Local Average Treatment Effects: Demand Curves

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Applied Econometrics

A Classic Example

- What is the effect of prices on quantity demanded?
- But a regression of $\log(q_t) = \beta_0 + \beta_1 \log(p_t) + u_t$ is going to be flawed.
- For one thing, how do we know that relationship represents supply or demand?
- Imagine an instrument $z_t \in \{0, 1\}$ that reduces supply but does not effect demand.
- What about?
  - Monotonicity?
  - Heterogeneity in treatemnt effects?
  - Exclusion Restriction?

LATE at the Fulton Fish Market (Graddy 1995)

Table 2

Ordinary Least Squares and Instrumental Variable Estimates of Demand Functions with Stormy Weather as an Instrument

	Ordinary least squares (dependent variable: log quantity)		*Instrumental variable*	
Variable	*(1)*	*(2)*	*(3)*	*(4)*
Log price	−0.54	−0.54	−1.08	−1.22
	(0.18)	(0.18)	(0.48)	(0.55)
Monday		0.03		−0.03
		(0.21)		(0.17)
Tuesday		−0.49		−0.53
		(0.20)		(0.18)
Wednesday		−0.54		0.58
		(0.21)		(0.20)
Thursday		0.09		0.12
		(0.20)		(0.18)
Weather on shore		−0.06		0.07
		(0.13)		(0.16)
Rain on shore		0.07		0.07
		(0.18)		(0.16)
R^2	0.08	0.23		
No. of Obs.	111	111	111	111

Source: The data used in these regressions are available by contacting the author.
Note: Standard errors are reported in parentheses.

Wald Estimator

Focus on the simple case:

- $z_t \in \{0, 1\}$ where 1 denotes "stormy at sea" and 0 denotes "calm at sea"
- Idea is that offshore weather makes fishing more difficult but doesn't change onshore demand.
- Ignore x (for now at least) or assume we condition on each value of x.

$$\hat{\alpha}_{1,0} \rightarrow^{p} \frac{\mathbb{E}[q_t|z_t = 1] - \mathbb{E}[q_t|z_t = 0]}{\mathbb{E}[p_t|z_t = 1] - \mathbb{E}[p_t|z_t = 0]} \equiv \alpha_{1,0}$$

- If we have homogenous α then any IV gives us a consistent estimate of α_1
- If we are in complicated (nonlinear, heterogeneous) then $\alpha_{1,0}$ the object we recover, is not an estimator of a structural parameter.
- Moreover, this is at best a LATE, and thus it differs depending on which instrument we use!

AIG: Assumptions

1. Regularity conditions on $q_t^d, q_t^s, p_t, z_t, w_t$ first and second moment and is stationary, etc.
   - $q_t^d(p, z, x)$, $q_t^s(p, z, x)$ are continuously differentiable in p.
2. $z_t \in \{0, 1\}$ is a valid instrument in q_t^d
   - Exclusion: for all p, t

     $$q_t^d(p, z = 1, x_t) = q_t^d(p, z = 0, x_t) \equiv q_t^d(p, x_t)$$

     ie: conditioning on p_t means no dependence on z_t
   - Relevance: for some period t: $q_t^s(p_t, 1, x_t) \neq q_t^s(p_t, 0, x_t)$.
     ie: z_t actually shifts supply somewhere!
   - Independence: ϵ_t, η_t, z_t are mutually independent conditional on x_t.

AGI: Structural Interpretation

In order to interpret the Wald estimator $\alpha_{1,0}$ we make some additional economic assumptions on the structure of the problem:

1. Observed price is market clearing price $q_t^d(p_t) = q_t^s(p_t, z_t)$ for all t. (This means no frictions!).
2. "Potential prices": for each value of z there is a unique market clearing price

$$\forall z, t : \tilde{p}(z,t) \text{ s.t. } q_t^d(\tilde{p}(z,t)) = q_t^s(\tilde{p}(z,t), z).$$

$\tilde{p}(z,t)$ is the potential price under any counterfactual (z,t)

AGI: Structural Interpretation

- Just like in IV we need denominator to be nonzero so that $E[p_t|z_t = 1] \neq E[p_t|z_t = 0]$.
- Other key assumption is the familiar monotonicity assumption
    - $\tilde{p}(z, t)$ is weakly increasing in z.
    - Just like in program evaluation this is the key assumption. There it rules out "defiers" here it allows us to interpret the average slope as $\alpha_{1,0}$.
    - Assumption is untestable because you do not observe both potential outcomes $\tilde{p}(0, t)$ and $\tilde{p}(1, t)$ (same as in program evaluation).
    - Any story about IV is just a story! (Always the case!) unless we have repeated observations on the same individual.
- Monotonicity makes extending to multiple product case difficult/impossible.

