Code Optimization

Code Optimization

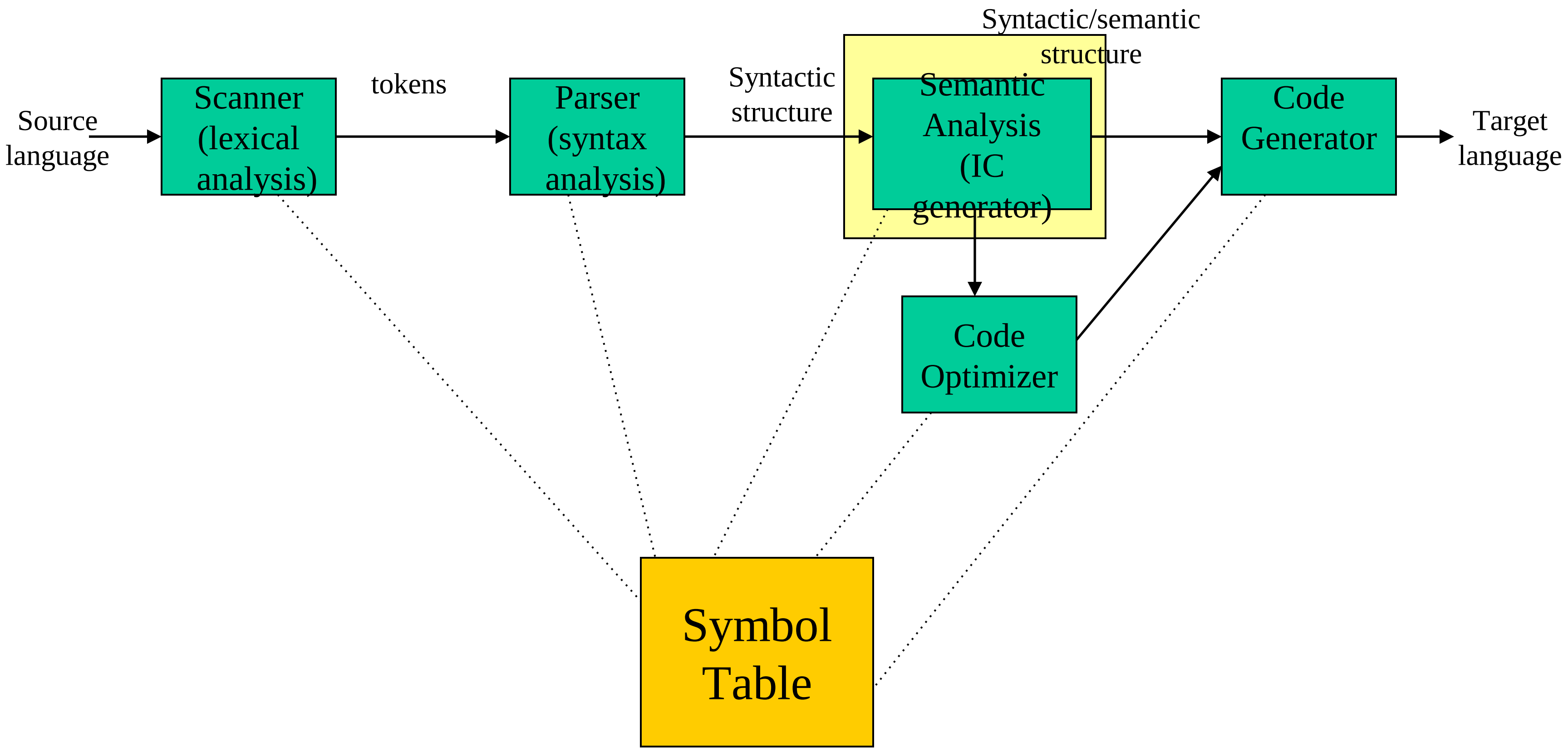
REQUIREMENTS:

- Meaning must be preserved (correctness)
- Speedup must occur on average.
- Work done must be worth the effort.

OPPORTUNITIES:

- Programmer (algorithm, directives)
- Intermediate code
- Target code

Code Optimization



Levels

- Window – peephole optimization
- Basic block
- Procedural – global (control flow graph)
- Program level – intraprocedural (program dependence graph)

Peephole Optimizations

- Constant Folding

```
x := 32              becomes    x := 64
x := x + 32
```

- Unreachable Code

```
goto L2
x := x + 1               □     unneeded
```

- Flow of control optimizations

```
goto L1              becomes     goto L2
 …
L1: goto L2
```

Peephole Optimizations

- Algebraic Simplification

  `x := x + 0` □ unneeded

- Dead code

  `x := 32` □ where x not used after statement

  `y := x + y` □ `y := y + 32`

- Reduction in strength

  `x := x * 2` □ `x := x + x`

Peephole Optimizations

- Local in nature
- Pattern driven
- Limited by the size of the window

Basic Block Level

- Common Subexpression elimination
- Constant Propagation
- Dead code elimination
- Plus many others such as copy propagation, value numbering, partial redundancy elimination, …

Simple example: a[i+1] = b[i+1]

- t1 = i+1
- t2 = b[t1]
- t3 = i + 1
- a[t3] = t2

- t1 = i + 1
- t2 = b[t1]
- t3 = i + 1 □ *no longer live*
- a[t1] = t2

Common expression can be eliminated

Now, suppose i is a constant:

- i = 4
- t1 = i+1
- t2 = b[t1]
- a[t1] = t2

- i = 4
- t1 = 5
- t2 = b[t1]
- a[t1] = t2

- i = 4
- t1 = 5
- t2 = b[5]
- a[5] = t2

Final Code:

- i = 4
- t2 = b[5]
- a[5] = t2

Control Flow Graph - CFG

CFG = < V, E, Entry >, where

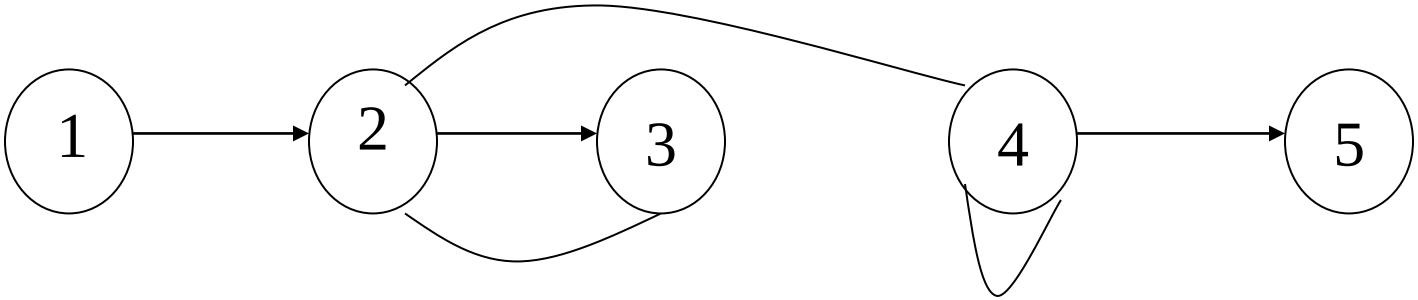
V = vertices or nodes, representing an instruction or basic block (group of statements).

E = (V x V) edges, potential flow of control

Entry is an element of V, the unique program entry

Two sets used in algorithms:

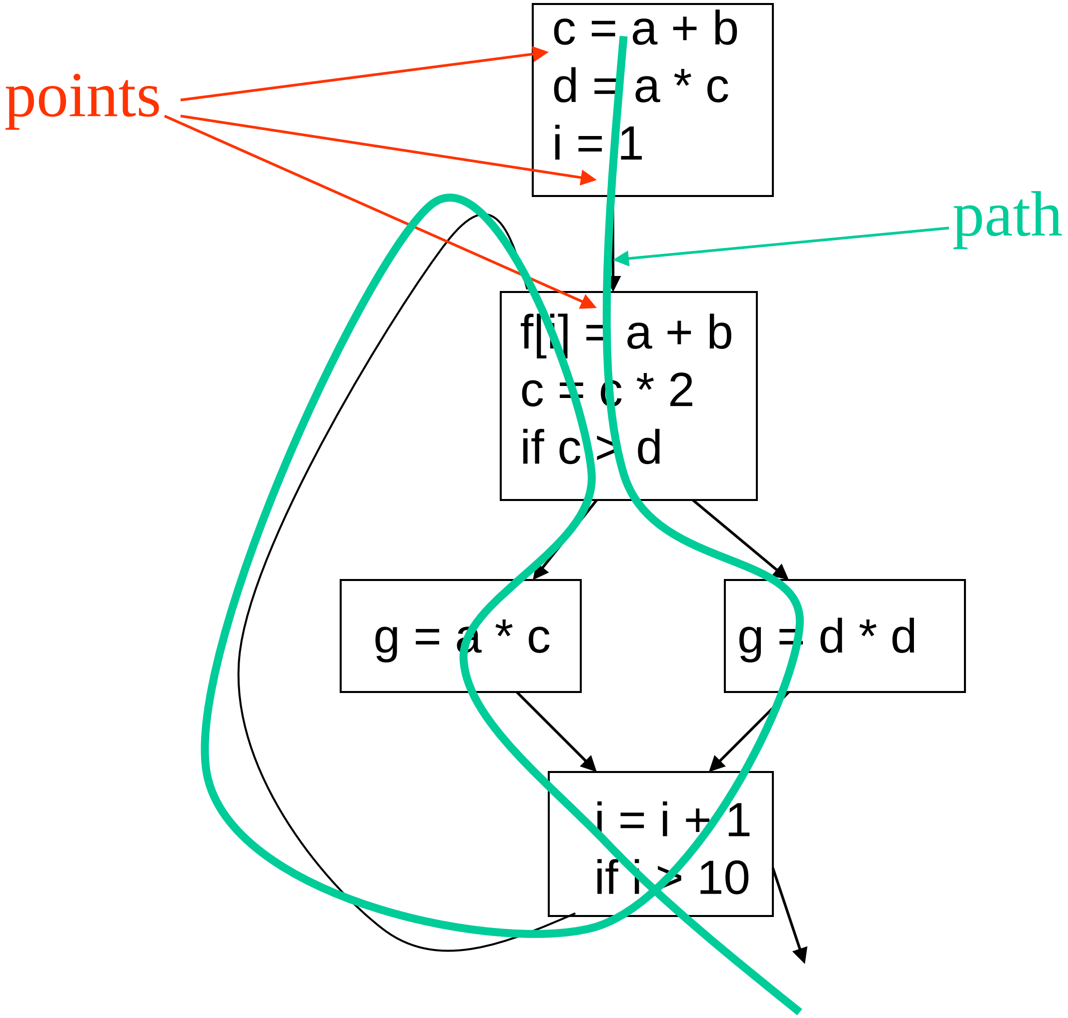
- Succ(v) = {x in V| exists e in E, e = v □ x}
- Pred(v) = {x in V| exists e in E, e = x □ v}



