# AI1103 Project

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# Design and BER Performance Analysis of MIMO and Massive MIMO Networks under Perfect and Imperfect CSI


## Abstract

With upcoming 5G networks, higher data rate and higher capacity are required for a commercial wireless communication system. After sternly affecting bit error rate of communication system, multipath fading in wireless communication system also gives weak signal strength. Multi input- multi output (MIMO) and Massive MIMO system is used to overcome this drawback. This research paper presents the salient features of MIMO and massive MIMO networks and investigates its BER performance under AWGN ( white Gaussian noise) and Rayleigh fading communication channel under the effects of perfect and imperfect channel state information (CS I) modes, along with the consideration of trials and prospects.

# Challenges for wireless communication system.

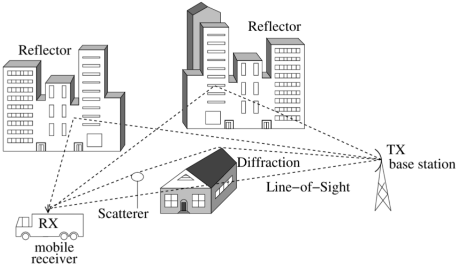


Figure: Multipath propagation

- No guiding medium between transmitter and receiver.
- Multiple signals superpose at the receiver. As a result of destructive interference, strength of signal fades(weakens). This effect is known as fading.

# MIMO

**MIMO - Multiple Input Multiple Output**

The MIMO system leads to a significant increase in the data rates.
It is a combination of Multiple Transmit Antennas at the transmitter and Multiple Receive Antennas at the receiver.
A MIMO system is a collection of a large number of fading Channels one between each transmit antenna and each receive antenna pair.

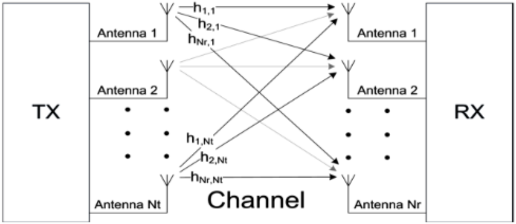


Figure: MIMO BLOCK DIAGRAM

# Massive MIMO

Massive MIMO — which is an extension of MIMO — expands beyond the legacy systems by adding a much higher number of antennas on the base station. The “massive” number of antennas helps focus energy, which brings drastic improvements in throughput and efficiency.

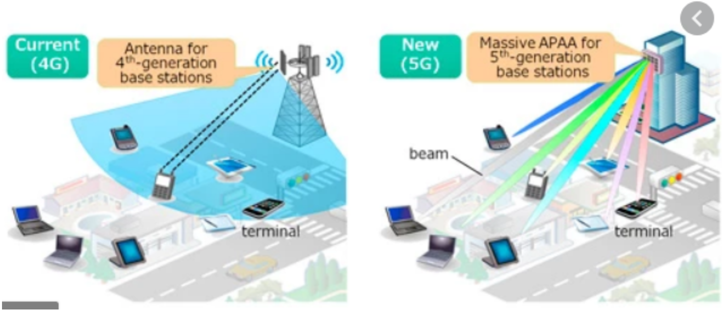


Figure: 4G v/s 5G

# Channel state information

Channel state information (CSI) refers to known channel properties of a communication link. This information describes how a signal propagates from the transmitter to the receiver.
There are basically two levels of CSI, namely instantaneous CSI and statistical CSI.

- **Instantaneous CSI (Perfect)** means that the current channel conditions are known.This can be used to optimize the received signal to achieve low bit error rates.
- **Statistical CSI (Imperfect)** means that a statistical characterization of the channel is known. This description can include, for example, the type of fading distribution.

The CSI acquisition is practically limited by how fast the channel conditions are changing.

# Rayleigh Distribution

The probability density function of the Rayleigh distribution is

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, \quad x \geq 0, \tag{1}$$

where $\sigma$ is the scale parameter of the distribution.

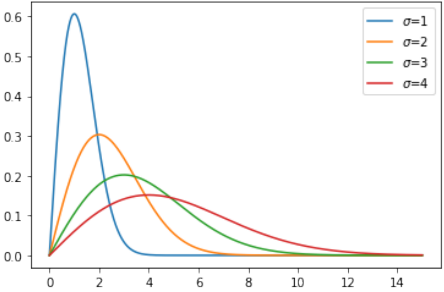


Figure: Pdf of Rayleigh Distribution for different values of $\sigma$

# Rayleigh fading Channel

The fading channel coefficient h depends on factors like attenuation ($a_i$) and time delay($\tau_i$) associated with the channel.

