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Lab 5

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Deliverables

1. Choose an input frequency for your supply to work at (one high enough that your inductors and resistors have the same order of magnitude of impedance)
2. Make a clearly defined "load network" using at least one inductor and at least two resistors (remembering to include the series resistance of your "inductor"), placing components in parallel or series as you like, so that the power factor is $< .9$ and lagging (inductive load), and have a separate small "transmission line resistance" like in the sample lab. Calculate what the power factor is, showing your work.
3. Analytically find the capacitance that *would* correct the power factor of the load network, showing your work
4. You probably can't create this capacitance with the ceramic capacitors you have on hand, so go back and change your question (either your supply frequency, your load network, or both) until you can. Redefine this as your new load network power factor correction problem and find the new power factor for this load network.
5. For the new problem, *demonstrate* the power factor correction by making your tri-solve objective to:
    a. Find the voltage and current (both amplitude and phase for each) and the complex power (both real and reactive power), both before and after adding the parallel capacitor to correct the power factor.
        i. Hint: position the small "transmission line resistance resistor" like in the sample lab so you can use it to measure the source current
    b. i.e., demonstrate that it worked for all 3 methods (analytical, multisim, and physical measurement), by calculating or measuring the source voltage and current phasors before and after adding the capacitor(s) and calculating the complex power.
6. Analytical: compare the three methods

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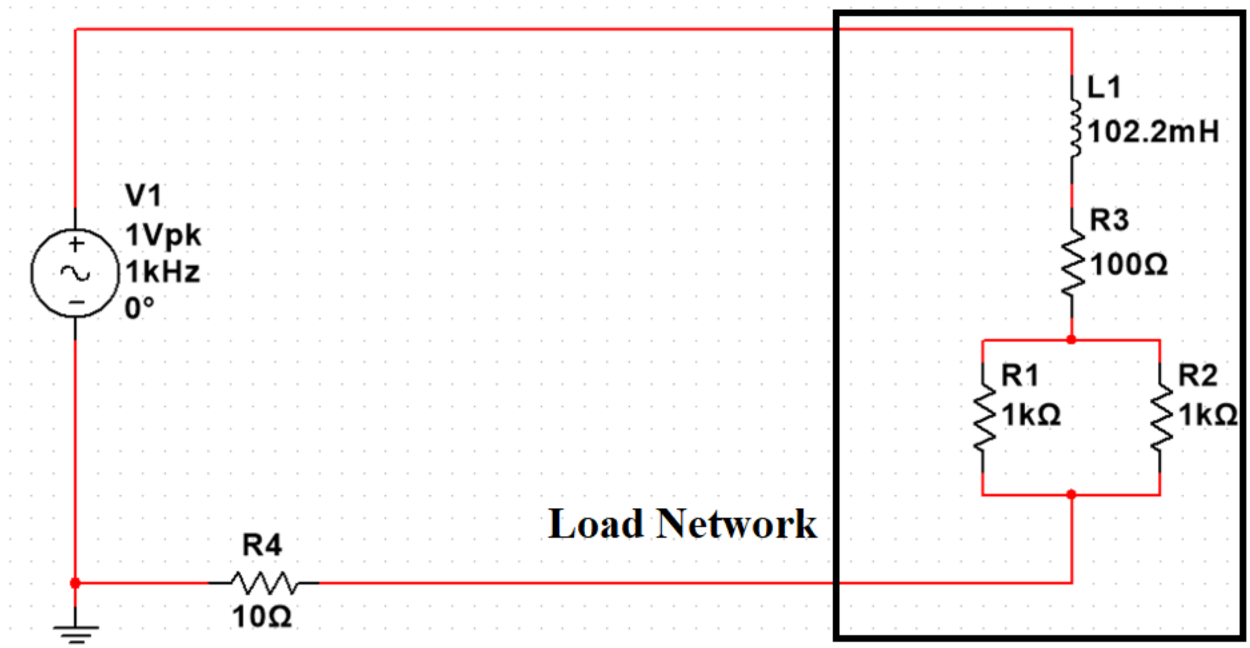
Introduction

In this lab, I will be tri-solving an AC power supply circuit which will consist of a load network and a transmission line. I will be tri-solving the circuit analytically, digitally, and physically. For my analysis, I will use Maple to find out variables (like voltage of the load) and will also determine the exact capacitance which will correct the power factor. For my digital solution, I shall replicate the circuit in Mulitsim and use the Tektronix Oscilloscope to measure the voltage amplitudes and the phase between the curves. Lastly, I shall build my circuit physically on a breadboard and use the Hantek Oscilloscope to repeat the same thing.



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Analytical Solution

I will first perform calculations and use equations to determine the voltages, current various forms of power seen through the circuit. Also, note that I have included the Maple calculations in my explanation in order to show the calculated variable in each step of the analysis.

Voltage divider

To find out the apparent power, first we shall consider finding out the voltages and currents between the load and R4. The total impedance of the load can be seen below:

$$Z_{load} = Z_L + R_3 + R_{par}$$
$$\text{where } R_{par} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

Then using Ohm's law, we shall calculate the value of the total current in the circuit.

$$I = \frac{V}{Z_{load} + R4}$$

Using this value, the voltages of the load and R4 can be easily found out:

$$V_{R4} = I_{load} \cdot R_4$$
$$V_{load} = V_{emf} - V_{R4}$$

Maple Input:

```
f:=1e3: omega:=2*3.14159*f: j:=I:
L:=102.2e-3:
V:=1:  R1:=1e3: R2:=1e3: R3:=100: R4:=10:
ZL:=j*omega*L:
Zpar:=(1/R1+1/R2)^(-1):
Z_load:=ZL+R3+Zpar;
I_load:=V/(Z_load+R4); polar(%);
V_load:=I_load*Z_load; polar(%);
V_R4:=abs(I_load*R4); polar(%);
```

Maple Output:

$$Z\_load := 600.0000000 + 642.140996\,I$$
$$I\_load := 0.0007776197876 - 0.0008185926966\,I$$
$$\text{polar}(0.001129064452, -0.8110613593)$$
$$V\_load := 0.9922238021 + 0.008185926959\,I$$
$$\text{polar}(0.9922575688, 0.008249894054)$$
$$V\_R4 := 0.01129064452$$
$$\text{polar}(0.01129064452, 0.)$$

Forms of Power and Power Factor

In DC circuits the power can be represented as an average value which is the product of the voltage and current. In AC supplies, because there is a continuous change in voltage and current, power representation takes many forms.

The complex power in a circuit can be represented as:

$$S = \frac{V \cdot I^*}{2}$$

In this equation, I* means the complex conjugate of the current phasor. This equation excludes the use of RMS values of voltage and current thus we get the division by 2 ($\sqrt{2} \cdot \sqrt{2} = 2$). The derivation of the complex power in terms of its time and phasor domain can be seen which results in a value containing the sum of a real and imaginary number.

$$S = \left(V_{rms} e^{j\theta_V}\right)\left(I_{rms} e^{-j\theta_I}\right) = V_{rms} I_{rms} e^{j\left(\theta_V - \theta_I\right)}$$
$$S = V_{rms} I_{rms}\left(\cos\theta_Z + j\sin\theta_Z\right)$$
$$S = P_{avg} + jQ$$

In this equation, P_{avg} (or the real power) represents the real component of the complex power.

$$P_{avg} = Re\{S\}$$

Q represents the reactive power which is the imaginary part of the complex power.

$$Q = Im\{S\}$$

The apparent power is the absolute value of the complex power which gives us a positive, real float. Because of its inclusion of the real and imaginary components of complex power, the apparent power is what the electricity-supplying companies use to bill us.

$$S_{app} = \sqrt{(P_{avg})^2 + (Q)^2}$$

And finally, the power factor is the represented by:

$$pf = \frac{P_{avg}}{S_{app}}$$

Later, when relating the phase angle and the power-factor, we shall be using this equation:

$$pf = \cos(\theta_V - \theta_I) = \cos(\theta_Z)$$

Using the above equations, we can solve for the complex power and power factor in Maple which gives us:

Maple Input

```
S:=(V*I_load)/2;
S_real:=Re(S);
S_app:=abs(S);
pf:=S_real/S_app;
```

Maple Output

$$S := 0.0003888098938 - 0.0004092963483\,\mathrm{I}$$

$$S\_real := 0.0003888098938$$

$$S\_app := 0.0005645322260$$

$$pf := 0.6887293159$$

Finding the value of Capacitance

As seen above, the value of the complex power included an imaginary number. This number arises because of the inductor which causes the current to lag. Because of this imaginary number, we get a higher value for the apparent power causing the power factor to decrease. In a purely resistive network, we should have a power factor of 1.

Currently the power factor with the circuit shown above is around 0.689 which is less than 1. Thus, in order to achieve a power factor of 1, we must perform power factor correction. With this method, we shall include a capacitor in parallel with the load network.

The physics behind this is that the capacitor causes the voltage to lead so when it is connected parallel to an inductor (which causes the voltage to lag), it will cause the voltage and current to not have a phase shift causing the imaginary value to equal to zero. This will thus give a power factor of 1 (as $\cos(0°) = 1$). This shall be visualized later in the digital and physical solutions.

We now need to find the exact value of the capacitor that causes the imaginary component of the load to go to zero. Because the imaginary component of the complex power stems only from the impedances, we need that to go to zero which will automatically cause the rest of the values to be real.

