Discrete

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$\neg(p \vee \neg q) \rightarrow \neg p$

p	q	$\neg q$	$p \vee \neg q$	$\neg(p \vee \neg q)$	$\rightarrow \neg p$
T	T	F	T	F	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	T	F	T

$p \oplus q$ $\quad p \rightarrow q \rightarrow r$

$p \rightarrow (q \rightarrow r)$ $\quad (p \rightarrow q) \rightarrow r$

8) b) c)

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \rightarrow q$	$(p \rightarrow q) \rightarrow r$
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	F	T	T	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

d) c)

p	q	$(p \rightarrow q)$ = x	$(q \rightarrow p)$ = y	$x \rightarrow y$	$p \wedge (p \rightarrow q)$ = x	$x \rightarrow q$
T	T	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	F	T
F	F	T	T	T	F	T

f) g)

p	q	$\overset{x}{p \wedge q}$	$x \rightarrow p$	$\neg p$	$\neg q$	$(\neg p \overset{x}{\vee} \neg q)$	$q \leftrightarrow x$
T	T	T	T	F	F	F	F
T	F	F	T	F	T	T	F
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	F

h)

p	q	r	$\overset{x}{p \rightarrow q}$	$\overset{y}{q \rightarrow r}$	$\overset{z}{x \wedge y}$	$\overset{a}{p \rightarrow r}$	$z \rightarrow a$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

12)

a) $\underbrace{[(p \wedge q) \wedge r]}_{T} \rightarrow \underbrace{(s \vee t)}_{F} = F$

$p = T \quad q = T \quad r = T \qquad s = F \quad t = F$

b) $\underbrace{[p \wedge (q \wedge r)]}_{T} \rightarrow \underbrace{(s \vee t)}_{F} = F$

$p, q, r = T \qquad s = T/F \quad t = T/F$

13) $\underbrace{(\underset{1}{q} \rightarrow [(\neg p \vee r) \wedge \neg s])}_{0} \wedge \underbrace{[\neg s \rightarrow (\neg r \wedge \underset{1}{q})]}_{0} = 1$ Given: $q = 1$

$\underset{1}{q} \rightarrow [\overset{1}{(\neg p_0 \vee r_0)} \wedge \neg s] = 1 \qquad \overset{1}{\neg s} \rightarrow \overset{0}{(\neg r \wedge q)}$

$1 \quad 1 \quad 1 \qquad 1 \quad 1 \quad 1$

$r = 0$

$s = 0$

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2.20 $(p \vee q) \wedge \neg(\neg p \wedge q)$

$(p \vee q) \wedge (\neg\neg p \vee \neg q)$ Demorgan

$(p \vee q) \wedge (p \vee \neg q)$ Double negation

$p \vee (q \wedge \neg q)$ Distributive

$p \vee F_0$ Inverse

p Identity

2.21 $\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$

$\neg\neg[(p \vee q) \wedge r] \wedge \neg\neg q$ Demorgan

$[(p \vee q) \wedge r] \wedge q$ Double negation

$[(p \wedge r) \vee (q \wedge r)] \wedge q$ Distributive

$\cancel{(p \wedge r) \vee (q \wedge r) \wedge}$

$((p \wedge r) \wedge q) \vee ((q \wedge r) \wedge q)$ ~~Absorption~~ Cummatative

$((r \wedge p) \wedge q)$ $((r \wedge q) \wedge q)$ Associative

$(r \wedge (p \wedge q)) \vee (r \wedge (q \wedge q))$

$(r \wedge (p \wedge q)) \vee (r \wedge q)$ Idempotent

$((p \wedge q) \vee q) \wedge r$ Distributive

p $q \wedge r$ Absorption

2.22 $(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r) \iff (p \vee r \wedge (t \vee \neg q))$

$p \vee [(q \vee r) \wedge (t \vee \neg q) \wedge (\neg t \vee r)]$ Distributive

Commutative

$p \vee [(q \vee r) \wedge (\neg t \vee r) \wedge (t \vee \neg q)]$

Distributive

$p \vee [(r \vee (q \wedge \neg t)) \wedge (t \vee \neg q)]$

$p \vee [(r \vee (q \wedge \neg t)) \wedge (\neg\neg t \vee \neg q)]$ Double Negation

