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Relation

Set : A well defined unordered collection of district elements

$A = \{1, 2, 3, 4\}$ $B = \{3, 2, 1, 4\}$ $C = \{1, 1, 2, 2, 3, 4\}$ = All are same

Null set / Empty set : A set with no elements.

Denoted as ϕ (Null), $\{\}$

$\{\phi\}$ = This not an empty set, It is set containing null elem

Subset : If every elements of set A is also element of set B then A is subset of B

Ex: $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3, 4, 5, 6\}$ $A \subseteq B$ $\subseteq$ (subset)

$C = \{2, 3\}$ $C \subseteq A$

Trivial subset : A is subset of A. Every set is subset of itself

Proper subset : Any subset of A which is not a trivial subset of A

Ex: $A = \{1, 2, 3, 4\}$ $A' = \{1, 2, 3\}$ $B = \{1\}$

$A' \subset A$ $B \subset A$

2^3, power set

Cardinality : Total no/. of elements in a set

Power set : If 'A' is finite set then set of all subsets of A

If $A = \{1, 2, 3\}$ $P(A) = ?$

$\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

set A
$|A| = n$ $P(A) = 2^n$

Cartesian Product or Cross Product : Let $A = \{a, b\}$ $B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$B \times A = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

$(A \times B) \neq (B \times A)$

$|A| = m$ $|B| = n$ $|A \times B| = m \times n$

Relation: Let A & B are 2 sets then every relation from A to B is subset of $A \times B$

Ex: $A = \{a, b\}$ $B = \{1, 2, 3\}$

$A \times B = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$

$R = \{(a,1)\ (a,2)\}$

$\{(b,1)\ (b,2)\ (b,3)\ (a,3)\}$

$A = m$

$B = n$

$A \times B = m \times n$

Relations $= 2^{m \times n}$

Reflexive Relation: A relation 'R' on set 'A' is said to be reflexive if

$(x, x) \in R \ \forall\ x \in A$

Ex: $A = \{1, 2, 3\}$ $B = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ — ✓

$R_2 = \{(1,1)\ (3,3)\}$ — X Not reflexive $\forall\ x \in A$ is not satisfied

$R_3 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\}$ — ✓ $\forall\ x \in A$ satisfied

diagonal

$A = \{1, 2\}$ then $A \times A = \begin{Bmatrix} (1,1) & (1,2) \\ (2,1) & (2,2) \end{Bmatrix}$

2

2

$R_1 = \{(1,1)\ (2,2)\}$

$R_2 = \{(1,1)\ (2,2)\ (1,2)\}$

$R_3 = \{(1,1)\ (2,2)\ (2,1)\}$

$R_4 = \{(1,1)\ (2,2)\ (2,1)\ (1,2)\}$

$A = \{1, 2, 3\}$ then $A \times A = \begin{Bmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{Bmatrix}$

$A = n$

$A \times A = n^2$

Diagonal $= n$

Relative $= n^2 - n$

Reflexive Relation $= 2^{n^2 - n}$

Not reflexive Relation = Total Relations − Reflexive Relation

Irreflexive Relation: A relation R on a set 'A' is said to be irreflexive if $x \not R\, x$ {x is not related to x} $\forall\ x \in A$ i.e $(x, x) \notin R$

Ex: $A = \{1, 2, 3\}$ 4

$A \times A = \begin{Bmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{Bmatrix}$

Irreflexive $R = \{(1,1)\ (2,2)\}$ = Not Reflexive

$R = \{\}$ Irreflexive

Irreflexive Relation $= 2^{n^2 - n}$

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Symmetric Relation: A Relation R on set A is said to be symmetric if xRy then yRx $\forall x, y \in A$ i.e, if $(x,y) \in R$ then $(y,x) \in R$ $\forall (x,y) \in A$

Ex: $A = \{1, 2, 3, 4\}$

Reflexive is defaultly symmetric

$R_1 = \{(1,2)\ (2,1)\ (2,2)\ (3,1)\ (1,3)\}$ = Symmetric

$R_2 = \{\}$ = Symmetric

Antisymmetric Relation: A relation R is said to be antisymmetric if xRy & yRx then $x = y$ $\forall x, y \in A$

i.e, if $(x,y) \in R$ then $(y,x) \in R$ only if $x = y$ $\forall x, y \in A$

Ex: $A = \{1, 2, 3\}$

$R = \{(1,2)\ (2,2)\ (2,1)\}$ = Symmetric, Not Antisymmetric

$R = \{(1,1)\ (2,2)\ (1,3)\}$ = Antisymmetric, Not Symmetric

$R = \{\}$ = Antisymmetric

Asymmetric Relation: A relation ~~R~~ ~~is~~ said to be asymmetric if xRy & $y \not R x$ then $x = y$ $\forall x, y \in A$

i.e, if $(x,y) \in R$ then $(y,x) \notin R$ $\forall x, y \in A$

Ex: $A = \{1, 2, 3\}$

$R_1 = \{(1,2)\ (1,3)\ (1,1)\}$ Sym = ✗ Anti = ✓ Asy = ✗

$R_2 = \{(1,2)\ (1,3)\ (2,3)\}$ Sym = ✗ Anti = ✓ Asy = ✓

$R_3 = \{\}$ Asymmetric

Note: Every Asymmetric are Antisymmetric but not viceversa

Transitive Relation: A relation R on a set 'A' is said to be transitive if (xRy) & (yRz) then (xRz) $\forall$ $x, y, z \in A$

i.e, If $(x,y) \in R$ & $(y,z) \in R$ then $(x,z) \in R$

$A \to B \quad B \to C$

Ex: $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\}$ — ✓

$R_2 = \{(1,2)\ (2,3)\}$ — ✗

$R_3 = \{\ \}$ = ✓

$R_4 = \{(1,1)\ (1,2)\ (2,1)\}$ ✓

Equivalence Relation: A Relation R on a set 'A' is said to be equivalence if it is reflexive, symmetric & transitive

Ex: $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$ R = ✓ S = ✓ T = ✓

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (2,1)\ (1,2)\}$ R = ✓ S = ✓ T = ✓

$R_3 = \{(1,1)\ (2,2)\ (3,3)\ (3,2)\ (1,3)\}$ R = ✓ S: ✗ T = ✓

$R_4 = \{\ \}$ R ✗ S = ✓ T = ✓

Partial Ordring Relation / Composite: Should be reflexive, Antisymmetric & Transitive

Partial Ordering set (POSET): A set 'A' with Partial ordering relation 'R' defined on 'A' is called POSET, Denoted by $[A, R]$.