Definitions

- point - any location between adjacent statements and before and after a basic block.
- A path in a CFG from point p_1 to p_n is a sequence of points such that ∀ j, 1 <= j < n, either p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that statement in the same block, or p_i is the end of some block and p_{i+1} is the start of a successor block.

CFG



Optimizations on CFG

- Must take control flow into account
  - Common Sub-expression Elimination
  - Constant Propagation
  - Dead Code Elimination
  - Partial redundancy Elimination
  - …
- Applying one optimization may create opportunities for other optimizations.

Redundant Expressions

An expression **x op y** is redundant at a point p if it has already been computed at some point(s) and no intervening operations redefine **x** or **y**.

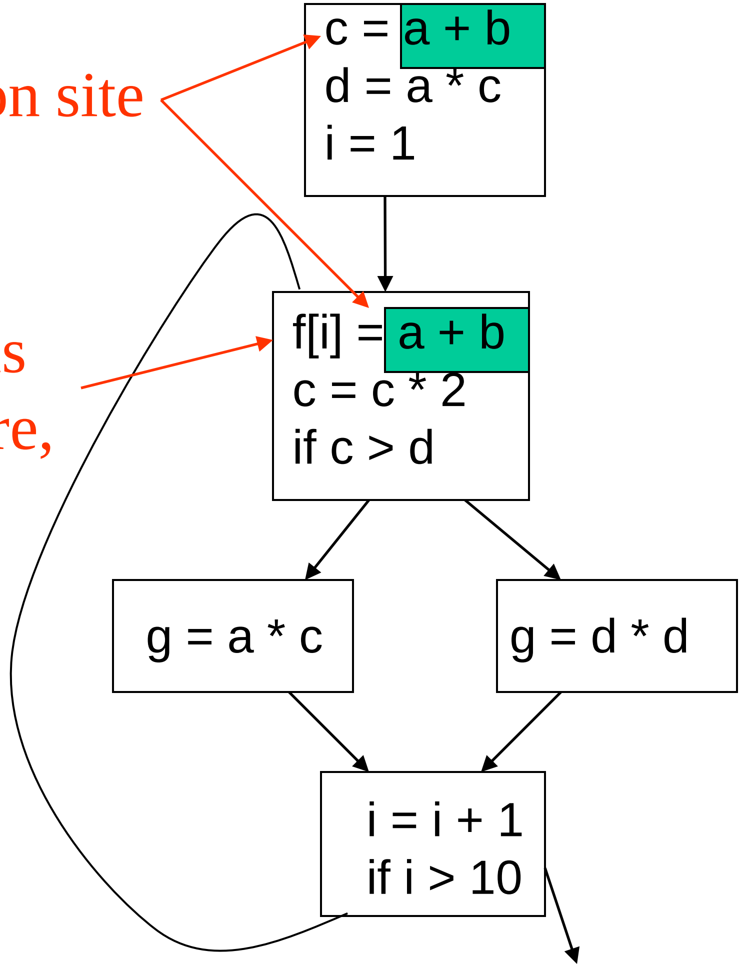
```
m = 2*y*z          t0 = 2*y          t0 = 2*y
                   m = t0*z          m = t0*z
n = 3*y*z          t1 = 3*y          t1 = 3*y
                   n = t1*z          n = t1*z
o = 2*y-z          t2 = 2*y
                   o = t2-z          o = t0-z
```

redundant

Redundant Expressions

Definition site

Since a + b is available here, □ redundant!



c = a + b
d = a * c
i = 1

f[i] = a + b
c = c * 2
if c > d

g = a * c

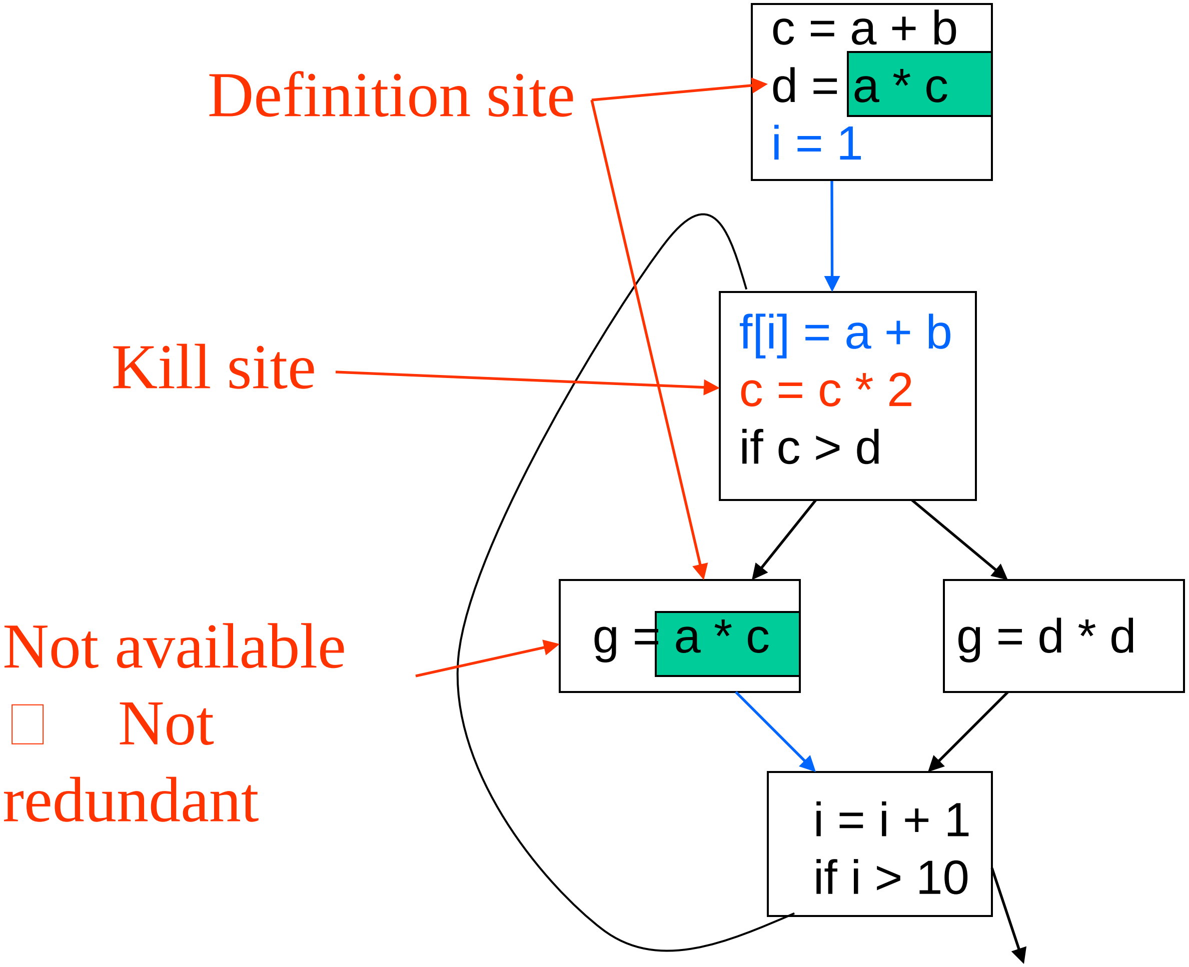
g = d * d

i = i + 1
if i > 10

Candidates:

🟩 a + b
a * c
d * d
c * 2
i + 1

Redundant Expressions



Definition site

Kill site

Not available
□ Not redundant

```
c = a + b
d = a * c
i = 1
```

```
f[i] = a + b
c = c * 2
if c > d
```

```
g = a * c
```

```
g = d * d
```

```
i = i + 1
if i > 10
```

