NYU, Tandon School of Engineering
Extended Bridge to CS — Summer 2021

Homework #1
Due by Friday 7/16 11:55pm

Submission instructions:

1. For this assignment you should turn in a '.pdf' file with your answers.
   Name your file 'YourNetID_hw1.pdf'

2. **Each question should start on a new page.**

3. **Typing your solutions would grant you 5 extra points.**

4. **You should submit your homework in the Gradescope system.** Note that when submitting the pdf file, you would be asked to assign the pages from your file to their corresponding questions.

5. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**

6. **You are expected to justify all your answers (not just to give the final answer).** As a rule of thumb, for questions taken from zyBooks, the format of your answers, should be like the format demonstrated in the sample solutions we exposed.

Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 =$

$1 \cdot 2^{0} + 1 \cdot 2^{1} + 0 \cdot 2^{2} + 1 \cdot 2^{3} + 1 \cdot 2^{4} + 0 \cdot 2^{5} + 0 \cdot 2^{6} + 1 \cdot 2^{7} =$
155_{10}

2. $456_7 = 6 \cdot 7^{0} + 5 \cdot 7^{1} + 4 \cdot 7^{2} = 237_{10}$
3. $38A_{16} = 10 \cdot 16^{0} + 8 \cdot 16^{1} + 3 \cdot 16^{2} = 906_{10}$
4. $2214_5 = 4 \cdot 5^{0} + 1 \cdot 5^{1} + 2 \cdot 5^{2} + 2 \cdot 5^{3} = 309_{10}$

B. Convert the following numbers to their binary representation:

1. $69_{10} = 1000101_2$
2. $485_{10} = 111100101_2$
3. $6D1A_{16} = 0110110100011010_2$

C. Convert the following numbers to their hexadecimal representation:

1. $1101011_2 = 6b_{16}$
2. $895_{10} = 37f_{16}$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

$$
\begin{array}{r}
{\scriptstyle 1\ 1\ 1\ 1} \\
7566_8 \\
+ \quad 4515_8 \\
\hline
14303_8
\end{array}
$$

2. $10110011_2 + 1101_2 = 11000000_2$

$$
\begin{array}{r}
{\scriptstyle 1\,1\,1\,1\,1\ 1} \\
10110011_2 \\
+ \quad 1101_2 \\
\hline
11000000_2
\end{array}
$$

3. $7A66_{16} + 45C5_{16} = c02b_{16}$

$$
\begin{array}{r}
{\scriptstyle 1\,1} \\
7A66_{16} \\
+ \quad 45C5_{16} \\
\hline
c02b_{16}
\end{array}
$$

4. $3022_5 - 2433_5 = 3\ 4_5$

$$
\begin{array}{r}
{\scriptstyle 4\ 6} \\
{\scriptstyle 2\ \cancel{5}\ \cancel{1}\,7} \\
\cancel{3}0\cancel{2}2_5 \\
- \quad 2433_5 \\
\hline
3\ 4_5
\end{array}
$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} = (01111100)_{8\text{ bit 2's comp}}$

			1	1	1	1	1	0	0
2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
||	||	||	||	||	||	||	||	||	||
512	256	128	64	32	16	8	4	2	1

124	60	28	12	4
- 64	- 32	- 16	- 8	- 4
-----------------------	-----------------------	-----------------------	-----------------------	-----------------------
60	28	12	4	0

2. $-124_{10} = (10000100)_{8\text{ bit 2's comp}}$

```
 11 1 1 1 1
 01111100
+10000100
------------------------------
100000000
```

3. $109_{10} = (01101101)_{8\text{ bit 2's comp}}$

1	1	1	1	1	0	0			
2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
||	||	||	||	||	||	||	||	||	||
512	256	128	64	32	16	8	4	2	1

124	60	28	12	4
- 64	- 32	- 16	- 8	- 4
-----------------------	-----------------------	-----------------------	-----------------------	-----------------------
60	28	12	4	0

