CISC 102 (Fall 20)
Homework #2: Logic (25 Points)

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Solutions are due before 11:59 PM on **Friday Midnight October 4, 2020** .

1. (4 pts) Let p, q, and r be the propositions
   p: You have the flu, q: You miss the final examination, and r: You pass the course.
   Express each of these propositions as an English sentence.

   (a) $q \rightarrow \neg r$
   You miss the final examination, and so you do not pass the course.

   (b) $p \vee q \vee r$
   You have the flu. or you miss the final examination, or you pass the course.

   (c) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
   You have the flu and do not pass the course, or you miss the final examination and do not pass the course.

   (d) $(p \wedge q) \vee (\neg q \wedge r)$
   You have the flu and you miss the final examination, or you do not miss the exam and pass the course.

2. (2 pts) Write each of these statements in the form "if p, then q" in English.

   (a) I will remember to send you the address only if you send me an e-mail message.
   $p \rightarrow q$
   If you send me an email message then I will remember to send you the address.

   (b) The Red Wings will win the Stanley Cup if their goalie plays well.
   $p \rightarrow q$
   If the Red Wings' goalie plays well, then they will win the Stanley Cup.

3. (3 pts) State the converse, contrapositive, and inverse of each of these conditional statements.

   (a) If it snows tonight, then I will stay at home.
   Converse: If I stay home then it will snow tonight.
   Contrapositive: If I don't stay home then it will not snow tonight.
   Inverse: If it doesn't snow tonight then I will not stay home.

   (b) I go to the beach whenever it is a sunny summer day.
   Converse: If I go to the beach then it is a sunny summer day.
   Contrapositive: If I don't go to the beach then it is not a sunny summer day.
   Inverse: If it is not a sunny summer day then I will not go to the beach.

   (c) When I stay up late, it is necessary that I sleep until noon.
   Converse: If I need to sleep until noon then I stay up late.
   Contrapositive: If I don't need to sleep until noon then I didn't stay up late.
   Inverse: If I didn't stay up late then I don't need to sleep until noon.

4. (2 pts) Are the two logical expressions $P \wedge \neg Q$ and $\neg(\neg P \vee Q)$ logically equivalent? Answer this question using a truth table (or tables)

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \vee Q$	$\neg(\neg P \vee Q)$
T	T	F	F	**F**	T	**F**
T	F	F	T	**T**	F	**T**
F	T	T	F	**F**	T	**F**
F	F	T	T	**F**	T	**F**

$P \wedge \neg Q$ and $\neg(\neg P \vee Q)$ are logically equivalent, as shown by the truth table outcomes.

5. (2 pts) Is $\neg(P \to Q)$ logically equivalent to $P \wedge \neg Q$?

Solving Using logic laws:
$\neg(P \to Q) = \neg(\neg P \vee Q)$
$= \neg(\neg P) \wedge \neg Q$
$= P \wedge \neg Q$

Solving Using Truth Tables:

P	Q	$\neg Q$	$P \to Q$	$\neg(P \to Q)$	$P \wedge \neg Q$
T	T	F	T	**F**	**F**
T	F	T	F	**T**	**T**
F	T	F	T	**F**	**F**
F	F	T	T	**F**	**F**

$(P \to Q)$ is logically equivalent to $(P \wedge \neg Q)$, as shown by the logic laws and truth table outcomes.

6. (2 pts)

   (a) Prove that $A \subseteq B \to A \cap \bar{B} = \varnothing$

   Proof by contradiction:
   $A \subseteq B = \forall x \{x \in A \to x \in B\} = p$
   $A \cap \bar{B} = \exists x \{x \in A \wedge x \notin B\} = q$
   $A \subseteq B \to A \cap \bar{B} = \varnothing$ can be written as $p \to q = \varnothing$
   Assume that $\neg(p \to q)$ is true.
   $\neg p = \exists x \{x \in A \wedge x \notin B\} = A \nsubseteq B$
   $\neg q = \forall x \{x \in A \vee x \in B\} = A \cup B$
   $A \nsubseteq B \to A \cup B = \forall x \{x \in \wedge x \in B\} = A \cup B$
   $A \cup B \neq \varnothing$
   This proves that $\neg(p \to q) \neq \varnothing$, therefore $p \to q = \varnothing$
   Therefore it must be true that $A \subseteq B \to A \cap \bar{B} = \varnothing$

   (b) Prove that $A \cap \bar{B} = \varnothing \to A \subseteq B$
   Need to prove: If A intersect $\bar{B}$ is equal to nothing, then A must be a subset of B.
   $A \cap \bar{B} = \varnothing$ then A and $\bar{B}$ must not share elements.
   $A \cap \bar{B} = p$
   This means that $p = \exists x \left\{x \in A \vee x \in \bar{B}\right\}$
   $A \subseteq B = q$
   This means that $q = \forall x \{x \in A \to x \in B\}$
   $p = \varnothing \to q$
   If $\exists x \left\{x \in A \vee x \in \bar{B}\right\} = \varnothing$, this does not have any values. Therefore $A \subseteq B$ is true, as this proves that the complete A set must be a subset of B.
   Therefore $A \subseteq B \to A \cap \bar{B} = \varnothing$ is true.