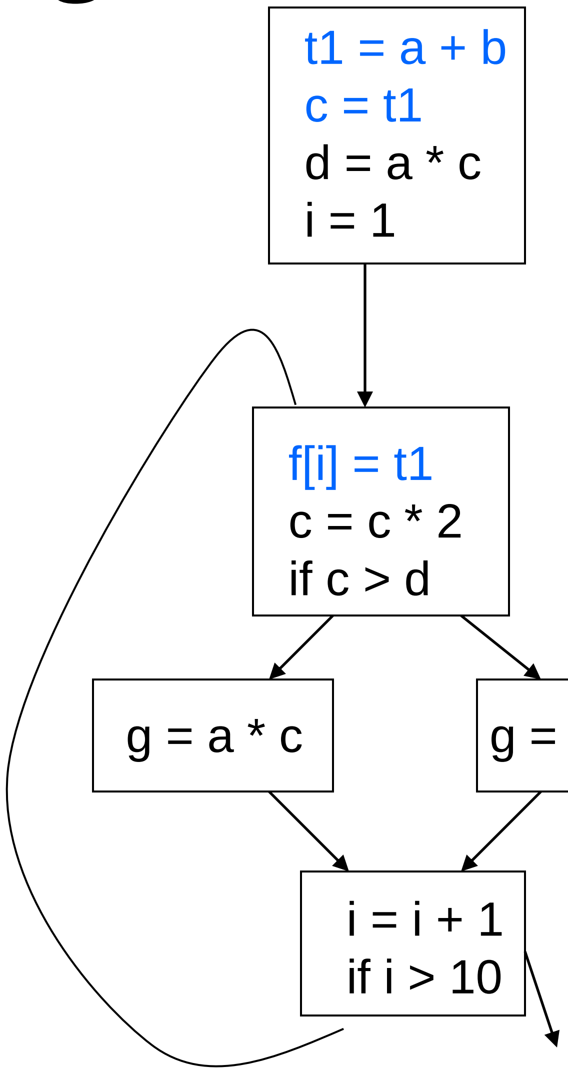
Candidates:

a + b
a * c
d * d
c * 2
i + 1

Redundant Expressions

- An expression *e* is defined at some point *p* in the CFG if its value is computed at *p*. (definition site)
- An expression *e* is killed at point *p* in the CFG if one or more of its operands is defined at *p*. (kill site)
- An expression is ***available*** at point *p* in a CFG if every path leading to *p* contains a prior definition of *e* and *e* is not killed between that definition and *p*.

Removing Redundant Expressions



```
t1 = a + b
c = t1
d = a * c
i = 1
```

```
f[i] = t1
c = c * 2
if c > d
```

```
g = a * c
```

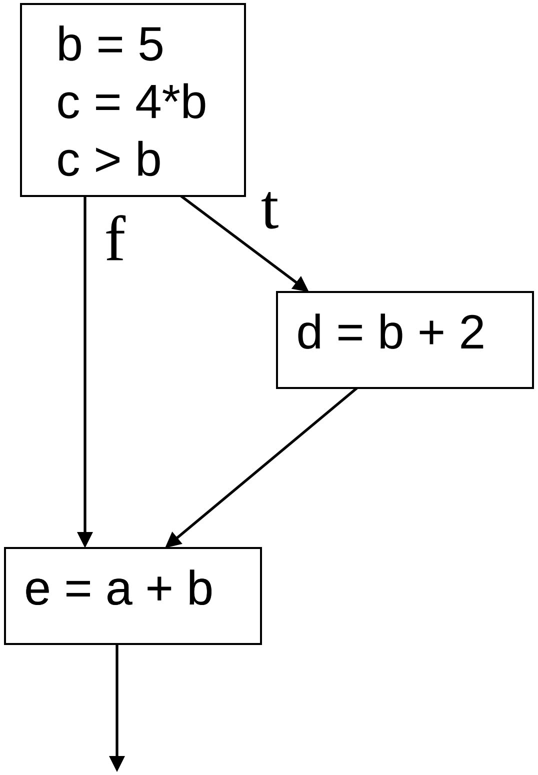
```
g = d*d
```

```
i = i + 1
if i > 10
```

Candidates:

