NYU, Tandon School of Engineering
Extended Bridge to CS — Summer
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Homework #5

Submission instructions:

1. For this assignment, you should turn in 3 files:
   - Two '.cpp' files, one for each question 1 and 2.
     Name your files 'YourNetID_hw5_q1.cpp', and 'YourNetID_hw5_q2.cpp'.
   - A '.pdf' file with your answers for questions 3-5.
     Each question should start on a new page!
     Name your file 'YourNetID_hw5_q3to5.pdf'

2. You must type all your solutions. We will take off points for submissions that are handwritten.

3. **You should submit your homework in the Gradescope system.**

4. **You can work and submit in groups of up to 4 people. If submitting as a group, make sure to associate all group members to the submission on gradescope.**

5. Pay special attention to the style of your code. Indent your code correctly, choose meaningful names for your variables, define constants where needed, choose the most appropriate control flow statements, etc.

6. For the math questions, you are expected to justify all your answers, not just to give the final answer (unless explicitly asked to).
   As a rule of thumb, for questions taken from zyBooks, the format of your answers, should be like the format demonstrated in the sample solutions we exposed.

Question 3:

Solve the following questions from the Discrete Math zyBook:

A.Exercise 4.1.3, sections b, c

Which of the following are functions from R to R? If f is a function, give its range.

(b) $f(x) = 1/(x^2 - 4)$

Here f is not a function , when x=2 or x= -2, The function f is not well-defined for f=1/0 .

(c) $f(x) = \sqrt{x^2}$

Here f is a function, The function is well defined. The range is {0 , R+}

B. Exercise 4.1.5, sections b, d, h, i, l

Express the range of each function using roster notation.

(b) Let A = {2, 3, 4, 5}. f: A → **Z** such that f(x) = x^2.

{4, 9, 16, 25}

(d) f: {0,1}^5 → **Z**. For x ∈ {0,1}^5, f(x) is the number of 1's that occur in x.

{0, 1, 2, 3, 4, 5 }

(h) Let A = {1, 2, 3}. f: A × A → **Z**×**Z**, where f(x,y) = (y, x).

A × A = {(1, 1),(1, 2),(1, 3),(2, 1),(2, 2),(2, 3),(3, 1),(3, 2),(3, 3)}
Range = {(1, 1),(2, 1),(3, 1),(1, 2),(2, 2),(3, 2),(1, 3),(2, 3),(3, 3)}, (3^3=9)

(i) Let A = {1, 2, 3}. f: A × A → **Z**×**Z**, where f(x,y) = (x,y+1).

A × A = {(1, 1),(1, 2),(1, 3),(2, 1),(2, 2),(2, 3),(3, 1),(3, 2),(3, 3)}
Range = {(1, 2),(1, 3),(1, 4),(2, 2),(2, 3),(2, 4),(3, 2),(3, 3),(3, 4)}

(l) Let A = {1, 2, 3}.f: P(A) → P(A). For X ⊆ A, f(X) = X - {1}.

P(A) = {∅, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3}}
Process: ∅={1}- {1}, {2}={1, 2}- {1}, {3}= {1, 3}- {1}, {2, 3} = {1, 2, 3}- {1}.
Range = {∅, {2}, {3}, {2, 3}}

Question 4:

I. Solve the following questions from the Discrete Math zyBook:

A. Exercise 4.2.2, sections

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(c) . h: **Z** → **Z**. h(x) = x^3

Not onto.

One-to-one

Target of y is integer, it is not equal to the range y, which is a perfect cube.

Let y be an integer but not perfect cube, there's no x in the integers such that x^3=y.

Eg: There's no x in the integers such that x^3=2 or x^3=4.

(g). f: **Z** × **Z** → **Z** × **Z**, f(x, y) = (x+1, 2y)

Not onto.

One-to-one

Target is not equal to range.

Let all the ordered pairs in the target with y as odd integers, there's no element from the domain map them.

Eg: There's no y from Z such that 2y=3, since 2y is only even integer.

(k)f :$z^+ \times z^+ \rightarrow z^+$. f(x, y) = 2^x + y

Neither one-to-one nor onto.

1.f(2,1)=5, f(1, 3) $=$ 5 .(2,1) $\neq$ (1, 3), but 5=5, so f is not one-to-one. And f(2, 2) = f(1, 4).

2.There's no (x,y) in z^+, such that 2^x + y=1 or 2. So f is not onto.

B. Exercise 4.2.4, sections

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(b) f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.

Not one-to-one : f(010)=f(110)=110.
Not onto. There's no x in the domain such that y =001.
While 001 is in the target but not range.

(c) f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110.

Both one-to-one and onto.

(d) f: $\{0, 1\}^3 \to \{0, 1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

Not onto. There's no x in the domain such that y =1000.
One-to-one.

