Volatility arbitrage indices – a primer

Keith Loggie, director global research & design at Standard & Poor's Index Services, looks at the instruments and their context

In broad terms, volatility arbitrage can be used to describe trading strategies based on the difference in volatility between related assets – for instance, the implied volatility of two options based on the same underlying asset. However, the term is most commonly used to describe strategies that take advantage of the difference between the forecasted future volatility of an asset and the implied volatility of options based on that asset. For more detail on volatility see 'box' on page 67.

This strategy is often implemented through a delta-neutral portfolio consisting of an option and its underlying asset. The return on such a portfolio will be based not on the future returns of the underlying asset, but rather on the volatility of its future price movements.

Buying an option and selling the underlying asset results in a long volatility position, while selling an option and buying the underlying asset results in a short volatility position. A long volatility position will be profitable to the

extent that the realised volatility on the underlying is ultimately higher than the implied volatility on the option at the time of the trade.

While delta-neutral options-based trades provide a means for investing based on a view of future volatility, they do present some drawbacks. Since the delta of an option changes as the price of the underlying asset changes over time, a portfolio consisting of an option and its underlying asset that is initially delta-neutral will soon no longer be so.

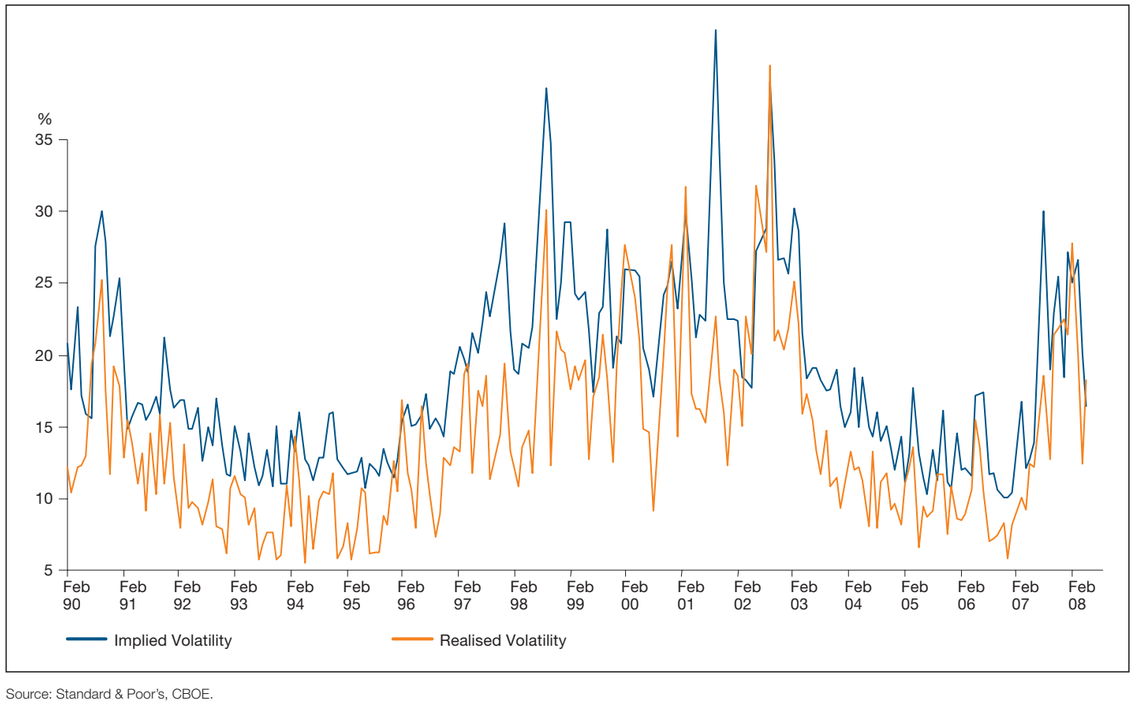
At this point, the performance of the portfolio is no longer based solely on volatility of the underlying asset but also on the performance of the underlying asset. This can be prevented by continuous delta hedging, or rebalancing of the portfolio to ensure that it is delta neutral. However, this not only creates transaction costs, but also it is not feasible for traders as they cannot constantly alter their hedge. Thus the position will generally not be solely dependent on volatility and is therefore not an ideal means of trading volatility.

Variance swaps

An alternative to options-based volatility trading is to use variance swaps. In a variance swap, one leg is valued based on the realised variance (volatility squared) of the underlying asset, as measured by logarithmic returns, while the other leg, the strike, is set at the inception of the swap and is based on the squared amount of the implied volatility of the underlying asset at the time the swap is struck.

