ASSIGNMENT 4

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Download all python codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−4/codes/Assignment−4.py

and latex-tikz codes from

https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment−4/Assignment−4.tex

1 Linear forms 2.36

Find the coordinates of the point where the line through $\begin{pmatrix}5\\1\\6\end{pmatrix}$ and $\begin{pmatrix}3\\4\\1\end{pmatrix}$ crosses the YZ-plane.

2 SOLUTION

The equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{2.0.1}$$

where

$$\mathbf{A} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{B} = \begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \tag{2.0.3}$$

The equation of the plane can be represented as

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.4}$$

Where

$$\mathbf{n} = \begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} \tag{2.0.5}$$

$$\tag{2.0.6}$$

The point of intersection of the line and the plane satisfies the plane equation and is given by

$$\begin{aligned} c = n_1(a_1 + \lambda(b_1 - a_1)) \\ + n_2(a_2 + \lambda(b_2 - a_2)) \\ + n_3(a_3 + \lambda(b_3 - a_3)) \end{aligned} \tag{2.0.7}$$

$$\implies \lambda = \frac{c - n_1a_1 - n_2a_2 - n_3a_3}{n_1(b_1 - a_1) + n_2(b_2 - a_2) + n_3(b_3 - a_3)} \tag{2.0.8}$$

The point of intersection is then given by

$$\mathbf{x} = \begin{pmatrix}a_1 + \lambda b_1\\a_2 + \lambda b_2\\a_3 + \lambda b_3\end{pmatrix} \tag{2.0.9}$$

For the given problem,

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} = \begin{pmatrix}5\\1\\6\end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} = \begin{pmatrix}2\\-3\\5\end{pmatrix} \tag{2.0.11}$$

$$\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} \tag{2.0.12}$$

$$c = 0 \tag{2.0.13}$$

Solving the above we get

$$\lambda = \frac{-5}{2} \tag{2.0.14}$$

Substituting the value of λ we have the point of contact as

$$\mathbf{x} = \begin{pmatrix}5\\1\\6\end{pmatrix} - \frac{5}{2}\begin{pmatrix}2\\-3\\5\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\17\\-13\end{pmatrix} \tag{2.0.15}$$

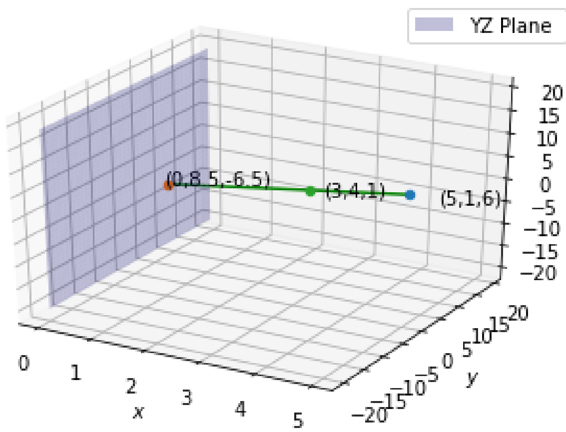

Fig. 0: Line and point of intersection