In order to concisely determine the capacitance, we need to first define admittance. Admittance is basically the opposite of impedance; it is defined as its reciprocal. It will be used in place of impedance to make the calculations easier.

$$Y_{load} = \frac{1}{Z_{load}}$$

We shall do the following calculations to determine the admittance of the load and equate the imaginary part to zero to find the exact value of the capacitance.

Maple Input

```
#finding the capacitance
Y_load:=1/Z_load;
YC:=(j*omega*C);
Y_load_new:=Y_load+YC;
c:=solve(Im(Y_load_new)=0);
```

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Maple Output

$$Y\_load := 0.0007768548438 - 0.0008314172386\,\mathrm{I}$$
$$YC := 6283.18\,\mathrm{I}\,C$$
$$Y\_load\_new := 0.0007768548438 - 0.0008314172386\,\mathrm{I} + 6283.18\,\mathrm{I}\,C$$
$$c := 1.323242751 \times 10^{-7} + 0.\,\mathrm{I}$$

The calculations suggest that the capacitance should be approximately 132 nF for the power factor to equal to 1.

Power and Voltage in Transmission Line

Because the transmission line (R4) is in series with the load network, we can make the conclusion that the current across R4 is the same as the current across the load network.

We will perform the following calculations to determine the value of the voltage and complex power.

Maple Input

```
#finding power before capacitor is added
VR4:=I_load*R4; polar(%);
SR4:=VR4*conjugate(I_load)/2;
```

Maple Output

$$VR4 := 0.007776197876 - 0.008185926966\,\mathrm{I}$$
$$\mathrm{polar}(0.01129064452, -0.8110613593)$$
$$SR4 := 6.373932685 \times 10^{-6} + 0.\,\mathrm{I}$$

After capacitor added

After the capacitor is added in parallel to the load network, we will repeat the same process above but this time with the impedance of the capacitor added.

Maple Input

```
> #finding power after capacitor is added
ZC:=1/(j*omega*c):
Zload_new:=(1/Z_load+1/ZC)^(-1);
I_load_new:=V/(Zload_new); polar(%);
VR4_after:=I_load_new*R4; polar(%);
```

```
V_load_after:=V-VR4_after;polar(%);
S_after:=V*conjugate(I_load_new)/2;
Sapp_after:=abs(S_after);
S_real_after:=Re(S_after);
pf:=S_real_after/Sapp_after;
SR4:=VR4_after*conjugate(I_load_new)/2;
```

Maple Output

$$Zload\_new := 1287.241765 - 3.313982721 \times 10^{-7}\, \mathrm{I}$$

$$I\_load\_new := 0.0007768548436 + 1.999999999 \times 10^{-13}\, \mathrm{I}$$

$$\mathrm{polar}(0.0007768548436, 2.574483529 \times 10^{-10})$$

$$VR4\_after := 0.007768548436 + 1.999999999 \times 10^{-12}\, \mathrm{I}$$

$$\mathrm{polar}(0.007768548436, 2.574483529 \times 10^{-10})$$

$$V\_load\_after := 0.9922314516 - 1.999999999 \times 10^{-12}\, \mathrm{I}$$

$$\mathrm{polar}(0.9922314516, -2.015658741 \times 10^{-12})$$

$$S\_after := 0.0003884274218 - 9.999999995 \times 10^{-14}\, \mathrm{I}$$

$$Sapp\_after := 0.0003884274218$$

$$S\_real\_after := 0.0003884274218$$

$$pf := 1.000000000$$

$$SR4 := 3.017517240 \times 10^{-6} + 0.\, \mathrm{I}$$

As we can see, the value of the complex power (S_after) has a zero imaginary number (it is slightly higher than zero because of the floating-point error in Maple). This causes the power factor to go to exactly 1 as seen above which means that there is no phase shift, and all the power is real.

Time domain and Phasor form

The method for finding out the time domain and phasor form is shown below.

I will use the current before the capacitor added as an example. First, we shall find the amplitude of the current and the value of omega.

$$A = 0.00113\ A$$

$$\omega = 2\pi f = 2 \times \pi \times 1000 = 2000\pi \text{ Hz}$$

$$\phi = -0.811\ radians$$

For time domain:

$$i(t) = A\cos(\omega t + \phi)$$

$$i(t) = 0.00113\cos(2000\pi t - 0.811)$$

For phasor domain:

$$i(t) = Ae^{j\phi}$$
$$i(t) = 0.00113e^{-0.811j}$$

Analytical Results Table (I)

Analytical Results (Voltage and Currents)		
Parameter	Time Domain	Phasor Domain
Current before capacitor added (A)	$0.00113\cos(2000\pi t - 0.811)$	$0.00113e^{-0.811j}$
Voltage of Load Network before capacitor added (V)	$0.992\cos(2000\pi t + 0.00825)$	$0.992e^{0.00825j}$
Voltage of Transmission Line before capacitor added (V)	$0.0113\cos(2000\pi t - 0.811)$	$0.0113e^{-0.811}$
Current after capacitor added (A)	$0.000777\cos(2000\pi t)$	0.000777
Voltage of Load Network after capacitor added (V)	$0.992\cos(2000\pi t)$	0.992
Voltage of Transmission Line after capacitor added (V)	$0.00777\cos(2000\pi t)$	0.00777

Analytical Results (Power)	
Apparent Power (before capacitor)	564 μW
Real Power (before capacitor)	389 μW
Reactive Power (before capacitor)	- 409 μW
Power Factor (before capacitor)	0.689
Apparent Power (after capacitor)	388 μW
Real Power (after capacitor)	388 μW
Reactive Power (after capacitor)	0 μW
Power Factor (after capacitor)	1.00
Power Consumed by R4 (before capacitor)	6.37 μW
Power Consumed by R4 (after capacitor)	3.02 μW

Repeating with a different frequency

The problem with the above calculation is that we can not replicate that physically. This is because in my lab kit, there is no 132 nF capacitor (according to the above calculation). So now, I will measure the capacitance of the ceramic capacitor that I will use (see physical section) which turned out to be 39.18 nF. With this capacitance in mind, I tried to plug in frequencies (in Maple) which will give me the exact same capacitance when calculating its value. After trying this trial-and-error method, the frequency which gave me 39.18 nF (more decimal places not necessary as multimeter read only two decimal points) was a frequency of 2.3351 kHz.

With this new frequency and capacitance, I repeated the exact same analysis as shown below:

Maple Input

```
> f:=2.3351e3: omega:=2*3.14159*f: j:=I:

L:=102.2e-3:
V:=1:  R1:=1e3: R2:=1e3: R3:=100: R4:=10:
ZL:=j*omega*L:
Zpar:=(1/R1+1/R2)^(-1):
Z_load:=ZL+R3+Zpar:
I_load:=V/(Z_load+R4); polar(%);
V_load:=I_load*Z_load; polar(%);
V_R4:=abs(I_load*R4); polar(%);
S:=(V*I_load)/2;
S_real:=Re(S);
S_app:=abs(S);
pf:=S_real/S_app;

#finding the capacitance
Y_load:=1/Z_load;
YC:=(j*omega*C);
Y_load_new:=Y_load+YC;
c:=solve(Im(Y_load_new)=0);

#finding power before capacitor is added
VR4:=I_load*R4; polar(%);
SR4:=VR4*conjugate(I_load)/2;

#finding power after capacitor is added
ZC:=1/(j*omega*c):
Zload_new:=(1/Z_load+1/ZC)^(-1);
I_load_new:=V/(Zload_new); polar(%);
VR4_after:=I_load_new*R4; polar(%);
V_load_after:=V-VR4_after;polar(%);
S_after:=V*conjugate(I_load_new)/2;
Sapp_after:=abs(S_after);
```

