NYU, Tandon School of Engineering
Extended Bridge to CS — Summer 2022 yl10952

Homework #2
Due by Friday 1/21 11:59pm

Submission instructions:

1. For this assignment you should turn in 5 files:
   - Four '.cpp' files, one for each question 1 to 4.
     Name your files 'YourNetID_hw2_q1.cpp', 'YourNetID_hw2_q2.cpp', etc.
   - A '.pdf' file with your answers for questions 5-9. **Each question should start on a new page!** Name your file 'YourNetID_hw2_q5to9.pdf'

2. **Typing your solutions would grant you 5 extra points.**

3. **You should submit your homework in the Gradescope system.** Note that when submitting the pdf file, you would be asked to assign the pages from your file to their corresponding questions.

4. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**

5. For the coding questions, pay special attention to the style of your code. Indent your code correctly, choose meaningful names for your variables, define constants where needed, etc.

6. For the math questions, you are expected to justify all your answers, not just to give the final answer (unless explicitly asked to).
   As a rule of thumb, for questions taken from zyBooks, the format of your answers should be like the format demonstrated in the sample solutions we exposed.

Question 5:

a) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.12.2, sections b e

B.

$p \rightarrow (q \wedge r)$

$\neg q$

$\therefore \neg p$

1	$p \rightarrow (q \wedge r)$	Hypothesis
2	$p \rightarrow q$	Simplification, 1
3	$\neg q$	Hypothesis
4	$\neg p$	Modus tollens, 2, 3

E.

$p \vee q$

$\neg p \vee r$

$\neg q$

$\therefore r$

1	$p \vee q$	Hypothesis
2	$\neg p \vee r$	Hypothesis
3	$q \vee r$	Resolution, 1, 2
4	$\neg q$	Hypothesis
5	r	Disjunctive syllogism, 3, 4

2. Exercise 1.12.3, section c

c.

$p \lor q$
$\neg p$

$\therefore q$

1	$p \lor q$	Hypothesis
2	$\neg(\neg p) \lor q$	Double negation, 1
3	$\neg p \to q$	Conditional identity, 2
4	$\neg p$	Hypothesis
5	q	Modus ponens, 3, 4

3. Exercise 1.12.5, sections c, d

C:

I will buy a new car and a new house only if I get a job.
I am not going to get a job.

∴ I will not buy a new car.

j: I will get a job
c: I will buy a new car
h: I will buy a new house

The form of the argument is：

$$\begin{array}{l} (c \land h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

The argument is not valid.
When c = T, and h = j = F,
The hypotheses are both true and the conclusion ¬c is false.

D。

I will buy a new car and a new house only if I get a job.
I am not going to get a job.
I will buy a new house.
∴ I will not buy a new car.

j: I will get a job c: I will buy a new car h: I will buy a new house

The form of the argument is

$$
\begin{array}{l}
(c \land h) \to j \\
\neg j \\
h \\
\hline
\therefore \neg c
\end{array}
$$

The argument is valid.

1	$(c \land h) \to j$	Hypothesis
2	$\neg j$	Hypothesis
3	$\neg(c \land h)$	Modus tollens 1， 2
4	$\neg c \lor \neg h$	De morgans law 3
5	$\neg h \lor \neg c$	Commutative law 4
6	h	Hypothesis
7	$\neg\neg h$	Double negation law,6
8	$\neg c$	Disjunctive syllogism,57

Logic Proof 2:

1. $(c \land h) \to j$	Hypothesis
2. $\neg(c \land h) \lor j$	Conditional identity, 1
3. $j \lor \neg(c \land h)$	Commutative law, 2
4. $\neg j$	Hypothesis
5. $\neg(c \land h)$	Disjunctive syllogism, 3, 4
6. $\neg c \lor \neg h$	De morgans law, 5
7. $\neg h \lor \neg c$	Commutative law, 6
8. h	Hypothesis
9. $\neg\neg h$	Double negation law, 8
10. $\neg c$	Disjunctive syllogism, 7, 9

b) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.13.3, section b

$\exists x\ (P(x) \lor Q(x))$
$\exists x\ \neg Q(x)$

$\therefore \exists x\ P(x)$

	P	Q
a	F	T
b	F	F

The hypothesis $\exists x\ (P(x) \lor Q(x))$ is true when x = a; because Q(x) is T.
the hypothesis $\exists x\ \neg Q(x)$ is true when x = b, because $\neg Q(x)$ is T.
However, then P(a) , P(b) are all F, so the conclusion is false.

