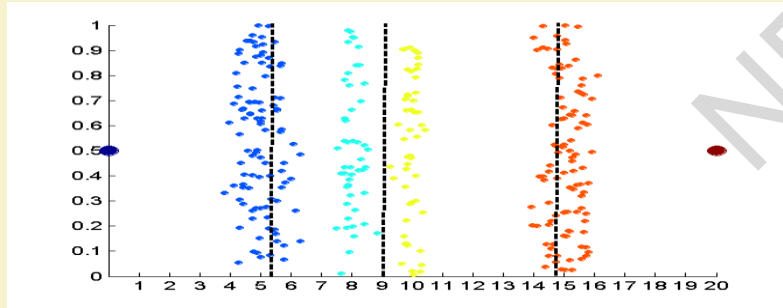 - Others: supervised and non-linear techniques
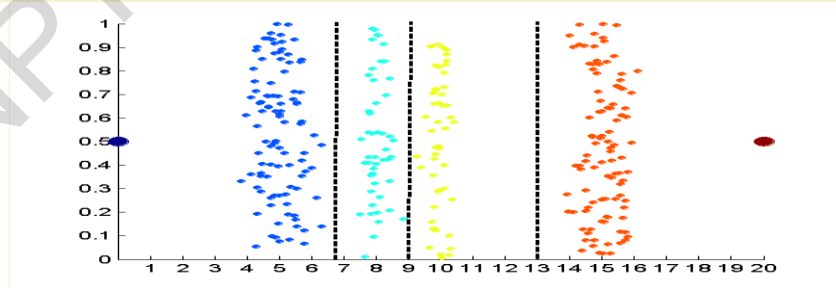
Discretization
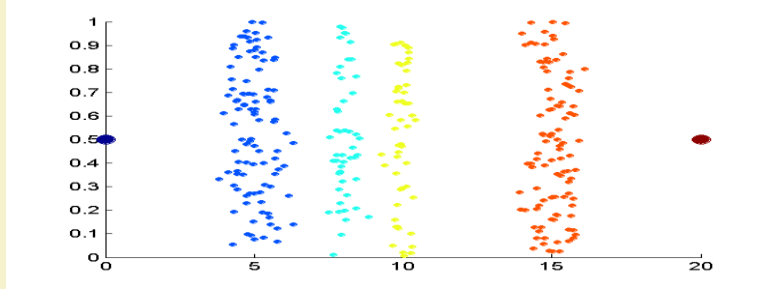
Data
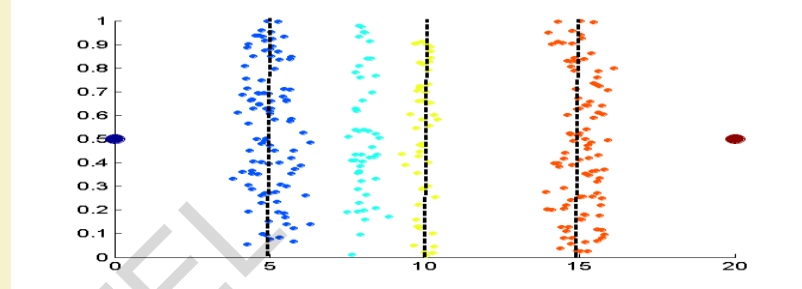
Equal interval width

Equal frequency

K-means

Attribute Transformation

- A function that maps the entire set of values of a given attribute to a new set of replacement values such that each old value can be identified with one of the new values
  - Simple functions: x^k, $\log(x)$, e^x, $|x|$
  - Standardization and Normalization

Similarity and Dissimilarity

- Similarity
  - Numerical measure of how alike two data objects are.
  - Is higher when objects are more alike.
  - Often falls in the range [0,1]
- Dissimilarity
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

Similarity/Dissimilarity for Simple Attributes

p and q are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{|p-q|}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{|p-q|}{n-1}$
Interval or Ratio	$d = |p - q|$	$s = -d,\ s = \frac{1}{1+d}$ or $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