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```
S_real_after:=Re(S_after);
pf:=S_real_after/Sapp_after;
SR4:=VR4_after*conjugate(I_load_new)/2;
```

Maple Output

$$I\_load := 0.0002327808381 - 0.0005722071415\,\mathrm{I}$$

$$\mathrm{polar}(0.0006177442281, -1.184431205)$$

$$V\_load := 0.9976721916 + 0.005722071364\,\mathrm{I}$$

$$\mathrm{polar}(0.9976886007, 0.005735359440)$$

$$V\_R4 := 0.006177442281$$

$$\mathrm{polar}(0.006177442281, 0.)$$

$$S := 0.0001163904190 - 0.0002861035708\,\mathrm{I}$$

$$S\_real := 0.0001163904190$$

$$S\_app := 0.0003088721141$$

$$pf := 0.3768239789$$

$$Y\_load := 0.0002300268979 - 0.0005748615393\,\mathrm{I}$$

$$YC := 14671.85362\,\mathrm{I}\,C$$

$$Y\_load\_new := 0.0002300268979 - 0.0005748615393\,\mathrm{I} + 14671.85362\,\mathrm{I}\,C$$

$$c := 3.918124827 \times 10^{-8} + 0.\,\mathrm{I}$$

$$VR4 := 0.002327808381 - 0.005722071415\,\mathrm{I}$$

$$\mathrm{polar}(0.006177442281, -1.184431205)$$

$$SR4 := 1.908039657 \times 10^{-6} + 0.\,\mathrm{I}$$

$$Zload\_new := 4347.317679 - 1.889917101 \times 10^{-6}\,\mathrm{I}$$

$$I\_load\_new := 0.0002300268979 + 1.000000000 \times 10^{-13}\,\mathrm{I}$$

$$\mathrm{polar}\left(0.0002300268979, 4.347317679 \times 10^{-10}\right)$$

$$VR4\_after := 0.002300268979 + 1.000000000 \times 10^{-12}\,\mathrm{I}$$

$$\mathrm{polar}\left(0.002300268979, 4.347317679 \times 10^{-10}\right)$$

$$V\_load\_after := 0.9976997310 - 1.000000000 \times 10^{-12}\,\mathrm{I}$$

$$\mathrm{polar}\left(0.9976997310, -1.002305572 \times 10^{-12}\right)$$

$$S\_after := 0.0001150134490 - 5.000000000 \times 10^{-14}\,\mathrm{I}$$

$$Sapp\_after := 0.0001150134490$$

$$S\_real\_after := 0.0001150134490$$

$$pf := 1.000000000$$

$$SR4 := 2.645618688 \times 10^{-7} + 0.\,\mathrm{I}$$

As one can see, the capacitance is very much near the actual capacitance, meaning that this frequency is perfect. Like before, we can see a zero imaginary number for the complex power (or slightly above zero due to floating-point error). We can also see that the power factor is exactly 1 and the apparent power has been decreased.

Analytical Results Table (II)

Analytical Results (Voltage and Currents)		
Parameter	Time Domain	Phasor Domain
Current before capacitor added (A)	$0.000618\cos(4670\pi t - 1.18)$	$0.000618e^{-1.18j}$
Voltage of Load Network before capacitor added (V)	$0.998\cos(4670\pi t + 0.00574)$	$0.998e^{0.00574j}$
Voltage of Transmission Line before capacitor added (V)	$0.00618\cos(4670\pi t - 1.18)$	$0.00618e^{-1.18j}$
Current after capacitor added (A)	$0.000230\cos(4670\pi t)$	0.000230
Voltage of Load Network after capacitor added (V)	$0.998\cos(4670\pi t)$	0.998
Voltage of Transmission Line after capacitor added (V)	$0.00230\cos(4670\pi t)$	0.00230

Analytical Results (Power)	
Apparent Power (before capacitor)	309 μW
Real Power (before capacitor)	116 μW
Reactive Power (before capacitor)	286 μW
Power Factor (before capacitor)	0.377
Apparent Power (after capacitor)	115 μW
Real Power (after capacitor)	115 μW
Reactive Power (after capacitor)	0 μW
Power Factor (after capacitor)	1.00
Power Consumed by R4 (before capacitor)	1.91 μW
