

Lab 4

Contents

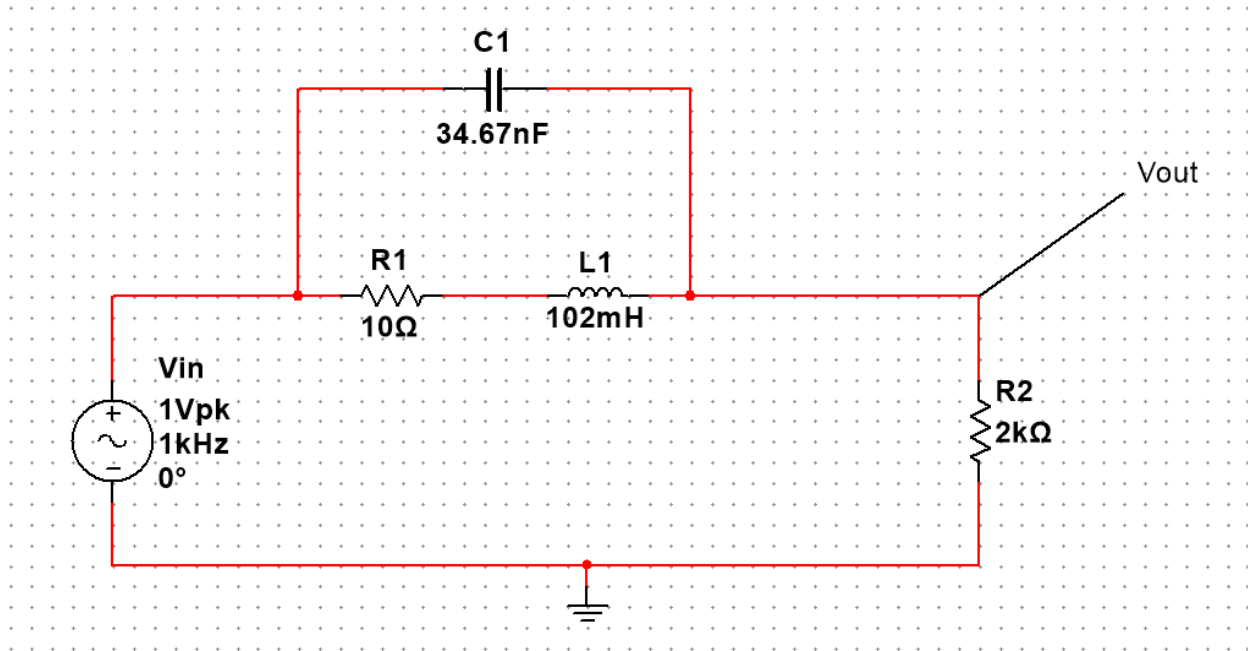
Deliverables	2
Introduction.....	2
Analytical Solution	3
Voltage divider (V_{out})	3
Voltage gain	4
Transfer function	4
Types of filters	6
High frequency	6
Low frequency.....	7
Bandstop filter and Resonance	8
-3db frequencies	9
Table of Analytical Solution Results	10
Maple Analysis.....	11
Digital Solution.....	14
AC Sweep (Magnitude).....	14
AC Sweep (Phase).....	16
Summary of AC Sweep Results	17
Bode Plotter (Magnitude).....	18
Bode Plotter (Phase).....	20
Summary of Bode Plotter Results	22
Table of Digital Solution Results	22
Physical Solution	23
Table of Physical Solution Results	31
Summary of Results	31
Reflection	32

Deliverables

1. Build a circuit similar to the ones tackled below in the example problems which has an input AC voltage source with one side at ground and a clearly defined output node relative to ground. Incorporate an interesting combination of R, L, and C elements so you can demonstrate your mastery of the topic.
 - a. Make sure that your power supply's current stays below 100 mA at all frequencies of interest.
2. Analytical: calculate the transfer function, create a Bode plot describing the circuit, determine which type of filter it is, and calculate the "centre frequency" (or the frequency with the most or least gain, as appropriate for your filter), and the -3dB frequency (or frequencies) in Hz.
3. Simulate your circuit in Multisim and create a bode plot. Use it to measure the same frequencies (centre and -3dB) you found analytically, as appropriate for your circuit.
4. Physically build the circuit and measure the output amplitude gain and phase shift (relative to the input) at several frequencies to obtain the datapoints necessary to create a bode plot and compare with your analytical and simulation work. It will be time consuming to take the measures to produce a detailed plot, so choose only 8-10 points that will give you a good idea of the important features of your frequency response curve.
5. Analysis: as usual

Introduction

This lab involves dealing with the response of a circuit to a range of frequencies and the workings and types of filters. I shall tri-solve the circuit shown below analytically, digitally, and physically. The analytical part of the solution involves calculating the transfer function using the voltage divider equation, determining the type of filter, calculating the center and -3dB frequencies and creating a Bode plot. After that I will make the circuit in Multisim and use the AC sweep and Bode plotter to make Bode plots and use those to measure the same frequencies I found analytically. Then I shall create a physical circuit and measure the signal at a range of frequencies and use that information to create a Bode plot.



Analytical Solution

Voltage divider (Vout)

To find the transfer function, we first need to calculate the output voltage. This shall be done using the voltage divider equation.

$$V_{out} = \frac{R_2}{Z_{par} + R_2} \cdot V_{in}$$

Z_{par} represents the effective impedance of R1, L1 and C1 where $(R_1 + L_1) \parallel C1$.

$$Z_{par} = \left(\frac{1}{R_1 + Z_L} + \frac{1}{Z_C} \right)^{-1}$$

Voltage gain

By knowing the output voltage and input voltage, we can deduce a formula for the voltage gain which is defined as the ratio of the output and input voltage.

$$G = \frac{V_{out}}{V_{in}} = \frac{\frac{R_2}{Z_{par} + R_2} \cdot V_{in}}{V_{in}} = \frac{R_2}{Z_{par} + R_2}$$

Transfer function

The transfer function shows the frequency response of the system as it indicates how the circuit responds to a range of input frequencies. It is similar to voltage gain but sometimes transfer functions can take in more than one type of variable (which voltage gain can not).

First, we shall define the impedances of the inductor and capacitor.

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$

We can then substitute in the values of the output voltage and input voltage in the transfer function equation.

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{R_2}{Z_{par} + R_2}$$

Solving the equation, we get:

$$H(j\omega) = \frac{1}{\frac{Z_{par}}{R_2} + 1}$$

$$H(j\omega) = \frac{1}{\frac{\left(\frac{1}{R_1 + Z_L} + \frac{1}{Z_C}\right)^{-1}}{R_2} + 1} = \frac{1}{\frac{\left(\frac{Z_C + R_1 + Z_L}{(R_1 + Z_L) \cdot Z_C}\right)^{-1}}{R_2} + 1}$$

$$H(j\omega) = \frac{1}{\frac{(R_1 \cdot Z_C) + (Z_L \cdot Z_C)}{R_2 \cdot (Z_C + R_1 + Z_L)} + 1}$$

Next, we will plug in the values for R1 and R2.

$$H(j\omega) = \frac{1}{\frac{10\Omega \cdot j\omega C + \frac{j\omega L}{j\omega C}}{2000\Omega \cdot \left(\frac{1}{j\omega C} + 10\Omega + j\omega L\right)} + 1}$$

$$H_1(j\omega) = \frac{1}{\frac{10j\omega C + \frac{L}{C}}{\frac{2000}{j\omega C} + 20000 + 2000j\omega L} + 1}$$

After using Maple to simplify the expression, we get our transfer function:

$$H_2(j\omega) = \frac{2000(CL\omega^2 - 10j\omega C - 1)}{2000CL\omega^2 - 20000j\omega C - j\omega L - 2010}$$

I also found another version of the transfer function which has the capacitance and inductance plugged in:

$$H_{\text{further simplified}} = \frac{2000 + 0.0006934j\omega - 7.07268 \times 10^{-6}\omega^2}{2010 + 0.1026934j\omega - 7.07268 \times 10^{-6}\omega^2}$$

Types of filters

The type of filter of a circuit depends on its ability (or inability) to take in a range of frequencies. The filter which can take in only high frequencies and reject low frequencies is called a high-pass filter. Under high frequencies, a capacitor will have zero impedance (leading to a short) and an inductor will have infinite impedance (leading to an open circuit).

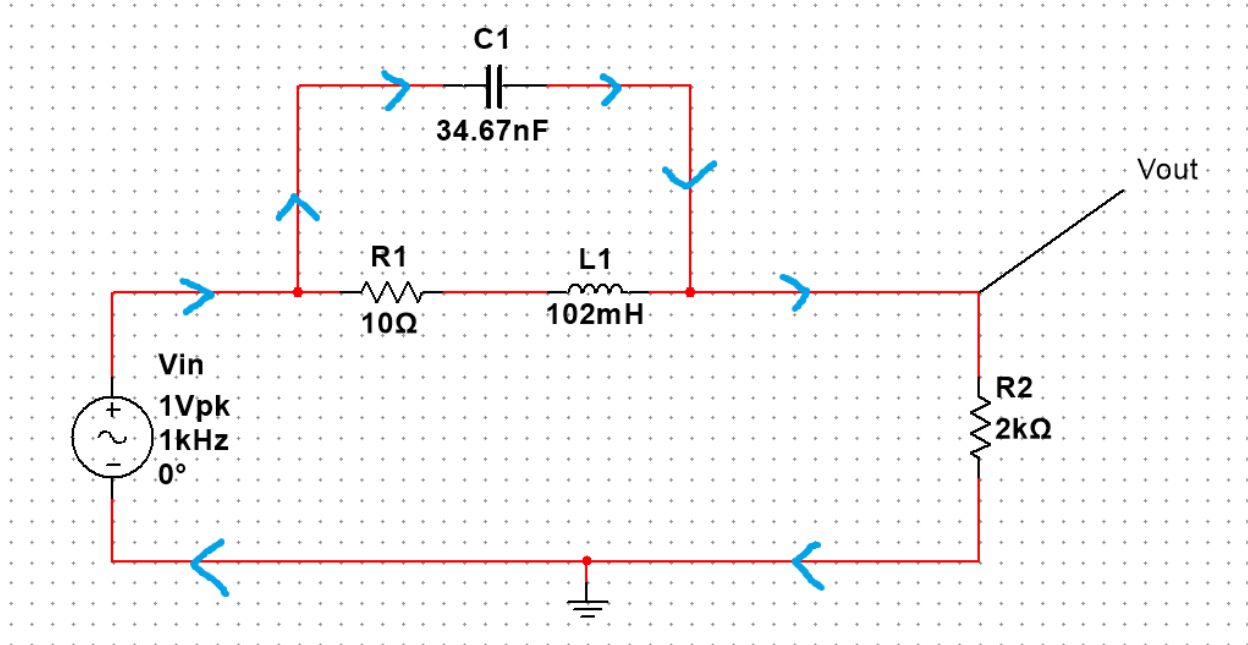
Conversely, if a filter can only take in low frequencies and reject high frequencies, it is called a low-pass filter. Under low frequencies, a capacitor will have infinite impedance (leading to an open circuit) and an inductor will have zero impedance (leading to a short).