2. Exercise 1.13.5, sections d, e

D.

Every student who missed class got detention.
Penelope is a student in the class.
Penelope did not miss class.

Penelope did not get a detention

$M(x)$: x missed class $D(x)$: x got a detention

The form of the argument is

$$\begin{array}{l} \forall x\,(M(x) \rightarrow D(x)) \\ \text{Penelope , a student in the class} \\ \neg M(\text{Penelope}) \\ \hline \therefore \neg D(\text{Penelope}) \end{array}$$

The argument is invalid.
Hypothesis: Penelope is a student in the class.
and let M(Penelope) = F and D(Penelope) = T. Then the hypotheses are all true and the conclusion is false.

Penelope didn't miss the class, and could have got a detention.

e)

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

Penelope did not get a detention.

M(x): x missed class
A(x): x received an A
D(x): x got a detention

The form of the argument is

$\forall x\,((M(x) \vee D(x)) \rightarrow \neg A(x))$
Penelope , a student in the class
A(Penelope)

$\therefore \neg D(\text{Penelope})$

The argument is valid.

1	Penelope is a student in the class	Hypothesis
2	$\forall x\,(M(x) \vee D(x) \rightarrow \neg A(x))$	Hypothesis
3	$M(\text{Penelope}) \vee D(\text{Penelope}) \rightarrow \neg A(\text{Penelope})$	Universal instantiation, 1,2
4	A(Penelope)	Hypothesis
5	$\neg(\neg A(\text{Penelope}))$	Double negation law, 4
6	$\neg(M(\text{Penelope}) \vee D(\text{Penelope}))$	Modus tollens, 3, 5
7	$\neg M(\text{Penelope}) \wedge \neg D(\text{Penelope})$	De Morgan's law, 6
8	$\neg D(\text{Penelope}) \wedge \neg M(\text{Penelope})$	Commutative Law , 7
9	$\neg D(\text{Penelope})$	implication, 7

Question 6:

Solve Exercise 2.4.1, section d;
Prove each of the following statements using a direct proof
D)The product of two odd integer is an odd integer.

Proof. Let x and y be two odd integers.

We should prove that xy is an odd integer.
Since x is odd, there is an integer m such that $x = 2m + 1$.
Since y is odd, there is an integer n such that $y = 2n+1$.

$xy = (2m + 1)(2n + 1)$
$= 4mn + 2m + 2n + 1$
$= 2(2mn + m + n) + 1$

Since m and n are integers, $2mn + m + n$ is also an integer.
As $xy = 2k + 1$, where $k = 2mn + m + n$ is an integer, xy is odd

Exercise 2.4.3, section b, from the Discrete Math zyBook:

B.
If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$

Proof. Let x be a real number and we shall prove $12 - 7x + x^2 \geq 0$

$12 - 7x + x^2$ can be broken down into $(x - 3)(x - 4)$.

This gives $$(x - 3)(x - 4) \geq 0$$

Since $x \leq 3$, we can subtract 3 from both sides of the inequality to get

$$(x - 3) \leq 0$$

Subtracting 1 from both sides of the inequality gives

$$(x - 4) \leq -1$$

Since $(x - 3)$ and $(x - 4)$ are both negative real numbers,
product of two negative numbers is at least greater than zero.
Therefore, $$12 - 7x + x^2 \geq 0$$

Question 7:

Solve Exercise 2.5.1, section d;

D)

For every integer n if $n^2 - 2n + 7$ is even then n is odd.

