Discrete Maths

Unit-2 = Fundamental of logic

Statements / Propositions

A declarative sentence that is either true or false (but not both)

Exclamation and commands are not considered as statements

Types : Primitive or Simple
Compound

$x = x+3$ = This is predection

Negation = $\neg p$ or $\sim p$

Connectives	Symbols
Conjunction	$\wedge$ (and)
Disjunction	$\vee$ (or)
Implication	$\rightarrow$ (if p then q)
Biconditional	$\leftrightarrow$ (p iff q)
Exclusive or	$\underline{\vee}$

p, q are statements

iff = if and only if

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$	$\neg p$
T	T	T	T	F	T	T	F
T	F	F	T	T	F	F	F
F	T	F	T	T	T	F	T
F	F	F	F	F	T	T	T

Implication - Equivalent forms

If p then q
p implies q
If p, q
q if p
($p \rightarrow q$)

q whenever p
p is sufficient condition for q
q is necessary condition for p
p only if q

Problems

1) $(p \wedge q) \rightarrow p$

p	q	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

A compound statement is T_0, if all values are true.

Tautology T_0

2) $\neg(p \vee \neg q) \rightarrow \neg p$

p	q	$\neg p$	$\neg q$	$(p \vee \neg q)$	$\neg(p \vee \neg q)$	$\neg(p \vee \neg q) \rightarrow \neg p$
T	T	F	F	T	F	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	T	F	T

Tautology

3) $q \leftrightarrow (\neg p \vee \neg q)$

p	q	$\neg p$	$\neg q$	$(\neg p \vee \neg q)$	$q \leftrightarrow (\neg p \vee \neg q)$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	T	F
F	F	T	T	T	F

If a compound statement is false for all assignments then it is called F_0

Contradiction

4) $p \wedge (\neg p \wedge q)$

p	q	$\neg p$	$(\neg p \wedge q)$	$p \wedge (\neg p \wedge q)$
T	T	F	F	F
T	F	F	F	F
F	T	T	T	F
F	F	T	F	F

Contradiction

If a compound statement produces combination of false and true then it is called Absurd

Logical Equivalent

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\equiv$

$[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$. Verify if this is T_0

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$p \to r$	$[(p \to q) \to (p \to r)]$	Full
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Problem

If statement q has the truth value 1, determine all truth value assignments for the primitive statements, p, r, s for which the truth value of the statement.

$$(q \rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \rightarrow (\neg r \wedge q)] \text{ is } 1.$$

$q = T/1$ $\quad$ $p = 0/F$ $\quad$ $r = F/0$ $\quad$ $s = 0/F$

② Logical Equivalent : $(p \leftrightarrow q) \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ ✓

p	q	$p \leftrightarrow q$	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

③ $(p \leftrightarrow q) \equiv (\neg p \vee q) \wedge (\neg q \vee p)$ ✓

p	q	$p \leftrightarrow q$	$\neg p \vee q$	$\neg q \vee p$	$(\neg p \vee q) \wedge (\neg q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

④ $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$ ✓

p	q	r	$(q \wedge r)$	$p \rightarrow (q \wedge r)$	$(p \rightarrow q)$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	

F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

Laws of logic

Law of Double Negation $\neg\neg p \Leftrightarrow p$

DeMorgan's law $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

Commutative law $p \vee q \Leftrightarrow q \vee p$

$p \wedge q \Leftrightarrow q \wedge p$

Associative law $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$

$p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

Distributive law $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Identity law $p \wedge T_0 \Leftrightarrow p$

$p \vee F_0 \Leftrightarrow p$

Idempotent law $p \wedge p \equiv p$

$q \vee q \equiv q$

Inverse law or Negation law $p \wedge \neg p \equiv F_0$

$p \vee \neg p \equiv T_0$

Domination law $p \wedge F_0 = F_0$

$p \vee T_0 = T_0$

Absorption law $p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Imp terms

Implication $p \to q$

Converse $q \to p$

Inverse $\neg p \to \neg q$

Contrapositive $\neg q \to \neg p$

$p \rightarrow q \Leftrightarrow \neg p \vee q$

$p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$

$(p \rightarrow q) \wedge (p \rightarrow r) \Leftrightarrow p \rightarrow (q \wedge r)$

