Part 8: Policy Evaluation- Regression Discontinuity

Chris Conlon

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Applied Econometrics

RDD

Regression Discontinuity Design

- Another popular research design is the Regression Discontinuity Design.
- In some sense this is a special case of IV regression. (RDD estimates a LATE).
- Most of this is taken from the JEL Paper by Lee and Lemieux (2010).

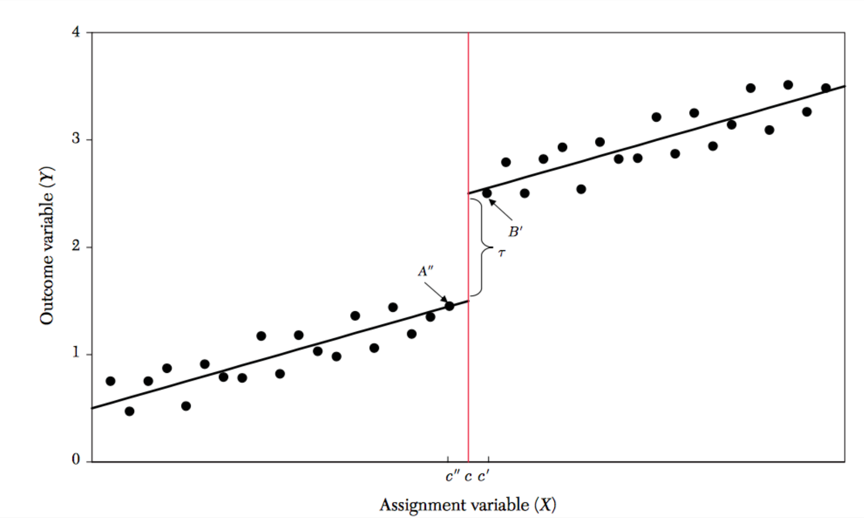
RDD: Basics

- We have a running or forcing variable x such that

$$\lim_{x \to c^+} P(T_i | X_i = x) \neq \lim_{x \to c^-} P(T_i | X_i = x)$$

- The idea is that there is a discontinuous jump in the probability of being treated.
- For now we focus on the sharp discontinuity: $P(T_i | X_i \geq c) = 1$ and $P(T_i | X_i < c) = 0$
- There is no single x for which we observe treatment and control. (Compare to Propensity Score!).
- The most important assumption is that of no manipulability $\tau_i \perp D_i$ in some neighborhood of c.
- Example: a social program is available to people who earned less than $25,000.
    - If we could compare people earning $24,999 to people earning $25,001 we would have as-if random assignment. (MAYBE)
    - But we might not have that many people...

RDD: In Pictures



RDD: Sharp RD Case

RDD uses a set of assumptions distinct from our LATE/IV assumptions. Instead it depends on continuity.

- We need that $E[Y^{(1)}|X]$ and $E[Y^{(0)}|X]$ both be continuous at $X = c$.
- People just to the left of c are a valid control for those just to the right of c.
- This is not a testable assumption $\rightarrow$ draw pictures!
- We could run the regression where $D_i = \mathbf{1}[X_i > c]$.

$$Y_i = \beta_0 + \tau D_i + X_i \beta + \epsilon_i$$

- This puts a lot of restrictions (linearity) on the relationship between Y and X.
- Also (without additional assumptions) we only learn about τ_i at the point $X = c$.

RDD: Nonlinearity

First thing to relax is assumption of linearity.

$$Y_i = f(x_i) + \tau D_i + \epsilon_i$$

This is known as partially linear model.

- Two options for $f(x_i)$:
    1. Kernels: Local Linear Regression
    2. Polynomials: $Y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \cdots + \beta_p x^p + \tau D_i + \epsilon_i$.
        - Actually, people suggest different polynomials on each side of cutoff! (Interact everything with D_i).
- Same objective. Want to flexibly capture what happens on both sides of cutoff.
- Otherwise risk confusing nonlinearity with discontinuity!

