

GATE ASSIGNMENT 3

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-3/codes/GATE_Assignment-3.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/GATE_Assignment-3/GATE_Assignment-3.tex

1 GATE EC 2005 Q.5

The function $x(t)$ is shown in figure. Even and

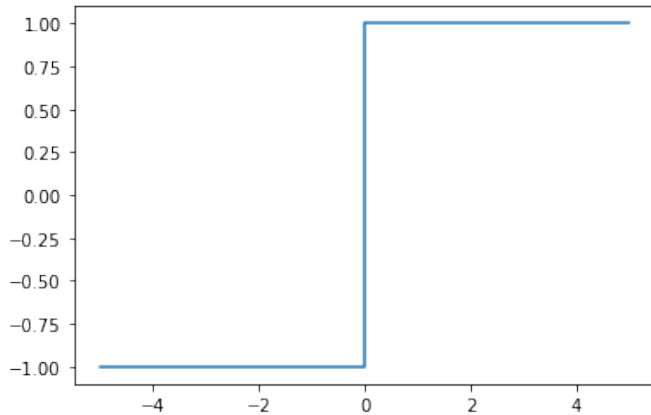


Fig. 0: Plot of $x[t]$

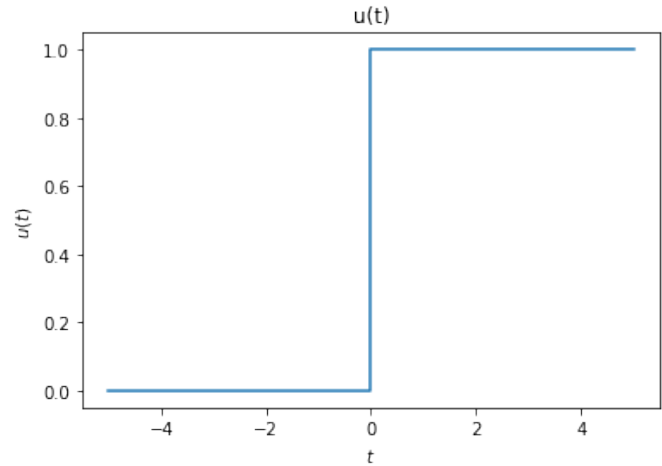


Fig. 4: Plot of $u[t]$

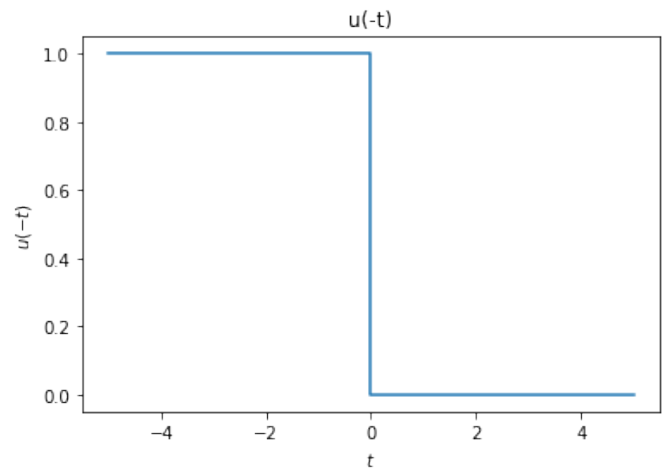


Fig. 4: Plot of $u[t]$

odd parts of a unit step function $u(t)$ are given by

- 1) $\frac{1}{2}, \frac{x(t)}{2}$
- 2) $\frac{-1}{2}, \frac{x(t)}{2}$
- 3) $\frac{1}{2}, \frac{-x(t)}{2}$
- 4) $\frac{-1}{2}, \frac{-x(t)}{2}$

2 SOLUTION

$$x(t) = \begin{cases} -1 & x \leq 0 \\ 0 & 0 \leq x = 0 \\ 1 & x \geq 0 \end{cases} \quad (2.0.1)$$

