Assignment 6

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Download all python codes from

```
https://github.com/AmulyaTallamraju/Assignment
    -6/blob/main/Assignment6/codes/Assignment
    -6.py
```

and latex-tikz codes from

```
https://github.com/AmulyaTallamraju/Assignment
    -6/blob/main/Assignment6Assignment-6.tex
```

GATE 2016 EC - Q.28

Two random variables X and Y are distributed according to

$$f_{XY}(x,y) = \begin{cases} x+y & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & otherwise \end{cases} \tag{0.0.1}$$

The probability $P(X+Y \le 1)$=

Solution

Finding the marginal Pdf of X and Y

$$f_X(x) = \int_0^1 f_{X,Y}(x,y)\, dy \tag{0.0.2}$$

$$= \int_0^1 (x+y)\, dy \tag{0.0.3}$$

$$f_Y(y) = \int_0^1 f_{X,Y}(x,y)\, dx \tag{0.0.4}$$

$$= \int_0^1 (x+y)\, dx \tag{0.0.5}$$

$$\tag{0.0.6}$$

We get

$$f_X(x) = \begin{cases} x+1/2 & 0 \le x \le 1 \\ 0 & otherwise \end{cases} \tag{0.0.7}$$

$$f_Y(y) = \begin{cases} y+1/2 & 0 \le y \le 1 \\ 0 & otherwise \end{cases} \tag{0.0.8}$$

Finding the marginal Cdf of Y

$$F_Y(y) = \int_0^y f_Y(t)\, dt \tag{0.0.9}$$

$$= \int_0^y \left(t + \frac{1}{2}\right) dt \tag{0.0.10}$$

$$\tag{0.0.11}$$

We get

$$F_Y(y) = \begin{cases} 0 & y \le 0 \\ \frac{y^2+y}{2} & 0 \le y \le 1 \\ 1 & otherwise \end{cases} \tag{0.0.12}$$

$$\Pr(X+Y \le 1) = \Pr(Y \le 1-X) \tag{0.0.13}$$

$$= \int_0^1 \Pr(Y \le 1-x | X=x)\, f_X(x)\, dx \tag{0.0.14}$$

$$= \int_0^1 F_Y(1-x) \times f_X(x)\, dx \tag{0.0.15}$$

$$= \int_0^1 F_Y(x) \times f_X(1-x)\, dx \tag{0.0.16}$$

$$= \int_0^1 \left(\frac{x+x^2}{2}\right)\left(\frac{3-2x}{2}\right) dx \tag{0.0.17}$$

$$= \int_0^1 \left(\frac{3x+x^2-2x^3}{4}\right) dx \tag{0.0.18}$$

$$= \frac{1}{4}\left[\frac{3x^2}{2} + \frac{x^3}{3} - \frac{2x^4}{4}\right]_0^1 \tag{0.0.19}$$

$$= \frac{1}{3} \tag{0.0.20}$$