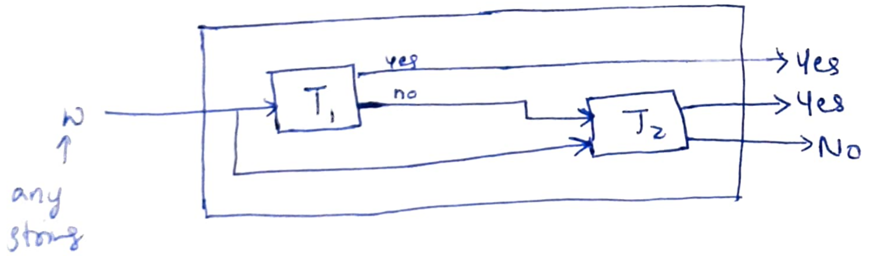
Recursively enumerable v/s Recursive lang

Recursively enumerable	Recursive lang
Any string $w \in L$ (Recursively enumerable) then M (TM) halts in final state.	$w \in L$ then M halts in final state
$w \notin L$ then M halts in non final state or infinite loop	$w \notin L$ then M halts in non-final state
Referred as Turing acceptable	Referred as Turing decidable

Recursive lang is subset of recursively enumerable lang.

1. Every recursive lang is recursively enumerable
2. If $L \subseteq \Sigma^*$ is accepted by TM T that halts on every input string, then L is recursive
3. If L_1 and L_2 are recursive ~~enumerable~~ langs over Σ then $L_1 \cup L_2$ & $L_1 \cap L_2$ are also ~~REL~~ RL

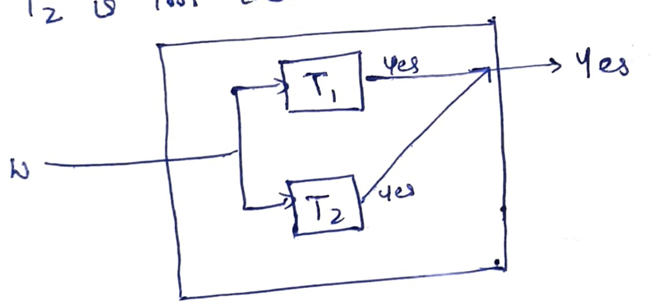
Proof: If T_1 is TM that accepts L_1, $L_1 = L(T_1)$
T_2 is TM that accepts L_2 $L_2 = L(T_2)$



(4) If L_1 & L_2 are REL over Σ then $L_1 \cup L_2$ & $L_2 \cap L_1$ are also REL

Proof: If T_1 is TM that accepts L_1 $L_1 = L(T_1)$

If T_2 is TM that accepts L_2 $L_2 = L(T_2)$



Cases:

If any one of TM accepts w.

If one TM rejects then we abandon it & continue with other.

If both ~~reject~~ TM rejects, we reject & halt.

If ~~TM~~ both TM's loop forever ~~then~~ occurs when $w \notin L_1 \cup L_2$

Draw for each if you want

$L(T_3) = L(T_1) \cup L(T_2)$

$L(T_3)$ is subset of $L(T_1)$ & $L(T_2)$

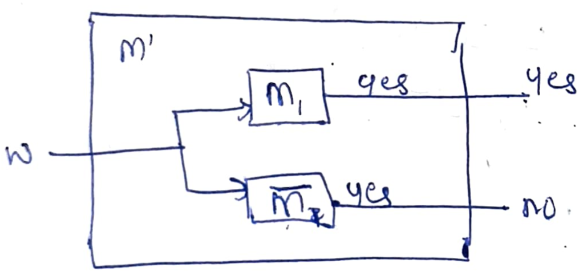
$L(T_1)$ & $L(T_2)$ is subset of $L(T_3)$

$w \in L_1 \cap L_2$ is accepted when both TM's accept w.