Power Consumed by R4 (after capacitor)	0.265 μW

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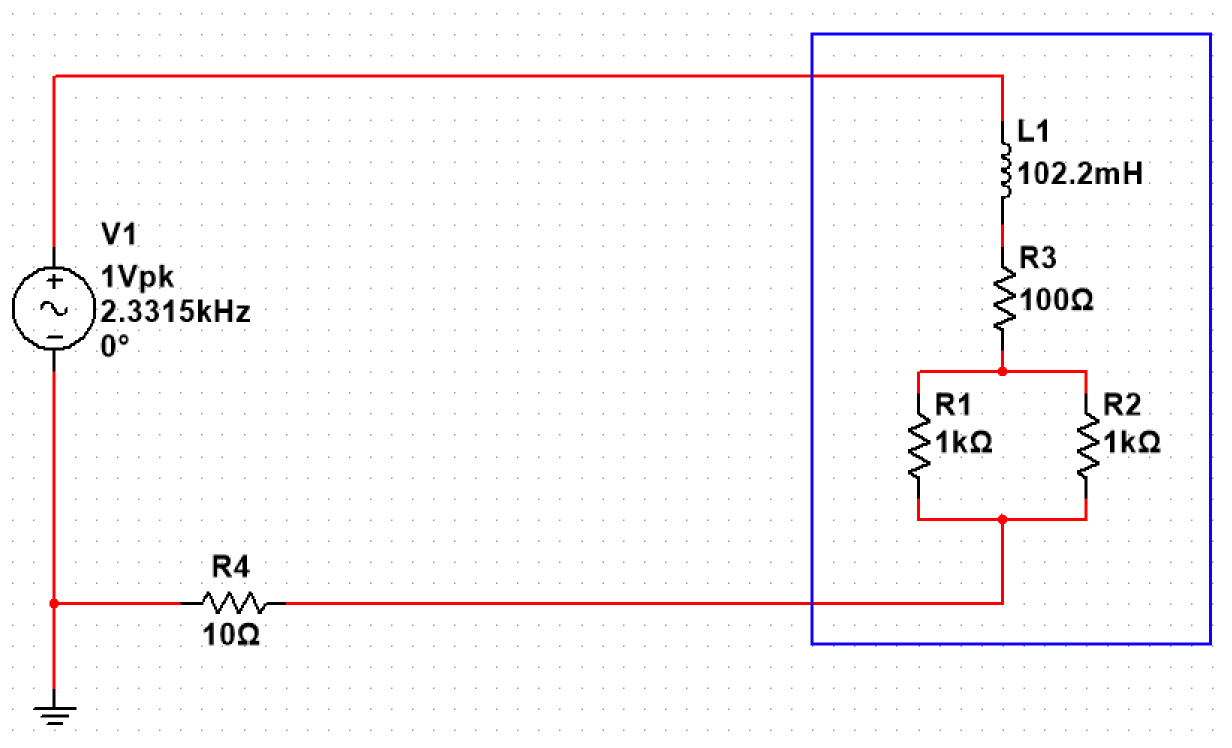
Digital Solution

For my digital solution, I divided it into two parts. For the first one, I will not include the capacitor and for the second one I shall connect the capacitor in parallel to the load. I designed this circuit in Mulitsim and perform the measurements on the Tektronix Oscilloscope. In addition, I also used some probes to read off the RMS value of current and the average power in the transmission line.

Without the Capacitor

Using the component values from the second solution, I replicated the circuit in Multisim.



Notice that I used the exact same frequency I deduced in the second analytical solution. Our aim now is to measure the voltage across the load and the transmission line. In addition, we shall also measure the time difference between the curves which will allow us to deduce the power factor. I have also included the current and power in R4 to compare its values with the analytical solution.

Using the Tektronix oscilloscope, I connected the G probe to the ground wire. I then connected channel 1 and channel 2 probes before and after the load network.

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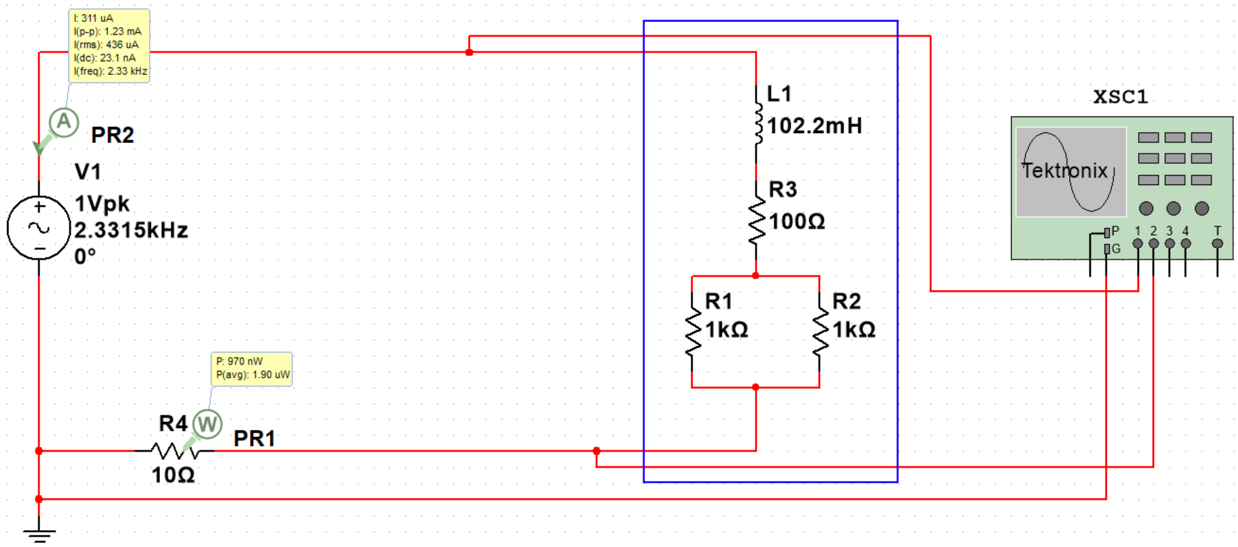
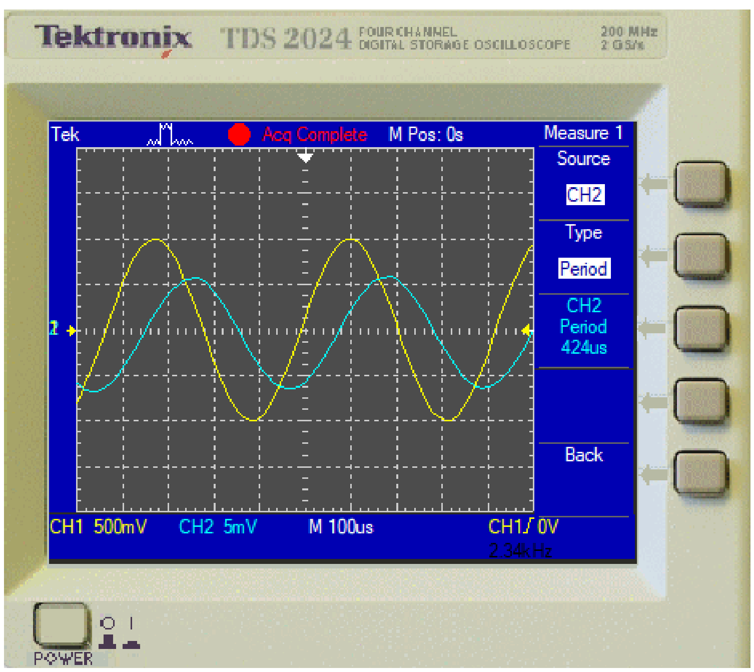


I then turned on the oscilloscope and the following waveform was produced:



The blue curve represents channel 2 whereas the yellow one represents channel 1. Using the “measure tool” I read the time period to be 424 μs. This will be used later in calculating the phase.

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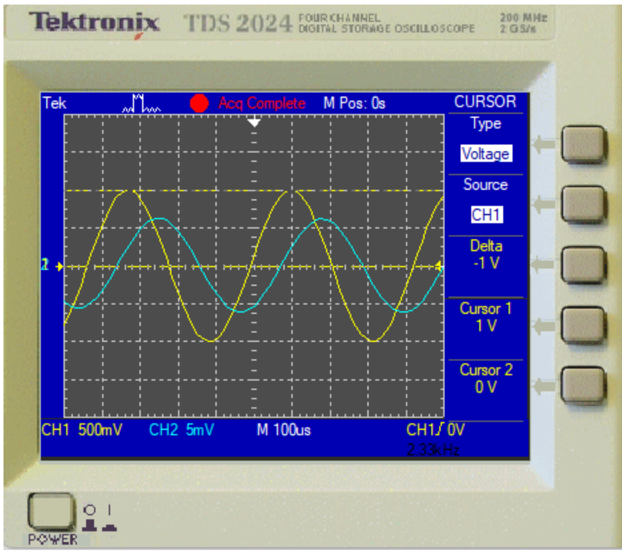
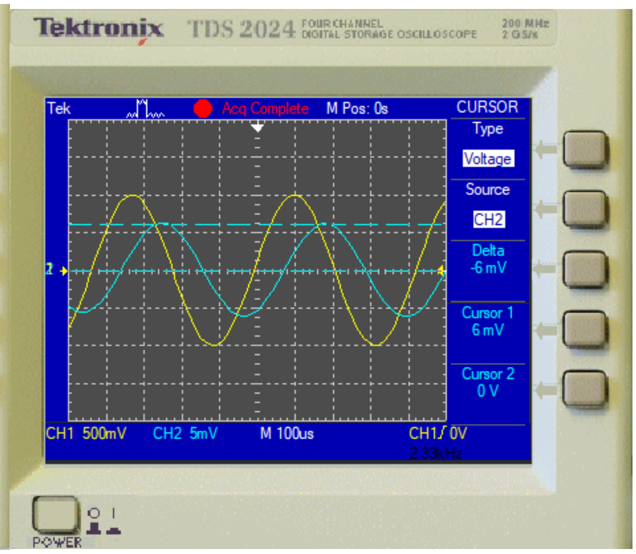
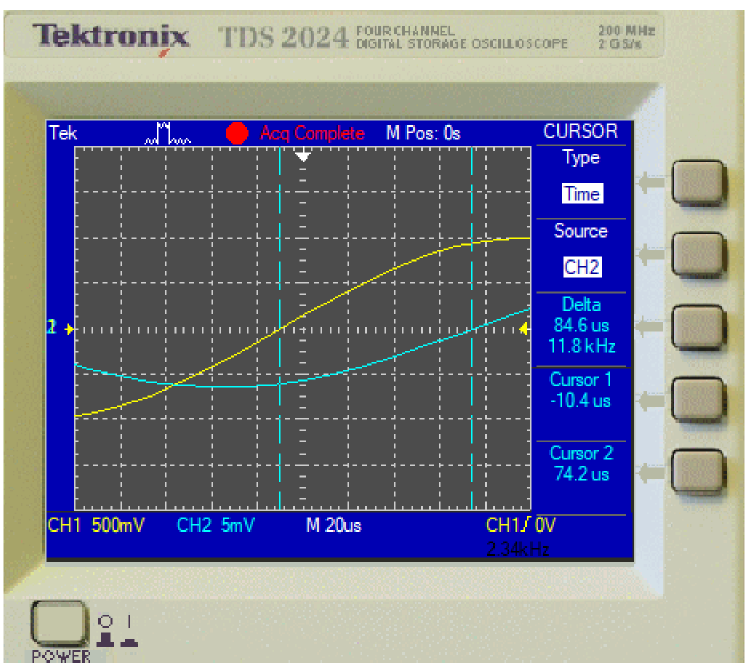
Now I used the voltage probes to measure the voltage amplitude of both the curves. Notice that the channel 2 voltage is equal to the voltage of the transmission line and the channel 1 voltage is equal to the voltage sum of the load and the transmission.





Next, I changed the type of the probe to “time” and measured the time difference between the two waves. I also increased the seconds/division to more accurately determine the point where each of the waves cross the x-axis.



With all these values, I can now calculate the voltage of the load, the voltage of transmission line and the power factor.

The voltage across R4 is simply the channel 2 voltage amplitude.

Moreover, the voltage across the load is going to be:

$$V_{load} = V_{emf} - V_{R4}$$

We will now calculate the power factor and phase. We have the time difference between the waves so we will use this formula to calculate the phase.

$$Phase = \frac{Time\ difference}{Time\ period} \cdot 360$$

This will give the phase in degrees, but we need it in radians as Maple performs trigonometry on radian angles.

$$Angle_{Radians} = Angle_{Degrees} \cdot \frac{\pi}{180}$$

With this value, we shall substitute it in our power factor equation (see analytical solution).

$$pf = \cos(\theta_V - \theta_I) = \cos(\theta_Z)$$

Maple Input

