Entry by Merger:

Estimates from a Two-Sided Matching Model with Externalities*

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Abstract

As firms often acquire incumbents to enter new markets, presence of desirable targets affects entry and merger decisions simultaneously. We study these decisions jointly by considering a two-sided matching model with externalities and estimate it using data on commercial banks to investigate the effect of entry deregulation. Our estimation strategy exploits the lattice structure of stable allocations to construct moment inequalities and partially identify payoffs including potential (dis)synergies. We find significantly low entry barriers for entry by merger and larger synergies between smaller potential entrants and larger incumbents. We then quantify the bias resulting from ignoring the entry-by-merger option, and consider the role of competition externalities in merger markets. Lastly, by prohibiting de novo entry, our counterfactual quantifies the effect of the deregulation.

Keywords: Entry, merger, two-sided matching, partial identification

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1 Introduction

Firms often use mergers and acquisitions to enter new markets.[1] In such cases, presence of desirable acquisition targets affects not only merger decisions but also entry decisions at the same time. A firm may choose not to enter a market if it cannot find a good target incumbent for acquisition. In some markets, entry barriers for *de novo* entry can be so high that acquiring an incumbent may be the only profitable way to enter. Thus, entry and merger decisions are joint decisions, and should not be separately studied in those markets. Moreover, as entry by merger impacts the competitive structure of markets, studying entry by merger may have important policy implications.[2]

In this paper, we study entry and merger decisions jointly in the U.S. commercial banking industry where entry by merger is prevalent. In particular, we investigate the effect of the state-level entry regulation. We focus on 7 states that deregulated the *intra*-state de novo entry restriction during the period between 1995 and 2000. Using data on bank behavior in the regional markets of these 7 states for the period right after the deregulation, we study how the entry regulation affected entry and merger decisions.

To study the banks' behavior, we consider a model in which the "with whom"decision of the merger and the post-entry (and post-merger) competition are addressed simultaneously. We do so by combining a standard entry model (Bresnahan and Reiss (1991), and Berry (1992)) with a two-sided matching model with contracts (Hatfield and Milgrom (2005)). Regarding the "with whom"decision, two features are particularly important: i) the payoff from merger depends on potential (dis)synergy, which significantly differs across pairs of firms (we specify a

[1]A significant fraction of market entry is reported to be by merger. Yip (1982) shows that more than one-third of entries in 31 product markets in the United States over the period 1972-1979 were by acquisition. Among 558 market entries into the United States by Japanese companies during 1981-1989, Hennart and Park (1993) found that entry by merger accounted for more than 36%.

[2]Entry by merger has been an important antitrust issues. The Federal Trade Commission (FTC) explicitly considers "potential new competitors" in its Merger Guidelines. A classic case is the FTC's decline of a merger attempt by Procter & Gamble (P&G) and the Crolox Corporation in 1967. FTC argued that P&G was the most likely potential entrant in the household bleach industry, and that P&G's acquisition of Clorox would eliminate P&G as a potential competitor, which would substantially reduce the competitiveness of the industry.

synergy function to quantify potential (dis)synergies), and ii) the merger decision is not unilateral (the target firm must agree to the merger contract). Reflecting these features, we adopt a matching model. Specifically, we consider a one-to-one two-sided matching model in which firms are partitioned into two sides, the sides of incumbents ($\mathscr{I}$) and potential entrants ($\mathscr{E}$) given that the vast majority of the mergers in our data are mergers between one incumbent and one potential entrant.[3]

Regarding how we incorporate the effect of post-entry competition into the matching model, we follow a standard entry model (Bresnahan and Reiss (1991) and Berry (1992)) in which the effect of competition on profit is modeled as a decreasing function of the number of operating firms. A firm considers this effect of competition on profit (negative externalities), and chooses the best option out of the three types of options {*Enter with merger, Enter without merger, Do not enter*}.[4] Because the profit of a firm depends not only on the firm's matching (merger partner) but also on the merger and entry decisions of other firms (e.g., whether another incumbent is acquired by a potential entrant), the model we consider becomes a two-sided matching model *with externalities*.

Considering externalities in a matching model is not straightforward (see, e.g., Sasaki and Toda (1996) and Hafalir (2008)). This is because, with externalities, payoffs depend not only on matching but also on the entire assignment of who match with whom. The solution concept in such a case thus has to take into consideration, for each deviation, what the entire assignment would be in addition to with whom the deviating players would match. The literature takes two approaches to deal with this issue. One approach is to consider "far-sighted"stabililty that incorporate what the assignment would be considering rematches among the rest of agents after each deviation (Sasaki and Toda (1996), and Hafalir (2008)). The model with general form

[3]This fact reflects the types of market we observe: our data is from small regional markets where average number of incumbents are less than 5 (see Section 4 for the detail), thus mergers between incumbents or mergers involving more than two incumbents tend to be infeasible due to antitrust concerns.

[4]As we consider a matching model, the first option requires the consent of the matching partner for the matching to be stable, while the last two options can be chosen unilaterally (See Section 3 for more detail). Our model differs from a standard two-sided matching model in that there are two outside options, i.e., firms can be unmatched in two ways by either entering without merger or by not entering the market.

of externalities is complex and the existence of a stable matching requires strong conditions under this approach. Alternative approach, which Pycia and Yenmez (2017) call "standard" in the sense that deviation does not lead to reaction of remaining agents, guarantee existence of stable matching by restricting preferences. We adopt this latter approach as it corresponds well with our model in which externalities take a particular form—it depends only on the aggregate number of operating firms negatively (i.e., negative network externalities).

Estimating a two-sided matching model posits an econometric challenge: the model typically has multiple equilibria and the parameter cannot be point-identified without imposing an equilibrium selection rule. Instead of imposing any equilibrium selection mechanism, we adopt the partial-identification approach. We propose a novel estimation approach that can be applied to many of two-sided matching models with non-transferable utility in general. In particular, we exploit the lattice structure of the set of equilibria. Though the econometrician cannot tell the equilibrium selection rule in the observed data, the lattice property provides upper and lower bounds for the equilibrium payoffs for each firm. To be more precise, all incumbents have the highest equilibrium payoff in incumbent($\mathscr{I}$)-optimal equilibrium, and the lowest equilibrium payoff in potential entrant($\mathscr{E}$)-optimal equilibrium. All other equilibrium payoffs are bounded by these two equilibrium payoffs. Hence, the payoff corresponding to the observed outcome is bounded above and below by these extremal equilibrium payoffs, from which we construct moment inequalities for incumbents. Similarly, all potential entrants obtain the highest payoff in $\mathscr{E}$-optimal equilibrium and so on. Thus, we can construct moment inequalities using these equilibrium characterizations without the knowledge of the equilibrium selection rule.

The identified set can be reduced further by considering other equilibrium properties on top of the moment inequalities in payoffs. In all equilibria, the model predicts that the identity of the firms that enter without merger (the result is analogous to the *lone wolf theorem* and the*rural hospitals theorem*. See, e.g., Roth (1986) and Hatfield and Milgrom (2005)). This implies that the payoff of the firms entering without merger is the same across all equilibrium.

Using this property, we construct moment equalities on the payoff for firms entering without merger in addition to the moment inequalities on payoffs for firms entering with merger.

Based on the moment equalities and inequalities discussed above, we estimate the model using Andrews and Soares (2010)'s generalized moment selection. In computing the sample analogue of the moments, we run a version of the generalized Gale-Shapley algorithm order to obtain both $\mathscr{I}$-optimal and $\mathscr{E}$-optimal equilibria for each simulation draw for each market.

We find a significant difference in entry barriers for incumbents, potential entrants, and entry by merger. The entry barrier for potential entrants is much higher than that of the incumbents as well as that of entry by merger, causing a significant fraction of entry by potential entrants to take the form of merger. Concerning the (dis)synergies from mergers between different types of firms, bank characteristics affect (dis)synergy differently across the sides of incumbents and potential entrants. We find that synergy between potential entrants with a smaller asset size and lower equity ratio and incumbents with a larger asset size tends to be much higher. Furthermore, a pair with closer headquarter locations have a higher synergy.

To see the role of mergers as a way to enter a new market, we estimate a standard entry model à la Berry (1992) and compare the estimation results. We find that the effect of competition is significantly underestimated if one treats observed entry by merger in the data as a regular entry (without merger). This is because the standard model without merger ignores the positive effect of potential synergies of mergers and this effect is instead attributed to smaller magnitude of the estimate of competition externalities coefficient. Hence, it is essential to incorporate entry by merger explicitly when one is interested in inferring the degree to which competition affects entry.

Moreover, we examine the role of negative externalities due to competition. In other words, we consider what if firms do not take into account the impact of any strategic interactions when they make entry decisions. This simulation highlights the importance of externalities in merger markets, which has not been well understood in the existing structural empirical literature on merger and acquisitions. The simulation results indicate that a lot more firms enter by themselves rather than entering by merger. This is because entry without merger becomes a less costlier way to enter the market and finding a merger partner with mutual interests would be still hard even without any negative externalities.

Lastly, we conduct a counterfactual policy simulation in order to assess the effect of the entry regulation. We find the number of banks operating in a market would have decreased if de novo entry by potential entrants had not been deregulated by the deregulation. Prohibition of de novo entry provides stronger incentive to enter by merger for potential entrants, and accordingly we find that the number of entry by merger increases if de novo entry were prohibited.

After discussing the related literature in Section 2, we present a two-sided matching model in Section 3. We document the data in Section 4, and then provide the econometric specification, identification, and estimation procedure in Section 5. Section 6 reports the results of the estimation and the counterfactual experiment. Finally, we conclude in Section 7.

2 Related Literature

Our paper adds to several strands of literature. The first strand of the related literature is the literature on estimating matching models. We add to this literature by proposing a new approach to estimate a matching model with *non-transferrable* utility.[5] Only a few papers estimate matching models with non-transferrable utility. Gordon and Knight (2009) consider merger of school districts, and their specification on match quality is similar to our synergy function, though they consider a one-sided matching model (roommate problem). Also their paper is close to ours as they run an algorithm to find a stable matching in their estimation. Sorensen (2007) studies the matching between venture capitalists and entrepreneurs, and estimates the model under the assumption that players have aligned preferences. Boyd, Lankford, Loeb, and Wyckoff (2013) similarly estimate a two-sided matching model between teach-

[5]Although there is a transfer, our model is a model with non-transferrable utility. This is because we allow players to have some components which are not transferable.

ers and schools by running the Gale-Shapley algorithm with the assumption that the school-optimal stable matching is realized. Similarly with a known equilibrium selection mechanism, Uetake and Watanabe (2012) proposes an estimation strategy for matching model with non-transferable utility using Adachi (2000)'s prematching mapping. Agarwal (2015) also proposes an empirical framework to estimate many-to-one two-sided matching models under a vertical preference restriction on one side and apply it to the data from the medical matching. In the environments with only aggregate-level data available, Echenique, Lee, Shum, and Yenmez (2013) study testable implications of stable matchings. In a similar environment, Hsieh (2011) proposes a modified deferred acceptance algorithm to study identification and estimation. These two papers differs from ours as they consider aggregate-level data while we use individual-level data. Our paper also differs from these papers in that we consider matching with contracts. Finally, a number of papers estimate matching models with *transferrable* utility.[6] Among these, Akkus, Cookson, and Hortacsu (2016) and Park (2013) are close to ours in that they study the merger of banks and mutual funds, respectively. Our paper adds to these papers by explicitly considering the effect of post-merger competition.[7]

The second strand of the related literature is the large and growing theoretical literature on matching.[8] Our paper builds on Hatfield and Milgrom (2005) (hereafter denoted as HM), who study a matching model with contract. In fact, our solution is derived using the HM's generalized Gale-Shapley algorithm. HM characterize the set of stable allocations for the matching model with contract, and show that the set of stable allocations in two-sided matching with contracts has lattice property and proves the existence using Tarski's fixed-point theorem.[9] We follow their approach and incorporate two additional features; we allow externalities and participation decisions. In matching models, a player's individual rationality condition is about

[6] See, e.g., Choo and Siow (2006), Fox (2018), Fox, Yang, and Hsu (2018), Galichon and Salanié (2009), Baccara, Imrohoroglu, Wilson, and Yariv (2012), Chiappori, Salanié, and Weiss (2017), and Dai (2016).