The strike price of the swap is determined by the implied volatility of the options currently traded in the market based on the underlying asset. Thus a variance swap position is equivalent to a portfolio of options on the underlying asset and can be hedged in such a manner.

Exhibit 1: **Implied and Realised Volatility of S&P 500**



Source: Standard & Poor's, CBOE.

A long variance swap position will profit if the realised variance of the underlying asset is greater than the implied variance at the time the swap is struck. A variance swap provides pure exposure to volatility, as, unlike options prices, its value is based solely on changes to volatility.

The payoff of a variance swap is equal to the difference between realised variance and implied variance, multiplied by the number of contract units. The number of contract units is set such that if the realised volatility is one volatility point above the strike, the payoff to the receiver of realised volatility will be equal to the notional value of the contract.

The variance swap market has grown exponentially over the past decade and is among the most liquid equity derivatives contracts in the over-the-counter markets. While the most liquid underlying index is the S&P 500, there are also markets in EuroStoxx 50, FTSE 100 and Nikkei 225 contracts as well.

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An interesting characteristic of volatility is that for most traded assets, implied volatility tends to be higher than actual realised volatility. This is due to the fact that options are often used as a hedge or insurance. An investor will purchase a put option to hedge against large downward price movements or a call option to hedge against large upward price movements.

The options market is in many ways similar to the home insurance market. Investors using options buy protection from changes in prices, much as a homeowner purchases insurance against fire, flood damage, etc. In both cases, the provider of the insurance is paid a premium to compensate for the risk that is being taken.

Just as insurance companies price their policies so that they expect to make a profit based on estimates of their eventual payouts, options writers will only be willing to write options if they can expect a sufficient profit to compensate for the risks they are assuming. Thus, options and options-based structures such as variance swaps will tend to be priced at a higher implied volatility than is actually expected to be realised.

Based on the above discussion, writers of options and investors who take a short volatility position in a variance swap or similar product should profit over time. Historical data bears this out.

As noted above, the most popular underlying index for variance swaps is the S&P 500 and the implied volatility of the S&P 500 is measured by VIX, the CBOE Volatility Index. VIX measures the implied volatility of the S&P 500 based on the entire strip of S&P 500 options contracts and uses the near-term and next-term options to calculate the implied volatility of the S&P 500 over the next 30 calendar days.

Exhibit 1 tracks the implied volatility of the S&P 500 as measured by VIX against the actual realised volatility for the following one-month period. The start and end dates of each monthly period are determined by the expiration date of VIX options, generally the third Friday of each month. One can see that in general the implied volatility is higher than the realised volatility.

The trend for implied volatility to be higher than realised volatility is quite consistent. Exhibit 2 shows that implied volatility exceeds realised volatility in 86 per cent of the months. Furthermore, the average and median differences where implied exceeds realised is more than the months where realised exceeds implied.

Indexation of volatility arbitrage

Over the past year, transparent indexation of volatility arbitrage has been

Exhibit 2: **One-Month Implied versus Realised Volatility**

	Implied Exceeds Realised	Realised Exceeds Implied
Number (%) of Observations	190 (86.36%)	30 (13.64%)
Average Monthly Difference	5.34%	3.63%
Median Monthly Difference	5.00%	3.13%

Source: Standard & Poor's. Data from February 1990 to June 2008.

"The choice of vega is an important determinant, and probably the principal qualitative element in design of volatility arbitrage indices."

introduced to serve as benchmarks for volatility arbitrage strategies and to serve as underlyings for index-linked products. The S&P 500 Volatility Arbitrage Index measures the performance of a variance swap strategy that consists of receiving the implied variance of the S&P 500 and paying the realised variance of the S&P 500.

