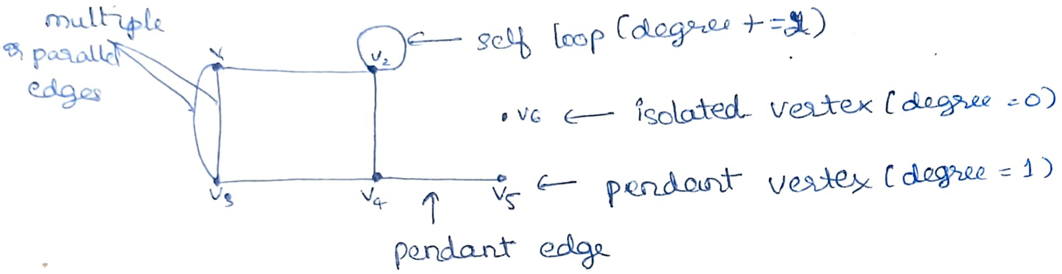
Graph Theory

Konigsberg Bridge Problem

Whether we can leave home, cross every bridge ~~every~~ exactly once & return home.

Graph is triple consisting of vertex set, edge set, relation b/w edge & pairs of vertices



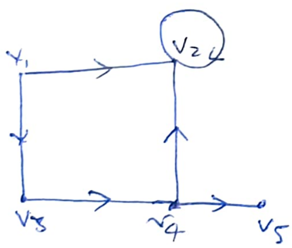
degree of vertex = No. of edges from that vertex $\qquad \sum_{i=0} v_i = 2|E|$

Sum of degrees of all vertex in a graph is even & twice the no. of edges

Graph Type = 1) Simple Graph [Graph with no multiple edges or self loop)

2) Multiple (Multi) Graph [Graph with multiple edges]

3) Pseudo Graph [Graph with self loop and multi edges]



in-degree = $v_1 = 0$, $v_2 = 3$, $v_3 = 1$, $v_4 = 1$, $v_5 = 1$.

out-degree = $v_1 = 2$, v.

Regular Graph = Every vertex has same degree

2-Regular graph =

3-regular =

Complete graph = Every vertex is connected to every other vertex $\triangledown$ ⊠

~~IDEA~~ - Every complete graph is a regular graph

Repeate Vertex	Repeate Edge	Open	Closed	
Y	Y	Y	–	Walk
Y	Y	–	Y	Closed Walk
Y	N	Y	–	Trial
Y	N	–	Y	Circuit

N	N	Y	–	Path
N	N	–	Y	Cycle

(i) Closure law

$\forall a, b \in G, \; a * b \in G$

(ii) Associative law

$\forall a, b, c \in G \quad a*(b*c) = (a*b)*c$

Complete Graph : Every vertices are connected to each other.

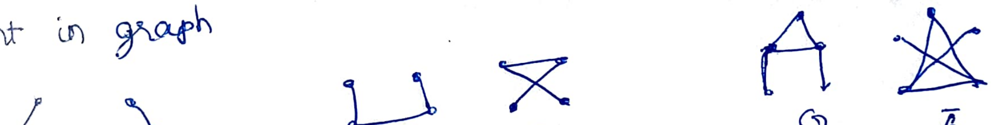
Representation : K_1, K_2

K_n n is no. of verties

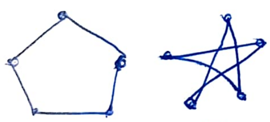
Complements of graph

It consists of all the vertices but those edges that are not present in graph

Ex:



g $\bar{G}$ G $\bar{G}$ G $\bar{G}$



G $\bar{G}$

Note : $G \cup \bar{G}$ should ~~be~~ form complete graph.