[7] Vissing (2017) also estimates a non-transferrable utility model of two-sided matching using the data from the oil and gas leasing industry. She assumes that firms in one side of the market always propose to those in the other side to avoid multiple stable allocation.

[8] See, e.g., a survey by Roth (2008).

[9] See also Adachi (2000), Echenique and Oviedo (2006), and Ostrovsky (2008) for characterizations of the set of stable matchings using similar techniques in various matching environments.

whether he has incentive to be matched with others, but not about his participation to the matching market. We explicitly consider the incentive to "participate,"and make the preference dependent on the number of "participants,"which is the externalities we consider.

Incorporating externalities into matching model is not straightforward. Analyzing matching models with externality is difficult because preference is defined over the set of assignments rather than over matchings. There are two approaches to address the extra complexity due to externalities. The literature started with the estimation function approach (Sasaki and Toda (1996) and Hafalir (2008)) under which "far-sighted"stability that incorporates reaction by all remaining agents to an agent's deviation based on the concept of "estimation function."Estimation functions specify the expectations on the assignment (i.e., what the rematching among all players would be) after each deviation. They prove that a strong requirement on the estimation function is necessary in order to guarantee the existence of stable matching. The second approach considers standard (or "myopic") stability under which a deviation does not lead to further re-matching among the remaining agent (Pycia and Yenmez (2017)). By restricting types of externalities and preferences, existence of stable matchings are shown in this strand of literature (Mumcu and Saglam (2010), Bando (2012), Branzei, Michalak, Rahwan, Larson, and Jennings (2012), and Pycia and Yenmez (2017)). Externalities in our paper only depends on the number of operating firms, and we follow this approach of using the standard stability concept.

The third strand of the literature is the literature on estimating entry models following Bresnahan and Reiss (1991) and Berry (1992), where the firms' underlying profit functions are inferred from the observed entry decisions.[10] We add to this literature by examining the entry and merger decisions jointly. We do so by combining two-sided matching model with the entry model. Our approach is similar to Ciliberto and Tamer (2009) in using a set estimator to address multiplicity of equilibria. Also, our identification argument builds on their identification results. Other related studies include Jia (2008) and Nishida (2015) that characterize

[10]Other recent contributions include Mazzeo (2002) and Seim (2006).

the equilibrium of an entry model with correlated markets as a fixed point in lattice and solve for it to estimate the model. Our paper differs from theirs in that ours make no equilibrium selection assumption and construct moment inequalities exploiting the lattice properties.

Among papers in entry literature, Perez-Saiz (2015) is closest paper to ours in terms of research question. He models firm's decision as a three stage extensive form game, and estimate it using the data of the U.S. cement industry. After choosing whether to enter by merger, enter without merger, or not entering, firms entering with merger engages in bidding for merger contracts, active firms play a Cournot game in the third stage. In his model, merger decisions are conditional on entry decisions, while these decisions are simultaneous in our model. He finds that ignoring the possibility of entry by merger significantly overestimates the entry cost.

The fourth strand is the literature on horizontal merger decisions. In spite of the large literature considering the effects of mergers, studies on the endogenous horizontal merger decision itself are limited.[11] Kamien and Zang (1990) show limits to monopolization through mergers, and Qiu and Zhou (2007) point out the importance of firm heterogeneity in horizontal mergers. Gowrisankaran (1999) develops a computable dynamic industry competition model with an endogenous merger decision. Following Gowrisankaran (1999), Igami and Uetake (2017) estimate a dynamic model of endogenous merger using the data on the hard dirk drive industry. They model merger and acquisitions by specifying a particular bargaining process or take-it-leave-it offer. Pesendorfer (2005) finds the relationship between market concentration and the profitability of mergers using a repeated game with merger decision. In a model with de novo foreign direct investment and cross-border merger and acquisitions, Nocke and Yeaple (2007) show the importance of firm heterogeneity as key determinants. Our paper adds to this literature by empirically investigating the role of firm heterogeneity in merger decisions.

Finally, our paper is also related to the literature studying banks' branching decisions. Ruffer and Holcomb (2001) use data from California and investigate the determinants of a bank's

[11]There are also a few papers that study the relationship between merger decisions and merger review policy. See, e.g., Nocke and Whinston (2010).

expansion decision by building a new branch and acquiring an existing branch, respectively. Their results show that a large bank would be likely to enter a new market by acquisition, but not through building a new branch, which is consistent with our result that larger potential entrants have higher synergy ceteris paribus. Wheelock and Wilson (2000) study the determinants of bank failures and acquisitions using a competing-risks model. Consistent with our finding, they show that less capitalized banks are more likely to be acquired for the period between 1984 and 1993. Cohen and Mazzeo (2007) estimate an entry model with vertical differentiation among retail depository institutions, and find evidence of product differentiation depending on market geography. Relatedly, Aguirregabiria, Clark, and Wang (2016) estimate a model of banks' branch location choice to consider the diversification of geographic risk. Our paper studies the role of mergers as the bank's strategy to expand its branch network.

3 Model

We model the entry and merger decisions as a two-sided matching problem with externalities. We build the model combining models of entry (Bresnahan and Reiss (1991) and Berry (1992)) and two-sided matching with contracts (Hatfield and Milgrom (2005)). Then, we provide characterizations of the stable outcomes: the set of stable outcomes forms a complete lattice, and the two extremal points of the set can be obtained by running a deferred acceptance algorithm that we propose. We use these characterizations for partial-identification and estimation of the model.

3.1 A Matching Model of Entry and Merger

We consider a static entry model in which firms can use merger and acquisition as a form of entry in addition to regular entry without merger. In particular we integrate entry and merger decisions into a two-sided matching model between an incumbent firm (denoted by $i \in \{1,...,N_{\mathscr{I}}\} \equiv \mathscr{I}$) and a potential entrant (denoted by $e \in \{1,...,N_{\mathscr{E}}\} \equiv \mathscr{E}$). The model is a static one and being an incumbent is simply a characteristic of a firm. In other words, being an incumbent does not have particular dynamic implications.

We adopt a two-sided matching model because mutual consent between the two parties is required for a merger. In particular, we consider one-to-one two-sided matching instead of coalition formation, one-to-many matching, or many-to-many matching for the reason that the vast majority of mergers in our data are one-to-one mergers between an incumbent and a potential entrant.[12] The modelling choice we make reflects the types of market we observe: our data is from small regional markets where average number of incumbents are less than 5 (see Section 4 for the detail), thus mergers between incumbents or mergers involving more than two incumbents tend to be infeasible due to antitrust concerns. Moreover, given that we study these particular markets, the vast majority of the mergers takes the form that an entrant acquires an incumbent. Finally, we consider a matching model instead of a specific extensive form game because the details about individual merger process (such as how both sides negotiate the merger contract, etc.) are not observable.

Firms on both sides have three types of choices. Potential entrant e can choose not to enter (denoted by $\{o\}$), enter by itself (denoted by $\{e\}$), or merge with incumbent i with a merger contract k_{ei} (denoted by $\{k_{ei}\}$). Merger contracts are bilateral ones between a potential entrant and an incumbent, and each firm can sign only one merger contract with a firm on the other side. A contract $k_{ei} = (e, i, p_{ei})$ specifies a potential entrant and an incumbent pair and the terms of the merger, $p_{ei} \in P$, where the set of merger terms P is finite as in HM. Similarly to potential entrants, incumbent i has three types of choices: it can choose not to enter (denoted by $\{o\}$), enter by itself (denoted by $\{i\}$), or merge with potential entrant e with a merger

[12] One can also think about a dynamic matching model in which agents consider matching each period in stead of static matching. We choose not to model dynamics as dynamic issues such as merger waves are limited in our data. Also, theoretical characterization of dynamic matching models is not much known, and computational cost of such models would be extremely high.
A potential bias resulting from not addressing dynamics could be as follows. If there is a significant economy of scale, for example, and banks may have incentive to conduct acquisition that is unprofitable by itself, but create larger synergy if such acquisition enables profitable acquisition in the future. In such a case, synergies estimated in the static model would be biased.

contract k_{ei} (denoted by $\{k_{ei}\}$). We denote the payoff as follows:

$$\begin{aligned}
&U_i(k_{ei}, k_{-i}) = u_i(k_{ei}, k_{-i}) + \varepsilon_{ei} && \text{if firm } i \text{ merges with firm } e \text{ with contract } k_{ei}, \\
&U_i(i, k_{-i}) = u_i(i, k_{-i}) + \varepsilon_{ii} && \text{if firm } i \text{ enters without merger,} \\
&U_i(o, k_{-i}) = 0 && \text{if firm } i \text{ does not enter,}
\end{aligned}$$

where $u_i(\cdot, k_{-i})$ is the payoff of firm i with other firms choosing k_{-i}, and ε_{ei} and ε_{ii} are idiosyncratic shocks unobservable to the econometrician. As we assume ε_{ei} and ε_{ee} to be smoothly distributed, the preferences are strict generically. The payoff of not entering is normalized to zero. Similarly, we can write potential entrant e's payoff as

$$\begin{aligned}
&U_e(k_{ei}, k_{-e}) = u_e(k_{ei}, k_{-e}) + \varepsilon_{ei} && \text{if firm } e \text{ merges with firm } i \text{ with contract } k_{ei}, \\
&U_e(e, k_{-e}) = u_e(e, k_{-e}) + \varepsilon_{ee} && \text{if firm } e \text{ enters without merger,} \\
&U_e(o, k_{-e}) = 0 && \text{if firm } e \text{ does not enter,}
\end{aligned}$$

where $u_e(\cdot, k_{-e})$ is the payoff of firm e with other firms choosing k_{-e}. The ε_{ei} ($\neq \varepsilon_{ie}$) and ε_{ee} are idiosyncratic shocks unobservable to the econometrician. We do not impose $\varepsilon_{ei} = \varepsilon_{ie}$ because acquiring and target firms may have heterogenous preferences on the same potential merger.

We consider both the payoff function $u_i(\cdot,\cdot)$ and $u_e(\cdot,\cdot)$ and shocks, ε_{ei}, ε_{ee}, ε_{ie} and ε_{ii} to include non-pecuniary components reflecting the fact that factors not directly measured by monetary profit may have varying importance across firms in merger and entry decisions. Such differences may result from variations in the degree to which managerial and shareholder interests are misaligned as in Jensen and Meckling (1976). Other sources could be differences in CEOs' overconfidence on merger decisions (Malmendier and Tate (2008)) and variations in manager's strategic ability on entry decisions (Goldfarb and Xiao (2011)).

