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Coding Theory

$r = c + e$

r = recieved ~~bit~~ word c = coding word e = error word

- When we know the position of error $p^k (1-p)^{n-k}$
- When we know the no. of errors $({}^nc_k)(p^k)(1-p)^{n-k}$

① c = 1010110 e = 0101101

$$\begin{array}{rl} & 1010110 \\ (\text{XOR}) & 0101101 \\ \hline r = & 1111011 \end{array}$$

Compare c & r, To get k

k = 4 errors

$p^k (1-p)^{n-k}$ p = 0.05

$(0.05)^4 (1-0.05)^3$

= 7.289 x 10^{-6}

= 0.000007289

② c = 1010110 p = 0.02 r = 1011111

$$\begin{array}{rl} c = & 1010110 \\ r = & 1011111 \\ \hline e = & 0001001 \end{array}$$

K = 2 errors

To get e, check the changes in c & r, whichever is changed will be marked 1

= $(0.02)^2 (1-0.02)^{7-2}$

= 0.00036

③ p = 0.05 c = 011011101 k = 1

n = 9 $\binom{9}{1}(0.05)(1-0.05)^8$

0.2985

111011100

111011110

~~[illegible]~~ Encode

Decode

$D(x) = D(x_1 x_2 \ldots x_m x_{m+1} \ldots x_{2m}, x_{2m+1} \ldots x_{3m})$

$\cdot\ S_1 S_2 \ldots S_m$

$S_i = \begin{cases} 1 & \text{if } x_i\, x_{i+m}\, x_{i+2m} \text{ has majority of 1's} \\ 0 & \text{has majority of 0's} \end{cases}$

1) E(00) E(01) = 00000 11111 = 5

E(00) E(10) = 5

E(00) E(11) = 10

E(01) (10) = 10

E(01) (11) = 5

E(10) (11) = 5

min = 5

ED = 4

EC = 2

1) $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Decode

$H = \begin{bmatrix} 1+0+1+1+0+0 \\ 1\ 1\ 0\ 0\ 0\ 0 \\ 0\ 1\ 1\ 0\ 0\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

3 x 3   6 x 1

110

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

1+0+1+1+0+0

1 1 0 0 0 0

0 1 1 0 0 1

= 1+0+0+0
1+1+0+1+0
0+1+0+0+1

3   1

1 1 1 1 0 1 1 0 0

C + E

S_1 = 11 41 71 = 1

S_2 = 21 50 80 = 0

S_3 = 31 61 09 = 1

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Date: / /

Coding Theory

① $c = 10110$ Z_2^5 $(Z_2, +)$

$p = 0.05$

$r = 00110$

$r = c + e$

$r - c = e$

$$\begin{array}{r} 00110 \\ 10110 \\ \hline 10000 \end{array}$$

$k = 1$

$= p^k (1-p)^{n-k}$

$= (0.05)(1-0.05)^4$

$=$

② $c = 10110$ Z_2^5 $p = 0.05$

i) 2 differ

$\binom{n}{k} p^k (1-p)^{n-k}$

$= \binom{5}{2} (0.05)^2 (1-0.05)^3$

ii) atmost 2

$\binom{5}{0}(0.05)^0(1-0.05)^5 + \binom{5}{1}(0.05)^1(1-0.05)^4 + \binom{5}{2}(0.05)^2(1-0.\ldots$

iii) atleast 3

$\binom{5}{3}(0.05)^3(1-0.05)^2 + \binom{5}{4}(0.05)^4(1-0.05)^1 + \binom{5}{5}(0.05)^5$

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Date: / /

6.15.7

3)

~~(9 → 3)~~ $Z_2^3 \longrightarrow Z_2^9$

a)