RDD: Kernel Boundary Problem

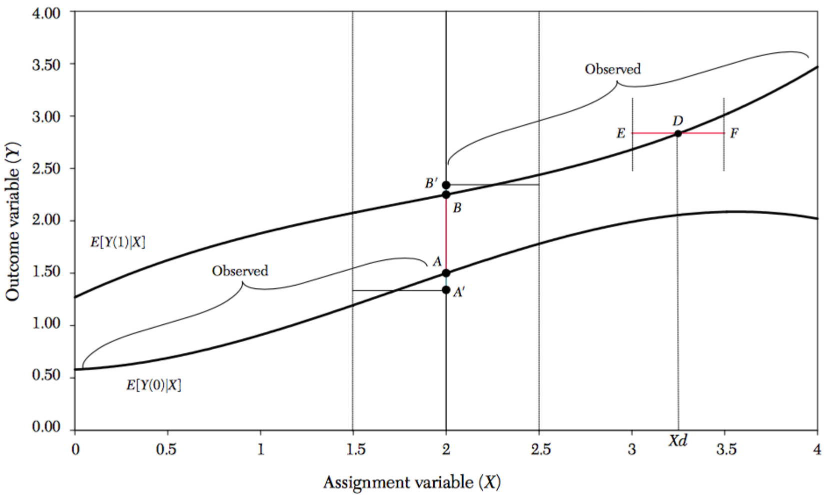


Figure 2. Nonlinear RD

RDD: Polynomial Implementation Details

To make life easier:

- replace $\tilde{x}_i = x_i - c$.
- Estimate coefficients β: $(1, \tilde{x}, \tilde{x}^2, \ldots, \tilde{x}^p)$ and $\tilde{\beta}$: $(D_i, D_i\tilde{x}, D_i\tilde{x}^2, \ldots, D_i\tilde{x}^p)$.
- Now treatment effect at c just the coefficient on D_i. (We can ignore the interaction terms).
- If we want treatment effect at $x_i > c$ then we have to account for interactions.
    - Identification away from c is somewhat dubious.
- Lee and Lemieux (2010) suggest estimating a coefficient on a dummy for each bin in the polynomial regression $\sum_k \phi_k B_k$.
    - Add polynomials until you can satisfy the test that the joint hypothesis test that $\phi_1 = \cdots \phi_k = 0$.
    - There are better ways to choose polynomial order...

RDD: Checklist

Most RDD papers follow the same formula (so should yours)

- Plot of $P(D|X)$ so that we can see the discontinuity
- Plot of $E[Y|X]$ so that we see discontinuity there also
- Plot of $E[W|X]$ so that we don't see a discontinuity in controls.
- Density of X (check for manipulation).
- Show robustness to different "windows"
- The OLS RDD estimates
- The Local Linear RDD estimates
- The polynomial (from each side) RDD estimates
- An f-test of "bins" showing that the polynomial is flexible enough.

Read Lee and Lemieux (2010) before you get started.

Application: Lee (2008)

Looked at incumbency advantage in the US House of Representatives

- Running variable was vote share in previous election
  - Problem of naive approach: good candidates get lots of votes!
  - Compare outcomes of districts with barely D to barely R.
- First we plot bin-scatter plots and quartic (from each side) polynomials.
- Discussion about how to choose bin-scatter bandwidth (CV).

Lee (2008)

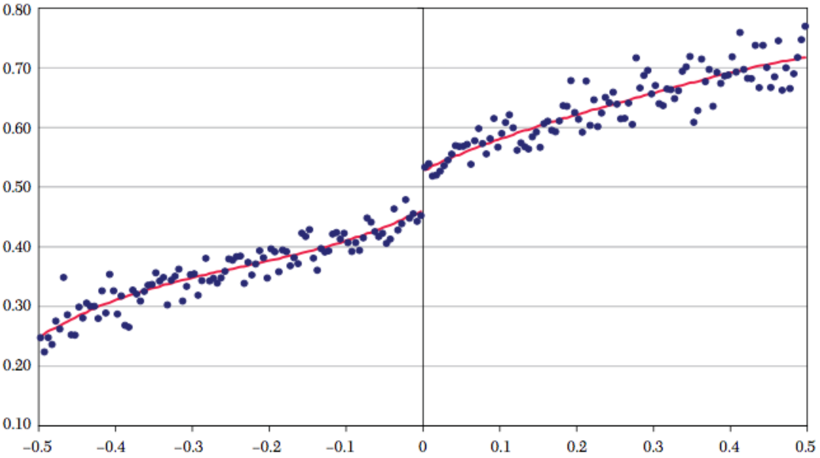


Figure 8. Share of Vote in Next Election, Bandwidth of 0.005 (200 bins)