$p \vee [(r \vee (q \wedge \neg t)) \wedge \neg(\neg t \wedge q)]$ Demorgan's

$p \vee [(r \vee (q \wedge \neg t)) \wedge \neg(q \wedge \neg t)]$ Commutative

~~Associative~~

$\cancel{p \vee [r \vee ((q \wedge \neg t) \wedge \neg(q \wedge \neg t))]}$

$\cancel{p \quad \wedge \quad \neg p}$

$\cancel{p \vee [r \vee F_0]}$ ~~Inverse~~

$\cancel{p \vee [(r \vee (q \wedge \neg t)) \wedge (\neg(q \wedge \neg t) \vee r)]}$ ~~Distributive~~

$\cancel{p \quad \wedge \quad \neg p}$

$\cancel{p \vee}$

$p \vee [((q \wedge \neg t) \wedge r) \vee (\underbrace{\neg(q \wedge \neg t)}_{\neg p} \wedge \underbrace{(q \wedge \neg t)}_{p})]$ Distributive

$p \vee [(\underbrace{\neg(q \wedge \neg t)}_{p} \wedge r) \vee F_0]$ - Inverse

$p \vee F_0$

$= T$

$p \vee [\neg(q \wedge \neg t) \wedge r]$ Identity

$p \vee [r \wedge (\neg q \vee \neg\neg t)]$ Commutative & Demorgan

$p \vee [r \wedge (\neg q \vee t)]$ Double Negation

Commutative

$p \vee [r \wedge (t \vee \neg q)]$

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$p \to q$
$\neg p \vee q$

1) b) $[p \to (q \vee r)] \Leftrightarrow [(p \wedge \neg q) \to r]$

	$\neg p \vee (q \vee r)$	$p \to q \Leftrightarrow \neg p \vee q$
~~$(\neg p \vee (q \vee r))$~~	$(\neg p \vee q) \vee r$	Associtative
~~$(\neg p \vee (\neg q \to r))$~~	$\neg(\neg p \vee q) \to r$	$p \to q \Leftrightarrow \neg p \vee q$
~~$(\neg q \to r) \vee \neg p$~~	$(\neg\neg p \wedge \neg q) \to r$	Demorgan
	$(p \wedge \neg q) \to r$	Double Negation

3) a) $[\underbrace{p \vee (q \wedge r)}_{p}] \vee \neg[\underbrace{p \vee (q \wedge r)}_{p}]$ ~~b) $[(p \vee q) \to r] \Leftrightarrow [\neg r \to \neg(p \vee q)]$~~

T_0 Inverse

b) $[(p \vee q) \to r] \leftrightarrow [\neg r \to \neg(p \vee q)]$

$= \cancel{[[(p \vee q) \to r] \to [\neg r \to \neg(p \vee q)]]} \wedge$ $p \leftrightarrow q$ ~~$\Leftrightarrow (p \to q) \wedge (q \to p)$~~

$[\neg r \to \neg(p \vee q)$

$= [\neg(p \vee q) \vee r] \leftrightarrow [\neg\neg r \vee \neg(p \vee q)]$ $p \to q \Leftrightarrow \neg p \vee q$

$= [(\neg p \wedge \neg q) \vee r] \leftrightarrow [r \vee (\neg p \wedge \neg q)]$ Demorgan & Double Negation

$= [(\neg p \wedge \neg q) \vee r] \leftrightarrow [(\neg p \wedge \neg q) \vee r]$ Commutative

$= [[(\neg p \wedge \neg q) \vee r] \to [(\neg p \wedge \neg q) \vee r]] \wedge [[(\neg p \wedge \neg q) \vee r] \to [(\neg p \wedge \neg q) \vee r]]$

$= [\neg[\underbrace{(\neg p \wedge \neg q) \vee r}_{\neg p}] \vee [\underbrace{(\neg p \wedge \neg q) \vee r}_{p}]] \wedge [\neg[\underbrace{(\neg p \wedge \neg q) \vee r}_{\neg p}] \vee [\underbrace{(\neg p \wedge \neg q) \vee r}_{p}]]$

$= \cancel{(p \wedge q) \vee \neg r}$ $T_0 \wedge T_0$ Inverse

$= T_0$

4) $[[[(p\wedge q)\wedge r)] \vee [(p\wedge q)\wedge \neg r]] \vee \neg q] \to s$

$\neg[[[(p\wedge q)\wedge r] \vee [(p\wedge q)\wedge \neg r]] \vee \neg q] \vee s$ — $p \to q \Leftrightarrow \neg p \vee q$

