

Part 8: Policy Evaluation- Regression Discontinuity

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Applied Econometrics

RDD

Regression Discontinuity Design

- Another popular research design is the Regression Discontinuity Design.
- In some sense this is a special case of IV regression. (RDD estimates a LATE).
- Most of this is taken from the JEL Paper by Lee and Lemieux (2010).

RDD: Basics

- We have a **running or forcing variable** x such that

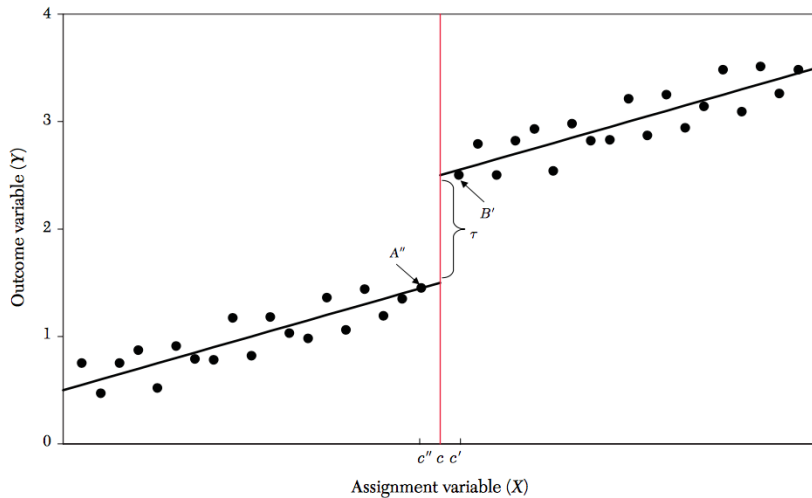
$$\lim_{x \rightarrow c^+} P(T_i | X_i = x) \neq \lim_{x \rightarrow c^-} P(T_i | X_i = x)$$

- The idea is that there is a **discontinuous jump** in the **probability of being treated**.
- For now we focus on the **sharp discontinuity**:

$$P(T_i | X_i \geq c) = 1 \text{ and } P(T_i | X_i < c) = 0$$

- There is no single x for which we observe treatment and control. (Compare to Propensity Score!).
- The most important assumption is that of **no manipulability** $\tau_i \perp D_i$ in some neighborhood of c .
- Example: a social program is available to people who earned less than \$25,000.
 - If we could compare people earning \$24,999 to people earning \$25,001 we would have as-if random assignment. (MAYBE)
 - But we might not have that many people...

RDD: In Pictures



RDD: Sharp RD Case

RDD uses a set of assumptions distinct from our LATE/IV assumptions. Instead it depends on **continuity**.

- We need that $E[Y^{(1)}|X]$ and $E[Y^{(0)}|X]$ both be continuous at $X = c$.
- People just to the left of c are a valid control for those just to the right of c .
- **This is not a testable assumption** \rightarrow draw pictures!
- We could run the regression where $D_i = \mathbf{1}[X_i > c]$.

$$Y_i = \beta_0 + \tau D_i + X_i \beta + \epsilon_i$$

- This puts a lot of restrictions (linearity) on the relationship between Y and X .
- Also (without additional assumptions) we only learn about τ_i at the point $X = c$.

RDD: Nonlinearity

First thing to relax is assumption of linearity.

$$Y_i = f(x_i) + \tau D_i + \epsilon_i$$

This is known as **partially linear model**.

- Two options for $f(x_i)$:
 1. Kernels: Local Linear Regression
 2. Polynomials: $Y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \dots + \beta_p x_i^p + \tau D_i + \epsilon_i$.
 - Actually, people suggest different polynomials on each side of cutoff! (Interact everything with D_i).
- Same objective. Want to flexibly capture what happens on both sides of cutoff.
- Otherwise risk confusing nonlinearity with discontinuity!

