Canonical Research Designs II: Difference-in-Differences II: Event Studies, Synthetic Control, and Synthetic DinD

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Today’s Topics

- Today, touching on two (related) topics
- First, finishing conversation on standard diff-in-diff, focusing on *event studies*
    - How do event studies generate a counteractual control unit
    - Issue: dynamic effects **plus** staggered timing **plus** heterogeneity
- Second, discuss synthetic control (and dind) methods
    - Not completely new methods, but big upswing in research

Event study

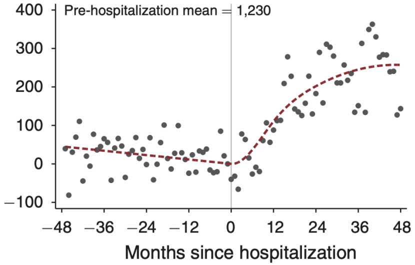
- Two important cases with these staggered timing dind (event studies)
    - There exists a never-treated group who is a potential control group
    - Everyone is treated eventually (No group is a "pure control")

- The older approach to estimate this model was:

$$Y_{it} = \alpha_i + \gamma_t + \sum_{s=L_0, s\neq -1}^{L_1} 1(t - T_i = s)\mu_s$$

- Without a true control group, can't have both time fe, unit fe, and the full set of relative effects
- Need to exclude both the baseline period AND at least some periods outside the treatment window

Panel B. Collection balances



- Dobkin et al. (2018)
- Comparison is between those not yet hospitalized and those hospitalized

Event study continued

- The necessary assumptions are the same (or similar) what we discussed last class
- Parallel trends

$$E(Y_{i,t}(\infty) - Y_{i,t'}(\infty) | G_i = g) = E(Y_{i,t}(\infty) - Y_{i,t'}(\infty) | G_i = g), \forall g, g', \text{and} t, t' \tag{1}$$

- Turns out, all of the groups need to be parallel.
- That might be a bad assumption (e.g. very far apart from one another)
  - Can be weakened in some cases, but only partially
- No anticipation:

$$Y_{it}(g) = Y_{it}(\infty) \forall t < g \tag{2}$$

Contamination Bias in event studies

- Sun and Abraham (2021) show that if the dynamic path of treatment is the same across cohorts (g), then the coefficient from the TWFE model will correctly estimate the period ATT

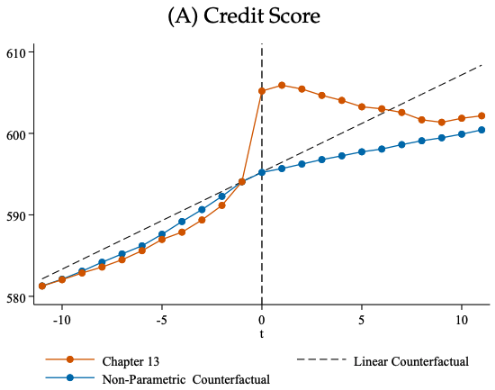
$$\tau_{it}(g) = \sum_{s \geq 0} \tau_s 1(t - g = s)$$

- If not, then there is g specific heterogeneity in paths. This creates issues:
  - Violate the pre-trend test as the use of "excluded" periods potentially contaminates pre-periods
  - Mismeasure the dynamic effects

- Additional untestable assumptions are required as we allow for more types of heterogeneity

Aside in event studies

- A key factor in how you construct your counterfactual (and what assumptions you find plausible) are a function of how far into the future you want to estimate outcomes
- An extremely short-run counterfactual could potentially just be a linear extrapolation
  - This assumes that the underlying model is locally linear, rather than globally
  - Construct a counterfactual from just a single time series, but highly non-robust
- Example from a robustness check in my own work (Dobbie et al. 2020)



Constructing a counterfactual is the key goal

- Issue in event study was the attempt to get a “free lunch” – we always need a control group

