Assignment 5

Amulya Tallamraju - AI20BTECH11003

Download all python codes from

https://github.com/AmulyaTallamraju/Assignment−5/blob/main/Assignment5/codes/Assignment−5.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/Assignment−5/blob/main/Assignment5/Assignment−5.tex

0.1 Using Definition

0.1.1. Let $X_1 \sim \mathcal{N}(0,1)$ and $X_2 \sim \mathcal{N}(0,1)$. Plot the CDF and PDf of

$$V = X_1^2 + X_2^2 \tag{0.1.1.1}$$

Solution: The CDF of V is plotted in 0.1.1

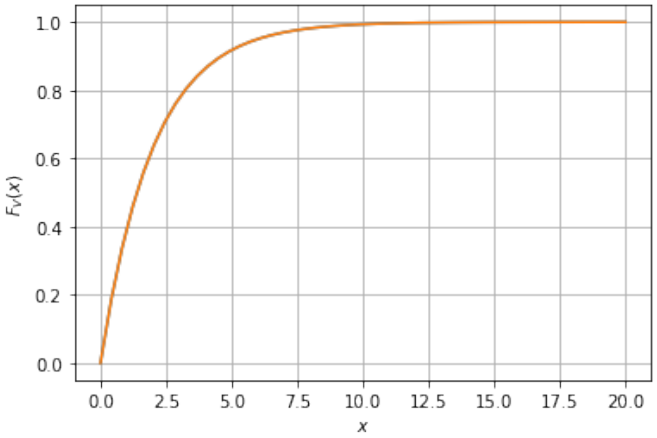

Fig. 0.1.1.1: CDF of V

using the code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.1_CDF.py

The PDf of V is plotted in 0.1.1.2 using the code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.1_PDf.py

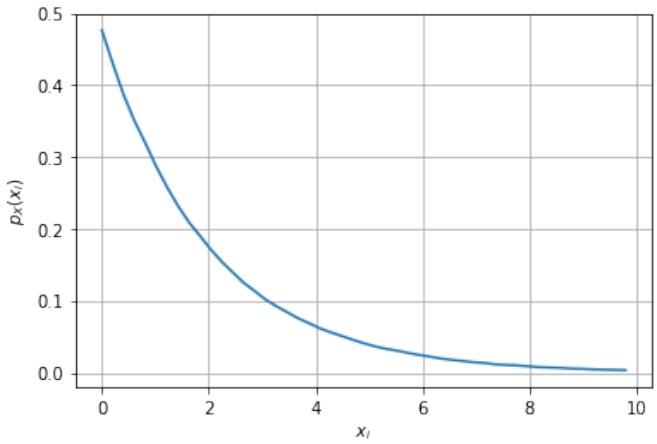

Fig. 0.1.1.2: PDf of V

0.1.2. If

$$F_V(x) = \begin{cases} 1 - e^{-\alpha x} & x \geq 0 \\ 0 & x < 0, \end{cases} \tag{0.1.2.1}$$

find α.

Solution: For the value $\alpha = 0.5$, the theory matches the simulation. The CDF of V is

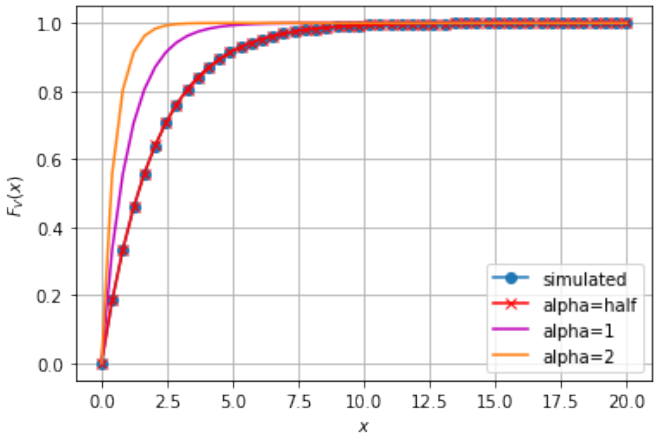

Fig. 0.1.2.1: CDF of V

plotted in 0.1.2.1 using the code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.2.py

