The motion planning of nonholonomic ROBOT

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Contents

1. Introduction

- **Nonholonomic robot**

  There are constrains on the velocity or acceleration of the robot which cannot be integrated.

- **Nonholonomic motion planning**

  Design an appropriate bounded input to steer the nonholonomic system from an initial configuration to a desired final configuration over finite time.

1. Introduction

❖ Difficulties

Motion coupling: cannot be expressed by a set of independent generalized coordinates

Nonlinear system: cannot use feedback linearization method

❖ Chained form system

Controllable nonholonomic system

Simple structure

Easy to integrate

$$
\begin{cases}
\dot{z}_1 = v_1 \\
\dot{z}_2 = v_2 \\
\dot{z}_3 = z_2 \cdot v_1 \\
\vdots \\
\dot{z}_n = z_{n-1} \cdot v_1
\end{cases}
$$

1. Introduction

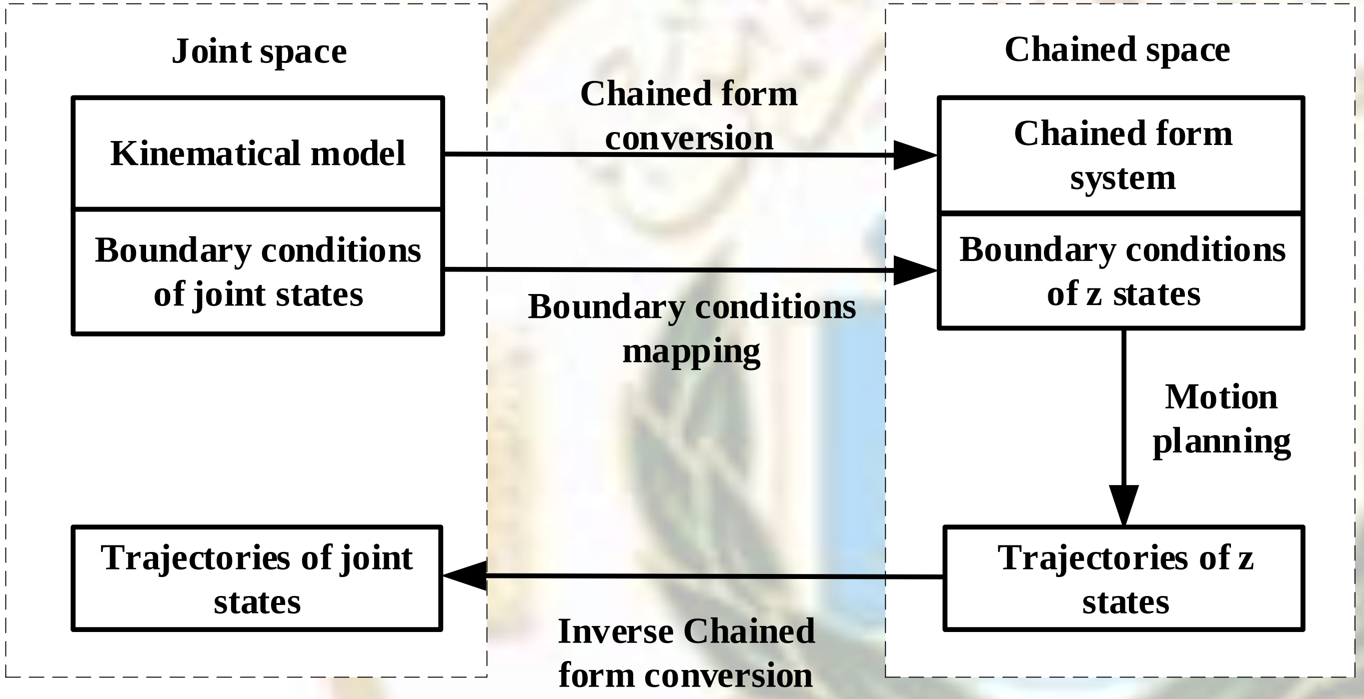


Fig. Motion planning schematic diagram

Motion planning of chained form system

- Cosine Switch Control algorithm
- Polynomial Control algorithm
- Sinusoidal control algorithm
- Piecewise constant algorithm