## Modelling the distribution of the fading channel coefficient

$$h = \sum_{i=0}^{L-1} a_i \exp\left(-j2\pi f_c \tau_i\right) \tag{2}$$

$$= \sum_{i=0}^{L-1} a_i \cos\left(2\pi f_c \tau_i\right) - j\sum_{i=0}^{L-1} a_i \sin\left(2\pi f_c \tau_i\right) \tag{3}$$

$$= X + jY \tag{4}$$

where X=$\sum_{i=0}^{L-1} a_i \cos\left(2\pi f_c \tau_i\right)$ and Y=$-\sum_{i=0}^{L-1} a_i \sin\left(2\pi f_c \tau_i\right)$

As X and Y are the sums of a large number of independent random variables, by the central limit theorem X and Y can be assumed to be Gaussian distributed random variables.

## Joint Pdf of X and Y

$$X, Y \sim \mathcal{N}(\mu, \sigma^2) \tag{5}$$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-(x-\mu)^2}{2\sigma^2} \tag{6}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi\sigma^2}} exp \frac{-(y-\mu)^2}{2\sigma^2} \tag{7}$$

Assuming X and Y are independent random variables and substituting $\mu = 0$ for simplification

$$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2} exp\Big(-(x^2 + y^2)\Big) \tag{8}$$

## Distribution of the fading channel in terms of its Amplitude and phase using Jacobian

$$h = x + jy = ae^{j\phi} \tag{9}$$

$$a = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x} \tag{10}$$

$$f_{A,\Phi}(a,\phi) = f_{XY}(x,y)\left|J_{XY}\right| \tag{11}$$

$$\left|J_{XY}\right| = \begin{vmatrix} \frac{\partial x}{\partial a} & \frac{\partial y}{\partial a} \\ \frac{\partial x}{\partial \phi} & \frac{\partial y}{\partial \phi} \end{vmatrix} = \begin{vmatrix} \cos\phi & \sin\phi \\ -a\sin\phi & a\cos\phi \end{vmatrix} \tag{12}$$

from (8),(10) and (12) we get

$$f_{A,\Phi}(a,\phi) = \frac{a}{2\pi\sigma^2}\exp\left(\frac{-a^2}{2\sigma^2}\right) \tag{13}$$

## Marginal distribution of A

$$f_A(a) = \int_{-\pi}^{\pi} f_{A,\Phi}(a, \phi)\, d\phi \tag{14}$$

$$= \int_{-\pi}^{\pi} \frac{a}{2\pi\sigma^2} \exp\left(\frac{-a^2}{2\sigma^2}\right) d\phi \tag{15}$$

$$= \frac{a}{\sigma^2} \exp\left(\frac{-a^2}{2\sigma^2}\right) \tag{16}$$

Thus the coefficient follows the Rayleigh distribution and is fading in nature. It is therefore called as a Rayleigh fading channel.

# AWGN

Additive white Gaussian noise (AWGN) is a basic noise model used in information theory to mimic the effect of many random processes that occur in nature. The modifiers denote specific characteristics:

- Additive because it is added to any noise that might be intrinsic to the information system.
- White refers to the idea that the noise has the same power distribution at every frequency.
- Gaussian because it has a normal distribution in the time domain with an average time domain value of zero.

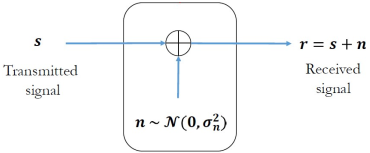


Figure: AWGN Channel

# BPSK Modulation

## BPSK

BPSK stands for binary phase shift keying in which the information symbol 0 is modulated with an amplitude level $\sqrt{P}$ and the information symbol 1 is modulated with an amplitude level - $\sqrt{P}$. So, we have 2 voltage levels $\sqrt{P}$ and - $\sqrt{P}$ . So, there are 2 phases the phase of $\sqrt{P}$ is 0 and the phase of - $\sqrt{P}$ is 180 degrees. At the receiver:

If received symbol $\geq 0 \rightarrow 0$

If received symbol $< 0 \rightarrow 1$

# BER

Bit error rate (BER) is defined as the percentage of bits that have errors relative to the total number of bits received in a transmission.