4. $-79_{10} = (10110001)_{8\text{ bit 2's comp}}$

```
 11  1 1 1 1 1 1
  01001111
+ 10110001
------------------------------
100000000
```

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}} = 30_{10}$

2. $11100110_{8\text{ bit 2's comp}} = -26_{10}$

1 1 1 1 1 1 1

11100110

\+ 00011010

—------------------------------

100000000

$00011010 = 26_{10}$

3. $00101101_{8\text{ bit 2's comp}} = 45_{10}$

4. $10011110_{8\text{ bit 2's comp}} = -98_{10}$

11 1 1 1 1 1

10011110

\+ 01100010

—------------------------------

100000000

$01100010 = 98_{10}$

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

$\neg(p \lor q)$

p	q	$\neg(p \lor q)$
T	T	F
T	F	F
F	T	F
F	F	T

(c) $r \lor (p \land \neg q)$

p	q	r	$r \lor (p \land \neg q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	F
F	F	T	T
F	F	F	F

2. Exercise 1.3.4, sections b, d

(b) $(p \to q) \to (q \to p)$

p	q	$(p \to q) \to (q \to p)$
T	T	T
T	F	T
F	T	F
F	F	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T
T	F	T
F	T	T
F	F	T

Question 5:
Solve the following questions from the Discrete Math
zyBook:
1. Exercise 1.2.7, sections b, c

(b)The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
$(B \land D) \lor (B \land M) \lor (D \land M)$

(c)Applicant must present either a birth certificate or both a driver's license and a marriage license.
$B \lor (D \land M)$

2. Exercise 1.3.7, sections b – e

(b) $(s \lor y) \rightarrow p$

A person can park in the school parking lot if they are a senior or at least seventeen years of age.

(c) $p \rightarrow y$

Being 17 years of age is a necessary condition for being able to park in the school parking lot.

(d)$p \leftrightarrow (s \land y)$

A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

(e)$p \rightarrow (s \lor y)$

Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

3. Exercise 1.3.9, sections c, d

(c) $c \rightarrow p$

The applicant can enroll in the course only if the applicant has parental permission.

(d)$c \rightarrow p$

Having parental permission is a necessary condition for enrolling in the course.

<u>Question 6:</u>

Solve the following questions from the Discrete Math
zyBook:

1. Exercise 1.3.6, sections b - d

(b)

Maintaining a B average is necessary for Joe to be eligible for the honors program.

: If Joe is eligible for the honors program, then he is maintaining a B average.

(c)

Rajiv can go on the roller coaster only if he is at least four feet tall.

: If Rajiv can go on the roller coaster, then he is at least four feet tall.

(d)

Rajiv can go on the roller coaster if he is at least four feet tall.

: If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

(c) $(p \lor r) \leftrightarrow (q \land r)$

r	$(p \lor r) \leftrightarrow (q \land r)$
T	F
F	F

$(p \lor r) \leftrightarrow (q \land r)$ is false.

(d)$(p \land r) \leftrightarrow (q \land r)$

r	$(p \land r) \leftrightarrow (q \land r)$
T	F
F	T

$(p \land r) \leftrightarrow (q \land r)$ is unknown, depends on r's value.

(e)$p \to (r \lor q)$

r	$p \to (r \lor q)$
T	T
F	F

$p \to (r \lor q)$ is unknown, depends on r's value.

(f)$(p \land q) \to r$

r	$(p \land q) \to r$
T	T
F	T

$(p \land q) \to r$ is true.

Question 7:

Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

(b) $\neg j \rightarrow (l \lor \neg r)$

$(r \lor \neg l) \rightarrow j$

If Sally did not get the job, then she was late for her interview or did not update her resume.

If Sally updated her resume and was not late for her interview, then she got the job.

(c) $j \rightarrow \neg l$

$\neg j \rightarrow l$

If Sally got the job then she was not late for her interview.