Ex: $A = \{1, 2, 3\}$

$R_1 = \{(1,1)\ (2,2)\ (3,3)\}$ R = ✓ A = ✓ T = ✓

$R_2 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,3)\ (1,3)\}$ R = ✓ A = ✓ T = ✓

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① If $A = \{1, 2, 3, 4, 5\}$

a) reflexive, symmetric & not transitive

$\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ \cancel{(2,1)}\ (1,2)\ (2,1)\ (1,5)\ (5,1)\}$

b) reflexive, transitive & not symmetric

$\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (1,2)\ (3,1)\ (5,4)\}$

c) symmetric, transitive & not reflexive

$\{(1,2)\ (2,1)\ (1,1)\ (5,4)\ (4,5)\ (5,5)\}$

Exercise 7.1

④ a) Relation R is reflexive $\forall x \in A,\ (x,x) \in R \quad R \subseteq A$

Relation R is symmetric $\forall x, y \in A \quad (x,y) \in R \Rightarrow (y,x) \in R \quad R \subseteq A$

Relation R is transitive $\forall x, y, z \in A \quad (x,y)\ (y,z) \in R \Rightarrow (x,z) \in R \quad R \subseteq A$

Relation R is antisymmetric $\forall x, y \in A \quad (x,y) \in R,\ (y,x) \quad x = y \quad R \subseteq A$

b) Relation R is not reflexive $\exists x \in A \quad (x,x) \notin R \quad R \subseteq A$

Relation R is not symmetric $\exists x \in A \quad (x,y) \in R \Rightarrow (y,x) \notin R$

Relation R is not transitive $\exists x \in A \quad (x,y)\ (y,z) \in R \Rightarrow (x,z) \notin R$

Relation R is not antisymmetric $\exists x \in A \quad (x,y),\ (y,x) \in R \quad x \neq y$

⑤ a) $R \subset Z^+ \times Z^+$ where aRb if $a | b$

$A = \{1, 2\} \quad B = \{3, 4\}$

$R = \{(2,4)\ (2,6)\ (2,2)\ (3,3)\ (1,1)\ (4,4)\}$

Reflexive, Antisymmetric, Transitive

Totally Ordered set : Poset $[A,R]$ is called a totally ordered set if every pair of elements in A are comparable.

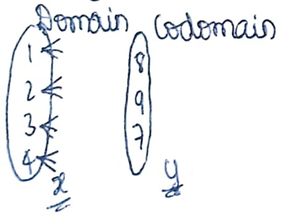
$aRb \quad bRA \quad \forall a,b \in A$

Every TOS is POSET but POSET may not be TOS

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Transitive
Cardinality of smallest relation	n	ϕ	ϕ	ϕ	ϕ	ϕ
Cardinality of largest relation	n^2	n^2-n	n^2	$\frac{n(n+1)}{2}$	$\frac{n^2-n}{2}$	n^2
How many relation	2^{n^2-n}	2^{n^2-n}	$2^n \cdot 2^{\frac{n^2-n}{2}}$	$2^n \cdot 3^{\frac{n(n-1)}{2}}$	$3^{\frac{n(n-1)}{2}}$	←

Functions : A relation 'f' from a set A to a set B is called function if to each element $a \in A$, we can assign unique element of B

$f : A \to B$



Relation $= A \times B = 4 \times 3 = 12$

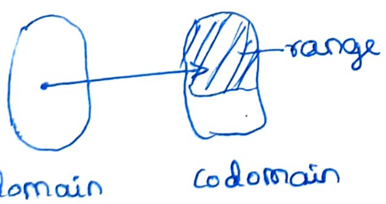
$2^{12} = 2^{xy}$

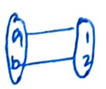
Functions $= 3 \times 3 \times 3 \times 3 = 81 = 3^4$

$y \times y \times y \ldots\ldots x \text{ times} = y^x$

Not Function $=$ Relation $-$ Function

$= 2^{xy} - y^x$



 codomain = range

$\frac{1}{2}$

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Determine R, S, A, T

...-1-2012...

(5) b) R is relation on $\mathbb{Z}$ where a R b if a | b

$\{(-1,-1)\ (-2,-2)\ (1,1)\ (2,2)$

$(1,2)\}$

$\therefore$ Transitive & Antisymmetric

c) For a given universe $\mathcal{U}$ & a fixed subset C of $\mathcal{U}$, define R on $P(\mathcal{U})$ as follows: For $A, B \subseteq \mathcal{U}$ we have ARB if $A \cap C = B \cap C$

$\mathcal{U} = \{1,2,3,4,5\}$ $A = \{2,6,3\}$ $B = \{2,3\}$ $C = \{3,4,5\}$

$A \cap C = 3$ $A \cap B = ✓$

$B \cap C = 3$

(10) $A = \{w, x, y, z\}$

a) reflexive $= 2^{4^2-4} = 2^{12}$

b) symmetric $= 2^4 . 2^{\frac{12}{2}} = 2^{10}$

c) reflexive & symmetric $= 2^{\frac{12}{2}} = 2^6$

d) reflexive & contains (x,y) $= 2^{16-4-1} = 2^{11}$

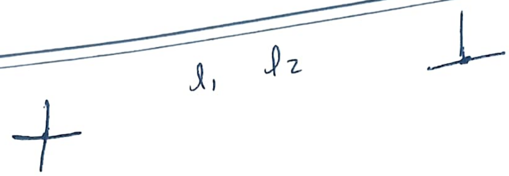
e) symmetric & contains $(x,y) = 2^4 . 2^{\frac{16-4}{2}-1} = 2^4 . 2^5 = 2^9$

f) antisymmetric $= 2^4 . 3^{\frac{16-4}{2}} = 2^4 . 3^6$

g) antisymmetric and contains $(x,y) = 2^4 . 3^{\frac{16-4}{2}-1} = 2^4 . 3^5$

h) symmetric and antisymmetric $= 2^4$

i) reflexive, symmetric and antisymmetric $= 1$

⑤ d) 

