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$A_1 = 1, 2, 3$ $A_1 \cap A_2 = 1$

$A_2 = -1, -2, 1$

Theorems

1) let $f: A \to B$ with $A_1, A_2 \subseteq A$ then

b) $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

$\to$ For each $b \in B$,

$b \in f(A_1 \cap A_2)$

$f(A_1 \cap A_2) \Rightarrow b$ $\quad b \Rightarrow f(a)$ for some $a \in A_1 \cap A_2$

$b = f(a)$ for some $a \in A_1$

$b = f(a)$ for some $a \in A_2$

$b \in f(A_1)$ and $b \in f(A_2) \Rightarrow b \in f(A_1) \cap f(A_2)$

~~$b \in f(A_1) \cap f(A_2)$~~

So, $f(A_1 \cap A_2) \subseteq f(A_1) \cap f(A_2)$

$A_1 = 1, 2, 3$ $\quad A_2 = -1, -2, 1$

a) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

$\to$ For each $x \in B$

$x \in f(A_1 \cup A_2)$

$f(A_1 \cup A_2) \Rightarrow x$ $\quad$ ~~$x_2, x_1 \in x$~~

$x = f(a)$ for some $a \in A_1$ or $a \in A_2$ ~~$x_1 = f(A_1)$ for each $x_1 \in f(A_1)$~~

~~$x_2 = f(A_2)$ for each $x_2 \in f(A_2)$~~

$x \in f(A_1)$ or $x \in f(A_2) \Rightarrow x \in f(A_1) \cup f(A_2)$

So, $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

c) ~~let~~ $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ when f is one to one

$\to$ ~~For each $x \in B$, $x \in f(A_1 \cap A_2)$, $x = f(A_1 \cap A_2)$~~

~~$x = f(a)$ for some $a \in A_1 \cap A_2$~~

~~$x = f(a)$ for some $a_1 \in A_1$~~

~~$x = f(a_2)$ for some $a_2 \in A_1$~~

~~$a_1 = a_2$ are equal bcoz one to one~~

$y \in f(A_1) \cap f(A_2)$

~~$y = f(x)$ for some x from $A_1 \cap A_2$~~

$y = f(x_1)$ for some x_1 from A_1

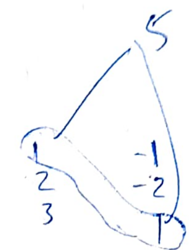
$y = f(x_2)$ for some $x_2 \in A_2$

$x_1 = x_2$ to satisfy one to one function

$y = f(x_1)$

$y \in f(A_1 \cap A_2)$

So, $f(A_1) \cap f(A_2) = f(A_1 \cap A_2)$



Exc 5.9

(14) a) $a_2 = 2a_{\lfloor 2/2 \rfloor} = 2a_1 = 2 \times 1 = 2.$

$a_3 = 2a_{\lfloor 3/2 \rfloor} = 2a_1 = 2$

$a_4 = 2a_{\lfloor 4/2 \rfloor} = 2a_2 = 2 \times 2 = 4$

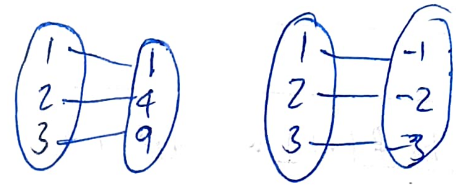
$a_5 = 2a_{\lfloor 5/2 \rfloor} = 2a_2 = 4$

$a_6 = 2a_3 = 4$

$a_7 = 2a_3 = 4$

$a_8 = 2a_4 = 8$

(b8)

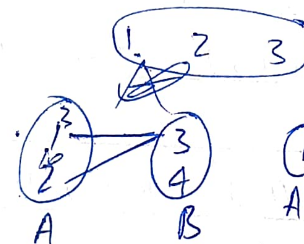


$\neq$

$\subseteq$

$f(A_1 \cap A_2)$

$\subseteq$



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A_1 A_2

$f(A_1 \cap A_2) = \emptyset$

$f(A_1) = 3$ $f(A_2) = 3$ $\neq$

$3 \cap 3 = \{3\}$.

(23) a) $a_{ij} = (i-1)12 + j$ b) $a_{ij} = (i-1)10 + j$

2) Let $f: A \to B$ & $g: B \to C$

a) If f & g are one to one then $g \circ f$ is one to one

Let $a_1, a_2 \in A$ $g \circ f : A \to C$

WRT $g \circ f(a_1) = g \circ f(a_2)$

$g(f(a_1)) = g(f(a_2))$

Since g is one to one $f(a_1) = f(a_2)$

f is one to one $a_1 = a_2$

Implies $g \circ f$ is one to one

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b) If f & g are onto, then $g \circ f$ is onto

→ $g \circ f : A \to C$ $\qquad z \in C$

Since g is onto $y \in B$ maps to z

$g(y) = z$

Since f is onto & $y \in B$, there exists some $x \in A$ that maps to y

$f(x) = y$

Since $z = g(y) = g(f(x)) = (g \circ f)(x)$

So, the range of $g \circ f$ $= C =$ Codomain $g \circ f$

$\therefore$ $g \circ f$ is onto.