a + b
~~a * c~~
~~d * d~~
~~c * 2~~
~~i + 1~~

Constant Propagation

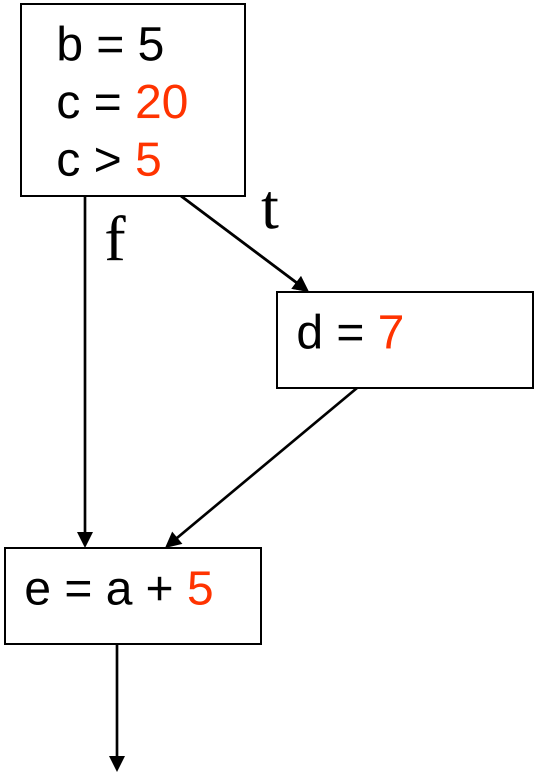

b = 5
c = 4*b
c > b

f

t

d = b + 2

e = a + b

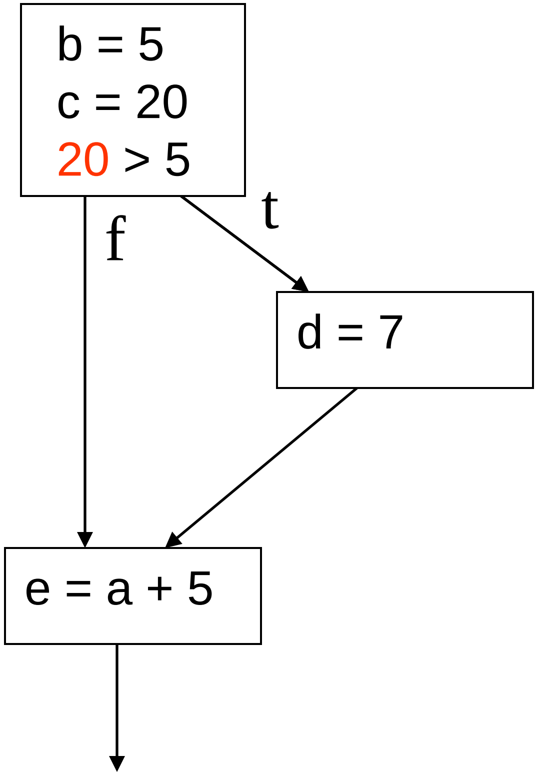

b = 5
c = 20
c > 5

f

t

d = 7

e = a + 5


b = 5
c = 20
20 > 5

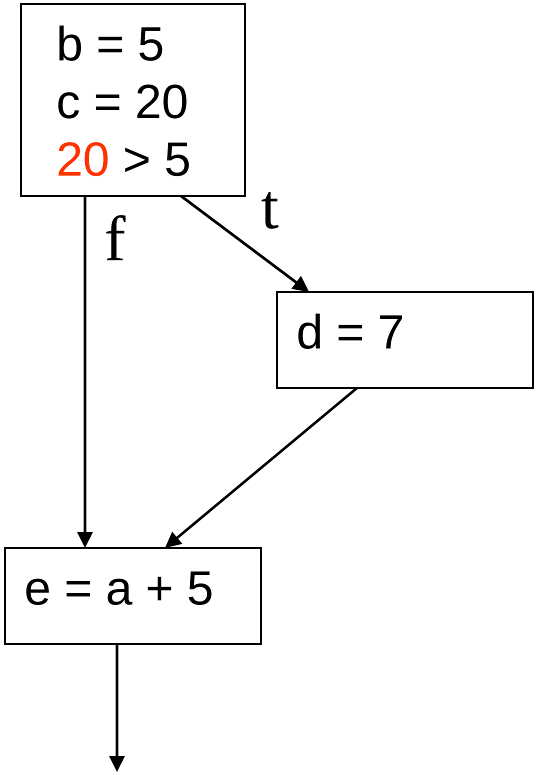
f

t

d = 7

e = a + 5

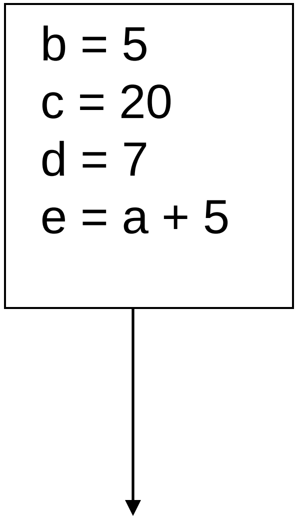
Constant Propagation



b = 5
c = 20
20 > 5

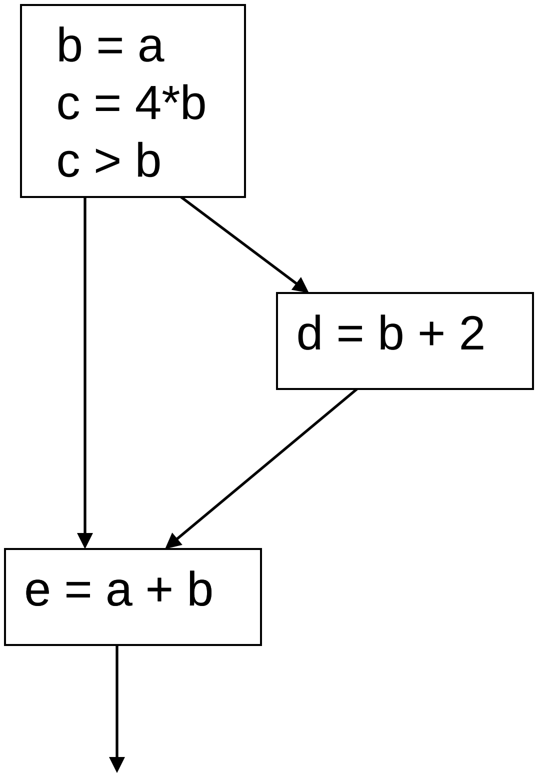
d = 7

e = a + 5



b = 5
c = 20
d = 7
e = a + 5

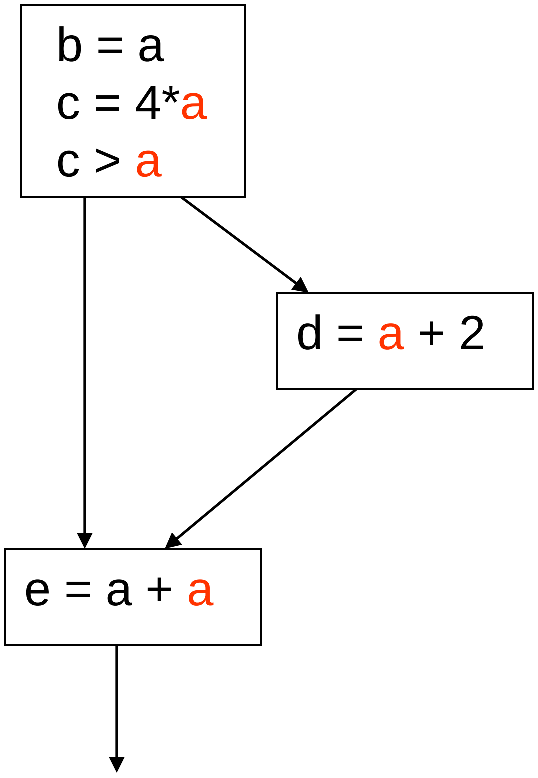
Copy Propagation



b = a
c = 4*b
c > b

d = b + 2

e = a + b



b = a
c = 4*a
c > a

d = a + 2

e = a + a

Simple Loop Optimizations: Code Motion

```
while (i <=  limit - 2)
```

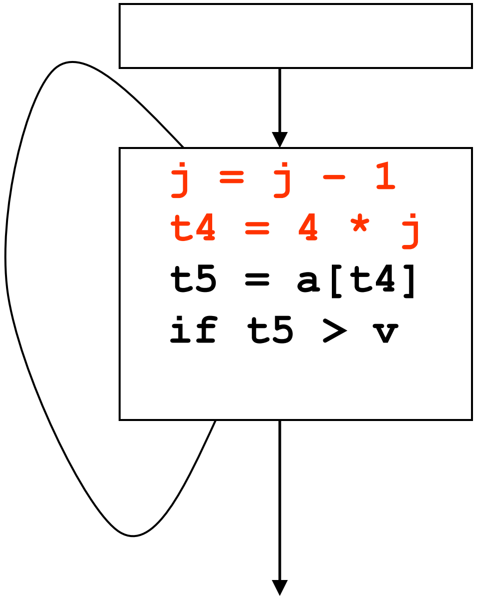
```
L1:
    t1 = limit - 2
    if (i > t1) goto L2
    body of loop
    goto L1
L2:
```

```
t := limit - 2
   while (i <= t)
```

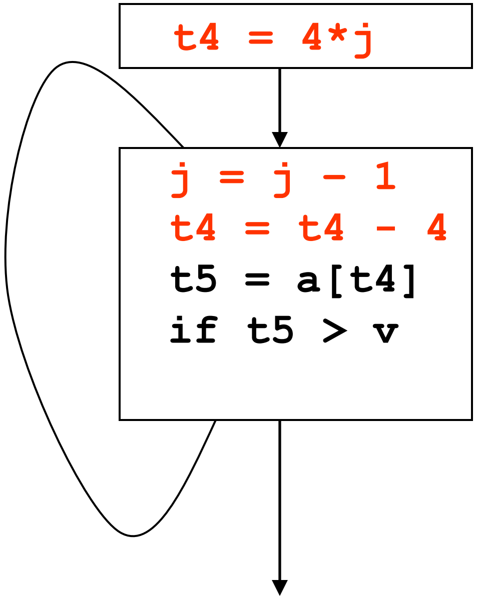
```
    t1 = limit - 2
L1:
    if (i > t1) goto L2
    body of loop
    goto L1
L2:
```

Simple Loop Optimizations: Strength Reduction

- Induction Variables control loop iterations



```
j = j - 1
t4 = 4 * j
t5 = a[t4]
if t5 > v
```



```
t4 = 4*j
```

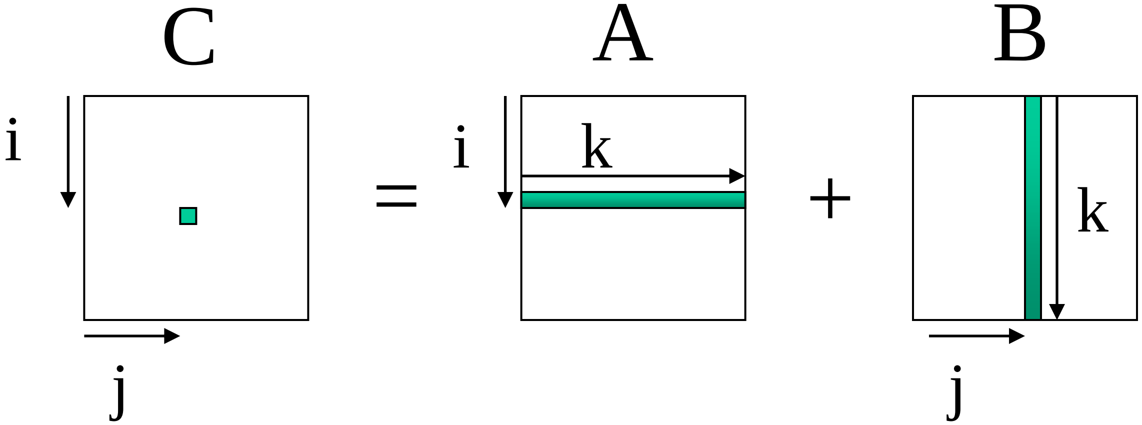
```
j = j - 1
t4 = t4 - 4
t5 = a[t4]
if t5 > v
```

Simple Loop Optimizations

- Loop transformations are often used to expose other optimization opportunities:
  - Normalization
  - Loop Interchange
  - Loop Fusion
  - Loop Reversal
  - …

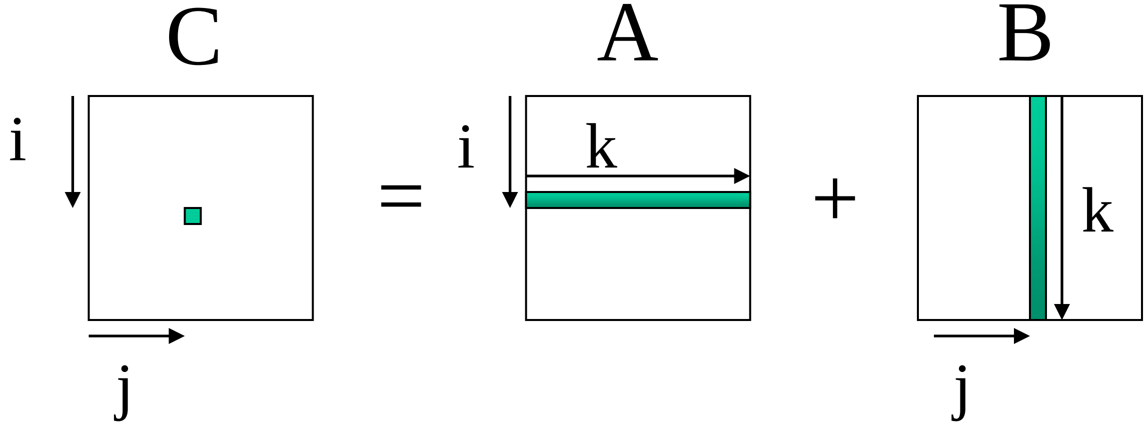
Consider Matrix Multiplication

```
for i = 1 to n do
  for j = 1 to n do
     for k = 1 to n do
        C[i,j] = C[i,j] + A[i,k] + B[k,j]
     end
  end
end
```