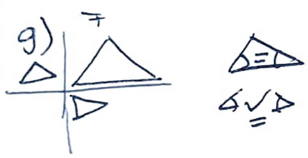
Reflexive.

e) $R = \{ (2,3)\ (3,2)\ (1,0)\ (-1,-2)\ (-2,-1) \}$

Symmetric.

f) $R = \{ (5,5)\ (4,4)\ (3,3)\ (2,2)\ (1,1)\ (0,0)\ (-1,-1)$
$(5,3)\ (3,5)$

Reflexive, Symmetric, Transitive

g) 

~~(7,2)(2,7~~,

h) ~~R_1~~ $A = \{ (1,1)\ (2,2)\ (5,5)\ (1,2)\ (2,5)\ \cancel{(1,5)}\ (5,1)\ (5,2)(2,1) \}$

$B = \{ (5,6)\ \cancel{(6,5)}\ (5,5)\ (6,6) \}$

$R_1 = \{ (1,1)\ \cancel{(2,2)}\ (5,5)\ \cancel{(1,2)}\ (2,5)\ (1,5)\ (5,1)\ (5,2)\ (2,1) \}$

Reflexive, Symmetric, Transitive

$A = \{ (1,1)\ (2,2)\ (6,6)$
$(1,6)\ (1,2)\ (2,1)$
$(6,1)\ (6,2)\ (2,6) \}$

$B = \{ (7,7)\ (2,2)$
$(2,7)\ (7,2) \}$

$R = \{ (1,1)\ (2,2)\ (1,6)\ (1,2)\ (2,1)\ (2,6) \}$

Reflexive, Transitive

⑥ Partial order = a,

Equivalence relation = f,

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(7) $R_1 = \{(1,1)\ (2,2)\ (1,2)\ (2,1)\}$ $R_1 \cap R_2 = \{(2,2)\}$

$R_2 = \{(2,2)\ (3,3)\ (2,3)\ (3,2)\}$

R_1, R_2 reflexive $\Rightarrow R_1 \cap R_2$ reflexive

$x \in A$ $(x,x) \in R_1, R_2$

$(x,x) \in R_1 \cap R_2$

i) Symmetric

$(x,y) \in A$ $(x,y)\ (y,x) \in R_1, R_2$

$(x,y)\ (y,x) \in R_1 \cap R_2$

ii) $(x,y)\ (y,x) \in \cancel{A}$ $x=y$

$(x,y) \in R_1\ R_2$

$(x,y) \in R_1 \cap R_2$

iii) $(x,y)\ (y,z) \in \cancel{R_1 \cap R} R_1 R_2\ (x,z) \in R_1 R_2$

$(x,y)\ (y,z) \in R_1 \cap R_2$ $(x,z) \in R_1 \cap R_2$

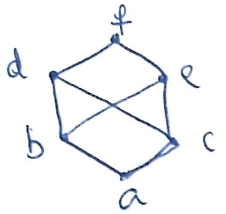
(8) $(x,y) \in R_1 R_2$ $(x,y) \in R_1 \cup R_2$

$(x,y)\ (y,x) \in R_1 R_2$ $(x,y)\ (y,x) \in R_1 \cup R_2$

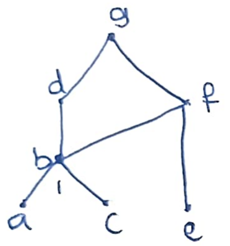
$(x,y)\ (y,x) \in R_1 R_2$ $x=y$ $(x,y)\ (y,x) \notin R_1 \cup R_2$

$(x,y)\ (y,z) \in R_1 R_2$ $(x,z) \in R_1 R_2$ $(x,y)\ (y,z) \notin R_1 \cup R_2$ $(x,z) \notin R_1 \cup R_2$

Minimal Element : An element x of set S is called minimal element if there is no $y \in S$ such that yRx (or $(y, x) \notin R$) and $y \neq x$



minimal : a
maximal : f
least : a
greatest : f



minimal : a, c, e
maximal : g
least : None
greatest : g

Maximal Element : An element x of set S is called maximal element if there is no $y \in S$ such that xRy (or $(x, y) \notin R$) and $x \neq y$

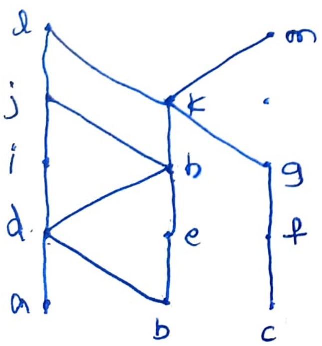
→ A poset can have more than one maximal element.

least element (minimum): An element $x \in S$ is called least if ~~for~~ $\forall y \in S$, xRy. Least element is unique if it exists

greatest element (maximum): An element $x \in S$ is - - - if $\forall y \in S$, yRx
Greatest element is unique if it exists

upper bound : An element $x \in S$ is upper if $\forall y \in T$ $(y, x) \in R$

lower bound : An element $x \in S$ is lower if $\forall y \in T$ $(x, y) \in R$



minimal : a, b, c
maximal : l, m
least : None
greatest : None
lower bounds of {a, b, c} : None
upper bounds of {a, b, c} : k, l, m
lower bound of {f, g, h} : None

LUB of {a, b, c} : k
GLB of {f, g, h} : $\emptyset$

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least ~~lo~~ upper bound (LUB or Supremum or Join or V):

let U is set of all upper bounds of set T. then, an element $x \in U$ is least upper bound if $\forall y \in U$ $(x, y) \in R$.