1. Introduction

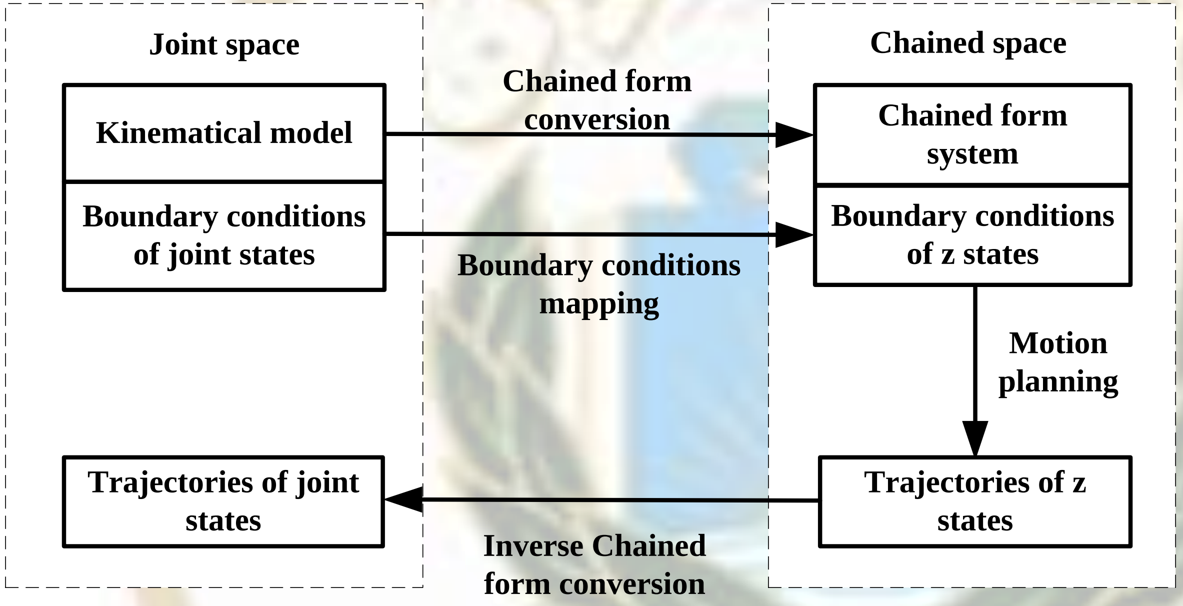


Fig. Motion planning schematic diagram

Whether the system can be converted into a chained form system

How to realize the motion planning of chained form system

Whether the inverse conversion is feasible

?

n-Dimensional Chained Form System

By considering the n-dimensional chained form system $z = \left[z_1 \; z_2 \; z_3 \ldots \; z_n \right]$

with two inputs $\left[v_1 \; v_2 \right]$

$$\dot{z}_1 = v_1 \;, \; \dot{z}_2 = v_2 \;, \; \dot{z}_3 = z_2 . v_1 \;, \; \cdots \;, \; \dot{z}_n = z_{n-1} . v_1$$

A chained form system is a system of the form:

$$\dot{z} = \begin{bmatrix} 1 \\ 0 \\ Z_2 \\ Z_3 \\ \vdots \\ Z_{n-1} \end{bmatrix} v_1 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} v_2 \qquad z = \left[z_1 \; z_2 \; z_3 \ldots \; z_n \right]^T$$

Cosine switch control

For n-dimensional chained form system, cosine switch control can steer it from a given

initial configuration z(0) to a desired configuration z(T) through $2(n - 2) + 1$ times of intervals mostly.

$$\varepsilon = T / [2(n - 2) + 1]$$

$2(n - 2)$ times of input switch

Cosine switch control

In odd time intervals, i.e., when:

$$t \in [2i.\varepsilon,(2i+1).\varepsilon] \quad , \quad (i = 0,1,2 \cdots n-2)$$

the control inputs are represented by:

$$v_1 = 0$$

$$v_2 = c_{2i+1}(1-\cos wt)$$

Where, c_{2i+1} are undetermined coefficients, w is the angular frequency and $w = 2\pi / \varepsilon$

we can solve the undetermined coefficients by substituting boundary conditions.

