

Homework #5
Due by Friday 8/13 11:59pm

Submission instructions:

1. For this assignment, you should turn in 3 files:
 - Two '.cpp' files, one for each question 1 and 2.
Name your files 'YourNetID_hw5_q1.cpp', and 'YourNetID_hw5_q2.cpp'.
 - A '.pdf' file with your answers for questions 3-5.
Each question should start on a new page!
Name your file 'YourNetID_hw5_q3to5.pdf'
2. You must type all your solutions. We will take off points for submissions that are handwritten.
3. **You should submit your homework in the Gradescope system.**
4. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**
5. Pay special attention to the style of your code. Indent your code correctly, choose meaningful names for your variables, define constants where needed, choose the most appropriate control flow statements, etc.
6. For the math questions, you are expected to justify all your answers, not just to give the final answer (unless explicitly asked to).
As a rule of thumb, for questions taken from zyBooks, the format of your answers, should be like the format demonstrated in the sample solutions we exposed.

Question 3:

Solve the following questions from the Discrete Math zyBook:

A.Exercise 4.1.3, sections b, c

$$(b) f(x) = 1/(x^2 - 4)$$

Here f is not a function, when $x=2$ or $x=-2$, The function f is not well-defined for $f=1/0$.

$$(c) f(x) = \sqrt{x^2}$$

Here f is a function, The function is well defined. The range is $\{0, \mathbb{R}^+\}$

B. Exercise 4.1.5, sections b, d, h, i, l

(b) Let $A = \{2, 3, 4, 5\}$. $f: A \rightarrow \mathbb{Z}$ such that $f(x) = x^2$.

$\{4, 9, 16, 25\}$

(d) $f: \{0,1\}^5 \rightarrow \mathbb{Z}$. For $x \in \{0,1\}^5$, $f(x)$ is the number of 1's that occur in x .

$\{0, 1, 2, 3, 4, 5\}$

(h) Let $A = \{1, 2, 3\}$. $f: A \times A \rightarrow \mathbb{Z} \times \mathbb{Z}$, where $f(x,y) = (y, x)$.

$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Range = $\{(1, 1), (2, 1), (3, 1), (1, 2), (2, 2), (3, 2), (1, 3), (2, 3), (3, 3)\}$

(i) Let $A = \{1, 2, 3\}$. $f: A \times A \rightarrow \mathbb{Z} \times \mathbb{Z}$, where $f(x,y) = (x, y+1)$.

$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Range = $\{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

(l) Let $A = \{1, 2, 3\}$. $f: P(A) \rightarrow P(A)$. For $X \subseteq A$, $f(X) = X - \{1\}$.

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$

Process: $\emptyset = \{1\} - \{1\}$, $\{2\} = \{1, 2\} - \{1\}$, $\{3\} = \{1, 3\} - \{1\}$, $\{2, 3\} = \{1, 2, 3\} - \{1\}$.

Range = $\{\emptyset, \{2\}, \{3\}, \{2, 3\}\}$

Question 4:

I. Solve the following questions from the Discrete Math zyBook:

A. Exercise 4.2.2, sections

(c) . $h: \mathbf{Z} \rightarrow \mathbf{Z}. h(x) = x^3$

Not onto.

One-to-one

Target of y is integer, it is not equal to the range y, which is perfect cube.

Let y be an integer but not perfect cube, there's no x in the integers such that $x^3 = y$.

Eg: There's no x in the integers such that $x^3 = 2$ or $x^3 = 4$.

(g). $f: \mathbf{Z} \times \mathbf{Z} \rightarrow \mathbf{Z} \times \mathbf{Z}, f(x, y) = (x+1, 2y)$

Not onto.

One-to-one

Target is not equal to range.

Let all the ordered pairs in the target with y as odd integers, there's no element from domain map them.

Eg: There's no y from Z such that $2y=3$, since 2y is only even integer.

(k) $f: \mathbf{Z}^+ \times \mathbf{Z}^+ \rightarrow \mathbf{Z}^+. f(x, y) = 2^x + y$

Neither one-to-one nor onto.

1. $f(2,1)=5, f(1,3) = 5. (2,1) \neq (1,3)$, but $5=5$, so f is not one-to-one. And $f(2,2) = f(1,4)$.

2. There's no (x,y) in \mathbf{Z}^+ , such that $2^x + y = 1$ or 2. So f is not onto.

B. Exercise 4.2.4, sections

- (b) $f: \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, $f(001) = 101$ and $f(110) = 110$.

Not one-to-one : $f(010)=f(110)=110$.

Not onto. There's no x in domain such that $y = 001$.

While 001 is in the target but not range.

- (c) $f: \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example $f(011) = 110$.

Both one-to-one and onto.

- (d) $f: \{0, 1\}^3 \rightarrow \{0, 1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, $f(100) = 1001$.

Not onto. There's no x in domain such that $y = 1000$.

One-to-one.