Next, let us first define $n(\cdot,\cdot)$, which is the number of operating firms given firm i's choice

k_i and other firms choices k_{-i} as

$$n(k_i, k_{-i}) = \frac{1}{2}\left[N_{all} - \sum_{j\in\mathscr{E}\cup\mathscr{I}} 1\{k_j = o\} + \sum_{j\in\mathscr{E}\cup\mathscr{I}} 1\{k_j = j\}\right],$$

where $N_{all} = N_{\mathscr{E}} + N_{\mathscr{I}}$. Though we write payoff of j as a function of k_j and k_{-j}, it affects payoff only through the number of operating firms $n(k_j, k_{-j})$. By denoting $N = n(k_j, k_{-j})$, we abuse notation as $U_j(k_j, N) = U_j(k_j, k_{-j})$ when our argument is about the degree of externalities.

3.2 Stable Outcome

We denote the set of merger contracts by $\mathscr{K} \equiv \mathscr{E} \times \mathscr{I} \times P$. Note that the set of merger contracts does not include the case where a potential entrant or incumbent does not enter the market or the case where they enter by themselves. Let the set of merger contracts in which entrant e is involved be K_e and in which incumbent i is involved be K_i, i.e.,

$$K_e = \bigcup\nolimits_{i\in\mathscr{I}} k_{ei}, \text{ and } K_i = \bigcup\nolimits_{e\in\mathscr{E}} k_{ei}.$$

We also define the set of available choices for e and i as

$$\overline{K}_e = K_e \cup \{e\} \cup \{o\} \text{ and } \overline{K}_i = K_i \cup \{i\} \cup \{o\}.$$

For simplifying the notation, we define $\bar{\mathscr{K}} = \bigcup_{j\in\mathscr{E}\bigcup\mathscr{I}} \bar{\mathscr{K}}_j$.

Considering externalities in a two-sided matching model is difficult in general (Sasaki and Toda (1996), Hafalir (2008)). This is because preference is dependent not only on one's own matching (as in the case without externalities), but also on how other players are matched with each other (i.e., the entire assignment). In a matching model with externalities, if a pair of players deviates (dissolves the match), each member of the pair has to think not only about their own matching but also about how other players (including his/her previous partner) are reacting to the deviation because preferences are defined over assignment rather than matching. To illustrate this point, suppose Players A and X are currently matched. If Player A deviates to form a blocking pair with Player Y, Player A has to consider not only that Player Y has incentive to be matched with A, but also what other players, including Player X, would do after the deviation, because the entire outcome (rather than just whom Player A is matched with) affects Player A. Thus, Player A's expectations about the possible entire outcomes after the deviation are crucial. In order to describe the way each player expects how other players are matched with each other, Sasaki and Toda (1996) and Hafalir (2008) proposes *estimation function*, which describe the expectation on how the re-matching among other agents happens.

We restrict our attention to myopic estimation function for technical tractability as in other papers in the literature (e.g., Sasaki and Toda (1996); Baccara, Imrohoroglu, Wilson, and Yariv (2012); Mumcu and Saglam (2010); Bando (2012), Pycia and Yenmez (2017)). Pycia and Yenmez (2017) calls stability concept adopting myopic estimation "standard"in the sense that a deviation does not lead to further re-matching among the remaining agents. Estimation about other players' behavior is assumed to be fixed under this assumption as strategies in Nash equilibrium are. Second, we define the estimation function as a mapping from the set of choices to the set of *estimated* number of operating firms (instead of the set of entire assignment), i.e., $\mathscr{N}_j : \overline{K}_j \rightarrow \mathbb{R}_+$. This reflects the fact that in our environment only the number of operating firms in the market affects payoffs instead of the entire assignment.[13]

Now we describe the choices by firms given the estimation function. In order to represent the choice by a potential entrant given the set of available merger contracts K_e and the estimation function $\mathscr{N}_e$, we define the *chosen set from merger contracts* $C_e(K_e, \mathscr{N}_e)$ for e as the

[13]This assumption can be relaxed in some dimensions such as the case that the firms are vertically differentiated as in Mazzeo (2002). Suppose each firm has its *type* based on some observable firm specific characteristics such as high, medium, and low quality or for- and non-profit. Denote the number of total operating firms for each type by $N_s, s = 1, 2, ..., S$. Then, the payoff can depend on the total number of operating firms for each *type*: $\pi_j(\{k\}, (N_s)_{s=1}^S)$, and similar analysis can be extended. In fact, we estimate the model with this specification ("type"is based on asset size) in addition to the base model without vertical differentiation. We present the results in Section 6.

following:

$$C_e(K_e,\mathcal{N}_e)=\begin{cases}\varnothing \text{ if } \left\{k\in K_e \mid \operatorname{argmax}_{k\in\overline{K}_e}\{U_e(k,\mathcal{N}_e)\}\right\}=\varnothing \\ \operatorname{argmax}_{k\in\overline{K}_e}\{U_e(k,\mathcal{N}_e)\} \text{ otherwise}\end{cases},$$

where $U_e(k,\mathcal{N}_e)\equiv U_e(k,\mathcal{N}_e(k))$ by suppressing k from $\mathcal{N}_e(k)$. This set is the best available merger contract given the available set of contracts K_e, which can be a null set if no merger contract is more attractive than de novo entry and no entry. Similarly, we define the chosen set from merger contracts for incumbent i given the available set of contracts K_i as follows:

$$C_i(K_i,\mathcal{N}_i)=\begin{cases}\varnothing \text{ if } \left\{k\in K_i \mid \operatorname{argmax}_{k\in\overline{K}_i}\{U_i(k,\mathcal{N}_i)\}\right\}=\varnothing \\ \operatorname{argmax}_{k\in\overline{K}_i}\{U_i(k,\mathcal{N}_i)\} \text{ otherwise}\end{cases},$$

which also can be a null set if no merger contract is more attractive than de novo entry and no entry.

In addition to the chosen set from merger contracts, we also need to track the choices of firms including no entry and entry without merger. We denote the *chosen set* for incumbent i by $\overline{C}_i(\overline{K}_i,\mathcal{N}_i)$ and for potential entrant e by $\overline{C}_e(\overline{K}_e,\mathcal{N}_e)$, which is either the most preferred merger contract available to firm j in K_j, entry without merger, or no entry, i.e.,

$$\begin{aligned}\overline{C}_i(\overline{K}_i,\mathcal{N}_i) &= \operatorname{argmax}_{k\in\overline{K}_i}\{U_i(k,\mathcal{N}_i)\},\\ \overline{C}_e(\overline{K}_e,\mathcal{N}_e) &= \operatorname{argmax}_{k\in\overline{K}_e}\{U_e(k,\mathcal{N}_e)\}.\end{aligned}$$

The difference between the *chosen set from merger contracts* and the *chosen set* is the following. $C_i(K,\mathcal{N}_i)$ and $C_e(K,\mathcal{N}_e)$ take the null set if no entry or entry without merger is preferred to any merger contract in K, while $\overline{C}_i(\overline{K}_i,\mathcal{N}_i)$ and $\overline{C}_e(\overline{K}_e,\mathcal{N}_e)$ will never be a null set. This is because $\overline{C}_i(\overline{K}_i,\mathcal{N}_i)$ and $\overline{C}_e(\overline{K}_e,\mathcal{N}_e)$ specify the optimal choice among $\overline{K}_j$ for each player, which may include no entry or entry without merger.

Now, we define stability as our solution concept.[14] Before defining stability, let us define

[14]We slightly abuse notation when the chosen set $\overline{C}_j(\overline{K},\mathcal{N}_j)$ is not a singleton. In such cases, the payoff to firm

the set of all merger contracts included in the set of outcomes $\overline{K}$ by $K = \bigcup_{j \in \mathscr{E} \cup \mathscr{I}} \overline{K}_j \backslash (\{o\} \cup \{j\})$.

Definition 1 (Stability)**.** *A set of available choices and estimation functions* $(\overline{K}^*, \mathscr{N}^*)$ *is a stable outcome if*

1 (Individual rationality) $\forall j \in \mathscr{E} \cup \mathscr{I}, \nexists \widetilde{k} \in \overline{K}_j$ *s.t.*

$$U_j(\widetilde{k}, \mathscr{N}_j^*) \geq U_j(\overline{C}_j(\overline{K}_j^*, \mathscr{N}_j^*), \mathscr{N}_j^*).$$

2 (No blocking contracts) $\nexists \widetilde{k} \subset \mathscr{K}$ *s.t.* $\widetilde{k} \neq K$ *and*

$$\widetilde{k} = \bigcup_{i \in \mathscr{I}} C_i(K_j^* \cup \widetilde{k}, \mathscr{N}_i^*) = \bigcup_{e \in \mathscr{E}} C_e(K_j^* \cup \widetilde{k}, \mathscr{N}_e^*).$$

A few comments are in order. First, we slightly modify the standard definition of stability as we allow two outside options: entry without merger and no entry. In the standard definition of individual rationality for matching models, players compare being unmatched to matching with someone. In such cases players can unilaterally choose to stay in the market alone. In our case, the players choose one of the better outside options if they were not matched, and they can make this decision *unilaterally* as in the first condition.

Second, the *no-blocking-contract* condition is a standard one to define stability in a two-sided matching model with contracts (see, e.g., HM). It requires that there exist no merger contracts to which firms from both sides would be willing to deviate.

Lastly, our stability definition does not require conditions similar to Sasaki and Toda (1996)'s φ-admissibility, a condition that requires some sort of "consistency "between estimation and realized assignment. This is because we focus on myopic estimation function to guarantee existence of stable outcomes under which players treat other players' choices as fixed in their estimation.[15]

The following ($\mathscr{E}$-proposing) deferred acceptance algorithm by HM finds a stable outcome

j is exactly the same regardless of the choices in $\overline{C}_j(\overline{K}, \mathscr{N}_j)$.

[15]Note that externalities exists though other players' choices are fixed in the estimation. A firm's choice changes the number of operating firms in the market, and affects the payoff of all other players.

given $\mathscr{N}^*$ as we consider a myopic estimation.

1. Initialize $K_{\mathscr{E}} = \mathscr{K}$, $K_{\mathscr{I}} = \emptyset$.
2. All $e \in \mathscr{E}$ choose $\{e\}, \{o\}$, or make the offer that is most favorable to e from $K_{\mathscr{E}}$ to members of $\mathscr{I}$.
3. All $i \in \mathscr{I}$ consider $\{i\}, \{o\}$, and all available offers, then hold the best, and reject the others.
4. Update $K_{\mathscr{E}}$ by removing offers that have been rejected. Update $K_{\mathscr{I}}$ by including newly made offers.
5. If there is no change to $K_{\mathscr{E}}$ and $K_{\mathscr{I}}$, terminate. Otherwise, return to Step 2.

We can also consider $\mathscr{I}$-proposing algorithm, which would entail making the following changes above: substituting $K_{\mathscr{I}} = \mathscr{K}$, $K_{\mathscr{E}} = \emptyset$ with $K_{\mathscr{E}} = \mathscr{K}$, $K_{\mathscr{I}} = \emptyset$ in (1); e with i in (2); i with e in (3); and $K_{\mathscr{E}}$ with $K_{\mathscr{I}}$ and $K_{\mathscr{I}}$ with $K_{\mathscr{E}}$ in (4). Note that this algorithm finds a stable outcome for any given myopic estimation, $\mathscr{N}^*$. If we consider a rational (non-myopic) estimation such as Sasaki and Toda (1996)'s φ-admissibility and Hafalir (2008)'s rational expectation, guaranteeing existence of a stable outcome becomes extremely difficult as Sasaki and Toda (1996) and Hafalir (2008) showed.