- Think back to cross-sectional setting with ATT
  - We always knew $Y_i(1)$. Key issue is an estimator for $Y_i(0)$.
  - Event study approaches had issues by ignoring this point and hoping regression would solve problem
  - Notably, this problem disappears if we have full homogeneity + no anticipation and only exclude pre-periods

- Point of emphasis – we need parallel trends to hold to construct a counterfactual in these settings. Why? $Y_{jt}(0) - Y_{j,t-1}(0)$ needs to be a good approximator of $Y_{i,t}(0) - Y_{i,t-1}(0)$.
  - Since we imposed $Y_{it} = \alpha_i + \gamma_t + D_{it}\tau$, the first differencing makes them good approximations

Generalizing the Dind approach

- Pivoting slightly: instead of imposing the parallel trends assumption directly through the linear model, we could construct a combination of units to approximate $Y_{it}(0)$
  - This is what one does in the cross-sectional setting with a pscore method! E.g. consider the ATT:

$$\tau_{ATT} = \underbrace{Y(1)}_{\text{Fully observed}} - \underbrace{\hat{Y}(0)}_{\text{Constructed}}$$

- How would one pick? Recall that with p-score methods or regression, weights effectively reweight based on comparability to treated group
  - With panel data, can use pre-treatment data to construct these weights
  - This method is known as synthetic control (and its various descendents)

Synthetic Control example - (Abadie et al. (2010))

- Consider following problem: California bans smoking in 1989. What does that do to smoking?
  - Define estimand: $\tau_{ban,CA} = Y_{california,post}(1) - Y_{california,post}(0)$
  - This is the effect of the *California* smoking ban
  - How can we get at it?

- We need a “synthetic California” as our control

- In an ideal world, the average of the other states would work – however, not clear empirically that they are a good counterfactual

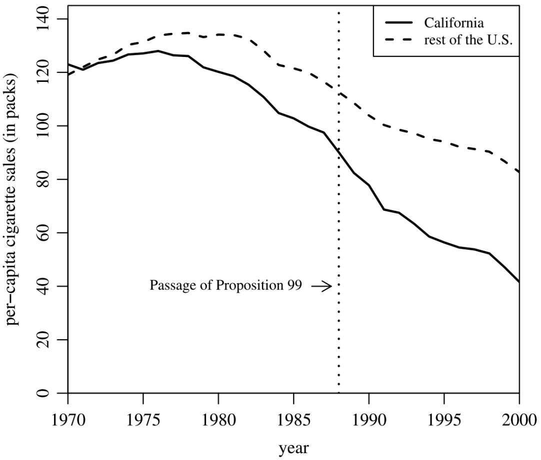


Figure 1. Trends in per-capita cigarette sales: California vs. the rest of the United States.

Generalized setup (Doudchenko and Imbens (2018))

- Consider the following general problem

- We have a panel with T time periods and $N+1$ units. Intervention D_{it} at time T_0 for one unit (unit $i=0$)

- Potential outcomes $Y_{it}(D_{it})$, and we only observe one of the potential outcomes (as per usual)
  - Fundamental problem of causal inference
  - We can also have fixed characteristics X_{it}

- Let $\mathbf{Y}_{a,b}$ denote the vector (or matrix in control case) for $a \in \{\text{treatment}, \text{control}\}$ and $b \in \{\text{pre}, \text{post}\}$ for the treated and control groups in the pre or post period.

- Then, we have observations (analogous setup for the covariates):

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

Generalized panel setup

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

- To estimate $\tau_i = Y_{t,post}(1) - Y_{t,post}(0)$, we need an estimate for $Y_{t,post}(0)$
- What if we just had the cross-section?
  - Note that if D_{it} were randomly assigned, we can derive an estimate using our p-score or regression methods
  - Even without random assignment, one could use covariates to match
  - Our main concern with p-score matching is bias
- Diff-in-diff exploited the panel structure by asserting a particular functional form

$$Y_{it} = \alpha_i + \gamma_t + D_{it}\tau + \epsilon_{it}$$

- Is there something particularly special about this linear additive factor structure?