If Sally did not get the job, then she was late for her interview.

(d) $(r \lor \neg l) \rightarrow j$

$j \rightarrow (r \lor \neg l)$

If Sally updated her resume or she was not late for her interview, then she got the job.

If Sally got the job, then she updated her resume and was not late for her interview.

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2, sections c, f, i

(c)$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$
$(\neg p \lor q) \land (\neg p \lor r)$
$\neg p \lor (q \land r)$
$p \to (q \land r)$

(f)$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$	De Morgan's laws
$\neg p \land \neg(\neg p \land q)$	De Morgan's laws
$\neg p \land (\neg\neg p \lor \neg q)$	Double negation law
$\neg p \land (p \lor \neg q)$	Distributive laws
$(\neg p \land p) \lor (\neg p \land \neg q)$	Complement laws
$F \lor (\neg p \land \neg q)$	Identity laws
$\neg p \land \neg q$	

(i)$(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$	Conditional identities
$\neg(p \land q) \lor r$	De Morgan's laws
$(\neg p \lor \neg q) \lor r$	Commutative laws
$(\neg q \lor \neg p) \lor r$	Associative laws
$\neg q \lor (\neg p \lor r)$	Associative laws
$(\neg p \lor r) \lor \neg q$	Conditional identities
$\neg(\neg p \lor r) \to \neg q$	De Morgan's laws

$(p \land \neg r) \rightarrow \neg q$	

2. Exercise 1.5.3, sections c, d

(c) $\neg r \lor (\neg r \rightarrow p)$

r	p	$\neg r \lor (\neg r \rightarrow p)$
T	F	T
F	T	T

$\neg r \lor (\neg r \rightarrow p)$ $\neg r \lor (\neg\neg r \lor p)$
$\neg r \lor (r \lor p)$
$(\neg r \lor r) \lor p$
$(r \lor \neg r) \lor p$
$T \lor p$
$p \lor T$
T

(d) $\neg(p \rightarrow q) \rightarrow \neg q$

p	q	$\neg(p \rightarrow q) \rightarrow \neg q$
T	F	T
F	T	T

$\neg(p \rightarrow q) \rightarrow \neg q$ $\neg(\neg p \lor q) \rightarrow \neg q$

$\neg\neg(\neg p \lor q) \lor \neg q$

$(\neg p \lor q) \lor \neg q$

$\neg p \lor (q \lor \neg q)$

$\neg p \lor T$

T

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3, sections c, d

(c) $\exists x(x = x^2)$

There is a number that is equal to its square.

(d) $\forall x(x \leq x^2 + 1)$

Every number is less than or equal to its square plus 1.

2. Exercise 1.7.4, sections b - d

(b) $\forall x(\neg S(x) \land W(x))$

Everyone was well and went to work yesterday.

(c) $\forall x(S(x) \land \neg W(x))$

Everyone who was sick yesterday did not go to work.

(d) $\exists x(S(x) \land W(x))$

Yesterday someone was sick and went to work.

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9, sections c - i

(c) $\exists x((x = c) \rightarrow P(x))$	F
(d) $\exists x(Q(x) \land R(x))$	T
(e) $Q(a) \land P(d)$	T
(f) $\forall x ((x \neq b) \rightarrow Q(x))$	T
(g) $\forall x (P(x) \lor R(x))$	F
(h) $\forall x (R(x) \rightarrow P(x))$	T
(i) $\exists x(Q(x) \lor R(x))$	T

	P(x)	Q(x)	R(x)
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

2. Exercise 1.9.2, sections b - i

(b) $\exists x\ \forall y\ Q(x, y)$

T

(c) $\exists y\ \forall x\ P(x, y)$

T

(d) $\exists x\ \exists y\ S(x, y)$

F

(e) $\forall x\ \exists y\ Q(x, y)$

F

(f) $\forall x\ \exists y\ P(x, y)$

T

(g) $\forall x\ \forall y\ P(x, y)$

F

(h) $\exists x\ \exists y\ Q(x, y)$

T

(i) $\forall x\ \forall y\ \neg S(x, y)$

T

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4, sections c - g

(c) $\exists x \exists y(x + y = xy)$

There are two numbers whose sum is equal to their product.

