

ASSIGNMENT 2

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Download all python codes from

<https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment-3/codes/Assignment-3.py>

and latex-tikz codes from

<https://github.com/AmulyaTallamraju/EE3900/blob/main/Assignment-3/Assignment-3.tex>

where

$$d = e \times \left(\frac{\sin \left(\delta - \sin^{-1} \left[\sin \alpha \times \left(\frac{b}{e} \right) \right] \right)}{\sin (360^\circ - (\alpha + \theta + \delta))} \right) \times \quad (2.0.15)$$

and

$$e = \sqrt{a^2 + b^2 - 2 \times a \times b \cos \alpha} \quad (2.0.16)$$

1 CONSTRUCTION 2.10

Construct a quadrilateral MORE where $MO = 6$, $OR = 4.5$, $\angle M = 60^\circ$, $\angle O = 105^\circ$ and $\angle R = 105^\circ$.

2 SOLUTION

Given

$$\angle M = 60^\circ = \theta \quad (2.0.1)$$

$$\angle O = 105^\circ = \alpha \quad (2.0.2)$$

$$\angle R = 105^\circ = \delta \quad (2.0.3)$$

$$\|\mathbf{O} - \mathbf{M}\| = 6 = a, \quad (2.0.4)$$

$$\|\mathbf{R} - \mathbf{O}\| = 4.5 = b, \quad (2.0.5)$$

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \quad (2.0.6)$$

Let

$$\|\mathbf{R} - \mathbf{E}\| = c \quad (2.0.7)$$

$$\|\mathbf{M} - \mathbf{E}\| = d \quad (2.0.8)$$

$$\|\mathbf{M} - \mathbf{R}\| = e \quad (2.0.9)$$

$$\theta = \theta_1 + \theta_2 \quad (2.0.10)$$

$$\delta_1 = \angle ERM \quad (2.0.11)$$

$$\delta_2 = \angle ORM \quad (2.0.12)$$

$$\gamma = \angle E \quad (2.0.13)$$

$$\gamma = 360^\circ - (\alpha + \theta + \delta) \quad (2.0.17)$$

Now, using cosine formula in $\triangle MOR$ we can find RM:

$$e^2 = a^2 + b^2 - 2 \times a \times b \cos \alpha \quad (2.0.18)$$

Using sine rule,

$$\frac{\sin \alpha}{e} = \frac{\sin \delta_2}{b} \quad (2.0.19)$$

$$\delta_2 = \sin^{-1} \left[\sin \alpha \times \left(\frac{b}{e} \right) \right] \quad (2.0.20)$$

$$(2.0.21)$$

Now in $\triangle MER$,

$$\delta_1 = \delta - \delta_2 \quad (2.0.22)$$

Using sine law of triangle,

$$\frac{\sin \gamma}{e} = \frac{\sin \delta_1}{d} \quad (2.0.23)$$

$$\Rightarrow d = e \times \left(\frac{\sin \delta_1}{\sin \gamma} \right) \quad (2.0.24)$$

From the above equations, we get

$$d = e \times \left(\frac{\sin \left(\delta - \sin^{-1} \left[\sin \alpha \times \left(\frac{b}{e} \right) \right] \right)}{\sin (360^\circ - (\alpha + \theta + \delta))} \right) \times \quad (2.0.25)$$

where

$$e = \sqrt{a^2 + b^2 - 2 \times a \times b \cos \alpha} \quad (2.0.26)$$

Lemma 2.1. If three angles and two sides of a quadrilateral are known, then the coordinates of the vertices can be expressed as

$$\mathbf{R} = \mathbf{O} + b \times \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \mathbf{E} = d \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \quad (2.0.14)$$

Calculating the co-ordinates of **R**:

Putting the values in the above equation we get,

$$\Rightarrow \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + 4.5 \begin{pmatrix} \cos 105^\circ \\ \sin 105^\circ \end{pmatrix} \quad (2.0.27)$$

$$\Rightarrow \mathbf{R} = \begin{pmatrix} 6 \\ 0 \end{pmatrix} + \begin{pmatrix} 1.16 \\ 4.35 \end{pmatrix} \quad (2.0.28)$$

$$\mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix} \quad (2.0.29)$$

Calculating the co-ordinates of **E**:

Putting the values in the above equation we get,

$$\Rightarrow \mathbf{E} = 7.34 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} \quad (2.0.30)$$

$$\Rightarrow \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix} \quad (2.0.31)$$

Now, we have the coordinate of vertices **M,O,R,E** as,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{O} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 7.16 \\ 4.35 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 3.67 \\ 6.36 \end{pmatrix}. \quad (2.0.32)$$

On constructing the given quadrilateral on python we get:

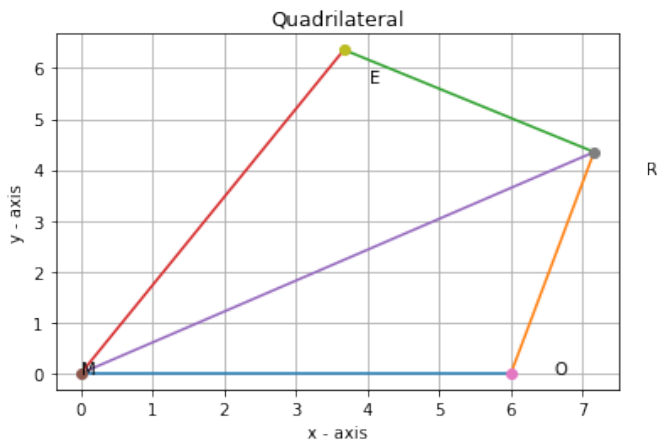


Fig. 0: Quadrilateral MORE