High frequency

We will first observe the behavior of the second transfer function ($H_2(j\omega)$) under high frequencies and deduce if the circuit is a high-pass filter or not.

$$\begin{aligned} H(j\omega) &= \frac{2000(CL\omega^2 - 10j\omega C - 1)}{2000CL\omega^2 - 20000j\omega C - j\omega L - 2010} \\ H(j\omega) &= \frac{2000CL\omega^2 - 20000j\omega C - 2000}{2000CL\omega^2 - 20000j\omega C - j\omega L - 2010} \\ H(j\omega) &= \frac{\left(\frac{1}{\omega^2}\right)(2000CL\omega^2 - 20000j\omega C - 2000)}{\left(\frac{1}{\omega^2}\right)(2000CL\omega^2 - 20000j\omega C - j\omega L - 2010)} \\ H(j\omega) &= \frac{\frac{2000CL\omega^2}{\omega^2} - \frac{20000j\omega C}{\omega^2} - \frac{2000}{\omega^2}}{\frac{2000CL\omega^2}{\omega^2} - \frac{20000j\omega C}{\omega^2} - \frac{j\omega L}{\omega^2} - \frac{2010}{\omega^2}} \\ \lim_{\omega \rightarrow \infty} H(j\omega) &= \frac{2000CL}{2000CL} = 1 \end{aligned}$$

Because $H(j\omega) \neq 0$, we can say that the circuit is indeed a high-pass filter. This makes sense because the inductor would reach infinite impedance and the capacitor will reach zero impedance causing all the current to take the path shown below.



Low frequency

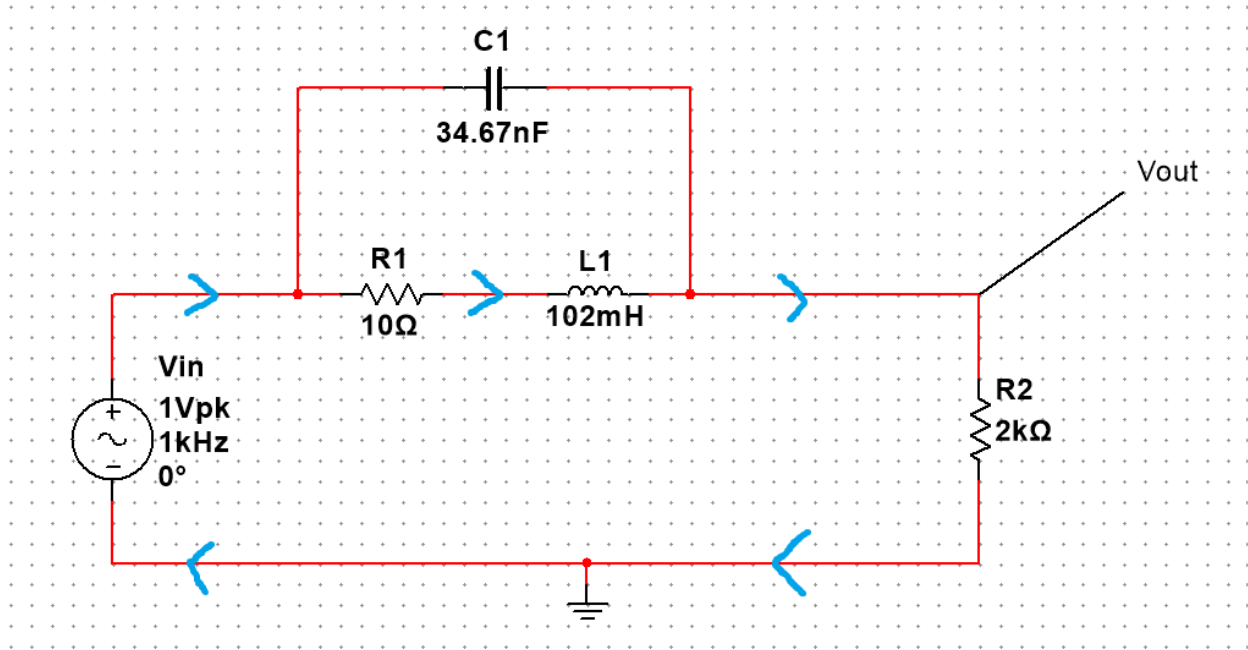
We also need to examine the circuit at low frequencies to either: confirm that it is a high-pass filter or to deduce that it is a bandstop filter.

$$H(j\omega) = \frac{2000(CL\omega^2 - 10j\omega C - 1)}{2000CL\omega^2 - 20000j\omega C - j\omega L - 2010}$$

$$\lim_{\omega \rightarrow 0} H(j\omega) = \frac{-2000}{-2010} = 0.995 \approx 1$$

We have taken the approximation to be 1 for a very important reason. The 10Ω resistor is a renegade resistor. This means that its value is so small as compared to the impedances of the capacitor and inductor that we can assume that it is not even there. As we shall see later, the resistance of the 10Ω resistor does not even affect the resonance effect. Thus, it is a good approximation.

Because $H(j\omega) \neq 0$, we can say that the circuit is also a low pass filter. This also makes sense because the capacitor would reach infinite impedance and the inductor would have a zero impedance which will cause all the current to move through the inductor branch only as shown below.



Bandstop filter and Resonance

All the conclusions from the analysis from high and low frequencies lead to the fact that the circuit is a bandstop filter circuit. This is because of its behaviours at high and low frequencies which tells us that it allows all frequencies except for those around resonance.

Again, we shall ignore the renegade resistor in our calculations. We shall take the inductor and capacitor in parallel and derive the formula for resonance frequency. From there we shall calculate the center frequency.

$$Z_C \parallel Z_L = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{-j}{\omega C - \frac{1}{\omega L}}$$

$$Z_{par} \rightarrow \infty \text{ when } \omega C - \frac{1}{\omega L} = 0$$

$$\therefore \omega = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(102 \times 10^{-3}) \cdot (34.67 \times 10^{-9})}}$$

$$\omega_0 = 2676 \text{ kHz}$$

The above derivation tells us that in order for Z_{par} to approach infinity, we must take the denominator to be zero. That denominator will give us the frequency at which resonance will occur. At this point Z_{par} will be infinite and we will have no current flowing, thus giving us the drop in our Bode plot.

In order to analyse why we need to ignore the 10Ω we shall do the resonance calculations with the renegade resistor. Without the 10Ω resistor we get 2676 Hz where:

$$|Z_L| = |Z_C| = \sqrt{\frac{L}{C}} = 1715 \Omega$$

This means that adding the resistor only accounts for only $\left(\frac{10}{1715} \times 100\right)\%$ which equates to only 0.583%. Thus, it is a good assumption to ignore the renegade resistor.

-3db frequencies

Because our circuit is a notch filter, we know that the value of $|H|_{max} = 1$ from our above calculations on how the transfer function responds to high and low frequencies. To find out the -3 dB frequencies, we will first find the location of the frequencies using:

$$|H| = \frac{|H|_{max}}{\sqrt{2}}$$

$$\therefore |H| = \frac{1}{\sqrt{2}}$$

By equating this to the first transfer function ($H_1(j\omega)$), we get the equation:

$$\frac{1}{10j\omega C + \frac{L}{C}} = \frac{1}{\sqrt{2}}$$

$$\frac{2000}{j\omega C} + 20000 + 2000j\omega L$$

$$\frac{10j\omega C + \frac{L}{C}}{\frac{2000}{j\omega C} + 20000 + 2000j\omega L} = \sqrt{2} - 1$$

Solving the equation in Maple gives us the following results for omega:

$$-22048.57837I, -12670.42254I$$

When we take the magnitude of these values and divide by 2π we get the -3 dB frequencies:

$$f_1 = 3509 \text{ Hz}$$

$$f_2 = 2017 \text{ Hz}$$

Table of Analytical Solution Results

Parameter	Value
Center Frequency	2.676 kHz
-3 dB frequency 1	2.017 kHz
-3 dB frequency 2	3.509 kHz