$LUB(T) = \text{minimum}\{UB(T)\}$

greatest lower bound (GLB or Infimum or Meet or $\wedge$):

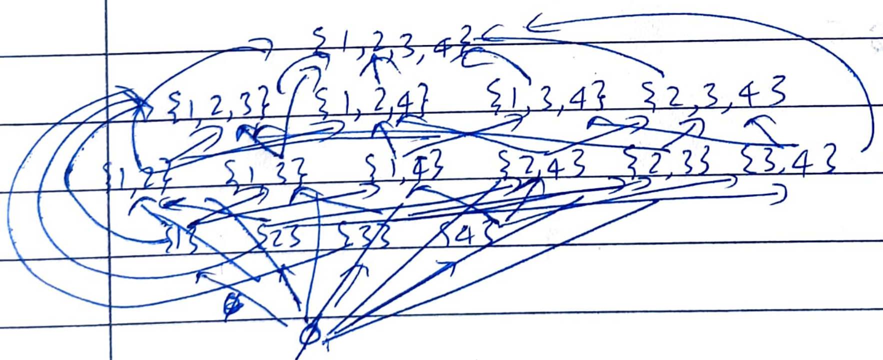
let L is set of all lower bounds of set T. Then an element $x \in U$ is ~~gr~~ GLB if $\forall y \in L$ $(y, x) \in R$.

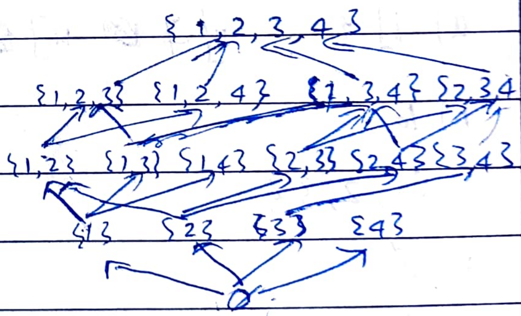
$GLB(T) = \text{maximum}\{LB(T)\}$

7.3 (1) $U = \{1, 2, 3, 4\}$ $(P(U), \subseteq)$

2^4

ϕ, {1} {2} {3} {4} {1,2} {1,3} {1,4} {2,4} {2,3} {3,4} {1,2,3} {1,2,4} {1,3,4} {2,3,4} {1,2,3,4}

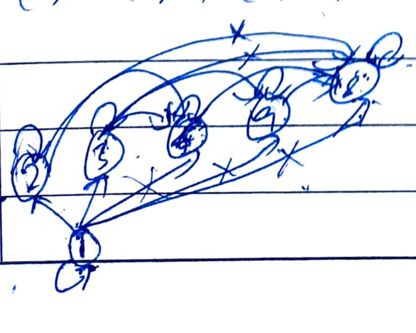


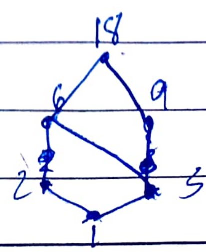


(2) $A = \{1, 2, 3, 6, 9, 18\}$ $x | y$

(1,2) (1,1) (1,3) (1,6) (1,9) (1,18) (2,6) (2,18) (3,6) (3,9) (3,18)

(6,18) (9,18) (18,18) (2,2) (3,3) (6,6) (9,9) (18,18)

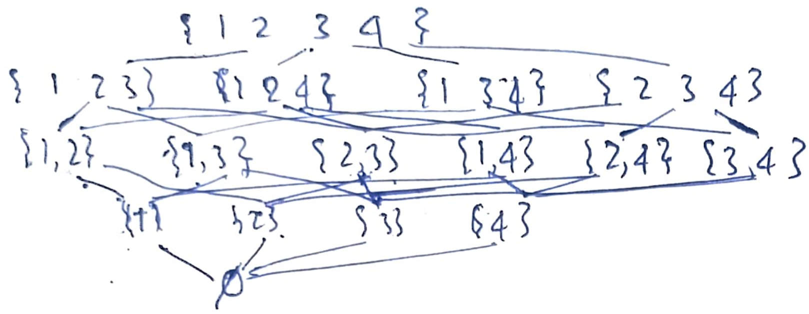




PO = R, A, T

(3) ~~A = { . . . } B = { . . . }~~

(17) U = {1, 2, 3, 4}



(a) {{1} {2}} = LUB = {1,2}
GLB = ∅

(b) B = {{1} {2} {3} {1,2}} LUB = {1,2,3}
GLB = ∅

(c) {∅ {1} {2} {1,2}} LUB = {1,2}
GLB = ∅

(d) {1} {1,2} {1,3} {1,2,3} LUB = {1,2,3}
GLB = {1}

(e) {1} {2} {3} {1,2} {1,3} {2,3} LUB = {1,2,3}
GLB = ∅

(18)

a) i] 1 ii] 4 iii] 5

b) 16

c) {1, 2, 3}

d) ∅

e) ∅

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(1) a) $y = x^2 + 7$ $x = 1 \quad y = 8$

$x = -1 \quad y = 8$

Function, Range = Set of +ve int

b) $y^2 = x$ $y = \sqrt{x}$ $x = 4 \quad y = \pm 2$

Relation

c) $y = 3x + 1$ Function, Range = Set of Real no.: ex:- 1/3

d) $y^2 = 1 - x^2$ $x = \frac{2}{3}$ $y^2 = \frac{9-4}{9} = \frac{5}{9}$

$y = \sqrt{\frac{5}{9}}$ $\pm \frac{\sqrt{5}}{3}$

Relation, ~~Range~~

e) No. of elements

$|A| = 5$ $|B| = 6$ $|R| = 6$ Relation

(2) $f(x) = \frac{1}{(x^2 - 2)}$

if $R \to R$ $x = \frac{-4}{2} = -2$ $f(x) = \frac{1}{4-2} = \frac{1}{2}$ $x = \frac{3}{2}$ $f(x) = \frac{1}{\frac{9-8}{4}} = 4$