```
> V:=1: R4:= 10: VR4:= 6e-3;
V_load:=V-VR4;
I_load:=VR4/R4;
T:= 424e-6: #time period
td:=84.6e-6: #time difference
phase:=360*td/T;
phase_rad:= phase*3.14159/180;
power_factor:=cos(phase_rad);
Vrms:=0.7071: Irms:=436e-6:
S:=power_factor*Vrms*Irms;
```

```
Sapp:=Vrms*Irms
SR4:=VR4*I_load/2;
```

Maple Output

$$VR4 := 0.006$$

$$V\_load := 0.994$$

$$I\_load := 0.0006000000000000$$

$$phase := 71.83018868$$

$$phase\_rad := 1.253672236$$

$$power\_factor := 0.3118353486$$

$$S := 0.00009613746590$$

$$Sapp := 0.0003082956$$

$$SR4 := 1.800000000 \times 10^{-6}$$

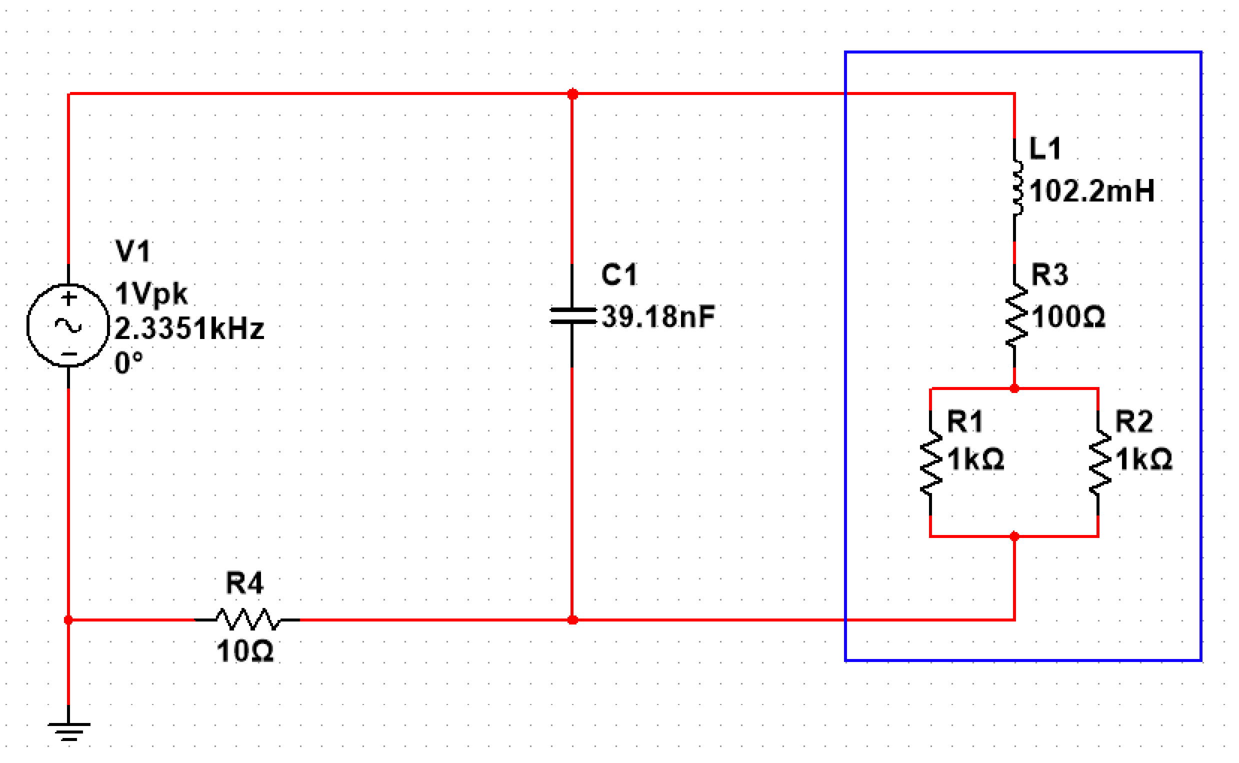
Digital Results Table (Without Capacitor)

Note for the digital solution the phasor form for the current is $0.0006e^{-1.25j}$. This clearly shows the lagging current. I have not included it in the table as I do not have the value of the phase of the transmission line (not possible to do so given the constraints of the digital software used).

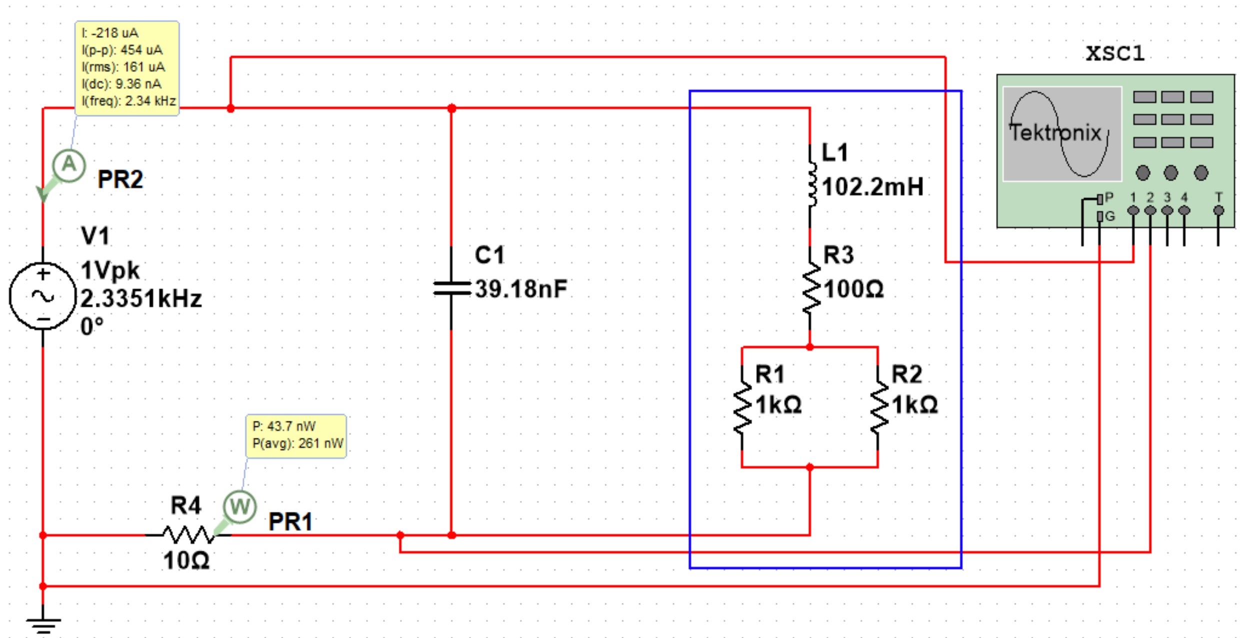
Digital Results (Without Capacitor)	
Current (A)	0.0006
Voltage of Load Network (V)	0.994
Voltage of Transmission Line (V)	0.006
Apparent Power (μW)	308
Real Power (μW)	96.1
Reactive Power (μW)	292
Power Factor	0.312
Power Consumed by R4 (μW)	1.80

With the capacitor

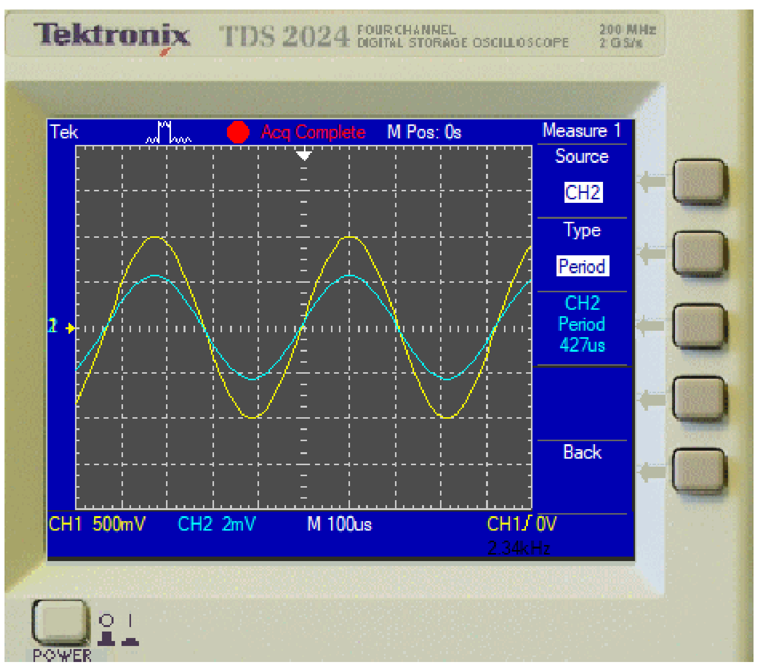
In order to increase the power factor to 1, we shall add a capacitor in parallel to the load network.



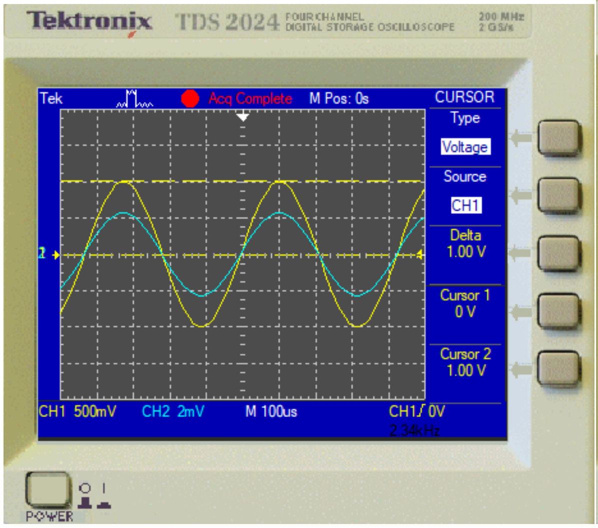
We will repeat the same process as we did for the circuit without the capacitor connected. The circuit is connected to the oscilloscope as shown below:

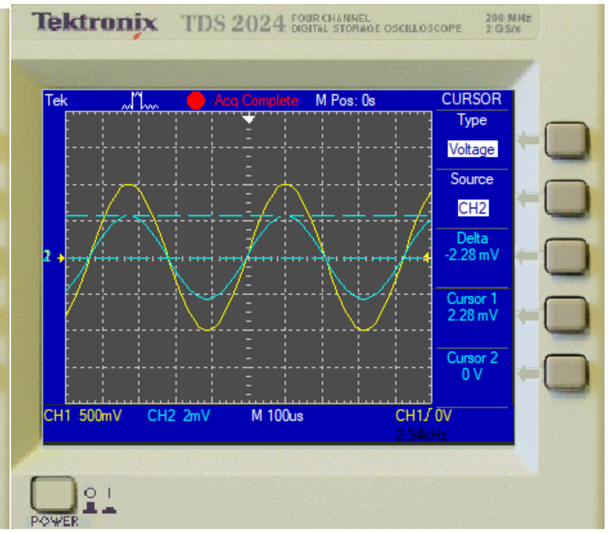


We get the following curves of channel 1 (yellow) and channel 2 (blue):

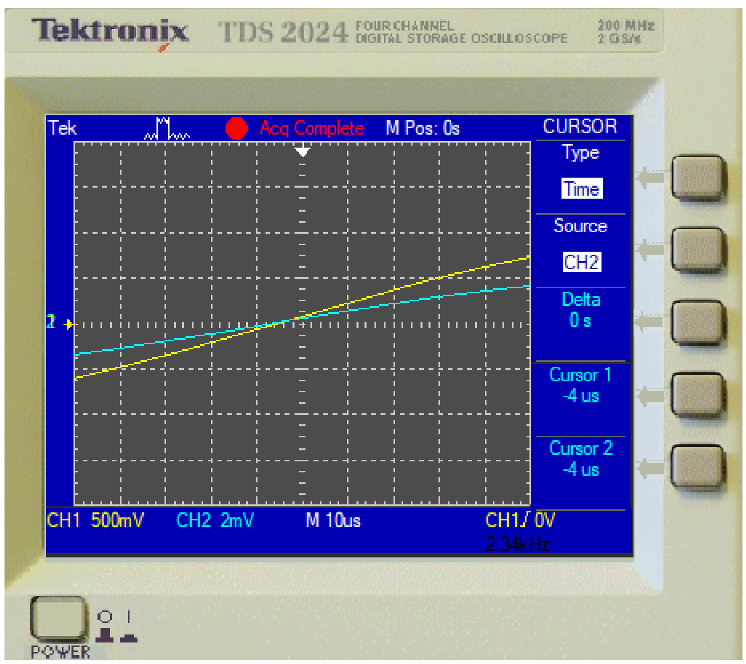


The voltage differences of channel 1 and channel 2 are measured.





Then we shall measure the time difference between the curves:



As seen above the time difference is 0 which means that the capacitor has successfully done power factor correction.

Now we shall repeat the same calculations as seen above:

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Maple Input