$[\neg\{[(p\wedge q)\}\wedge r] \vee [(p\wedge q)\wedge \neg r]] \wedge \neg\neg q] \vee s$ — Demorgan's

$[\neg[(p\wedge q)\wedge r] \wedge \neg[(p\wedge q)\wedge \neg r] \wedge q] \vee s$ — Demorgan & Double Negation

$[[\neg(p\wedge q) \vee \neg r] \wedge [\neg(p\wedge q) \vee \neg\neg r] \wedge q] \vee s$ — Demorgan

$[[(\neg p \vee \neg q) \vee \neg r] \wedge [(\neg p \vee \neg q) \vee r]] \wedge q] \vee s$ — Demorgan, Double Negation

$[[(\neg p \vee \neg q) \vee (\neg r \wedge r)] \wedge q] \vee s$ — Distributive

0 0 ∧ 1
0 1 ∧ 0

$[[(\neg p \vee \neg q) \vee F_0] \wedge q] \vee s$ — ~~Identity~~ Inverse

1 ∨ 0
0 ∨ 0

$= [(\neg p \vee \neg q) \wedge q] \vee s$ — Identity

$= [(\neg p \wedge q) \vee (\neg q \wedge q)] \vee s$ — Distributive

$= [(\neg p \wedge q) \vee F_0] \vee s$ — Inverse

$= (\neg p \wedge q) \vee s$ — Identity

$A\bar{B} + \bar{A}B$

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7) a) $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \iff p \wedge q$

$(\neg p \vee q) \wedge p$ Absorption

$(p \wedge \neg p) \vee (p \wedge q)$ Distribution

~~Inverse~~

$\cancel{(\neg p \vee q) \wedge ((p \wedge p) \wedge q)}$ ~~Associative~~

$\cancel{(\neg p \vee q) \wedge (p \wedge q)}$ ~~Identity~~

$\cancel{\neg p \wedge p}$ ~~Distributive~~

$\cancel{F \wedge p}$

$((\neg p \vee q) \wedge p) \wedge (p \wedge q)$ Associative

$((\neg p \wedge p) \vee (q \wedge p)) \wedge (p \wedge q)$ Distributive

10
01 $F_0 \vee (q \wedge p) \wedge (p \wedge q)$ Inverse, Commutative

$(p \wedge q)^{p} \wedge (p \wedge q)$ Identity

$p \wedge q$ Idempotent

b) $p \vee q \iff (\neg p \wedge q) \vee (p \vee (p \vee q))$

8) a) $q \rightarrow p$
$= \neg q \vee p$
$= \neg q \wedge p$

b) $p \rightarrow (q \wedge r)$
$= \neg p \vee (q \wedge r)$
$= \neg p \wedge (q \vee r)$

c) $p \leftrightarrow q$
$(p \rightarrow q) \wedge (q \rightarrow p)$
$(\neg p \wedge q) \vee (\neg q \wedge p)$

d) $p \veebar q$
$(p \vee \neg q) \wedge (\neg p \vee q)$

9) a) $p : 0+0 = 0$ b) $p : -1 < 3$ $q = 3+7 = 10$

$q = 1+1 = 1$ $r : \sin \frac{3\pi}{2} = -1$

$T \to F = F$	Implication	p	$(p \wedge q) \to r$
$F \to T = T$	Converse	$T \to T$	$= T$
$F \to T = T$	Inverse	$\cancel{F} \to \cancel{F}$	$= T$
$T \to F = F$	Contrapositive	$F \to F$	$= T$
		$F \to F$	$= T$

10) a) $q \to pq$ $p \to q$ True

b) $q \to p$ $\neg q \to \neg p$ True

c) $q \leftrightarrow p$ $\neg p \to \neg q$ True

11) $p \to (q \to r)$

Contrapositive a) $\neg(q \to r) \to \neg p$ b) $(\neg q \vee r) \vee \neg p$

$\neg\neg(q \to r) \vee \neg p$ Demorgan's

$(q \to r) \vee \neg p$

Contine

14) $(p \vee q) \to [q \to q]$

$(p \vee q) \to (\neg q \vee q]$ $p \to q \iff \neg p \vee q$

$(p \vee q) \to T_0$ $\therefore$ Inverse

$\underline{\neg(p \vee q)} \vee T_0$ $p \to q \iff \neg p \vee q$

p

T_0 Domination law

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19) a) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$