$$\begin{aligned}
z_1(t_{2i+1}) &= z_1(t_{2i}) \\
z_2(t_{2i+1}) &= c_{2i+1}.\varepsilon + z_2(t_{2i}) \\
z_3(t_{2i+1}) &= z_3(t_{2i}) \\
&\vdots \\
z_n(t_{2i+1}) &= z_n(t_{2i})
\end{aligned}$$

Cosine switch control

In even time intervals, i.e., when $t \in [(2j+1).\varepsilon, (2j+2).\varepsilon] \;\;, \;\; (j = 0,1,2 \cdots n-3)$

the control inputs are represented by:

$$v_1 = c_{2j+2}(1-\cos wt)$$

$$v_2 = 0$$

$$z_1(t_{2j+2}) = c_{2j+2}.\varepsilon + z_1(t_{2j+1})$$

$$z_2(t_{2j+2}) = z_2(t_{2j+1})$$

$$z_3(t_{2j+2}) = c_{2j+2}.z_2(t_{2j+1}).\varepsilon + z_3(t_{2j+1})$$

$$\vdots$$

$$z_n(t_{2j+2}) = \sum_{k=1}^{n-2} \frac{\left(c_{2j+2}.\varepsilon\right)^k \cdot z_{n-k}(t_{2j+1})}{k!} + z_n(t_{2j+1})$$

Cosine switch control

The final configuration at T can be calculated by iterative operation via:

$$z_1(T)=\sum_{j=0}^{n-3} c_{2j+2}\cdot\varepsilon+z_1(0)$$

$$z_2(T)=\sum_{i=0}^{n-2} c_{2i+1}\cdot\varepsilon+z_2(0)$$

$$z_3(T)=\sum_{i=0}^{n-3}\left(\sum_{j=i}^{n-3} c_{2j+2}\cdot\varepsilon\right)\cdot c_{2i+1}\cdot\varepsilon+\sum_{j=0}^{n-3} c_{2j+2}\cdot\varepsilon\cdot z_2(0)+z_3(0)$$

$$\vdots$$

$$z_n(T)=\sum_{i=0}^{n-3}\frac{\left(\sum_{j=i}^{n-3} c_{2i+2}\cdot\varepsilon\right)^{n-2}}{(n-2)!}\cdot c_{2i+1}\cdot\varepsilon+\sum_{k=1}^{n-2}\frac{\left(\sum_{j=0}^{n-3} c_{2j+2}\cdot\varepsilon\right)^{k}}{k!}\cdot z_{n-k}(0)+z_n(0)$$

Specify a set of coefficients c_{2j+2}, and they must be satisfied with:

$$\sum_{j=0}^{n-3} c_{2j+2}=\frac{z_1(T)-z_1(0)}{\varepsilon}\quad\Longrightarrow\quad c_{2i+1}$$