⑤ If L is RL over Σ, then its complement $\bar{L}$ is also Recursive

⑥ If L is REL & its complement $\bar{L}$ is also REL then L is Recursive (Therefore $\bar{L}$ is recursive)

Proof: let M1 & M2 be TMs such that $L = L(M1)$, $\bar{L} = L(M2)$



$L(M') = L$

$L(M')$ is subset of L and viceversa

M' is TM for L

M' always halts since either M1 or M2 halts for any given string w

M' shows that L is recursive

~~If follows~~ $x \in L$ or $x \in \bar{L}$

If one TM halts then M' behaves like recursive

∴ L is Recursive

By above theorem, we know that "If L is RL then its $\bar{L}$ is also RL"

∴ $\bar{L}$ is Recursive

Unrestricted Grammar

If production $u \to v$ where u, v are strings of variables & terminals

$u = (V \cup T)^+$ $\quad v = (V \cup T)^*$

① $L = \{a^n b^n c^n, n \ge 0\}$

$S \to FS1 \mid \varepsilon$
$S1 \to ABCS \mid ABC$

$CA \to AC$
$BA \to AB$
$CB \to BC$ $\qquad bB \to bb$
$FA \to a$ $\qquad bC \to bc$
$aA \to aa$ $\qquad cC \to cc$
$aB \to ab$

aabbcc

$S \to FS1$
$\to FABCS$
$\to FABCABC$
$\to FABACBC$
$\to FAABBCC$
$\to aABBCC$

② $L = \{ww \mid w \in a, b\}$

$S \to S'Z$ $\qquad Z \to \varepsilon$
$S' \to aS'A \mid bS'B \mid \varepsilon$
$AZ \to XZ$ $\qquad BZ \to YZ$
$AX \to XA$ $\qquad AY \to YA$
$BX \to XB$ $\qquad BY \to YB$
$aX \to aa$ $\qquad aY \to ab$
$bX \to ba$ $\qquad bY \to bb$

aa, abab, babbbabb

③ Identify lang generated

$S \to S_1B$
$S_1 \to aS_1B$
$bB \to bbbB$
$aS_1B \to aa$
$B \to \varepsilon$

$S \to S_1B$
$\to aS_1BbB$
$\to aaS_1bbB$
$a^n S_1 b^n B$
ε $\quad bbB$
$a^n S b^{n+k} B$

④ $L = \{a(aa)^n \mid n \geq 0\}$ $\qquad$ aaaaa

$S \to aS_1$ $\qquad$ $S \to AS \mid aT$ $\qquad$ $S \to aS$

$aS_1 \to aaaS_1$ $\qquad$ $AT \to T$ $\qquad$ $S \to aaaS_1$

$S_1 \to \epsilon$ $\qquad$ $T \to \epsilon$

$Aa \to aaaA$

$S \to AS \to AaT \to aaaA \to aaa$

⑤ $L = \{a^{2k} \mid k \in N\}$ — Natural no.

$(a^2)^k = a^{2k} \quad | \; k \geq 1$ $\qquad$ aaaaaaaa

a^2, a^4, a^6, a^8 $\qquad$ $S \to aaS_1$ $\qquad$ $S \to$

aa $\quad$ aaaa $\qquad$ $S_1 \to \epsilon \mid aaS_1$

⑥ lang generated

$S \to TDE$ $\qquad$ $T \to ABCT \mid \epsilon$ $\qquad$ ABCABCDE

$AB \to BA$ $\qquad$ $BA \to AB$ $\qquad$ ABACBDCE

$CA \to AC$ $\qquad$ $CB \to BC$ $\qquad$ ABCDE $\qquad$ AABBCDCE

$CD \to DC$ $\qquad$ $CE \to Ea$ $\qquad$ BADCE $\qquad$ AABBDCCE $\qquad$ $L = \{a^n b^n a^n \mid n \geq 0\}$