Table 5.1. Similarity and dissimilarity for simple attributes
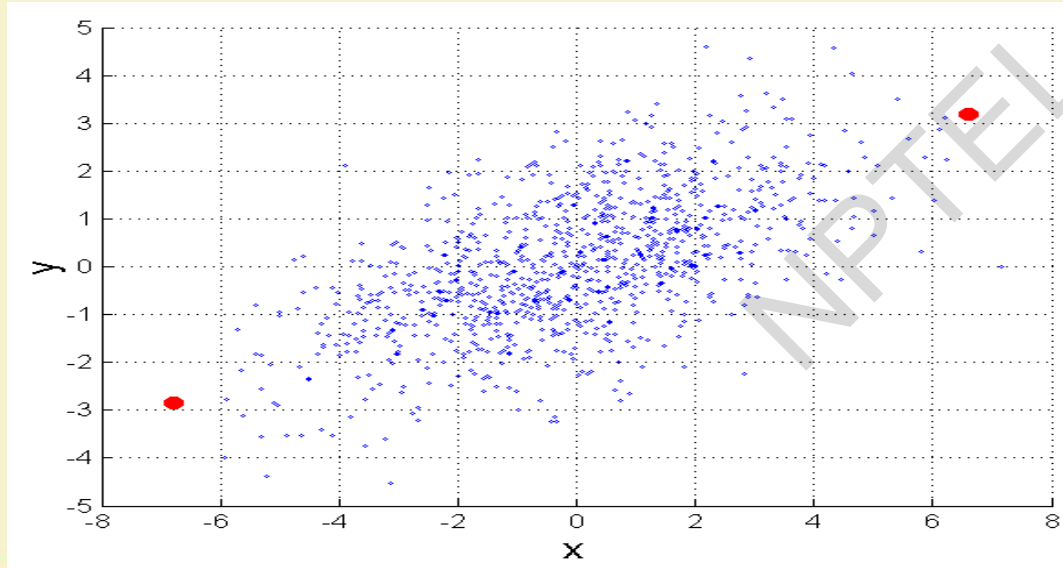
Euclidean Distance

- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n}(p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects p and q.

- Standardization is necessary, if scales differ.

Mahalanobis Distance

$$mahalanobis(p,q) = (p-q)\Sigma^{-1}(p-q)^T$$

Σ is the covariance matrix of the input data X

$$\Sigma_{j,k} = \frac{1}{n-1}\sum_{i=1}^{n}(X_{ij}-\overline{X}_j)(X_{ik}-\overline{X}_k)$$

For red points, the Euclidean distance is 14.7, Mahalanobis distance is 6.

Cosine Similarity

- If d_1 and d_2 are two document vectors, then

$$\cos(d_1, d_2) = (d_1 \bullet d_2) / ||d_1|| \ ||d_2|| \ ,$$

 where $\bullet$ indicates vector dot product and $|| \ d \ ||$ is the length of vector d.

- Example:

  $d_1 = $ **3 2 0 5 0 0 0 2 0 0**
  $d_2 = $ **1 0 0 0 0 0 0 1 0 2**

 $d_1 \bullet d_2 = $ 3*1 + 2*0 + 0*0 + 5*0 + 0*0 + 0*0 + 0*0 + 2*1 + 0*0 + 0*2 = 5

 $||d_1|| = $ (3*3+2*2+0*0+5*5+0*0+0*0+0*0+2*2+0*0+0*0)$^{0.5}$ = (42) $^{0.5}$ = 6.481

 $||d_2|| = $ (1*1+0*0+0*0+0*0+0*0+0*0+0*0+1*1+0*0+2*2) $^{0.5}$ = (6) $^{0.5}$ = 2.245

  $\cos(d_1, d_2) = $ .3150

Similarity Between Binary Vectors

- Common situation is that objects, *p* and *q*, have only binary attributes

- Compute similarities using the following quantities
  M_{01} = the number of attributes where p was 0 and q was 1
  M_{10} = the number of attributes where p was 1 and q was 0
  M_{00} = the number of attributes where p was 0 and q was 0
  M_{11} = the number of attributes where p was 1 and q was 1

- Simple Matching and Jaccard Coefficients
  SMC = number of matches / number of attributes
  $$= (M_{11} + M_{00}) / (M_{01} + M_{10} + M_{11} + M_{00})$$