11-5

Graph Theory

$G = (V, E)$

Trivial = One vertex graph

Walk : It is finite sequence

Any x-y walk where x=y is => Closed walk

=> Open walk

Walk can have repeated edges and vertices

Trail = If no edge in x-y walk is repeated

Circuit = Closed trail

Path = No vertex in walk is repeated

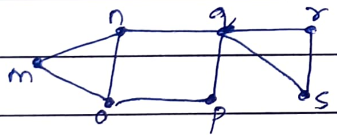
Cycle = Closed Path

For any Graph G, the number of components is denoted by $\kappa(G)$

Multigraph = Graph with multi-edges or parallel edges or two or more edges are formed between two vertices

Ex 11.1

2)



a) Walk from n to p not trail

$\{n,q\}, \{q,r\} \{r,s\} \{s,q\} \{q,n\}, \{n,m\} \{m,o\} \{o,p\}$

b) n-p tail not path

$\{n,q\} \{q,r\} \{r,s\} \{s,q\} \{q,p\}$

c) path n to p

$\{n,o\} \{o,p\}$

d) closed walk n-n not circuit

$\{n,q\} \{q,r\} \{r,s\} \{s,q\} \{q,n\}$

e) circuit n-n not cycle

$\{n,q\} \{q,r\} \{r,s\} \{s,q\} \{q,p\} \{p,o\} \{o,n\}$

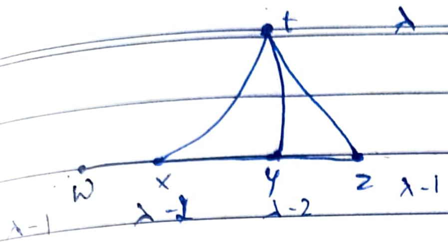
f) cycle from n to n

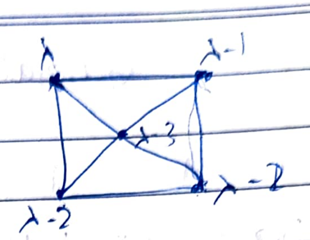
$\{n,o\} \{o,p\} \{p,q\} \{q,n\}$

Chromatic

116

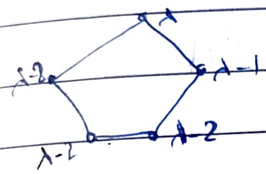
1)





$\lambda(\lambda-1)^2(\lambda-2)^2$

$\lambda(\lambda-1)(\lambda-2)^2(\lambda-3)$



$\lambda(\lambda-1)(\lambda-2)^3$

2) f : 4 g : 3 k : 4 j : 3 i : 3 h : 3
u : 4 v : 3 z : 4 y : 2 x : 3 w : 4

Euler circuit = If a circuit traverses every edge of graph exactly once.

Hamilton cycle = ~~If~~ A cycle that contains every vertex in V. every vertex should be visited once

Hamilton path = Path ~~&~~ in graph that contains each vertex papergrid

Date: / /

Trees

$|V| = |E| + 1$ To find $2|E| = \sum_{v \in V} deg(v)$

If forest has k trees, then

$v = e + k$

Exercise

(2) $|V_1| = |E_1| + 1 = 17 + 1 = 18$

$|V_2| = 2|V_1| = 2 \times 18 = 36$

$|E_2| = |V_2| - 1 = 36 - 1$

(3) a) $e_1 = 40$ $k = 7$

$v_1 = 40 + 7 = 47$

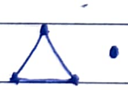
b) $v - e = k$

$62 - \cancel{40}\,51 = k$

$k = 11$

(4) $v = e + k$

(5) A —— B => path has exactly 2 pendant vertices

(6) 

(8) a) $2|E| = x + 4(2) + 1(3) + 2(4) + 1(5)$

$= x + 24$

$|E| = |V| - 1 = x + 4 + 1 + 2 + 1 - 1 = x + 7$

$2(x+7) = x + 24$

$2x + 14 = x + 24$

$\boxed{x = 10}$

b)

papergrid

Date: / /

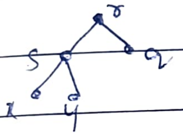
Basic minimum edges needs to make trees.

Spanning tree = It should not have cycle

Non-isomorphic = Different shapes with same vertices & edges

Tree = Acyclic

$m - n + 1 = |E| - |V| + 1$ are edges to removed to make spanning tree.



id (r) = 0

id (s or q) = 1

in-degree = parent count

0 – root node

1 – other nodes

od ( ) = out-degree = no. of childrens

If od (v) ≤ m then its m-ary tree

If ~~od~~ ≤ 2 then its binary tree

If od (v) = 0 or m then complete m-ary tree

If ~~od (v)~~ m = 2 then its complete binary tree

Decomposition theorem ~~for spanning~~ tree