Memory Usage

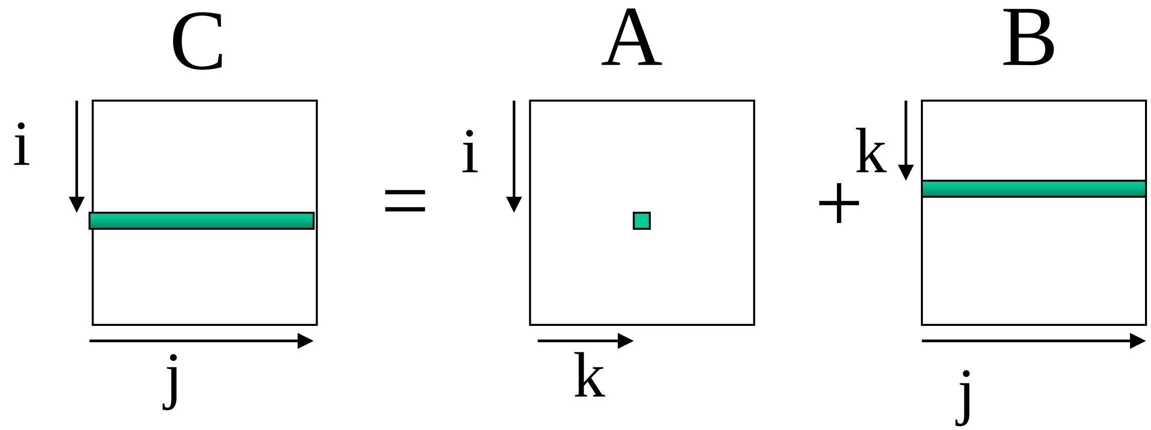
- **For A:** Elements are accessed across rows, spatial locality is exploited for cache (assuming row major storage)
- **For B:** Elements are accessed along columns, unless cache can hold all of B, cache will have problems.
- **For C:** Single element computed per loop – use register to hold



Matrix Multiplication Version 2

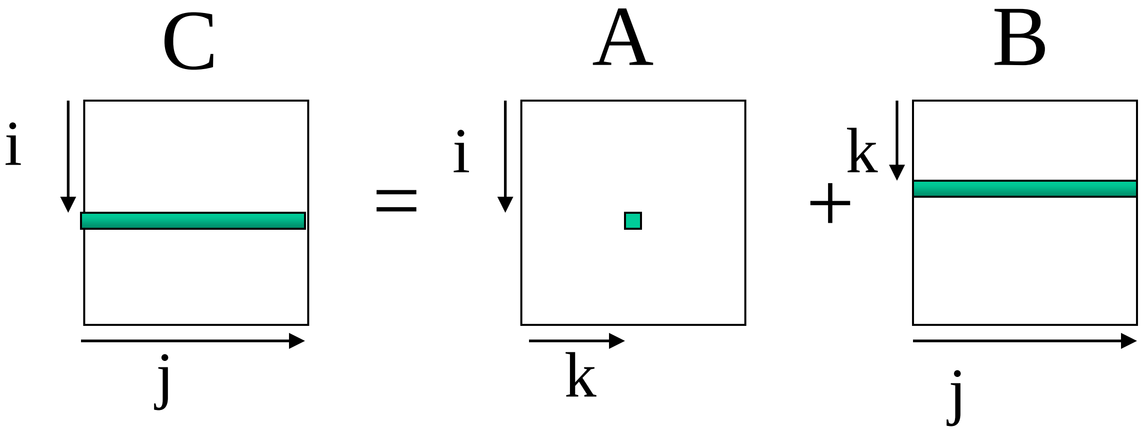
```
for i = 1 to n do
  for k = 1 to n do
     for j = 1 to n do
        C[i,j] = C[i,j] + A[i,k] + B[k,j]
     end
  end
end
```

loop interchange



Memory Usage

- **For A:** Single element loaded for loop body
- **For B:** Elements are accessed along rows to exploit spatial locality.
- **For C:** Extra loading/storing, but across rows



Simple Loop Optimizations

- How to determine safety?
  - Does the new multiply give the same answer?
  - Can be reversed??
    - **`for (I=1 to N) a[I] = a[I+1]`** – can this loop be safely reversed?

Data Dependencies

- Flow Dependencies - write/read

  ```
  x := 4;
  y := x + 1
  ```

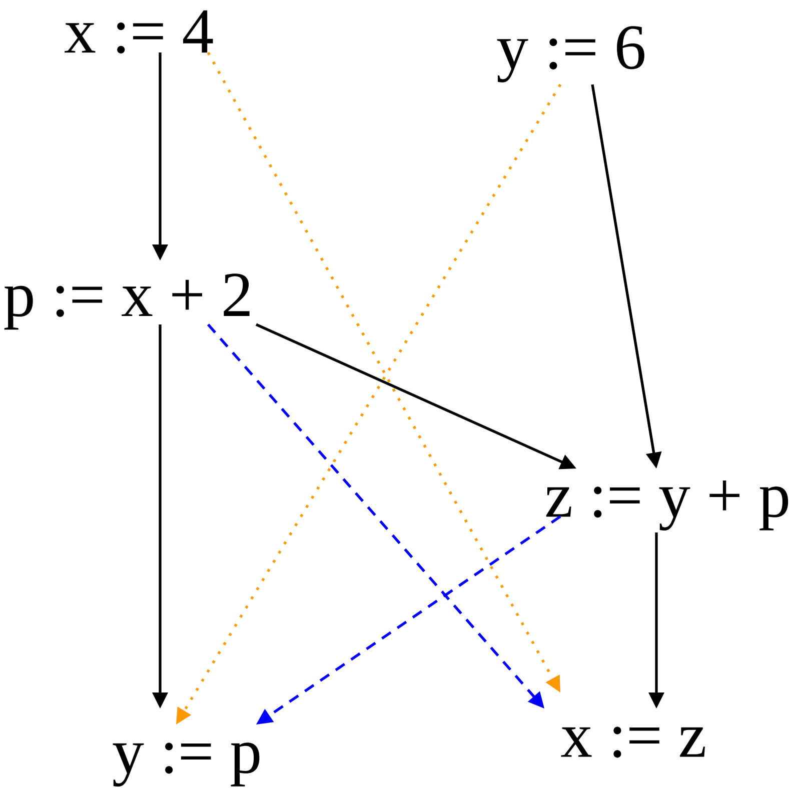
- Output Dependencies - write/write

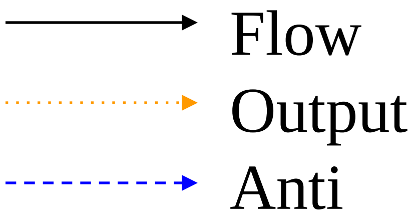
  ```
  x := 4;
  x := y + 1;
  ```

- Antidependencies - read/write

  ```
  y := x + 1;
  x := 4;
  ```

```
x := 4
y := 6
p := x + 2
z := y + p
x := z
y := p
```





Global Data Flow Analysis

Collecting information about the way data is used in a program.

- Takes control flow into account
- HL control constructs
  - Simpler – syntax driven
  - Useful for data flow analysis of source code
- General control constructs – arbitrary branching

Information needed for optimizations such as: constant propagation, common sub-expressions, partial redundancy elimination …

Dataflow Analysis: Iterative Techniques

- First, compute local (block level) information.
- Iterate until no changes

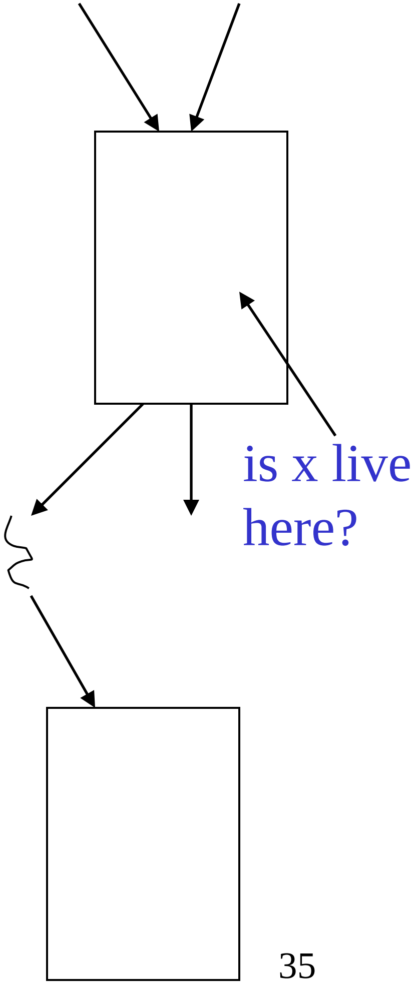
```
while change do
    change = false
    for each basic block
        apply equations updating IN and OUT
        if either IN or OUT changes, set change to true
end
```

Live Variable Analysis

A variable **x** is *live* at a point p if there is some path from p where **x** is used before it is defined.

Want to determine for some variable x and point p whether the value of x *could* be used along some path starting at p.