MOBILE ROBOT KINEMATICS

$$V = \frac{V_L + V_R}{2}$$

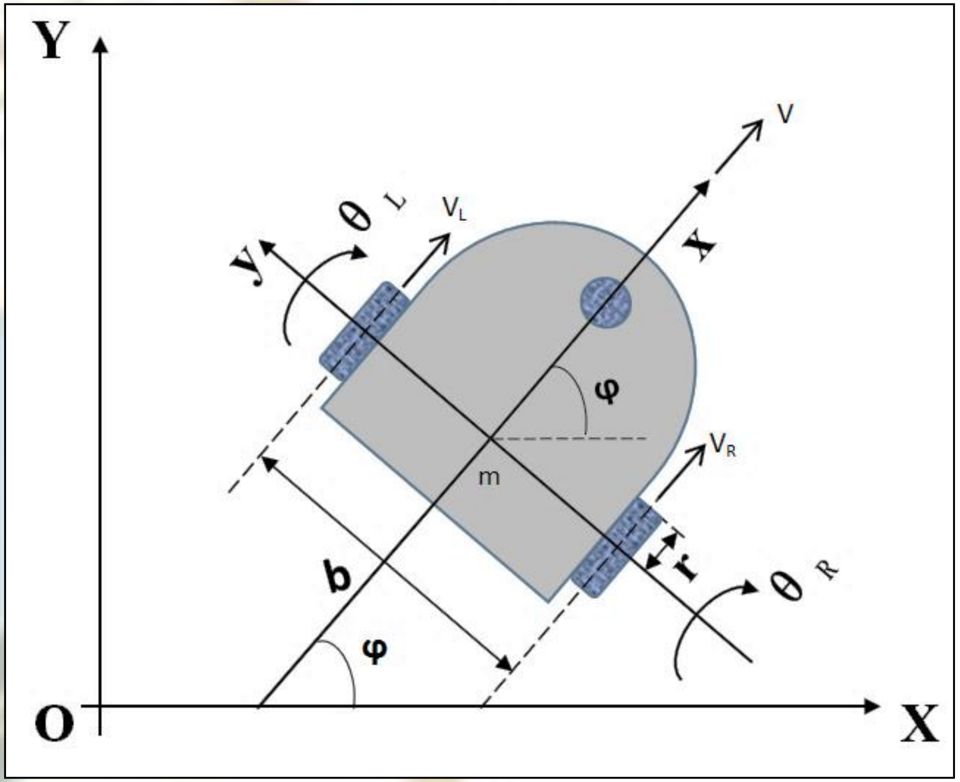
$$V_L = r \cdot \dot{\theta}_L$$

$$V_R = r \cdot \dot{\theta}_R$$

$$V = \frac{r\dot{\theta}_L + r\dot{\theta}_R}{2} = \frac{r}{2}\left(\dot{\theta}_L + \dot{\theta}_R\right)$$

$$\dot{\varphi} = \frac{V_R - V_L}{b}$$

$$\dot{\varphi} = \frac{r}{b}\left(\dot{\theta}_R - \dot{\theta}_L\right)$$



MOBILE ROBOT KINEMATICS

$$\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \frac{r\cos\varphi}{2} & \frac{r\cos\varphi}{2} \\ \frac{r\sin\varphi}{2} & \frac{r\sin\varphi}{2} \\ \frac{r}{b} & -\frac{r}{b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} \frac{r\cos\varphi}{2}\left(\dot{\theta}_R + \dot{\theta}_L\right) \\ \frac{r\sin\varphi}{2}\left(\dot{\theta}_R + \dot{\theta}_L\right) \\ \frac{r}{b}\left(\dot{\theta}_R - \dot{\theta}_L\right) \end{bmatrix} = \begin{bmatrix} V\cos\varphi \\ V\sin\varphi \\ \frac{V_R - V_L}{b} \end{bmatrix}$$

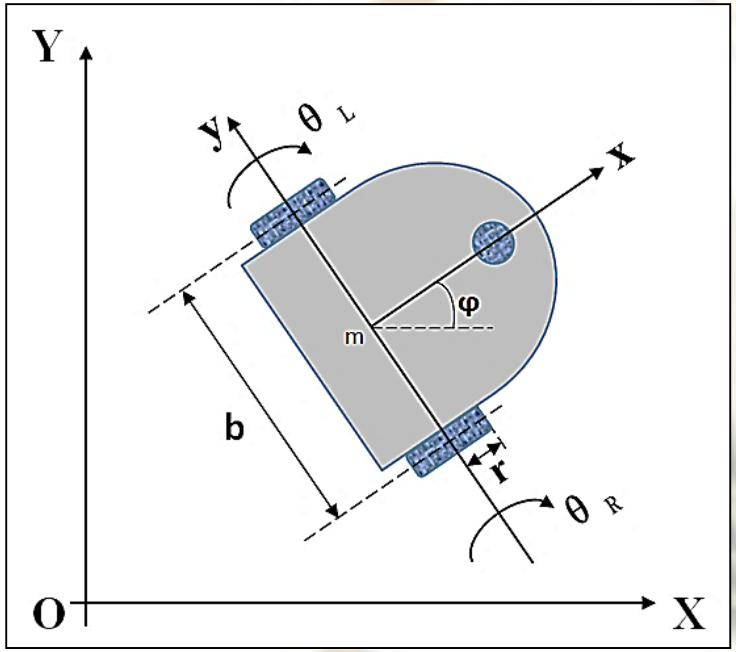
$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} V \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{b} & -\frac{r}{b} \end{bmatrix} \begin{bmatrix} \dot{\theta}_R \\ \dot{\theta}_L \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\left(\dot{\theta}_R + \dot{\theta}_L\right) \\ \frac{r}{b}\left(\dot{\theta}_R - \dot{\theta}_L\right) \end{bmatrix}$$

u_1 refers to the forward velocity of car V, u_2 stands for the steering velocity of car.

