# QUIZ2

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/QUIZ2/codes/QUIZ2.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/QUIZ2/2.tex

## 1 3.22 (b)

Consider an LTI system that is stable anf for which $H(z)$, the $z-$Transform of the impulse response is given by

$$H(z) = \frac{3}{1+\frac{1}{3}z^{-1}} \tag{1.0.1}$$

Suppose $x[n]$, the input to the system, is a unit step sequence.

1) Find the output $y[n]$ by computing the inverse $z-$ transform of $Y(z)$.

## 2 Solution

**Theorem 2.1** (Convolution Theorem)**.** *Let $f$ and $g$ be two functions with convolution $f * g$. Let $F$ be the Fourier transform operator. Then*

$$F(f * g) = F(f) \cdot F(g) \tag{2.0.1}$$
$$F(f \cdot g) = F(f) * F(g) \tag{2.0.2}$$

$$y[n] = h[n] * x[n] \tag{2.0.3}$$

Using 2.1

$$Y(z) = H(z)X(z) \tag{2.0.4}$$

$\mathcal{Z}$ transform of $x[n]$ is given by

$$X(z) = \mathcal{Z}(x[n]) \tag{2.0.5}$$
$$= \sum_{-\infty}^{\infty} (z^{-1})^n u[n] \tag{2.0.6}$$
$$= \sum_{0}^{\infty} (z^{-1})^n \tag{2.0.7}$$

The ROC of $X(z)$ is $|z| > 1$ and in that region

$$X(z) = \frac{1}{1-z^{-1}} \tag{2.0.8}$$
$$\implies Y(z) = \frac{3}{1+\frac{1}{3}z^{-1}} \frac{1}{1-z^{-1}} \tag{2.0.9}$$
$$= \frac{\frac{3}{4}}{1+\frac{1}{3}z^{-1}} + \frac{\frac{9}{4}}{1-z^{-1}} \tag{2.0.10}$$

The $\mathcal{Z}$ transform for a sequence of the form $a^n u[n]$ is given by

$$\sum_{n=-\infty}^{\infty} \left(az^{-1}\right)^n u[n] = \left((1-az^{-1})^{-1}\right) \tag{2.0.11}$$

where $|a| < |z|$ denotes the ROC. Thus, Inverse $\mathcal{Z}$ transform of $Y(z)$] is given by

$$y[n] = \frac{3}{4}\left(\frac{-1}{3}\right)^n u[n] + \frac{9}{4}u[n] \tag{2.0.12}$$
$$= \frac{9}{4}\left(1 - \left(\frac{-1}{3}\right)^{n+1}\right)u[n] \tag{2.0.13}$$

Using countour integral The inverse Z-transform of X(z) is defined as

$$x[n] = \frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz \tag{2.0.14}$$

where c is a counter clockwise contour in the ROC of X(z) encircling the origin. $\implies x[n] = \sum$[residues of $X(z)z^{n-1}$ at the poles inside c]

$$Y(z)z^{n-1} = z^{n-1}\frac{9z^2}{(3z+1)(z-1)} \tag{2.0.15}$$

There are two poles at $z = \frac{-1}{3}$ and $z = 1$

$$R(z = 1) = (z-1)z^{n-1}\frac{9z^2}{(3z+1)(z-1)} \qquad = \frac{9}{4} \tag{2.0.16}$$

$$R\left(z = \frac{-1}{3}\right) = \left(z + \frac{1}{3}\right)z^{n-1}\frac{9z^2}{(3z+1)(z-1)} \tag{2.0.17}$$

$$= \frac{3}{4}\left(\frac{-1}{3}\right)^n \tag{2.0.18}$$

Thus, for $n \geq 0$

$$Y(z) = \frac{3}{4}\left(\frac{-1}{3}\right)^n + \frac{9}{4} \tag{2.0.19}$$

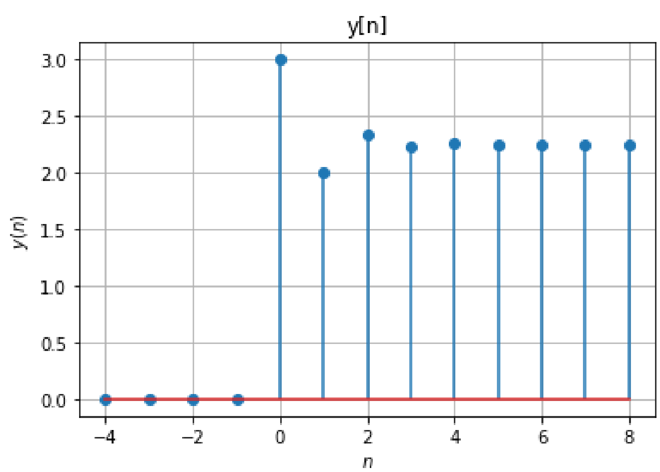


Fig. 1: Plot of y[n]