Conlon Mortimer RJE (2021)

Wald Estimator: Redux

Consider the following Wald estimator:

$$\text{Wald}\left(p_j, p_j', x\right) = \frac{q_k\left(p_j', x\right) - q_k\left(p_j, x\right)}{-\left(q_j\left(p_j', x\right) - q_j\left(p_j, x\right)\right)}$$

$$\lim_{p_j' \to p_j} \text{Wald}\left(p_j, p_j', x\right) \to \frac{\frac{\partial q_k}{\partial P_j}\left(p_j, x\right)}{-\frac{\partial q_j}{\partial P_j}\left(p_j, x\right)} \equiv D_{jk}\left(p_j, x\right)$$

We call $D_{jk}\left(p_j, x\right)$ the Diversion Ratio.

As we increase P_j and people leave what fraction switch to k?

Wald Estimator: Relation to IV Case

To solidify the connection with the quasi-experimental LATE framework, recognize that our hypothetical price change experiment can be interpreted using the following definitions:

Outcome $Y_i \in \{0,1\}$ denotes the event that consumer i purchases product k: $d_{ik}(P_j) = 1$.

Treatment $T_i \in \{0,1\}$ denotes the event that consumer i does **not** purchase product j. In other words $T_i = 0$ implies $d_{ij}(P_j) = 1$ and $T_i = 1$ implies $d_{ij}(P_j) = 0$.

Instrument $Z_i = P_j$ the price of j induces consumers into not purchasing j.

Diversion: Why do we care?

- Related to cross price elasticity $D_{jk} = \frac{\frac{\partial q_k}{\partial P_j}(p_j, x)}{-\frac{\partial q_j}{\partial P_j}(p_j, x)} = -\frac{\epsilon_{jk}}{\epsilon_{jj}} \times \frac{q_k}{q_j}$
- Related to multi-product differentiated Bertrand FOC $(MR = MC)$:

$$p_j(\mathbf{p}) \left[1 + \frac{1}{\epsilon_{jj}(\mathbf{p})} \right] = mc_j + \sum_k (p_k - mc_k) \cdot D_{jk}$$

- $D_{jk} \in [0,1]$ and $\sum_k D_{jk} = 1$ and high-diversion ratios indicate close substitutes.

Diversion: Assumptions

Like in multiple discrete choice lectures assume that:

1. Consumers make mutually exclusive and exhaustive discrete choice.
2. Can be guaranteed by presence of an outside option.
3. Utility $u_{ij}(x)$ can be deterministic or stochastic.
4. x contains all covariates that don't change (other prices and characteristics).
5. Could be mixed logit but doesn't need to be...

$$d_{ij}(p_j, x) = \begin{cases} 1 & u_{ij}(p_j, x) > u_{ij'}(p_j, x) \text{ for all } j' \in \mathcal{J} \text{ and } j' \neq j. \\ 0 & o.w. \end{cases}$$

6. $s_{ij}(x) = \int d_{ij}(x) dF_i$ (share of individuals choosing j) and $q_j(x) = s_j(x) \cdot M$.

Diversion as LATE

Analogue to LATE Theorem (Imbens Angrist (1994))
Under the following conditions:

(a) Mutually Exclusive and Exhaustive Discrete Choice: $d_{ij} \in \{0,1\}$ and $\sum_{j \in \mathcal{J}} d_{ij} = 1$.

(b) Exclusion: $u_{ik}(p_j, x) = u_{ik}(p'_j, x)$ for all $k \neq j$ and any (p_j, p'_j);

(c) Monotonicity: $u_{ij}(p'_j, x) \leq u_{ij}(p_j, x)$ for all i and any $(p'_j > p_j)$; and

(d) Existence of a first-stage: $d_{ij}(p_j, x) = 1$ and $d_{ij}(p'_j, x) = 0$ for $(p'_j > p_j)$ for some i;

(e) Random Assignment: $(u_{ij}(P_j, x), u_{ik}(P_j, x)) \perp P_j$.

then the Wald estimator

$$\frac{q_k(p'_j, x) - q_k(p_j, x)}{-\Big(q_j(p'_j, x) - q_j(p_j, x)\Big)} = \mathbb{E}[D_{jk,i}(x) | d_{ij}(p_j, x) > d_{ij}(p'_j, x)]$$

Proof in Conlon Mortimer (2021).

