

# Assignment 6

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Download all python codes from

<https://github.com/AmulyaTallamraju/Assignment-6/blob/main/Assignment6/codes/Assignment-6.py>

and latex-tikz codes from

<https://github.com/AmulyaTallamraju/Assignment-6/blob/main/Assignment6Assignment-6.tex>

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Two random variables  $X$  and  $Y$  are distributed according to

$$f_{XY}(x, y) = \begin{cases} x + y & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.1)$$

The probability  $P(X + Y \leq 1) =$

SOLUTION

We know that

$$P((X, Y) \in A) = \iint_A f_{XY}(x, y) dx dy \quad (0.0.2)$$

$$0 < X + Y \leq 1 \implies 0 < Y \leq 1 - X \quad (0.0.3)$$

so using (0.0.2)

$$\Pr(X + Y \leq 1) = \Pr(Y \leq 1 - X) \quad (0.0.4)$$

$$= \int_0^1 \int_0^{1-x} f_{XY}(x, y) dy dx \quad (0.0.5)$$

$$= \int_0^1 \int_0^{1-x} (x + y) dy dx \quad (0.0.6)$$

$$= \int_0^1 \left[ \left( \frac{y^2}{2} + xy \right) \right]_0^{1-x} dx \quad (0.0.7)$$

$$= \int_0^1 \left( \frac{1 - x^2}{2} \right) dx \quad (0.0.8)$$

$$= \left[ \left( \frac{x}{2} - \frac{x^3}{6} \right) \right]_0^1 dx \quad (0.0.9)$$

$$= \frac{1}{3} \quad (0.0.10)$$