In each round each potential entrant offers a merger contract to an incumbent or chooses either of the outside options in step (b), and each incumbent holds the best contract offered or either of the outside options in step (c). The set of available contracts for $\mathscr{I}$, $K_{\mathscr{I}}$, starts with an empty set and expands *monotonically* as more offers are (cumulatively) made each round. The set of available contracts for $\mathscr{E}$, $K_{\mathscr{E}}$, starts with the entire set of contracts in step (a), and it *monotonically* shrinks as offers are rejected each round. HM show the existence of stable allocation and the characterization using this algorithm as follows.

Theorem 3.1 (Hatfield and Milgrom (2005))**.** *Given $\mathscr{N}^*$,* *(i) The set of stable contracts $\overline{K}^{*\mathscr{E}}$ is unanimously the most preferred set of contracts among all of stable contracts for $\mathscr{E}$ and unanimously the least preferred for $\mathscr{I}$, and vice versa for $\overline{K}^{*\mathscr{I}}$, i.e.,*

$$U_e(\overline{K}^{*\mathscr{E}}_e, \mathscr{N}^*) \geq U_e(\overline{K}^*_e, \mathscr{N}^*) \geq U_e(\overline{K}^{*\mathscr{I}}_e, \mathscr{N}^*) \quad \forall \overline{K}^*_e, \forall e \in \mathscr{E}, \tag{3.1}$$

$$U_i(\overline{K}^{*\mathscr{E}}_i, \mathscr{N}^*) \geq U_i(\overline{K}^*_i, \mathscr{N}^*) \geq U_i(\overline{K}^{*\mathscr{I}}_i, \mathscr{N}^*) \quad \forall \overline{K}^*_i, \forall i \in \mathscr{I}. \tag{3.2}$$

(ii) The $\mathscr{E}$-proposing algorithm converges to the $\mathscr{E}$-optimal stable outcome, $\overline{K}^{*\mathscr{E}}$, and the $\mathscr{I}$-proposing algorithm converges to the $\mathscr{I}$-optimal stable allocation, $\overline{K}^{*\mathscr{I}}$;

(iii) An unmatched firm in a stable outcome is also unmatched in any stable outcome.

We exploit this result in our estimation. First, part (i) of the theorem shows that there exists upper and lower bounds for the equilibrium payoffs of all firms. Though we do not know the equilibrium selection mechanism, equilibrium payoffs are always bounded above and below by these bounds. We use this result to construct moment inequalities in our estimation.

Second, to compute these bounds, we used part (ii) of the theorem in our estimation. Given parameter values and simulated draws of the shocks, we can run both the $\mathscr{E}$- and $\mathscr{I}$-proposing deferred acceptance algorithm with contracts to obtain these bounds.

Finally, part (iii) of the theorem implies that the set of firms that are unmatched is the same for all stable outcomes.[16] This is the so-called "rural hospitals theorem" (Roth (1986)). Moreover, this implies that the payoffs of the unmatched players are the same because their

[16]Hatfield and Milgrom (2005)) show a variation of the rural-hospitals theorem in the case of many-to-one matching with contract. They show that every hospital signs exactly the same number of contracts at every point in the set of stable allocations if the hospitals' preferences satisfy what they call the law of aggregate demand and substitutability. This result implies that the set of hospitals that cannot fill the capacity in a stable allocation cannot fill it in any stable allocation. It does not necessarily imply the same set of doctors is hired in all stable allocations, though the number of unmatched doctors remains the same.

However, in our case of one-to-one matching with contract, preferences of both sides satisfy these two conditions. Therefore, we can show that the set of unmatched firms is identical in any stable outcome given N.

outcome is unique in any equilibrium, i.e.„$\forall i$ s.t. $\overline{C}_i(\overline{K}_i^*, \mathcal{N}^*) = i$ and $\forall e$ s.t. $\overline{C}_e(\overline{K}_e^*, \mathcal{N}^*) = e$

$$U_e(\overline{K}_e^{*\mathcal{E}}, \mathcal{N}^*) = U_e(\overline{K}_e^{*\mathcal{I}}, \mathcal{N}^*) = U_e(\overline{K}_e^*, \mathcal{N}^*) \quad \forall \overline{K}_e^*, \tag{3.3}$$

$$U_i(\overline{K}_i^{*\mathcal{E}}, \mathcal{N}^*) = U_i(\overline{K}_i^{*\mathcal{I}}, \mathcal{N}^*) = U_i(\overline{K}_i^*, \mathcal{N}^*) \quad \forall \overline{K}_i^*. \tag{3.4}$$

Thus, we can construct moment equalities in terms of the payoffs using the above equation for the firms that are unmatched.

4 Data

Before presenting the estimation and identification, let us discuss the data we use in the estimation. The banking industry in the U.S. provides us with an interesting and important change in entry regulation. At the beginning of 1995, intrastate de novo branching was not permitted in 12 states, and branching by merger was the only way to enter new markets in these states.[17] The other 38 states fully permitted intrastate branching at that time. Then, in the following five years by the end of 2000, 7 of those 12 states have deregulated intrastate de novo branching. Focusing on these seven states allows us to study the effect of the intrastate branching regulation on the market structure.

We use the data on commercial banks in the U.S. from the local markets of the seven states in which intrastate branching was fully deregulated between 1995 and 2000.[18] We use the data of three year periods right after the deregulation in each state. We obtain our main data on the branching of commercial banks from the Institutional Directory of the Federal Deposit Insurance Cooperation (FDIC). We augment the financial data of the banks with data from the Central Data Repository of the Federal Financial Institutions Examination Council. The data we construct contains information on the location of all branches and the financial statistics of every FDIC insured bank that had at least one branch in one of the seven states during the

[17] The only exception was Iowa, which deregulated entry by merger in 1997.

[18] The 7 states are Arkansas, Colorado, Georgia, Montana, North Dakota, Oklahoma, and Wyoming. Our data do not include New Jersey because there was no county with population less than 50,000 in the state.

Market (# of obs = 507)	Mean	Std. Dev.	Min	Max
Population (thousand)	15.5	11.0	0.6	49.0
Income (per capita)	19,306.5	3,872.4	9,154	52,723
Total banks per market	29.5	13.1	9	93
Potential entrants per market	25.6	12.5	5	90
Incumbents per market	3.9	2.3	1	24
Operating banks per market	3.8	2.1	0	12
Number of mergers per market	0.7	0.9	0	4

Table 1: Descriptive Statistics — Market Level

data period. The data also keeps track of the banks' mergers and acquisitions. Data on merger contracts are obtained from the data set of SNL Financial, which reports deal values for each merger.[19]

Markets in the banking industry are known to be local in nature.[20] Existing works as well as antitrust analysis use geographic area as the definition of a market for the banking industry. Following Cohen and Mazzeo (2007) we focus our attention on rural markets, and we use a county as a market. This is because the typical market definition for urban areas (such as the Metropolitan Statistical Area) is likely to include submarkets within it. For this reason we exclude counties with a population greater than 50,000 from our data.[21] In such markets, consumers are also very less likely to use banks in other markets.

As discussed in the model section, we classify banks into incumbents and potential entrants in each market. We define banks that have operated in the market at the time of the deregulation as incumbents. Regarding potential entrants, banks that have operated in a contiguous market during the data period are defined as potential entrants. There is a small number of banks that have entered though they are not identified as potential entrants according

[19]Merger data of SNL Financial do not necessarily have the same bank names as in FDIC data for the buyer and target banks. About a third of the mergers are matched by the FDIC certification number for both buyer and target banks. Another one third of the mergers are matched by the FDIC certification number of one side, and the holding company name and the FDIC holding company number. For the rest of the mergers, we matched manually using bank and holding company information from both data sets, and other sources such as regulatory filings. If a merger was still unmatched, we interpolated the transfer using buyer and target characteristics.

[20]See, e.g., Ruffer and Holcomb (2001), Ishii (2005), and Cohen and Mazzeo (2007).

[21]The number of markets does not change much if we use the criteria with a population less than 100,000.

to this definition. Hence, we added these banks to the set of potential entrants as well, which also includes a small number of newly established banks that account for 0.1% of the potential entrants. We define a firm entry if the firm exists at the end of our sample period.

Table 1 reports the summary statistics of the market-level information. On average, there are 3.9 incumbent banks and 25.6 potential entrants in a market. There is substantial variation across markets for the number of incumbents and potential entrants. Among those firms, the number of operating banks is 3.8 on average. The average number of mergers per market is 0.7.[22]

One of the assumptions of our empirical analysis is that the entry and merger decisions are independent across markets. Regarding this point, about 80% of the incumbents were present only in one market, and about 95% of the incumbents were present in less than three markets. Conditioning on entry, both the incumbents and potential entrants enter only one market in more than 75% of the cases and less than three markets in 95% of the cases for incumbents and 92% for potential entrants. Conditioning on entry with merger, both types of banks enter less than three markets in 87% of the cases. Thus, in the vast majority of our data, banks do not overlap across markets, and we treat markets independently. The fact that our data is mostly from small regional banks in small regional markets helps us on the independence assumption.

Table 2 reports the summary statistics of the bank-level information. The incumbents' mean size of assets is much smaller than that of potential entrants at $6.45 billion. This may reflect the fact that we define potential entrants as banks in contiguous markets, which tend to be larger than the market we consider. Descriptive statistics on the equity ratio are roughly the same for incumbents and potential entrants with a mean of 10%.

Looking at the breakdown by the modes of entry, the incumbents that enter without merger are on average smaller than incumbents that enter with merger or do not enter. Potential en-

[22]During our sample period in our sample markets, there were no bank failure cases with open bank assistance by FDIC, while there were five bank failures where purchase and assumption (P&A) transactions were made. Since we observe such a small number of P&A transactions and cannot identify parameters specific to P&A, we treated these cases in the same way as other regular mergers.

		Mean	Std. Dev.	Min	Max
Incumbent					
Overall					
	Asset ($1B)	6.45	38.2	0.004	580
	Deposit ($1B)	4.10	25.4	0.002	390
	Equity Ratio	0.10	0.03	0.02	0.51
Entry by merger					
	Asset ($1B)	7.78	35.2	0.02	320
	Deposit ($1B)	5.33	20.4	0.02	180
	Equity Ratio	0.08	0.01	0.05	0.12
Entry w/o merger					
	Asset ($1B)	3.75	26.6	0.004	580
	Deposit ($1B)	2.62	18.1	0.004	390
	Equity Ratio	0.10	0.03	0.05	0.51
No entry					
	Asset ($1B)	7.08	27.6	0.007	220
	Deposit ($1B)	4.79	18.9	0.006	150
	Equity Ratio	0.09	0.03	0.05	0.18
Potential Entrant					
Overall					
	Asset ($1M)	17.0	78.0	0.002	580
	Deposit ($1M)	11.1	50.9	0	390
	Equity Ratio	0.10	0.05	0.02	0.97
Entry by merger					
	Asset ($1B)	52.1	144.0	0.02	570
	Deposit ($1B)	34.0	93.2	0.01	370
	Equity Ratio	0.09	0.02	0.05	0.19
Entry w/o merger					
	Asset ($1B)	32.8	118.0	0.01	570
	Deposit ($1B)	21.7	76.5	0.01	370
	Equity Ratio	0.10	0.03	0.05	0.30
No entry					
	Asset ($1B)	16.3	75.9	0.002	580
	Deposit ($1B)	10.6	49.5	0	390
	Equity Ratio	0.10	0.05	0.02	0.97
Merger Payment					
	($1B)	0.89	3.75	0.001	21.2

Table 2: Descriptive Statistics – Bank Characteristics by Incumbent and Potential Entrant, and by Mode of Entry: We report summary statistics of bank characteristics conditional on modes of entry, for amount of asset, amount of deposit, and equity ratio.

trants that enter by merger have the largest mean asset and deposit amount, while banks that do not enter have the smallest mean asset and deposit amount. Potential entrants that enter without merger is in-between them for both asset and deposit at $32.8 billion and $21.7 billion, respectively. Table 2 also reports the merger payment from buyer to target banks, which includes not only cash payment but also payment by share.