  J = number of 11 matches / number of not-both-zero attributes values
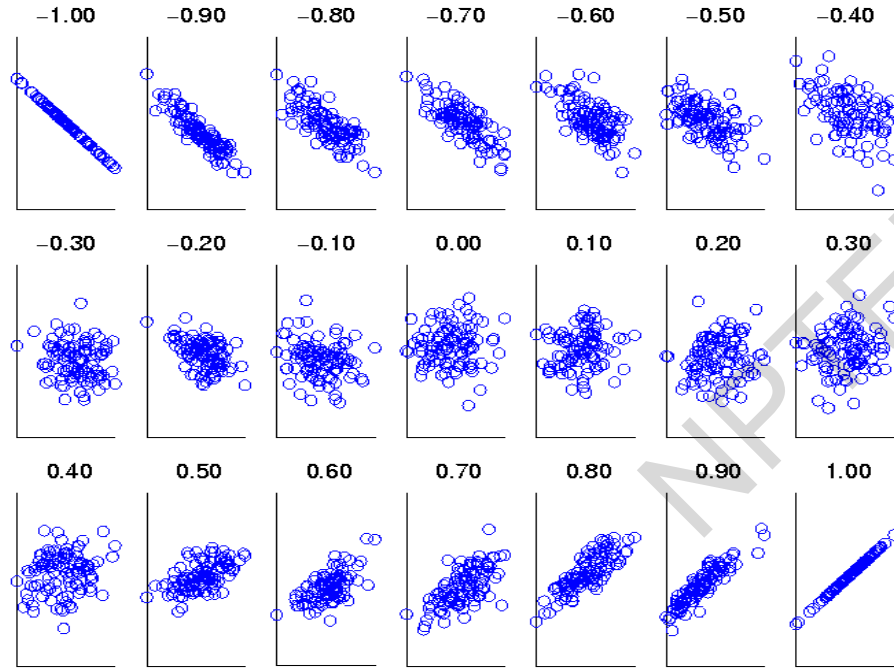 $$= (M_{11}) / (M_{01} + M_{10} + M_{11})$$

Correlation

- Correlation measures the linear relationship between objects
- To compute correlation, we standardize data objects, p and q, and then take their dot product

$$p'_k = (p_k - mean(p)) / std(p)$$

$$q'_k = (q_k - mean(q)) / std(q)$$

$$correlation(p, q) = p' \bullet q'$$

Visually Evaluating Correlation

Scatter plots showing the similarity from –1 to 1.

End of Data Preprocessing

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Data Mining
Association Rules

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Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

TID	Items
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- **Association Rule**
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    {Milk, Diaper} $\rightarrow$ {Beer}
- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:
$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds
  - ⇒ Computationally prohibitive!

