GATE ASSIGNMENT 1

Amulya Tallamraju
AI20BTECH11003

Download all python codes from

```
https://github.com/AmulyaTallamraju/EE3900/blob/
    main/GATE_Assignment−1/codes/
    GATE_Assignment−1.py
```

and latex-tikz codes from

```
https://github.com/AmulyaTallamraju/EE3900/blob/
    main/GATE_Assignment−1/
    GATE_Assignment−1.tex
```

1 GATE EC 2021 Q.41

Consider the signals $x[n] = 2^{n-1}u[-n+2]$ and $y[n] = 2^{-n+2}u[n+1]$, where $u[n]$ is the unit step sequence. Let $X(e^{j\omega})$ and $Y(e^{j\omega})$ the discrete-time Fourier transform of $x[n]$ and $y[n]$, respectively. The value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})\,d\omega \tag{1.0.1}$$

is

2 Solution

Theorem 2.1 (Convolution Theorem)**.** *Let f and g be two functions with convolution $f * g$. Let F be the Fourier transform operator. Then*

$$F(f * g) = F(f) \cdot F(g) \tag{2.0.1}$$

$$F(f \cdot g) = F(f) * F(g) \tag{2.0.2}$$

If the DTFT of $y[n]$ is $Y(e^{j\omega})$ then using the time reversal property, the DTFT of $y[-n]$ is

$$Y\left(\frac{1}{e^{j\omega}}\right) = Y(e^{-j\omega}) \tag{2.0.3}$$

Let

$$z[n] = x[n] * y[-n] \tag{2.0.4}$$

Let $Z(e^{j\omega})$ be the DTFT of $z[n]$. Using Theorem 2.1 we get

$$Z(e^{j\omega}) = X(e^{j\omega})Y(e^{-j\omega}) \tag{2.0.5}$$

By applying IDTFT, we can write:

$$z[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})e^{j\omega n}\,d\omega \tag{2.0.6}$$

Putting $n = 0$, we get

$$z[0] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})\,d\omega \tag{2.0.7}$$

which is the required value. From (2.0.4) we have

$$z[n] = x[n] * y[-n] \tag{2.0.8}$$

$$= \sum_{k=-\infty}^{\infty} x[k]y[-n+k] \tag{2.0.9}$$

$$= \sum_{k=-\infty}^{\infty} 2^{k-1}u[-k+2]2^{n-k+2}u[-n+k+1] \tag{2.0.10}$$

$$= \sum_{k=-\infty}^{\infty} 2^{n+1}u[-k+2]u[-n+k+1] \tag{2.0.11}$$

The unit step function is defined as

$$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases} \tag{2.0.12}$$

From (2.0.11), putting $n = 0$, we get

$$z[0] = = \sum_{k=-\infty}^{\infty} 2^1 u[-k+2]u[k+1] \tag{2.0.13}$$

$$= \sum_{k=-1}^{2} 2 \tag{2.0.14}$$

$$= 8 \tag{2.0.15}$$

Therefore, the value of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})Y(e^{-j\omega})\,d\omega = 8 \tag{2.0.16}$$

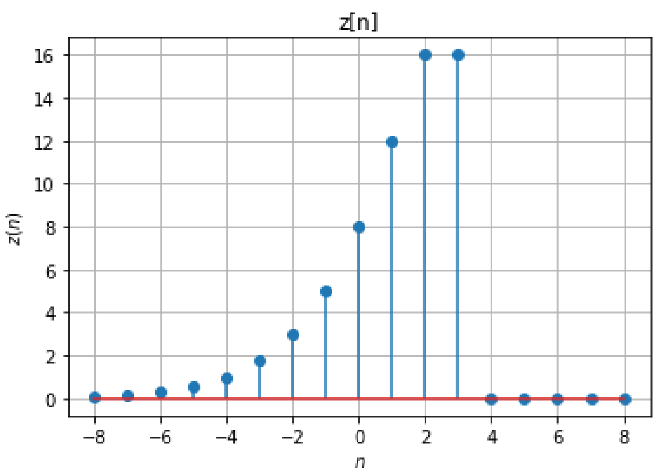

Fig. 0: Plot of z[n]