7.2

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0.1 Using Definition

0.1. Plot

$$P_e = \Pr\left(\hat{X} = -1|X = 1\right) \tag{0.1.1}$$

for

$$Y = AX + N, \tag{0.1.2}$$

where A is Raleigh with $E\left[A^2\right] = \gamma, N \sim \mathcal{N}(0,1), X \in (-1,1)$ for $0 \le \gamma \le 10$ dB.

0.2. Assuming that N is a constant, find an expression for P_e. Call this $P_e(N)$

Solution: The estimated value $\hat{X}$ is given by

$$\hat{X} = \begin{cases} +1 & Y > 0 \\ -1 & Y < 0 \end{cases} \tag{0.2.1}$$

For $X = 1$,

$$Y = A + N \tag{0.2.2}$$

$$P_e = \Pr\left(\hat{X} = -1|X = 1\right) \tag{0.2.3}$$

$$= \Pr(Y < 0|X = 1) \tag{0.2.4}$$

$$= \Pr(A < -N) \tag{0.2.5}$$

$$= F_A(-N) \tag{0.2.6}$$

$$= \int_{-\infty}^{-N} f_A(x)dx \tag{0.2.7}$$

By definition

$$f_A(x) = \begin{cases} \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right) & x \ge 0 \\ 0 & otherwise \end{cases} \tag{0.2.8}$$

If $N > 0, f_A(x) = 0$. Then,

$$P_e = 0 \tag{0.2.9}$$

If $N < 0$. Then,

$$P_e(N) = \int_{-\infty}^{-N} f_A(x)dx \tag{0.2.10}$$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{-N} f_A(x)dx \tag{0.2.11}$$

$$= \int_{0}^{-N} \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right)dx \tag{0.2.12}$$

$$= 1 - \exp\left(-\frac{N^2}{2\sigma^2}\right) \tag{0.2.13}$$

Therefore,

$$P_e(N) = \begin{cases} 1 - \exp\left(-\frac{N^2}{2\sigma^2}\right) & N < 0 \\ 0 & otherwise \end{cases} \tag{0.2.14}$$

0.3. For a function g,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)p_X(x)\,dx \tag{0.3.1}$$

Find $P_e = E[P_e(N)]$.

0.4. Plot P_e in problems 0.1 and 0.5 on the same graph w.r.t γ. Comment.

0.5. For a function g,

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)p_X(x)\,dx \tag{0.5.1}$$

Find $P_e = E[P_e(N)]$.

0.6. Plot P_e in problems 0.1 and 0.5 on the same graph w.r.t γ. Comment.