(g) Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8} and let B = {1}. f: P(A) → P(A). For $X \subseteq A$, f(X) = X - B. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.
Neither.
Not onto. There's no x in the domain such that y= {1}. {1} is in the target but not in range.
Not One-to-one. Because f({1,2}) = f({2})={2}; while {1,2} ≠ {2}.

II. Give an example of a function from the set of integers to the set of positive integers that is:

f:Z—>z^{+}

A. one-to-one, but not onto.

f(x)=3x+1, when x>=0; f(x)=3|x|, when x<0.

There's no x ∈ Z, such that 3x+1 or 3|x| =2, The range of y is not equal to target.

B. onto, but not one-to-one.

f(x)=|x| + 1 : f(1)=f(-1)=2.

C. one-to-one and onto.

f(x)=2x+1, when x>=0, (all odd) ; f(x)=2|x|, when x<0. (all even)

D. neither one-to-one nor onto

f(x)=$x^2 + |x|$

Not one-on-one: f(1)=f(-1)=2

Not onto: There's no x ∈ Z, such that $x^2 + |x|$=3. The range of y is not equal to target.

Question 5:

Solve the following questions from the Discrete Math zyBook:

For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship of f^-1

A Exercise 4.3.2, sections

(c) $f: \mathbb{R} \to \mathbb{R}$. $f(x) = 2x + 3$

The function has a well defined inverse.

$f^{-1}(x) = \frac{x-3}{2}$

(d) Let A be defined to be the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$
$f: P(A) \to \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
For $X \subseteq A$, $f(X) = |X|$. Recall that for a finite set A, $P(A)$ denotes the power set of A which is the set of all subsets of A.

f({1})= f({2})=|{1}|=|{2}|=1, f is not one-to-one, f does not has a well defined inverse
(f is onto. Because for every element in y, there's an x in A such that f(x)=y=|x|. Cardinity)

(g) $f: \{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example, $f(011) = 110$.

The function has a well defined inverse.

Here f is a bijection from $\{0, 1\}^3$ to $\{0, 1\}^3$, f has a inverse f^{-1}.

Then for every x $\in\{0, 1\}^3$, y$\in\{0, 1\}^3$, f(x) = y if and only if, f^{-1}(y) = x.

$f^{-1} = f, \; for\; x \in \{0, 1\}^3$.

(i) $f: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$, $f(x, y) = (x + 5, y - 2)$

The function has a well defined inverse.

f^{-1}(x, y) = (x − 5, y + 2)

B Exercise 4.4.8, sections

The domain and target set of functions f, g, and h are **Z**. The functions are defined as:

- f(x) = 2x + 3
- g(x) = 5x + 7
- h(x) = x2 + 1

(c) f o h

f o h(x) = $2x^2$ + 5

(d) h o f

h o f(x) = $4x^2$ + 12x+10

C Exercise 4.4.2, sections

Consider three functions f, g, and h, whose domain and target are **Z**. Let

$$f(x) = x^2 \qquad g(x) = 2^x \qquad h(x) = \left\lceil \frac{x}{5} \right\rceil$$

(b)Evaluate (f o h)(52)

f o h(52) = $\left(\left\lceil \frac{52}{5} \right\rceil\right)^2$ 11^2=121

(c)Evaluate (g o h o f)(4)

g o h of(4) = g o h (16)=g (4)=16

(d)Give a mathematical expression for h o f.

f(x) =x^2

h o f(x)= $\left\lceil \frac{x^2}{5} \right\rceil$

D Exercise 4.4.6, sections

Define the following functions f, g, and h:

- f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.
- g: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of g is obtained by taking the input string and reversing the bits. For example, g(011) = 110.
- h: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of h is obtained by taking the input string x, and replacing the last bit with a copy of the first bit. For example, h(011) = 010.

(c) What is (h o f)(010)?

f(010) = 110 h o f(010) = 111

(d)What is the range of h o f?

Range = {101, 111}

(e)What is the range of g o f?

Range = {001, 011, 101, 111}

E Extra Credit: Exercise 4.4.4, sections

Let f: X → Y and g: Y → Z be two functions.

(c) Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

No. If f is not one-to-one, then f(x1)=f(x2)=y1, x1≠ $x2$.
Then g(y1)=z1.
Then g o f (x1)= g(y1)=z1, g o f (x2)= g(y1)=z1,
then x1≠ $x2$, g o f (x1)=g o f (x2)=z1,
So g o f is not one-to-one.

(d) Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

Yes. X is{1,2,3}
Y is {a ,b, c}
Z is {A, B}
g domain is {a ,b ,c}.
If g is not one-to-one, then g(a)=A, g(b)=g(c)=B. b≠c

Here f range is {a,b,c},
If f is one-to-one, then f(1)=a, f (2)=b, (f (3)=b);
Then g o f(1)=A,g o f(2)=B, 1≠2, and A≠B.
So g o f is one-to-one.
