ASSIGNMENT 3

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−3/codes/Assignment−3.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−3/Assignment−3.tex

1 Construction 2.10

Construct a quadrilateral MORE where $MO = 6, OR = 4.5, \angle M = 60°, \angle O = 105°$ and $\angle R = 105°$.

2 Solution

Lemma 2.1. *Given*

$$MO = 6 \tag{2.0.1}$$
$$OR = 4.5 \tag{2.0.2}$$
$$\angle M = 60^\circ \tag{2.0.3}$$
$$\angle O = 105^\circ \tag{2.0.4}$$
$$\angle R = 105^\circ \tag{2.0.5}$$

Let

$$\angle M = \theta \tag{2.0.6}$$
$$\angle O = \alpha \tag{2.0.7}$$
$$\angle R = \delta \tag{2.0.8}$$
$$\|\mathbf{O} - \mathbf{M}\| = a, \tag{2.0.9}$$
$$\|\mathbf{R} - \mathbf{O}\| = b, \tag{2.0.10}$$
$$\|\mathbf{R} - \mathbf{E}\| = c \tag{2.0.11}$$
$$\|\mathbf{M} - \mathbf{E}\| = d \tag{2.0.12}$$
$$\|\mathbf{M} - \mathbf{R}\| = e \tag{2.0.13}$$
$$\theta = \theta_1 + \theta_2 \tag{2.0.14}$$
$$\delta_1 = \angle ERM \tag{2.0.15}$$
$$\delta_2 = \angle ORM \tag{2.0.16}$$
$$\gamma = \angle E \tag{2.0.17}$$

If three angles and two sides of a quadrilateral are known, then the coordinates of the vertices can be expressed as

$$\mathbf{R} = \mathbf{O} + b \times \begin{pmatrix} \cos(180^\circ - \alpha) \\ \sin(180^\circ - \alpha) \end{pmatrix} \tag{2.0.18}$$
$$\mathbf{E} = d \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.19}$$

Where

$$d = e \times \left(\frac{\sin\left(\delta - \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right]\right)}{\sin(360^\circ - (\alpha + \theta + \delta))} \right) \tag{2.0.20}$$
$$e = \sqrt{a^2 + b^2 - 2 \times a \times b\cos\alpha} \tag{2.0.21}$$

Proof. Using angle sum rule of quadrilaterals

$$\gamma = 360^\circ - (\alpha + \theta + \delta) \tag{2.0.22}$$

Now, using cosine formula in $\triangle MOR$ we can find e:

$$e^2 = a^2 + b^2 - 2 \times a \times b\cos\alpha \tag{2.0.23}$$

Using sine rule,

$$\frac{\sin\alpha}{e} = \frac{\sin\delta_2}{b} \tag{2.0.24}$$
$$\delta_2 = \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right] \tag{2.0.25}$$
$$\tag{2.0.26}$$

Now in $\triangle MER$,

$$\delta_1 = \delta - \delta_2 \tag{2.0.27}$$

Using sine law of triangle,

$$\frac{\sin\gamma}{e} = \frac{\sin\delta_1}{d} \tag{2.0.28}$$
$$\implies d = e \times \left(\frac{\sin\delta_1}{\sin\gamma}\right) \tag{2.0.29}$$

From the above equations, we get

$$d = e \times \left(\frac{\sin\left(\delta - \sin^{-1}\left[\sin\alpha \times \left(\frac{b}{e}\right)\right]\right)}{\sin\left(360^\circ - (\alpha + \theta + \delta)\right)} \right) \tag{2.0.30}$$

where

$$e = \sqrt{a^2 + b^2 - 2 \times a \times b \cos\alpha} \tag{2.0.31}$$

□

Calculating the co-ordinates of **R**:
Putting the values in the above equation we get,

$$\implies \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + 4.5 \begin{pmatrix} -\cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \tag{2.0.32}$$

$$\implies \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.16 \\ 4.35 \end{pmatrix} \tag{2.0.33}$$

$$\mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix} \tag{2.0.34}$$

Calculating the co-ordinates of **E**:
Putting the values in the above equation we get,

$$\implies \mathbf{E} = 7.34 \begin{pmatrix} \cos 60 \\ \sin 60 \end{pmatrix} \tag{2.0.35}$$

$$\implies \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix} \tag{2.0.36}$$

Now, we have the coordinate of vertices M,O,R,E as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix}. \tag{2.0.37}$$

On constructing the given quadrilateral on python we get:

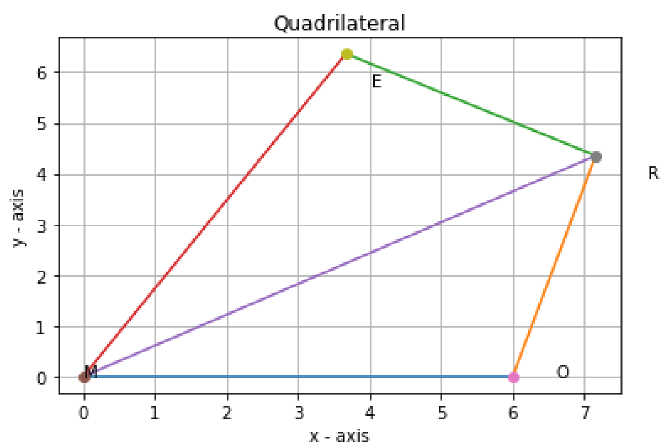

Fig. 0: Quadrilateral MORE