NYU, Tandon School of Engineering
Extended Bridge to CS — Summer 2021

Homework #3
Due by Friday 7/30, 11:59pm

Submission instructions:

1. For this assignment you should turn in 7 files:
   - Six '.cpp' files, one for each question 1 to 6.
     Name your files 'YourNetID_hw3_q1.cpp', 'YourNetID_hw3_q2.cpp', etc.
   - A '.pdf' file with your answers for questions 7-11.
     Each question should start on a new page!
     Name your file 'YourNetID_hw3_q7to11.pdf'

2. **Typing your solutions would grant you 5 extra points.**

3. **You should submit your homework in the Gradescope system.** Note that when submitting the pdf file, you would be asked to assign the pages from your file to their corresponding questions.

4. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**

5. For the coding questions, pay special attention to the style of your code. Indent your code correctly, choose meaningful names for your variables, define constants where needed, choose the most appropriate control flow statements, etc.

6. For the math questions, you are expected to justify all your answers, not just to give the final answer (unless explicitly asked to).
   As a rule of thumb, for questions taken from zyBooks, the format of your answers, should be like the format demonstrated in the sample solutions we exposed.

Question 7:

Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.1.1, sections a-g

3.1.1: Set membership and subsets - true or false.

Use the definitions for the sets given below to determine whether each statement is true or false:

A = { x ∈ **Z**: x is an integer multiple of 3 }
B = { x ∈ **Z**: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }

An integer x is a perfect square if there is an integer y such that x = y^2.

(a) 27 ∈ A

T : 27=3*9

(b) 27 ∈ B

F :25=5^2, 27 is not a perfect square of any number.

(c)100 ∈ B.

T :100=10^2

(d)E ⊆ C or C ⊆ E.

F : E $\neq C$

(e)E ⊆ A

T : 9=3*3, 6=3*2, 3=3*1, E⊂A, E≠A

(f)A ⊂ E

F: 12 ∈ A, 12 $\notin E$,

(g)E ∈ A

F: E is not an element, E is a Null set.

b) Exercise 3.1.2, sections a-e

A = { x ∈ **Z**: x is an integer multiple of 3 }
B = { x ∈ **Z**: x is a perfect square }
C = { 4, 5, 9, 10 }
D = { 2, 4, 11, 14 }
E = { 3, 6, 9 }
F = { 4, 6, 16 }
An integer x is a perfect square if there is an integer y such that x = y^2.

(a)15 ⊂ A

F

(b){15} ⊂ A

T

(c)∅ ⊂ A

T

(d)A ⊆ A

T

(e)∅ ∈ B

F

c) Exercise 3.1.5, sections b, d

(b) {3, 6, 9, 12, . . . }

Let B = {3, 6, 9, 12, . . . }

B = {x ∈ Z : x is an integer multiple of 3 and x ≥ 3}

(d) {0, 10, 20, 30 . . . 1000}

Let D = {0, 10, 20, 30 . . . 1000}

D = {x ∈ Z : x is an integer multiple of 10 and 0 ≤ x ≤ 1000}

The set is finite. Hence |D| = 101

d) Exercise 3.2.1, sections a-k

(a) $2 \in X$: T
(b) $\{2\} \subseteq X$: T
(c) $\{2\} \in X$: F
(d) $3 \in X$: F
(e) $\{1, 2\} \in X$: T
(f) $\{1, 2\} \subseteq X$: T
(g) $\{2, 4\} \subseteq X$: T
(h) $\{2, 4\} \in X$: F
(i) $\{2, 3\} \subseteq X$: F
(j) $\{2, 3\} \in X$: F
(k) $|X| = 7$: F

<u>Question 8:</u>
Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

(b) Let A = {1, 2, 3}. What is {X ∈ P(A) : 2 ∈ X}
P(A) = {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}
Let B = {X ∈ P(A) : 2 ∈ X}
B = {{2}, {1, 2}, {2, 3}, {1, 2, 3}}