5 Identification and Estimation

In this section, we propose an estimation strategy for two-sided matching models based on moment inequalities and equalities. One of the major issues of estimating two-sided matching models is addressing the multiplicity of stable matchings. Our estimation strategy uses moment inequalities to deal with the issue of multiple equilibria similar to recent studies estimating noncooperative games by a set estimator (Ciliberto and Tamer (2009), Ho (2009), Kawai and Watanabe (2013)). We do so by exploiting the lattice structure of the set of equilibria (or stable outcomes, to be more precise).

5.1 Specification

Before discussing our identification strategy, we start from specifying the model. We write the profit and synergy functions for incumbents and potential entrants. We first specify the profit function for incumbent i for entry without merger and no entry as

$$\begin{aligned}
\pi_i(i, k_{-i}) &= \alpha n(i, k_{-i}) + z\beta_0 + x_i\beta_{1I} + \beta_{2I} + \xi, \\
\pi_i(o, k_{-i}) &= 0,
\end{aligned}$$

where $\alpha < 0$ is the degree of the negative externalities due to competition. z denotes market characteristics affecting the market size such as population and income, x_i denotes the characteristics of firm i such as asset size and equity ratio, and β_0 and β_{1I} denote the effects of these characteristics on profits. β_{2I} is a constant term for incumbents entering without merger capturing the entry barriers. The last term, ξ, denotes market-level profit shock, which follows a distribution independently. If the firm does not enter the market, the profit is zero. Similarly, we write the profit function for entrant e (for entry without merger and no entry) as

$$\begin{aligned}\pi_e(e, k_{-e}) &= \alpha n(e, k_{-e}) + z\beta_0 + x_e\beta_{1E} + \beta_{2E} + \xi, \\ \pi_e(o, k_{-e}) &= 0,\end{aligned}$$

where x_e denotes firm e's characteristics, and β_{2E} is a constant term for potential entrants capturing entry barriers, which we allow to differ from β_{2I} for incumbents.

Next, in order to write the profit for the case of entry with merger, we start with the profit of the merged entity. We specify the profit of the merged entity to depend on negative externality of competition, market characteristics, and market-level shock in the same way as entry without merger. In addition, we have another term, $f(x_i, x_e, x_{ie})$, which we call as (dis)synergy function, in order to capture the (dis)synergies generated given the characteristics of both firms (x_i and x_e) and the merger-specific characteristics, x_{ie}. The profit after merger between firms e and i given the (dis)synergy function f and the number of operating firms is written as

$$\begin{aligned}\pi(k_{ei}, k_{-i}) &= \alpha n(k_{ei}, k_{-i}) + z\beta_0 + f(x_i, x_e, x_{ie}) + \xi, \\ f(x_i, x_e, x_{ie}) &= \beta_{2M} + x_i\beta_3 + x_e\beta_4 + x_i x_e\beta_5 + x_{ie}\beta_6,\end{aligned}$$

where β_{2M} is a constant term for (dis)synergy, β_3 and β_4 are the effects of the incumbent's and potential entrant's characteristics on (dis)synergy of the merger, respectively, and β_5 captures the effect of the interaction terms of both firms' characteristics on (dis)synergy. β_6 measures how the match-specific characteristics affect the (dis)synergies.

Because the terms of merger contracts take cash and stock as medium of payment, we consider the space of the term of trade P as $P = T \times R$, where $T = \{\underline{t}, ..., 0, ..., \overline{t}\}$ corresponds to a finite set of cash transfers, and $R = \{0, ..., 1\}$ corresponds to a finite set of stock shares between e and i after merger.[23] Now we can write the profit function for the case of mergers with contract k_{ei} for firms e and i as

$$\begin{aligned}
\pi_e(k_{ei}, k_{-e}) &= r_{ei}\pi(k_{ei}, k_{-e}) - t_{ei}, \\
\pi_i(k_{ei}, k_{-i}) &= (1 - r_{ei})\pi(k_{ei}, k_{-i}) + t_{ei},
\end{aligned}$$

where $t_{ei} \in T$ denotes a cash transfer from e to i and $r_{ei} \in R$ denotes a payment in stock shares of the merged entity from e to i.[24] Note that both r_{ei} and t_{ei} are observable for the realized merger contracts as data.

The exact specification of the payoff function for incumbent i is

$$\begin{aligned}
\pi_i(i, N) &= \alpha N + \mathbf{z}[\beta_0^{pop}, \beta_0^{income}] + \mathbf{x}_i[\beta_{1I}^{size}, \beta_{1I}^{eq_ratio}] + \beta_{2I} + \xi_m, \\
\pi_i(k_{ei}, N) &= (1 - r_{ei})\big(\alpha N + \mathbf{z}[\beta_0^{pop}, \beta_0^{income}] + f(\mathbf{x}_i, \mathbf{x}_e, \mathbf{x}_{ie}) + \xi_m\big) + t_{ei},
\end{aligned}$$

where $\xi_m \sim N(0,1)$, and for potential entrant e is

$$\begin{aligned}
\pi_e(e, N) &= \alpha N + \mathbf{z}[\beta_0^{pop}, \beta_0^{income}] + \mathbf{x}_e[\beta_{1E}^{size}, \beta_{1E}^{eq_ratio}] + \beta_{2E} + \xi_m, \\
\pi_e(k_{ei}, N) &= r_{ei}\big(\alpha N + \mathbf{z}[\beta_0^{pop}, \beta_0^{income}] + f(\mathbf{x}_i, \mathbf{x}_e, \mathbf{x}_{ie}) + \xi_m\big) - t_{ei},
\end{aligned}$$

where the synergy function in the payoff is specified as

$$\begin{aligned}
f(\mathbf{x}_i, \mathbf{x}_e, \mathbf{x}_{ie}) &= \beta_{2M} + \mathbf{x}_i[\beta_3^{size}, \beta_3^{eq_ratio}] + \mathbf{x}_e[\beta_4^{size}, \beta_4^{eq_ratio}] \\
&\quad + \mathbf{x}_i^1 \mathbf{x}_e^1 \beta_5^{size} + \mathbf{x}_i^2 \mathbf{x}_e^2 \beta_5^{eq_ratio} + d_{ie}\beta_{6,dist} + d_{ie}^2 \beta_{6,dist2} + h_{ie}\beta_{6,bhc},
\end{aligned}$$

where $\mathbf{x}_{ie}$ consists of the distance between the headquarter of i and e, d_{ie}, and the indicator variable for the same bank holding company, h_{ie}, i.e. $\mathbf{x}_{ie} = [d_{ie}, h_{ie}]'$. $\mathbf{z}$ is a vector of the market

[23] We discretize the terms of trade by setting the highest observed amount as upper bound, and then create 20 bins that have the same width in log, and use the mean value of each bin as t.

[24] Set T can include negative values for the cases where i acquires e.

characteristics (log of population and log of per capita income), $\mathbf{x}_i$ is incumbent i's characteristics (log of total asset size and equity ratio), $\mathbf{x}_e$ is potential entrant e's characteristics (log of total asset size and equity ratio), $\mathbf{x}_i^1\mathbf{x}_e^1$ is an interaction term of incumbent i and potential entrant j's asset size, $\mathbf{x}_i^2\mathbf{x}_e^2$ is an interaction term of incumbent i and potential entrant j's equity ratio, and r_{ei} and t_{ei} are the terms of the merger contract k_{ei} (payment by stock and cash). Note that β_{2I} and β_{2E} capture the size of entry barriers of incumbents and potential entrants for entry without merger, respectively, and β_{2M} measures the cost of entry by merger. Finally, we specify $U_j(\pi) = log(\pi + 1)$ if $\pi > 0$ and $U_j(\pi) = -log(-\pi + 1)$ if $\pi < 0$ as this specification provides better fit.

5.2 Identification

5.2.1 Identified Set

Our identification is based on the restrictions provided by the inequalities (3.1) and (3.2) in Theorem 3.1 and the equalities (3.3) and (3.4). Theorem 3.1 shows that the payoffs in the two extremal equilibria (or stable outcomes to be more precise), $\overline{K}^{*\mathscr{E}}$ and $\overline{K}^{\mathscr{I}*}$, provide the upper and lower bounds of any equilibrium payoff. Hence they constitute the bounds of the payoffs corresponding to the observed matching and merger contract. Furthermore, equations (3.3) and (3.4) show that, in any stable outcome, the payoffs of firms that do not merge are the same. This result leads us to construct moment equalities regarding the payoffs of non-merging firms.

Let us first revisit the result of Theorem 3.1 (more specifically, equations (3.1) and (3.2)): the two extremal stable outcomes provide the highest payoff to one side of the banks and the lowest payoff to the other side, i.e., (suppressing the dependence on $\mathscr{N}$ for notational convenience) for any stable outcome $\overline{K}^*$,

$$U_e(\overline{K}^{*\mathscr{E}};\theta) \geq U_e(\overline{K}^{*};\theta) \geq U_e(\overline{K}^{*\mathscr{I}};\theta), \forall e \in \mathscr{E},$$
$$U_i(\overline{K}^{*\mathscr{I}};\theta) \geq U_i(\overline{K}^{*};\theta) \geq U_i(\overline{K}^{*\mathscr{E}};\theta), \forall i \in \mathscr{I}.$$

In other words, given θ and $\mathscr{N}$, e's payoff in any stable outcome is bounded above by the payoff in the $\mathscr{E}$-optimal stable outcome, $U_e(\overline{K}^{*\mathscr{E}};\theta)$, and bounded below by the payoff in the $\mathscr{I}$-optimal stable outcome, $U_i(\overline{K}^{*\mathscr{I}};\theta)$. Similarly the payoffs for all incumbents are bounded above and below by $U_i(\overline{K}^{*\mathscr{I}};\theta)$ and $U_i(\overline{K}^{*\mathscr{E}};\theta)$, respectively. Observe that we can compute these bounds given θ and $\mathscr{N}$ (and shocks) using the algorithm shown in Section 3.2 as shown in Theorem 3.1 (ii). We will come back to how we use the observed data to pin down $\mathscr{N}$ in Step 3 in our estimation procedure in Section 5.3.