Lee (2008)

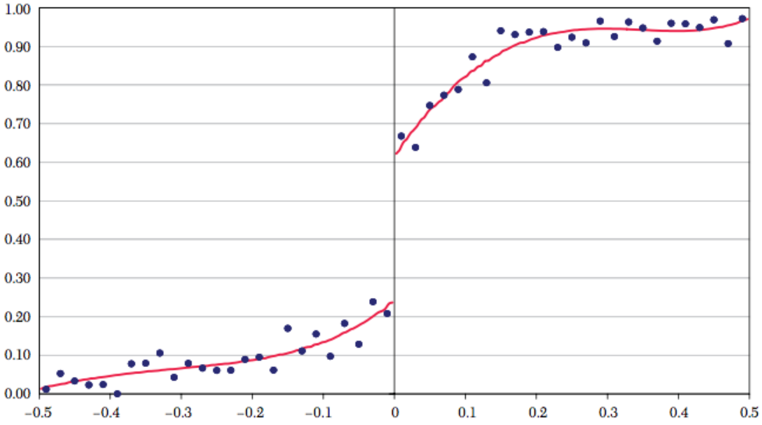


Figure 9. Winning the Next Election, Bandwidth of 0.02 (50 bins)

Lee (2008)

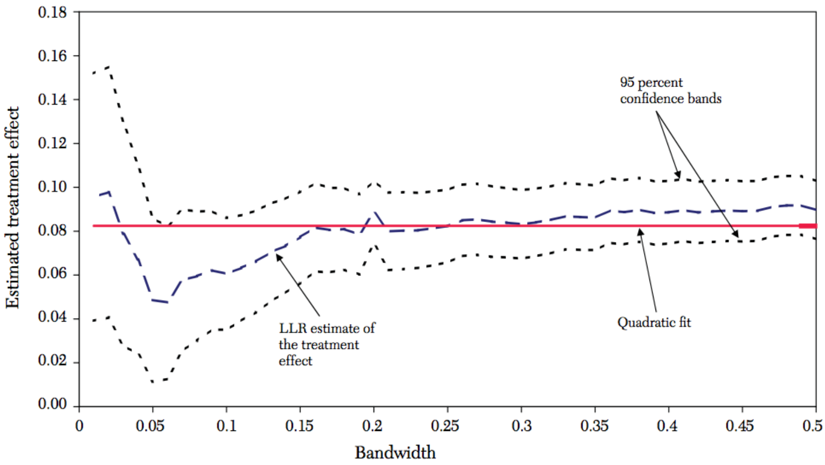


Figure 18. Local Linear Regression with Varying Bandwidth: Share of Vote at Next Election

Other Examples

Luca on Yelp

- Have data on restaurant revenues and yelp ratings.
- Yelp produces a yelp score (weighted average rating) to two decimals ie: 4.32.
- Score gets rounded to nearest half star
- Compare 4.24 to 4.26 to see the impact of an extra half star.
- Now there are multiple discontinuities: Pool them? Estimate multiple effects?

Fuzzy RD

An important extension in the Fuzzy RD. Back to where we started:

$$\lim_{x \to c^+} P(T_i | X_i = x) \neq \lim_{x \to c^-} P(T_i | X_i = x)$$

- We need a discontinuous jump in probability of treatment, but it doesn't need to be $0 \to 1$.

$$\tau_i(c) = \frac{\lim_{x \to c^+} P(Y_i | X_i = x) - \lim_{x \to c^-} P(Y_i | X_i = x)}{\lim_{x \to c^+} P(T_i | X_i = x) - \lim_{x \to c^-} P(T_i | X_i = x)}$$

- Under sharp RD everyone was a complier, now we have some always takers and some never takers too.
- Now we are estimating the treatment effect only for the population of compliers at $x = c$.