Question 9:
Solve the following questions from the Discrete Math zyBook:
a) Exercise 3.3.1, sections c-e

(c) $A \cap C$
$A \cap C = \{-3, 1, 17\}$

(d) $A \cup (B \cap C)$
$B \cap C = \{-5, 1\}$
$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

(e) $A \cap B \cap C$
$B \cap C = \{-5, 1\}$
$A \cap (B \cap C) = \{1\}$

b) Exercise 3.3.3, sections a, b, e, f

(a)
$$\bigcap_{i=2}^{5} Ai = A2 \cap A3 \cap A4 \cap A5$$
$A2 = \{1, 2, 4\}$
$A3 = \{1, 3, 9\}$
$A4 = \{1, 4, 16\}$
$A5 = \{1, 5, 25\}$
$A2 \cap A3 \cap A4 \cap A5 = \{1\}$

(b)
$$\bigcup_{i=2}^{5} Ai = A2 \cup A3 \cup A4 \cup A5$$
$A2 = \{1, 2, 4\}$
$A3 = \{1, 3, 9\}$
$A4 = \{1, 4, 16\}$
$A5 = \{1, 5, 25\}$
$A2 \cup A3 \cup A4 \cup A5 = \{1, 2, 3, 4, 5, 9, 16, 25\}$

(e)

$$\bigcap_{i=1}^{100} Ci = C1 \cap C2 \cap C3 \cap \cdots \cap C100$$

C1 = {x ∈ R : − 1 ≤ x ≤ 1}

C2 = {x ∈ R :$\frac{-1}{2} \leq x \leq \frac{1}{2}$}

C3 = {x ∈ R :$\frac{-1}{3} \leq x \leq \frac{1}{3}$}

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C100 = {x ∈ R:$\frac{-1}{100} \leq x \leq \frac{1}{100}$}

C1 ∩ C2 ∩ C3 ∩ · · · ∩ C100 = {x ∈ R :$\frac{-1}{100} \leq x \leq \frac{1}{100}$}

Because the x values between $\frac{-1}{100}$ $and \leq \frac{1}{100}$ is part of all the C in C1, C2, C3, . . . , C100

(f)

$$\bigcup_{i=1}^{100} Ci = C1 \cup C2 \cup C3 \cup \cdots \cup C100$$

C1 = {x ∈ R : − 1 ≤ x ≤ 1}

C2 = {x ∈ R :$\frac{-1}{2} \leq x \leq \frac{1}{2}$}

C3 = {x ∈ R :$\frac{-1}{3} \leq x \leq \frac{1}{3}$}

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C100 = {x ∈ R:$\frac{-1}{100} \leq x \leq \frac{1}{100}$}

C1 ∪ C2 ∪ C3 ∪ · · · ∪ C100 = {x ∈ R :$\frac{-1}{100} \leq x \leq \frac{1}{100}$}

Because the x values between $-\ 1\ and 1$ covers all the values in C1, C2, C3, . . . , C100

c) Exercise 3.3.4, sections b, d

(b) P(A ∪ B)

A ∪ B = {a, b, c}

P(A ∪ B) = {∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}

(d) P(A) ∪ P(B)

P(A) = {∅, {a}, {b}, {a, b}}

P(B) = {∅, {b}, {c}, {b, c}}

P(A) ∪ P(B) = {∅, {a}, {b}, {c}, {a, b}, {b, c}}

Question 10:

Solve the following questions from the Discrete MathzyBook:

a) Exercise 3.5.1, sections b, c

(b) Write an element from the set B × A × C.
B × A × C = (foam, tall, non-fat)