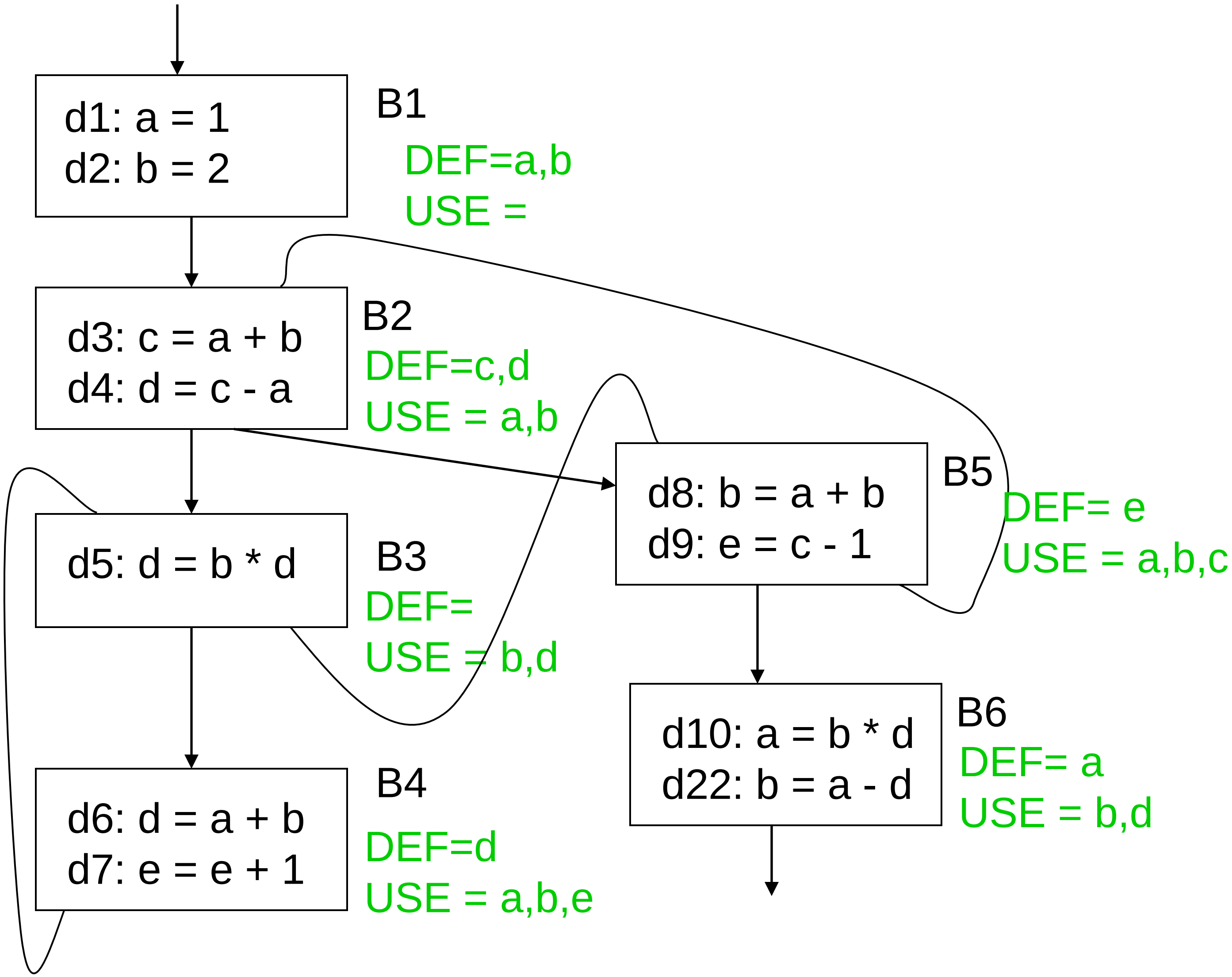
- Information flows backwards
- May – ‘along some path starting at p’



Global Live Variable Analysis

Want to determine for some variable *x* and point *p* whether the value of *x* could be used along some path starting at *p*.

- DEF[B] - set of variables assigned values in B prior to any use of that variable
- USE[B] - set of variables used in B prior to any definition of that variable
- OUT[B] - variables live immediately after the block
  OUT[B] - ∪IN[S] for all S in succ(B)
- IN[B] - variables live immediately before the block
  IN[B] = USE[B] + (OUT[B] - DEF[B])



d1: a = 1
d2: b = 2

B1

DEF=a,b
USE =

d3: c = a + b
d4: d = c - a

B2

DEF=c,d
USE = a,b

d5: d = b * d

B3

DEF=
USE = b,d

d6: d = a + b
d7: e = e + 1

B4

DEF=d
USE = a,b,e

d8: b = a + b
d9: e = c - 1

B5

DEF= e
USE = a,b,c

d10: a = b * d
d22: b = a - d

B6

DEF= a
USE = b,d

Global Live Variable Analysis

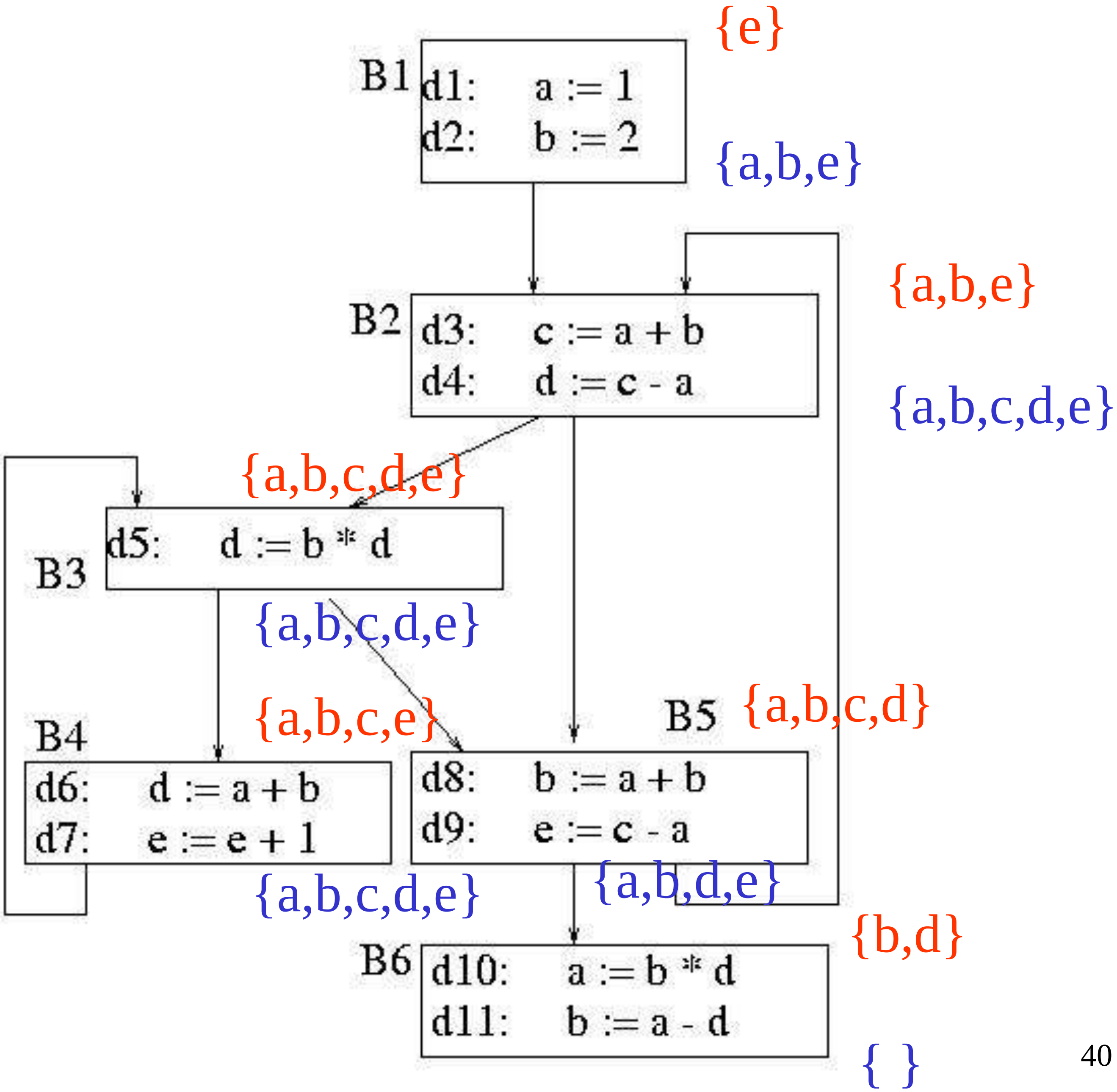
Want to determine for some variable *x* and point *p* whether the value of *x* could be used along some path starting at *p*.

- DEF[B] - set of variables assigned values in B prior to any use of that variable
- USE[B] - set of variables used in B prior to any definition of that variable
- OUT[B] - variables live immediately after the block
  OUT[B] - ∪ IN[S] for all S in succ(B)
- IN[B] - variables live immediately before the block
  IN[B] = USE[B] ∪ (OUT[B] - DEF[B])

	IN	OUT	IN	OUT	IN	OUT
B1	∅	a,b	∅	a,b	e	a,b,e
B2	a,b	a,b,c,d	a,b,e	a,b,c,d ,e	a,b,e	a,b,c,d,e
B3	a,b,c,d e	a,b,c,e	a,b,c,d,e	a,b,c,d,e	a,b,c,d,e	a,b,c,d,e
B4	a,b,c,e	a,b,c,d,e	a,b,c,e	a,b,c,d,e	a,b,c,e	a,b,c,d,e
B5	a,b,c,d	a,b,d	a,b,c,d	a,b,d,e	a,b,c,d	a,b,d,e
B6	b,d	∅	b,d	∅	b,d	∅

OUT[B] = ∪ IN[S] for all S in succ(B)
IN[B] = USE[B] + (OUT[B] - DEF[B])

Block	DEF	USE
B1	{a,b}	{ }
B2	{c,d}	{a,b}
B3	{ }	{b,d}
B4	{d}	{a,b,e}
B5	{e}	{a,b,c}
B6	{a}	{b,d}