Proof. Assume n is an even integer and we will show $n^2 - 2n + 7$ is odd.
So $n = 2k$ for some integer k

$$\begin{aligned} n^2 - 2n + 7 &= (2k)^2 - 2(2k) + 7 \\ &= 4k^2 - 4k + 7 \\ &= 2(2k^2 - 2k + 3) + 1 \end{aligned}$$

Since k is a integer, $2k^2 - 2k + 3$ is also an integer.
Therefore, $n^2 - 2n + 7$,
where $m = 2k^2 - 2k + 3$ is an integer.
We can say $n^2 - 2n + 7$ is an odd integer

■

Exercise 2.5.4, sections a, b;

a.)

For every pair of real number x and y, if $x^3 + xy^2 \leq x^2 y + y^3$ then $x \leq y$

Proof. Assume for every pair of real number x and y, and $x > y$ we will show that $x^3 + xy^2 > x^2 y + y^3$

Since x and y are real numbers and $x > y$, either $x or\ y \neq 0$.

Thus,$x2 + y2 \neq 0$ and $x2 + y2 > 0$.

Multiply both side of the equation $x > y$ by $x2 + y2$ gives

$x * (x2 + y2) > y * (x2 + y2)$

$x^3 + xy^2 > x^2 y + y^3$

$x^3 + xy^2 > x^2 y + y^3$

Since $x2 + y2$ must be positive, the inequality holds. ■

b) For every pair of real number x and y, if $x + y > 20$ then $x > 10$ or $y > 10$

Proof. Assume for every pair of real number x, y and $x \leq 10$, and $y \leq 10$.
We will show that $x + y \leq 20$

We assign x and y to their maximum value and we get

$x = 10$
$y = 10$

we can rewrite $x + y \leq 20$:

$x + y \leq 20$
$10 + 10 \leq 20$
$20 \leq 20$

Exercise 2.5.5, section c, from the Discrete Math zyBook:

c)
For every non -zero number x, if x is irrational, then $\frac{1}{x}$ is also irrational

Proof. Let x be non-zero real number.

Assume $\frac{1}{x}$ is not an irrational number. we will show x is not irrational

Since every real number is either rational or irrational,

$\frac{1}{x}$ is a real number and not irrational,

then $x \neq 0$ so $\frac{1}{x}$ is rational.

Thus $\frac{1}{x} = \frac{b}{a}$ where a and b is an integer and $b \neq 0$

Switching reciprocal on both side $x = \frac{b}{a}$ is also rational.

Question 8:

Solve Exercise 2.6.6, sections c, d, from the Discrete Math zyBook:

C) The average of three real numbers are greater than or equal to at least one of the numbers.

Proof. Let x, y, z be real number.
Assume average of these three number is less than all of the three numbers

$$x\frac{x+y+z}{3} < y\frac{x+y+z}{3} < z\frac{x+y+z}{3} < x + y + z$$
$$\frac{3x+3y+3z}{3} < x + y + z$$

$Divide\ by$ 3 = $x + y + z < x + y + z$
= $x + y + z < x + y + z$

Since sum of three number is not less than the sum, this contradict the assumption.

The average of three real number is greater than or equal to at least one of the number.

■

d) There is no smallest integer

Proof. Assume there is smallest integer x

when we subtract x by 1 = $x - 1 < x$

This contradict assume that x is the smallest integer ■

Question 9:

Solve Exercise 2.7.2, section b, from the Discrete Math zyBook:

b)
If integers x and y have the same parity, then $x + y$ is even.
The parity of a number tells whether the number is odd or even.
If x and y have the same parity they are either both even or both odd.

Proof. Proof by case
Case 1: x and y are both even. Thus $x = 2m$ and $y = 2n$ for some integers of m and n

We have : $x + y = 2m + 2n = 2(m + n)$

Since m and n are both integer, $m + n$ is also an integer.
And $x + y$ can be expressed as 2 times an integer, which makes $x + y$ even

Case 2: x and y are both odd. Thus $x = 2m + 1$ and $y = 2n + 1$ for some integers of m and n

We have : $x + y = (2m + 1) + (2n + 1) = 2(m + n + 1)$

Since m and n are both integers, $m + n + 1$ is also an integer.
And $x + y$ can be expressed as 2 times an integer, which makes $x + y$ even

■