if $2 \to R$ $x = 1$ $f(x) = \frac{1}{-1} = -1$

$x = \sqrt{2} = \frac{1}{2-2} = \frac{1}{0} = \infty$

③ a) $\{(1,x)\ (2,y)\ (3,z)\ (4,z)\}$

$\{(2,x)\ (1,x)\ (3,x)\ (4,x)\}$

$\{(2,y)\ (1,y)\ (3,y)\ (4,y)\}$

$\{(2,z)\ (1,z)\ (3,z)\ (4,z)\}$

$\{(1,y)\ (2,y)\ (3,z)\ (4,z)\}$

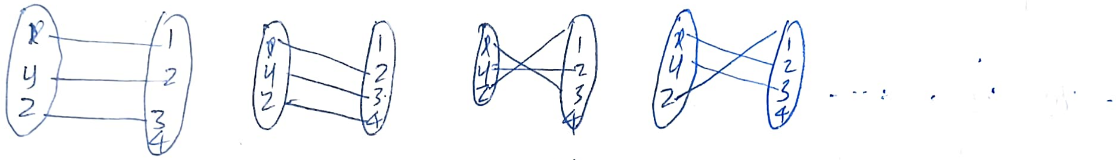
b) Codomain (Domain)

3^4

c) No

d) 4^3

e)



$P(\text{codomain}, \text{domain}) = P(4,3) = \frac{4!}{(4-3)!} = 4! = 24$

f) c^{d-1} ~~for~~ $3^{4-1} = 3^3$

g) $3^{4-2} = 3^2$

h) $3^{4-2} = 3^2$

④ functions = 2187 $\quad |B| = 3$ — no of elements in B $\quad |A| = ?$

$3^x = 2187$ $\qquad 3^{7} = 2187$

~~$\ln x = \ln 2187$~~

$$\begin{array}{r|l} 3 & 2187 \\ 3 & 729 \\ 3 & 243 \\ 3 & 81 \\ 3 & 27 \\ 3 & 9 \\ 3 & 3 \\ & 1 \end{array}$$

7

⑤ $A = \{(x,y) \mid y = 2x+1\}$, $B = \{(x,y) \mid y = 3x\}$ $\quad C = \{(x,y) \mid x - y = 7\}$

a) $A \cap B$

$2x + 1 = 3x$

$\boxed{1 = x}$

$\boxed{y = 3}$ Satisfies

b) $B \cap C$

$x - 7 = 3x$

$-7 = 2x$

$\boxed{x = -\frac{7}{2} \quad y = -\frac{21}{2}}$

Satisfies

c) $\overline{\bar{A} \cup \bar{C}} = \bar{\bar{A}} \cap \bar{\bar{C}} = A \cap C$

$2x + 1 = x - 7$

$\boxed{x = -8}$

$\boxed{y = -15}$

d) $\overline{B} \cup \overline{C} = \overline{B \cap C} \quad x \neq -\frac{7}{2} \quad y \neq -\frac{21}{2}$

⑥ same a) a) ✓ $\quad$ b) ✗ $\quad$ c) ✓ $\quad$ d) ✓

b) a) ✓ $\quad$ b) ✗ $\quad$ c) ✗ $\quad$ d) ✓

$x = n$ $\lfloor x \rfloor = n \le x < n+1$

$\lceil x \rceil = n-1 < x \le n$

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(7) a) $\lfloor 2.3 - 1.6 \rfloor = \lfloor 0.9 \rfloor = 0$

b) $\lfloor 2.3 \rfloor - \lfloor 1.6 \rfloor = 2 - 1 = 1$

c) $\lceil 3.4 \rceil \lfloor 6.2 \rfloor = 4 \times 6 = 24$

d) $\lfloor 3.4 \rfloor \lceil 6.2 \rceil = 3 \times 7 = 21$

e) $\lfloor 2\pi \rfloor = \lfloor 6.28 \rfloor = 6$

f) $2 \lceil \pi \rceil = 2 \lceil 3.1 \rceil = 2 \times 4 = 8$

(8) a) $7 \lfloor x \rfloor = \lfloor 7x \rfloor$

let $\lfloor x \rfloor = n$, $n \le x < n+1$

$7\lfloor x \rfloor = 7n$, $7n \le 7x < \cancel{7(n+1)}\ 7n+1$

$\cancel{7n \le 7x < 7n+7}$

$n \le x < n + \frac{1}{7}$

$\therefore$ x must lie in the interval $(n, n+\frac{1}{7})$

$\{ \ldots (-2, \frac{-13}{7}), (-1, \frac{-6}{7}), (0, \frac{1}{7}), (1, \frac{8}{7}), (2, \frac{15}{7}) \ldots \}$

b) $\lfloor 7x \rfloor = 7$

~~let $\lfloor x \rfloor = n$~~ $\lfloor x \rfloor = n$

$\lfloor 7x \rfloor = 7$, wkt $7x$ is between 7 & 8

$7 \le 7x < 8 \Rightarrow 1 \le x < 8/7$

$x \in (1, 8/7)$

c) $\lfloor x + 7 \rfloor = x + 7$

~~$x+7=n$~~ $\lfloor x+7 \rfloor$ will return integer

So, $x+7$ should also be integer

$\therefore$ x can be anything from Z

Open interval

$[a,b]$ includes a & b

$(-\infty, b]$ includes b not $-\infty$

$[a,\infty)$ includes a not ∞

d) $\lfloor x+7 \rfloor = \lfloor x \rfloor + 7$

~~$x = A$~~ R

(10) $\lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor = \lfloor 2x \rfloor$

(15) a) $f(0) = 1$ $f(1) = 3$ $f(-1) = -1$ (-2)

~~Not~~ one to one Range = Set of odd Integer

b) $f(-\frac{1}{2}) = 2$ $f(-1) = -1$ $f(0) = 1$ $f(\frac{3}{2}) = 4$ $f(\frac{-2}{3}) = \frac{-1}{3}$