(d) $\forall x \forall y(((x > 0) \wedge (y > 0)) \rightarrow (x/y > 0))$

The ratio of every two positive numbers is also positive.

(e) $\forall x(((x > 0) \wedge (x < 1)) \rightarrow (1/x > 1))$

The reciprocal of every positive number less than one is greater than one.

(f) $\neg \exists x \forall y(x < y)$

There is no smallest number.

(g) $\forall x \exists y((x\ 6= 0) \rightarrow (xy = 1))$

Every number other than 0 has a multiplicative inverse.

2. Exercise 1.10.7, sections c - f

(c) $\forall x \exists y((x \ 6= 0) \to (xy = 1))$

There is at least one new employee who missed the deadline.

(d) $\forall y(D(y) \to P(Sam, y))$

Sam knows the phone number of everyone who missed the deadline.

(e) $\forall y(D(y) \to P(Sam, y))$

There is a new employee who knows everyone's phone number.

(f) $\exists x \forall y((N(x) \land D(x)) \land (((x \ 6= y) \land N(y)) \to \neg D(y)))$

Exactly one new employee missed the deadline.

3. Exercise 1.10.10, sections c – f

(c) $\forall x \exists y(T(x, y) \land (y \ 6= M\ ath101))$

Every student has taken at least one class other than Math 101.

(d) $\exists x \forall y((y \ 6= M\ ath101) \to T(x, y))$

There is a student who has taken every math class other than Math 101.

(e) $\forall x \exists y \exists z(((x \ 6= Sam)) \to ((y \ 6= z) \land T(x, y) \land T(x, z)))$

Everyone other than Sam has taken at least two different math classes.

(f) $\forall x \exists y \exists z(((x \ 6= Sam)) \to ((y \ 6= z) \land T(x, y) \land T(x, z)))$

Sam has taken exactly two math classes.

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2, sections b – e

(b)Every patient was given the medication or the placebo or both.

$\forall x(D(x) \lor P(x))$

$\neg\forall x(D(x) \lor P(x))$

$\exists x(\neg D(x) \land \neg P(x))$

Every patient was either not given the medication or not given placebo.

(c)There is a patient who took the medication and had migraines.

$\exists x(D(x) \land M(x))$

$\neg\exists x(D(x) \land M(x))$

$\forall x(\neg D(x) \lor \neg M(x))$

Every patient was not given the medication or not had migraines or both.

(d)Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \rightarrow q \equiv \neg p \lor q$.)

$\forall x(P(x) \rightarrow M(x))$

$\neg\forall x(P(x) \rightarrow M(x))$

$\exists x(P(x) \land \neg M(x))$

There is a patient who took the placebo and not had migraines.

(e)There is a patient who had migraines and was given the placebo.

$\exists x(M(x) \land P(x))$
$\neg\exists x(M(x) \land P(x))$
$\forall x(\neg M(x) \lor \neg P(x))$

Every patient did not had migraines or did not given placebo or both.

2. Exercise 1.9.4, sections c - e

(c) $\exists x\ \forall y\ (P(x, y) \to Q(x, y))$

$\forall x \exists y (P(x, y)\ \land\ \neg Q(x, y))$

(d) $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

$\forall x \exists y ((P(x, y)\ \land\ \neg P(y, x))\ \lor\ (P(y, x)\ \land\ \neg P(x, y)))$

(e) $\exists x\ \exists y\ P(x, y)\ \land\ \forall x\ \forall y\ Q(x, y)$

$\forall x \forall y \neg P(x, y)\ \lor\ \exists x \exists y \neg Q(x, y)$