$p \vee [p$ — Absorption

p — Idempotent

b) $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$

$\underbrace{p \vee q}_{t} \vee (\neg \underbrace{(p \vee q)}_{t} \wedge r)$ — Demorgan

$t \vee (\neg t \wedge r)$

$\underbrace{(p \vee q)}_{t} \underset{\vee}{\vee} \underbrace{\neg(p \vee q)}_{\neg t} \wedge (p \vee q \vee r)$ — Distributive

$1 \vee 0 = 1$, $0 \; 1$, $1 \wedge 0$

$T_0 \wedge (p \vee q \vee r)$ — Inverse

$p \vee q \vee r$ — Identity

c) $[(\neg p \vee \neg q) \rightarrow (p \wedge q \wedge r)] \Leftrightarrow p \wedge q$

$\neg(\neg p \vee \neg q) \vee (p \wedge q \wedge r)$ — $p \rightarrow q \Leftrightarrow \neg p \vee q$

$\underbrace{(p \wedge q)}_{t} \vee (\underbrace{p \wedge q}_{t} \wedge r)$ — Demorgan's & Double Negation

~~Distributive~~

$p \wedge q$ — Absorption

$0 \; 1$, $1 \; 0$, $0 \; 1 \; 1$

$r \vee T_0$

20) a) $[p \wedge (\neg r \vee q \vee \neg q)] \vee [(r \vee t \vee \neg r) \wedge \neg q]$

$[p \wedge (\neg r \vee T_0)] \vee [(T_0 \vee t) \wedge \neg q]$ — Inverse

$[p \wedge T_0] \vee [T_0 \wedge \neg q]$ — Domination

$p \vee \neg q$ — Identity

b) $[p \vee (\underline{p \wedge q}) \vee (\underline{p \wedge q} \wedge \neg r)] \wedge [(\underbrace{p \wedge r \wedge t}_{s}) \vee t)]$

$[p \vee p] \wedge [t]$ Absorption law

$p \wedge t$ Idempotent

6) a) ~~No~~ $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

$\neg p \vee \neg(q \vee r) \vee \neg(\neg p \vee \neg q \vee r)$ Negation

$\neg p \wedge (\neg q \wedge \neg r) \wedge (p \wedge q \wedge \neg r)$ Demorgan's

$(\neg p \wedge \neg q) \wedge \underline{\neg r} \wedge (p \wedge q \wedge \underline{\neg r})$ Associative

Distributive

$(\neg p \wedge \neg q)$

$p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

$\neg p \wedge \neg(q \vee r) \wedge \neg(\neg p \vee \neg q \vee r)$ Negation

$\neg p \wedge (\neg q \wedge \neg r) \wedge (p \wedge q \wedge \neg r)$ Demorgan, Double Negation

$(\neg p \wedge \neg q) \wedge \underline{\neg r} \wedge (p \wedge q \wedge \underline{\neg r})$

$p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

$\neg(p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r))$ Negation

$\neg p \vee \neg(q \vee r) \vee \neg(\neg p \vee \neg q \vee r)$ Demorgan

$\neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$ Demorgan, Double Negation

$(\neg p \vee \neg q) \wedge \underline{\neg r} \vee (p \wedge q \wedge \underline{\neg r})$ ~~Associative~~

$(\neg p \wedge \neg r) \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$

~~$\neg p \vee \neg q$~~

$(\neg p \vee \neg q) \wedge \neg r$ Absorption

$\neg(p \wedge q) \wedge \neg r$ Demorgan

$\neg[(p \wedge q) \vee r]$

$(\neg p \vee \neg q \vee (p \wedge q)) \wedge \neg r$

$(\neg(p \wedge q) \vee (p \wedge q)) \wedge \neg r$

$\neg p \vee p$

p

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~~$(\neg p \vee \neg q) \wedge (\neg p \vee \neg r)$~~ $\vee (p \wedge q \wedge \neg r)$ Distributive

b) $(p \wedge q) \rightarrow r$

$\neg[(p \wedge q) \rightarrow r]$ Negation

$\neg[\neg(p \wedge q) \vee r]$ $p \rightarrow q \Longleftrightarrow \neg p \vee q$

~~$(p \wedge q) \vee \neg r$~~ ~~Double Negation~~

$\neg\neg(p \wedge q) \wedge \neg r$ Demorgan

$(p \wedge q) \wedge \neg r$ Double Negation

c) $p \rightarrow (\neg q \wedge r)$

$\neg[p \rightarrow (\neg q \wedge r)]$ Negation

$\neg[\neg p \vee (\neg q \wedge r)]$ $p \rightarrow q \Longleftrightarrow \neg p \vee q$