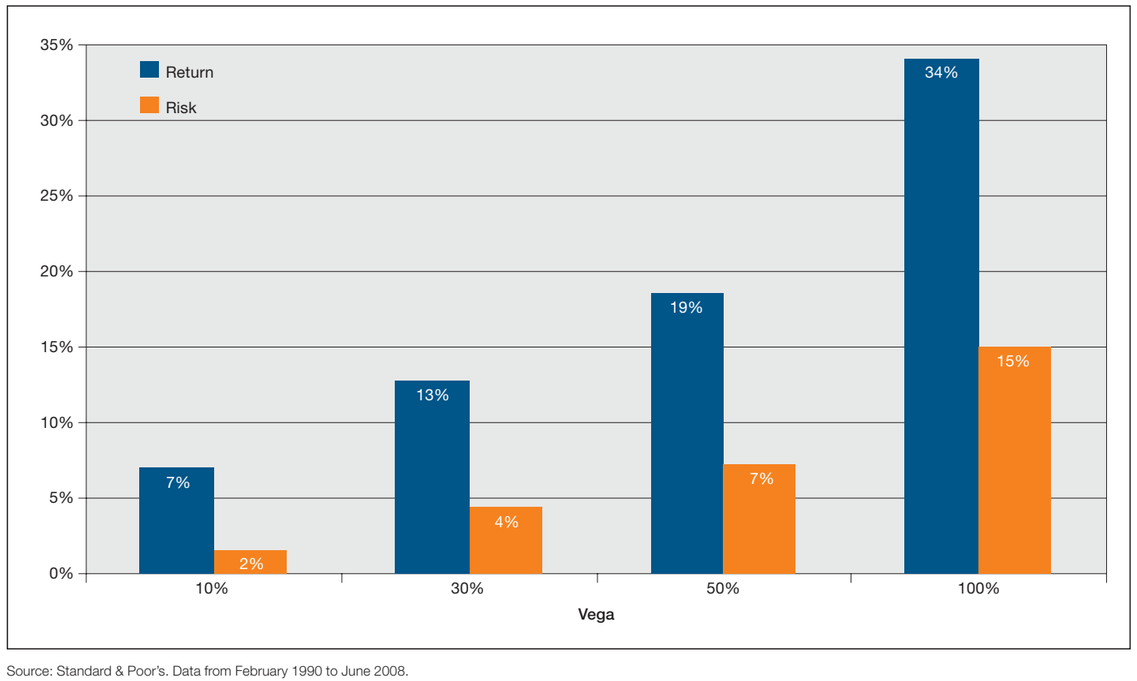
The index assumes a one-month variance swap is entered into on the third Friday of a given month and is held until the third Friday of the following month, at which time the position is rolled over. The index is calculated on a price return (unfunded) and total return (funded) basis. The total return index includes interest accrual on the notional value of the index based on the one-month US dollar Libor rate.

The implied variance used for the index, equivalent to the strike price of a corresponding variance swap, is measured based on the average level of VIX between 12.00pm and 1.00pm on the roll date, less 1 per cent to account for implementation slippage. The performance of the index is calculated based on the difference in the implied variance less the realised variance of the S&P 500 from the prior rebalancing date through the current date.

This difference is multiplied by the variance notional, equal to vega (a volatility exposure modifier) divided by twice the strike price (which converts volatility points into dollar amounts). For more details see the 'box' on page 65.

The choice of vega is an important determinant, and probably the principal qualitative element in design of volatility arbitrage indices. As Exhibit 3 shows, the higher the vega, the higher the expected return and risk from the index. For the S&P 500 Volatility Arbitrage index, the vega is set at a level of 30 per cent.

Exhibit 3: Impact of Vega on Volatility Arbitrage Index Returns



Source: Standard & Poor's. Data from February 1990 to June 2008.

Index methodology

As noted in the main text, the S&P 500 Volatility Arbitrage Index is rebalanced on the third Friday of each month. The methodology approximates a one-month variance swap position that receives implied variance on the S&P 500, as measured by the VIX, the Chicago Board Option Exchange Volatility Index, and pays realised variance on the S&P 500.

The total return version of the index incorporates interest accrual on the notional value of the index. Interest accrues based on the one-month US dollar Libor rate.

Index calculations

On any rebalancing date, t, the index is calculated as follows:

$$IndexValue_t = IndexValue_{t-1} * (1 + VolArbStrategy_t) \quad (1)$$

where:

$IndexValue_{t-1}$ = The index value as of the last rebalancing date (t-1).

$VolArbStrategy_t$ = A percentage, as determined by the following formula:

$$VolArbStrategy_t = VarianceNotional_{t-1} * \left[(IVS_{t-1})^2 - (RVS_{t-1,t})^2\right] \quad (2)$$

where:

$t\text{-}1$ = The last rebalancing date

IVS_t = Implied Volatility Strike. On any rebalancing date, t, it is represented by the equally-weighted average of the levels of VIX, the CBOE Volatility Index as published by the CBOE, minus 1 per cent. The average is calculated using the index levels as published by the CBOE every five minutes from, and including, 12:00pm to, and including, 01:00pm (ET), divided by 100. For index history prior to January 18 2007, IVS_t is represented by the average of the high value and low value of VIX as published by the CBOE, divided by 100, minus 1 per cent.