```
V:=1: R4:= 10: VR4:= 2.28e-3;
V_load:=V-VR4;
I_load:=VR4/R4;
T:= 427e-6: #time period
td:=0: #time difference
phase:=360*td/T;
phase_rad:= phase*3.14159/180;
power_factor:=cos(phase_rad);
Vrms:=0.7071: Irms:=161e-6:
S:=power_factor*Vrms*Irms;
Sapp:=Vrms*Irms;
SR4:=VR4*I_load/2;
```

Maple Output

$$VR4 := 0.00228$$

$$V\_load := 0.99772$$

$$I\_load := 0.0002280000000$$

$$phase := 0.$$

$$phase\_rad := 0.$$

$$power\_factor := 1.$$

$$S := 0.0001138431$$

$$Sapp := 0.0001138431$$

$$SR4 := 2.599200000 \times 10^{-7}$$

Digital Results Table (With Capacitor)

Digital Results (With Capacitor)	
Current (A)	0.000228
Voltage of Load Network (V)	0.998
Voltage of Transmission Line (V)	0.00228
Apparent Power (μW)	114
Real Power (μW)	114
Reactive Power (μW)	0
Power Factor	1.00
Power Consumed by R4 (μW)	0.260

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Analytical and Digital Solution Comparison

Note that I have given only the absolute value (or amplitudes) of the solution. For the time domain and phasor form, please check the solution-specific table.

	Analytical Solution	Digital Solution
Current before capacitor (A)	0.000618	0.0006 ± 0.00003
Voltage of Load Network before capacitor (V)	0.998	0.99400 ± 0.00002
Voltage of Transmission Line before capacitor (V)	0.00618	0.006 ± 0.00002
Current after capacitance (A)	0.000230	0.00028 ± 0.00003
Voltage of Load Network after capacitor (V)	0.998	0.998 ± 0.00002
Voltage of Transmission Line after capacitor (V)	0.00230	0.0028 ± 0.00002
Apparent Power before capacitor (μW)	309	308 ± 3
Real Power before capacitor (μW)	116	91.6 ± 5
Reactive Power before capacitor (μW)	286	292 ± 1
Power Factor before capacitor	0.377	0.312 ± 0.02
Apparent Power after capacitor (μW)	115	114 ± 4
Real Power after capacitor (μW)	115	114 ± 2
Reactive Power after capacitor (μW)	0	0 ± 2
Power Factor after capacitor	1.00	1.00 ± 0.01
Power Consumed by R4 before capacitor (μW)	1.91	1.80 ± 0.4
Power Consumed by R4 after capacitor (μW)	0.265	0.260 ± 0.1

Physical Solution

For my physical solution, I built my circuit on a breadboard. The red wire represents the positive supply whereas the black wire represents the negative wire. I attached a probe (channel 1) to the positive supply as a reference probe. The green wire extending from the circuit connects to the other probe (channel 2) which measures the voltage at that point (between the load network and the transmission line).

Components Measurement

I measured the values of the capacitance and resistance of all the components.

Note that this is only one of the six values I measured for the capacitance. The value I used in my solution (39.18 nF) is the average of these six values because of a fluctuating reading.

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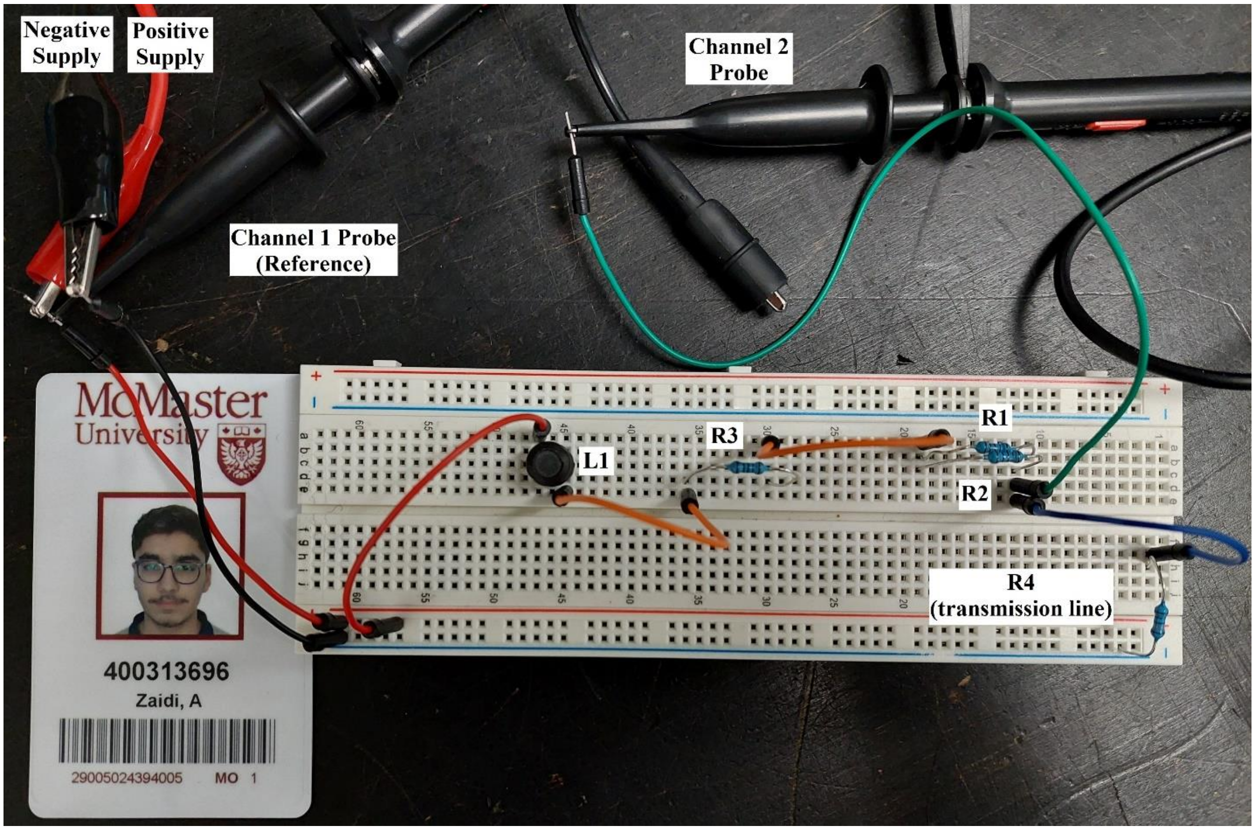
Without the capacitor

Below is the image of my circuit which is without the capacitor.



I used the Hantek Oscilloscope to take my readings and visualize the curves measured through channel 1 and channel 2. The scope reading for the circuit shown above is shown below:

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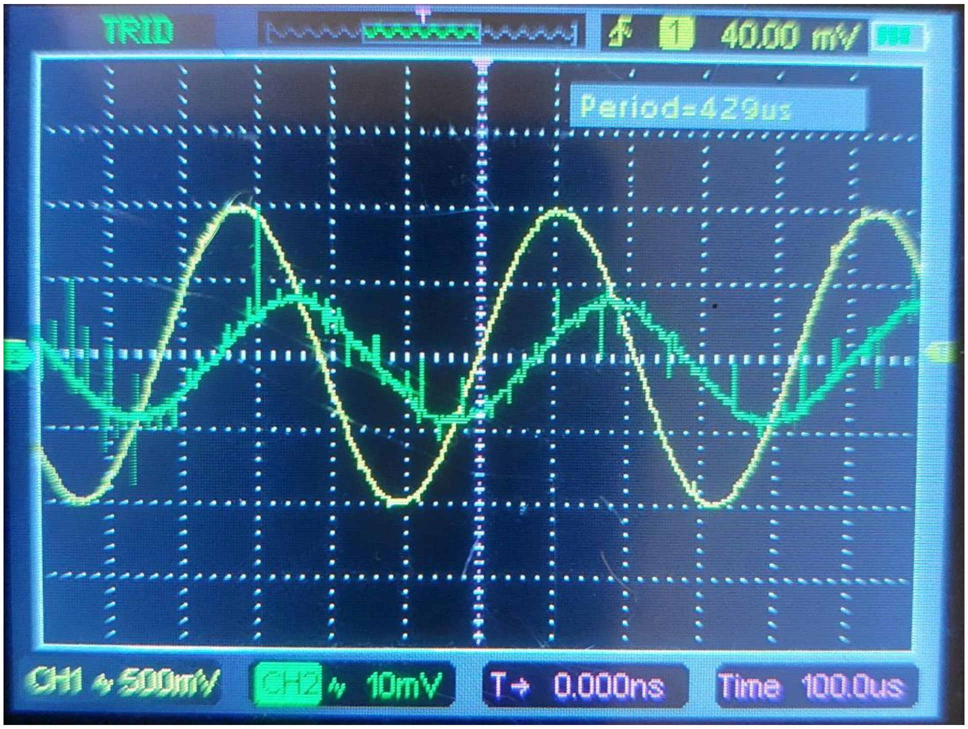
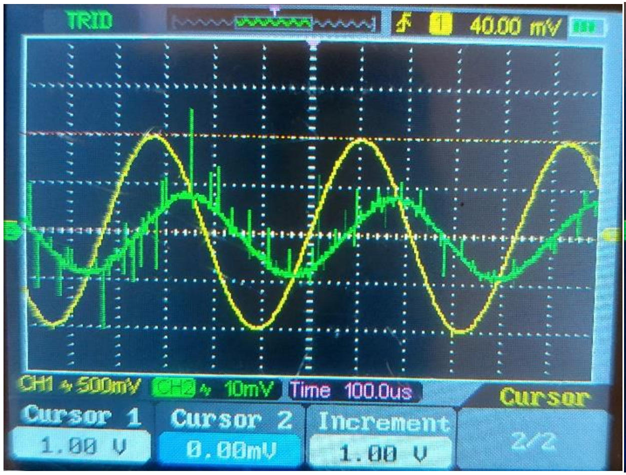
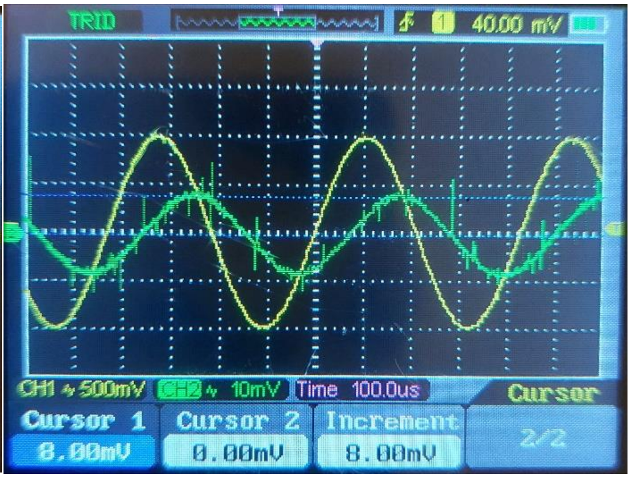


I turned on the measurement tool to measure the time period of channel 1. The reason why I used channel 1 is because the period of both the waves are the same and channel 2 has a lot of noise in its curve which results in a fluctuating reading. Thus, it was better to use channel 1 for a more stable and accurate reading.

After that, I read off the voltage amplitudes of channel 1 and channel 2 waves using the voltage cursors.





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This will allow me to calculate the voltages of the load and the transmission line. Next, I increased the time/division of the oscilloscope in order to measure the time difference (lag) between the waves. This was done to increase accuracy and reduce error in the time cursor reading.

I then performed the calculations to determine the power factor and voltages of the load network and the transmission line.

Maple Input