$\neg\neg p \wedge \neg(\neg q \wedge r)$ Demorgan

$p \wedge (q \vee \neg r)$ Demorgan, Double Negation

~~d) $p \vee q \vee (\neg p \wedge \neg q \wedge r)$~~

~~d) $p \vee q \vee (\neg p \wedge \neg q$~~

a) $p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r)$

$\neg(p \wedge (q \vee r) \wedge (\neg p \vee \neg q \vee r))$ Negation

$\neg p \vee \neg(q \vee r) \vee \neg(\neg p \vee \neg q \vee r)$ Demorgan

$\neg p \vee (\neg q \wedge \neg r) \vee (p \wedge q \wedge \neg r)$ Demorgan , Double Negation

$\cancel{(\neg p \vee \neg q) \wedge (\neg p \vee \neg r) \vee (p \wedge q \wedge \neg r)}$ Distributive

Commutative

$\cancel{(\neg p \vee \neg q) \wedge (\neg p \vee \neg r) \vee (}$

$\cancel{\neg p \vee \neg}$

$\neg p \vee [(\neg q \vee (p \wedge q)) \wedge \neg r]$ Distributive

$\neg p \vee [(\neg q \vee q \wedge p) \wedge \neg r]$ Commutative

$\neg p \vee (T_0 \wedge p) \wedge \neg r$ Inverse

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$p \to r$	(1) $p \to r,\ r \to s$	premises
$r \to s$	(2) $p \to s$	(1) law of syllogism
$t \vee \neg s$	(3) $t \vee \neg s$	premise
$\neg t \vee u$	(4) $\neg s \vee t$	Commutative
$\neg u$	(5) $s \to t$	(4) $s \to t \Leftrightarrow \neg s \vee t$
$\therefore \neg p$	(6) $\neg t \vee u$	premise
	(7) $t \to u$	(6) $t \to u \Leftrightarrow \neg t \vee u$
	(8) $s \to u$	(5) (7) law of syllogism
	(9) $p \to u$	(2) (8) —"—
	(10) $\neg u$	premise
	(11) $\neg p$	Modus Tollens

$p \to (q \wedge r)$	(1) $p \to (q \wedge r)$	premise
$r \to s$	(2) $p \to q$	(1) Conjunctive Simplication
$\neg(q \wedge s)$	(3) $r \to s$	premise
$\therefore \neg p$	(4) $\neg(q \wedge s)$	premise
	(5) $\neg q \vee \neg s$	(4), Demorgan
	(6) $\neg p \vee \neg r$	(2) (3) (5), Destructive Dilemma
	(7) $\cancel{\neg p\ \neg q \to \neg s}$	~~Conclusion~~
	(8)	

10]

d) $p \to q$
$r \to \neg q$
r
$\therefore \neg p$

$\neg q$ $\quad \neg p$

e) $p \to (q \to r)$
$\neg q \to \neg p$
p
$\therefore r$

$q \to r$ $\quad q$ $\quad r$

f) $p \wedge q$
$p \to (r \wedge q)$
$r \to (s \vee t)$
$\neg s$
$\therefore t$

p Conjunctive Simplication
$r \wedge q$ Modus Ponnes
r Conjunctive Simplication
$s \vee t$ Modus Ponnes
Disjunction Syllogism
t

g) $p \to (q \to r)$
$p \vee s$
$t \to q$
$\neg s$
$\therefore \neg r \to \neg t$

Commutative $s \vee p$
p Disjunction Syllogism
Modus Ponnes $q \to r$
~~$q \to r$~~ ~~t~~ ~~Converse~~
Law of Syllogism $r \to t$
$\neg r \to \neg t$
Inverse

h) $p \vee q$
$\neg p \vee r$
$\neg r$
$\therefore q$

Commutative $r \vee \neg p$
$\neg p$ Disjunctive Syllogism
~~$\neg p \to q$~~
~~Modus Ponnes~~ q

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a) $p \wedge \neg q$ — Conjunctive Simplification → p — Contraduction → $p \wedge r$

r

$\therefore (p \wedge r) \vee q$

Commutative: $\neg q \wedge p$; $q \to p$; $p \to q$; $\neg p \vee q$; $q \vee \neg p$