Maple Analysis

```
> restart;
Vin:=1:
R1:=10: R2:=2000:
ZL:=I*omega*L: ZC:=1/(I*omega*C):
Zpar:= (1/(R1+ZL)+1/ZC)^(-1);
H:=simplify(R2/(Zpar+R2));
C:=34.67e-9: L:=102e-3:
H_further_simplified:=simplify(R2/(Zpar+R2));
cent_omega:=1/sqrt(L*C);
cent_freq:=cent_omega/(2*3.14159);
solve((H_further_simplified)=(1/sqrt(2)))
```

$$Z_{par} := \frac{1}{\frac{1}{10 + I\omega L} + I\omega C}$$

$$H := \frac{2000 (CL\omega^2 - 10I\omega C - 1)}{2000 CL\omega^2 - 20000I\omega C - I\omega L - 2010}$$

$$H_{further_simplified} := \frac{2000. + 0.0006934000000I\omega - 7.07268 \times 10^{-6} \omega^2}{2010. + 0.1026934000I\omega - 7.07268 \times 10^{-6} \omega^2}$$

$$cent_omega := 16816.01141$$

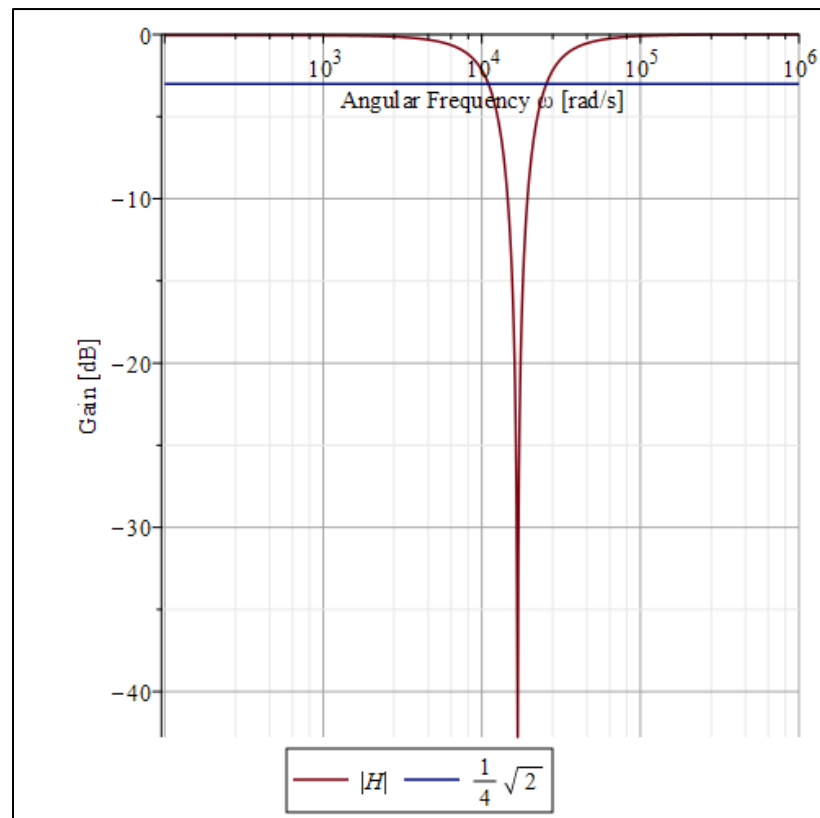
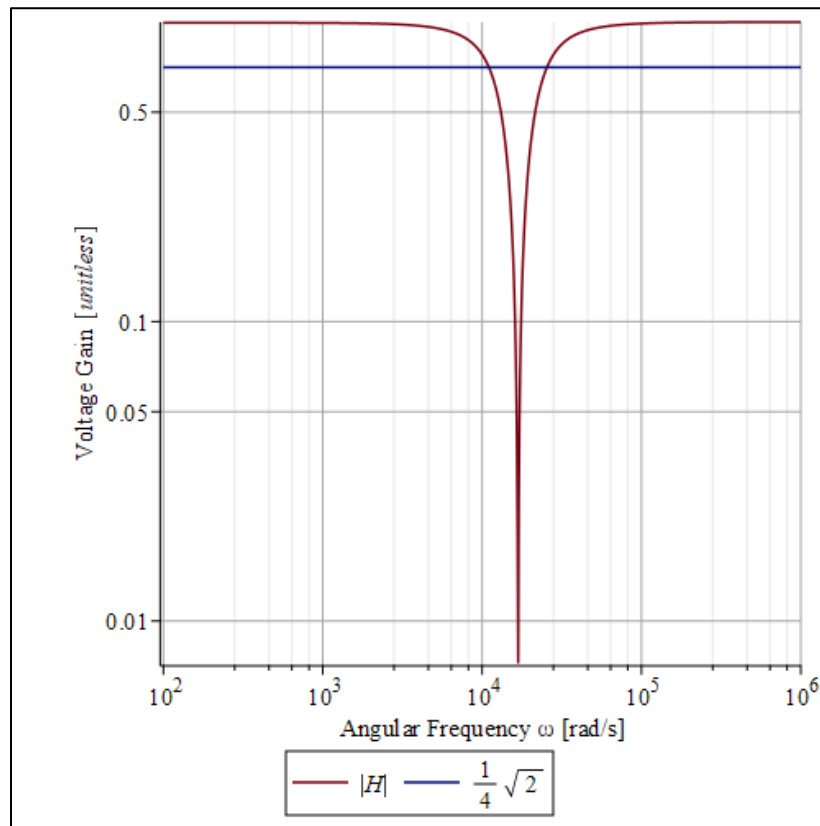
$$cent_freq := 2676.353599$$

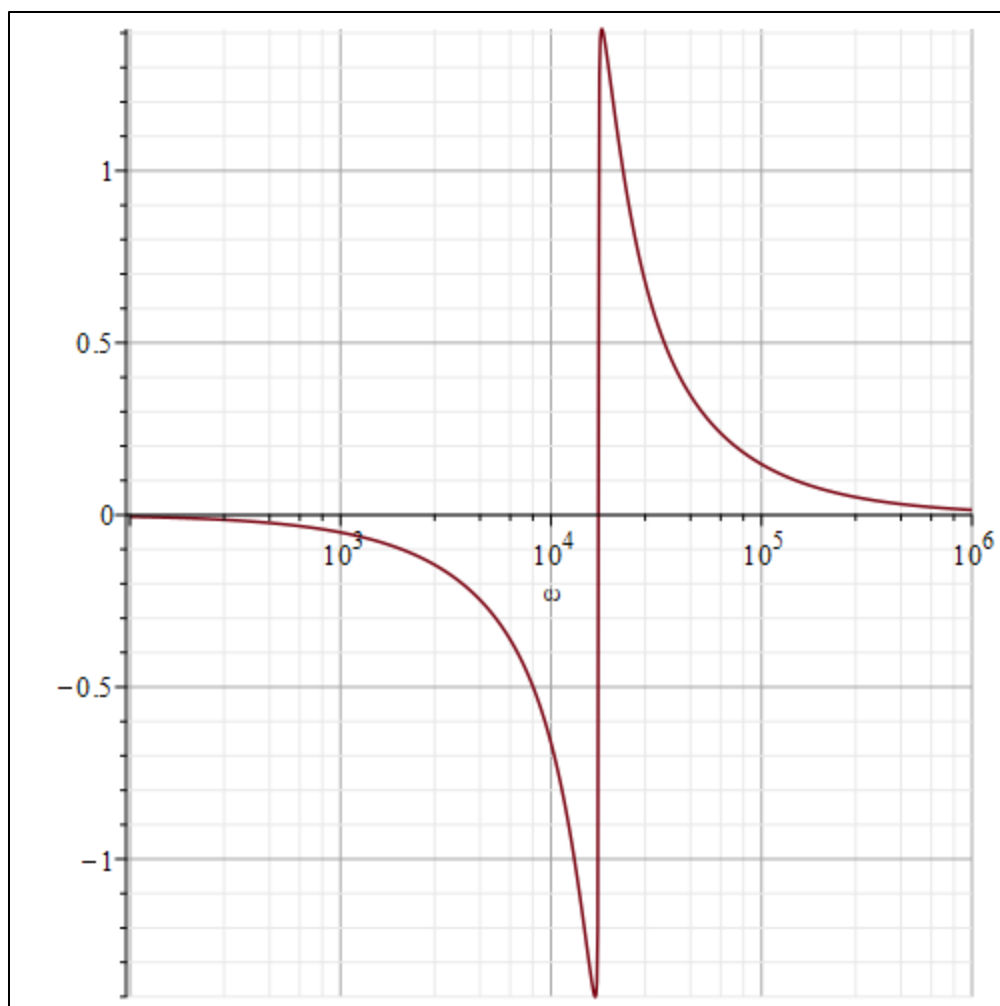
$$-22048.57837I, -12670.42254I$$

```
> with(plots):
loglogplot([abs(H_further_simplified), 1/sqrt(2)],
omega=100...1e6, numpoints = 1000, gridlines =
true, labels=[typeset("Angular Frequency ", omega, " [rad/s]"),
typeset("Voltage Gain ", [unitless])],
legend=[abs('H'), (1/2)/sqrt(2)],
labeldirections=[horizontal, vertical]);

semilogplot([20*log10(abs(H_further_simplified)), 20*log10(1/sqrt(2))], omega=100...1e6, numpoints = 1000, gridlines = true,
labels=[typeset("Angular Frequency ", omega, " [rad/s]"),
typeset("Gain [dB]"), legend=[abs('H'), (1/2)/sqrt(2)],
labeldirections=[horizontal, vertical]);

semilogplot([argument(H_further_simplified)], omega=100...1e6,
numpoints = 1000, gridlines = true);
```





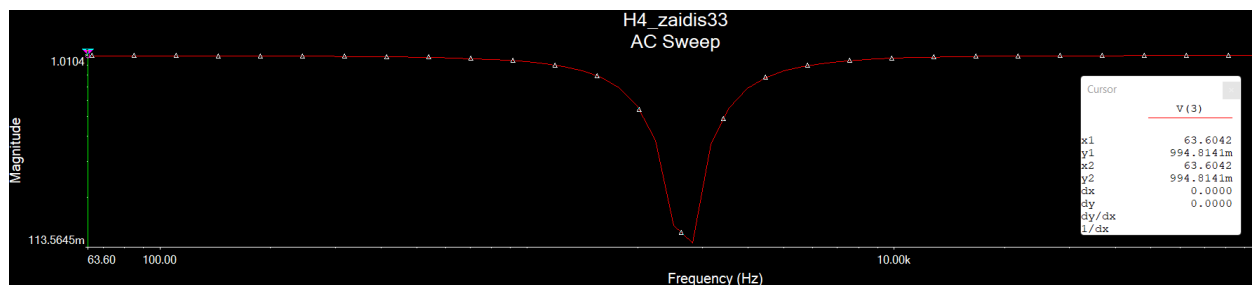
Digital Solution

For my digital solution, I created the circuit in Multisim and simulated it using two methods: AC sweep and Bode plotter.

