Context free Grammar and Context free Language

Regular Language → Finite Automata
Context free Language → Pushdown Automata

Applications

* Parser
* XML , DFD

Context free Grammar

Defined with 4 tuples

$G = (V, T, P, S)$

where, V = finite set of variable / non-terminal
T = finite set of terminals
P = Set of rules or productives
$A \rightarrow \alpha$, where $A \in V$ and $\alpha \in (V \cup T)^*$
S = Start variable , $S \in V$.

Eg: $G = (\{S\}, \{a, b\}, P, S)$

where, $P = \{ S \rightarrow \epsilon$
$S \rightarrow aSb \}$

NOTE:
String can only be sequence of terminals.

$S \rightarrow \epsilon$
$S \rightarrow aSb$ $\Rightarrow \{ \epsilon, ab, aabb, aaabbb, --- a^n b^n \mid n \geq 0 \}$

$L = \{ a^n b^n \mid n \geq 0 \}$

* Write CFG for $L = \{ a^n b^n \mid n \geq 1 \}$

$S \rightarrow ab$
$S \rightarrow aSb$ $\{ ab, aabb, aaabbb, --- \}$

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2) $L = \{a^n b^{n+1} \mid n \geq 0\}$

$S \to b$
$S \to aSb$ $\{b, abb, aabbb, ----\}$

3) $L = \{a^{2n} \mid n \geq 0\}$

$S \to \epsilon$
$S \to aSa$ $\{aa, \epsilon, aaaa, ----\}$

4) $L = \{a^{2n+1} \mid n \geq 0\}$

$S \to a$
$S \to aSa$ $\{a, aaa, aaaaa, ----\}$

<u>Derivation</u>

* $G = (\{S, A, B\}, \{a, b\}, P, S)$

$P = \{ S \to aAB \to ①$
$A \to bBb \to ②$
$B \to A / \epsilon$ ③ ④

<u>abbbb</u>

<u>Left most Derivation</u>

$S \underset{1}{\Rightarrow} a\underline{A}B \underset{2}{\Rightarrow} ab\underline{B}bB \underset{4}{\Rightarrow} ab\epsilon bB \Rightarrow abb\underline{B} \underset{3}{\Rightarrow} abb\underline{A} \underset{2}{\Rightarrow} abbb\underline{B}b \underset{4}{\Rightarrow}$
$abbb\epsilon b \underset{4}{\Rightarrow} abbbb$

<u>Derivation</u>: The process of obtaining a string from the start variable of G by applying one or more production is called Derivation.

'$\Rightarrow$' represent derivation process.

① Left most Derivation
② Right most Derivation

<u>For abbbb</u>

<u>Right most Derivation</u>

$S \underset{1}{\Rightarrow} aA\underline{B} \underset{3}{\Rightarrow} aA\underline{A} \underset{2}{\Rightarrow} aAb\underline{B}b \underset{4}{\Rightarrow} aAb\epsilon b \Rightarrow a\underline{A}bb \underset{2}{\Rightarrow} ab\overset{\epsilon}{\underline{B}}bbb \underset{4}{\Rightarrow} abbbb$

* Top down Parser → Left most derivation 'C'
* Bottom up Parser → Right most derivation 'C++'

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Left most Derivation

A derivation is said to be left most, if in each step, the left most variable in the sentencial form, is replaced by its production.

$$S \overset{*}{\underset{lm}{\Longrightarrow}} abbbb$$

Right most derivation

A derivation is said to be right most, if in each step, the right most variable in the sentencial form, is replaced by its production.

$$S \overset{*}{\underset{rm}{\Longrightarrow}} abbbb$$

Write CFG for the following

1) $L = \{ ww^R \mid w \in \{a,b\}^* \}$ $\qquad w^R \rightarrow$ Reverse of w.

2) $L = \{ 0^n w w^R 1^n \mid n \geq 1, w \in \{a,b\}^* \}$

3) $L = \{ |w| \bmod 3 = 0, w \in \{a,b\}^* \}$

4) $L = \{ a^m b^m c^n \mid m \geq 1, n \geq 0 \}$

Answers

1) $S \rightarrow \epsilon$ $\qquad \{\epsilon, aa, bb, abba, baab, \ldots\}$ $\qquad$ asa, aεa, aa

$S \rightarrow aSa$

$S \rightarrow bSb$ $\qquad S \rightarrow \epsilon / asa / bSb$

2) ~~$(S \rightarrow \epsilon / 0S1$~~ $\qquad \{01, 0aa1, 0bb1, 0abba1, 0baab1, \ldots\}$

~~$\rightarrow \epsilon / asa / bSb)$~~ $\qquad$ 0abba

$S \rightarrow 0S1$

$A \rightarrow \epsilon / 0A1 / B$

$B \rightarrow aAa / bAb$

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2) {ε, aaa, bbb, aab, aba, baa, bba, bab, abb, aaabbb, bbbaaa ---}

$S \to \epsilon / aaas / bbbs / aabs / abas / baas / bbas / babs / abbs$

$S \to \epsilon / aAb$
$A \to bSa$

baaba
babsba
bba

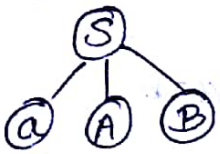
4) {ab, abc, aabbc, abcc, aabbcc, ----}

$S \to aAc$
$A \to bSb / B$
$B \to aSa / bSb / cSc$

Derivation tree

It is an ordered tree, in which nodes are labelled with left side of productions in which the children of a node represent its corresponding right side.