$(p \rightarrow q) \vee (p \rightarrow r) \Leftrightarrow p \rightarrow (q \vee r)$

Imp
logical equivalents

Dual (s^d)

If s (statement) contains no logical connectives other than $\wedge$, $\vee$, T_0 then replace with $\vee$, $\wedge$, F_0 respectively.

$$s = (p \wedge q) \wedge (\neg p \vee T_0)$$

$$s^d = (p \vee q) \vee (\neg p \wedge F_0)$$

Pr ① $s : q \rightarrow p$

$s^d : \neg q \wedge p$

② $s : p \leftrightarrow q$

$\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$(\neg p \vee q) \wedge (\neg q \vee p)$

$(\neg p \wedge q) \vee (\neg q \wedge p)$

Solve using substitution rules

ⓐ $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$

	Reasons
$(p \vee q) \wedge \neg(\neg p \wedge q)$	
$(p \vee q) \wedge [\neg\neg p \vee \neg q]$	Demorgan's law
$(p \vee q) \wedge (p \vee \neg q)$	Double Negation law
$p \vee (q \wedge \neg q)$	Distributive law
$p \vee F_0$	Inverse law
$= p$	

(b) $\neg[\neg[(p \vee q) \wedge r)] \vee \neg q] \iff q \wedge r$

$\neg[\neg[(p \vee q) \wedge r)] \vee \neg q]$

~~$\neg[\neg[(p \wedge r) \vee (q \wedge r)] \vee \neg q]$~~

~~$\neg[(\neg(p \vee q) \vee \neg r) \vee \neg q]$~~ Demorgan's law

$[\neg\neg[(p \vee q) \wedge r] \wedge \neg\neg q]$ demorgan's law ~~law of double~~ negation

$[(p \vee q) \wedge r] \wedge q$ law of double negation

$(p \vee q) \wedge (r \wedge q)$ Associative law

$(p \vee q) \wedge (q \wedge r)$ Commutative law

$[(p \vee q) \wedge q] \wedge r$ Associative law

$q \wedge r$ Absorption law

Write the reasons for following steps

(a) $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)]$

$(p \rightarrow q) \wedge \neg q$ Commutative & Absorption law

$(\neg p \vee q) \wedge \neg q$ $p \rightarrow q \iff \neg p \vee q$

$\neg q \wedge (\neg p \vee q)$ Commutative law

$(\neg q \wedge \neg p) \vee (\neg q \wedge q)$ Distributive law

$(\neg q \wedge \neg p) \vee F_0$ Inverse law

$\neg q \wedge \neg p$ Identity law

$\neg(q \vee p)$ Demorgan's law

Negate and Express

(a) Norma is doing her homework and Karen is practing her piano lessons

$\neg(p \wedge q) \iff \neg p \vee \neg q$

Norma is not doing her homework or Karen is not practing her piano lessons.

Write the converse, inverse and contrapositive of each implications. And determine its truth table

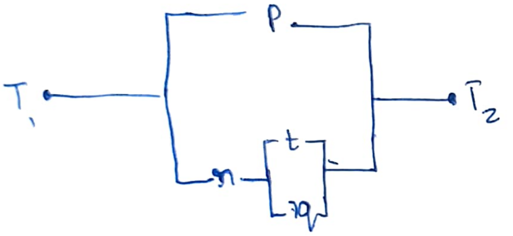
ⓐ If $-1 < 3$ and $3+7 = 10$, then $\sin\left(\frac{3\pi}{2}\right) = -1$. $(T \wedge T) \to T = T$

Sol

Converse = If $\sin\frac{3\pi}{2} = -1$, then $-1<3$ and $3+7=10$ $T \to (T \wedge T) = T$

Inverse = If $-1 \geq 3$ or $3+7 \neq 10$, then $\sin\frac{3\pi}{2} \neq -1$ $(F \vee F) \to F = T$

Contrapositive = If $\sin\frac{3\pi}{2} \neq -1$, then $-1 \geq 3$ or $3+7 \neq 10$ $F \to (F \vee F) = T$