AC Sweep (Magnitude)

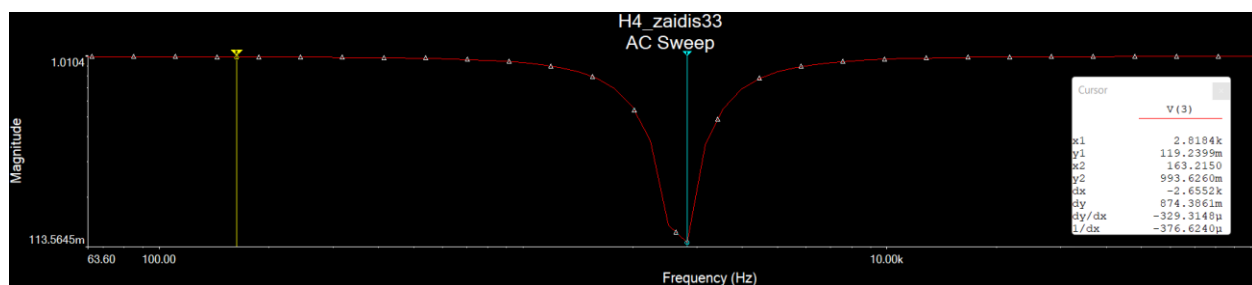
In the AC sweep I had Vout as my output variable. The domain of the plot was from 1 Hz to 1 GHz. I used 20 points per decade and had a logarithmic scale to show the Bode plot more clearly.

The magnitude vs. frequency bode plot is shown below:



I enabled the “show all cursors” option to measure the center and -3dB frequencies.

For measuring my center frequency, I right-clicked on the cursor and selected the “Y-min” option. This moved the cursor to the point where the gain is the lowest thus giving me the center frequency.



Cursor	
V (3)	
x1	2.8184k
y1	119.2399m
x2	163.2150
y2	993.6260m
dx	-2.6552k
dy	874.3861m
dy/dx	-329.3148μ
1/dx	-376.6240μ

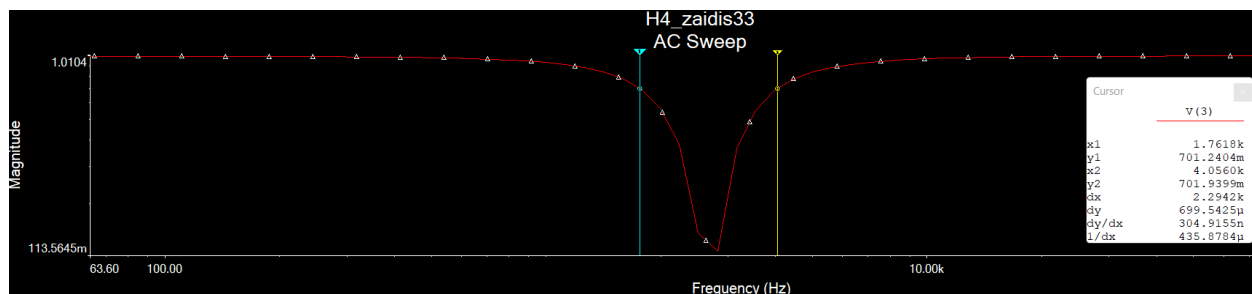
I also found out the maximum gain by right clicking on the second cursor and choosing the “Y-max” option. With this, I can now calculate the gain where the output is half (-3dB frequencies).

$$|H| = \frac{|H|_{max}}{\sqrt{2}}$$

$$|H|_{max} = 0.993$$

$$\therefore |H| = \frac{0.993}{\sqrt{2}} = 0.702$$

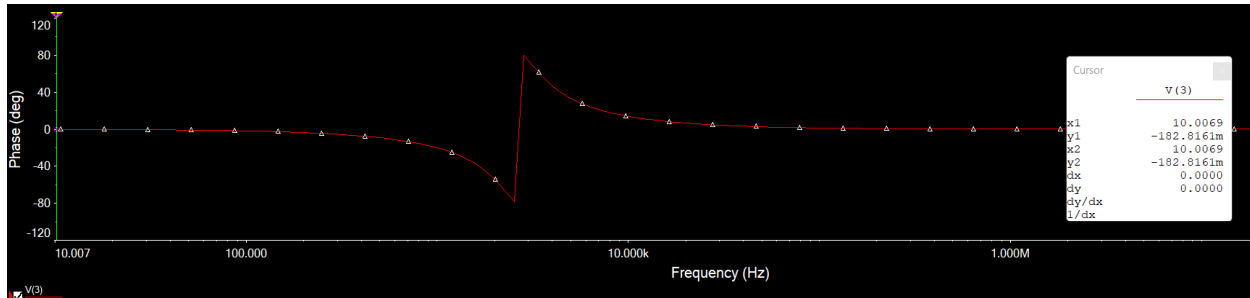
I now set my y-values to be exactly 0.702 which will give me the -3dB frequencies.



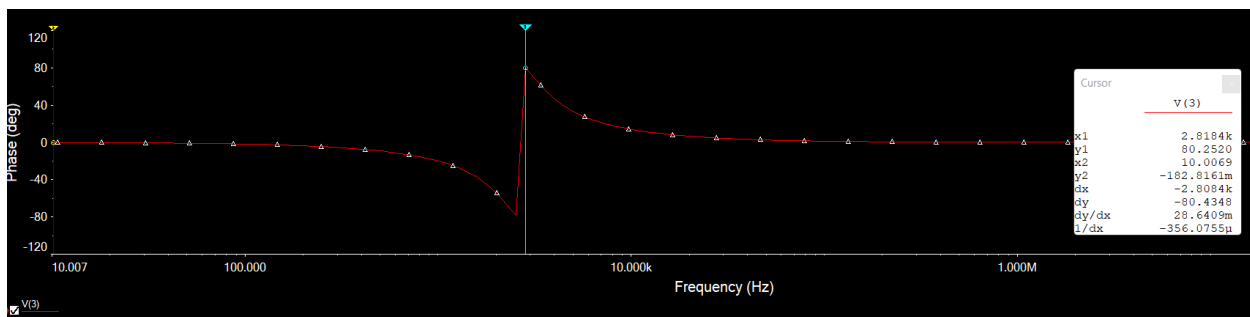
Cursor	
V (3)	
x1	1.7618k
y1	701.2404m
x2	4.0560k
y2	701.9399m
dx	2.2942k
dy	699.5425μ
dy/dx	304.9155n
1/dx	435.8784μ

AC Sweep (Phase)

The AC sweep produced two plots, one for magnitude and the other for phase. For phase, the Bode plot produced is:



I used my center frequency calculated previously to find out the phase at that point.

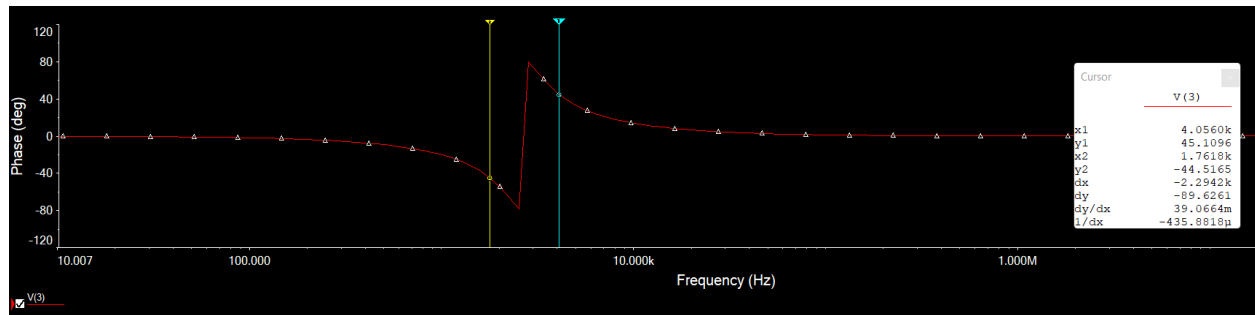


Cursor

V (3)

x1	2.8184k
y1	80.2520
x2	10.0069
y2	-182.8161m
dx	-2.8084k
dy	-80.4348
dy/dx	28.6409m
1/dx	-356.0755u

After that, I used my -3dB frequencies to determine the phase at those points.