Related Idea: Kinks

A related idea is that of kinks.

- Instead of a discontinuous jump in the outcome there is a discontinuous jump in β_i on x_i.
- Often things like tax schedules or government benefits have a kinked pattern.

One quantity to rule them all: MTE

Heckman and Vytlacil provide a unifying non-parametric framework to categorize treatment effects. Their approach is known as the marginal treatment effect or MTE

- The MTE isn't a number it is a function.
- All of the other objects (LATE, ATE, ATT, etc.) can be written as integrals (weighted averages) of the MTE.
- The idea is to bridge the treatment effect parameters (stuff we get from running regressions) and the structural parameters: features of $f(\beta_i)$.

One quantity to rule them all: MTE

- Consider a treatment effect $\beta_i = Y_i(1) - Y_i(0)$.
- Think about a single-index such that $T_i = 1(v_i \leq Z_i'\gamma)$.
- Think about the person for whom $v_i = Z_i'\gamma$ (just barely untreated).

$$\Delta^{MTE}(X_i, v_i) = E[\beta_i | X_i, v_i = Z_i'\gamma]$$

- MTE is average impact of receiving a treatment for everyone with the same $Z'\gamma$.
- For any single index model we can rewrite

$$T_i = 1(v_i \leq Z_i'\gamma) = 1(u_{is} \leq F(Z_i'\gamma)) \text{ for } u_s \in [0, 1]$$

- F is just the cdf of v_i
- Now we can write $P(Z) = Pr(T = 1|Z) = F(Z'\gamma)$.

MTE: Derivation

Now we can write,

$$\begin{aligned} Y_0 &= \gamma_0' X + U_0 \\ Y_1 &= \gamma_1' X + U_1 \end{aligned}$$

$P(T = 1|Z) = P(Z)$ works as our instrument with two assumptions:

1. $(U_0, U_1, u_s) \perp P(Z)|X$. (Exogeneity)
2. Conditional on X there is enough variation in Z for $P(Z)$ to take on all values $\in (0, 1)$.
    - This is much stronger than typical relevance condition. Much more like the special regressor method we will discus next time.

MTE: Derivation

Now we can write,

$$\begin{aligned} Y &= \gamma_0' X + T(\gamma_1 - \gamma_0)' X + U_0 + T(U_1 - U_0) \\ E[Y|X, P(Z) = p] &= \gamma_0' X + p(\gamma_1 - \gamma_0)' X + E[T(U_1 - U_0)|X, P(Z) = p] \end{aligned}$$

Observe $T = 1$ over the interval $u_s = [0, p]$ and zero for higher values of u_s. Let $U_1 - U_0 \equiv \eta$.

$$\begin{aligned} E[T(U_1 - U_0)|P(Z) = p, X] &= \int_{-\infty}^{\infty} \int_0^p (U_1 - U_0) f((U_1 - U_0)|U_s = u_s) du_s d(U_1 - U_0) \\ E[T(\eta)|P(Z) = p, X] &= \int_{-\infty}^{\infty} \int_0^p \eta f(\eta|U_s = u_s) d\eta d u_s \end{aligned}$$

$$\begin{aligned} \Delta^{MTE}(p) &= \frac{\partial E[Y|X, P(Z) = p]}{\partial p} = (\gamma_1 - \gamma_0)' X + \int_{-\infty}^{\infty} \eta f(\eta|U_s = p) d\eta \\ &= (\gamma_1 - \gamma_0)' X + E[\eta|u_s = p] \end{aligned}$$

What is $E[\eta|u_s = p]$? The expected unobserved gain from treatment of those people who are on the treatment/no-treatment margin $P(Z) = p$.