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

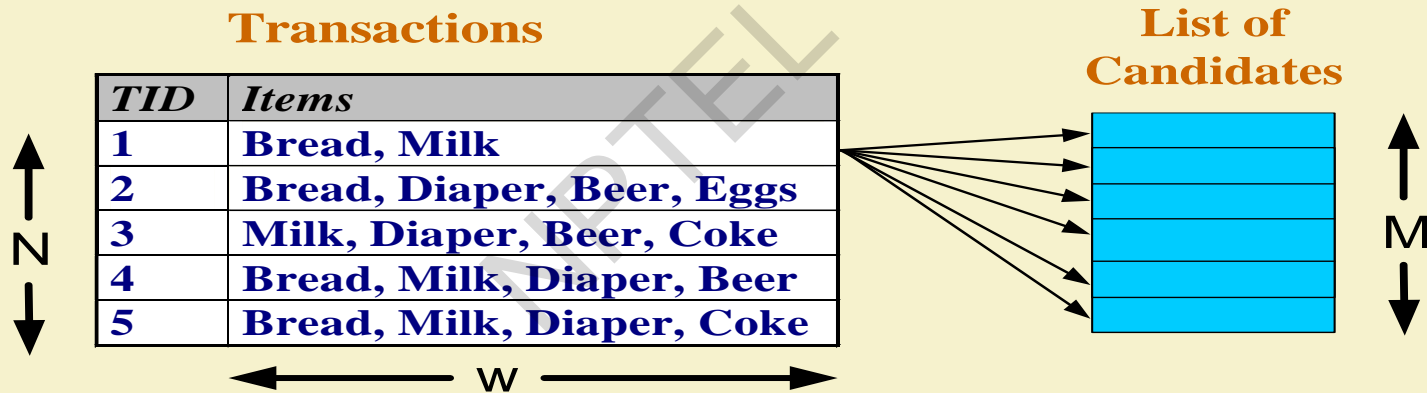- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation
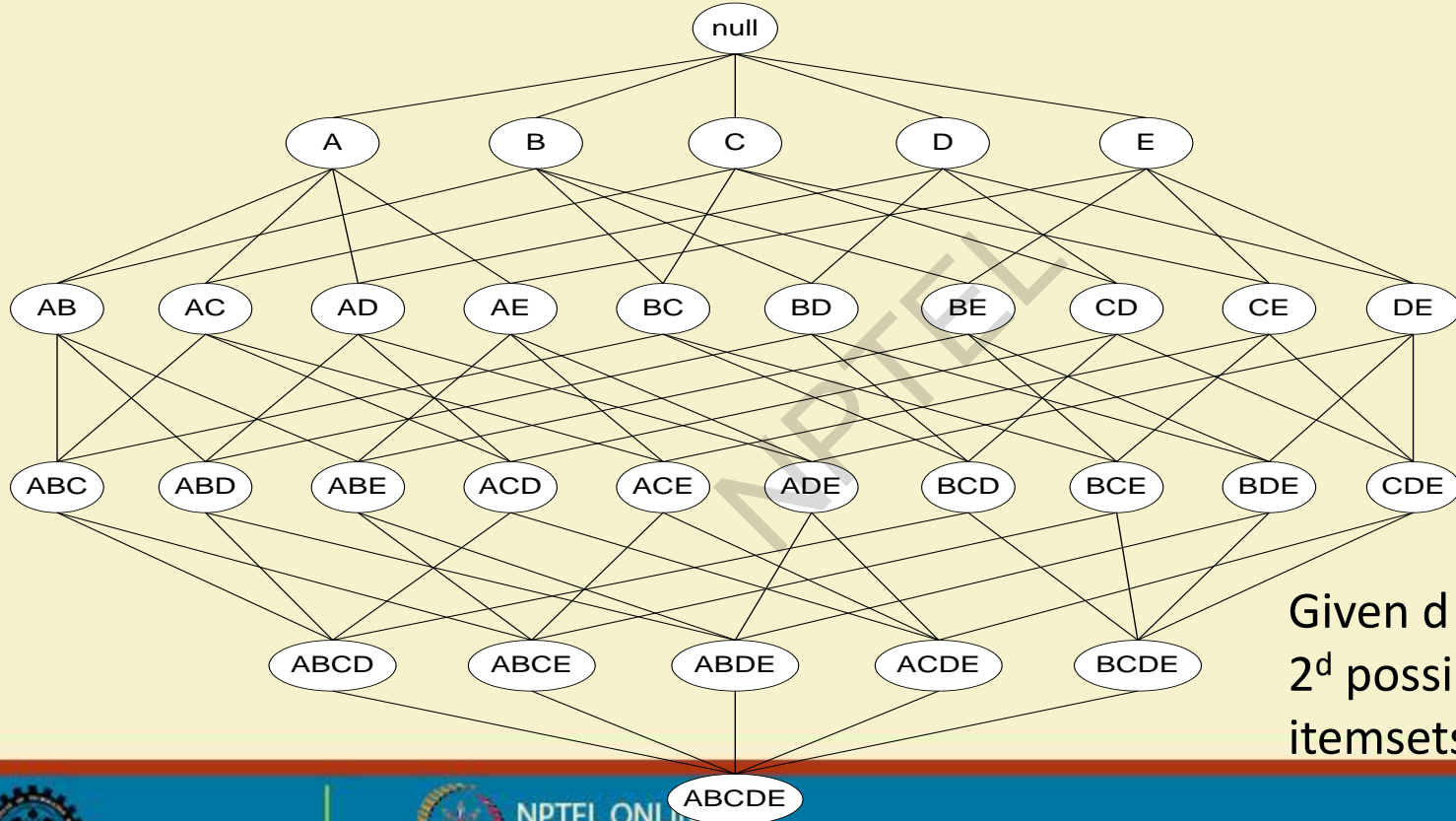
Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

Transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

N

w

List of Candidates

M

  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since M = 2^d !!!

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
  - Used by DHP and vertical-based mining algorithms

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
 - No need to match every candidate against every transaction

Reducing Number of Candidates

- **Apriori principle**:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Longrightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