## AWGN Channel

The symbols 0 and 1 are equiprobable hence it suffices to calculate the bit error rate as the probability that the received signal is 1 when the transmitted signal is 0.

$$y = x + n \tag{17}$$

$$P_e = \Pr\left(\sqrt{P} + n < 0\right) \tag{18}$$

$$= \Pr\left(\sqrt{P} < -n\right) = \Pr\left(\sqrt{P} < n\right) \tag{19}$$

Since n has a symmetric pdf. Let $w \sim \mathcal{N}(0,1)$.Then $n = \sigma w$

$$P_e = \Pr\left(\sqrt{\frac{P}{\sigma^2}} < w\right) \tag{20}$$

## BER of AWGN in terms of Q function

$$P_e = \Pr\left(\sqrt{\frac{P}{\sigma^2}} < w\right) \tag{21}$$

$$= Q\left(\sqrt{\frac{P}{\sigma^2}}\right) \tag{22}$$

$$= Q\left(\sqrt{SNR}\right) \tag{23}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du. \tag{24}$$

$$SNR = \frac{P_{signal}}{E[N^2]} = \frac{P}{\sigma^2} \tag{25}$$

# BER Analysis for Rayleigh Fading Channel

For a wireless communication system

$$y = hx + n \tag{26}$$

where h is the fading channel coefficient following Rayleigh distribution and n is awgn.
Received signal power = $|h|^2 P$ but $h = ae^{j\phi}$. Therefore, we have Received power = $a^2 P$

$$SNR_F = \frac{a^2 P}{\sigma^2} = a^2 SNR \tag{27}$$

From (23)

$$P_e(a) = Q(\sqrt{SNR_F}) \tag{28}$$

$$= Q(\sqrt{a^2 SNR}) \tag{29}$$

## BER expression for rayleigh fading channel

But, because of the random nature of the fading channel coefficient $a^2SNR$ is also a random quantity. Therefore, average bit error rate is given by

$$P_e = E[P_e(A)] \tag{30}$$

$$= \int_0^\infty Q(\sqrt{a^2SNR}) f_A(a)\, da \tag{31}$$

$$= \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2 + SNR}}\right) \tag{32}$$

# BER Analysis

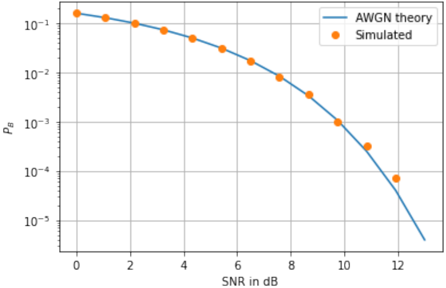


Figure: BER of AWGN under BPSK modulation

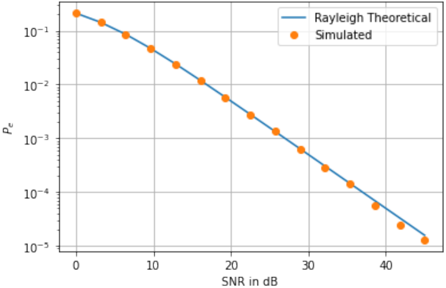


Figure: BER of Rayleigh fading channel with AWGN

# ZF and MRC

## Zero Forcing

This is used for the retrieval of the original signal by choosing $\bar{x}$ that minimises $||\bar{y} - H\bar{x}||^2$ . The main aim of Zero forcing is to improve the performance of the system by bringing the inter-symbol interference to zero.

## Maximum Ratio Combining

This technique is employed to maximise the SNR. The output is taken as the weighted output of various received signals. For maximising the SNR, the weight vector is taken to be proportional to the channel coefficients vector.

# Conclusion

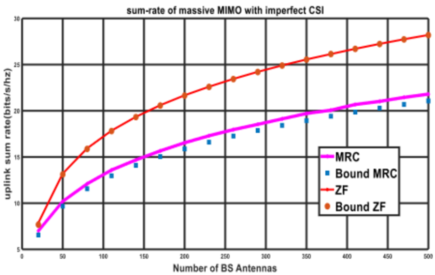


Figure: sum rates of massive mimo imperfect CSI

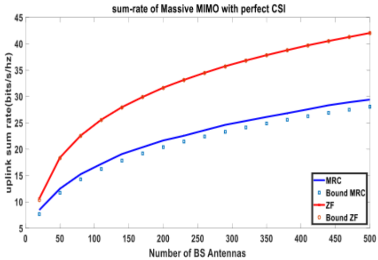


Figure: sum rates of massive mimo perfect CSI

MIMO system performance has increased under ZF mode when seen both under perfect and imperfect CSI.