How to Estimate an MTE

Easy

1. Estimate $P(Z) = Pr(T = 1|Z)$ nonparametrically (include exogenous part of X in Z).
2. Nonparametric regression of Y on X and $P(Z)$ (polynomials?)
3. Differentiate w.r.t. $P(Z)$
4. plot it for all values of $P(Z) = p$.

So long as $P(Z)$ covers $(0, 1)$ then we can trace out the full distribution of $\Delta^{MTE}(p)$.

Everything is an MTE

Calculate the outcome given (X,Z) (actually X and $P(Z)=p$).

- ATE : This one is obvious. We treat everyone!

$$\int_{-\infty}^{\infty} \Delta^{MTE}(p) = (\gamma_1 - \gamma_0)'X + \underbrace{\int_{-\infty}^{\infty} E(\eta|u_s)d\,u_s}_{0}$$

- LATE: Fix an X and $P(Z)$ varies from $b(X)$ to $a(X)$ and we integrated over the area between (compliers).

$$LATE(X) = \int_{-\infty}^{\infty} \Delta^{MTE}(p) = (\gamma_1 - \gamma_0)'X + \frac{1}{a(X)-b(X)}\int_{b(X)}^{a(X)} E(\eta|u_s)d\,u_s$$

- ATT

$$TT(X) = \int_{-\infty}^{\infty} \Delta^{MTE}(p)\frac{Pr(P(Z|X)>p)}{E[P(Z|X)]}d\,p$$

- Weights for IV and OLS are a bit more complicated. See the Heckman and Vytlacil paper(s).

Carneiro, Heckman and Vytlacil (AER 2010)

- Estimate returns to college (including heterogeneity of returns).
- NLSY 1979
- $Y = \log(wage)$
- Covariates X: Experience (years), Ability (AFQT Score), Mother's Education, Cohort Dummies, State Unemployment, MSA level average wage.
- Instruments Z: College in MSA at age 14, average earnings in MSA at 17 (opportunity cost), avg unemployment rate in state.

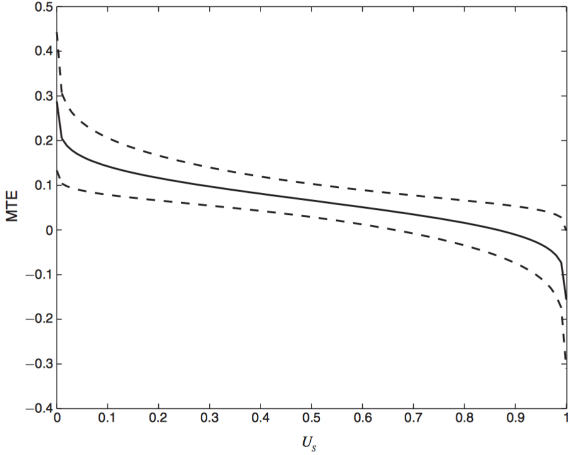


FIGURE 1. MTE ESTIMATED FROM A NORMAL SELECTION MODEL

Notes: To estimate the function plotted here, we estimate a parametric normal selection model by maximum likelihood. The figure is computed using the following formula:

TABLE 4—TEST OF LINEARITY OF $E(Y|\mathbf{X}, P = p)$ USING POLYNOMIALS IN P; AND TEST OF EQUALITY OF LATEs OVER DIFFERENT INTERVALS (H_0: $LATE^j\,(U_S^{L_j}, U_S^{H_j}) - LATE^{j+1}\,(U_S^{L_{j+1}}, U_S^{H_{j+1}}) = 0$)

Panel A. Test of linearity of $E(Y|\mathbf{X}, P = p)$ using models with different orders of polynomials in P[a]

Degree of polynomial for model	2	3	4	5
p-value of joint test of nonlinear terms	0.035	0.049	0.086	0.122
Adjusted critical value	0.057			
Outcome of test	Reject			

Panel B. Test of equality of LATEs (H_0: $LATE^j\,(U_S^{L_j}, U_S^{H_j}) - LATE^{j+1}\,(U_S^{L_{j+1}}, U_S^{H_{j+1}}) = 0$)[b]