$S \to aAB$
$A \to bBb$
$B \to \epsilon / A$



Formal definition

Let $G = (V, T, S, P)$ be a CFG, an ordered tree is a derivation tree with the following conditions.

1. The root is labelled with 'S'.
2. Every leaf has a label from $T \cup \{\epsilon\}$
3. Every interior node has a label from V (variable).
4. If we have a vertex labelled $A \in V$, and if children are labelled $a_1, a_2, \ldots a_n$, then P must contain a production of the form $A \to a_1 a_2 \ldots a_n$
5. The leaf labelled 'ε' has no siblings, ie., a vertex with a child labelled 'ε' can have no other children.

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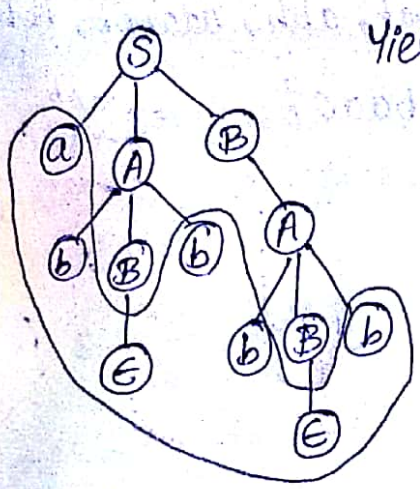
Yield of tree = abbbb



Problems

1) $S \rightarrow AB$

$A \rightarrow aaA / \epsilon$

$B \rightarrow Bb / \epsilon$

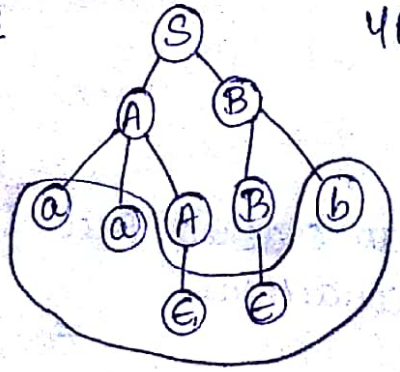
Write LMD, RMD, DT for input "aab"

LMD: $S \Rightarrow AB \Rightarrow aaAB \Rightarrow aa\epsilon B \Rightarrow aaB \Rightarrow aaBb \Rightarrow aa\epsilon b \Rightarrow aab$

RMD: $S \Rightarrow AB \Rightarrow ABb \Rightarrow A\epsilon b \Rightarrow Ab \Rightarrow aaAb \Rightarrow aa\epsilon b \Rightarrow aab$

DT:

Yield of tree = aab



2) $E \rightarrow +EE / *EE / -EE / x / y$. LMD, RMD, DT for i/p $+*-xyxy$

LMD: $E \Rightarrow +EE \Rightarrow +*EEE \Rightarrow +*-EEEE \Rightarrow +*-xEEE \Rightarrow +*-xyEE$
$\Rightarrow +*-xyxE \Rightarrow +*-xyxy$

RMD: $E \Rightarrow +EE \Rightarrow +Ey \Rightarrow +*EEy \Rightarrow +*Exy \Rightarrow +*-EExy \Rightarrow$
$+*-Eyxy \Rightarrow +*-xyxy$

DT:

Yield of tree = $+*-xyxy$

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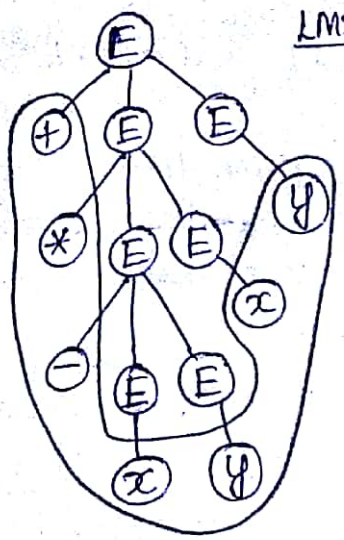
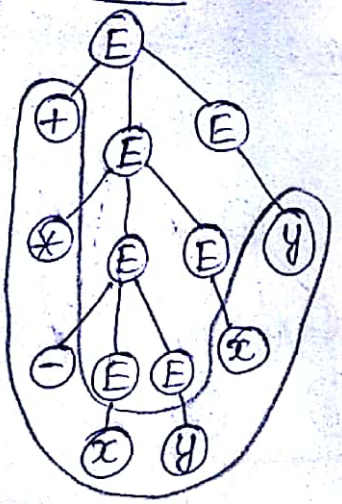
LMD



