

Assignment 2

Amulya Tallamraju - AI20BTECH11003

Download all python codes from

<https://github.com/AmulyaTallamraju/Assignment-2/blob/main/Assignment2/codes/Assignment-2.py>

and latex-tikz codes from

<https://github.com/AmulyaTallamraju/Assignment-2/blob/main/Assignment2/Assignment-2.tex>

GATE- PROBLEM NO.4

Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place)

SOLUTION

Let

$$X_1, X_2, X_3 \in \{1, 2, 3, 4, 5, 6\} \quad (0.0.1)$$

represent the three dice.

Since, all the three are fair dice, the probability of any dice showing a particular number is given by

$$\Pr(X = i) = \begin{cases} \frac{1}{6} & i=1,2,3,4,5,6 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.2)$$

If all the dice show a particular number i ,

$$\implies \Pr(X_1 = X_2 = X_3 = i) \quad (0.0.3)$$

Since the events are independent,

$$\begin{aligned} \Pr(X_1 = X_2 = X_3 = i) \\ = \Pr(X_1 = i) \Pr(X_2 = i) \Pr(X_3 = i) \end{aligned} \quad (0.0.4)$$

where $i=1,2,3,4,5,6$.

There are 6 faces on a cubical dice. Hence, there are six cases in which all the dice show the same number

$$\Pr(X_1 = X_2 = X_3) = \sum_{i=1}^6 \Pr(X_1 = X_2 = X_3 = i) \quad (0.0.5)$$

From (0.0.4), we have

$$\begin{aligned} \Pr(X_1 = X_2 = X_3) \\ = \sum_{i=1}^6 \Pr(X_1 = i) \Pr(X_2 = i) \Pr(X_3 = i) \end{aligned} \quad (0.0.6)$$

$$= \sum_{i=1}^6 \left(\frac{1}{6}\right) \left(\frac{1}{6}\right) \left(\frac{1}{6}\right) \quad (0.0.7)$$

$$= \frac{1}{36} \quad (0.0.8)$$

Therefore, the probability that all the three dice have the same number of dots on the face showing up is 0.028.