Ranges of U_S for LATE^j	(0, 0.04)	(0.08, 0.12)	(0.16, 0.20)	(0.24, 0.28)	(0.32, 0.36)	(0.40, 0.44)
Ranges of U_S for LATE^{j+1}	(0.08, 0.12)	(0.16, 0.20)	(0.24, 0.28)	(0.32, 0.36)	(0.40, 0.44)	(0.48, 0.52)
Difference in LATEs	0.0689	0.0629	0.0577	0.0531	0.0492	0.0459
p-value	0.0240	0.0280	0.0280	0.0320	0.0320	0.0520
Ranges of U_S for LATE^j	(0.48, 0.52)	(0.56, 0.60)	(0.64, 0.68)	(0.72, 0.76)	(0.80, 0.84)	(0.88, 0.92)
Ranges of U_S for LATE^{j+1}	(0.56, 0.60)	(0.64, 0.68)	(0.72, 0.76)	(0.80, 0.84)	(0.88, 0.92)	(0.96, 1)
Difference in LATEs	0.0431	0.0408	0.0385	0.0364	0.0339	0.0311
p-value	0.0520	0.0760	0.0960	0.1320	0.1800	0.2400
Joint p-value	0.0520					

Carneiro, Heckman and Vytlacil

TABLE 5—RETURNS TO A YEAR OF COLLEGE

Model		Normal	Semiparametric
$ATE = E(\beta)$		0.0670	Not identified
		(0.0378)	
$TT = E(\beta \mid S = 1)$		0.1433	Not identified
		(0.0346)	
$TUT = E(\beta \mid S = 0)$		−0.0066	Not identified
		(0.0707)	
MPRTE			
Policy perturbation	Metric		
$Z_\alpha^k = Z^k + \alpha$	$\lvert \mathbf{Z}\gamma - V \rvert < e$	0.0662	0.0802
		(0.0373)	(0.0424)
$P_\alpha = P + \alpha$	$\lvert P - U \rvert < e$	0.0637	0.0865
		(0.0379)	(0.0455)
$P_\alpha = (1 + \alpha)P$	$\lvert \frac{P}{U} - 1 \rvert < e$	0.0363	0.0148
		(0.0569)	(0.0589)
Linear IV (Using $P(\mathbf{Z})$ as the instrument)		0.0951	
		(0.0386)	
OLS		0.0836	
		(0.0068)	

Notes: This table presents estimates of various returns to college, for the semiparametric and the normal selection models: average treatment effect (ATE), treatment on the treated (TT), treatment on the untreated (TUT), and different versions of the marginal policy relevant treat-

Carneiro, Heckman and Vytlacil

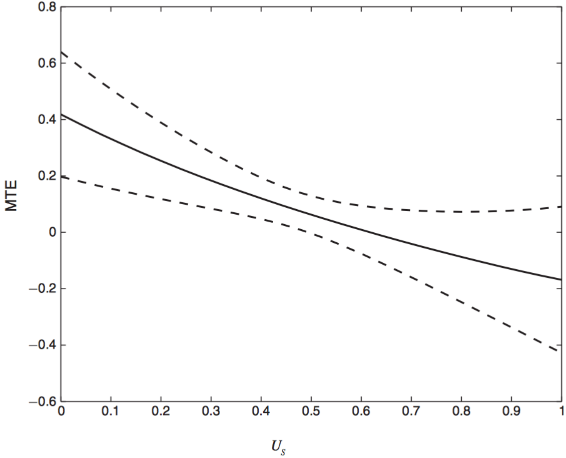


FIGURE 4. $E(Y_1 - Y_0 | \mathbf{X}, U_S)$ WITH 90 PERCENT CONFIDENCE INTERVAL— LOCALLY QUADRATIC REGRESSION ESTIMATES

Notes: To estimate the function plotted here, we first use a partially linear regression of log wages on polynomials