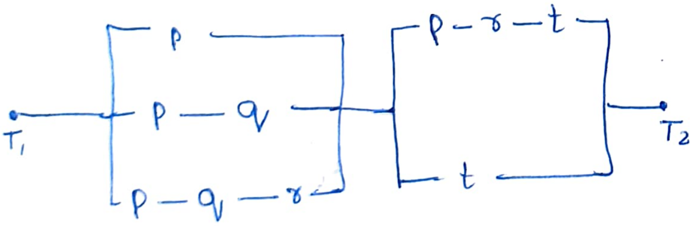
$\Rightarrow$ $p \vee (r \wedge (t \vee \neg q))$

Rule of Inference

$$\underbrace{(p_1 \wedge p_2 \wedge p_3 \wedge \dots \wedge p_n)}_{\text{premises}} \to \underbrace{q}_{\text{conclusion}} \text{ is } T_0$$

Problems

① Simplify



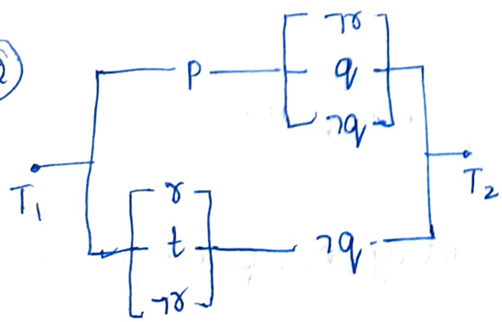
$[p \vee (p \wedge q) \vee (p \wedge q \wedge r)] \wedge [(p \wedge r \wedge t) \vee t]$

$[p \vee (p \wedge q \wedge r)] \wedge [(p \wedge r \wedge t) \vee t]$ Absorption law

$p \wedge [((p \wedge r) \wedge t) \vee t]$ —||—

$p \wedge t$ —||—

②



$[p \wedge (\neg r \vee q \vee \neg q)] \vee [(r \vee t \vee \neg r) \wedge \neg q]$

$[p \wedge (\neg r \vee T_0)] \vee [(t \vee T_0) \wedge \neg q]$ Inverse law

$(p \wedge T_0) \vee (T_0 \wedge \neg q)$ Domination law

$p \vee \neg q$ Identity law

③ $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$

$p \vee q \vee (\neg(\underline{p \vee q}) \wedge r)$ Demorgan's law

$\vee \neg p \wedge r$

$p \vee q \vee$ Absorption law

④ $[(\neg p \vee \neg q) \longrightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$

$\neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$ $p \rightarrow q \Leftrightarrow \neg p \vee q$

$(\neg\neg p \wedge \neg\neg q) \vee (p \wedge q \wedge r)$ Demorgan's law

$(p \wedge q) \vee (\underbrace{p \wedge q}_{p} \wedge \underbrace{r}_{q})$ law of double negation

Absorption law

$p \wedge q$

⑤ $(p \vee q) \longrightarrow [q \longrightarrow q]$

$p \vee q \longrightarrow (\neg q \vee q)$ $p \rightarrow q \Leftrightarrow \neg p \vee q$

$p \vee q \longrightarrow T_0$ Inverse law

$\neg(p \vee q) \vee \underbrace{T_0}_{q}$ $p \rightarrow q \Leftrightarrow \neg p \vee q$

T_0 Domination law

Rule of Inference	Related logical Implication	Name of Rule
p $p \to q$ $\therefore q$	$[p \wedge (p \to q)] \to q$	Rule of Detachment (Modus Ponens)
$p \to q$ $q \to r$ $\therefore p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Law of Syllogism
$p \to q$ $\neg q$ $\therefore \neg p$	$[(p \to q) \wedge \neg q] \to \neg p$	Modus Tollens.
p q $\therefore p \wedge q$		Rule of Conjunction
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \to q$	Rule of Disjunctive Syllogism
$\neg p \to F_0$ $\therefore p$	$(\neg p \to F_0) \to p$	Rule of Contradiction
$p \wedge q$ $\therefore p$	$(p \wedge q) \to p$	Rule of Conjunctive Simplification
p $\therefore p \vee q$	$p \to (p \vee q)$	Rule of Disjunctive Amplication
$p \wedge q$ $p \to (q \to r)$ $\therefore r$	$[(p \wedge q) \wedge [p \to (q \to r)]] \to r$	Rule of Conditional Proof
$p \to r$ $q \to r$ $\therefore (p \vee q) \to r$	$[(p \to r) \wedge (q \to r)] \to [(p \vee q) \to r]$	Rule for proof by cases

$$\begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline q \vee s \end{array}$$