One to one Range = Set of Q

c) $f(x) = x^3 - x$ $f(1) = 0$ $f(0) = 0$ ~~Not one~~ $f(-1) = 0$

Not one to one

d) $f(x) = e^x$ $f(0) = 1$ One to One Range = Set of +ve real

e) $f(x) = \sin x$ $-\frac{\pi}{2}(-90)$ to $\frac{\pi}{2}(90) \to R$ One to One Range = ~~R~~ $[-1, 1]$

f) $f(x) = \sin x$ $(0, \pi) \to R$ Not one to one

(16) $f(x) = x^2$

a) $f(A) = f\{2, 3\} = \{4, 9\}$ b) $f(-3, -2, 2, 3) = (9, 4, 4, 9) = \{4, 9\}$

c) $f(-3, 3) = \cancel{\{9\}}\ 9 \Rightarrow [0, 9]$ d) $f((-3, 2]) =$ ~~0, 9~~ $(9, 4)$ - Endpoints $f(x)$ decreases from 9 to 4 $, 9]$

e) $f([7, 2]) = [49, 4] \to$ It is just endpoints $[0, 49]$

(17) 4^3 to 4^5

Extended $= 4^5 - 4^3 = 4^2$

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(10) 6720 : Injective (one to one) function

$|A| = 5$ $|B| = ?$ Since it is injective $|B| \geq 5$

$P(|B|, |A|) = 6720 = \frac{|B|!}{(\cancel{5} - |B|)!}$ $B = 8$

(8) ~~(9)~~ a) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in Z$

True $\lceil 7 \rceil = \lfloor 7 \rfloor = 7$

b) $\lfloor a \rfloor = \lceil a \rceil$ for all $a \in R$

False $\lfloor 8.9 \rfloor = 8$ $\lceil 8.9 \rceil = 9$

c) $\lfloor a \rfloor = \lceil a \rceil - 1$ for all $a \in R - Z$

True $\lfloor 8.9 \rfloor = 8$ $\lceil 8.9 \rceil - 1 = 9 - 1 = 8$

d) $-\lceil a \rceil = \lceil -a \rceil$ for all $a \in R$

False $-\lceil 8.9 \rceil = -9$ $\lceil -8.9 \rceil = -8$

(11) a) $\lceil x \rceil = n$ $n-1 < x \leq n$

$3\lceil x \rceil = 3n$ $3n-1 < 3x \leq 3n$

$n - \frac{1}{3} < x \leq n$

x lies between $(n - \frac{1}{3}, n)$

$\ldots\ldots \frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}, \frac{2}{3}, \frac{8}{3} \ldots\ldots$

b) $\lceil x \rceil = k$ $k-1 < x \leq k$

$n\lceil x \rceil = nk$ $nk-1 < nx \leq nk$

$k - \frac{1}{n} < x \leq k$

~~x~~ $k \in Z^+$ (By Ques)

x lies between $(k - \frac{1}{n}, k)$

$\ldots\ldots \frac{-2n-1}{n}, \frac{-n-1}{n}, \frac{-1}{n}, \frac{n-1}{n}, \frac{2n-1}{n} \ldots\ldots$

(12) $\left\lceil \frac{n}{k} \right\rceil = \left\lfloor \frac{(n-1)}{k} \right\rfloor + 1 \qquad n, k \in Z^+$

If $n = 4 \quad k = 2$

$\left\lceil \frac{4}{2} \right\rceil = \left\lfloor \frac{3}{2} \right\rfloor + 1$

$\lceil 2 \rceil = \lfloor 1.5 \rfloor + 1$

$2 = \underline{\underline{2}}$

(27) a) $A(1,3) = 5$

$A(0, A(~~0~~1, 2))$ 4

$A(0, A(1,1))$ 4 3

~~$A(0, A(2, 0))$~~

$A(0, A(~~0~~1, 0))$ 3

$A(0, 1)$ 2

$A(2,3)$

128

$A(1, A(2,2))$ ~~20~~ 18

$A(0, A(1,17))$ 20

16

15

$A(1, A(2,1))$

$18 = 10 + 8$

$A(0, A(1,7))$

10

$A(0, A(1,6))$

9

$A(0, A(1,5))$ 8

$A(0, A(1,4))$ 7

$A(0, A(1,3))$ 6

$A(0, A(1,2))$ 5

$A(0, A(1,1))$ 4

$A(0, A(1,0))$ 3

$A(0,1)$ 2

$A(1, A(2,0))$

$8 = 5 + 3$

$A(0, A(~~1~~0,2))$ 5

$A(0, A(1,1))$ 4

$A(0, A(1,0))$ 3

$A(0, 1)$ 2

$A(1,1)$ 3

$A(0, A(1,0))$ 3

$A(0, A(1,0))$ 3

$A(0,1)$ 2

$A(0,1)$ 2

$A(1, A(2,0))$ 3

m, n

$A(0, A(1,2))$

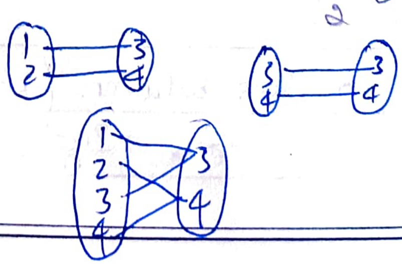
b) ~~$A(1,n) = n+2$ for all $n \in N$~~

~~$A(0, A(1, n-1))$~~

(21) $A = X \cup Y \qquad X \cap Y = \varnothing$

$f|_x$ = One to One $\qquad f|_y$ = One to One

A = Not One to One



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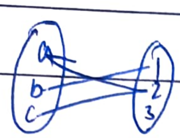
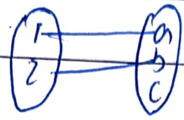
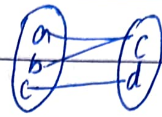
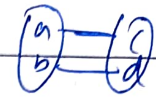
1. a)  b)  c)  d) 