Generalized panel setup

$$\mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{t,post} & \mathbf{Y}_{c,post} \\ \mathbf{Y}_{t,pre} & \mathbf{Y}_{c,pre} \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_{t,post}(1) & \mathbf{Y}_{c,post}(0) \\ \mathbf{Y}_{t,pre}(0) & \mathbf{Y}_{c,pre}(0) \end{pmatrix}$$

- Recall that our problem boils down to the estimate of an untreated “synthetic” unit
- Following Doudchenko and Imbens (2018), note estimators of the following form:

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

  - A constant μ allows for very different averages (common in diff-in-diff)
  - Weights are allowed to vary across i – a simple average would be diff-in-diff
- We can now consider deviations from diff-in-diff

The synthetic control method (Abadie et al. (2010)

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

- In ADH, they impose
  1. $\mu = 0$
  2. $\sum_i \omega_i = 1$
  3. $\omega_i \geq 0 \ \forall i$

- These three restrictions create a counterfactual California whose outcomes are within the support of the other states, and is a weighted sum of a subset of states

The synthetic control method (Abadie et al. (2010)

$$\hat{Y}_{t,post}(0) = \mu + \sum_{i \in c} \omega_i Y_{i,T}$$

- Formally, the ω_i need to be estimated, and are constructed by minimizing the distance between covariates in the pre-period:

$$||\boldsymbol{X}_{\text{treat}} - \boldsymbol{X}_{\text{control}} \boldsymbol{W}||$$

- The crucial piece tying this together: $\boldsymbol{X}$ can include both lagged outcomes, and covariates.

- Note we can now re-envision our panel data:
  - Observed outcomes: $\mathbf{Y}_{t,post}(1)$, $\mathbf{Y}_{c,post}(0)$
  - Observed covariates / predictors: $\mathbf{Y}_{t,pre}(0)$, $\mathbf{Y}_{c,pre}(0)$, $\mathbf{X}_t$, $\mathbf{X}_c$

- In many ways, this is just a matching problem using many characteristics!

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- Formally, the ω_i need to be estimated, and are constructed by minimizing the distance between covariates in the pre-period:

$$\{\hat{\omega}\}_i = \arg\min_{\boldsymbol{W}} ||\boldsymbol{X}_{\text{treat}} - \boldsymbol{X}_{\text{control}} \boldsymbol{W}||$$

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The synthetic control method (Abadie et al. (2010)

- This approach can be incredibly successful

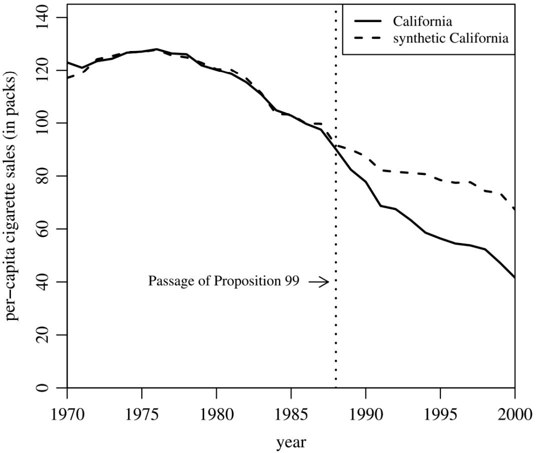

Figure 2. Trends in per-capita cigarette sales: California vs. synthetic California.

