Numbers : CISC221 Assignment 2 Part B

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1. Encode the following decimal numbers with 8-bit two's complement binary, or indicate that number would overflow the range:

a. 49_{10}

$\frac{49}{2} = 24R1$

$\frac{24}{2} = 12R0$

$\frac{12}{2} = 6R0$

$\frac{6}{2} = 3R0$

$\frac{3}{2} = 1R1$

Reading the remainders backwards
110001 however this is only 6-bit, so add 2 zeros to front.
This number is positive so no further action is needed.

8-bit two complement binary of $49_{10} = 00110001$

b-31_{10}

$\frac{31}{2} = 15R1$

$\frac{15}{2} = 7R1$

$\frac{7}{2} = 3R1$

$\frac{3}{2} = 1R1$

Reading the remainders backwards
11111 however this is only 5-bit, so add 3 zeros to front.

This number is negative, so we must find the complement.

$00011111 \rightarrow 11100001$

8-bit two complement binary of $-31_{10} = 11100001$

c. 120_{10}

$\frac{120}{2} = 60R0$

$\frac{60}{2} = 30R0$

$\frac{30}{2} = 15R0$

$\frac{15}{2} = 7R1$

$\frac{7}{2} = 3R1$

$\frac{3}{2} = 1R1$

Reading the remainders backwards
110001 however this is only 7-bit, so add 1 zero to front.

8-bit two complement binary of $120_{10} = 01111000$

d. -128_{10}

$\frac{128}{2} = 64R0$

$\frac{64}{2} = 32R0$

$\frac{32}{2} = 16R0$

$\frac{16}{2} = 8R0$

$\frac{8}{2} = 4R0$

$\frac{4}{2} = 2R0$

$\frac{2}{2} = 1R0$

Reading the remainders backwards
10000000. This is 8-bit so no zeros added to the front.

This number is negative, so we must find the complement.

$10000000 \rightarrow 10000000$

8-bit two complement binary of $-128_{10} = 10000000$

e 128_{10}

This is out of range, because the maximum positive integer that can be represented in 8-bit is 127.

2. Around 250 B.C., the Greek mathematician Archimedes proved that . Had he had access to a computer and the standard library ¡math.h¿ , he would have been able to $22371 < \pi < 227$ determine that the single-precision floating-point approximation of π has the hexadecimal representation 0x40490FDB . Of course, all of these are just approximations, since π is not rational.

a) What is the fractional binary number denoted by this floating-point value? This can be found first by converting the hexadecimal given:

0x40490FDB=

$4 = 0 + 4 + 0 + 0, 0 = 0 + 0 + 0 + 0, 4 = 0 + 4 + 0 + 0, 9 = 8 + 0 + 0 + 1, 0 = 0 + 0 + 0 + 0, F = 8 + 4 + 2 + 1, D = 8 + 4 + 0 + 1, B = 8 + 4 + 2 + 1$

Therefore 0100 0000 0100 1001 0000 1111 1101 1011 is hexadecimal 0x40490FDB binary value.

The breakdown of the binary is as follows:
Sign = 0 Fractional = 10010010000111111011011 Exponent = 10000000

We need to use the formula $M \times B^{E-127}$

The value of the exponent is 10000000 = 128
Therefore E-127 = 128
E = 1

So we move the decimal 1 left on the Fractional:
1.00100100001111111011011
Add the sign from the front of the exponent and the final fractional binary is:

$11.00100100001111111011011_2$

b) What is the fractional binary representation of $\frac{22}{7}$?

For this, first we convert to decimal.

$\frac{22}{7} = 3.142857142857143$

Take the integer:

$\frac{3}{2} = 1R1$
$= 11$

Next take the decimal and multiply it by 2, taking the 0 or 1 off the front:

$0.142857142857143 \times 2 = 0.28571428571428571$

$0.28571428571428571 \times 2 = 0.571428571428571$

$0.571428571428571 \times 2 = 1.142857142857143$

$0.142857142857143 \times 2 = 0.28571428571428571$

As we see, this repeats. Therefore if we do not specify if the representation is 8,16,32 etc bit, we just denote like this:

$11.0010010010010010..._2$

c) At what bit position do these two approximations to π diverge?

As seen on the two calculations above, they diverge on the 9^{th} bit to the right of the binary point.