2. a) $f(x) = x+7$

$f(-1) = 6$ $f(-5) = 2$ $f(-6) = 1$ $f(-8) = -1$

One to One & Onto

b) $f(x) = 2x-3$

$f(-1) = -5$ $f(0) = -3$ $f(1) = -1$ $f(2) = 1$ $f(-2) = -7$

One to One Range: Odd no/:

c) $f(x) = -x+5$

$f(-1) = 6$ $f(0) = 5$ $f(1) = 4$ $f(-3) = 8$ $f(3) = 2$

One to One & Onto

d) $f(x) = x^2$

$f(1) = 1$ $f(0) = 0$ $f(-1) = 1$ Neither one to one nor onto

e) $f(x) = x^2 + x$

$f(1) = 2$ $f(2) = 6$ $f(-1) = 0$ $f(0) = 0$ $f(-2) = 2$

Neither one to one nor onto

f) $f(x) = x^3$

One to one Range: Z or $\{ \ldots, -27, -8, -1, 0, 1, 8, 27 \ldots \}$

3. a) $g(x) = x+7$

$g(1) = 8$ $f(-5) = 2$ $f(-1) = 6$ $f(-6) = 1$ $f(-8) = -1$

One to One & Onto

b) $f(x) = 2x-3$

$g(\frac{4}{2}) = 1$ $g(0) = -3$ $g(\frac{-4}{2}) = -1$ One to One & Onto

c) $g(x) = -x+5$

$g(0)=5 \quad g(-1)=6 \quad g(1)=4$ One to One & Onto

$g(6)=-1 \quad g(4)=1$

d) $g(x)=x^2$ Not One to One & Onto

e) $g(x) = x^2+x$

$g(2)=6 \quad g(0)=0 \quad g(-1)=0$ Not One to One & Onto

f) $g(x)=x^3$

$g(-1)=-1 \quad g(1)=1 \quad g(0)=0 \quad g(-2)=-8$ Onto One

④ $|A| = 4 (m) \quad |B| = 6 - (n)$

a) $6^4 = 1296$, $\frac{6!}{(6-4)!} = \frac{6!}{2!} = 6\times5\times4\times3 = 360$

$= \sum_{k=0}^{n=6} (-1)^k \binom{n}{n-k}(n-k)^m$

$= (-1)^0\binom{6}{6}6^4 + (-1)\binom{6}{5}5^4 + (-1)^2\binom{6}{4}4^4 + (-1)^3\binom{6}{3}3^4 + (-1)^4\binom{6}{2}2^4 + (-1)^5\binom{6}{1}2^4 +$
$(-1)^6\binom{6}{0}0^4$

$= +1296 - 3750 + 3840 - 1620 + 240 - 6 + 0 = 0$

b) $|B| = m = 6 \quad |A| = n = 4$

$4^6 = 4096$ $\quad \cancel{\binom{4}{4-6} = \binom{4}{-2} = 0} \quad \frac{4!}{(4-6)!} = \frac{4!}{(-2)!} = 0$

$\sum_{k=0}^{4} (-1)^k \binom{n}{n-k}(n-k)^m = \binom{4}{4}(4)^6 + (-1)\binom{4}{3}3^6 + (-1)^2\binom{4}{2}2^6 + (-1)^3\binom{4}{1}1^6 + (-1)^4\binom{4}{0}0^6$

$= 4096 - 2916 + 384 - 4 + 0 = 1560$

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Ex:-

$$6^8 = \sum_{i=1}^{6} \binom{6}{i}(i!)\,S(8,i)$$

$$RHS = \binom{6}{1}(1!)S(8,1) + \binom{6}{2}(2!)S(8,2) + \binom{6}{3}(3!)S(8,3) + \binom{6}{4}(4!)S(8,4)$$
$$+ \binom{6}{5}(5!)S(8,5) + \binom{6}{6}(6!)S(8,6)$$

$S(8,1)$

$S(7,0) + 1\times S(7,1)$

$S(6,0) + 1\times S(6,1)$

$S(5,0) + 1\times S(5,1)$

$S(4,0) + 1\times S(4,1)$

$S(3,0) + 1\times S(3,1)$

$S(2,0) + 1\times S(2,1)$

$0 + S(1,0) + 1\times(1,1)$

1

$S(8,2) = 127$

$S(7,1) + 2 * S(7,2)$ $\quad 63*2$

$S(6,0)+1\times S(6,1) \quad S(6,1) + 2 * S(6,2)$

$1 \quad 1 \quad 31*2$

$S(5,1) + 2*S(5,2)$

$1 + 15*2$

$S(4,1) + 2*S(4,2)$

$1 + 2*7$

$S(3,1) + 2*(3,2)$

$1 + 2*3$

$S(2,1) + 2*(2,2)$

$1 + 2*1$

$= 6\times1\times1 + 15\times2\times127 + 20\times6\times966 + 15\times24\times1701 + 6\times120\times10500 +$
$+ 1\times720\times266 = 1679616 = 6^8 = LHS$

(5) $\displaystyle\sum_{k=0}^{n} (-1)^k \binom{n}{n-k}(n-k)^m = 0$ for $n=5$

$m=2$

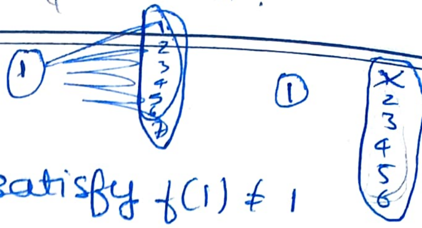
$0 - 80 + 90 - 40 + 5 = -25$ ×

$m=3 \quad 125 - 320 + 270 - 80 + 5 = 0$ ✓

$m=4 \quad 625 - 1280 + 810 - 160 + 5 = 0$ ✓

5.6

Bijective : $\frac{\text{Dom}!}{(\text{Com} - \text{Dom})!}$



① a) 7! bijective functions are there

6! ~~bijective~~ functions are there that satisfy $f(1) \neq 1$

$\therefore (7! - 6!) = 4320$

or

6! = choice for 1 and there 6 remaining elements

$6 \times 6! = 4320$

b) $\{1, 2, 3 \ldots\ldots n\} = \cancel{n! - (n-1)!}\ (n-1)(n-1)!$

② a) $f(x) = 2x + 4$ $\qquad g(x) = \frac{2x^2 - 8}{x+2}$

$\frac{14 \times 2}{28} - 8 = \frac{20}{9}$

~~$A = [-2, 7]$~~

~~$f(7) = 10 \qquad g(7) = \frac{20}{9}$~~

$g(x) = \frac{2(x^2 - 4)}{x+2} = \frac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}} = 2x - 4$