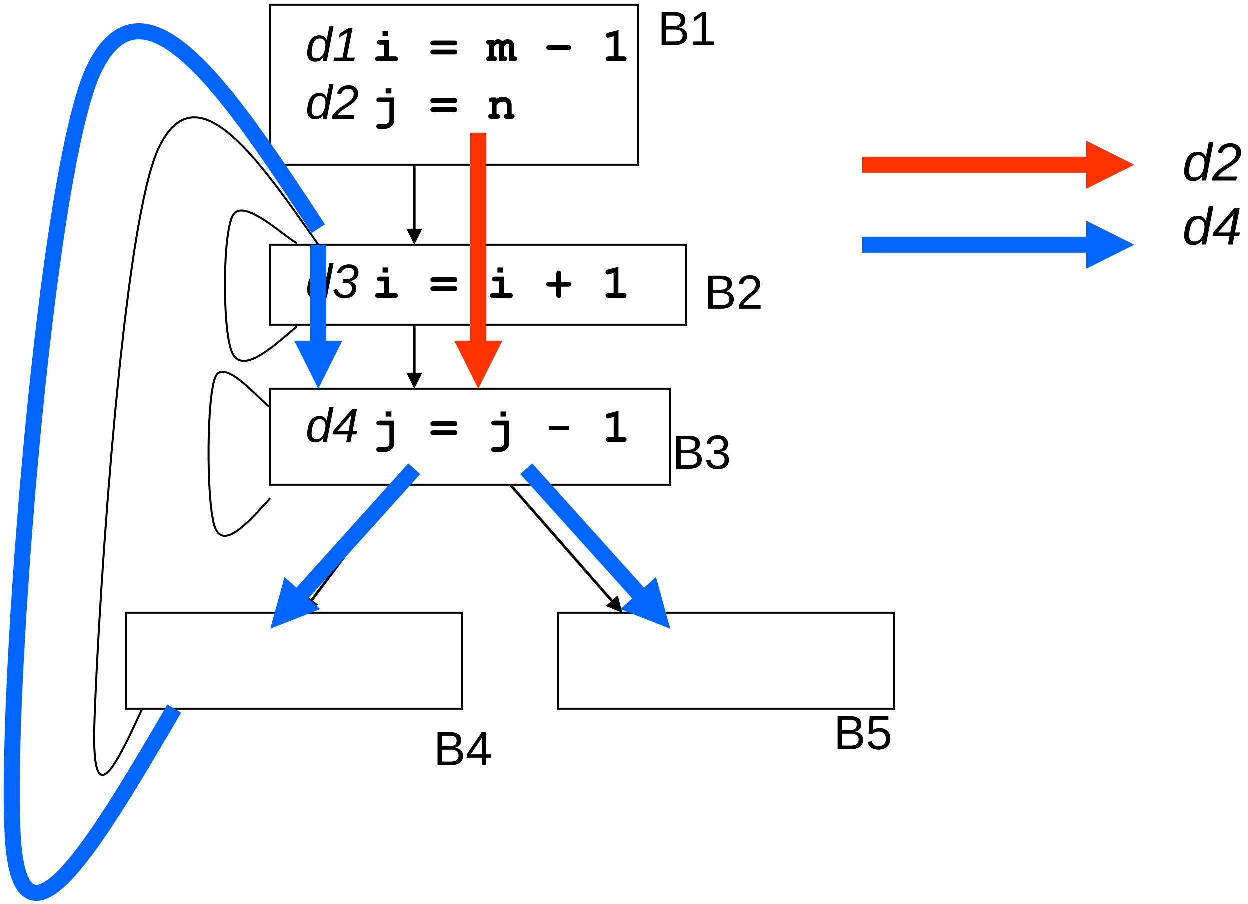


{e}
B1
d1: a := 1
d2: b := 2
{a,b,e}
{a,b,e}
B2
d3: c := a + b
d4: d := c - a
{a,b,c,d,e}
{a,b,c,d,e}
d5: d := b * d
B3
{a,b,c,d,e}
{a,b,c,e}
B4
d6: d := a + b
d7: e := e + 1
{a,b,c,d,e}
B5
{a,b,c,d}
d8: b := a + b
d9: e := c - a
{a,b,d,e}
{b,d}
B6
d10: a := b * d
d11: b := a - d
{ }

Dataflow Analysis Problem #2: Reachability

- A *definition of a variable x* is a statement that may assign a value to *x*.
- A definition may reach a program point *p* if there exists some path from the point immediately following the definition to *p* such that the assignment is not killed along that path.
- Concept: relationship between definitions and uses

What blocks do definitions d2 and d4 reach?



d1 `i = m - 1`
d2 `j = n`

d3 `i = i + 1`

d4 `j = j - 1`

Reachability Analysis: Unstructured Input

1. Compute GEN and KILL at block—level
2. Compute IN[B] and OUT[B] for B

   IN[B] = U OUT[P]   *where P is a predecessor of B*

   OUT[B] = GEN[B] U (IN[B] - KILL[B])

3. Repeat step 2 until there are no changes to OUT sets

Reachability Analysis: Step 1

For each block, compute local (block level) information = GEN/KILL sets

- GEN[B] = set of definitions generated by B
- KILL[B] = set of definitions that can not reach the end of B

This information does not take control flow between blocks into account.

Reasoning about Basic Blocks

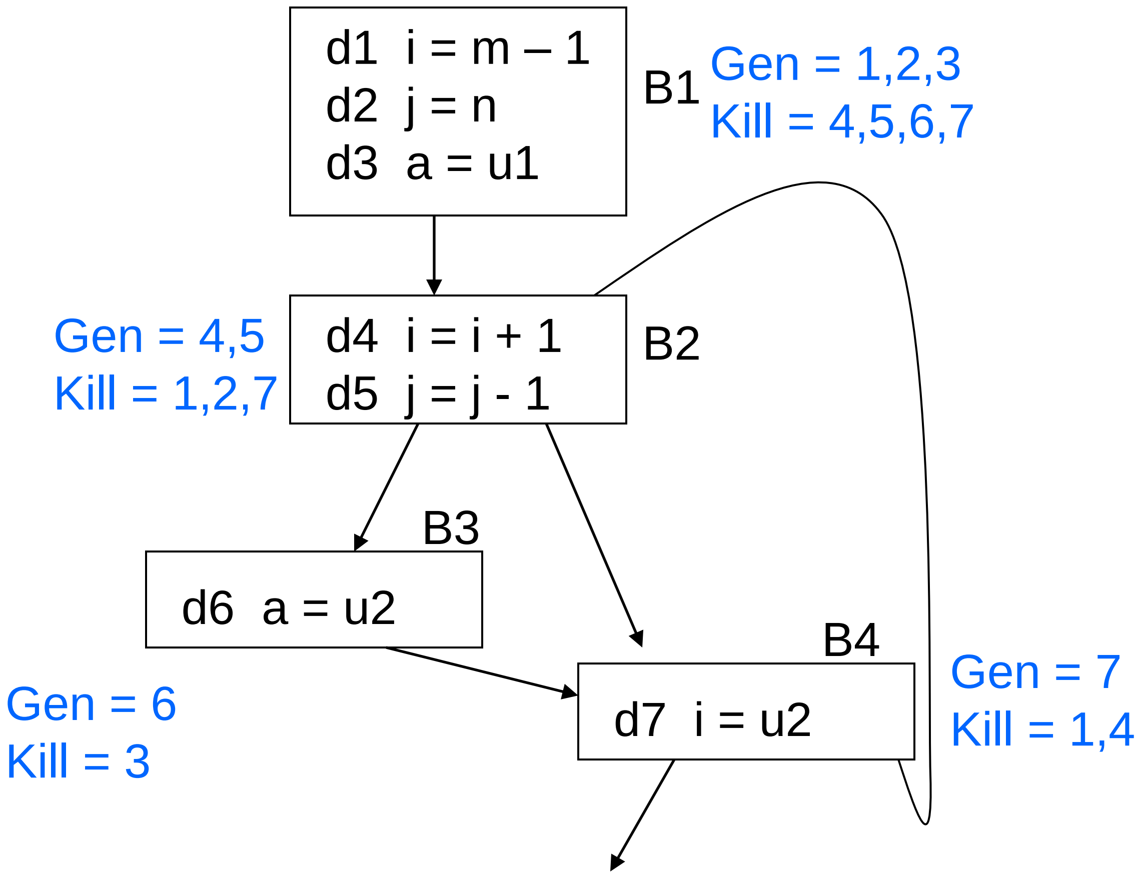
Effect of single statement: a = b + c

- Uses variables {b,c}
- **Kills all definitions of {a}**
- **Generates new definition (i.e. assigns a value) of {a}**

Local Analysis:

- Analyze the effect of each instruction
- Compose these effects to derive information about the entire block

Example


d1 i = m – 1
d2 j = n
d3 a = u1
B1
Gen = 1,2,3
Kill = 4,5,6,7
Gen = 4,5
Kill = 1,2,7
d4 i = i + 1
d5 j = j - 1
B2
B3
d6 a = u2
B4
d7 i = u2
Gen = 7
Kill = 1,4
Gen = 6
Kill = 3

Reachability Analysis: Step 2

Compute IN/OUT for each block in a forward direction. Start with IN[B] = Ø

- IN[B] = *set of defns reaching the start of B*

  = ∪ (out[P]) for all predecessor blocks in the CFG
- OUT[B] = *set of defns reaching the end of B*