(c) Write the set B × C using roster notation.
B × C = {(foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole)}

b) Exercise 3.5.3, sections b, c, e

(b) $Z^2 \subseteq R^2$: T

(c) $Z^2 \cap Z^3 = \emptyset$: T

(e) For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C : T

c) Exercise 3.5.6, sections d, e

(d) $\{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

$\{0\} \cup \{0\}^2 = \{0, 00\}$

$\{1\} \cup \{1\}^2 = \{1, 11\}$

Let $D = \{xy : \text{where } x \in \{0\} \cup \{0\}^2 \text{ and } y \in \{1\} \cup \{1\}^2\}$

D = {01, 011, 001, 0011}

(e) $\{xy : \text{where } x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

$\{a\} \cup \{a\}^2 = \{a, aa\}$

Let $E = \{xy : \text{where } x \in \{aa, ab\} \text{ and } y \in \{a\} \cup \{a\}^2\}$

E = {aaa, aaaa, aba, abaa}

d) Exercise 3.5.7, sections c, f, g

(c) $(A \times B) \cup (A \times C)\}$
$A \times B = \{ab, ac\}$
$A \times C = \{aa, ab, ad\}$
$(A \times B) \cup (A \times C) = \{aa, ab, ac, ad\}$

(f) $P(A \times B)$
$A \times B = \{ab, ac\}$
$P(A \times B) = \{\emptyset, \{ab\}, \{ac\}, \{ab, ac\}\}$

(g) $P(A) \times P(B)$. Use ordered pair notation for elements of the Cartesian product
$P(A) = \{\emptyset, \{a\}\}$
$P(B) = \{\emptyset, \{b\}, \{c\}, \{b, c\}\}$
$P(A) \times P(B) = \{(\emptyset, \emptyset),(\emptyset, \{b\}),(\emptyset, \{c\}),(\emptyset, \{b, c\}),(\{a\}, \emptyset),(\{a\}, \{b\}),$
$(\{a\}, \{c\}),(\{a\}, \{b, c\})\}$

Question 11:

Solve the following questions from the Discrete MathzyBook:

a) Exercise 3.6.2, sections b, c

(b) $(B \cup A) \cap (\overline{B} \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	Hypothesis
$(A \cup B) \cap (A \cup \overline{B})$	Commutative Law
$A \cup (B \cap \overline{B})$	Distribution Law
$A \cup \emptyset$	Complement Law A Identity Law

(c) $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}}$	Hypothesis
$\overline{A} \cup \overline{\overline{B}}$	De Morgan's Law
$\overline{A} \cup B$	Double Complement Law

b) Exercise 3.6.3, sections b, d

(b) $A - (B \cap A) = A$

Let $A = \{1, 2, 3, 4\}$

Let $B = \{3, 4, 5\}$

$(B \cap A) = \{3, 4\}$

$A - (B \cap A) = \{1, 2\}$ 6= $\{1, 2, 3, 4\}$

$\therefore A - (B \cap A)$ 6= A

$\therefore A - (B \cap A) = A$ is not a set identity.

(d) $(B - A) \cup A = A$

Let $A = \{1, 2, 3, 4\}$

Let $B = \{3, 4, 5, 6\}$

$B - A = \{5, 6\}$

$(B - A) \cup A = \{1, 2, 3, 4, 5, 6\}$ 6= $\{1, 2, 3, 4\}$

$\therefore (B - A) \cup A$ 6= A

$\therefore (B - A) \cup A = A$ is not a set identity.

c) Exercise 3.6.4, sections b, c

(b) $A \cap (B - A) = \emptyset$

$A \cap (B - A)$	Hypothesis
$A \cap (B \cap A)$	Subtraction Law
$A \cap (A \cap B)$	Commutative Law
$(A \cap A) \cap B$	Associative Law
$\emptyset \cap B$	Complement Law
$B \cap \emptyset$	Commutative Law
$\emptyset$	Domination Law

(c) $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	Hypothesis
$A \cup (B \cap A)$	Subtraction Law
$(A \cup B) \cap (A \cup A)$	Distribution Law
$(A \cup B) \cap U$	Complement Law
$(A \cup B)$	Identity Law