GATE ASSIGNMENT 3

Amulya Tallamraju
AI20BTECH11003

Download all python codes from

```
https://github.com/AmulyaTallamraju/EE3900/blob/
    main/GATE_Assignment-3/codes/
    GATE_Assignment-3.py
```

and latex-tikz codes from

```
https://github.com/AmulyaTallamraju/EE3900/blob/
    main/GATE_Assignment-3/
    GATE_Assignment-3.tex
```

1 GATE EC 2005 Q.5

The function $x(t)$ is shown in figure. Even and odd parts of a unit step function u(t) are given by

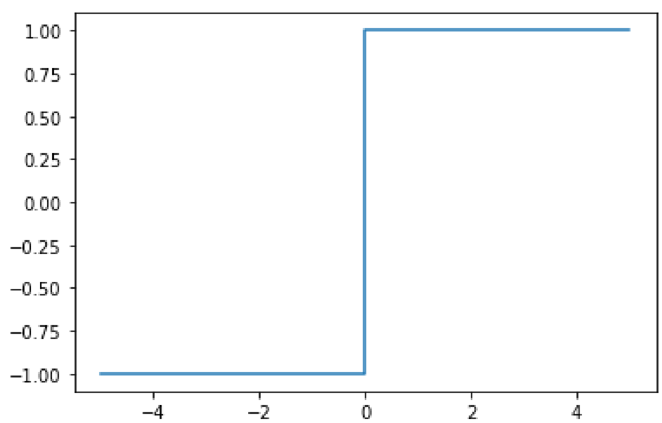

Fig. 0: Plot of $x[t]$

2 Solution

$$x(t) = \begin{cases} -1 & x \leq 0 \\ 0 & 0 \leq x = 0 \\ 1 & x \geq 0 \end{cases} \tag{2.0.1}$$

From the above definition of $x(t)$ we can see that it is the same as $sgn(t)$. Odd part of $u(t)$ is given by

$$\frac{u(t) - u(-t)}{2} \tag{2.0.2}$$

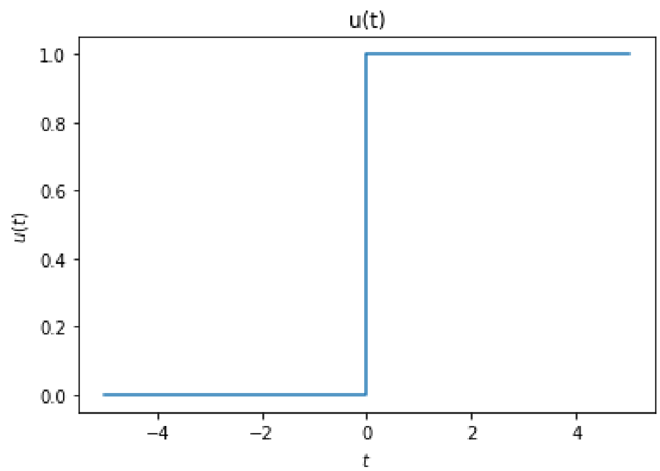

Fig. 0: Plot of $u[t]$

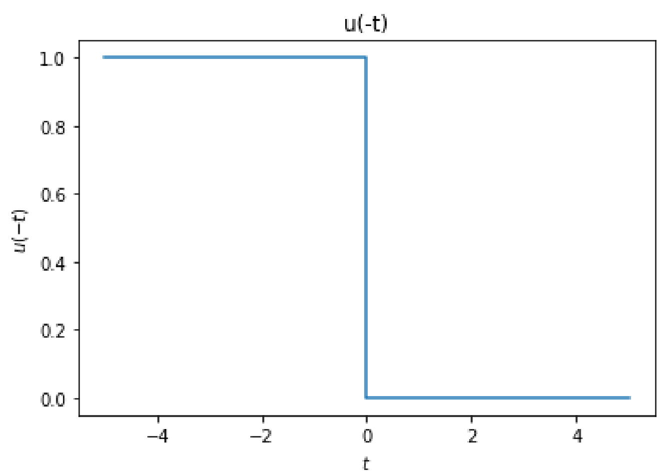

Fig. 0: Plot of $u[t]$

One observing the plots of $x(t), u(t), -u(-t)$ we can see that

$$x(t) = u(t) - u(-t) \tag{2.0.3}$$

Thus, the odd part of $u(t)$ is $\frac{x(t)}{2}$. The even part of u(t) is given by

$$\frac{u(t) + u(-t)}{2} = \frac{1}{2} \tag{2.0.4}$$

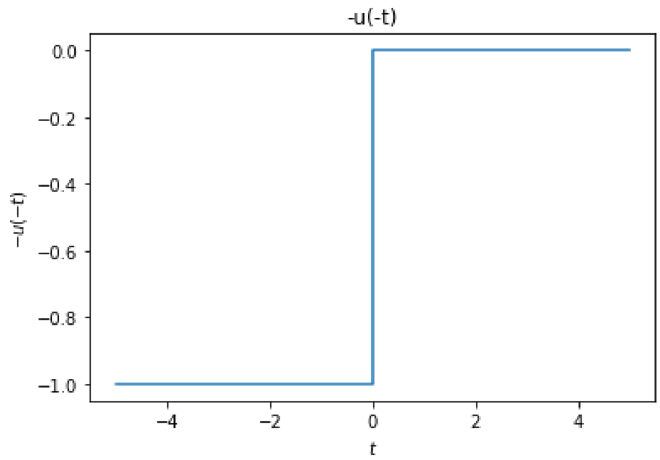

Fig. 0: Plot of $-u[-t]$

Thus the even and odd parts of the unit step signal are

$$\frac{1}{2}, \frac{x(t)}{2} \tag{2.0.5}$$

The fourier transform of $x(t)$ is given by

$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt \tag{2.0.6}$$

This signal is not absolutely integrable so we calculate Fourier Transform of $x(t)$ as a limiting case of the sum of exponential $e^{-at}u(t) - e^{at}u(t)$ as $a \to 0$.

$$\mathcal{F}\{x(t)\} = \lim_{a\to 0} \int_{-\infty}^{+\infty} \left(e^{-at}u(t) - e^{at}u(t)\right) e^{-2\pi f jt}dt \tag{2.0.7}$$

$$= \lim_{a\to 0} \left[\frac{1}{a + 2\pi f j} - \frac{1}{a - 2\pi f j}\right] \tag{2.0.8}$$

$$= \frac{1}{\pi f j} \tag{2.0.9}$$

The fourier transform of the unit step function can be found by realising that

$$u(t) = \frac{1}{2}(1 + \mathrm{sgn}(t)) \tag{2.0.10}$$

$$\implies \mathcal{F}\{u(t)\} = \frac{1}{2}\left(\mathcal{F}\{1\} + \mathcal{F}\{\mathrm{sgn}(t)\}\right) \tag{2.0.11}$$

$$= \pi\delta(f) + \frac{1}{2\pi f j} \tag{2.0.12}$$

The Double-sided Laplace transform of $x(t)$

$$\mathcal{L}\{x\}(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt. \tag{2.0.13}$$

$$= \frac{2}{s} \tag{2.0.14}$$