Assignment 1: Basic Programming in R

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The deadline is the **start time of February 18 class**.

Report the following results in html format using R markdown. In other words, replicate this document. You write functions in a separate R file and put in `R` folder in the project folder. Build the project as a package and load it from the R markdown file. The execution code sholuld be written in R markdown file.

You submit:

- R file containing functions.
- R markdown file containing your answers and executing codes.
- HTML report generated from the R markdown.

Simulate data

Consider to simulate data from the following model and estimate the parameters from the simulated data.

$$y_{ij} = 1\{j = \text{argmax}_{k=1,2} \beta x_k + \epsilon_{ik}\},$$

where ϵ_{ik} follows i.i.d. type-I extreme value distribution, $\beta = 0.2$, $x_1 = 0$ and $x_2 = 1$.

1. To simulate data, first make a data frame as follows:

```
df <- expand.grid(i = 1:1000, k = 1:2) %>%
  tibble::as.tibble() %>%
  dplyr::mutate(x = ifelse(k == 1, 0, 1)) %>%
  dplyr::arrange(i, k)
```

```
## Warning: `as.tibble()` is deprecated, use `as_tibble()` (but mind the new semantics).
## This warning is displayed once per session.
```

```
df
```

```
## # A tibble: 2,000 x 3
##        i     k     x
##    <int> <int> <dbl>
##  1     1     1     0
##  2     1     2     1
##  3     2     1     0
##  4     2     2     1
##  5     3     1     0
##  6     3     2     1
##  7     4     1     0
##  8     4     2     1
##  9     5     1     0
## 10     5     2     1
## # ... with 1,990 more rows
```

2. Second, draw type-I extreme value random variables. Set the seed at 1. You can use `evd` package to draw the variables. You should get exactly the same realization if the seed is correctly set.

```r
set.seed(1)
df <- df %>%
  dplyr::mutate(e = evd::rgev(dim(df)[1]))
df
```

```
## # A tibble: 2,000 x 4
##        i     k     x       e
##    <int> <int> <dbl>   <dbl>
##  1     1     1     0  0.281
##  2     1     2     1 -0.167
##  3     2     1     0  1.93
##  4     2     2     1  1.97
##  5     3     1     0  0.830
##  6     3     2     1 -1.06
##  7     4     1     0 -0.207
##  8     4     2     1  0.617
##  9     5     1     0  0.0444
## 10     5     2     1  1.92
## # ... with 1,990 more rows
```

3. Third, compute the latent value of each option to obtain the following data frame:

```r
beta <- 0.2
df <- df %>%
  dplyr::mutate(latent = beta * x + e)
df
```

```
## # A tibble: 2,000 x 5
##        i     k     x       e  latent
##    <int> <int> <dbl>   <dbl>   <dbl>
##  1     1     1     0  0.281   0.281
##  2     1     2     1 -0.167   0.0331
##  3     2     1     0  1.93    1.93
##  4     2     2     1  1.97    2.17
##  5     3     1     0  0.830   0.830
##  6     3     2     1 -1.06   -0.863
##  7     4     1     0 -0.207  -0.207
##  8     4     2     1  0.617   0.817
##  9     5     1     0  0.0444  0.0444
## 10     5     2     1  1.92    2.12
## # ... with 1,990 more rows
```

4. Finally, compute y by comparing the latent values of $k = 1, 2$ for each i to obtain the following result:

```r
df <- df %>%
  dplyr::group_by(i) %>%
  dplyr::mutate(y = ifelse(latent == max(latent), 1, 0)) %>%
  dplyr::ungroup()
df
```

```
## # A tibble: 2,000 x 6
##        i     k     x       e  latent     y
##    <int> <int> <dbl>   <dbl>   <dbl> <dbl>
##  1     1     1     0  0.281   0.281      1
##  2     1     2     1 -0.167   0.0331     0
##  3     2     1     0  1.93    1.93       0
##  4     2     2     1  1.97    2.17       1
```

```
## 5     3     1     0  0.830   0.830      1
## 6     3     2     1 -1.06   -0.863      0
## 7     4     1     0 -0.207  -0.207      0
## 8     4     2     1  0.617   0.817      1
## 9     5     1     0  0.0444  0.0444     0
## 10    5     2     1  1.92    2.12       1
## # ... with 1,990 more rows
```

Estimate the parameter

Now you generated simulated data. Suppose you observe x_k and y_{ik} for each i and k and estimate β by a maximum likelihood estimator. The likelihood for i to choose k ($y_{ik} = 1$) can be shown to be:

$$p_{ik}(\beta) = \frac{\exp(\beta x_k)}{\exp(\beta x_1) + \exp(\beta x_2)}.$$

Then, the likelihood of observing $\{y_{ik}\}_{i,k}$ is:

$$L(\beta) = \prod_{i=1}^{1000} p_{i1}(\beta)^{y_{i1}}[1 - p_{i1}(\beta)]^{1-y_{i1}},$$

and the log likelihood is:

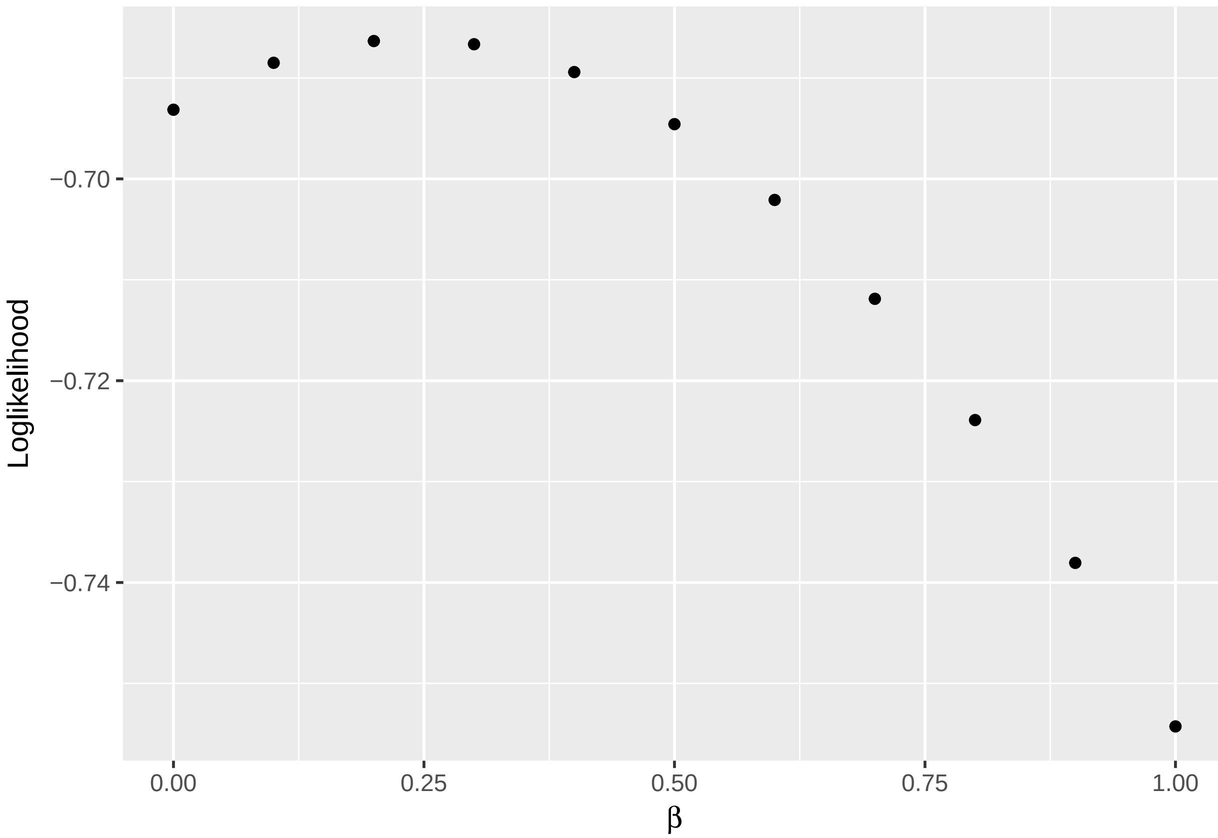
$$l(\beta) = \sum_{i=1}^{1000} \{y_{i1} \log p_{i1}(\beta) + (1 - y_{i1}) \log[1 - p_{i1}(\beta)\}.$$

1. Write a function to compute the livelihood for a given β and data and name the function `loglikelihood_A1`.

```r
library(EmpiricalIO)
loglikelihood_A1 <-
  function(b, df) {
  l <- df %>%
    dplyr::group_by(i) %>%
    dplyr::mutate(p = exp(b*x)/sum(exp(b*x))) %>%
    dplyr::ungroup() %>%
    dplyr::filter(y == 1)
  l <- mean(log(l$p))
  return(l)
}
```

1. Compute the value of log likelihood for $\beta = 0, 0.1, \cdots, 1$ and plot the result using `ggplot2` packages. You can use `latex2exp` package to use LaTeX math symbol in the label:

```r
b_seq <- seq(0, 1, 0.1)
output <-
  foreach (b = b_seq,
           .combine = "rbind") %do% {
  l <- loglikelihood_A1(b, df)
  return(l)
}
output <- data.frame(x = b_seq, y = output)
ggplot(output, aes(x = x, y = y)) +
  geom_point() + xlab(TeX("$\\beta$")) + ylab("Loglikelihood")
```



1. Find and report β that maximizes the log likelihood for the simulated data. You can use `optim` function to achieve this. You will use `Brent` method and set the lower bound at -1 and upper bound at 1 for the parameter search.

```
result <- optim(par = beta,
                fn = loglikelihood_A1,
                df = df,
                method = "Brent",
                lower = -1,
                upper = 1,
                control = list(fnscale = - 1))
result
```

```
## $par
## [1] 0.2371046
##
## $value
## [1] -0.6861689
##
## $counts
## function gradient
##       NA       NA
##
## $convergence
## [1] 0
##
## $message
## NULL
```