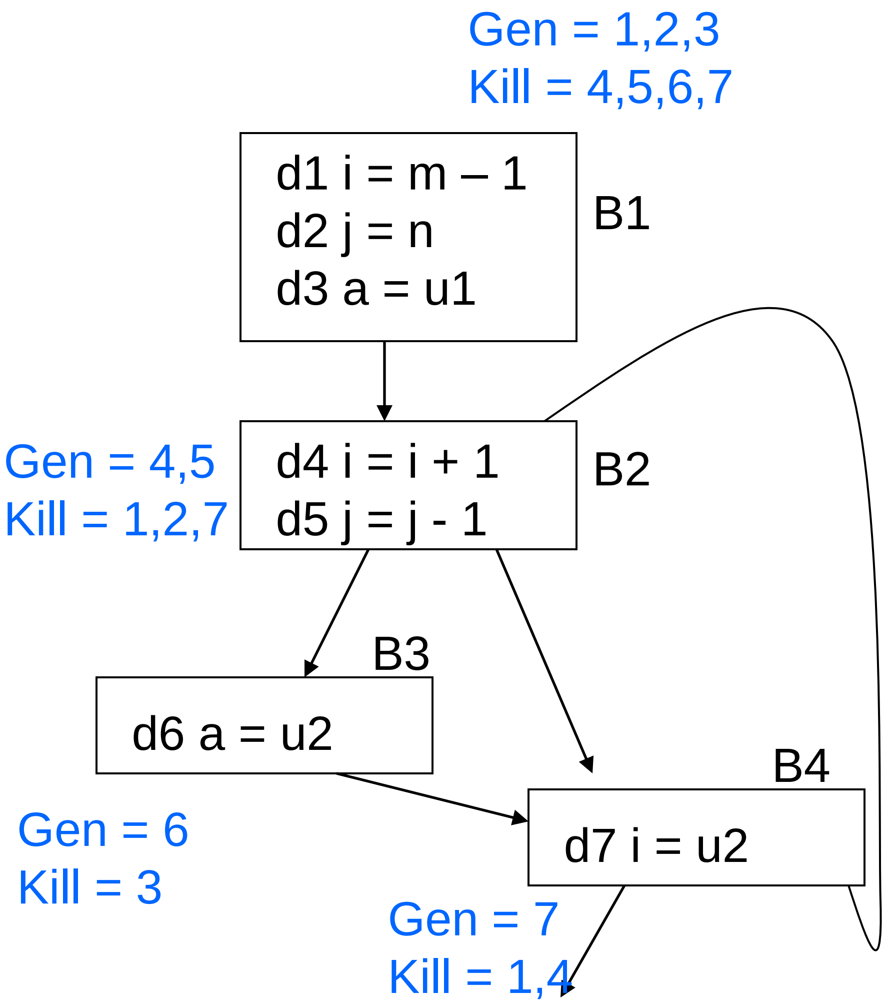
  = GEN[B] ∪ (IN[B] – KILL[B])

Keep computing IN/OUT sets until a fixed point is reached.

Reaching Definitions Algorithm

- Input: Flow graph with GEN and KILL for each block
- Output: in[B] and out[B] for each block.

```
For each block B do out[B] = gen[B], (true if in[B] = emptyset)
change := true;
while change do begin
          change := false;
          for each block B do begin
            in[B] := U out[P], where P is a predecessor of B;
            oldout = out[B];
            out[B] := gen[B] U  (in[B] - kill [B])
            if out[B] != oldout then change := true;
     end
end
```



Gen = 1,2,3
Kill = 4,5,6,7

B1
d1 i = m – 1
d2 j = n
d3 a = u1

B2
d4 i = i + 1
d5 j = j - 1

Gen = 4,5
Kill = 1,2,7

B3
d6 a = u2

Gen = 6
Kill = 3

B4
d7 i = u2

Gen = 7
Kill = 1,4

	IN	OUT		
B1	∅	1,2,3		
B2	∅	4,5		
B3	∅	6		
B4	∅	7		

IN[B] = ∪(out[P]) for all predecessor blocks in the CFG
OUT[B] = GEN[B] ∪ (IN[B] – KILL[B])

	IN	OUT	IN	OUT		
B1	Ø	1,2,3	Ø	1,2,3		
B2	Ø	4,5	OUT[1]+OUT[4] = 1,2,3,7	4,5 + (1,2,3,7 – 1,2,7) = 3,4,5		
B3	Ø	6	OUT[2] = 3,4,5	6 + (3,4,5 – 3) = 4,5,6		
B4	Ø	7	OUT[2]+OUT[3] = 3,4,5,6	7 + (3,4,5,6 – 1,4) = 3,5,6,7		

IN[B] = ∪(out[P]) for all predecessor blocks in the CFG

OUT[B] = GEN[B] + (IN[B] – KILL[B])

	IN	OUT	IN	OUT	IN	OUT
B1	∅	1,2,3	∅	1,2,3	∅	1,2,3
B2	∅	4,5	1,2,3,7	3,4,5	OUT[1] + OUT[4] = 1,2,3,5,6,7	4,5 + (1,2,3,5,6,7-1,2,7) = 3,4,5,6
B3	∅	6	3,4,5	4,5,6	OUT[2] = 3,4,5,6	6 + (3,4,5,6 – 3) = 4,5,6
B4	∅	7	3,4,5,6	3,5,6,7	OUT[2] + OUT[3] = 3,4,5,6	7+(3,4,5,6 – 1,4) = 3,5,6,7

IN[B] = ∪(out[P]) for all predecessor blocks in the CFG
OUT[B] = GEN[B] + (IN[B] – KILL[B])

Forward vs. Backward

Forward flow vs. Backward flow

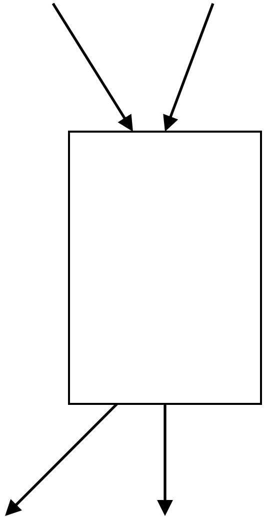
Forward: Compute OUT for given IN,GEN,KILL

- Information propagates from the predecessors of a vertex.
- Examples: Reachability, available expressions, constant propagation

Backward: Compute IN for given OUT,GEN,KILL

- Information propagates from the successors of a vertex.
- Example: Live variable Analysis

Forward vs. Backward Equations



Forward vs. backward

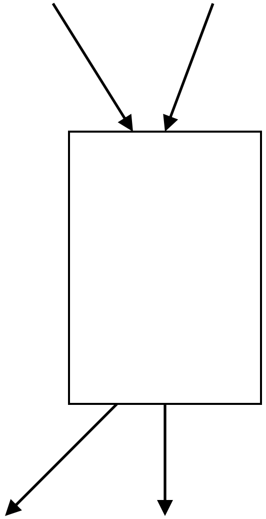
- Forward:
  - IN[B] - process OUT[P] for all P in predecessors(B)
  - OUT[B] = local U (IN[B] – local)
- Backward:
  - OUT[B] - process IN[S] for all S in successor(B)
  - IN[B] = local U (OUT[B] – local)

May vs. Must

- May vs. Must
  - Must – true on **all paths**
    - Ex: constant propagation – variable must provably hold appropriate constant on all paths in order to do a substitution
  - May – true on **some path**
    - Ex: Live variable analysis – a variable is live if it could be used on some path; reachability – a definition reaches a point if it can reach it on some path

May vs. Must Equations

- May vs. Must
  - May – IN[B] = ∪(out[P]) for all P in pred(B)
  - Must – IN[B] = ∩(out[P]) for all P in pred(B)



- Reachability
  - IN[B] = $\cup$(out[P]) for all P in pred(B)
  - OUT[B] = GEN[B] + (IN[B] – KILL[B])
- Live Variable Analysis
  - OUT[B] = $\cup$(IN[S]) for all S in succ(B)
  - IN[B] = USE[B] $\cup$ (OUT[B] - DEF[B])
- Constant Propagation
  - IN[B] = $\cap$(out[P]) for all P in pred(B)
  - OUT[B] = DEF_CONST[B] $\cup$ (IN[B] – KILL_CONST[B])

Discussion

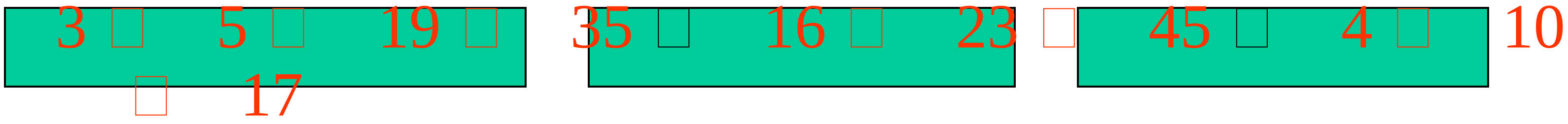
- Why does this work?
  - Finite set – can be represented as bit vectors
  - Theory of lattices
- Is this guaranteed to terminate?
  - Sets only grow and since finite in size …
- Can we find ways to reduce the number of iterations?

Choosing visit order for Dataflow Analysis

In forward flow analysis situations, if we visit the blocks in depth first order, we can reduce the number of iterations.

Suppose definition d follows block path 3 □ 5 □ 19 □ 35 □ 16 □ 23 □ 45 □ 4 □ 10 □ 17 where the block numbering corresponds to the preorder depth-first numbering.

Then we can compute the reach of this definition in 3 iterations of our algorithm.



3 □ 5 □ 19 □ 35 □ 16 □ 23 □ 45 □ 4 □ 10 □ 17