$BD \to Db$ $\qquad$ $A \to a$ $\qquad$ ABDCE $\qquad$ aaBDbcEa

$D \to \epsilon$ $\qquad$ $E \to \epsilon$ $\qquad$ ADbEa $\qquad$ aaDbbEaa

aba $\qquad$ aabbaa

Context sensitive lang

Production $u \to v$ where u, v are strings of variables & terminals where

$|u| \leq |v|$ $\qquad$ $u, v = (V \cup T)^+$

Ex: $aS \to aV$ $\qquad$ $|u| = 2$ $\qquad$ $|v| = 2$ ✓

$a \to aS$ $\qquad$ $|u| = 1$ $\qquad$ $|v| = 2$ ✓

$aS \to a$ $\qquad$ $|u| = 2$ $\qquad$ $|v| = 1$ ✗ Not CSG

① $L = \{a^n b^n c^n \mid n \geq 1\}$ $\qquad$ abc $\qquad$ aabbcc

$S \to FS_1$ $\qquad$ $S_1 \to ABCS_1 \mid ABC$ $\qquad$ FABCABC

$CA \to AC$ $\qquad$ $BA \to AB$ $\qquad$ $CB \to BC$ $\qquad$ aBACBC

$FA \to a$ $\qquad$ $aA \to aa$ $\qquad$ $aB \to ab$ $\qquad$ aABBCC

$bB \to bb$ $\qquad$ $bC \to bc$ $\qquad$ $cC \to cc$ $\qquad$ aaBBCC

aabbcc ✓

② $L = \{a^n b^m c^n a^m,\ m,n \geq 1\}$

abca   aabcca   aabbbccaaa

$S \rightarrow Aa$

$A \rightarrow aAc \mid B \mid b$

$B \rightarrow bBX \mid b$

$X_c \rightarrow cX$

$X_a \rightarrow aa$

Aa
aAca
abca

Aa
aAca
aaAcca
aabBXcca
aabbBXXcca
aabbbccXXa
aabbbccaaa

③ lang identified

$S \rightarrow aS'bX \mid abX$

$S' \rightarrow aS'bC \mid S'bC \mid S'C \mid C$

$Cb \rightarrow bC$

$CX \rightarrow XC$

$X \rightarrow c$

$S \rightarrow aS'bX$
aaS'bCbX
aaS'bCbCbX
aaCbCbCbX
aabCBCbCX
aabbCbCXc
aabbbCXcc
aabbbcccc

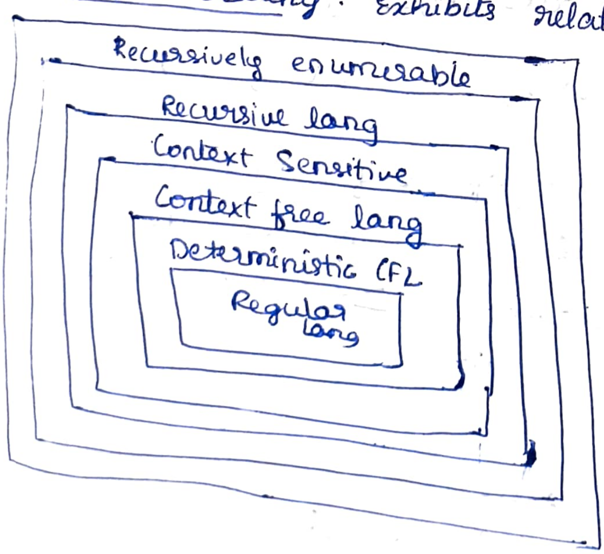
$S \rightarrow abX$
abc

$S \rightarrow aS'bX$
aCbX
abCX
abXc
abcc

$S \rightarrow aS'bX$
aS'CbX
aCbCX
abCXc
abccc

$L = \{a^i b^j c^k \mid 1 \leq i \leq j \leq k\}$

Chomsky hierarchy: Exhibits relationship among families



CH

Type0	Type 1	Type2	Type 3
RE	CSL	CFL.	RL.

Halting Problem of TM

The problems that run forever on TM are not solvable. In other words, there are some problem input instances for which TM will not halt on inputs that they do not accept. Those problems are called unsolvable or undecidable problems.

If lang L is not accepted by TM then lang is not RE.