Compliance Types

Compliance Type	$(d_{ij}(p_j,x), d_{ij}(p'_j,x))$	Description
Always Takers	$(0,0)$	Don't buy j at either price.
Never Takers	$(1,1)$	Buy j at either price
Compliers	$(1,0)$	Only buy j at lower price $p_j < p'_j$
Defiers	$(0,1)$	Only buy j at higher prices $p'_j > p_j$
Treatment Effects Parameter	**Abbreviation**	**Expression**
Average Treatment Effect	ATE	$\mathbb{E}[D_{jk,i}(x)]$
Average Treatment on the Treated	ATT	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=0]$
Average Treatment on the Untreated	ATUT	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=1]$
Local Average Treatment Effect	LATE	$\mathbb{E}[D_{jk,i}(x)\|d_{ij}(p_j,x)=1, d_{ij}(p'_j,x)=0]$

Diversion as LATE

What is the point?

1. *Ceteris paribus* price change $p_j \to p'_j$ gives the average diversion ratio among the compliers.
2. Compliers buy j at p_j but not at p'_j.
3. Choice of P_j is arbitrary could have been any z_j or $-z_j$ satisfying monotonicity:

$$u_{ij}(z'_j, x) \leq u_{ij}(z_j, x) \text{ for all } i \text{ and any } (z'_j > z_j).$$

4. Under pretty weak assumptions $ATT = \mathbb{E}[D_{jk,i}(x) | d_{ij}(p_j, x) = 0] = \frac{s_{ik}(x)}{1 - s_{ij}(x)}$

Diversion under RC Logit

We can always write any treatment effect parameter (LATE, ATE, ATUT, etc.) as weighted average over individual diversion ratios:

$$\mathbb{E}[D_{jk,i}(x)|?] = \int D_{jk,i}(x) w_{ij}(x) d\,F_i$$

The weights $w_{ij}(x)$ depend on the intervention:

- Price change
- Quality change
- Second-choice/Product Removal

Weighting for Mixed Logit

For mixed logit: $D_{jk,i}(x) = \frac{s_{ik}(x)}{1-s_{ij}(x)}$

	$w_{ij}(x) \propto$	$\widetilde{w}_{ij}(x) \propto$
second choice data	$s_{ij}(x)$	$\frac{s_{ij}(x)}{1-s_{ij}(x)}$
price change $\frac{\partial}{\partial p_j}$	$s_{ij}(x) \cdot (1 - s_{ij}(x)) \cdot \lvert\alpha_i\rvert$	$s_{ij}(x) \cdot \lvert\alpha_i\rvert$
characteristic change $\frac{\partial}{\partial x_j}$	$s_{ij}(x) \cdot (1 - s_{ij}(x)) \cdot \lvert\beta_i\rvert$	$s_{ij}(x) \cdot \lvert\beta_i\rvert$
small quality change $\frac{\partial}{\partial \xi_j}$	$s_{ij}(x) \cdot (1 - s_{ij}(x))$	$s_{ij}(x)$
finite price change $w_i(p_j, p_j', x)$	$\lvert s_{ij}(p_j', x) - s_{ij}(p_j, x)\rvert$	$\frac{\lvert s_{ij}(p_j', x) - s_{ij}(p_j, x)\rvert}{1-s_{ij}(x)}$
finite quality change $w_i(\xi_j, \xi_j', x)$	$\lvert s_{ij}(\xi_j', x) - s_{ij}(\xi_j, x)\rvert$	$\frac{\lvert s_{ij}(\xi_j', x) - s_{ij}(\xi_j, x)\rvert}{1-s_{ij}(x)}$
willingness to pay (WTP)	$= \frac{s_{ij}(x)}{\lvert\alpha_i\rvert \cdot s_{i0}(x)}$	$\frac{s_{ij}(x)}{\lvert\alpha_i\rvert \cdot s_{i0}(x)(1-s_{ij}(x))}$

Allows us to calculate any average diversion ratio we want!

Decomposition