RMD



Obtain CFG for following RE

1) $(a+b)^* ab$
2) $(ab)^* a (a+b)^*$
3) $(a+b)^*$

Answers

1) $G = \{ \underset{\uparrow ab,}{aab}, bab, abab, aaab, bbab \ldots \}$

$S \rightarrow Aab$

$A \rightarrow \epsilon \mid aA \mid bA$

2) $G = \{ a, abaa, abab, ababaaa, \ldots \}$

$S \rightarrow AaB$

$A \rightarrow \epsilon \mid abA$

$B \rightarrow \epsilon \mid aB \mid bB$

3) $G = \{ \epsilon, a, b, aa, bb, ab, \ldots \}$

$S \rightarrow \epsilon \mid Aa \mid Ab$

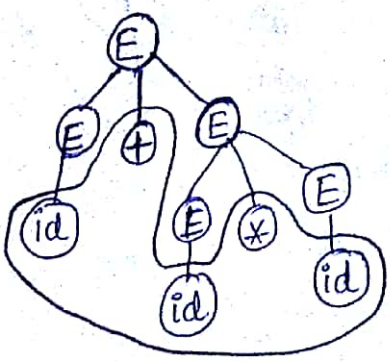
$A \rightarrow S$

Ambiguous Grammer

Eg) $E \rightarrow \overset{①}{E+E} \mid \overset{②}{E*E} \mid \overset{③}{id}$

Generate, $\underset{1}{id} + \underset{2}{id} * \underset{3}{id}$

LMD: $E \overset{①}{\Rightarrow} E+E \overset{③}{\Rightarrow} id+E \overset{②}{\Rightarrow} id+E*E \overset{③}{\Rightarrow} id+E*E \overset{③}{\Rightarrow} id+id*id$



$1+2*3 = 7$

multiplication first

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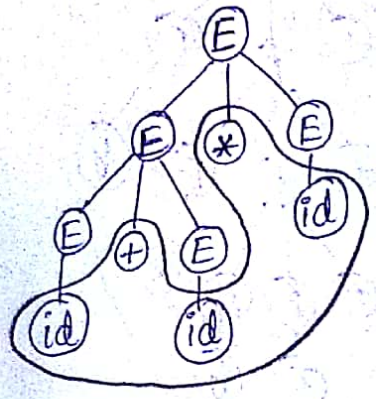
LMD: $E \overset{2}{\Rightarrow} E*E \overset{1}{\Rightarrow} E+E*E \overset{3}{\Rightarrow} id+E*E \overset{3}{\Rightarrow} id+id*E \overset{3}{\Rightarrow} id+id*id$



$1+2*3 = 9$ Addition first

Ambiguous Grammer

Let CFG, $G = (V, T, P, S)$ is ambiguous, if there is atleast one string W in T^*, for which we can find two different Parsetree, each with root labelled as an yield W.

Eg: 2) Check whether following grammer is ambiguous or not.

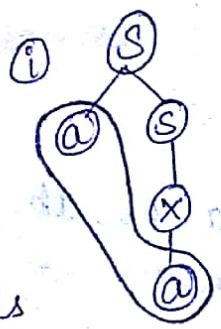
$S \rightarrow aS / X$ "String aa"

$X \rightarrow aX / a$

(i) $S \underset{lm}{\Rightarrow} aS \Rightarrow aX \Rightarrow aa$

(ii) $S \underset{lm}{\Rightarrow} X \Rightarrow aX \Rightarrow aa$

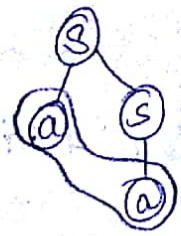
(i)



(ii)

The given grammer is ambigous as it generates one or more trees.

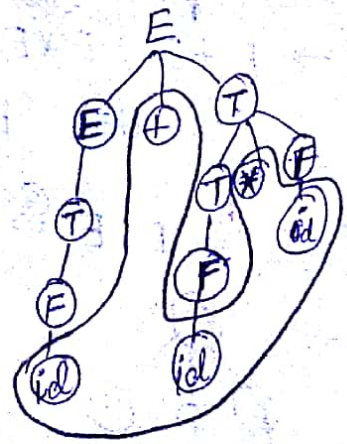
$S \rightarrow a / \cancel{aa} / aS$



$E \rightarrow E+T / T$

$T \rightarrow T*F / F$

$F \rightarrow id$



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Declaration Statement CFG

Data type variables_list Datatype → int, float, double, char

$D \to TV;$

$T \to int / float / double / char$

$V \to id / id.V$

1) Find LMD, RMD and DT for the string aaabbabbba using CFG.