$RVS_{t-1,t}$ = Realised Volatility Strike of the S&P 500 calculated using the following formula:

$$RVS_{t-1,t} = \sqrt{\frac{252 * \sum_{i=1}^{n} ln\left(\frac{Index_i}{Index_{i-1}}\right)^2}{n}} \quad (3)$$

where:

$Index_i$ = The closing price of the S&P 500 on day i.

n = The number of trading days from, and including, the prior rebalancing date, $t\text{-}1$, but excluding the current rebalancing date, t.

$$VarianceNotional_t = \frac{1}{2 * IVS_t} * Vega \quad (4)$$

where:

$Vega$ = A limit to the exposure on the spread. 30 per cent is used for this index.

Total Return Index

A total return version of the index is calculated, which includes interest accrual on the notional value of the index based on the one-month US dollar Libor rate, as follows:

$$TRIV_t = TRIV_{t-1} * \left(1 + LIBOR_{t-1} * (d_{t-1,t}/360) + VolArbStrategy_t\right)$$

where:

$TRIV_t$ = Total return index value as of the current rebalancing date, t.

$TRIV_{t-1}$ = Total return index value as of the last rebalancing date, $t\text{-}1$.

$d_{t-1,t}$ = Count of calendar days from the prior rebalancing date, $t\text{-}1$, to the current rebalancing date, t, but excluding the prior rebalancing date.

$LIBOR_{t-1}$ = 1 month US dollar Libor rate as of the last rebalancing date, $t\text{-}1$.

Base Date

The index base date is February 16 1990 at a base value of 100.

Index committee

The S&P Arbitrage Index Committee maintains the S&P 500 Volatility Arbitrage Index. The Index Committee meets regularly. At each meeting, the Index Committee reviews any significant market events. In addition, the Committee may revise index policy for timing of rebalancings or other matters.

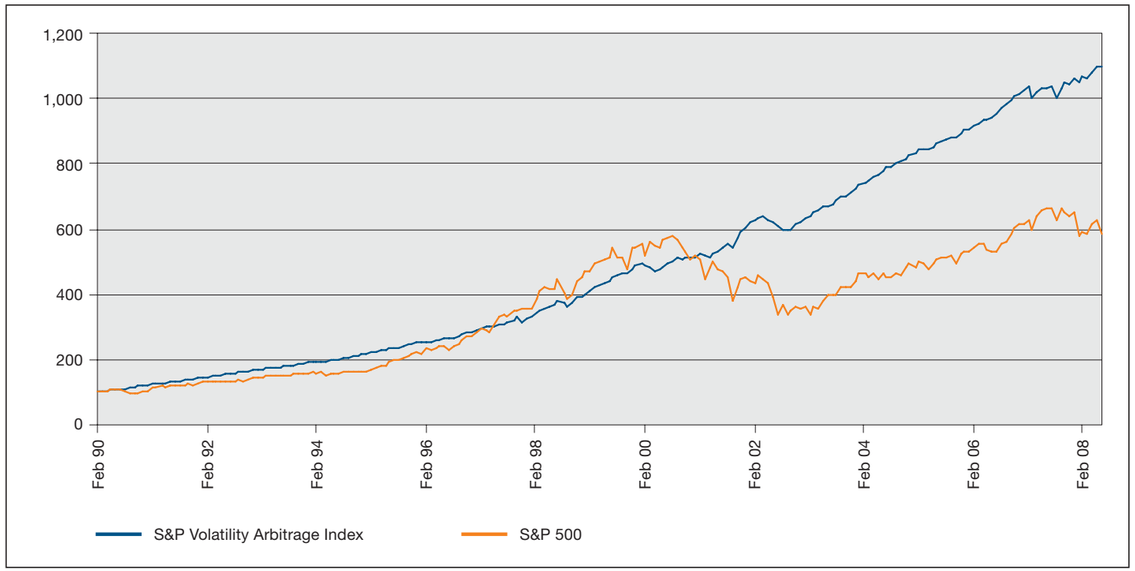
Standard & Poor's considers information about changes to its indices and related matters to be potentially market moving and material. Therefore, all Index Committee discussions are confidential.