Cursor

V (3)

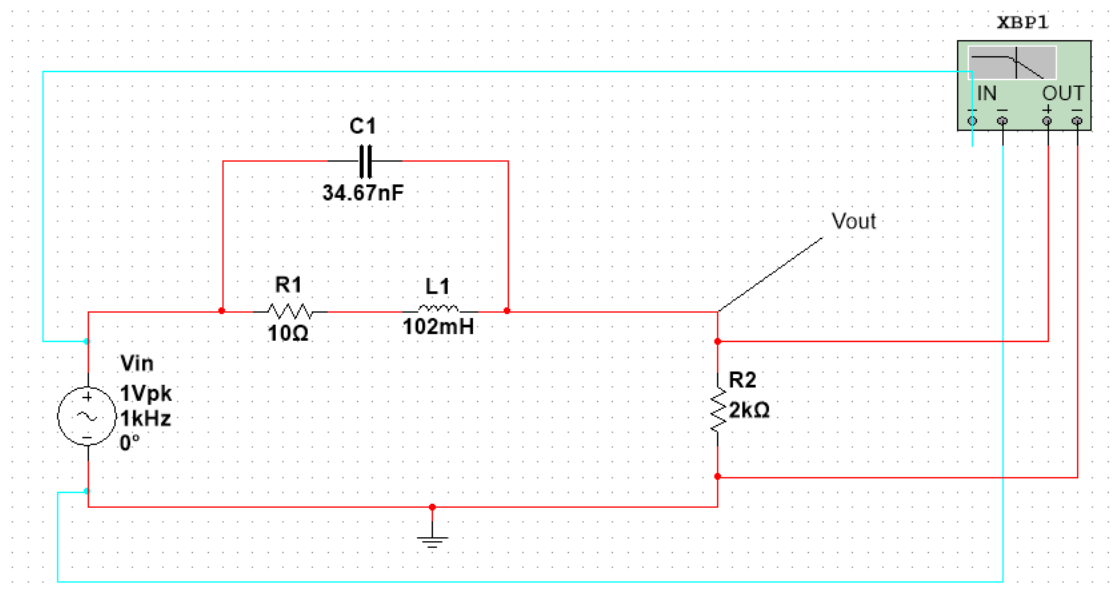
x1	4.0560k
y1	45.1096
x2	1.7618k
y2	-44.5165
dx	-2.2942k
dy	-89.6261
dy/dx	39.0664m
1/dx	-435.8818μ

Summary of AC Sweep Results

Parameter	Frequency (kHz)	Voltage Gain	Phase Shift (deg)
Center Frequency	2.818	0.119	80.25
-3dB Frequency 1	1.762	0.701	-44.52
3dB Frequency 2	4.056	0.701	45.11

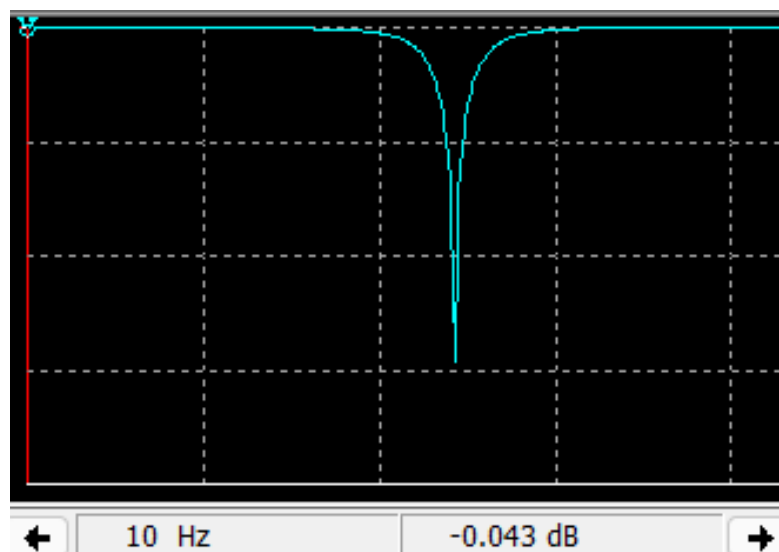
Bode Plotter (Magnitude)

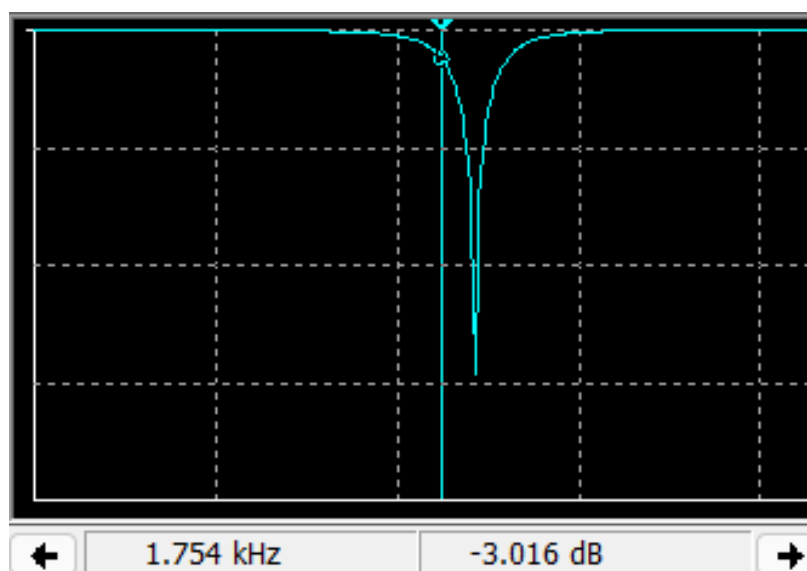
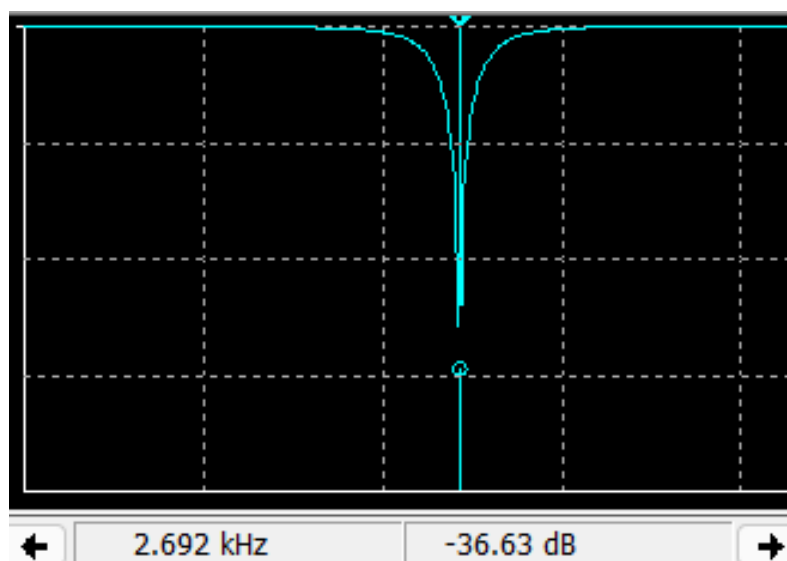
For the Bode plotter method, I had to attach the Bode plotter to the positive and negative sides of the supply and the output resistor (R2) as shown below:

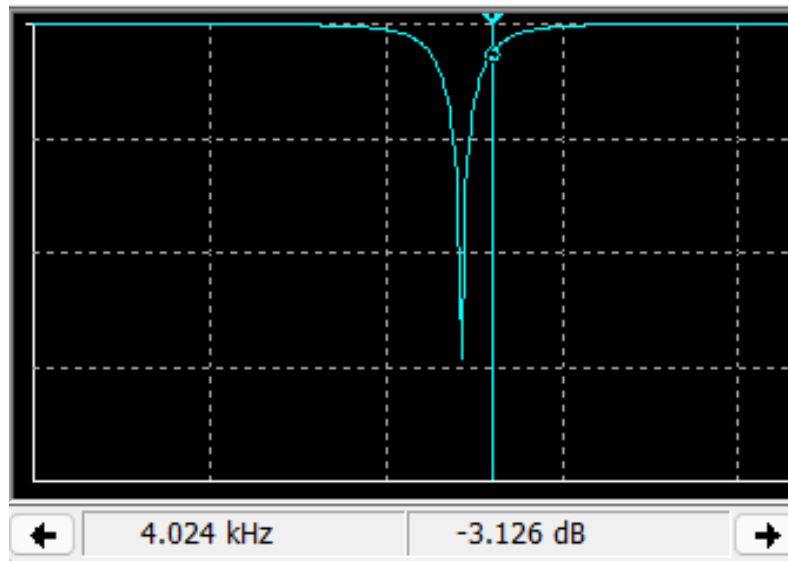


I repeated the digital process in order to confirm my results from the AC sweep and to take an average of the results if they slightly differ.

The process of finding the magnitude is the same as that of the AC sweep but the only difference was that there was one cursor so we could not measure the -3dB frequencies on the same plot.