Illustrating Apriori Principle

Found to be Infrequent

Pruned supersets
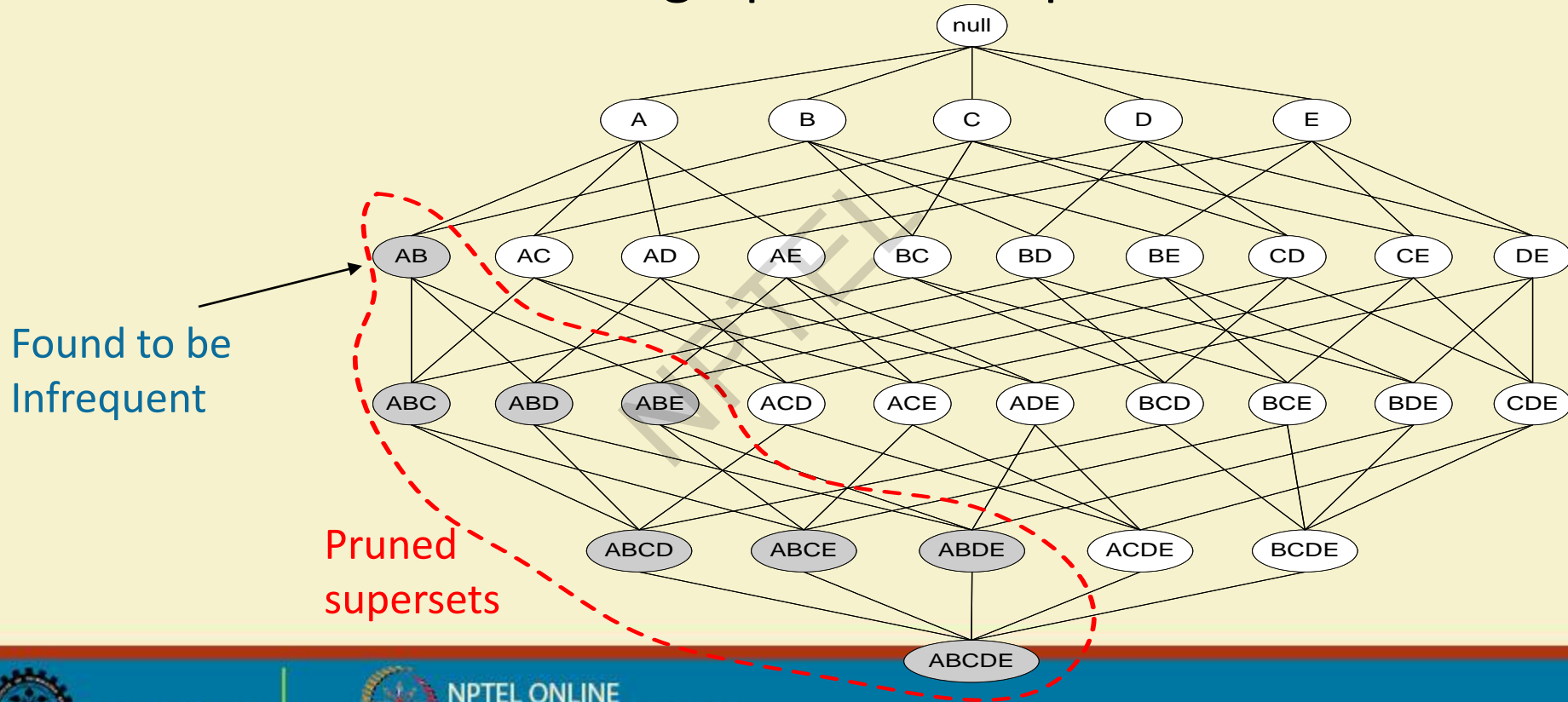
Illustrating Apriori Principle

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

If every subset is considered,
$$^6C_1 + {}^6C_2 + {}^6C_3 = 41$$
With support-based pruning,
$$6 + 6 + 1 = 13$$

Apriori Algorithm

- Method:

    - Let k=1
    - Generate frequent itemsets of length 1
    - Repeat until no new frequent itemsets are identified
        - Generate length (k+1) candidate itemsets from length k frequent itemsets
        - Prune candidate itemsets containing subsets of length k that are infrequent
        - Count the support of each candidate by scanning the DB
        - Eliminate candidates that are infrequent, leaving only those that are frequent

Factors Affecting Complexity

- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - more space is needed to store support count of each item
  - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
  - Apriori makes multiple passes, run time of algorithm increase with number of transactions
- Average transaction width
  - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

Rule Generation

- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property
    - $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., $L = \{A,B,C,D\}$:

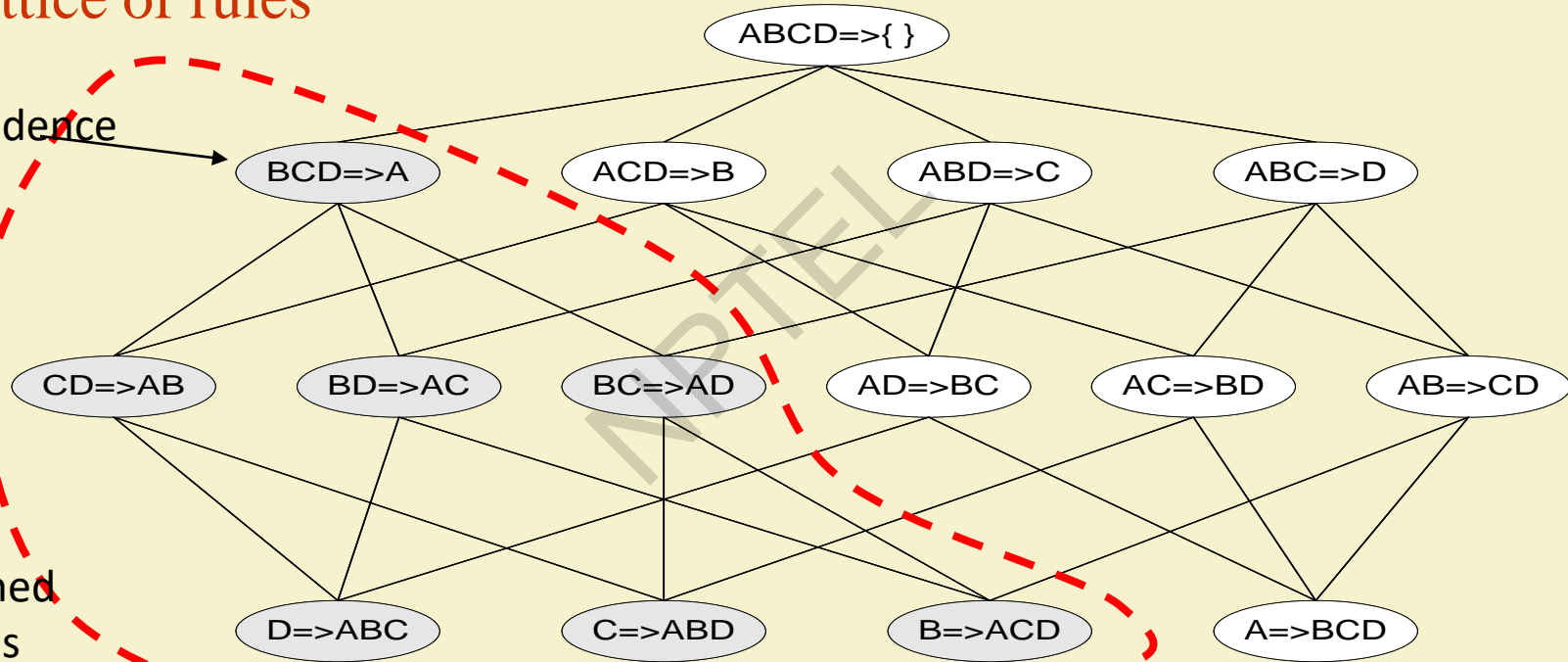 $$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

