SI 211: Numerical Analysis
Homework 1

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Deadline: Sep 26, 2018

1. We want to evaluate the function

$$f(x) = \frac{\sin(10^4 x)}{x}$$

for different values of x.

   (a) Evaluate the above function at $x = \pi$ by using `Matlab`, `Julia`, `C++` or any other programming language of your choice. How big is the numerical approximation error? Can you explain why you observe this error?

   (b) Evaluate the above function at $x = 10^{-10}$. How big is the numerical evaluation error?

2. Numeric differentiation based on central differences:

   (a) Implement a function (for example in Python, Julia, or Matlab) that uses numeric differentiation based on central differences,

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} \; .$$

   Here, the inputs of the differentiation routine are the scalar function f that we want to differentiate, the point x at which the derivative should be evaluated, and the finite perturbation $h > 0$. Use the syntax

```
diff(f,x,h) = ...
```

   (b) Evaluate the derivative of the function $f(x) = \exp(x)$ at $x = 0$ using the above routine `diff`. Plot the numerical differentiation error in dependence on $h \in [10^{-15}, 10^{-1}]$ and interpret the result. Use logarithmic scales on both axis!

3. In order to evaluate the factorable function $f(x) = \sin(\cos(x)) * \cos(x)^2$ we write an evaluation algorithm of the form

$$\begin{aligned} a_0 &= x \\ a_1 &= \cos(x) \\ a_2 &= a_1 * a_1 \\ a_3 &= \sin(a_1) \\ a_4 &= a_2 * a_3 \\ f(x) &= a_4 \; . \end{aligned}$$

   What is the corresponding algorithm for evaluating the derivative of $f(x)$ using the forward mode of algorithmic differentiation (AD)? What is the order of magnitude of the numerical error that is associated with evaluating the derivative of f at $x = 0$ using this AD code?