We also compute the payoffs for each firm j corresponding to the data, $U_j(\overline{K}^{*DATA};\theta)$, given θ (and shocks), where we denote the particular equilibrium selected in the observation as $\overline{K}^{*DATA}$. Though we cannot know which equilibrium the data-generating process corresponds to, the payoff corresponding to the equilibrium in the data must still be bounded by $U_j(\overline{K}^{*\mathscr{I}};\theta)$ and $U_j(\overline{K}^{*\mathscr{E}};\theta)$ for all j. Hence, we can consider the following types of inequalities:

$$E\left[U_e(\overline{K}^{*\mathscr{E}};\theta) - U_e(\overline{K}^{*DATA};\theta)|X\right] \geq 0, \tag{5.1}$$
$$E\left[U_i(\overline{K}^{*\mathscr{I}};\theta) - U_i(\overline{K}^{*DATA};\theta)|X\right] \geq 0, \tag{5.2}$$
$$E\left[U_e(\overline{K}^{*DATA};\theta) - U_e(\overline{K}^{*\mathscr{I}};\theta)|X\right] \geq 0, \tag{5.3}$$
$$E\left[U_i(\overline{K}^{*DATA};\theta) - U_i(\overline{K}^{*\mathscr{E}};\theta)|X\right] \geq 0, \tag{5.4}$$

where $X = \{(x_{i_1}, ..., x_{i_{N_\mathscr{I}}}), (x_{e_1}, ..., x_{e_{N_\mathscr{E}}}), z\}$ denotes firm characteristics of all firms in a market and market characteristics. Note that these inequalities are at the level of individual firms. However, our unit of observation is a market, and identity and number of incumbents and potential entrants differ across markets. Thus, we use moments based on these inequalities at market level in our estimation, which we describe in Appendix A (we construct 44 moment inequalities.).

Next, we discuss moment equalities resulting from equations (3.3) and (3.4), which state that unmatched (non-merging) firms earn exactly the same payoff in any stable outcome, i.e., for any j choosing either $\{j\}$ or $\{o\}$, we have

$$E\left[U_j(\overline{K}^{*DATA};\theta)|X\right]=E\left[U_j(\overline{K}^{*\mathscr{E}};\theta)|X\right]=E\left[U_j(\overline{K}^{*\mathscr{I}};\theta)|X\right]. \tag{5.5}$$

Same as moment inequalities, these equalities are at the individual firm level, and we describe the moments we use in our estimation based on these equalities in Appendix A (we construct 14 moment equalities.). Note that we cannot construct the moments exactly as in 5.1 because different market contains different set of firms.

Finally, we define the identified set Θ_{id} using both moment inequalities and equalities as $\Theta_{id}=\{\theta\in\Theta:$ inequalities (5.1)–(5.4) and equalities (5.5) are satisfied at $\theta\}$.

A few comments are in order. First, the identified set Θ_{id} is not a singleton unless all inequalities bind. Because two-sided matching models do not have unique stable outcomes, these inequalities do not bind in general. Hence the model is only partially identified. Second, putting it informally, equality constraints provide restrictions that identify the parameters in payoffs for entry without merger, and inequality constraints provide restrictions that partially identify the synergy function. As several variables are included in payoffs for both entry with and without merger, we use both types of restrictions to construct our identified set.

In Appendix D, we conduct a series of Monte Carlo simulations to show the validity of our estimation strategy. We consider a simple two-sided marriage matching model under different model specifications. We find that the true parameter is included in the identified set regardless of the specification and that the confidence interval based on the moment inequalities and the moment equalities is estimated to be smaller as the number of markets becomes large. Hence, we confirm that our estimation strategy works well to identify the parameters.

5.2.2 Additional Restrictions that Help Identification

There are some additional restrictions that may help identification of the model. Though our estimation strategy is robust to lack of point-identification, we discuss how these other restrictions practically improve the identification of the model under certain conditions, i.e., the identified set Θ_{id} becomes sharper to the extent that such conditions are satisfied in the data.

The basic idea of these additional restrictions is to consider two types of exclusion restrictions: the exclusion restriction employed in the regular entry model and the exclusion restriction for the synergy function. These two types of variables satisfying the exclusion restrictions help identification because x_i and x_e are included not only in the payoff of entering without merger but also in the payoff of entering with merger. Hence, those variables aid separating effects of x_i and x_e in payoffs of both cases.

The first type of exclusion restriction we consider is the same one as adopted in the literature on estimating entry models. As in Berry (1992) and Tamer (2003), we use a variable that affects a firm's profit but does not enter the other firms' profit functions. More precisely, a variable affecting fixed cost of entering a market but not affecting revenue and/or variable cost would satisfy such a requirement. In our setup, whether a firm is an incumbent or a potential entrant provides this variation. Though the discreteness of the incumbency dummy prevents us from making identification at infinity argument to achieve point-identification (see, e.g., Ciliberto and Tamer (2009)), variation of this incumbency dummy helps us make the identified set smaller.

The second type of exclusion restriction enhances identification of the (dis)synergy function, f. Note that the first type of exclusion restriction is not useful for identification of f if all exogenous variables in f are also included in $U_j(j,N)$ (payoff for entry without merger). Thus, we consider a merger-specific variable that enters the synergy function of the particular merger, but affects neither the synergy of any other combination of firms nor the payoffs of the two firms entering without merger.

For this purpose, we use the distance between the headquarters of the incumbent and the potential entrant, which is the first element of x_{ie}, $x_{ie}^{(1)}$. This variable affects the post-merger synergy for various reasons, such as communication between the workers of the target and acquiring banks, while it is unlikely to affect the payoff of mergers of any other bank pairs and that of entry without merger.

We provide informal argument how these two excluded variables make the identified set smaller in two steps, where we use each type of restrictions in each step. First, we use the second type of exclusion restriction to improve identification of parameters that are not included in the synergy function. The assumption we use is that the dissynergy becomes larger as the distance increases.[25] This implies that the payoff with merger becomes strictly less than that of entering without merger with such large distance.[26] Although the distance cannot go to infinity in our data, this intuition carries over as long as the payoff with merger becomes low enough if the distance increases, and the firms have no incentive to choose mergers.

Now, given that the firms have no incentive to choose mergers, our model is equivalent to a regular entry model. This is because we can ignore the choice of entry with merger in such a case (it gives strictly lower payoff than not entering). Therefore, the first type of exclusion restriction helps to further identify the payoff of entry without merger, $U_j(j,N)$, as shown in Berry (1992) and Tamer (2003).

Second, we informally discuss how the synergy function can be better identified. Given the argument for $U_j(j,N)$ in the previous paragraph, we can use the variation of outcome k_{ei} and that of characteristics x_i, x_e, and x_{ie} to improve identification of $U_i(k_{ei},N)$ and $U_e(k_{ei},N)$. Because the identified set on the effect of x_i and x_e on $U_j(j,N)$ becomes smaller by the first step, the effects of x_i and x_e on $U_i(k_{ie},N)$ and $U_i(i,N)$ are isolated better. The same argument applies to $U_e(k_{ie},N)$ and $U_e(e,N)$. Finally, as the effects of x_i and x_e on $U_i(k_{ie},N)$ and $U_e(k_{ie},N)$ are isolated better, the variation of k_{ie} and that of x_i, x_e, and x_{ie} improves the identification

[25]To be more precise, the dis-synergy goes to infinity as the distance becomes infinity, i.e., $f(x_i,x_e,x_{ie}) \to -\infty$ as $x_{ie}^{(1)} \to \infty$.

[26]More precisely, the payoff for entry with merger goes to negative infinity as the distance goes to infinity, i.e., $\Pi_e(k_{ei},N) \to -\infty$ and $\Pi_i(k_{ei},N) \to -\infty$ without changing $\Pi_j(j,N)$ as $x_{ie}^{(1)} \to \infty$.

about the synergy function f.

Note that the above discussion concerning these two types of exclusion restrictions helps the identified set to be smaller. We do not need the assumptions on the exclusion variables to be perfectly satisfied because our model are only partially identified and we estimate the model taking this fact into consideration. However, the degree to which these exclusion restrictions helps improves the identification of the model.

5.3 Estimation

Following the identification argument, we estimate the model using the moment inequality estimator developed by Andrews and Soares (2010). If an econometrician knew the equilibrium selection mechanism, a single outcome would correspond to one realization of the unobserved error terms (ξ, ε). However, as discussed in Section 4.2, the multiplicity of equilibria (stable outcomes in our case) implies that the model parameters are only partially identified: This makes the use of a set estimator more appropriate.

We denote 44 moment inequalities and 14 equalities discussed in Section 5.2 (and described in Appendix A) by

$$\begin{aligned} E\left[h_l(X;\theta)\right] &\geq 0, \quad l = 1, ..., 44 \\ E\left[h_l(X;\theta)\right] &= 0, \quad l = 45, ...58 \end{aligned}$$

Now we describe our estimation procedure using these moments. Our procedure solves the generalized Gale-Shapley algorithm with externalities for each set of simulated draws of unobserved shocks, and computes the sample analogue of moment inequalities and equalities in the following way.

1. Fix parameter θ. For each market $m = 1, ..., M$, obtain S draws of $\varepsilon_{\mathscr{I}}^{ms} = \{\{\varepsilon_{ie}^{ms}\}_{i=1}^{N_i^m}\}_{e=1}^{N_e^m}$, $\varepsilon_{\mathscr{E}}^{ms} = \{\{\varepsilon_{ei}^{ms}\}_{e=1}^{N_e^m}\}_{i=1}^{N_i^m}$, and ξ_m^s from distributions $g_{\mathscr{I}}$, $g_{\mathscr{E}}$, and g_m.

2. For each draw $\eta^{ms} = (\varepsilon_{\mathscr{I}}^{ms}, \varepsilon_{\mathscr{E}}^{ms}, \xi_m^s)$ in each market m, run an $\mathscr{E}$-proposing generalized

Gale-Shapley algorithm with externalities to obtain the $\mathscr{E}$-optimal $\mathscr{N}$-stable outcome. [27] Run also an $\mathscr{I}$-proposing generalized Gale-Shapley algorithm with externalities for the same draw $(\varepsilon_{\mathscr{I}}^{ms}, \varepsilon_{\mathscr{E}}^{ms}, \xi_m^s)$ in the same market m to obtain the $\mathscr{I}$-optimal stable outcomes.

3. For each market $m = 1, ..., M$, obtain S draws of $\eta^{ms,DATA} = (\varepsilon_{\mathscr{I}}^{ms,DATA}, \varepsilon_{\mathscr{E}}^{ms,DATA}, \xi_m^{s,DATA})$ which satisfies the requirement that the observed match $\overline{K}^{*DATA}$ is a stable outcome with observed number of operating firms (See Appendix A for details).

4. Construct a sample analogue of the moment inequalities and equalities using the $\mathscr{E}$-optimal and $\mathscr{I}$-optimal stable outcomes as well as the observed match $\overline{K}^{*DATA}$:

$$
\begin{aligned}
\frac{1}{MS}\sum_{m=1}^{M}\sum_{s=1}^{S} h_l(X^m, \eta^{ms}, \eta^{ms,DATA}; \theta) &\geq 0, \quad l = 1, ..., 44 \\
\frac{1}{MS}\sum_{m=1}^{M}\sum_{s=1}^{S} h_l(X^m, \eta^{ms}, \eta^{ms,DATA}; \theta) &= 0, \quad l = 45, ..., 58.
\end{aligned}
$$

5. Use the moment inequality estimator by Andrews and Soares (2010).

The specific functions we use to construct test statistics and a critical value in Andrews and Soares (2010) are $S = S_1$ and $\varphi_j = \varphi_j^{(4)}$ with the number of bootstrapping $R = 1000$. Because we cannot report a 21-dimensional confidence set, we compute min and max of the confidence set projected on each dimension and report it in the next section (see Appendix B for details).