1) 111101100

$S_1 = 1_1 \quad 1_4 \quad 1_7 \quad = 1$

$S_2 = 1_2 \quad 0_5 \quad 0_8 \quad = 0$

$S_3 = 1_3 \quad 1_6 \quad 0_9 \quad = 1$

2) 000100011

$S_1 = 0 \quad 1 \quad 0 \quad = 0$

$S_2 = 0 \quad 0 \quad 1 \quad = 0$

$S_3 = 0 \quad 0 \quad 1 \quad = 0$

3) 010011111

$S_1 = 0 \quad 0 \quad 1 \quad = 0$

$S_2 = 1 \quad 1 \quad 1 \quad = 1$

$S_3 = 0 \quad 1 \quad 1 \quad = 1$

$3 \xrightarrow{E} 9$

b) $D(r) = 000$

$S_1 = 0 \qquad S_2 = 0 \qquad S_3 = 0$

$\frac{0}{1} \frac{1}{2} \frac{1}{3} \frac{1}{1} \frac{0}{2} \frac{0}{3} \frac{0}{1} \frac{0}{2} \frac{0}{3}$

4. ~~E(W) = WWWWW~~

0 1 1 1 0 1

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Date: / /

$$\begin{array}{r} 010101 \\ 000001 \\ \hline 010100 \end{array}$$

1) $Z_2^2 \longrightarrow Z_2^6$

$E(00) = 000000$ min = 3

$E(10) = 101010$

$E(01) = 010101$

$E(11) = 111111$

Error detection = min − 1 = 3 − 1 = 2

Error correction = $\frac{min-1}{2} = \frac{3-1}{2} = 1$

Ex

2

1) $S(101010, 1)$

$\{101010, 101011, 101000, 101110, 100010, 111010, 001010\}$

2) a) 110101 2 → 6 b) 101011

$S_1 = 1 \quad 0 = 1 \quad 0$ $S_1 = 1 \quad 1 = 1$

$S_2 = 1 \quad 1 = 1 \quad 1$ $S_2 = 0 \quad 0 \quad 0$

3) ~~$|S(x,1)|$~~

a) $\sum_{i=0}^{k} \binom{n}{i}$ n = total bits length, k = errors

n = 10

$|S(x,1)|$ $\binom{10}{0} + \binom{10}{1} = 1 + 10 = 11$

$|S(x,2)|$ $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} = 56$

$|S(x,3)|$ $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} = 176$

b) $\sum_{i=0}^{k} \binom{n}{i}$

4) $Z^5 \to Z^{25}$ min = 9

ED = 8

EC = 4

Date: / /

⑤ a) $E: Z^2 \rightarrow Z^5$

$00 \rightarrow 00001$ $\quad$ $01 \rightarrow 01010$

$10 \rightarrow 10100$ $\quad$ $11 \rightarrow 11111$

$E(00)\ E(10) = 10101 = 3$

$E(00)\ E(01) = 01011 = 3$ $\qquad$ min = 3

$E(00)\ E(11) = 11110 = 4$ $\qquad$ ED = 2

$E(10)\ E(01) = 11110 = 4$ $\qquad$ EC = 1

$E(10)\ E(11) = 01011 = 3$

$E(01)\ E(11) = 10101 = 3$

⑥ a)

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

i) 111101

$$H \times r^T \quad \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+0+1+1+0+0 \\ 1+1+0+0+0+0 \\ 0+1+1+0+0+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

1101 $\qquad$ 110

viii)

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0+0+0+1+0+0 \\ 0+1+0+0+0+0 \\ 0+1+0+0+0+0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

~~Ans~~ 111 is not among the columns of H

If 111 = 110 + 001 then 110101 $\quad$ D(c) = 110

111 = 001 + 100 then ~~0110~~ 000000 $\quad$ D(c) = 000

111 = 101 + 010 then 011110 $\quad$ D(c) = 011

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Date: / /

v) $\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{matrix} 1+0+0+0+0+0 \\ 1+1+0 \\ 0+1+0+0+0+1 \end{matrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$= 110$

7. $G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$ I_2 A

$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ A^T

$E(00) = \underset{1\times 2}{[0\,0]} \underset{2\times 5}{\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}} = \underset{1\times 5}{[0\,0\,0\,0\,0]}$

$E(01) = [0\,1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [0\,1\,0\,1\,1]$

$E(10) = [1\,0] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [1\,0\,1\,1\,0]$

$E(11) = [1\,1] \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} = [1\,1\,1\,0\,1]$