

GATE ASSIGNMENT 1

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-1/codes/GATE_Assignment-1.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-1/GATE_Assignment-1.tex

1 VECTORS 2.3

Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega}) d\omega \quad (1.0.1)$$

is

2 SOLUTION

Theorem 2.1 (Convolution Theorem). *Let f and g be two functions with convolution $f * g$. Let F be the Fourier transform operator. Then*

$$F(f * g) = F(f) \cdot F(g) \quad (2.0.1)$$

$$F(f \cdot g) = F(f) * F(g) \quad (2.0.2)$$

If the DTFT of $y[n]$ is $Y(e^{j\omega})$ then using the time reversal property, the DTFT of $y[-n]$ is

$$Y\left(\frac{1}{e^{j\omega}}\right) = Y(e^{-j\omega}) \quad (2.0.3)$$

Let

$$z[n] = x[n] * y[-n] \quad (2.0.4)$$

Let $Z(e^{j\omega})$ be the DTFT of $z[n]$. Using Theorem 2.1 we get

$$Z(e^{j\omega}) = X(e^{j\omega})Y(e^{-j\omega}) \quad (2.0.5)$$

By applying IDTFT, we can write:

$$z[n] = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega n} d\omega \quad (2.0.6)$$

Putting $n = 0$, we get

$$z[0] = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega}) d\omega \quad (2.0.7)$$

which is the required value. From (2.0.4) we have

$$z[n] = x[n] * y[-n] \quad (2.0.8)$$

$$= \sum_{k=-\infty}^{\infty} x[k]y[-n+k] \quad (2.0.9)$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1}u[-k+2]2^{n-k+2}u[-n+k+1] \quad (2.0.10)$$

$$= \sum_{k=-\infty}^{\infty} 2^{n+1}u[-k+2]u[-n+k+1] \quad (2.0.11)$$

The unit step function is defined as

$$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases} \quad (2.0.12)$$

From (2.0.11), putting $n = 0$, we get

$$z[0] = \sum_{k=-\infty}^{\infty} 2^1 u[-k+2]u[k+1] \quad (2.0.13)$$

$$= \sum_{k=-1}^2 2 \quad (2.0.14)$$

$$= 8 \quad (2.0.15)$$

Therefore, the value of the integral

$$\frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega}) d\omega = 8 \quad (2.0.16)$$

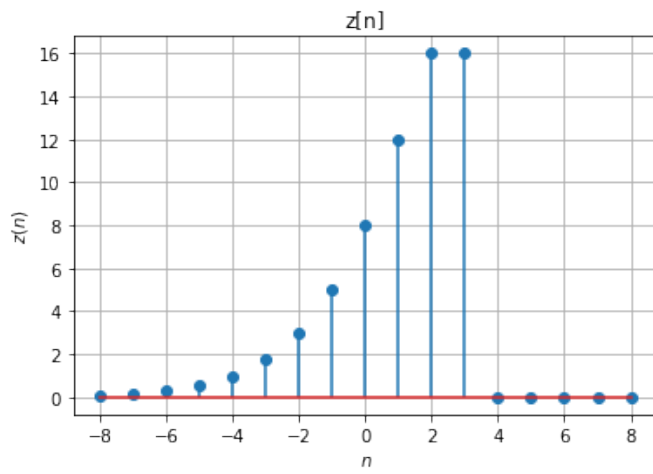


Fig. 0: Plot of $z[n]$