$S \to aB / bA$

$A \to aS / bAA / a$

$B \to bS / aBB / b$

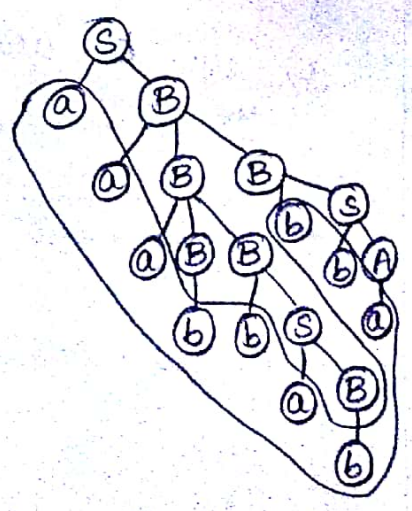
LMD: $S \overset{lm}{\Rightarrow} aS \Rightarrow aaS \Rightarrow aaaS \overset{aaa}{\Rightarrow} bA \Rightarrow aaabbAA \Rightarrow aaabbaA \Rightarrow$
$aaabbabAA \Rightarrow aaabbabbAAA \Rightarrow aaabbabbbAAAA \Rightarrow$

LMD: $S \overset{lm}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aaaBBB \Rightarrow aaabBB \Rightarrow aaabbSB \Rightarrow$
$aaabbaBB \Rightarrow aaabbabB \Rightarrow aaabbabbS \Rightarrow aaabbabbbA \Rightarrow$
$aaabbabbba$

RMD: $S \overset{rm}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aaBb \Rightarrow aaaBBb \Rightarrow aaaB$

$S \overset{rm}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBbaB \Rightarrow aaBbabS \Rightarrow aaBbabbA$
$\Rightarrow aaBbabbbA \Rightarrow aaBbabbba \Rightarrow aaaBBbabbba \Rightarrow aaaBbbabbb a$

✓ $S \overset{rm}{\Rightarrow} aB \Rightarrow aaBB \Rightarrow aaBbS \Rightarrow aaBbbA \Rightarrow aaBbba \Rightarrow aaaBBbba$
$\Rightarrow aaaBbbba \Rightarrow aaabSbbba \Rightarrow aaabbAbbba \Rightarrow aaabbabbba$



2) Is the following CFG is ambiguous?

① $S \to AB / aaB$

$A \to a / Aa$

$B \to b$

$S \overset{lm}{\Rightarrow} AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab$

$S \overset{lm}{\Rightarrow} aaB \Rightarrow aab$

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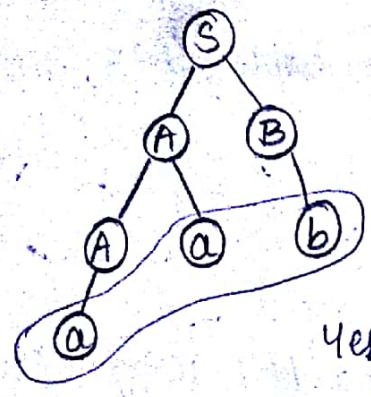
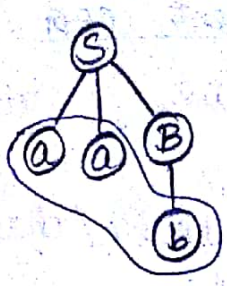




Text ①

Yes, it is ambiguous

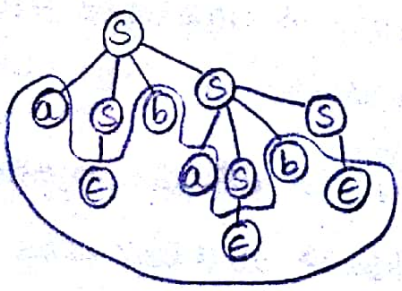
b) $S \to aSbS \mid bSaS \mid \epsilon$

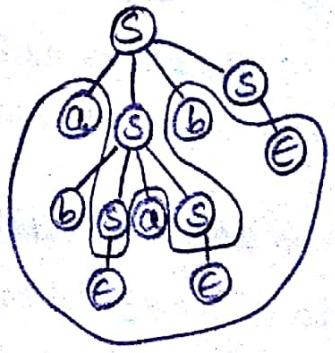
$S \overset{lm}{\Rightarrow} aSbS \Rightarrow abSaSbS \Rightarrow ab\epsilon aSbS \Rightarrow ab\epsilon a\epsilon bS \Rightarrow$
$ababS \Rightarrow ababaSaS \Rightarrow ababa\epsilon aS \Rightarrow ababaaS \Rightarrow$
$aababaa\epsilon \Rightarrow ababaa$

$S \overset{lm}{\Rightarrow} aSbS \Rightarrow$ ~~aSbaSbS~~ $abSaSbS \Rightarrow ab\epsilon aSbS \Rightarrow abaSbS \Rightarrow$
$ababSaSbS \Rightarrow abab$