The synthetic control method (Abadie et al. (2010)

- This approach can be incredibly successful

- By careful construction of a synthetic control, can calculate counterfactual impacts due to policy

- Still subject to same caveats from DinD – not invariant to some transformations (e.g. log and linear)

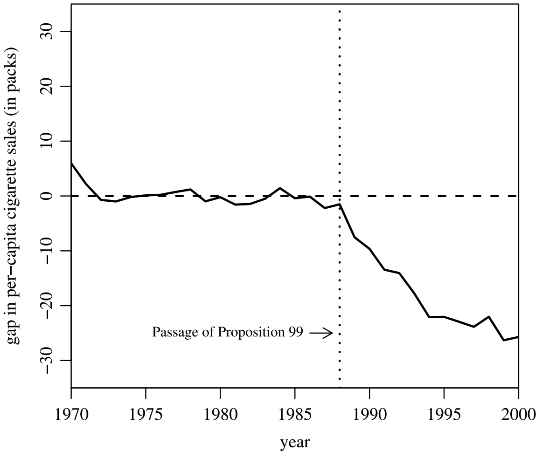

Figure 3. Per-capita cigarette sales gap between California and synthetic California.

Inference in the synthetic control method (Abadie et al. (2010)

- Inference for this method is slightly more complex, as there is only a single treated unit
  - Large sample asymptotics unlikely to work

- Placebo approach is standard: apply method to each potential control unit, and report effect in period

- Analogy here is to a randomization inference argument, comparing to a "null" effect

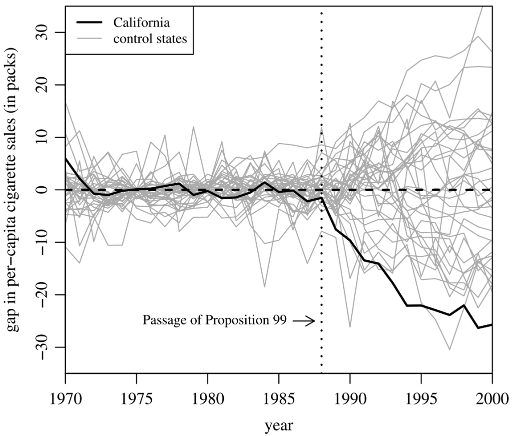

Figure 5. Per-capita cigarette sales gaps in California and placebo gaps in 34 control states (discards states with pre-Proposition 99 MSPE twenty times higher than California's).

Synthetic Diff-in-diff

- In Arkhangelsky et al. (2019), they show you can rewrite the synthetic control estimator as
$$(\hat{\mu}, \hat{\gamma}, \hat{\tau}) = \text{arg}\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \gamma_t - D_{it}\tau)^2 \hat{\omega}_i,$$
subject to the $\hat{\omega}_i$ chosen via the SC approach

- Contrast that with DID:
$$(\hat{\mu}, \hat{\alpha}, \hat{\gamma}, \hat{\tau}) = \text{arg}\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \alpha_i - \gamma_t - D_{it}\tau)^2$$

- They then propose a more robust approach, called Synthetic diff-in-diff, which estimates
$$(\hat{\mu}, \hat{\alpha}, \hat{\gamma}, \hat{\tau}) = \text{arg}\min_{\mu,\gamma,\tau} \sum_i \sum_t (Y_{it} - \mu - \alpha_i - \gamma_t - D_{it}\tau)^2 \hat{\omega}_i \hat{\lambda}_t$$

- This approach relaxes the parallel trends assumption by requiring parallel trends in an underlying approximate factor structure

Synthetic Diff-in-diff

- Key difference is twofold:
    1. Pre-trend means do not need to match "exactly"
    2. Weighting is not equivalent across all time periods