$\therefore f = g$

b) $f(-2) = \frac{2(4) - 8}{-2+2} = \frac{0}{0}$ = Indefinite

So $f \neq g$

③ $(g \circ f)(x) = 9x^2 - 9x + 3$

$(g(f(x))) = 9x^2 - 9x + 3$

$g(ax + b) = 9x^2 - 9x + 3$

$1 - ax + b + (ax+b)^2 = 9x^2 - 9x + 3$

$1 - ax + b + a^2x^2 + b^2 + 2abx = 9x^2 - 9x + 3$

$a^2x^2 - ax + 2abx + b^2 + b + 1 = 9x^2 - 9x + 3$

$(a^2)x^2 - (a - 2ab)x + (b^2 + b + 1) = 9x^2 - 9x + 3$

$a^2 = 9 \qquad a - 2ab = +9$

$a = \pm 3$

$a = 3$

$3 - 6b = 9$

$-6b = 6$

$b = -1$

$a = -3$

$-3 + 6b = 9$

$6b = 12$

$b = 2$

0.166

32

364

682

66

16384

768 - 4

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(4) $g \circ f = g(f)$

If $x=1$ $g(f(1)) = g(2) = 2\times 2 = 4$

$x=2$ $g(f(2)) = g(3) = 2\times 3 = 6$

$x=3$ $g(f(3)) = g(5) = 2\times 5 = 10$

$x=4$ $g(f(4)) = g(7) = 2\times 7 = 14$

(6) $(f\circ g)(x) = f(g(x)) = f(cx+d) = a(cx+d)+b$

$(g\circ f)(x) = g(f(x)) = g(ax+b) = c(ax+b)+d$

$f(g(x)) = g(f(x)) \leftarrow$ By Ques

$a(cx+d)+b = c(ax+b)+d$

$\cancel{acx} + ad + b = \cancel{cax} + cb + d$

$ad + b = cb + d$

(7) a) $f\circ g = f(g(x)) = f(3x) = 3x-1$

$g\circ f = g(f(x)) = g(x-1) = 3(x-1) = 3x-3$

$g\circ h = g(h(x)) = g\begin{cases}0\\1\end{cases} = \begin{cases}0, & \text{if } x \text{ even}\\ 3, & x \text{ odd}\end{cases}$

$h\circ g = h(g(x)) = h(3x) = \begin{cases}0 & \text{if } x \text{ even}\\ 1 & x \text{ odd}\end{cases}$

?? $f\circ(g\circ h) = f(g(h(x))) = f(g\begin{pmatrix}0 & \text{even}\\ 1 & \text{odd}\end{pmatrix}) = f\begin{Bmatrix}0\\3\end{Bmatrix} = \begin{cases}-1 & \text{even}\\ 2 & \text{odd}\end{cases}$

$(f\circ g)\circ h = h(3x-1) = \begin{cases}-1 & \text{even}\\ 2 & \text{odd}\end{cases}$

b) $f^2 = f\circ f = f(f) = f(x-1) = x-1-1 = x-2$

$f^3 = f\circ f^2 = f(f^2) = f(x-2) = x-2-1 = x-3$

$g^2 = g\circ g = g(3x) = 9x$

$g^3 = g\circ g^2 = g(9x) = 27x$

$h^2 = h\circ h = h\begin{pmatrix}0\\1\end{pmatrix} = \begin{matrix}0\\1\end{matrix}$ $\qquad h^3 = h^{500} = \begin{matrix}0\\1\end{matrix}$

(10) a) $f = \{(x,y) \mid 2x+3y=7\}$

$f^{-1} = \{(y,x) \mid 2x+3y=7\}$

$= \{(x,y \mid 2y+3x=7\}$

b) $f = \{(x,y) \mid ax+by=c,\ b\neq 0\}$

$f^{-1} = \{(y,x) \mid ax+by=c,\ b\neq 0\}$

$= \{(x,y) \mid ay+bx=c,\ b\neq 0, a\neq 0\}$

c) $f = \{(x,y) \mid y=x^3\}$ — $x = \sqrt[3]{y} = y^{\frac{1}{3}}$

$f^{-1} = \{(x,y) \mid x=y^3\}$

d) ~~$f = \{(x,y) \mid y = x^4+x\}$~~ X

~~$f^{-1} = \{(x,y) \mid x = y^4+y\}$~~

(12) a) $f^{-1}(\{2\}) = \{1\}$ b) $f^{-1}(\{6\}) = \{2,3,5\}$ c) $f^{-1}(\{6,8\}) = \{2,3,4,5,6)$

d) $f^{-1}(\{6,8,10\}) = \{2,3,4,5,6\}$ e) $f^{-1}(\{6,8,10,12\}) = \{2,3,4,5,6,7\}$ f) $f^{-1}(\{10,12\}) = \{7\}$

~~(13) a) $f^{-1}(-10) = -3$ $f^{-1}(0) = 7$ $f^{-1}(4) = 3$ $f^{-1}(6) = 5$ $f^{-1}(7) = 6$ $f^{-1}(8) = 7$~~

~~b) $f^{-1}([-5,-1]) =$~~

~~(17) a) $f(x) = x+1$~~

~~$f(0)=1$ $f(-1)=0$ $f(1)=2$ $f(2)=3$ $f(-2)=-1$~~

~~Range: $Z^+ - \{1\}$~~

~~b) Yes~~

(17) a) $f(x) = x+1$

$f(1)=2$ $f(2)=3$ Range $= Z^+ - \{1\}$

b) No

c) Yess

d) $g(x) = \max\{1, x-1\}$

$g(1)=1$ $g(2)=1$ $g(3)=2$ $g(4)=3$ Range $= Z^+$

e) Yes

f) No, bcoz $g(1)=g(2)=1$

g) $g(f(x)) = g(x+1) = \max\{1, x+\cancel{1}-\cancel{1}\} = \max\{1,x\} = x$

$\therefore\ g(f) = 1_{Z^+}$