✓ $S \overset{lm}{\Rightarrow} aSbS \Rightarrow a\epsilon bS \Rightarrow abS \Rightarrow abaSbS \Rightarrow aba\epsilon bS \Rightarrow$
$aba\epsilon b\epsilon \Rightarrow abab$

✓ $S \overset{lm}{\Rightarrow} aSbS \Rightarrow abSaSbS \Rightarrow ab\epsilon aSbS \Rightarrow aba\epsilon bS \Rightarrow abab\epsilon \Rightarrow$ aba





c) $ib + ib + aea$ Using $S \to ic+S \mid ic + SeS \mid a$
$C \to b$

$S \overset{lm}{\Rightarrow} ic + S \Rightarrow$

$ic + ic + SeS \Rightarrow ic + ic + Sea \Rightarrow ic + ic + aea \Rightarrow ib + ib + aea$

$S \overset{lm}{\Rightarrow} ic + SeS \Rightarrow ic + ic + SeS \Rightarrow ic + ic + Sea \Rightarrow ib + ib + aea$

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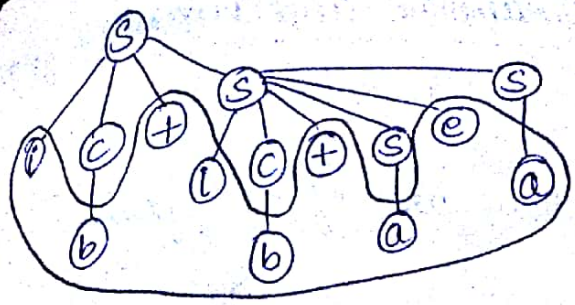

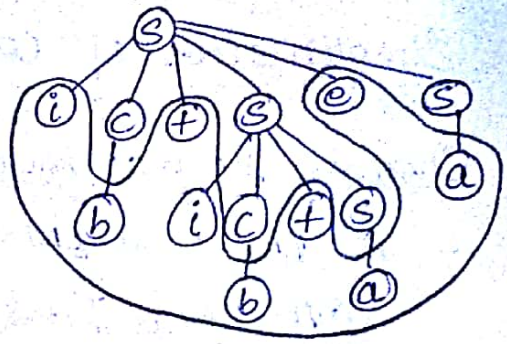

2) i) $L = \{a^n b^m c^k \mid k = n+m,\ n \geq 0,\ m \geq 0\}$

$P = \{ S \to \epsilon / B$
$A \to bAc \mid bc \mid \epsilon$
$B \to aBc \mid ac / A / \epsilon \}$

$S \to E / abc$
$aAbAc$
$A \to a/b/c$
$aAbAcA$
$A \to a/bc$

ii) $L = \{a^n b^m c^k \mid n + 2m = k$

$P = \{ S \to \epsilon / B$
$A \to bAcc \mid bcc / \epsilon$
$B \to aBc \mid ac / A / \epsilon \}$

iii) $L = \{a^n b^m c^k \mid n = m$ (or) $m \leq k\}$

$P = \{ S \to \epsilon / A$
$A \to aAbB / S$
$B \to Bc / \epsilon \}$

vi) $L = \{xy \mid x \in \{a,b\}^*$ and $y \in \{ab$ or $ba\}\}$

$P = \{ S \to SB / \epsilon / as / bs$
$B \to ab / ba \}$

v) $L = \{0^m 1^m 2^n \mid m \geq 1,\ n \geq 0\}$

$P = \{ S \to 0A1B$
$A \to 0A1 / \epsilon$
$B \to 2B / \epsilon \}$

vi) $L = \{a^i b^j \mid i \neq j,\ i, j \geq 0\}$

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Simplification of Context free Grammar and normal forms

Chomsky Normal form

A CFG is in CNF, if all productions are of the form.

$A \to BC$] ⇒ 2 variable

(or)

$A \to a$] ⇒ one terminal

where A, B, C are in V
'a' is in T.

Eg: The grammar,

$S \to AS / a$
$A \to SA / b$ is in chomsky normal form.

The grammar,

$S \to AS / ASS$
$A \to SA / bb$ is not in chomsky normal form

Simplifying CFG (Preliminary steps)

i) Eliminate useless symbols and production.
ii) Eliminate ε productions. (Eg:- $A \to \epsilon$)
iii) Eliminate unit productions. (Eg:- $A \to B$)

(i) EUS

The variables or terminals that do not appear in any derivation of terminal string from the start variable.

Def:- [A symbol X is useful for a grammar $G = (V, T, P, S)$ if there is some derivation of the form.]

$S \xrightarrow{*} \alpha X \beta \xrightarrow{*} w$, where w is in T^*.