- Conceptually – different ways to generate the counterfactual given a model

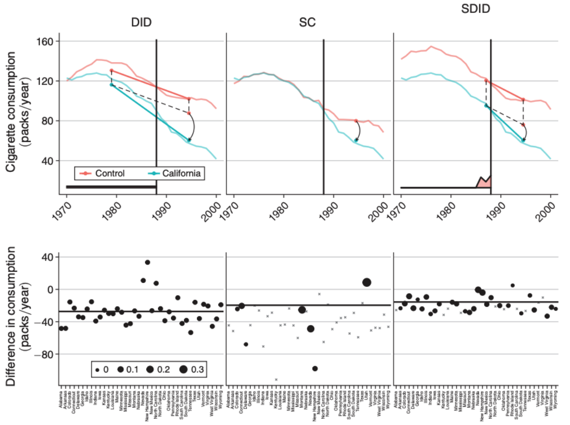

FIGURE 1. A COMPARISON BETWEEN DID, SC, AND SDID ESTIMATES FOR THE EFFECT OF CALIFORNIA PROPOSITION 99 ON PER-CAPITA ANNUAL CIGARETTE CONSUMPTION (IN PACKS/YEAR)

Synthetic Diff-in-diff

- Key difference is twofold:
  1. Pre-trend means do not need to match "exactly"
  2. Weighting is not equivalent across all time periods

- Conceptually – different ways to generate the counterfactual given a model

$$\hat{\tau} = \hat{\delta}_{tr} - \sum_{i=1}^{N_{co}} \hat{\omega}_i \hat{\delta}_i \qquad \text{where} \qquad \hat{\delta}_{tr} = \frac{1}{N_{tr}} \sum_{i=N_{co}+1}^{N} \hat{\delta}_i .$$

$$\hat{\delta}_i^{sc} = \frac{1}{T_{post}} \sum_{t=T_{pre}+1}^{T} Y_{it},$$

$$\hat{\delta}_i^{did} = \frac{1}{T_{post}} \sum_{t=T_{pre}+1}^{T} Y_{it} - \frac{1}{T_{pre}} \sum_{t=1}^{T_{pre}} Y_{it},$$

$$\hat{\delta}_i^{sdid} = \frac{1}{T_{post}} \sum_{t=T_{pre}+1}^{T} Y_{it} - \sum_{t=1}^{T_{pre}} \hat{\lambda}_t^{sdid} Y_{it}.$$

Synthetic Diff-in-diff

- So far, synth dind method discussion focused on single adoption period.
    1. Staggered adoption in synthetic control isn't meaningful
    2. How can you adopt it?
- Conceptually – split up the adoption timings a la Calloway & Sant'anna and others

$$
\mathbf{W} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}
\begin{array}{c}
\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}
\end{array}.
$$

Synthetic Diff-in-diff

- So far, synth dind method discussion focused on single adoption period.
    1. Staggered adoption in synthetic control isn't meaningful
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$$\mathbf{W}^1 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 5 \\ 6 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{array},$$

$$\mathbf{W}^2 = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}.$$

So what about synthetic methods?

- Both an old field, and a new one – lots of new methodological papers coming out
  - It is a very cool method!
- So far, limited application by researchers. Why?
- My thoughts:
  - These are strong structural assumptions, and not clear we have good tests yet
  - Despite concerns re: pre-trends in dind, the assumptions felt testable
- Researcher degrees of freedom seem multifold. True in DinD too, but perhaps more transparent?
  - More worrisome: dind is equally problematic, but we aren't aware of it
- If researchers are more willing to understand that DinD is sensitive to functional form, ML methods that construct counterfactual outcomes are a natural direction

My recommendation / takeaway

- Synthetic control is the ideal approach when faced with a single treatment
  - By far the most natural approach in this setting, and is a practical approach
  - Typcial approach – get a good synthetic control for a given treatment. If none exists, stop. Ben-Michael, Feller and Rothstein (2021) provide a better approach, which adjusts for imperfect pre-match.
- Synthetic DiD seems very promising as a generalization
  - Key question is convincing readers why this shoudl work better than traditional method
  - My view: empirical papers will first need to show how / why their method works with both diff-in-diff and synth diff-in-diff
- Key point: *all of this relies on a model of the control outcome*
- Three packages to explore: augsynth/tidysynth andsynthdid packages (original synth package is tough to use)

Next class

- Extensions: continuous treatments, multiple treatments, alternative approaches
- Checklist: What do you need to do?