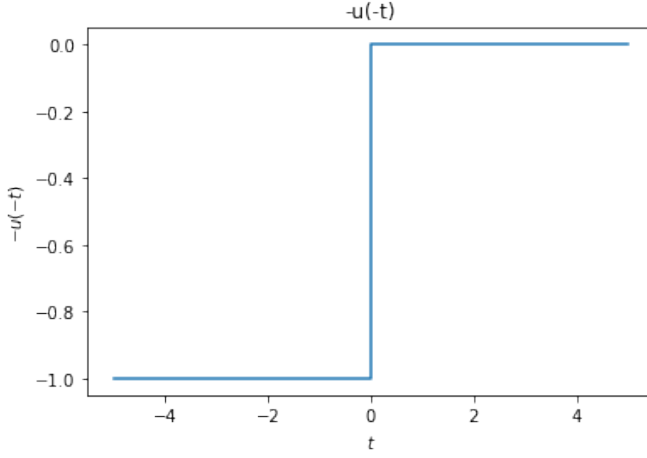
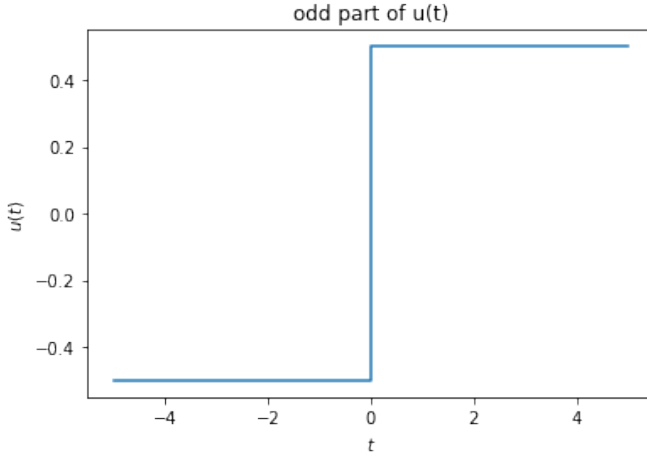
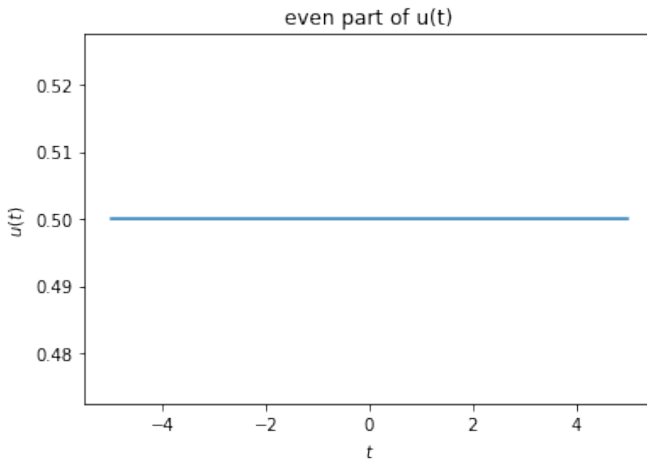
From the above definition of $x(t)$ we can see that it is the same as $\text{sgn}(t)$. Odd part of $u(t)$ is given by

$$\frac{u(t) - u(-t)}{2} \quad (2.0.2)$$

One observing the plots of $x(t), u(t), -u(-t)$ we can see that

$$x(t) = u(t) - u(-t) \quad (2.0.3)$$

Thus, the odd part of $u(t)$ is $\frac{x(t)}{2}$. The even part of

Fig. 4: Plot of $-u[-t]$ Fig. 4: Plot of odd part of $u[t]$ Fig. 4: Plot of even part of $u[t]$

Thus the even and odd parts of the unit step signal are

$$\frac{1}{2}, \frac{x(t)}{2} \quad (2.0.5)$$

The fourier transform of $x(t)$ is given by

$$\mathcal{F}\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt \quad (2.0.6)$$

This signal is not absolutely integrable so we calculate Fourier Transform of $x(t)$ as a limiting case of the sum of exponential $e^{-at}u(t) - e^{at}u(t)$ as $a \rightarrow 0$.

$$\mathcal{F}\{x(t)\} = \lim_{a \rightarrow 0} \int_{-\infty}^{+\infty} (e^{-at}u(t) - e^{at}u(t)) e^{-2\pi f j t} dt \quad (2.0.7)$$

$$= \lim_{a \rightarrow 0} \left[\frac{1}{a + 2\pi f j} - \frac{1}{a - 2\pi f j} \right] \quad (2.0.8)$$

$$= \frac{1}{\pi f j} \quad (2.0.9)$$

The fourier transform of the unit step function can be found by realising that

$$u(t) = \frac{1}{2}(1 + \text{sgn}(t)) \quad (2.0.10)$$

$$\Rightarrow \mathcal{F}\{u(t)\} = \frac{1}{2}(\mathcal{F}\{1\} + \mathcal{F}\{\text{sgn}(t)\}) \quad (2.0.11)$$

$$= \pi\delta(f) + \frac{1}{2\pi f j} \quad (2.0.12)$$

The Double-sided Laplace transform of $x(t)$

$$\mathcal{L}\{x\}(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt. \quad (2.0.13)$$

$$= \frac{2}{s} \quad (2.0.14)$$

$u(t)$ is given by

$$\frac{u(t) + u(-t)}{2} = \frac{1}{2} \quad (2.0.4)$$