Part of Definition:

If X is not useful, then it is useless

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Elimination steps

1) X is generating if $X \xrightarrow{*} w$ for some terminal string.
2) X is reachable, if there is a derivation
$S \xrightarrow{*} \alpha X \beta$ for some α and β

⇒ A symbol that is useful will be both generating and reachable.

⇒ First eliminate the signals that are not generative, then the ones that are not reachable.

Eg. ⇒ $S \to AB/a$ } useful
$A \to b$
$C \to c$ ⇒ not reachable (useless) but generating
$B \to d$ } ful

⇒ $S \to AB/a$
$A \to b/C$
$C \to D$
$B \to d$

Here C, D are not generating

Eliminate useless productions form

① $S \to a/aA/B$
$A \to aB/a$
$B \to Aa$
$C \to cCD$
$D \to ddd$

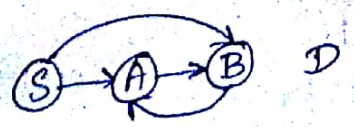
$G_1 = (V_1, T_1, P_1, S)$

P_1	V_1	T_1
$A \to a$	{A}	{a}
$D \to ddd$	{A, D}	{a, d}
$S \to a$	{A, D, S}	{ " }
$S \to aA$	{A, D, S}	{ " }
$B \to Aa$	{S, A, D, B}	{ " }
$S \to B$	{ " }	{ " }
$A \to aB$	{ " }	{ " }

(Generating)

$G_2 = (V_2, T_2, P_2, S)$

P_2	V_2	T_2
$S \to a/aA/B$	{SABD}	{a}
$A \to a/aB$	"	"
$B \to Aa$	"	"



(Reachable)

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(ii) $S \to aA / bA$
$A \to aA / a$
$B \to bB$
$D \to ab / Ea$
$E \to ac / d$

$G_1 \Rightarrow (V_1, T_1, P_1, S)$

P_1	V_1	T_1
$A \to a$	$\{A\}$	$\{a\}$
$D \to ab$	$\{A, D\}$	$\{a, b\}$
$E \to ac/d$	$\{A, D, E\}$	$\{a, b, c, d\}$
$S \to aA/bA$	$\{A, S, D, E\}$	$\{\}$
$A \to aA$	$\{\}$	$\{\}$
$D \to Ea$	$\{\}$	$\{\}$

$G_2 \Rightarrow (V_2, T_2, P_2, S)$

P_2	V_2	T_2
$S \to aA/bA$	$\{S, A\}$	$\{a, b\}$
$A \to a/aA$	$\{S, A\}$	$\{a, b\}$

S → A B D E

Steps

Generality

First add only the productions that have only terminal, then according to the variable set, add the productions with those variables also.

Reachable

Then start from starting variable & continue.

Eliminate ϵ - productions

Definition:- * Any production of a context free grammar of the form, $A \to \epsilon$ is called ϵ - production.

* Any variable A for which the derivation $A \xrightarrow{*} \epsilon$ is possible is called Nullable.

Computing Nullable Variable (V_N)

Let $G = (V, T, P, S)$ be a CFG. Find all the nullable variable of G by the following iterative algorithm,

Basic: If $A \to \epsilon$ is a production of G, then A is nullable.

Induction: If there is production $B \to C_1 C_2 \cdots C_k$, where each C_i is nullable then B is nullable.

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Theorem

Let G be any CFG with ϵ not in $L(G)$. Then, there exists an equivalent grammar G_1, having no ϵ productions.

Proof We first find the set V_n of all nullable variables of G, using the following steps,

1) Find all production $A \to \epsilon$, put A into V_n.
2) Repeat the following steps until no other variables are added to V_n ....

For all productions $B \to A_1 A_2 \cdots A_k$, where $A_1 A_2 \cdots A_k$ are in V_N, put B into V_n.

Eg: 1) Eliminate ϵ - production

a) $S \to AaB \mid aaB$

$A \to \epsilon$

$B \to bbA \mid \epsilon$

$\left.\begin{array}{l} A \to \epsilon \\ B \to \epsilon \end{array}\right\} \Rightarrow A, B$ are nullable variables

P_1	V_N
$A \to \epsilon$	$\{A\}$
$B \to \epsilon$	$\{A, B\}$

P_2

$S \to aB \mid Aa \mid a \mid aa \mid AaB \mid aaB$

$B \to bbA \mid bb$

2) $S \to ABaC$

$A \to BC$

$B \to b \mid \epsilon$

$C \to D \mid \epsilon$

$D \to d$

P_1

$\left.\begin{array}{l} A \to \epsilon\epsilon \\ B \to \epsilon \\ C \to \epsilon \end{array}\right\} \Rightarrow A, B, C$ are nullable

