CISC 102 (Fall 20)
Homework #1:Sets (20 Points)

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Solutions are due before 11:59 PM on **Friday Midnight September 25, 2020** .

1. (3 pts) ist the elements in the following sets:

   (a) $\{n \in \mathbb{Z} | n^2 < 7\}$
   $\{-2, -1, 0, 1, 2\}$

   (b) $\{x^2 | x \in \mathbb{N}_0 \wedge x < 5\}$
   $\{0, 1, 4, 9, 16\}$

   (c) $\{m \in \mathbb{Q} | m^2 = 7\}$.
   $\{2.64575131106\}$

2. (3 pts) Let $A = \{1, 2, 3\}$ and $B = \{1, 3\}$

   (a) List the elements of $A \times B \times A$
   $\{1,1,1\}, \{1,1,2\}, \{1,1,3\}, \{1,3,1\}, \{1,3,2\}, \{1,3,3\}, \{2,1,1\}, \{2,1,2\},$
   $\{2,1,3\}, \{2,3,1\}, \{2,3,2\}, \{2,3,3\}, \{3,1,1\}, \{3,3,2\}, \{3,3,3\}$

   (b) (3 pts) List the elements of $(A \times B) \cap (B \times A)$
   $\{1,1\}, \{1,3\}, \{3,1\}, \{3,3\}$

   (c) List the elements of $(A \times A) \setminus (A \times B)$
   $\{1,2\}, \{2,2\}, \{3,2\}$

3. (3 pts) For the following sets decide whether they are finite or infinite. If the set is finite, write down its size.

   (a) $\{x \in \mathbb{N}_0 | x > 10\}$
   This set is infinite.

   (b) $\{x \in \mathbb{N}_0 | x \leq 10\}$.
   This set is finite and has a size of 11.

(c) $\{4, \{4\}, \{4, \{4\}\}, \{\mathbb{N}\}\}$
This set is finite and has a size of 4.

4. (2 pts) Show that if A, B, C are sets with $A \subseteq B$ and $B \subset C$ then $A \subset C$.

$A \subseteq B \Rightarrow \forall x \in A : x \in B$
$B \subset C \Rightarrow \forall x \in B : x \in C$
Therefore $\forall x \in A : x \in C \Rightarrow A \subset C$
However $A \nsubseteq C$ because $B \nsubseteq C$

5. (4 pts) For each of the following statements about sets, decide whether they are true or false. Justify your assertion, either way.

(a) $\mathbb{Z}$ is finite
False. $\mathbb{Z}$ contains all negative and positive integers, therefore the set is infinite as this defines an infinite amount of elements.

(b) $\{\{\mathbb{Z}\}\}$ is finite
True. $\{\{\mathbb{Z}\}\}$ is finite as the set only contains one element, which is $\{\mathbb{Z}\}$. This is distinguishable because elements in a set are determined by placement in curly brackets.

(c) $\{x \in \mathbb{Q} | x^2 = 2\}$ is finite.
True. Following the notation, we can find that x is the square root of 2, and is a rational number. This means the set contains one number and therefore is finite.

(d) If A is a finite set then $|\mathcal{P}(A)| > |A|$
True. To find the cardinality of $|\mathcal{P}(A)|$ we use the formula $|\mathcal{P}(A)| = 2^{|A|}$.
In all situations it is true that $2^{|A|} > |A|$

6. (2 pts) Let $A = \{1, 2, 3, 4\}$. Write down $\mathcal{P}(A)$. What is $|\mathcal{P}(A)|$?

$\mathcal{P}(A)$=$\{\varnothing, \{1\}, \{, 2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \}$
$\{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$

$|\mathcal{P}(A)| = 2^4 = 16$

7. (3 pts) In a class of 65 students, 25 speak Spanish, 32 are excellent cooks, and 50 love dogs. Each student is in at least one of these categories.

There are 18 Spanish speakers who don't cook. There are 21 dog lovers who are excellent cooks. There are 4 cooks who speak Spanish and do not love dogs. Determine the number of students in the class in each of the following categories:

(a) Speak Spanish and love dogs
$S \cap D = 17$
There are 17 students that speak Spanish and love dogs.

(b) Love dogs and cannot cook
$D - C = 29$
There are 29 students that love dogs and cannot cook.

(c) Speak Spanish, are excellent cooks, and love dogs
$(S \cap C) \cap D = 3$
There are 3 students that speak Spanish, are excellent cooks, and love dogs.

See additional work here:

$S = 25 C = 32 D = 50$
$S - C = 18$
$D \cap C = 21$
$(S \cap C) - D = 4$
$S \cup C \cup D = 65$

Venn Diagram: