

GATE ASSIGNMENT 1

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-2/codes/GATE_Assignment-2.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-2/GATE_Assignment-2.tex

1 GATE EC 2005 Q.5

The function $x(t)$ is shown in figure. Even and odd parts of a unit step function $u(t)$ are given by

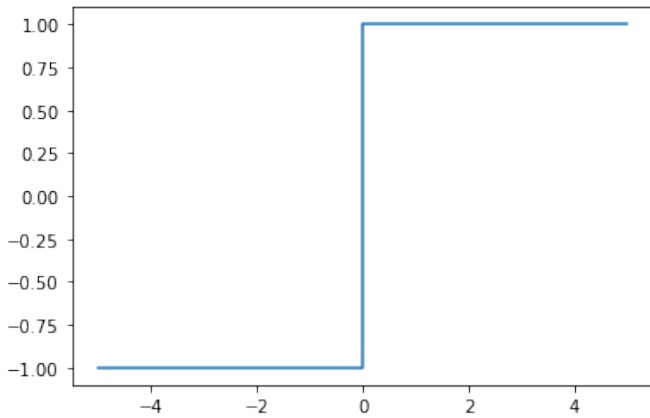


Fig. 0: Plot of $x[t]$

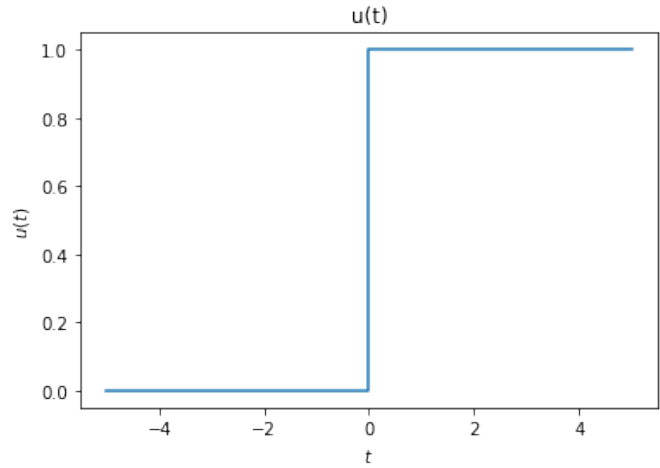


Fig. 0: Plot of $u[t]$

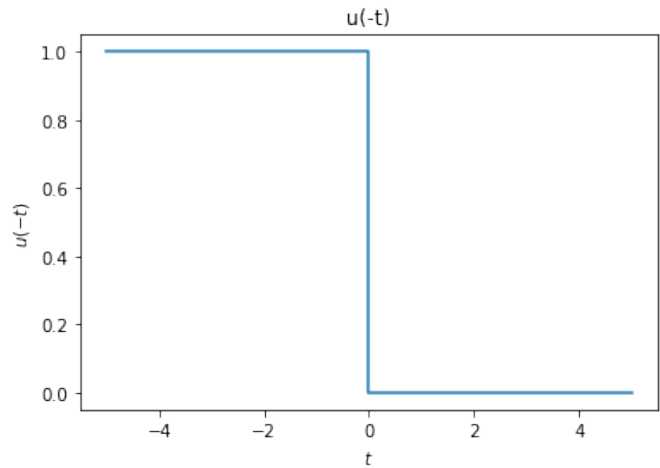


Fig. 0: Plot of $u[t]$

2 SOLUTION

$$x(t) = \begin{cases} -1 & x \leq 0 \\ 0 & 0 \leq x = 0 \\ 1 & x \geq 0 \end{cases} \quad (2.0.1)$$

From the above definition of $x(t)$ we can see that it is the same as $\text{sgn}(t)$. Odd part of $u(t)$ is given by

$$\frac{u(t) - u(-t)}{2} \quad (2.0.2)$$

One observing the plots of $x(t)$, $u(t)$, $-u(-t)$ we can see that

$$x(t) = u(t) - u(-t) \quad (2.0.3)$$

Thus, the odd part of $u(t)$ is $\frac{x(t)}{2}$. The even part of $u(t)$ is given by

$$\frac{u(t) + u(-t)}{2} = \frac{1}{2} \quad (2.0.4)$$

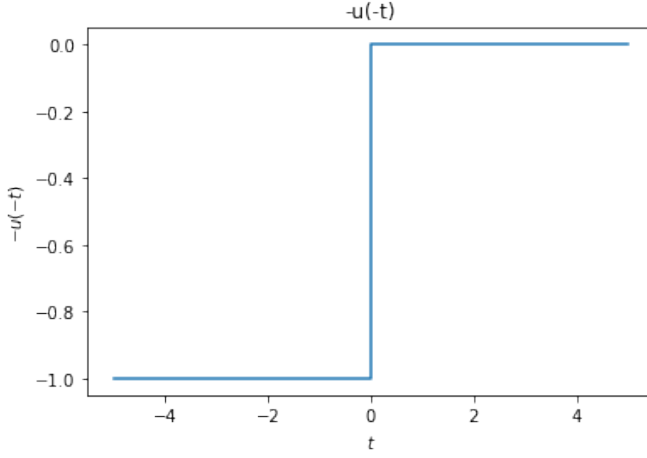


Fig. 0: Plot of $-u[-t]$

Thus the even and odd parts of the unit step signal are

$$\frac{1}{2}, \frac{x(t)}{2} \quad (2.0.5)$$

The fourier transform of $x(t)$ is given by

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt \quad (2.0.6)$$

This signal is not absolutely integrable so we calculate Fourier Transform of $x(t)$ as a limiting case of the sum of exponential $e^{-at}u(t) - e^{at}u(t)$ as $a \rightarrow 0$.

$$X(\omega) = \lim_{a \rightarrow 0} \int_{-\infty}^{+\infty} (e^{-at}u(t) - e^{at}u(t)) e^{-j\omega t} dt \quad (2.0.7)$$

$$= \lim_{a \rightarrow 0} \left[\frac{1}{a + j\omega} - \frac{1}{a - j\omega} \right] \quad (2.0.8)$$

$$= \frac{2}{j\omega} \quad (2.0.9)$$

Laplace transform of $x(t)$

$$\mathcal{L}\{x\}(s) = \int_0^{\infty} x(t)e^{-st} dt. \quad (2.0.10)$$

$$= \frac{1}{s} \quad (2.0.11)$$