- (g) Let A be defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ and let $B = \{1\}$. $f: P(A) \rightarrow P(A)$. For $X \subseteq A$, $f(X) = X - B$. Recall that for a finite set A , $P(A)$ denotes the power set of A which is the set of all subsets of A .

Not onto. There's no x in domain such that $y = \{1\}$. $\{1\}$ is in the target but not range.

One-to-one.

II. Give an example of a function from the set of integers to the set of positive integers that is:

$$f: \mathbb{Z} \rightarrow \mathbb{Z}^+$$

A. one-to-one, but not onto.

$$f(x) = 3x + 1, \text{ when } x \geq 0; \quad f(x) = 3|x|, \text{ when } x < 0.$$

There's no $x \in \mathbb{Z}$, such that $3x + 1$ or $3|x| = 2$, The range of y is not equal to target.

B. onto, but not one-to-one.

$$f(x) = |x| + 1 : f(1) = f(-1) = 2.$$

C. one-to-one and onto.

$$f(x) = 2x + 1, \text{ when } x \geq 0, \text{ (all odd)} ; \quad f(x) = 2|x|, \text{ when } x < 0. \text{ (all even)}$$

D. neither one-to-one nor onto

$$f(x) = x^2 + |x|$$

Not one-on-one: $f(1) = f(-1) = 2$

Not onto: There's no $x \in \mathbb{Z}$, such that $x^2 + |x| = 3$. The range of y is not equal to target.

Question 5:

Solve the following questions from the Discrete Math zyBook:

A Exercise 4.3.2, sections

(c) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 2x + 3$

The function has a well defined inverse.

$$f^{-1}(x) = \frac{x-3}{2}$$

(d) Let A be defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$
 $f : P(A) \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
 For $X \subseteq A$, $f(X) = |X|$. Recall that for a finite set A , $P(A)$ denotes the power set of A which is the set of all subsets of A .

The inverse function is not well defined. f^{-1} is not one-to-one.
 Here f doesn't have an inverse because f is not onto.

(g) $f : \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example, $f(011) = 110$.

The function has a well defined inverse.

Here f is a bijection from $\{0, 1\}^3$ to $\{0, 1\}^3$, f has an inverse f^{-1} .

Then for every $x \in \{0, 1\}^3, y \in \{0, 1\}^3$, $f(x) = y$ if and only if, $f^{-1}(y) = x$.
 $f^{-1} = f$

(i) $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z} \times \mathbb{Z}, f(x, y) = (x + 5, y - 2)$

The function has a well defined inverse.

$$f^{-1}(x, y) = (x - 5, y + 2)$$

B Exercise 4.4.8, sections(c) $f \circ h$

$$f \circ h(x) = 2x^2 + 5$$

(d) $h \circ f$

$$h \circ f(x) = 4x^2 + 12x + 10$$

C Exercise 4.4.2, sections(b) Evaluate $(f \circ h)(52)$

$$f \circ h(52) = \left(\left\lceil \frac{52}{5} \right\rceil \right)^2 - 11^2 = 121$$

(c) Evaluate $(g \circ h \circ f)(4)$

$$g \circ h \circ f(4) = g \circ h(16) = g(4) = 16$$

(d) Give a mathematical expression for $h \circ f$.

$$f(x) = x^2$$

$$h \circ f(x) = \left\lceil \frac{x^2}{5} \right\rceil$$

D Exercise 4.4.6, sections(c) What is $(h \circ f)(010)$?

$$f(010) = 110 \quad h \circ f(010) = 111$$

(d) What is the range of $h \circ f$?

$$\text{Range} = \{101, 111\}$$

(e) What is the range of $g \circ f$?

$$\text{Range} = \{001, 011, 101, 111\}$$

E Extra Credit: Exercise 4.4.4, sections

(c) Is it possible that f is not one-to-one and $g \circ f$ is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g .

No. If f is not one-to-one, then $f(x_1)=f(x_2)=y_1$, $x_1 \neq x_2$.

Then $g(y_1)=z_1$.

Then $g \circ f(x_1)=g(y_1)=z_1$, $g \circ f(x_2)=g(y_1)=z_1$,

then $x_1 \neq x_2$, $g \circ f(x_1)=g \circ f(x_2)=z_1$,

So $g \circ f$ is not one-to-one.

(d) Is it possible that g is not one-to-one and $g \circ f$ is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g .

Yes. X is $\{1,2\}$

Y is $\{a, b, c\}$

Z is $\{A, B\}$

g domain is $\{a, b, c\}$.

If g is not one-to-one, then $g(a)=A$, $g(b)=g(c)=B$.

Here f range is $\{a, b\}$,

If f is one-to-one, then $f(1)=a$, $f(2)=b$.

Then $g \circ f(1)=A$, $g \circ f(2)=B$, $1 \neq 2$, and $A \neq B$.

So $g \circ f$ is one-to-one.