Rule of Constructive Dilemma

$$\begin{array}{l} p \to q \\ r \to s \\ \neg q \vee \neg s \\ \hline \therefore \neg p \vee \neg r \end{array}$$

Rule of Destructive Dilemma

① a) Rita is baking a cake

b) If Rita is baking a cake, then she is not practising her flute

c) If Rita is not practicing her flute, then her ~~flute~~ father will not buy her a car

d) Therefore Rita's father will not buy her a car.

Sol: p: Rita is baking a cake

q: Rita is practising her flute

r: Rita's father will buy her a car

From Qu

$$\begin{array}{l} p \\ p \to \neg q \\ \neg q \to \neg r \\ \hline \therefore \neg r \end{array}$$

Steps	Rule
1) p	premise
2) $p \to \neg q$	premise
3) $\neg q$	(1), (2) Rule of detachment
4) $\neg q \to \neg r$	premise
5) $\neg r$	(3), (4) Rule of detachment

or

Steps	
$p \to \neg q$	premise
$\neg q \to \neg r$	premise
$p \to \neg r$	~~Rule~~ Syllogism
p	premise
$\neg r$	Rule of Detachment

② $[p \wedge (p \to q) \wedge (\neg q \vee r)] \to r$

Sol

$$\begin{array}{l} p \\ p \to q \\ \neg q \vee r \\ \hline \therefore r \end{array}$$

Steps	
1) $p, p \to q$	premise
2) q	(1) & Rule of Detachment
3) $\neg q \vee r$	premise
4) $q \to r$	(3) & $q \to r \Leftrightarrow \neg q \vee r$
5) r	(2) (4) & Rule of Detachment

b) $p \to q$

$r \to \neg q$

r

$\therefore \neg p$

Sol

(1)	$r,\ r \to \neg q$	premises
(2)	$\neg q$	(1) & Rule of Detachment
(3)	$p \to q$	premises
(4)	$\neg p$	(2), (3) & Modulus Tollens

* c) $p \to r$

$r \to s$

$t \vee \neg s$

$\neg t \vee u$

$\neg u$

$\therefore \neg p$

(1)	$p \to r,\ r \to s$	premises
(2)	$p \to s$	(1) & Law of Syllogism
(3)	$t \vee \neg s$	premises
(4)	$\neg s \vee t$	(3), Commutative law
(5)	$s \to t$	(4) & $\neg s \vee t \Leftrightarrow s \to t$
(6)	$p \to t$	(2), (5) & law of Syllogism
(7)	$\neg t \vee u$	premises
(8)	$t \to u$	(7) & $\neg t \vee u \Leftrightarrow t \to u$
(9)	$p \to u$	(6) (8) & law of Syllogism
(10)	$\neg u$	premises
(11)	$\neg p$	(9) (10) & Modulus Tollens

Predicates and Quantifiers

Predicates are the statements that contains variables.

Predicates became proposition if i) Assigned specific value
ii) Quantifier

Two types of Quantifier : Universal $\forall$ (for all) $P(x) = P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n)$

Existential $\exists$ (for some) $P(x) = P(x_1) \vee P(x_2) \vee \dots \vee P(x_n)$

Negation of Quantifier : $\sim \forall x\, P(x) \Leftrightarrow \exists x \sim P(x)$

