ASSIGNMENT 2

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−2/codes/Assignment−2.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−2/Assignment−2.tex

1 Matrices 2.4 (iv)

Let $A = \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix}$. Find AB

2 Solution

$$AB = \begin{pmatrix} (2\times 1) + (4\times(-2)) & (2\times 3) + (4\times 5) \\ (3\times 1) + (2\times(-2)) & (3\times 3) + (2\times 5) \end{pmatrix} \tag{2.0.1}$$

$$= \begin{pmatrix} -6 & 26 \\ -1 & 19 \end{pmatrix} \tag{2.0.2}$$

Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{2.0.3}$$

$$B = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \tag{2.0.4}$$

For the multiplication to be commutative,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{2.0.5}$$

$$\implies \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} = \begin{pmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{pmatrix} \tag{2.0.6}$$

From (2.0.6) we get

$$ae + bg = ae + cf \implies bg = cf \tag{2.0.7}$$

$$af + bh = be + df \tag{2.0.8}$$

$$ce + dg = ag + ch \tag{2.0.9}$$

$$cf + dh = bg + dh \implies cf = bg \tag{2.0.10}$$

Simplifying the above equations further we have

$$bg = cf \tag{2.0.11}$$

$$f(a-d) = b(e-h) \tag{2.0.12}$$

$$g(a-d) = c(e-h) \tag{2.0.13}$$

The above equations must hold true for the multiplication of the matrices to be commutative. We have three cases

1) $a! = d$ and $e! = h$. Dividing (2.0.12) by (2.0.13)

$$\frac{f}{g} = \frac{b}{c} \tag{2.0.14}$$

$$\implies fc = bg \tag{2.0.15}$$

2) $a = d$. Substituting in (2.0.12) and (2.0.13)

$$0 = b(e-h) \tag{2.0.16}$$

$$0 = c(e-h) \tag{2.0.17}$$

For the above equations to hold true, $e = h$ or $b = c = 0$

3) $e = h$. Substituting in (2.0.12) and (2.0.13)

$$0 = f(a-d) \tag{2.0.18}$$

$$0 = g(a-d) \tag{2.0.19}$$

For the above equations to hold true, $a = d$ or $f = g = 0$

Considering the given A and B,

$$b = 4, g = -2, c = 3, f = 3bg = -8! = 9 = cf \tag{2.0.20}$$

Hence, the multiplication of the matrices A and B is not commutative. This can be verified by observing that

$$BA = \begin{pmatrix} 11 & 10 \\ 11 & 2 \end{pmatrix} \tag{2.0.21}$$