3) If $f : A \to B$, $g : B \to C$, $h : C \to D$ then $(h \circ g) \circ f = h \circ (g \circ f)$

$LHS = ((h \circ g) \circ f)(x) = \cancel{h(g(f(x)))}\ (h \circ g)[f(x)]$ $\qquad x \in A$

$= (h \circ g)(y)$

$= h[g(y)]$

$= h(z) = s$

$RHS = (h \circ (g \circ f))(x) = h \circ (g \circ f(x)) = h \circ [g[f(x)]] = h \circ [g(y)] = h(z) = s$

$LHS = RHS$ proved

Since $A \to B$, $B \to C$ $\therefore$ $A \to C$

$A \to C$, $C \to D$ $\therefore$ $A \to D$

There is a function from $A \to B$ & $B \to C$. This implies that $A \to C$ has function $g \circ f$

4) If a $f: A \to B$ is invertible & $g: B \to A$ satisfies $g \circ f = 1_A$ & $f \circ g = 1_B$

then this function g is unique.

→ If g is not ~~function~~ unique

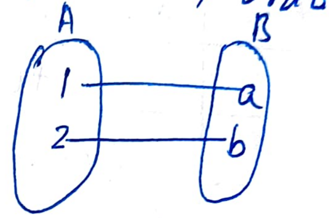
there is other function $h: B \to A$ with $h \circ f = 1_A$ and $\overset{f \circ h}{\cancel{f \circ g}} = 1_B$

$\therefore\ h = h \circ 1_B = h \circ (f \circ g) = (h \circ f) \circ g = 1_A \circ g = g$

$h = g$

$\therefore$ g is unique

5) A $f: A \to B$ is invertible if and only if it is one to one & onto



→ ~~Assuming $f: A \to B$ is invertible~~

$g: B \to A$ with $g \circ f = 1_A$, $f \circ g = 1_B$

Assuming $f: A \to B$ is invertible.

$g: B \to A$ with $g \circ f = 1_A$, $f \circ g = 1_B$

If $a_1, a_2 \in A$ with $f(a_1) = f(a_2)$

$$g(f(a_1)) = g(f(a_2)) \Rightarrow (g \circ f)(a_1) = (g \circ f)(a_2)$$

Since $g \circ f = 1_A$ it says that $a_1 = a_2$, so f is one to one

$b \in B$, $\cancel{(g \circ f)(b)}$ $b = 1_B(b)$

$= f \circ g\,(b)$

f is onto

$g(f(x))$

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6) If $f: A \to B$, $g: B \to C$ are invertible functions, then $g \circ f:$ $A \to C$ is invertible & $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

→ $f, g \to$ invertible functions

$f, g \to$ Both one to one & onto

$g \circ f$ is one to one & onto $=$ $g \circ f$ invertible

$(g \circ f) \circ (f^{-1} \circ g^{-1}) = 1_C$

$\cancel{(g \circ f) \circ (}$ $(f^{-1} \circ g^{-1}) \circ (g \circ f) = 1_A$

$$f^{-1} \circ g^{-1} = \frac{1}{(g \circ f)}$$

$$f^{-1} \circ g^{-1} = (g \circ f)^{-1}$$

7) If $f: A \to B$ & $B_1 B_2 \subseteq B$ then

(a) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

→ $a \in f^{-1}(B_1 \cap B_2)$

$f(a) \in B_1 \cap B_2$

$f(a) \in B_1$ and $f(a) \in B_2$

$a \in f^{-1}(B_1)$ and $a \in f^{-1}(B_2)$

$a \in f^{-1}(B_1)$ and $f^{-1}(B_2)$

$f^{-1}(B_1) \cap f^{-1}(B_2)$

b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

→ $a \in A$, $a \in f^{-1}(B_1 \cup B_2)$

$f(a) \in B_1 \cup B_2$

$f(a) \in B_1$ or $f(a) \in B_2$

$a \in f^{-1}(B_1)$ or $f^{-1}(B_2)$

$\cup$

(c) $f^{-1}(\overline{B_1}) = \overline{f^{-1}(B_1)}$.

$\rightarrow$ $a \in f^{-1}(\overline{B_1})$

$f(a) \in \overline{B_1}$ $\quad$ $f(a) \notin B_1$

$a \notin f^{-1}(B_1)$

$a \in \overline{f^{-1}(B_1)}$

8) let $f: A \to B$ for finite sets A & B, where $|A| = |B|$. following are equivalent

(a) f is one to one

Assuming f is not one to one

then elements $a_1, a_2 \in A$

$a_1 \neq a_2$ but $f(a_1) = f(a_2)$

$|A| > |f(A)| = |B|$

$\hookrightarrow$ This is contradicting given statement $|A| = |B|$

So, f is one to one

(b) f is onto

Assuming f is not onto

$|f(A)| < |B|$

Functions Formula

No. of functions = $|Codomain|^{|Domain|}$

No. of one to one functions = $\frac{n!}{(n-m)!}$ or $P(n,m)$

$n = |codomain|$
$m = |domain|$

No. of onto functions = $\sum_{k=0}^{n} (-1)^k \binom{n}{n-k} (n-k)^m$

Stirling No. = $n! \, S(m,n)$

$$S(m,n) = \frac{1}{n!} \sum_{k=0}^{n} (-1)^k \binom{n}{n-k} (n-k)^m$$

or

$$S(m,n) = S(m-1, n-1) + n\,S(m-1, n)$$