Equation shows that the output velocities are nonzero even if only one wheel is rotating, for this reason this type of platform has the ability to change its orientation on the spot.

Ex. Differential wheeled robot

Kinematic model



$$n = 3$$

$$X^{\bullet} = \cos\varphi \cdot u_1$$
$$Y^{\bullet} = \sin\varphi \cdot u_1$$
$$\varphi^{\bullet} = u_2$$

$$u_1 = V = \sqrt{X^{\bullet 2} + Y^{\bullet 2}}$$
$$u_2 = \varphi^{\bullet}$$

$$\begin{bmatrix} X^{\bullet} \\ Y^{\bullet} \\ \varphi^{\bullet} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}\cos\varphi & \frac{r}{2}\cos\varphi \\ \frac{r}{2}\sin\varphi & \frac{r}{2}\sin\varphi \\ \frac{r}{b} & -\frac{r}{b} \end{bmatrix} \begin{bmatrix} \theta_R^{\bullet} \\ \theta_L^{\bullet} \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{b} & -\frac{r}{b} \end{bmatrix} \begin{bmatrix} \theta_R^{\bullet} \\ \theta_L^{\bullet} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & 0 \\ \sin\varphi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

u_1 refers to the forward velocity of car V, u_2 stands for the steering velocity of car.

Ex. Differential wheeled robot chained form system model

$$\begin{cases} z_1 = X \\ z_2 = \tan\varphi \\ z_3 = Y \end{cases} \Rightarrow \begin{cases} z_1^{\bullet} = X^{\bullet} \\ z_2^{\bullet} = \dfrac{1}{\cos^2\varphi}\varphi^{\bullet} \\ z_3^{\bullet} = Y^{\bullet} \end{cases}$$

$$\begin{cases} \boxed{X^{\bullet} = \cos\varphi\cdot u_1} \\ Y^{\bullet} = \sin\varphi\cdot u_1 \\ \boxed{\varphi^{\bullet} = u_2} \end{cases}$$

$$\begin{cases} \boxed{z_1^{\bullet} = v_1} \ , \ \boxed{z_2^{\bullet} = v_2} \ , \ z_3^{\bullet} = z_2.v_1 \\ \boxed{z_1^{\bullet} = X^{\bullet}} \ , \ \boxed{z_2^{\bullet} = \dfrac{1}{\cos^2\varphi}\varphi^{\bullet}} \end{cases}$$

$$\begin{cases} \boxed{v_1 = \cos\varphi\cdot u_1} \\ \boxed{v_2 = \dfrac{1}{\cos^2\varphi}\cdot u_2} \end{cases}$$

$$\Rightarrow \begin{cases} z_3^{\bullet} = z_2.v_1 \\ z_2 = \boxed{\int z_2^{\bullet}.d\varphi = \int v_2.d\varphi = \int\left(\dfrac{1}{\cos^2\varphi}\cdot\varphi^{\bullet}\right).d\varphi = \tan\varphi} \end{cases}$$

so :

$$z_3^{\bullet} = z_2.v_1 = (\tan\varphi)X^{\bullet} = \frac{\sin\varphi}{\cos\varphi}.X^{\bullet} = \sin\varphi.u_1 = Y^{\bullet}$$

$$\begin{cases} z_1 = X \\ z_2 = \tan\varphi \\ z_3 = Y \end{cases} \Rightarrow \begin{cases} v_1 = \cos\varphi\cdot u_1 \\ v_2 = \dfrac{1}{\cos^2\varphi}\cdot u_2 \end{cases}$$