RDD: Kernel Boundary Problem

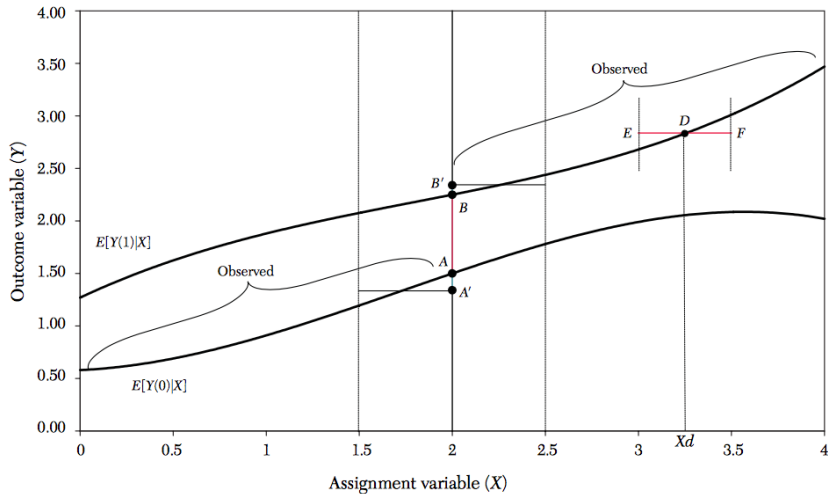


Figure 2. Nonlinear RD

RDD: Polynomial Implementation Details

To make life easier:

- replace $\tilde{x}_i = x_i - c$.
- Estimate coefficients $\beta: (1, \tilde{x}, \tilde{x}^2, \dots, \tilde{x}^p)$ and $\tilde{\beta}: (D_i, D_i\tilde{x}, D_i\tilde{x}^2, \dots, D_i\tilde{x}^p)$.
- Now treatment effect at c just the coefficient on D_i . (We can ignore the interaction terms).
- If we want treatment effect at $x_i > c$ then we have to account for interactions.
 - Identification away from c is somewhat dubious.
- Lee and Lemieux (2010) suggest estimating a coefficient on a dummy for each bin in the polynomial regression $\sum_k \phi_k B_k$.
 - Add polynomials until you can satisfy the test that the joint hypothesis test that $\phi_1 = \dots = \phi_k = 0$.
 - There are better ways to choose polynomial order...

RDD: Checklist

Most RDD papers follow the same formula (so should yours)

- Plot of $P(D|X)$ so that we can see the discontinuity
- Plot of $E[Y|X]$ so that we see discontinuity there also
- Plot of $E[W|X]$ so that we don't see a discontinuity in controls.
- Density of X (check for manipulation).
- Show robustness to different “windows”
- The OLS RDD estimates
- The Local Linear RDD estimates
- The polynomial (from each side) RDD estimates
- An f-test of “bins” showing that the polynomial is flexible enough.

Read Lee and Lemieux (2010) before you get started.

Looked at incumbency advantage in the US House of Representatives

- Running variable was vote share in previous election
 - Problem of naive approach: good candidates get lots of votes!
 - Compare outcomes of districts with barely D to barely R .
- First we plot bin-scatter plots and quartic (from each side) polynomials.
- Discussion about how to choose bin-scatter bandwidth (CV).

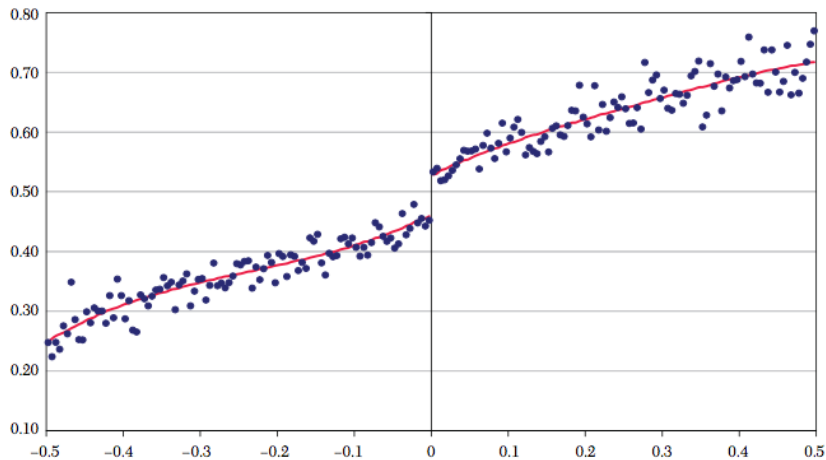


Figure 8. Share of Vote in Next Election, Bandwidth of 0.005 (200 bins)