Exhibit 4: **Return and Risk Characteristics of S&P 500 Volatility Arbitrage Index**

	S&P 500 Volatility Arbitrage		S&P 500	
	Annualised Return	Standard Deviation	Annualised Return	Standard Deviation
3 Year	8.26%	4.44%	4.69%	13.02%
5 Year	10.31%	3.66%	7.75%	11.94%
10 Year	11.49%	4.94%	3.49%	17.36%
Since Inception	12.72%	4.38%	9.21%	14.75%
Correlation with S&P 500	0.478		1.000	
Correlation with Lehman Aggregate	–0.029		0.013	
Per cent of Month Positive	85.91%		61.36%	
Largest Drawdown	6.35%		41.71%	

Source: Standard & Poor's. Data from February 1990 to June 2008.

Exhibit 5: **Performance of S&P Volatility Arbitrage since Inception**



Source: Standard & Poor's. Data from February 1990 to June 2008.

Alternative formulations of volatility arbitrage indices could use a different variance swap tenor (for example one-month instead of three-months), a different source of the strike (traders' variance swap quotes instead of VIX), and a different vega.

The S&P 500 Volatility Arbitrage Index has history back to February 1990. The strategy has outperformed the US equity market as measured by the S&P 500 by more than 300 basis points per year.

More importantly, the strategy has resulted in consistent positive returns. It has a very low annualised standard deviation of 4.38 per cent versus 14.75 per cent for the S&P 500. Out of 220 monthly observations, it has had a negative return in only 31 months. The strategy has yet negative return for any 12-month period and has a much lower maximum drawdown.

Conclusion

Volatility arbitrage is morphing from a niche institutional strategy to mass market, index-linked products. Volatility arbitrage indices, such as the S&P 500 Volatility Arbitrage Index, have shown consistent positive performance and low volatility. As with any arbitrage strategy that gains in popularity, increase in arbitrageurs may tighten the spreads between implied and realised volatilities in the future.

However, as long as there is an insurance premium in the options market, there will be room for volatility arbitrage. The key issue that will drive returns in the future will be the size of the protection-seeker market versus the size of the arbitrageur market. ■

Implied and realised volatility explained

Implied volatility

Implied volatilities are derived from options prices. The implied volatility of an option is the level of volatility consistent with the current market price of the option. Of the factors that determine the value of an option, strike price, stock price, time to expiration, volatility, expected cash flows and the risk-free interest rate, volatility is the only factor that is unknown.

The value of an option is dependent on the future volatility of the underlying asset. As future volatility is unknown, the price of an option in the market will be determined by investors' expectation of what the volatility of the asset will be. Thus, for a given options pricing model such as Black-Scholes, the implied volatility of an option can be inferred based on the current market price of the option.

For a given underlying asset there will often be numerous options traded on it, based on different strike prices and expiration dates. These options will generally not trade with the same implied volatility. For example, investors may expect a quite different volatility for an asset over the next three months than over the next year.

Thus options that expire in one year's time will be priced with a different implied volatility than those that expire in three months. This is referred to as the term structure of volatility. Also, options expiring at the same time but at different strike prices tend to be priced based on different implied volatilities. This is due to what is known as the volatility skew.

Options pricing models such as Black-Scholes make the simplifying assumption that returns of the underlying asset are normally distributed. However, in reality this is not generally the case. Stock returns, for example, tend to be long-tailed with more observations far from the mean than would be expected based on a normal distribution. Since the chances of an extreme observation are more likely than would be implied by a normal distribution, the implied volatility of far in the money and far out of the money options will tend to be higher than that of at the money options.

Because of the skew in the term structure of volatility there is not a single direct measure of implied volatility of an underlying asset but rather many measures based on the various different options available on it. A benchmark of implied volatility for an underlying asset is generally calculated by taking a weighted blend of multiple options prices. An example of this is the VIX calculation, a measure of implied volatility of the S&P 500.

Realised volatility

The realised volatility of an asset is the volatility of the actual historical returns of an asset. Thus, unlike implied volatility which is a forward-looking prediction of volatility, realised volatility is backward looking. For example, one could calculate the realised volatility of the S&P 500 of June 2008 by taking the standard deviation of the daily returns of the index within that month. For variance swaps, realised volatility is calculated based on logarithmic returns rather than arithmetic returns.

The difference between implied volatility and realised volatility is that implied volatility is a forecast of the future realised volatility of an asset. While implied volatility is measured based on the prices of options based on the underlying asset, realised volatility is calculated based on actual historical returns of the underlying asset. Implied and realised volatility will differ for a given asset over a given time period to the extent that investors' estimates of volatility as measured by options prices prior to the period in question differ from the actual volatility of the asset over that time.

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The S&P 500 Volatility Arbitrage Index was officially launched on January 17, 2008. The returns of the index from February 20, 1990 to January 17, 2008 are back-tested by applying the published index methodology.