Ex. Differential wheeled robot initial and final configurations

$$intial \quad configuration \begin{bmatrix} X(0)=0 \\ Y(0)=1 \\ \varphi(0)=0 \end{bmatrix} \quad , \; final \quad configuration \begin{bmatrix} X(T)=5 \\ Y(T)=0 \\ \varphi(T)=\pi/4 \end{bmatrix}$$

$$\begin{cases} z_1 = X \\ z_2 = \tan\varphi \\ z_3 = Y \end{cases}$$

$$intial \quad configuration \begin{bmatrix} z_1(0)=0 \\ z_2(0)=0 \\ z_3(0)=1 \end{bmatrix} \quad , \; final \quad configuration \begin{bmatrix} z_1(T)=z_1(30)=5 \\ z_2(T)=z_2(30)=\tan\dfrac{\pi}{4}=1 \\ z_3(T)=z_3(30)=0 \end{bmatrix}$$

$$n=3 \quad , \quad \varepsilon = \frac{T}{2(n-2)+1} = 10\ sec$$

Ex. Differential wheeled robot boundary conditions

$$interval\ \ 1: \qquad t_0 = 0 \rightarrow t_1 = 10 \qquad \text{C1}$$

$$interval\ \ 2: \qquad t_1 = 10 \rightarrow t_2 = 20 \qquad \text{C2}$$

$$interval\ \ 3: \qquad t_2 = 20 \rightarrow t_3 = 30 \qquad \text{c3}$$

$$\begin{cases} z_1(t_1) = z_1(t_0) \\ z_2(t_1) = c_1.\varepsilon + z_2(t_0) \\ z_3(t_1) = z_3(t_0) \end{cases} \Rightarrow \begin{cases} z_1(10) = z_1(0) = 0 \\ z_2(10) = c_1.\varepsilon + z_2(0) = 10c_1 \\ z_3(10) = z_3(0) = 1 \end{cases}$$

$$\begin{cases} z_1(t_2) = c_2.\varepsilon + z_1(t_1) \\ z_2(t_2) = z_2(t_1) \\ z_3(t_2) = c_2.z_2(t_1).\varepsilon + z_3(t_1) \end{cases} \Rightarrow \begin{cases} z_1(20) = c_2.\varepsilon + z_1(10) = 10c_2 \\ z_2(20) = z_2(10) = 10c_1 \\ z_3(20) = c_2.z_2(10).\varepsilon + z_3(10) = 100c_1c_2 + 1 \end{cases}$$

$$\begin{cases} z_1(t_3) = z_1(t_2) \\ z_2(t_3) = c_3.\varepsilon + z_2(t_2) \\ z_3(t_3) = z_3(t_2) \end{cases} \Rightarrow \begin{cases} z_1(30) = 5 = z_1(20) = 10c_2 \\ z_2(30) = 1 = c_3.\varepsilon + z_2(20) = 10c_3 + 10c_1 \\ z_3(30) = 0 = z_3(20) = 100c_1c_2 + 1 \end{cases} \Rightarrow \begin{cases} c_2 = 0.5 \\ c_1 = -0.02 \\ c_3 = 6/50 \end{cases}$$

Ex. Differential wheeled robot motion control

__the first interval [0,10]:__

$$\begin{bmatrix} z_1(0)=0 \\ z_2(0)=0 \\ z_3(0)=1 \end{bmatrix}, \begin{bmatrix} z_1(10)=0 \\ z_2(10)=-0.2 \\ z_3(10)=1 \end{bmatrix}, \begin{bmatrix} z_1(20)=5 \\ z_2(20)=-0.2 \\ z_3(20)=0 \end{bmatrix}, \begin{bmatrix} z_1(30)=5 \\ z_2(30)=1 \\ z_3(30)=0 \end{bmatrix}$$

$$\begin{array}{c} v_1=0 \\ v_1=u_1\cdot\cos\varphi \Rightarrow \\ \dot{z}_1=v_1 \end{array} \begin{cases} u_1=0 \\ \dot{z}_1=0 \Rightarrow z_1=A \begin{cases} A \text{ is constant, we get it from} \\ \text{the intial and final coditions of this interval} \\ z_1(0)=z_1(10)=0 \Rightarrow A=0 \\ z_1=0 \Rightarrow X(t)=0,\ t\in[0.10] \end{cases} \end{cases}$$