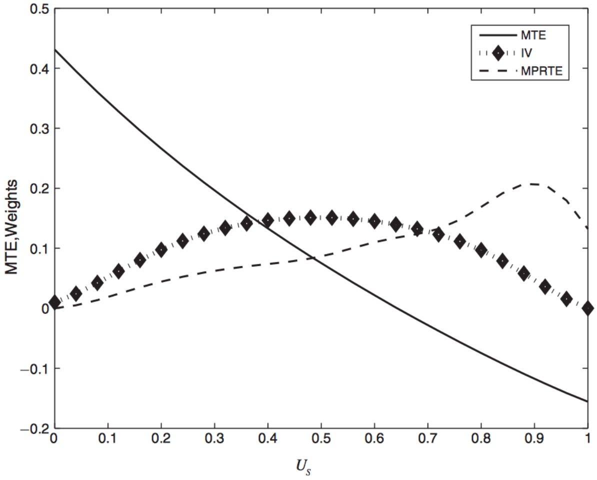


FIGURE 6. WEIGHTS FOR IV AND MPRTE

Note: The scale of the *y*-axis is the scale of the MTE, not the scale of the weights, which are scaled to fit the picture.

Diversion Example

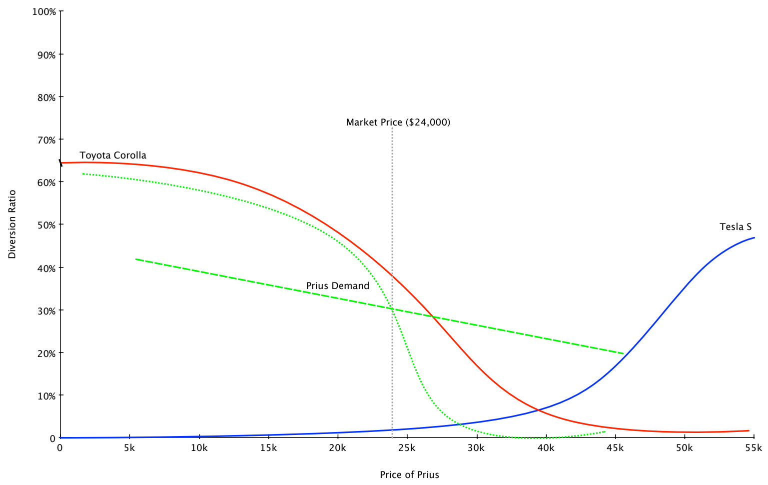
I have done some work trying to bring these methods into merger analysis.

- Key quantity: Diversion Ratio as I raise my price, how much do people switch to a particular competitor's product

$$D_{jk}(p_j, p_{-j}) = \left| \frac{\partial q_k}{\partial p_j}(p_j, p_{-j}) / \frac{\partial q_j}{\partial p_j}(p_j, p_{-j}) \right|$$

- We hold p_{-j} fixed and trace out $D_{jk}(p_j)$.
- The treatment is leaving good j.
- The Y_i is increased sales of good k.
- The Z_i is the price of good j.
- The key is that all changes in sales of k come through people leaving good j (no direct effects).

Diversion for Prius (FAKE!)



Diversion Example

$$\widehat{D_{jk}^{LATE}} = \frac{1}{\Delta q_j} \int_{p_j^0}^{p_j^0+\Delta p_j} \underbrace{\frac{\partial q_k(p_j, p_{-j}^0)}{\partial q_j}}_{\equiv D_{jk}(p_j, p_{-j}^0)} \left| \frac{\partial q_j(p_j, p_{-j}^0)}{\partial p_j} \right| dp_j$$

- $D_{jk}(p_j, p_{-j}^0)$ is the MTE.
- Weights $w(p_j) = \frac{1}{\Delta q_j} \frac{\partial q_j(p_j, p_{-j}^0)}{\partial p_j}$ correspond to the lost sales of j at a particular p_j as a fraction of all lost sales.
- When is $LATE \approx ATE$?
  - Demand for Prius is steep: everyone leaves right away
  - $D_{j,k}(p_j)$ is relatively flat.
  - We might want to think about raising the price to choke price (or eliminating the product from the consumers choice set) same as treating everyone!