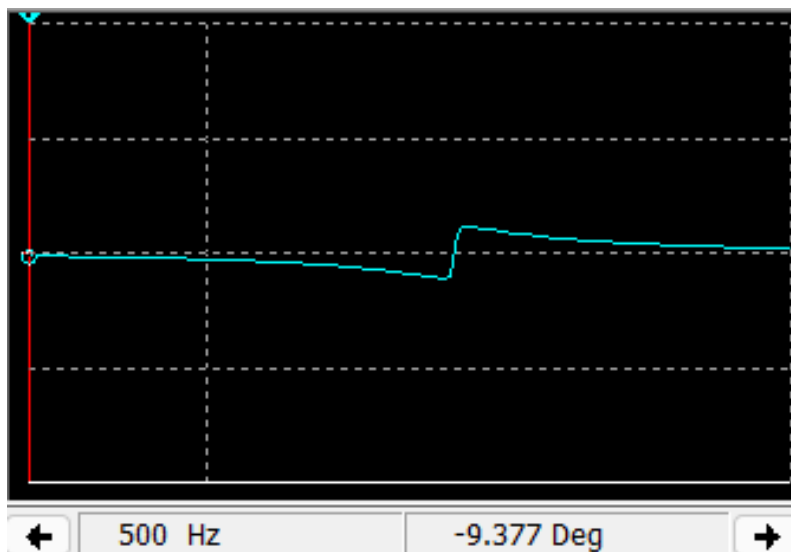


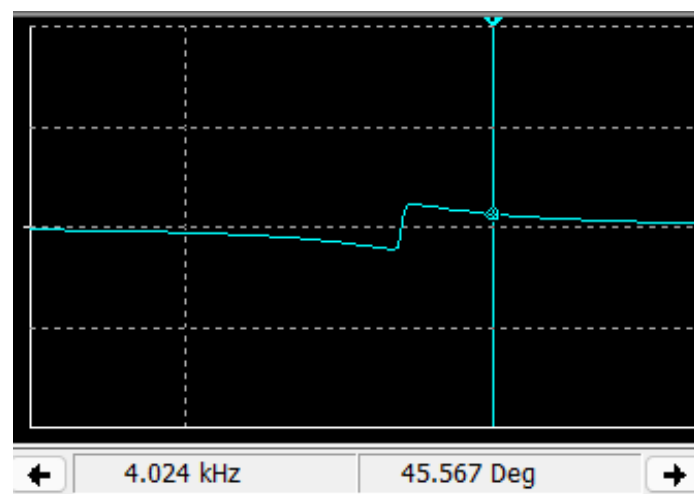
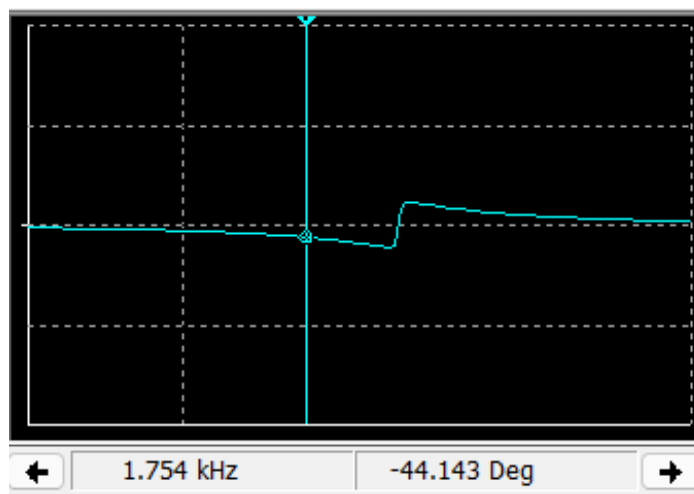
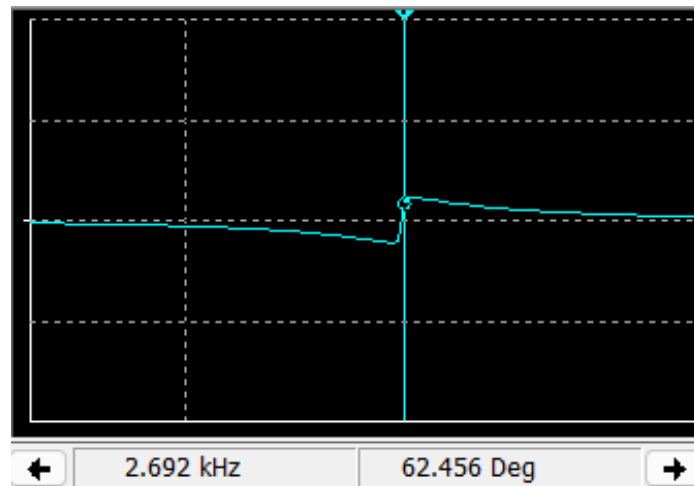




Bode Plotter (Phase)

Finding the phase at the center and -3dB frequencies is the same as that of the AC sweep but in the Bode Plotter, we only have one cursor thus we could not measure the -3dB frequencies on one axis.





Summary of Bode Plotter Results

Parameter	Frequency (kHz)	Phase Shift (deg)
Center Frequency	2.692	63.46
-3dB Frequency 1	1.754	-44.14
-3dB Frequency 2	4.024	45.57

Table of Digital Solution Results

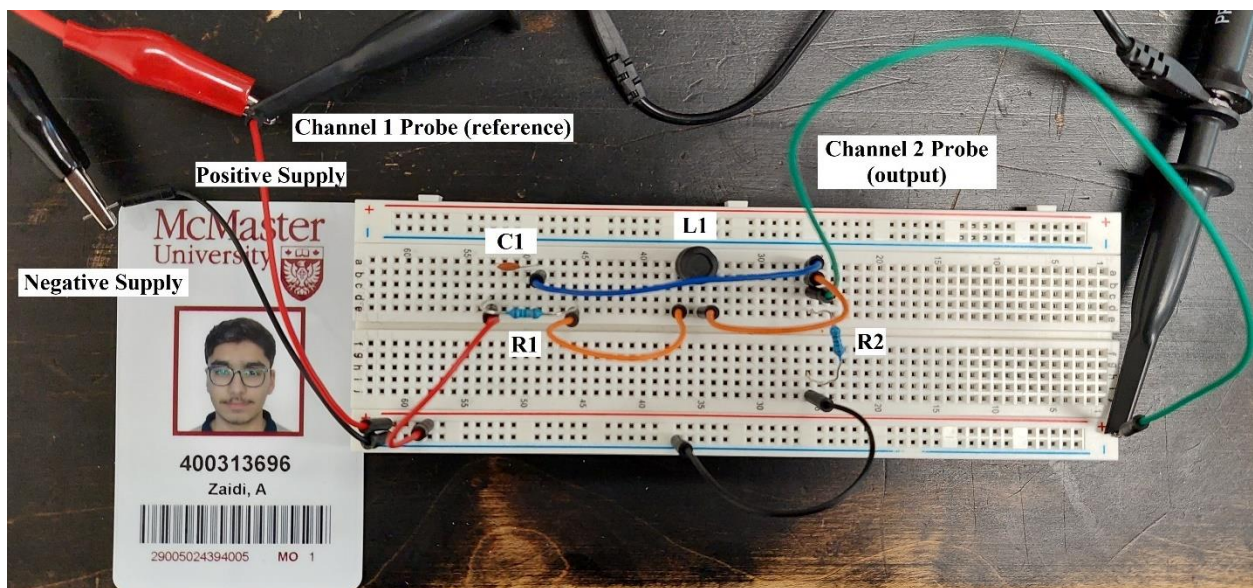
Now I will take the average of the AC sweep and Bode Plotter results.

Parameter	Frequency (kHz)	Voltage Gain	Phase Shift (deg)
Center Frequency	2.755	0.119	71.85
-3dB Frequency 1	1.758	0.701	-44.33
3dB Frequency 2	4.040	0.701	45.34

Physical Solution

For my physical circuit, I built the circuit on a breadboard and connected the components as shown below. I then used the Hantek Oscilloscope to measure the output. The red wire represents the positive signal path, and the black wire represents the negative signal path.

The channel 1 (reference probe) is connected to the positive supply wire. Channel 1 will be seen as the yellow curve on the Hantek. The channel 2 (output probe) can be seen as the green wire connected just after the resistor R2. It will be seen as the green curve on the Hantek.



I also measured the values of the components using the Digital Multimeter of the Hantek. Note that the value of the capacitor is just one of the three values I used to calculate the average which I used in my circuit.

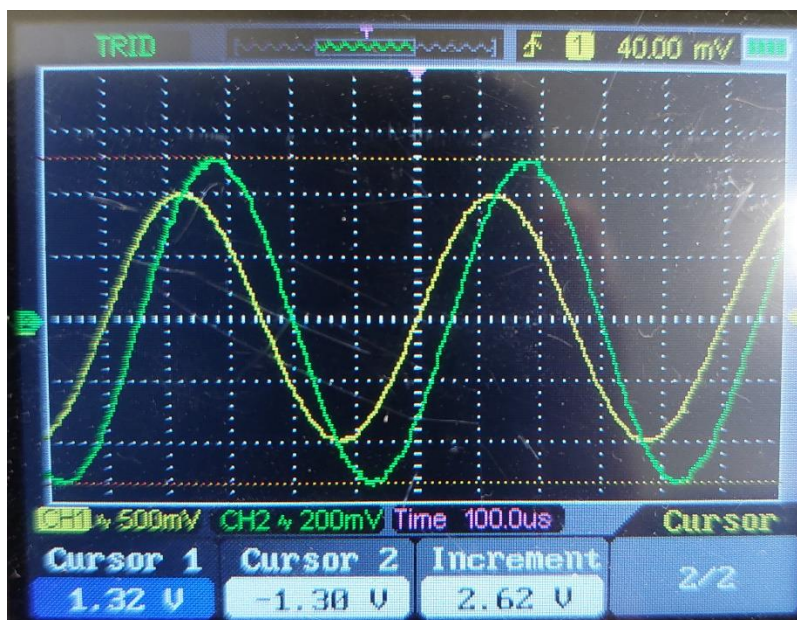


The constraint we have with the Hantek is that it can not create a Bode plot because it can only measure the output at a certain frequency only. Thus, I shall measure the output voltage and time

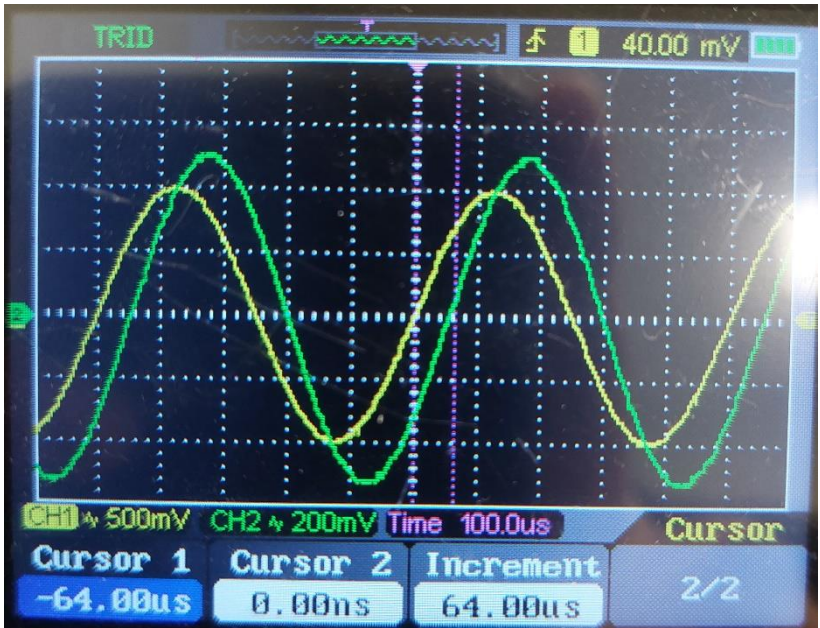
difference at 10 separate frequencies to get the voltage gain and phase at that point. With that I shall create a Bode plot on a graph plotter.