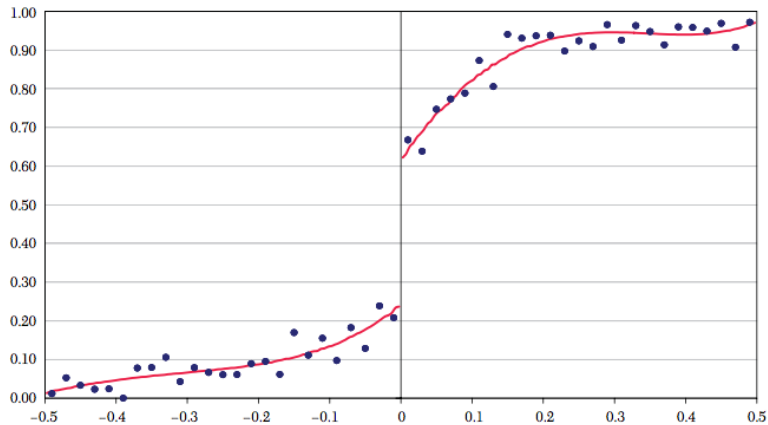


Figure 9. Winning the Next Election, Bandwidth of 0.02 (50 bins)

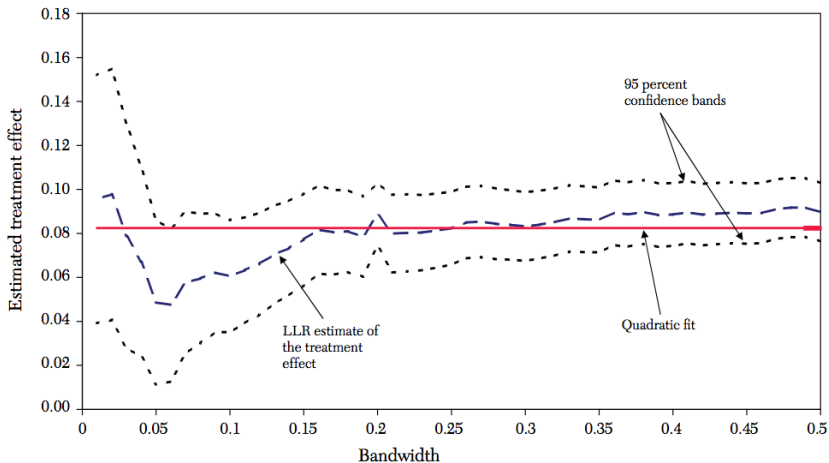


Figure 18. Local Linear Regression with Varying Bandwidth: Share of Vote at Next Election

Luca on Yelp

- Have data on restaurant revenues and yelp ratings.
- Yelp produces a yelp score (weighted average rating) to two decimals ie: 4.32.
- Score gets rounded to nearest half star
- Compare 4.24 to 4.26 to see the impact of an extra half star.
- Now there are multiple discontinuities: Pool them? Estimate multiple effects?

An important extension in the **Fuzzy RD**. Back to where we started:

$$\lim_{x \rightarrow c^+} P(T_i | X_i = x) \neq \lim_{x \rightarrow c^-} P(T_i | X_i = x)$$

- We need a discontinuous jump in probability of treatment, but it doesn't need to be $0 \rightarrow 1$.

$$\tau_i(c) = \frac{\lim_{x \rightarrow c^+} P(Y_i | X_i = x) - \lim_{x \rightarrow c^-} P(Y_i | X_i = x)}{\lim_{x \rightarrow c^+} P(T_i | X_i = x) - \lim_{x \rightarrow c^-} P(T_i | X_i = x)}$$

- Under sharp RD everyone was a **complier**, now we have some **always takers** and some **never takers** too.
- Now we are estimating the treatment effect only for the population of compliers at $x = c$.

Related Idea: Kinks

A related idea is that of **kinks**.

- Instead of a discontinuous jump in the outcome there is a discontinuous jump in β_i on x_i .
- Often things like tax schedules or government benefits have a kinked pattern.

One quantity to rule them all: MTE

Heckman and Vytlacil provide a unifying non-parametric framework to categorize treatment effects. Their approach is known as the **marginal treatment effect** or MTE

- The MTE isn't a number it is a **function**.
- All of the other objects (LATE, ATE, ATT, etc.) can be written as integrals (weighted averages) of the MTE.
- The idea is to bridge the treatment effect parameters (stuff we get from running regressions) and the structural parameters: features of $f(\beta_i)$.