$\sim \exists x\, P(x) \Leftrightarrow \forall x \sim P(x)$

1. $p(x) : x \leq 3$ $\quad q(x) : x+1$ is odd. Universal consists of all integers

a) $q(1)$ $\quad q(1) : 2 \times$ F

b) $\neg p(3)$ $\quad \neg(T) = F$

c) $p(7) \vee q(7)$ $\quad F \vee F = F$

d) $p(3) \wedge q(4)$ $\quad T \wedge T = T$

e) $\neg(p(-4) \vee q(-3))$ $\quad \neg(T \vee F) = \neg(T) = F$

f) $\neg p(-4) \wedge \neg q(-3)$ $\quad F \wedge T = F$

② $p(x): x \le 3$ $q(x)$: $x+1$ is odd $r(x): x > 0$

Universal of all integer

ⓐ $p(3) \vee [q(3) \vee \neg r(3)]$ $T \vee [F \vee F]$ $T \vee F = T$

ⓑ $p(2) \to [q(2) \to r(2)]$ $T \to [T \to T]$ T

ⓒ $[p(2) \wedge q(2)] \to r(2)$ $(T \wedge T) \to T$ T

ⓓ $p(0) \to [\neg q(-1) \leftrightarrow r(1)]$ $T \to [T \leftrightarrow T]$ $T \to \cancel{F}$ $= T$

③ $P(x): x^2 = 2x$

Universal of all integer

ⓐ $\exists x\, p(x)$

$x^2 - 2x = 0$ $x = 0$ or 2 $P(x) = T$

$x(x-2) = 0$

$x = 0$ or 2 $\in I$

ⓑ $\forall x\, p(x) = T$

④ $p(x): x^2 - 7x + 10 = 0$ Universal is all integer

$q(x): x^2 - 2x - 3 = 0$

$r(x): x < 0$

$5 > 2$

$5 < 3$

$5, 2$

~~I $\forall x\, [p(x) \to \neg r(x)]$~~

~~let $x = -1$ $(-1)^2 - 7(-1) + 10 = 0$~~ $\cancel{-1 < 0}$ ✓

$-1 + 7 + 10 = 0$

$16 \neq 0$

$\cancel{x^2 - 7x + 10 = 0}$

$\cancel{x^2 - 7x = -10}$

$x^2 - 7x + 10 = 0$ $x^2 - 2x - 3 = 0$ $x < 0$

$x = 2, 5$ $x = +3, -1$

$-3x^2$

$-3x + 1x$

$3, -1$

I] $\forall x\, [p(x) \to \neg r(x)]$

$x = 2$ or 5 $r(x) = F$ $\neg r(x) = T$

$x = 2$ $P(x) = 4 - 14 + 10 = 0$ $0 = 0$ ✓

$x = 5$ $P(5) = 25 - 35 + 10 = 0$ $0 = 0$ ✓

$T \to T = T$

$T \to T = T$

II] $\forall x\, [q(x) \to r(x)]$

~~T~~ F

~~T~~

III] $\exists x\, [q(x) \to r(x)]$

T

(1) Identify the bound variables and free variables in each of the following expressions. In both cases the universe comprises all real no/.

a] $\forall y \exists z\ [\cos(x+y) = \sin(z-x)]$

Bound variable = y, z = Variable which has Quantifiers

Free variable = x

b] $\exists x \exists y\ (x^2+y^2 = z)$

Bound variable = x, y

Free variable = z

Quantifiers: Definition and Proofs of theorems

Rule of Universal Specification

If an open statement becomes true for all replacements by the members in a given universe, then that open statement is true for each specific individual member in that universe.

Proof:

$\forall x\ [m(x) \rightarrow c(x)]$

$m(l)$

$\therefore c(l)$

(1) $\forall x[m(x) \rightarrow c(x)]$, ~~m(l)~~ — Premises

(2) $m(l)$ — 〃

(3) $m(l) \rightarrow c(l)$ — (1), Rule of Universal Specification

(4) $c(l)$ — (2)(3), Rule of Detachment

(2) $\neg r(c)$

$\forall t\ [p(t) \rightarrow q(t)]$

$\forall t\ [q(t) \rightarrow r(t)]$

$\therefore \neg p(c)$

(1) $\forall t\ [q(t) \rightarrow r(t)]$ — Premises

(2) $q(c) \rightarrow r(c)$ — (1), Rule of Universal Specification

(3) $\neg r(c)$ — Premises

(4) $\neg q(c)$ — (2)(3), Modus Tollens

(5) $\forall t\ [p(t) \rightarrow q(t)]$ — premises

(6) $p(c) \rightarrow q(c)$ — (5), Rule of Universal specification

(7) $\neg p(c)$ — (4)(6), Modus Tollens

$j(x)$: x is junior $\quad$ $s(x)$: x is senior

⑧ $p(x)$: x is enrolled in a physical education class

No junior or senior is enrolled in a physical education class

Mary is enrolled in PT class.