Table 3: Confidence Intervals of Structural Parameters

Parameter	Confidence Interval	Parameter	Confidence Interval
α	$[-2.502, -1.498]$	$\beta_3^{eq\_ratio}$	$[-3.868, 1.782]$
β_0^{pop}	$[0.001, 0.927]$	β_3^{size}	$[-5.602, 4.169]$
β_0^{income}	$[0.013, 1.561]$	$\beta_4^{eq\_ratio}$	$[-3.722, -0.200]$
$\beta_1^{eq\_ratio}$	$[3.421, 7.450]$	β_4^{size}	$[-3.788, -0.690]$
β_1^{size}	$[-0.875, -0.131]$	$\beta_5^{eq\_ratio}$	$[-5.107, 5.081]$
β_{2I}	$[8.603, 9.367]$	β_5^{size}	$[-4.550, -2.630]$
β_{2E}	$[-4.906, -0.523]$	$\beta_{6,dist}$	$[-4.870, -2.800]$
β_{2M}	$[-7.699, 6.839]$	β_{bhc}	$[0.010, 6.453]$
σ_I	$[0.211, 2.262]$		
σ_E	$[0.001, 0.705]$		

Note: We estimate the confidence intervals using the method proposed by Andrews and Soares (2010).

6 Results and Counterfactual Experiments

6.1 Parameter Estimates

Table 3 report the estimation results. First, we discuss parameter estimates of externality from competition, α, and the parameters of market characteristics. The effect of competition, α, has estimate of $[-2.502, -1.498]$, implying that there is negative externality from competition in the market. Concerning market characteristics, the coefficients for both per capita income and population are positive at $\beta_0^{pop} \in [0.001, 0.927]$ and $\beta_0^{income} \in [0.013, 1.561]$. This implies that banks' profits tend to be higher in markets with a larger population and higher income *ceteris paribus.*

Regarding bank characteristics, the estimates for the effect of equity ratio and asset size are $\beta_1^{eq\_ratio} \in [3.421, 7.450]$ and $\beta_1^{size} \in [-0.875, -0.131]$, implying that a healthy balance sheet has strong effects on profitability and that a larger bank tend to be less profitable in small regional markets our data is from.

[27] In our estimation, we discretize the set of contracts by creating bins for the terms of contracts. See specification on P in Section 5.1 for detail.

Constant terms for incumbents and potential entrants are estimated as $\beta_{2E} \in [-4.906, -0.523]$ and $\beta_{2I} \in [8.603, 9.367]$, while that for the entry by merger is $\beta_{2M} \in [-7.699, 6.839]$. These constant terms are interpreted as sunk costs of entry for different forms of entry (entry with or without merger) for potential entrants and incumbents. Potential entrants have much higher entry costs if they enter de novo, while the sunk cost for entry by merger can be either negative or positive, which partly explains why potential entrants tend to enter by merger rather than de novo. For incumbents, entry by merger involves much higher sunk cost compared with de novo entry. Comparing incumbents with potential entrants, incumbents have much lower entry costs for de novo entry, again consistent with much lower entry rate by the potential entrants.

Regarding the parameters of synergy function f, the estimates for the effect of asset size for incumbents and potential entrants are $\beta_3^{size} = [-5.602, 4.169]$ and $\beta_4^{size} = [-3.722, -0.690]$. That is, size of incumbent may or may not affect synergy, but smaller potential entrants have higher synergies. The interaction term for the incumbents' and potential entrants' asset size is $\beta_5^{size} = [-4.550, -2.630]$, which implies that when the incumbent and the potential entrant have different asset size, the synergy is larger. Thus, combined with the estimate of β_4^{size}, the synergy is larger when potential entrant is small and incumbent is large. The effects of the distance between two merging banks are $\beta_{6,dist} = [-4.870, -2.800]$, which indicates that the synergy becomes smaller as the distance between the merging firms becomes greater. The estimates imply that relatively smaller potential entrant with headquarters in closer location acquiring larger incumbents has higher synergy. The estimates for the synergy effect of the equity ratio is mixed. The results show that the smaller the equity ratio of the potential entrant, the larger the synergy is, while it is not conclusive with regard to the incumbent's equity ratio and the interaction term.

Lastly, the estimates of the standard deviation are $\sigma_I = [0.211, 2.262]$ and $\sigma_E = [0.001, 0.705]$ compared to the market-level shock that is normalized to have standard deviation of one.

6.2 Comparison with Berry (1992)

In order to highlight the role of considering mergers as a way to enter a new market, we compare the estimation results of our matching model with those of a traditional entry model based on Berry (1992). We treat entry by merger as entry by potential entrant, and estimate the model. In particular, we estimate the following model

$$\begin{aligned} \pi_i(i) &= \alpha n + z\beta_0 + x_i\beta_{1I} + \beta_{2I} + \xi, \\ \pi_i(o) &= 0, \end{aligned}$$

for incumbents and

$$\begin{aligned} \pi_e(e) &= \alpha n + z\beta_0 + x_e\beta_{1E} + \beta_{2E} + \xi, \\ \pi_e(o) &= 0, \end{aligned}$$

for potential entrants. As in the main model, $\alpha < 0$ is the degree of the negative externalities due to competition. z denotes market characteristics, x_i (or x_e) denotes the characteristics of firm i (or firm e), and β_0 and β_{1I} denote the effects of these characteristics on profits. β_{2I} (or β_{2E}) is a constant term for incumbents (potential entrants) entering the market. The last term, ξ, denotes market-level profit shock, which follows a normal distribution independently. If the firm does not enter the market, the profit is zero. Following Berry (1992), we estimate the parameters with Simulated Method of Moment. The standard errors are obtained by bootstrapping.

We report the estimation results in Table 4. Note that we normalize the variance of the market-level shock, ξ at one in the full model and this model. Hence, we can compare the parameters in these two models. We find that the estimation results show that the estimates from the entry-only model by Berry (1992) are consistent with the ones from the full model, i.e., the sign of each parameter is the same. However, the magnitude of the effect of competition, α,

Table 4: Estimates of Berry (1992) Specification

Parameter	Estimates
α	-0.654
	(0.316)
β_I	0.655
	(0.064)
β_E	-9.522
	(0.964)
β_{eq}	0.356
	(0.030)
β_{size}	-0.279
	(0.020)
β_{pop}	0.750
	(0.053)
β_{inc}	0.435
	(0.039)
σ_I	0.596
	(0.044)
σ_E	0.053
	(0.004)

Note: The parameters are estimated by Simulated Method of Moment and the standard errors are estimated by bootstrapping.

is estimated to be significantly different. In the main model, Table 3 shows that the coefficient on the number of competitors is $[-2.502, -1.498]$, while in the entry-only model, Table 4 shows that it is -0.654, i.e., the effect of competition is estimated to be smaller. In other words, one regards that the market is less competitive when we treat an entry by merger as just a regular entry by potential entrant. Note that the number of operating firms does not change when the firm chooses entry by merger, while it increases when the firm chooses entry by itself. Therefore, in order to explain the observed entry pattern, in which firms enter by mergers, the estimated effect of competition becomes smaller (in absolute value) if we treat entry by merger with a regular entry.

Table 5: Counterfactual Simulation: No Negative Externalities

	Mean	Std Dev	10%-tile	Median	90%-tile
Number of Operating Firms					
Data	3.9014	2.1683	1.2	3	7
Counterfactual	[5.191,20.47]	[5.001,13.314]	[1,10]	[4,24]	[10,42]
Number of Entry by Merger					
Data	0.627	0.823	0	0	2
Counterfactual	[0.03,0.160]	[0.584,0.602]	[0,0]	[0,0]	[0,1]

6.3 Role of Externalities

In addition to the role of matching, another key feature of our model is allowing negative externalities of competition. The effect of externalities has not been well understood in the literature that consider endogenous merger and acquisitions based on structural models (e.g.,**??**). In this subsection, we aim at demonstrating the effect of negative externalities on equilibrium market outcomes. We do so by simulating equilibrium outcomes when setting the effect of externalities on payoffs to 0, i.e., $\alpha = 0$. In this simulation, firms still can choose either entry with merger, entry without merger, and no entry, but they do not take strategic interactions into account.

Table 5 shows the results of the simulation. The table indicates that the number of operating firms increases significantly, while the number of mergers decreases. This is because the cost of entry without merger is significantly reduced by ignoring negative externalities. Although the cost of entry by merger is also reduced, the effect that the entry by merger reduces the number of operating firm by one no longer exist in this case, which makes entry by merger less attractive and reduced the number of entry by merger.

The simulation exercise clearly highlights the importance of incorporating negative externalities into the analysis when studying merger and acquisitions. If we ignore the effect of externalities, we observe unrealistically competitive market structure, which leads to wrong inference about the effect of competition.

Table 6: Counterfactual Policy Experiment: Effects of the Deregulation

	Mean	Std Dev	10%-tile	Median	90%-tile
Number of Operating Firms					
Data	3.901	2.168	1.2	3	7
Counterfactual	[1,854,3.022]	[1.317,1.741]	[0,1]	[2,4]	[3,5]
Number of Entry by Merger					
Data	0.627	0.823	0	0	2
Counterfactual	[0.630,0.659]	[1.297,1.393]	[0,0]	[0,0]	[2,3]

6.4 Counterfactual Policy Experiment: Effect of De Novo Entry Ban

Finally, we study the effects of the deregulation of entry restrictions by conducting a counterfactual experiment. The intrastate de novo entry had not been lifted in the 7 states we study by the end of 2000. Our data correspond to the period that is right after the deregulation of de novo entry. In our counterfactual, we simulate market structure by prohibiting de novo entry for potential entrants. The difference between the data and the predicted outcome of the counterfactual experiment presents the effect of the deregulation on the market structure.

Table 6 reports the results of the counterfactual experiment. We obtain our results by simulating the model without the choice of de novo entry for potential entrants (setting $\beta_{2E} = -\infty$) and letting the parameter values to move within the confidence set. Computational details are discussed in Appendix C. The results show that the number of operating banks would have been smaller if de novo entry was prohibited. As the last two rows of Table 6 show, the mean number of entry by merger is increased under the counterfactual. This is because prohibiting de novo entry by potential entrants increases the incentive for the potential entrants to enter by merger. However, the increase of entry by merger is not large enough to undo the decrease of de novo entry, resulting in smaller number of operating banks on average.

7 Conclusion

In this paper, we study entry and merger decisions of banks jointly. We show the existence of the stable outcome in a two-sided matching model with externalities by proposing an algorithm. Using data on commercial banks in the U.S., we then estimate the model with a moment-inequalities estimator based on the equilibrium characterization of the matching model without imposing an equilibrium selection mechanism. We find that entry barriers differ significantly across modes of entry and that synergy is larger when incumbent banks are smaller in asset size and potential entrants are larger in asset size with their headquarters closer to each other.

In order to highlight the role of matching, we estimate an alternative structural model via Berry (1992) using the same data, in which we treat entry by merger observed in the data as entry without merger. The estimation results show that the estimated degree of competition, a key variable for policymakers, is significantly smaller than the original estimate we obtained from the matching model. Moreover, we examine the role of negative externalities or competition by simulating equilibrium outcomes when the estimated parameter of competition is set at zero. We find significantly more entries, while fewer mergers. This is because it becomes much easier to enter the market without mergers.