7. (2 pts) Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

Solving using logic laws, we can see:
$(p \to q) \vee (p \to r)$
$(\neg p \vee q) \vee (\neg p \vee r)$

$\neg p \vee (q \vee r)$
$p \rightarrow (q \vee r)$
Therefore $(p \rightarrow q) \vee (p \rightarrow r) = p \rightarrow (q \vee r)$

Solving Using Truth Tables:

p	q	r	$(p \rightarrow q)$	$(p \rightarrow r)$	$(p \rightarrow q) \vee (p \rightarrow r)$	$(q \vee r)$	$p \rightarrow (q \vee r)$
T	T	T	T	T	**T**	T	**T**
T	T	F	T	F	**T**	T	**T**
T	F	T	F	T	**T**	T	**T**
T	F	F	F	F	**F**	F	**F**
F	F	F	T	T	**T**	F	**T**
F	T	F	T	T	**T**	T	**T**
F	T	T	T	T	**T**	T	**T**
F	F	T	T	T	**T**	T	**T**

Using logic laws and truth tables we can see that$(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

8. (3 pts) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

   (a) There is a person in your class who cannot swim.
   Domain: Students in class. $A(x) = x$ can swim.
   Therefore $\neg A(x)$
   $\exists x \neg A(x)$

   Domain: All People. $A(x) = x$ can swim.
   $B(x)$ = student in class.
   Therefore $B(x)$and $\neg A(x)$
   $\exists x (B(x) \wedge \neg A(x))$

   (b) All students in your class can solve quadratic equations.
   Domain: Students in class. $A(x) = x$ can solve quadratic equations.
   Therefore $A(x)$ applies to all
   $\forall x A(x)$

   Domain: All People. $A(x) = x$ can solve quadratic equations.
   $B(x)$ = student in class.
   Therefore $B(x)$ applies to all $A(x)$
   $\forall x (B(x) \rightarrow A(x))$

(c) Some student in your class does not want to be rich.

Domain: Students in class.
$A(x) = x$ wants to be rich.
Therefore $\neg A(x)$
$\exists x \neg A(x)$

Domain: All People. $A(x) = x$ wants to be rich.
$B(x) =$ student in class.
Therefore $B(x)$and $\neg A(x)$
$\exists x(B(x) \wedge \neg A(x))$

9. (3 pts) Express the negations of these propositions using quantifiers, and in English.

(a) Every student in this class likes mathematics.

Quantifier: $A(x) = x$ likes mathematics.
$\neg \forall x A(x)$
Negation $= \exists x \neg A(x)$

English: There exists a student student who does not like mathematics.

(b) There is a student in this class who has never seen a computer.

Quantifier: $A(x) = x$ has seen a computer.
$\exists x \neg A(x)$
Negation $= \forall x A(x)$

English: All students in class have seen a computer.

(c) There is a student in this class who has taken every mathematics course offered at this school.

Quantifier:
$x =$ a student in this class.
$y =$ a course at this school.
$B(x, y) = x$ has taken a course y.
$\exists x \forall y B(x, y)$
Negation $= \forall x \exists y \neg B(x, y)$

English: For every student in this class, there is a mathematics course at this school that the student has not taken.

10. (2 pts)

(a) Suppose that $Q(x)$ is the statement $x + 1 = 2x$. What are the truth values of $\forall x Q(x)$ and $\exists x Q(x)$?

$\forall x Q(x)$ where $x + 1 = 2x$
This false because this statement doesn't work for some x values.

$\exists x Q(x)$ where $x + 1 = 2x$
This is true because this statement works for $x = 1$.

(b) Let $P(m, n)$ be "n is greater than or equal to m" where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of $\exists n \forall m P(m, n)$ and $\forall m \exists n P(m, n)$?

$\exists n \forall m P(m, n)$ states that there is a nonnegative interger n such that for every nonnegative interger $m, P(m \leq n)$. This is true because so matter what value of n, there can be a m value that is less than or equal to said n value.

$\forall m \exists n P(m, n)$ states that for every nonnegative interger m there is a nonnegative interger $n, P(m \leq n)$. This is true because for every value of m there can be a larger or equal value of n.