Therefore Mary is not a senior

Sol: $\forall x [(j(x) \vee s(x)) \longrightarrow \neg p(x)]$

$p(m)$

$\therefore \neg s(m)$

(1) $\forall x [(j(x) \vee s(x)) \rightarrow \neg p(x)]$ premise

(2) $(j(m) \vee s(m)) \longrightarrow \neg p(m)$ (1), Rule of Universal Specification

(3) $\neg\neg p(m) \longrightarrow \neg(j(m) \vee s(m))$ (2), ~~Commutative~~ & Contrapositive $p(t) \rightarrow q(t)$, $\neg q(t) \rightarrow \neg p(t)$

(4) $p(m) \longrightarrow (\neg j(m) \wedge \neg s(m))$ (3), Law of Double negation & Demorgan's law

(5) $p(m) \longrightarrow \neg s(m)$ (4), Commutative & Rule of Conjunctive Simplification

(6) $p(m)$ premise

(7) $\neg s(m)$ Modus Ponens, (5)(6)

Rule of Universal Generalization

If an open statement $p(x)$ is proved to be true when x is replaced by any arbitrarily chosen element c from our universe, then the universally quantified statement $\forall x\, p(x)$ is true.

①
$\forall x [p(x) \rightarrow q(x)]$
$\forall x [q(x) \rightarrow r(x)]$
$\therefore \forall x [p(x) \rightarrow r(x)]$

(1) $\forall x [p(x) \rightarrow q(x)]$ premise

(2) $p(c) \rightarrow q(c)$ (1) Rule of Universal Specification

(3) $\forall x [q(x) \rightarrow r(x)]$ premise

(4) $q(c) \rightarrow r(c)$ (3), Rule of Universal Specification

(5) $p(c) \longrightarrow r(c)$ (2)(4), Law of Syllogism

(6) $\forall x [p(x) \longrightarrow r(x)]$ (5), Rule of Universal Generalization

Theorem is a valid logical assertion which can be proved using:

axioms = statements which are given to be true

rules of inference = logical rules allowing the deduction of conclusions from premises

Lemma is a pre-theorem or result which is needed to prove a theorem

Corollary is a post-theorem or result which follows directly from a theorem

Methods of Proof

$p \to q$ is conjecture until a proof is produced [T or F]

① Direct Proof

If p is true and q has to be true to prove $p \to q$ true

Ex: If n is odd int, then n^2 is odd

(n is odd) $\to$ (n^2 is odd)

$\exists k \; n = 2k+1$

Prove $n^2 = (2k+1)^2$

$= 4k^2 + 1 + 4k$

$= 2(2k^2 + 2k) + 1$

$= 2m + 1$

② Indirect Proof

Direct proof's contrapositive

Ex: ($3n+2$ is odd) $\to$ (n is odd)

contrapositive

(n is even) $\to$ ($3n+2$ is even)

$\exists k \; n = 2k$

$3n+2 = 3(2k) + 2$

$= 2(3k+1)$

$= 2(m)$

③ Vacuous Proof

If we know one of the hypotheses in p is false then $p \to q$ is vacuously true

$F \to F = T$

$F \to T = T$

Ex: ST P(0) is true where P(n): If $n > 1$, then $n^2 > n$

P(0): $(0 > 1) \to (0^2 > 0)$

F (By vacuous proof)

④ Trivial Proof

If we know q is true, then $p \to q$ is true

$F \to T$, $T \to T = T$

Ex: ST P(0) is true where P(n): If $a \geq b > 0$, then $a^n \geq b^n$

P(0) If $a \geq b > 0$ → $a^0 \geq b^0$

$\underbrace{1 \geq 1}_{T}$

True By trivial Proof

⑤ Proof by Contradiction

Ex: PT $\sqrt{2}$ is irrational

Try to ~~approve~~ prove ~p is false then P will true for given statement

⑥ Proof by case

⑦ Existence proof