Lastly, we conducted a counterfactual policy simulation. We simulate the market outcomes where de novo entries are prohibited and hence entry with merger is the only way to enter new markets. The simulation results show that the number of total operating firms slightly decrease while the number of entry by mergers slightly increases.

There are many issues left for a future research regarding firms' merger and entry decisions. One issue we could not address was how entry by merger affects industry dynamics. A natural step would be to consider a merger as an additional investment tool in a dynamic industry competition model. Another issue concerns the way the externalities of post-entry competition affect firms. One can extend the model to consider the market with vertical or horizontal differentiation as in Mazzeo (2002) and Seim (2006).

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Appendix A Moment Inequalities and Equalities

We describe the moment inequalities and equalities we use in our estimation. First, we have moment inequalities from mean of payoffs for potential entrants

$$E\left[\frac{1}{N_{\mathscr{E}}}\sum_{e\in\mathscr{E}}\Big[U_e(\overline{K}^{*\mathscr{E}};\theta)-U_e(\overline{K}^{*DATA};\theta)\Big]\right] \geq 0,$$
$$E\left[\frac{1}{N_{\mathscr{E}}}\sum_{e\in\mathscr{E}}\Big[U_e(\overline{K}^{*DATA};\theta)-U_e(\overline{K}^{*\mathscr{I}};\theta)\Big]\right] \geq 0,$$

and the same for incumbents as well. Also, we consider the conditional moments for the mean payoffs of the potential entrants,

$$E\left[\frac{1}{N_{\mathscr{E}}}\sum_{e\in\mathscr{E}}\Big[U_e(\overline{K}^{*\mathscr{E}};\theta)-U_e(\overline{K}^{*DATA};\theta)|X\Big]\right] \geq 0,$$
$$E\left[\frac{1}{N_{\mathscr{E}}}\sum_{e\in\mathscr{E}}\Big[U_e(\overline{K}^{*DATA};\theta)-U_e(\overline{K}^{*\mathscr{I}};\theta)|X\Big]\right] \geq 0,$$

as well as for the incumbents. In addition to the differences in mean payoff between the observed and upper (and lower) bounds, we consider quantiles as well. Denoting α-quantile among the player $\mathscr{E}$ by $Q_{\alpha,\mathscr{E}}(\cdot)$, we use

$$E\Big[Q_\alpha\Big(U_e(\overline{K}^{*\mathscr{E}};\theta)-U_e(\overline{K}^{*DATA};\theta)\Big)\Big] \geq 0,$$
$$E\Big[Q_\alpha\Big(U_e(\overline{K}^{*DATA};\theta)-U_e(\overline{K}^{*\mathscr{I}};\theta)\Big)\Big] \geq 0,$$

as well as for the incumbents. The quantiles we use for incumbents are 25%, 50%, and 75%. For potential entrants, we use 95%, and 99% because there is not much information for lower quantiles for potential entrants.

Regarding moment equalities, we have two types of equalities. The first type of equalities are regarding the number of operating firms, i.e.,

$$
\begin{aligned}
E\left[N_{merge}^{DATA}|X\right] &= E\left[\gamma(\overline{K}^{*\mathscr{E}}, N^{DATA};\theta)|X\right] = E\left[\gamma(\overline{K}^{*\mathscr{I}}, N^{DATA};\theta)|X\right],\\
E\left[N^{DATA}|X\right] &= E\left[\lambda(\overline{K}^{*\mathscr{E}}, N^{DATA};\theta)|X\right] = E\left[\lambda(\overline{K}^{*\mathscr{I}}, N^{DATA};\theta)|X\right].
\end{aligned}
$$

The second type of equalities are about the payoffs of unmatched firms (from Part (ii) of Corollary 1). We take mean of the payoff of unmatched firms to construct moment equalities. Denoting the set of unmatched firms in each market by UM and the number of the unmatched firms by N_{UM}, we have the following moment equalities;

$$
\begin{aligned}
E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*DATA};\theta)\right] &= E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*\mathscr{E}};\theta)\right],\\
E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*DATA};\theta)\right] &= E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*\mathscr{I}};\theta)\right],\\
E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*DATA};\theta)|X\right] &= E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*\mathscr{E}};\theta)|X\right],\\
E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*DATA};\theta)|X\right] &= E\left[\frac{1}{N_{UM}}\sum_{j\in UM} U_j(\overline{K}^{*\mathscr{I}};\theta)|X\right].
\end{aligned}
$$

In total, there are 44 moment inequalities and 14 moment equalities.

Regarding the third step, we use draws of $\eta^{ms,DATA} = (\varepsilon_{\mathscr{I}}^{ms,DATA}, \varepsilon_{\mathscr{E}}^{ms,DATA}, \xi_m^{s,DATA})$ which satisfies the requirement that the observed match $\overline{K}^{*DATA}$ and N^{DATA} is a stable outcome. As the draws, $\eta^{ms,DATA}$, need to satisfy this requirement, one approach would be to have a very large number of draws from the distributions $g_{\mathscr{I}}$, $g_{\mathscr{E}}$, and g_m, and throw away the draws that does not make $\overline{K}^{*DATA}$ and N^{DATA} as a stable outcome. This approach, however, is computationally inefficient when parameter are far way from the true value. Instead, we use the GHK simulator so that we can generate $\eta^{ms,DATA}$ effectively.

Appendix B Computation of Table 3

The model has 21 parameters, and the confidence set, which we denote as CS, is a 21-dimensional object. As we cannot present a 21-dimensional object in a convenient way, we present the min and max of the CS along each dimension in Table 3. In the following, we explain how we obtained the min and the max of the CS along each dimension.

Following the notation of Andrews and Soares (2010), a parameter value θ is included in CS if $T_n(\theta) \leq \widehat{c}_n(\theta, 1-\alpha)$ where $T_n(\theta)$ is the test statistic and $\widehat{c}_n(\theta, 1-\alpha)$ is the critical value. Denoting the j-th element of θ by θ^j, we report $\underline{\theta}^j = \min\{\theta^j | \theta \in CS\}$ and $\overline{\theta}^j = \max\{\theta j | \theta \in CS\}$. Though computing CS directly is extremely costly given that the CS has 21 dimensions, we can compute $\underline{\theta} j$ within manageable time by solving the following constrained optimization problem for each of j-th dimension;

$$\begin{aligned} &\min_{\theta} \theta^j \\ &s.t.\ T_n(\theta) \leq \widehat{c}_n(\theta, 1-\alpha), \end{aligned}$$

where θ^j is the j-th element of θ. By maximizing instead of minimizing θ^j, we can obtain $\overline{\theta}^j$. We repeat this for $j = 1, ..., 21$ and report the solutions in Table 3.

Appendix C Computation of Counterfactual Simulations

Though we cannot directly compute the 21-dimensional confidence set, CS, we can still conduct counterfactual policy experiments by imposing a restriction that the parameter values must satisfy the requirement for being included in CS. In computing the upper and lower bounds for an outcome of interest y (say, number of operating firms), we solve the following constrained optimization problem similar to the problem in Appendix C;

$$\min_{\theta} y(\theta)$$
$$s.t.\ T_n(\theta) \leq \widehat{c}_n(\theta, 1-\alpha),$$

where the notations are same as in Appendix C. We compute the solution to the problem to have the lower bound for the outcome of interest y, such as mean and median numbers of operating firms. We also solve the corresponding maximization problem to obtain the upper bound for the outcome of interest.

Appendix D Monte Carlo Simulation

In this section, we present the results of Monte Carlo simulations to demonstrate the validity of our estimator for two-sided matching models. We consider a simple two-sided matching model without externalities to make the presentation clear. In all of the specifications below, we fix the utility functions as follows.

$$\begin{aligned} U_{mw} &= -\theta_1 \sqrt{(x_m^1 - x_w^1)^2} + \theta_2 x_w^2 + \varepsilon_{mw}, \\ U_{wm} &= -\theta_1 \sqrt{(x_m^1 - x_w^1)^2} + \theta_2 x_m^2 + \varepsilon_{wm}, \end{aligned}$$

where $\theta = (\theta_1, \theta_2)$ is the vector of parameters, $x_m = (x_m^1, x_m^2)$ is the vector of m's characteristics, and $x_w = (x_w^1, x_w^2)$ is the vector of w's characteristics. The true parameter is $\theta_0 = (9, 7)$ and each of the observed characteristics $x = (x_m, x_w)$ is independently drawn from the normal distribution $N(2, 4)$. When we simulate the data, we assume that the men-optimal matching and the women optimal matching are randomly realized with equal probability in the data.

In the following simulations, we change the number of markets, the number of players in each market, and the distribution of the error term $\varepsilon = (\{\varepsilon_{mw}\}, \{\varepsilon_{wm}\})$. In the first set of simulations, we independently draw the number of players in each market from the uniform

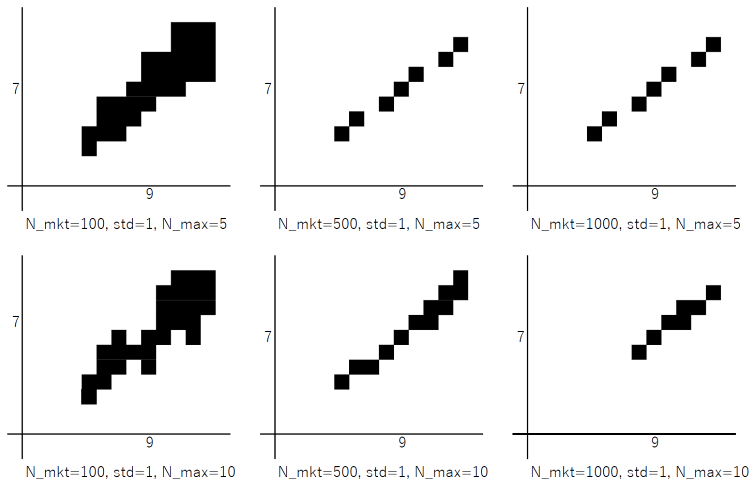

Figure A.1: Results of Monte Carlo Simulations

distribution of $U(1,6)$ and the error term from the normal distribution $\varepsilon_{mw} \sim N(0,1)$ (similarly $\varepsilon_{wm} \sim N(0,1)$), and change the number of markets $N = 100, 500, 1000$. In the second experiment, we independently draw the number of players from the uniform distribution $U(1,11)$, but fix the distribution of the error term ε. Then we compare three different market sizes $N = 100, 500, 1000$. In the estimation, we construct four moment inequalities based on equations (5.1) - (5.4), and one moment equality based on equation (5.5). As in the main model, we estimate confidence intervals using $S = S_1$ and $\varphi_j = \varphi_j^{(4)}$ in Andrews and Soares (2010).

The result is as in Figure A.1. The figure plots identified set in black the space of θ. The top three panels corresponds to the first set of the simulations, in which $\varepsilon_{mw} \sim N(0,1)$ and the number of players from $U(1,6)$, with the top left panel corresponding to $N = 100$, the top-middle panel $N = 500$, and the right panel $N = 1000$. As the number of markets increases, we find that the identified set shrinks. The true parameter value is always included in all three panels.

The bottom three panels corresponds to the second set of simulations in which $\varepsilon_{mw} \sim N(0,1)$ with number of players from $U(1,11)$. Same as the top three panels, the identified set shrinks as the number of markets increases, while the true value is always included in the identified set. The identified set is slightly larger for with the larger number of players comparing between the top and middle panels for each row. It is because multiple stable matching is generally more likely to occur in larger markets.