Conjunctive Simplification: $\neg q$

$(p \wedge r) \wedge \neg q$ — Negation — $\neg(p \wedge r) \vee \neg\neg q$

$\neg q$ Negation

Destructive Amplification $(p \wedge r) \vee q$

11) $[(p \wedge \neg q) \wedge [p \to (q \to r)]] \to \neg r$ $\quad r = T$

$\underbrace{(p \wedge \neg q)}_{T} \quad \underbrace{[p \to (q \to r)]}_{T} \quad \underbrace{\neg r}_{F}$ $\quad p = T$

T $\quad q = F$

$[[(\overset{F}{p} \wedge q) \to \overset{T}{r}] \wedge [\neg \overset{F}{q} \vee \overset{T}{r}]] \to p$ $\quad p = F \quad q = F/T$

$\underbrace{[(p \wedge q) \to r]}_{T} \quad \underbrace{[\neg q \vee r]}_{T} \quad \underbrace{p}_{F}$ $\quad r = T$

$[(\overset{T}{p} \leftrightarrow \overset{T}{q}) \wedge (\overset{T}{q} \to \overset{T}{r}) \wedge (\overset{T}{r} \vee \neg \overset{T}{s}) \wedge (\neg \overset{T}{s} \to \overset{T}{q})] \to s$

$\underbrace{(p \leftrightarrow q)}_{T} \quad \underbrace{(q \to r)}_{T} \quad \underbrace{(r \vee \neg s)}_{T} \quad \underbrace{(\neg s \to q)}_{T} \quad \underbrace{s}_{F}$

$[\overset{T}{p} \wedge (\overset{T}{p} \to \overset{T}{r}) \wedge [(\overset{T}{p} \to (\overset{T}{q} \vee \neg \overset{F}{r})] \wedge (\neg \overset{F}{q} \vee \neg \overset{T}{s})] \to \underbrace{s}_{F}$ $\quad r = T$, $p = T$, $q = T$, $s = F$

$\underbrace{p}_{T}$

12) a) $((p \wedge q) \to r)$ — law of Syllogism → $(p \wedge q) \to s$ — Modus Tollen → $\neg(p \wedge q) = \neg p \vee \neg q$ (De morgan)

$r \to s$

$\neg s$

$\therefore \neg p \vee \neg q$

b) $p \rightarrow q$

$r \rightarrow s$

$(q \vee s) \rightarrow t$

$\neg t$

$\therefore \neg p \wedge \neg r$

$q \rightarrow p$

$s \rightarrow r$

$\neg(q \vee s)$

$p \vee r \xrightarrow{\text{Negation}} \neg(p \vee r)$

$\neg p \wedge \neg r$

$p \rightarrow q$

$r \rightarrow s$

$(q \vee s) \rightarrow t$

$\neg t$

$\therefore \neg p \wedge \neg r$

$\neg r$

$\neg p \wedge \neg r$

$\neg s$ Commutative, conjunctive simplification.

$\neg(q \vee s)$ Modus Tollen

$\neg q \wedge \neg s$ Demorgan

$\neg q$ Conjunctive Simplification

$\neg p$ Modus Tollen

c) $(p \vee q) \rightarrow \neg r$

$s \rightarrow \neg p$

$s \wedge q$

$\therefore r$

$\neg p$

s

q

$\cancel{\neg q \rightarrow s}$

$\cancel{\neg p \rightarrow \neg q}$ $\cancel{(p \vee \neg q)}$

13) a) $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow \dfrac{(q \vee r)}{T}$

① ② IV ④ 9
2.4 bi

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2.4

① $p(x): x \leq 3$ $q(x): x+1$ is odd

a) $q(1)$ F

b) $\neg p(3)$ F

c) $p(7) \vee q(7)$ $F \vee F = F$

② b) $\underbrace{[p(x) \wedge q(x)]}_{T} \wedge \underbrace{r(x)}_{T} = T$

$x = 2$

⑧ $p(x) = x^2 - 8x + 15 = 0$ $q(x) =$

$x = +3, +5$

2.5] 12) a) k, l are both even, k + l is even

let $k = 2a$ $l = 2b$

then $k + l = 2a + 2b$

$= 2(a+b)$

$= 2m$

b) k, l even, kl is even

let $k = 2a$ $l = 2b$

$k \times l = 2a \times 2b = 4ab = 2(2ab)$ ~~False~~

$= 2(m)$

5)

a) $\forall x\,[a(x) \rightarrow r(x)]$ $\cancel{a(x)}$ $a(\pi) \rightarrow r(\pi)$ $\neg a(\pi)$