# Rule Generation for Apriori Algorithm

Lattice of rules

Low
Confidence
Rule

Pruned
Rules

ABCD=>{ }

BCD=>A ACD=>B ABD=>C ABC=>D

CD=>AB BD=>AC BC=>AD AD=>BC AC=>BD AB=>CD
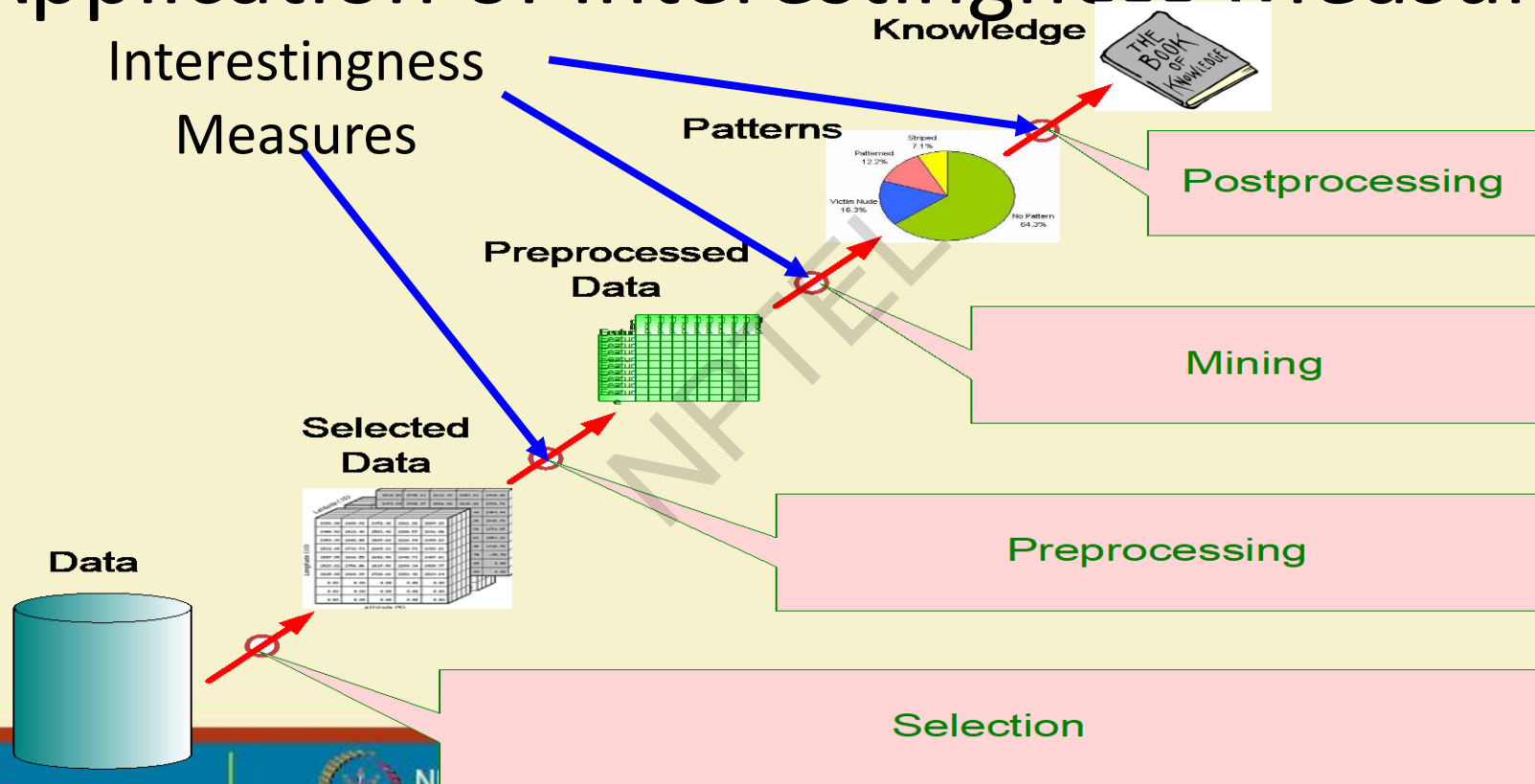
D=>ABC C=>ABD B=>ACD A=>BCD

Rule Generation for Apriori Algorithm

- Candidate rule is generated by merging two rules that share the same prefix in the rule consequent

- join(CD=>AB,BD=>AC) would produce the candidate rule D => ABC

- Prune rule D=>ABC if its subset AD=>BC does not have high confidence

```
     CD=>AB           BD=>AC


            D=>ABC
```
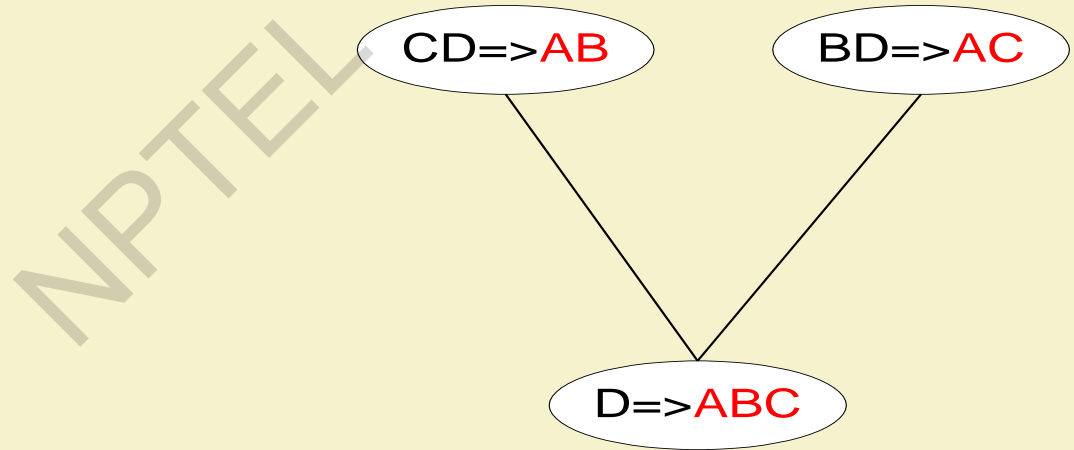
Pattern Evaluation

