Lecture 4: Bayesian Analysis

Chris Conlon

NYU Stern

February 24, 2019

- Suppose that we toss a coin several times with $x_i \in \{H, T\} = \{1, 0\}$
- $\mathbf{X} = \{H, T, H, H, \dots\}$.
- Suppose that the probability of heads $Pr(x_i = H) = p$.
- What is the likelihood of an observed sequence of $\mathbf{X}$? where x_i are I.I.D.

$$Pr(x_i|p) = p^{x_i}(1-p)^{1-x_i}$$
$$Pr(\mathbf{X}|p) = p^{\sum_i x_i}(1-p)^{\sum_i (1-x_i)}$$

Introduction: MLE for coin toss

- Can construct the log likelihood

$$\ell(\mathbf{X}|p) = \left(\sum_i x_i\right) \ln p + \left(N - \sum_i x_i\right) \ln(1-p)$$

$$\frac{\partial \ell(p)}{\partial p} = \left(\sum_i x_i\right) \frac{1}{p} - \left(N - \sum_i x_i\right) \frac{1}{1-p} = 0$$

$$\frac{1-p}{p} = \frac{\left(\frac{1}{N} \cdot N - \frac{1}{N} \cdot \sum_i x_i\right)}{\frac{1}{N} \cdot \sum_i x_i} \rightarrow \hat{p} = \frac{1}{N} \cdot \sum_i x_i$$