Post Correspondence Problem

Given 2 sequences of n-strings on some alphabet Σ say

$A = w_1 w_2 \dots w_n$

$B = v_1 v_2 \dots v_n$

then there exists PC solution for the pair (A, B) if there is a non-empty sequence of intes $i, j, \dots k$ such that $w_i w_j \dots w_k = v_i v_j \dots v_k$

(1)

	L_{w_i} A	L_{v_i} B
1	11	111
2	100	001
3	111	11

- If any ordering of PC if we get same value in both pairs then it has PC solution. Atleast one soln satisfied then it is PC solution.

$w_3 w_2 w_1 = v_3 v_2 v_1$ ← ordering of w & v should match

$111\,10011 = 11001111$

$w_1 w_2 w_3 = v_1 v_2 v_3$

$11100111 = 11100111$

(2)

00	0
001	11
1000	011

~~Total length is not matching~~

∴ No Solution

③ A = {1, 10111, 10}
B = {111, 10, 0}

→ ~~No~~ Solution
2 1 1 3

④ A: {100, 101, 110}
B: {10, 01, 1010}

→ 1 2 . . 1
10010110 1100
1001 01 10

No Solution

⑤
	i	ii
1	1	10
2	0	10
3	010	01
4	11	1

~~1010~~ 1010

~~10~~ 1001

4 2

~~110~~ ~~1010~~ ~~0110~~ 1010

~~110~~ ~~1001~~ ~~0101~~ 1010

→ 1 2 1 3 3 4

Time and Space Complexity of TM

Time C = How long computation takes to execute ?
i.e, in TM this could be measured as no of moves which are required to perform computation.
i.e, no of m/c cycle.

Space C = no. of bytes used.

1) Identify lang. generated by unrestricted grammar

a) $S \to ABCS \mid ABC$

$AB \to BA$ $AC \to CA$

$BC \to CB$ $BA \to AB$

$CA \to AC$ $CB \to BC$

$A \to a$ $B \to b$ $C \to c$

$S \to ABC$
abc

$S \to ABC$
$\to BAC$
$\to bac$

$S \to ABCABC$
$\to ACBBAC$
$\to CABBCA$
$\to cabbca$

$n_a(w) = n_b(w) = n_c(w)$

b) $S \to LaR$ $L \to LD \mid LT \mid \varepsilon$

$Da \to aaD$

$Ta \to aaaT$ $DR \to R$

$TR \to R$ $R \to \varepsilon$

$S \to LaR \to a$
$\to \cancel{L}DaR$
$\to LaaDR$
$\to LTaaR$
$\to LaaaaTR$
$\to aaaa$

LaR
$LTaR$
$LaaaTR$
aaa

LaR
$LDaR$
$LaaDR$
aa

LaR
$LDaR$
$LTDaR$
$LTTDaR$
$LaaaaaaR$

$L = \{a^n \mid n > 0\}$

c) $S \to LaMR$ $ER \to \varepsilon$

$L \to LT \mid E$ $Ta \to aT$

$TM \to aaMT$ $TR \to aMR$

$Ea \to aE$ $EM \to E$

$\to LaMR$
$\to LTaMR$
$LaTMR$
$EaTMR$
$aEaaMTR$
$aaaEaMR$
$aaaa$

$\to LaMR$
$\to aEMR$
$\to a$

$\to LaMR$
$LTTaMR$
$ETaTMR$
$EaTaaMTR$
$aEaaTMTR$
$aaaEaaMTaMR$
$aaaaaEaaaMTR$
$aaaaaaaaEaMR$
$aaaaaaaaa$

$L = \{a^{3n} \mid n \geq 0\}$

d) $S \to TD_1D_2$

$T \to ABCT \mid \varepsilon$

$AB \to BA$ $BA \to AB$

$CA \to AC$ $CB \to BC$

$CD_1 \to D_1C$ $CD_2 \to D_2a$

$BD_1 \to D_1b$ $A \to a$

$D_1 \to \varepsilon$ $D_2 \to \varepsilon$