ليس حلا أن يكون التجيب
مساويا للصفر لأنه يؤثر
على v2
So u1=0

$$\begin{array}{c} v_2=c_1(1-\cos wt) \\ \dot{z}_2=v_2 \\ v_2=\dfrac{u_2}{\cos^2\varphi} \end{array} \Rightarrow \begin{cases} \dfrac{u_2}{\cos^2\varphi}=c_1(1-\cos wt) \\ \dot{z}_2=c_1(1-\cos wt) \end{cases}$$

$$\Rightarrow z_2=c_1t-\frac{c_1}{w}\sin wt+B$$

B is constant, we get it from the intial and final coditions of this interval :

$$z_2(0)=0 \ \& \ z_2(10)=-0.2$$

$$\Rightarrow \begin{cases} B=0 \\ z_2=c_1t-\dfrac{c_1}{w}\sin wt \end{cases}$$

$$z_2 = \tan\varphi \Rightarrow \varphi = \arctan\left(c_1 t - \frac{c_1}{w}\sin wt\right), t \in [0,10], w = \frac{2\pi}{\varepsilon}$$

$$t = 0 \Rightarrow \varphi = 0$$

$$t = 10 \Rightarrow \varphi = \arctan(10c_1) = \arctan(-0.2) \approx -11.3^\circ$$

$$u_2 = \varphi^{\cdot} \Rightarrow u_2 = \frac{d}{dt}\left(\arctan\left(c_1 t - \frac{c_1}{w}\sin wt\right)\right)$$

$$v_2 = \frac{1}{\cos^2\varphi}u_2 \Rightarrow v_2 = \frac{1}{\cos^2\left(\arctan\left(c_1 t - \frac{c_1}{w}\sin wt\right)\right)}\frac{d}{dt}\left(\arctan\left(c_1 t - \frac{c_1}{w}\sin wt\right)\right)$$

$$\frac{d}{dx}arctan\, x = \frac{1}{1+x^2}$$

$$\left\{\begin{matrix} z_3^{\bullet} = z_2.v_1 \\ v_1 = 0 \end{matrix}\right\} \Rightarrow z_3 = C$$

C is constant, we get it from the intial and final coditions of this interval :

$$\begin{bmatrix} z_3(0) = 1 \\ z_3(10) = 1 \end{bmatrix} \Rightarrow C = 1 \Rightarrow z_3(t) = 1 \Rightarrow Y(t) = 1$$

Ex. Differential wheeled robot motion control

the second interval [10,20]:

$$w = \frac{2\pi}{\varepsilon}, \varepsilon = 10, \; c_2 = 0.5, c_1 = -0.02, c_3 = 6/50$$

$$\begin{cases} X(t) = c_2 t - \dfrac{c_2}{w}\sin wt - 10c_2 \\ Y(t) = -0.2c_2 t - \dfrac{0.2c_2}{w}\sin wt + 2 \\ \varphi(t) = \arctan(10c_1) = \arctan(-0.2) \approx -11.3^\circ \\ v_1 = 0.5 - 0.5\cos wt, v_2 = 0 \end{cases}$$

Ex. Differential wheeled robot motion control

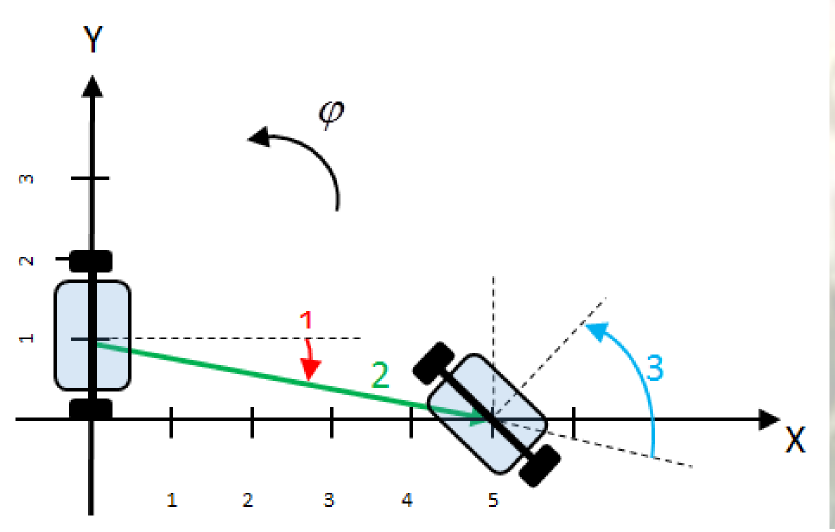
the third interval [20,30]:

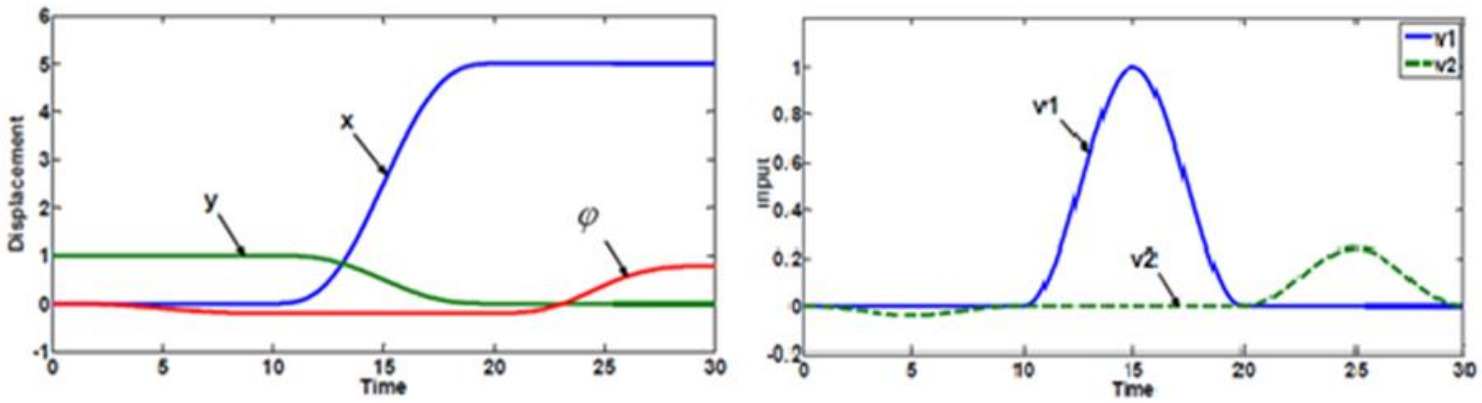
$$w = \frac{2\pi}{\varepsilon}, \varepsilon = 10,\ c_2 = 0.5, c_1 = -0.02, c_3 = 6/50$$

$$\begin{cases} X(t) = 5 \\ Y(t) = 0 \end{cases}$$

$$\begin{cases} \varphi(t) = \arctan\left(c_3 t - \frac{c_3}{w}\sin wt + 10c_1 - 20c_3\right) \begin{cases} t = 20 \Rightarrow \varphi(t) \approx -11.3^\circ \\ t = 30 \Rightarrow \varphi(t) = 45^\circ \end{cases} \\ v_1 = 0, v_2 = \frac{1}{\cos^2\varphi} u_2 \end{cases}$$

Ex. Differential wheeled robot simulation results





(a) Displacement versus time

(b) Input versus time

Conclusion

- The control inputs switch between two different modes to accomplish the cosine switch control.
- Cosine functions with unknown coefficients are taken as control inputs.
- After integrating operation and obtaining the expression of terminal configuration, we can solve the undetermined coefficients by substituting boundary conditions.
- Cosine function is used to avoid the mutations of velocity and acceleration at switching time.

Homework

- Modeling, simulation and Robot motion Animation of the proposed method using MATLAB, C++ or python. Visualize the results and graphs.
- Write a code in PICC compiler or Arduino IDE to make two dc motors rotate in specific velocity values (setpoints) for a specific time. You can detect these values from your simulation results and store it in a vector.

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