$\neg r(\pi)$ $\cancel{\neg r(y)}$ $\neg r(\pi)$

b) $\cancel{\forall x[e(x) \rightarrow c(x)]}$

$\cancel{m(y) \rightarrow e(y)}$

$\cancel{m(y) \rightarrow c(y)}$

c)

$\forall x\,[q(x) \rightarrow d(x)]$

$\cancel{s(p) \rightarrow}$ $q(s)$

$\cancel{s(p) \rightarrow}$ $d(s)$

d) $\forall x\,[r(x) \rightarrow e(x)]$

$\neg e(y)$

$\neg r(y)$

14) If n is odd then n^2 is odd

let $= n = 2k+1$

$n^2 = (2k+1)^2$

$= 4k^2+1+2k$

$= 4k^2+2k+1$

$= 2(2k^2+k)+1$

$= 2m+1$

Proof by Contradiction

15) If n^2 is odd, then n is odd

let n^2 is odd & n is even

$n = 2a$

$n^2 = 2a \times 2a$

$= 2(2a^2)$

$= 2a$

So if n is even then n^2 is also even

$\therefore$ n^2 is odd $=$ n is odd.

16) n^2 is even if & only if n is even

$\cancel{n^2}$ is $n = 2a$ $\cancel{n^2 = 2a}$ $n = 2$

$n^2 = 2a \times 2a$ $\cancel{n = \sqrt{2a}}$

$= 2(2a^2)$

$= 2m$

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1) $\binom{6}{2}$ ab ba db bd ec ce fc cf ef bf
ca ac dc cd ed de fd df de a/a
cb bc ea ae fa af fe ef df
da ad eb be fb bf f dc

2) $\binom{12}{5}$ 3) 210 792 91 3003

Recurrence Relation = Given a relation regarding how a_n relates to its predecessors

Ex: Fibonacci Sequence

~~General~~

~~First order~~ First order

$$a_n = c a_{n-1} + f(n)$$

If $f(n) = 0$ Homogeneous

$f(n) \neq 0$ Non-homogeneous

First order $a_n = c^n a_0$

$a_n = c^n a_0$
$a_n = 4^n 3$

Ex: $a_{n+1} = 4a_n$ for $n \geq 0$, $a_0 = 3$

$c = 4$ $a_0 = 3$

Given $a_{n+1} = 4a_n$ — (1)

General solution $a_n = c^n a_0$ — (2)

If $a_n = c^n$ then $a_{n+1} = c^{n+1}$ — (3)