P_2

$S \to Bac \mid Aac \mid ABa \mid ABac \mid ac \mid a \mid Aa \mid Ba$

$B \to b$

$C \to D$

$D \to d$

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3) $S \to BAAB$

$A \to 0A2 \mid 2A0 \mid \epsilon$

$B \to AB \mid 1B \mid \epsilon$

P_1	V_n
$S \to \epsilon\epsilon\epsilon\epsilon$	$\{S\}$
$A \to \epsilon$	$\{S, A\}$
$B \to \epsilon$	$\{S, A, B\}$

$S, A, B \to$ Nullable Variables

P_2

$S \to BAAB \mid AAB \mid BAB \mid BAA \mid AB \mid BA \mid AA \mid BB \mid B \mid A$

$A \to 0A2 \mid 02 \mid 2A0 \mid 20$

$B \to AB \mid 1B \mid 1 \mid A \mid B$

Elimination of Unit Productions

Def:- Any production of a Context free grammar of the form $A \to B$, where $A, B \in V$ is called Unit-Production.

Basis & Induction

Problems:-

1) $S \to Aa \mid B$

$B \to A \mid bb$

$A \to a \mid bc \mid B$

Unit productions	Non unit productions
$S \to B$	$S \to Aa$
$B \to A$	$B \to bb$
$A \to B$	$A \to a \mid bc$

$S \overset{*}{\Rightarrow} B$ (S B)

$S \Rightarrow A$ (S A)

$B \overset{*}{\Rightarrow} A$ (BA)

$A \overset{*}{\Rightarrow} B$ (AB)

$\xrightarrow{(\because \text{ by applying } B \to A)}$

$S \to bb$

$S \to a \mid bc$

$B \to a \mid bc$

$A \to bb$

Merge both

$S \to Aa \mid bb \mid a \mid bc$

$B \to bb \mid a \mid bc$

$A \to a \mid bc \mid bb$

Steps

① Write Unit & non-unit productions separately.

② Write unit pairs like (SB) and replace B with non-unit terms

③ Merge Non-unit productions & newly generated non-unit productions.

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1) $S \to AB$
$A \to a$
$B \to C \mid b$
$C \to D$
$D \to E \mid bC$
$E \to d \mid Ab$

Unit prod

$B \to C$
$C \to D$
$D \to E$

Non-unit prod

$S \to AB$
$A \to a$
$B \to b$
$D \to bC$
$E \to d / Ab$

$B \to C$ (BC)
$B \to D$ (BD)
$B \to E$ (BE)
$C \to D$ (CD)
$C \to E$ (CE)
$D \to E$ (DE)

$B \to C \to D \to E$

$B \to bC$ as $C \to D$
$B \to bC$

$B \to d \mid Ab$
$C \to d \mid Ab$
$D \to d \mid Ab$

$S \to AB$
$A \to a$
$B \to b \mid d \mid Ab$
$C \to d \mid Ab$
$D \to d \mid Ab \mid bC$
$E \to d \mid Ab$

* G
$\downarrow$
ϵ production
$\downarrow$
G_1
$\downarrow$
Unit production
$\downarrow$
G_2
$\downarrow$
Useless production
$\downarrow$
G_3

1) $S \to aA \mid aBB$
$A \to aaA \mid \epsilon$
$B \to bB \mid bbC$
$C \to B$

P: $A \to \epsilon$

V_n: $\{A\}$

G_1
$S \to a \mid aBB \mid aA$
$A \to aa \mid aaA$
$B \to bB \mid bbC$
$C \to B$

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1)

Unit production

$C \to B$

Non-unit production

$S \to aBB \mid aA \mid a$
$A \to aa \mid aaA$
$B \to bB \mid bbC$
$C \to B$

G_2

$S \to aBB \mid aA \mid a$
$A \to aa \mid aaA$
$B \to bB \mid bbC$
$C \to bB \mid bbC$

Useless production

Generating

P_1	V_1	T_1
$S \to a$	$\{S\}$	$\{a\}$
$A \to aa$	$\{S, A\}$	$\{a\}$
$S \to aA$	$\{S, A\}$	$\{a\}$
$A \to aaA$	$\{S, A\}$	$\{a\}$

Reachable

P_2	V_2	T_2
$S \to aA \mid a$	$\{S, A\}$	$\{a\}$
$A \to aa \mid aaA$	$\{S, A\}$	$\{a\}$

$S \longrightarrow A$

2) $S \to abAB$
$A \to bAB \mid \epsilon$
$B \to BAa \mid A \mid \epsilon$

ϵ - Production

$A \to \epsilon$
$B \to \epsilon$

G_1

$S \to abAB \mid abB \mid abA \mid ab$
$A \to bAB \mid bA \mid bB \mid b$
$B \to BAa \mid Aa \mid Ba \mid a \mid A$

Unit-production

$B \to A$

Non-Unit production

$S \to abAB \mid abB \mid abA \mid ab$
$A \to bAB \mid bA \mid bB \mid b$
$B \to BAa \mid Aa \mid Ba \mid a \mid A$