I shall demonstrate the method with the 2 kHz frequency. First, I will calibrate the Hantek as it might give an incorrect reading at low frequencies. Then I will connect the channel 2 probe to the green wire as seen in the figure above. I will then set the wave as a sine wave, the coupling to be AC and the frequency to be 2 kHz. This will give me the wave shown below.

I will then use my voltage cursors to measure the peak-to-peak voltage which will be equal to the output voltage at that specific frequency.

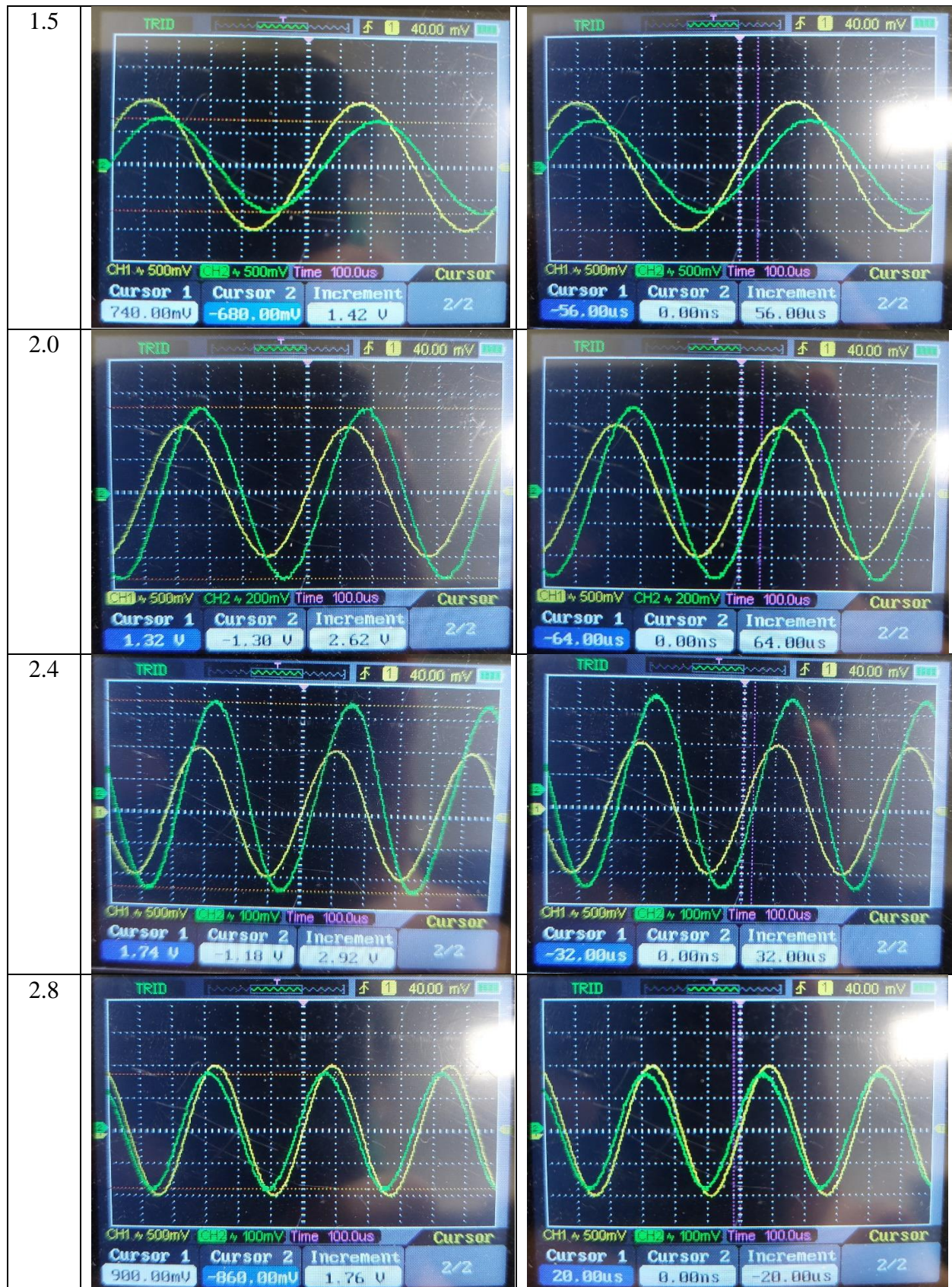


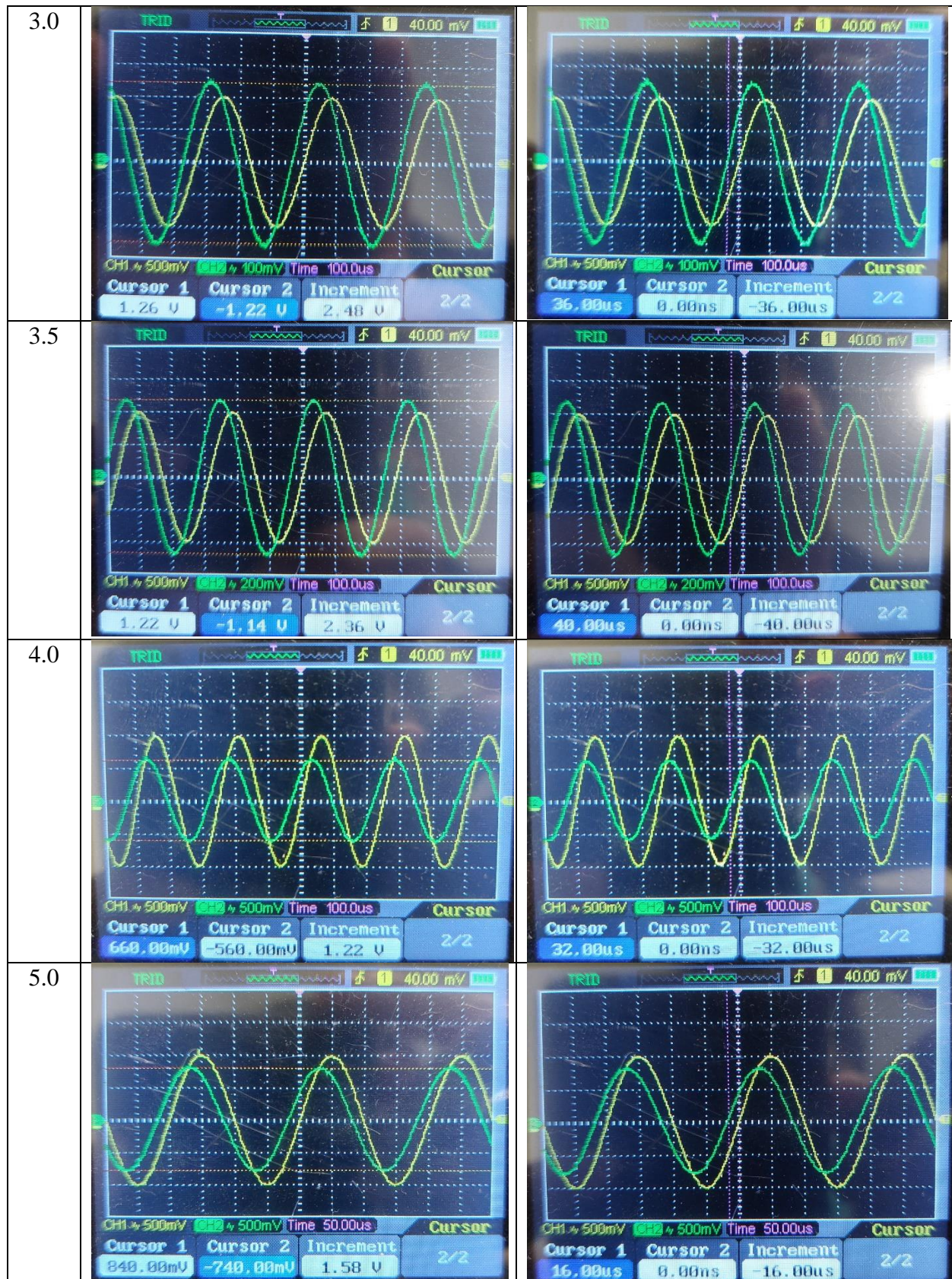
Then I will enable my time cursors and measure the time difference between the input and output wave at that specific frequency.



Repeating the same procedure for 9 other frequencies (concentrated mostly around the center frequency) we will get the following results.

Freq / kHz	Voltage difference	Time difference
1.0	<p>The screenshot shows an oscilloscope display with two sine waves, CH1 (yellow) and CH2 (green). The CH1 signal has a peak-to-peak voltage of 500mV, and the CH2 signal has a peak-to-peak voltage of 500mV. The time scale is set to 100.0us. The cursor settings show a voltage difference of 1.60 V between the two signals. The display also shows a time difference of 56.00us.</p>	<p>The screenshot shows an oscilloscope display with two sine waves, CH1 (yellow) and CH2 (green). The CH1 signal has a peak-to-peak voltage of 500mV, and the CH2 signal has a peak-to-peak voltage of 500mV. The time scale is set to 100.0us. The cursor settings show a time difference of 56.00us between the two signals. The display also shows a voltage difference of 56.00us.</p>







I collected all my terms in two tables, one for the gain and the other for phase.