0.1.3. Plot the CDF and PDf of

$$A = \sqrt{V} \tag{0.1.3.1}$$

The CDF of A is plotted in 0.1.4.1 using the

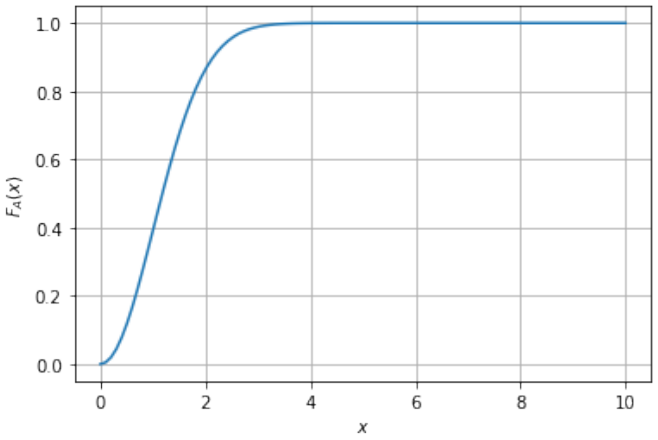

Fig. 0.1.3.1: CDF of A

code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.3_CDF.py

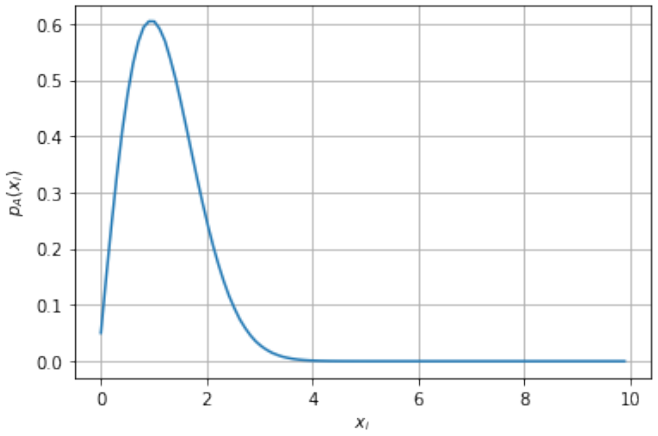

Fig. 0.1.3.2: PDf of V

The PDf of V is plotted in 0.1.1.2 using the code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.3_PDf.py

0.1.4. Find an expression for $F_A(x)$ using the definition. Plot this expression and compare with the result of problem 0.1.3.

Solution:

$$F_A(x) = \Pr(A \le x) = \Pr\left(\sqrt{V} \le x\right) \tag{0.1.4.1}$$

$$= \Pr\left(V \le x^2\right) = F_V\left(x^2\right) \tag{0.1.4.2}$$

From (0.1.2.1),

$$F_V\left(x^2\right) = \begin{cases} 1 - e^{-\alpha x^2} & x \ge 0 \\ 0 & x < 0, \end{cases} \tag{0.1.4.3}$$

Substituting

$$\alpha = \frac{1}{2} \tag{0.1.4.4}$$

$$F_V\left(x^2\right) = \begin{cases} 1 - e^{-\frac{x^2}{2}} & x \ge 0 \\ 0 & x < 0, \end{cases} \tag{0.1.4.5}$$

The CDF of A is plotted in 0.1.4.1 using the

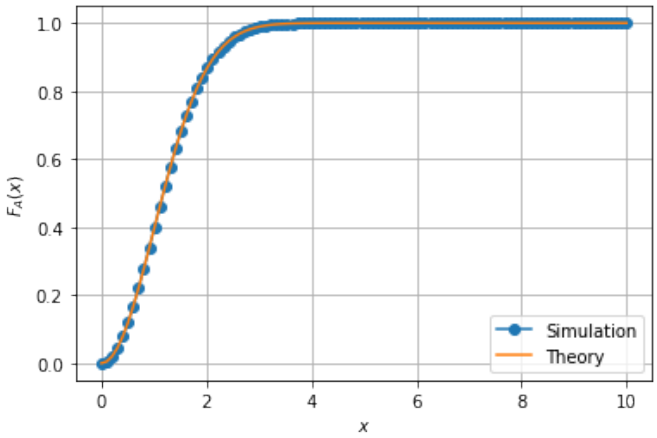

Fig. 0.1.4.1: CDF of A

code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.4.py

0.1.5. Find an expression for $p_A(x)$.

Solution: The PDf is obtained as

$$f_V\left(x^2\right) = \frac{d}{dx} F_V\left(x^2\right) \tag{0.1.5.1}$$

$$= \begin{cases} xe^{-\frac{x^2}{2}} & x \ge 0 \\ 0 & x < 0, \end{cases} \tag{0.1.5.2}$$

The PDf of A is plotted in 0.1.5.1 using the code below.

https://github.com/AmulyaTallamraju/AI1103/blob/main/probman/codes/6.1.5.py

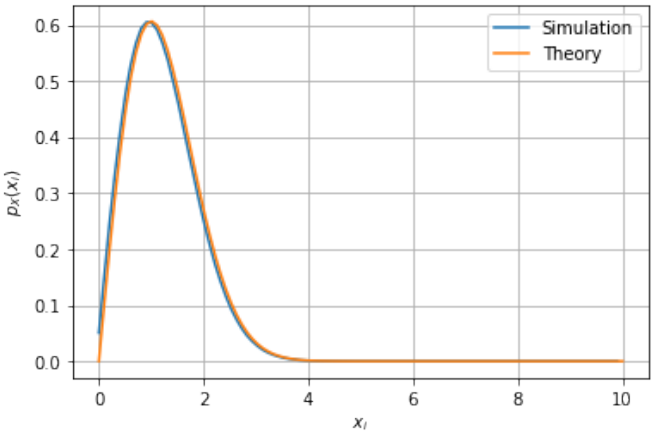

Fig. 0.1.5.1: PDf of A