G_2

$S \to abAB \mid abB \mid abA \mid ab$
$A \to bAB \mid bA \mid bB \mid b$
$B \to BAa \mid Aa \mid Ba \mid a \mid bAB \mid bA \mid bB \mid b$

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Problem

1) $S \to 0A \mid 1B$
$A \to 0AA \mid 1S \mid 1$
$B \to 1BB \mid 0S \mid 0$

Step I

$S \to S_1A \mid S_2B$
$S_1 \to 0$
$S_2 \to 1$
$A \to S_1AA \mid S_2S \mid 1$
$B \to S_2BB \mid S_1S \mid 0$

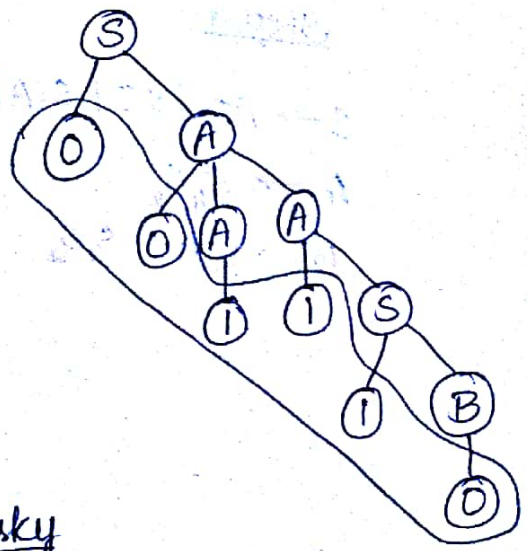
Step 2

$S \to S_1A \mid S_2B$
$S_1 \to 0$
$S_2 \to 1$
$A \to S_1A_1 \mid S_2S \mid 1$
$A_1 \to AA$
$B \to S_2B_1 \mid S_1S \mid 0$
$B_1 \to BB$

Get string 001110

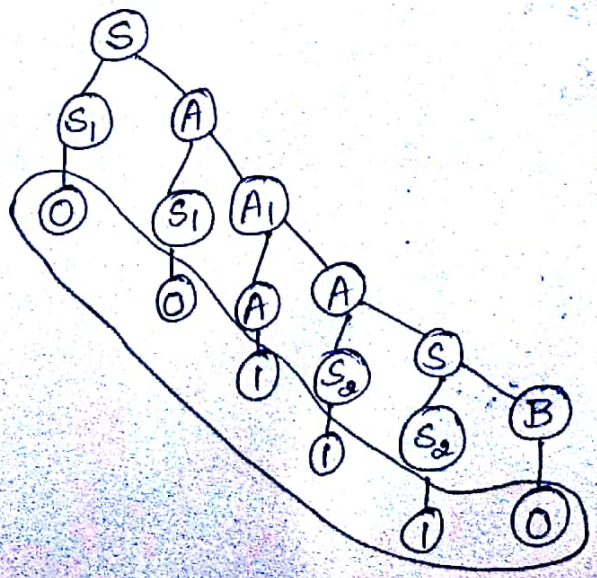
CFG

LMD $\Rightarrow$ $S \underset{lm}{\Rightarrow} 0A \Rightarrow 00AA \Rightarrow 0011S \Rightarrow 00111B \Rightarrow 001110$



Chomsky

$S \Rightarrow S_1A \Rightarrow 0A \Rightarrow 0S_1A_1 \Rightarrow 00A_1 \Rightarrow 00AA \Rightarrow 001A \Rightarrow 001S_2S \Rightarrow$
$0011S \Rightarrow 0011S_2B \Rightarrow 00111B \Rightarrow 001110$



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Q) $S \to AB \mid aB$
$A \to aab \mid \epsilon$
$B \to bbA$

P	V_n	
$A \to \epsilon$	$\{A\}$	$\to$ Nullable

$S \to AB \mid aB \mid B$
$A \to aab$
$B \to bbA \mid bb$

<u>Unit production</u>

$S \to B$

<u>Non-unit production</u>

$S \to AB \mid aB \mid B$
$A \to aab$
$B \to bbA \mid bb$

$S \to AB \mid aB \mid bbA \mid bb$
$A \to aab$
$B \to bbA \mid bb$

<u>Chomsky</u>

<u>Step 1</u>

$S \to AB \mid S_1B \mid S_2S_2A \mid S_2S_2$
$S_1 \to a$
$S_2 \to b$
$S \to AB \mid S_1B \mid S_3A \mid S_2S_2$
$S_3 \to S_2S_2$
$A \to S_1S_1S_2$
$A \to S_4S_2$
$S_4 \to S_1S_1$
$B \to S_2S_2A \mid S_2S_2$
$B \to S_3A \mid S_2S_2$

<u>Step 2</u>

$S \to AB \mid S_1B \mid S_3A \mid S_2S_2$
$A \to S_4S_2$
$B \to S_3A \mid S_2S_2$

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