Frequency / kHz	V max / V	V min / V	V amp / V	V Gain	Gain / dB
1.0	0.84	-0.76	1.60	1.60	4.08
1.5	0.74	-0.68	1.42	1.42	3.05
2.0	1.32	-1.20	2.62	2.62	8.37
2.4	1.74	-1.18	2.92	2.92	9.31
2.8	0.90	-0.86	1.76	1.76	4.91
3.0	1.26	-1.22	2.48	2.48	7.89
3.5	1.22	-1.14	2.36	2.36	7.46
4.0	0.66	-0.56	1.22	1.22	1.73
5.0	0.84	-0.74	1.58	1.58	3.97
10.0	1.00	-0.90	1.90	1.90	5.58

Frequency / kHz	Time difference / μs	Phase (deg)
1.0	56	20.16
1.5	56	30.24
2.0	64	46.08
2.4	32	27.65
2.8	-20	-20.16
3.0	-36	-38.88
3.5	-40	-50.40
4.0	-32	-46.08
5.0	-16	-28.80
10.0	-4	-14.40

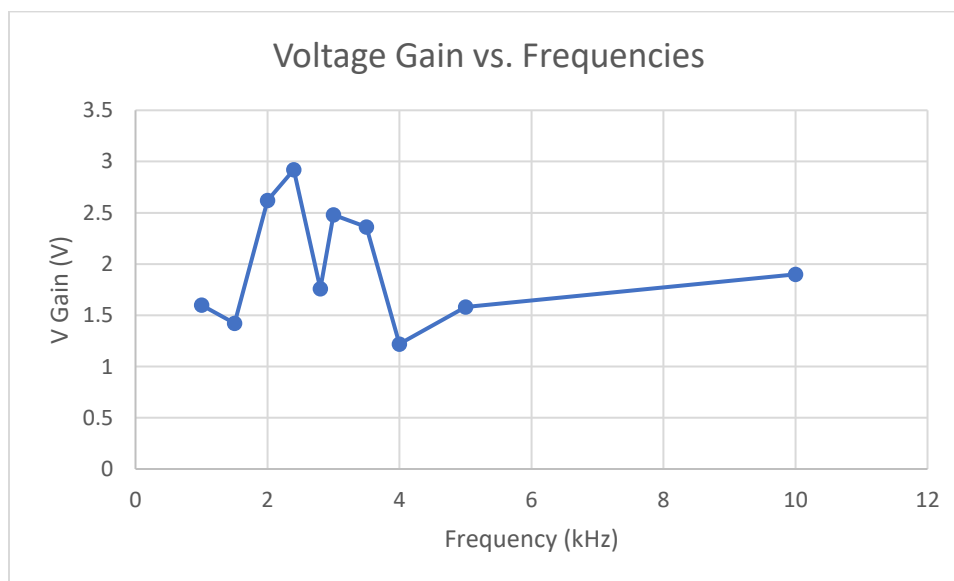
I found out the value of the amplitude, gain and phase by using the following formulae:

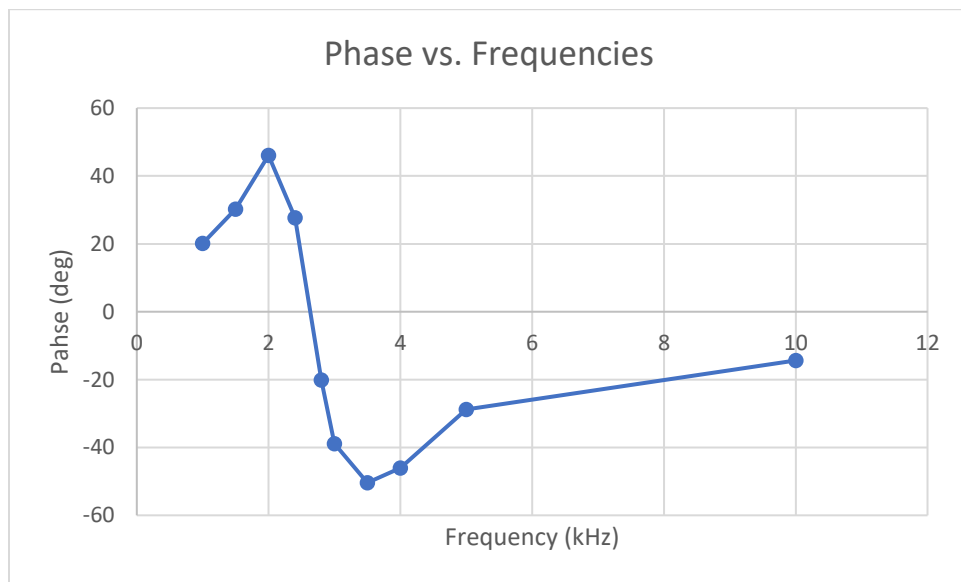
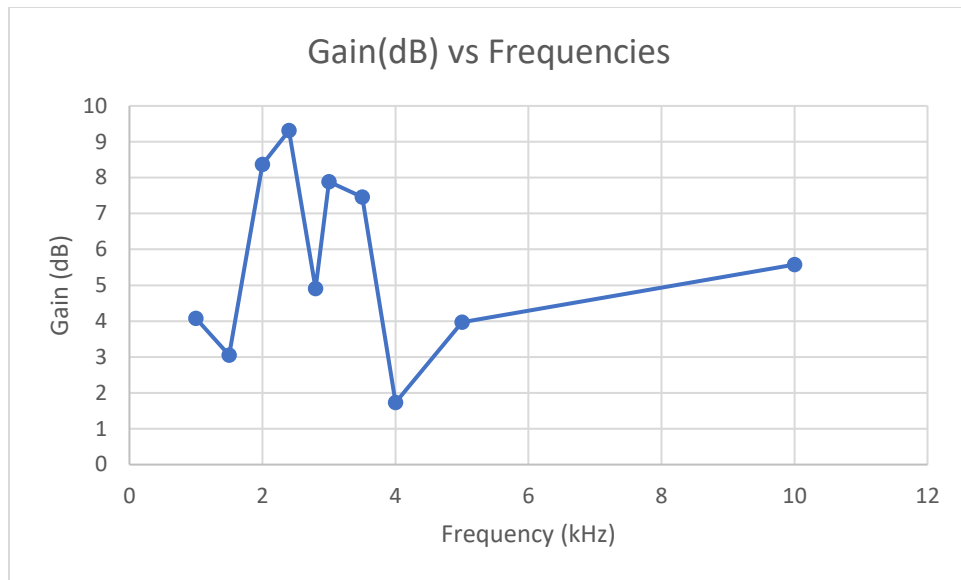
V Amplitude	$\frac{(V_{max} - V_{min})}{2}$
V Gain	$\frac{V_{amp}}{V_{in}}$
Gain	$20 \cdot \log_{10} \left(\frac{V_{amp}}{V_{in}} \right)$
Phase	$360 \cdot \left(\frac{\frac{time\ difference}{1}}{frequency} \right)$

```
> v:=1.90;
Gain_dB:=20*(log(V)/log(10)): evalf(%)

> f:=10000;
t:=-4e-6;
Phase:=360*(t/(1/f))
```

I then used Microsoft Excel to plot my graphs and the results are shown below.





The gain vs. frequency graphs have a bit of anomaly in points but one can clearly see the “dip” at 2.8 kHz. The phase graph, however, was very close to the Maple graph in the analytical solution.

Table of Physical Solution Results

Parameter	Frequency (kHz)	Voltage Gain	Phase Shift (deg)
Center Frequency	2.8 ± 4	1.76 ± 1	-20.16
-3dB Frequency 1	1.5 ± 3	0.707	-46.08
3dB Frequency 2	4.0 ± 1	0.707	30.24

Summary of Results

Analytical Solution

Parameter	Value
Center Frequency	2.676 kHz
-3 dB frequency 1	2.017 kHz
-3 dB frequency 2	3.509 kHz

Digital Solution

Parameter	Frequency (kHz)	Voltage Gain	Phase Shift (deg)
Center Frequency	2.755	0.119	71.85
-3dB Frequency 1	1.758	0.701	-44.33
3dB Frequency 2	4.040	0.701	45.34

Physical Solution

Parameter	Frequency (kHz)	Voltage Gain	Phase Shift (deg)
Center Frequency	2.8 ± 4	1.76 ± 1	-20.16
-3dB Frequency 1	1.5 ± 3	0.707	-46.08
3dB Frequency 2	4.0 ± 1	0.707	30.24

Reflection

The results from my analytical and digital solution were very close. However, my physical solution results were relatively inaccurate to the true values. The main problem I had was with the cursors. In the digital solution, when using the Bode Plotter, I could not get the cursor to be at exactly -3dB thus I just took the closest value to it. That was the reason why I also used the AC sweep to produce another set of results and averaged it to produce a very accurate digital solution.

My physical solution had a lot of errors. This was mainly because of the low number of readings I took. Even though ten readings seemed enough, I found out that some of my readings were anomalous which resulted in error. If the Hantek could produce a Bode plot, then I'm sure that there would be far less error as my setup seemed fine.

Even after all the errors, I still learnt a lot about how a range of frequencies affect the circuit. I learnt the capacitors and inductors response to high and low frequencies (see Types of filters). I learnt about the different types of filters and how they use the theory behind capacitors and inductors to allow (or disallow) frequencies of high and low values.

To challenge myself and to show my knowledge on this topic, I chose the bandstop filter (which was a combination of the low-pass and high-pass filter). I used a renegade resistor to show its negligible effect on resonance and then found out the center and -3dB frequencies.

Overall, this lab was a combination of being both challenging and fun. I learnt a lot about bandstop filters when I was researching their effects on the internet. I learnt that they were used in audio systems to remove interfering frequencies. This makes sense because if we create our circuit which has a resonance frequency of the interfering frequency then its gain would be, theoretically, zero thus removing all unwanted signals.