By (1) (3) $4a_n = c^{n+1}$

$4\cancel{c^n} = \cancel{c^n} \times c$

$\boxed{4 = c}$

Put c in (3) $a_n = 4^n 3$

① $a_n = 7a_{n-1}$ where $n \geq 1$ & $a_2 = 98$

$c = 7$

$a_2 = 7^2 \times a_0$

$a_0 = \frac{98}{49} = 2$

$a_n = 7^n \, 2$

$a_2 = c^2 \times a_0$

$98 = 49 \times a_0$

$a_0 = 2$

② $a_0 = 1000$

$a_n = a_n + 0.005a_n = 1.005a_n$

$c = \frac{6\%}{12} = 0.005$

$a_n = c^n a_0$

$a_n = (1.005)^n \, 1000$

$a_{12} = (1.005)^{12} \, 1000 = 1061.677 \ldots$

18
18
36
54

① $a_{n+1} = 5a_n$ $a_0 = 2$

$a_{n+1} = (-3) \cdot a_n$ $a_0 = 6$ $n \geq 1$

$a_n = \frac{2}{5} a_{n-1}$

$\frac{14}{5} \times 7$

① $a_{n+1} = 1.5a_n$

$a_n = \left(\frac{5}{4}\right)^n a_{n-1}$

$a_{n+1} = 1.5^n a_n$

$a_{n+1} = \left(\frac{4}{3}\right)^n a_0$ $a_1 = 5 = \frac{4}{3} a_0$

$= \cancel{\frac{4}{3} \times \frac{15}{4}}$ $\leftarrow \frac{15}{4}$

$= \left(\frac{4}{3}\right)^n \frac{15}{4}$

$a_n = \frac{3}{2} a_{n-1}$

$= \left(\frac{3}{2}\right)^n 16$

$81 = \left(\frac{3}{2}\right)^4 a_0$

$a_0 = 16$

③ $a_{n+1} = da_n$ $a_3 = 153/49$ $a_5 = 1377/2401$

$a_3 = c^3 a_0$

$\frac{153}{} = 49 a_0 c^3$ ① $1377 = 2401 c^5 a_0$ ②

$\frac{153 = 49 \cancel{a_0} c^3}{1377 = 2401 \cancel{a_0} c^{5}}$

$c^2 = \frac{9}{49}$ $c = \frac{3}{7}$

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(5) $i = \frac{6\%}{4} = \frac{0.06}{4} = 0.015$

$a_0 = 100 \quad a_n = 200$

$200 = 100 \times (1.015)^n$

$2 = (1.015)^n$

$n = 47$

$47 \times 3 = 141$ months

(6) $i = \frac{8\%}{4} = 0.02 \quad a_{15} = 7218.27 \quad c = 1.02$

$7218.27 = (1.02)^{15} \times a_0$

$5363.28 = a_0$

$7218.27 = (1.32)^{15} \times a_0$

$a_0 = 112.156$

Second order

Case 1 : Roots are distinct & real $\quad a_n = Ak_1^n + Bk_2^n$

Case 2 : Roots are equal & real $\quad a_n = (A + Bn)k^n$

Case 3 : Roots are imaginary $p \pm iq$ $\quad a_n = r^n (A\cos n\theta + B\sin n\theta)$

$\theta = \tan^{-1}(q/p) \quad r = \sqrt{p^2+q^2}$

(1) (a) ~~$k_1 = 3 \quad k_2 = 2$~~

~~$a_n = A3^n + B2^n$~~

~~$a_0 = 1 = A + B \quad a_1 = 3 = A3 + B2 \quad A = 1 \quad B = 0$~~

(b) $k_1 = 5 \quad k_2 = \frac{1}{2}$

$a_0 = 2 = A + B \quad a_1 = -6 = 10A + \frac{1}{2}B \quad A = -2 \quad B = 4$

$n+2-n$

① $a_{n+2} + a_n = 0 \quad n \geq 0 \quad a_0 = 0 \quad a_1 = 3$

$a_{n+2} + 0\,a_{n+1} + a_n = 0$

$k^2 + 0k + 1 = 0$

$k = 1i + 0, \; 0 - 1i$

$r = \sqrt{1^2 + 0^2} = \sqrt{1} = 1 \qquad \theta = \tan^{-1}(q/p) = \tan^{-1}\infty = 90$

$(n-1)^3$

$n^3 - 1 \div 3(n)(n)$

$3n(n+1)$

$= 3n^2 + 3n$

$a_n = r^n (A\cos n\theta + B\sin n\theta)$.

$a_0 = 1^0 (A\cos 0^\circ + B\sin 0^\circ) = 1(A\times 1 + B\times 0) = A$

$\boxed{0 = A}$

$= (3)(n)(1)$

$= 3n(n+2)$

$a_1 = 1^{(1)} (A\cos 90^\circ + B\sin 90^\circ) = 1(A\times 0 + \cancel{\sin}\, B\times 1) = B \qquad = 3n^2 + 6n$

$\boxed{3 = B}$

16

2, 3, 2, 5, 4

$(2)^3 (-3)^2 (2)^5$

$a_n = r^n (\overset{0\times 2}{0\cos n\theta} + 3\sin n\theta)$

$= 3 \sin n\theta$

② $a_n + 5a_{n-1} + 5a_{n-2} = 0 \qquad a_0 = 0 \quad a_1 = 2\sqrt{5}$

$k^2 + 5k + 5 = 0$

$k = \frac{-5+\sqrt{5}}{2}, \; \frac{-5-\sqrt{5}}{2}$

$a_n = 2\left(\frac{-5+\sqrt{5}}{2}\right)^n - 2\left(\frac{-5-\sqrt{5}}{2}\right)^n$

case 1

$a_n = AK_1^n + BK_2^n$

$a_0 = A + B = 0$

$a_1 = A\left(\frac{-5+\sqrt{5}}{2}\right) + B\left(\frac{-5-\sqrt{5}}{2}\right)$

$\cancel{2\sqrt{5}}$

$A = -16.944 \qquad B = 16.944 \qquad A = 2$

$2.118 \qquad 2.118 \qquad B = 2$

2.15