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if {A,B,C} $\rightarrow$ {D} and {A,B} $\rightarrow$ {D} have same support & confidence

- Interestingness measures can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

Application of Interestingness Measure

Interestingness Measures

Knowledge

Patterns

Preprocessed Data

Selected Data

Data

Postprocessing

Mining

Preprocessing

Selection

Computing Interestingness Measure

- Given a rule X $\rightarrow$ Y, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for supports X $\rightarrow$ Y

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$|T|$

Used to define various measures

◆ support, confidence, lift, Gini, J-measure, etc.

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

  - $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
  - $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
  - $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

- take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$

Example: Lift/Interest

	Coffee	Coffee̅	
Tea	15	5	20
Tea̅	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it

#	Measure	Formula
1	ϕ-coefficient	$\dfrac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\dfrac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\dfrac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\dfrac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)}=\dfrac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\dfrac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}=\dfrac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\dfrac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\dfrac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i),-\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\left(P(A,B)\log(\frac{P(B|A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}|A)}{P(\overline{B})}),\right.$ $\left. P(A,B)\log(\frac{P(A|B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}|B)}{P(A)})\right)$
9	Gini index (G)	$\max\left(P(A)[P(B|A)^2+P(\overline{B}|A)^2]+P(\overline{A})[P(B|\overline{A})^2+P(\overline{B}|\overline{A})^2]\right.$ $-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A|B)^2+P(\overline{A}|B)^2]+P(\overline{B})[P(A|\overline{B})^2+P(\overline{A}|\overline{B})^2]$ $\left.-P(A)^2-P(\overline{A})^2\right)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B|A),P(A|B))$
12	Laplace (L)	$\max\left(\frac{NP(A,B)+1}{NP(A)+2},\frac{NP(A,B)+1}{NP(B)+2}\right)$
13	Conviction (V)	$\max\left(\frac{P(A)P(\overline{B})}{P(A\overline{B})},\frac{P(B)P(\overline{A})}{P(B\overline{A})}\right)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B)-P(A)P(B)$
17	Certainty factor (F)	$\max\left(\frac{P(B|A)-P(B)}{1-P(B)},\frac{P(A|B)-P(A)}{1-P(A)}\right)$
18	Added Value (AV)	$\max(P(B|A)-P(B),P(A|B)-P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})}\times\frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B|A)-P(B),P(A|B)-P(A))$

Subjective Interestingness Measure

- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

- Need to model expectation of users (domain knowledge)
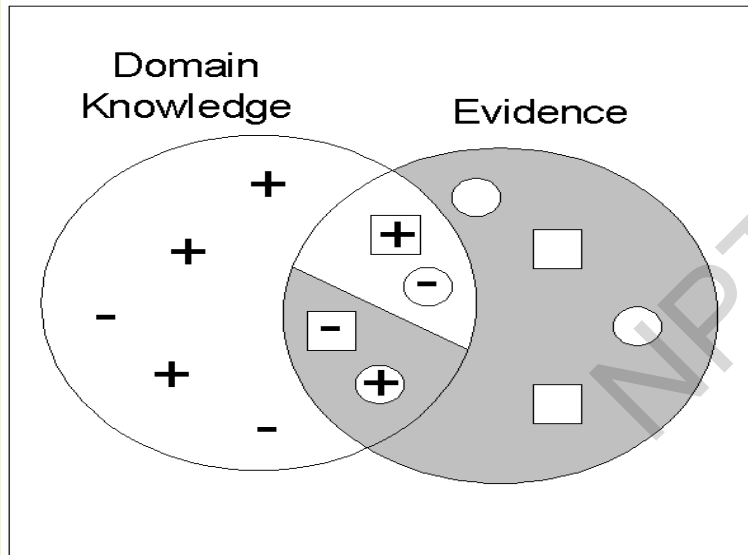
+ Pattern expected to be frequent

- Pattern expected to be infrequent

▭ Pattern found to be frequent

◯ Pattern found to be infrequent

⊞ ⊖ Expected Patterns

⊟ ⊕ Unexpected Patterns

- Need to combine expectation of users with evidence from data (i.e., extracted patterns)

End of Association Rule