```
> f:=2.3351e3: omega:=2*3.14159*f: j:=I:
L:=102.2e-3:
V:=1:  VR4:= 8e-3: R1:=1e3: R2:=1e3: R3:=100: R4:=10:
ZL:=j*omega*L:
Zpar:=(1/R1+1/R2)^(-1):
Z_load:=ZL+R3+Zpar;
I_load:=V/(Z_load+R4); polar(%);
V_load:=V-VR4;
Vrms:=evalf((V_load)/sqrt(2));
Irms:= Vrms/Z_load;

T:=429e-6:
td:=84e-6: #time difference
```

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```
phase:=360*td/T;
phase_rad:= phase*3.14159/180;
power_factor:=cos(phase_rad);

S_real:=abs(power_factor*Vrms*Irms);
S_reactive:=abs(Vrms*Irms*sin(phase_rad));
Sapp:=Vrms*Irms; polar(%);

VrmsR4:=evalf((VR4)/sqrt(2)):
IrmsR4:=VrmsR4/R4:
SR4:=power_factor*VrmsR4*IrmsR4;
Sapp:=abs(SR4);
```

Maple Output

$$Z\_load := 600.0000000 + 1499.463440\,I$$

$$I\_load := 0.0002327808381 - 0.0005722071415\,I$$

$$\mathrm{polar}(0.0006177442281, -1.184431205)$$

$$V\_load := 0.992$$

$$Vrms := 0.7014499268$$

$$Irms := 0.0001613523507 - 0.0004032365847\,I$$

$$phase := 70.48951049$$

$$phase\_rad := 1.230273007$$

$$power\_factor := 0.3339804086$$

$$S\_real := 0.0001017485026$$

$$S\_reactive := 0.0002871608714$$

$$Sapp := 0.0001131805946 - 0.0002828502728\,I$$

$$\mathrm{polar}(0.0003046541052, -1.190166564)$$

$$SR4 := 1.068737307 \times 10^{-6}$$

$$Sapp := 1.068737307 \times 10^{-6}$$

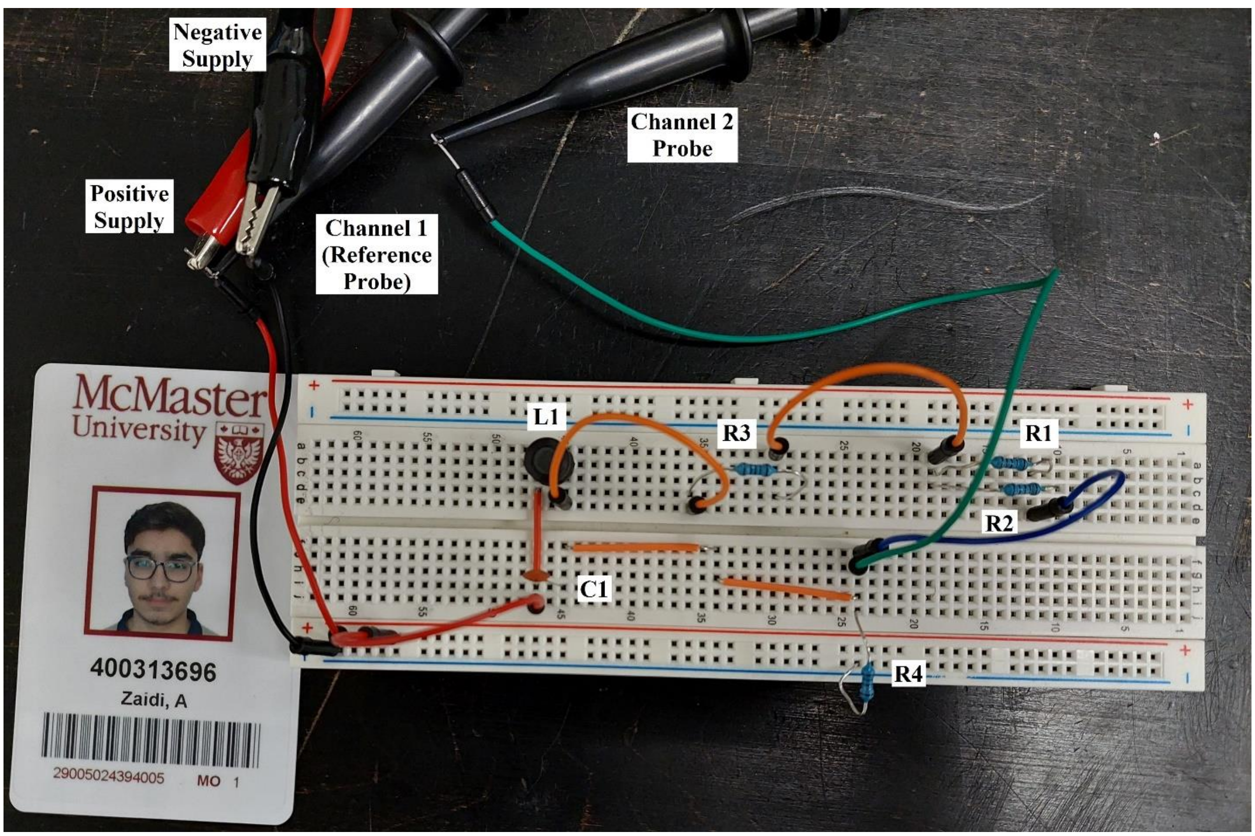
Physical Results Table (Without Capacitor)

Note for the physical solution the phasor form for the current is $0.000618e^{-1.18j}$. This clearly shows the lagging current. I have not included it in the table as I do not have the value of the phase of the transmission line (not possible to do so given the constraints of the physical solution method).

Physical Results (Without Capacitor)	
Current (A)	0.000618 A
Voltage of Load Network (V)	0.992 V
Voltage of Transmission Line (V)	0.008 V
Apparent Power (μW)	305 μW
Real Power (μW)	102 μW
Reactive Power (μW)	287 μW
Power Factor	0.334
Power Consumed by R4 (μW)	1.07 μW

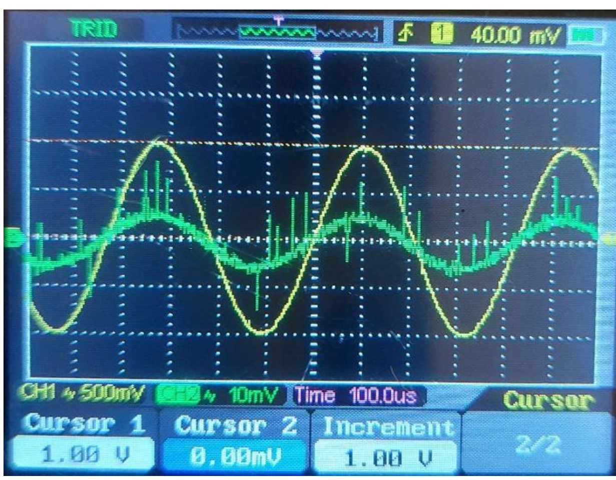
With capacitor

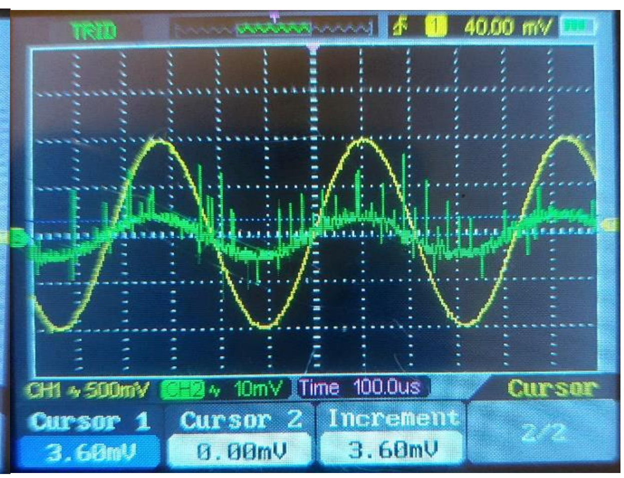
By connecting the capacitor in parallel to the load network, we get the following circuit:



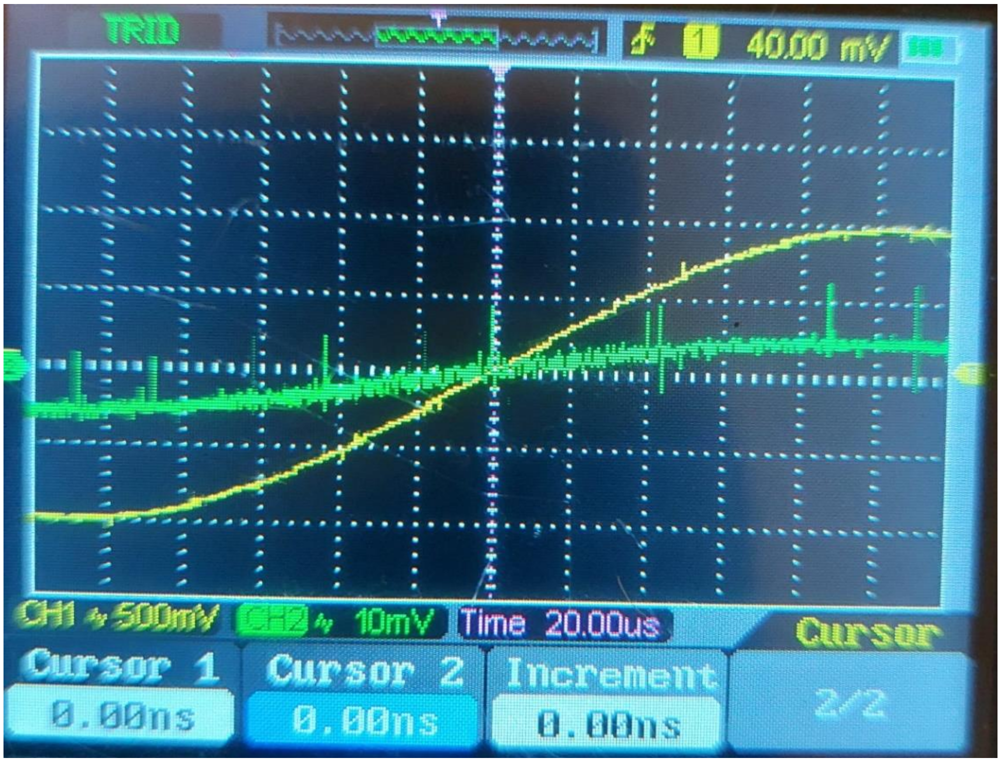
The scope gives us the following curves:

Now, as before, we shall measure the voltage amplitudes of the waves.





Next, we will measure the time difference between the waves.



Performing the calculations gives us the following results:

Maple Input

```
> f:=2.3351e3: omega:=2*3.14159*f: j:=I:
L:=102.2e-3: c:=39.18e-9:
V:=1:  VR4:= 3.6e-3: R1:=1e3: R2:=1e3: R3:=100: R4:=10:
ZL:=j*omega*L:
ZC:=1/(j*omega*c):
Zpar:=(1/R1+1/R2)^(-1):
Z_load:=ZL+R3+Zpar;
Zload_new:=(1/Z_load+1/ZC)^(-1);
I_load:=V/(Zload_new+R4); polar(%);
V_load:=V-VR4;
Vrms:=evalf((V_load)/sqrt(2));
Irms:= Vrms/Zload_new;

T:=428e-6:
td:=0: #time difference
phase:=360*td/T;
phase_rad:= phase*3.14159/180;
power_factor:=cos(phase_rad);

S:=power_factor*Vrms*Irms; polar(%);
Sapp:=Vrms*Irms; polar(%);
```

```
SR4:= abs(VR4*I_load/2)
```

Maple Output

$$Z\_load := 600.0000000 + 1499.463440\,I$$

$$Zload\_new := 4347.317652 + 0.3461269753\,I$$

$$I\_load := 0.0002294989885 - 1.823043374 \times 10^{-8}\,I$$

$$\mathrm{polar}(0.0002294989892, -0.00007943579107)$$

$$V\_load := 0.9964$$

$$Vrms := 0.7045611966$$

$$Irms := 0.0001620680264 - 1.290361558 \times 10^{-8}\,I$$

$$phase := 0.$$

$$phase\_rad := 0.$$

$$power\_factor := 1.$$

$$S := 0.0001141868426 - 9.091386834 \times 10^{-9}\,I$$

$$\mathrm{polar}(0.0001141868430, -0.00007961851478)$$

$$Sapp := 0.0001141868426 - 9.091386834 \times 10^{-9}\,I$$

$$\mathrm{polar}(0.0001141868430, -0.00007961851478)$$

$$SR4 := 4.130981806 \times 10^{-7}$$

Physical Results Table (With Capacitor)

Physical Results (With Capacitor)	
Current (A)	0.000229 A
Voltage of Load Network (V)	0.996 V
Voltage of Transmission Line (V)	0.0036 V
Apparent Power (μW)	114 μW
Real Power (μW)	114 μW
Reactive Power (μW)	0 μW
Power Factor	1.00
Power Consumed by R4 (μW)	0.414 μW

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Summary of Solutions

Note that I have given only the absolute value (or amplitudes) of the solution. For the time domain and phasor form, please check the solution-specific table.

	Analytical Solution	Digital Solution	Physical Solution
Current before capacitor (A)	0.000618	0.0006 ± 0.00003	0.000618 ± 0.0002
Voltage of Load Network before capacitor (V)	0.998	0.99400 ± 0.00002	0.992 ± 0.0004
Voltage of Transmission Line before capacitor (V)	0.00618	0.006 ± 0.00002	0.008 ± 0.0004
Current after capacitance (A)	0.000230	0.00028 ± 0.00003	0.000229 ± 0.0002
Voltage of Load Network after capacitor (V)	0.998	0.998 ± 0.00002	0.996 ± 0.0004
Voltage of Transmission Line after capacitor (V)	0.00230	0.0028 ± 0.00002	0.0036 ± 0.0004
Apparent Power before capacitor (μW)	309	308 ± 3	305 ± 4
Real Power before capacitor (μW)	116	91.6 ± 5	102 ± 7
Reactive Power before capacitor (μW)	286	292 ± 1	287 ± 5
Power Factor before capacitor	0.377	0.312 ± 0.02	0.334 ± 0.03
Apparent Power after capacitor (μW)	115	114 ± 4	114 ± 6
Real Power after capacitor (μW)	115	114 ± 2	114 ± 7
Reactive Power after capacitor (μW)	0	0 ± 2	0 ± 3
Power Factor after capacitor	1.00	1.00 ± 0.01	1.00 ± 0.03
Power Consumed by R4 before capacitor (μW)	1.91	1.80 ± 0.4	1.07 ± 0.6
Power Consumed by R4 after capacitor (μW)	0.265	0.260 ± 0.1	0.414 ± 0.9

Sources of Error

Did not take internal resistance of inductor	I did not take the internal resistance of the inductor in consideration because my answers agreed without it, so it was ignored for making the solution easier and more relevant to the load network and transmission line.
Did not measure the inductance of the inductor	The inductance of the inductor was measured in the lab and its value was given to us as 102.2 mH. This could indeed cause many inaccuracies as not all inductors could have possessed that value
Error in using cursors (in both digital and physical)	The cursors were placed between the curves in order to measure the voltage amplitudes and time difference. There was an error associated with the cursors (in the physical solution) which was 0.4 mV with the voltage and 0.2 μs with the time difference.
Could not use "measure" tool as too much noise in physical circuit	Ideally, to avoid placing the cursors incorrectly, the measure tool would have been perfect for giving an exact true value. However, because of the noise produced by channel 2, it was impossible to read off a value as it was changing constantly.
Physical solution systemic errors	Discussed in Analysis

Analysis

As seen above, the values from my analytical, digital, and physical solutions are within the error range provided. An exception to this is seen in only one or two values which are anomalous because of the errors discusses above.

Because my solutions agree with each other, I can say that the most accurate representation of the true values is from the analytical solution. This is because the equations I used in my analytical solution have a theoretical background behind them and are fully correct which means that they do not have any error. Their correctness is proved by my digital and physical solution.

My digital solution had the error of cursors associated with it but overall, they are second-most reliable solution. Overall software is reliable, but I still had issues with the current probe. When measuring the RMS current, I was getting a different value at first and then it changed to a normal value.

The physical solution is the least reliable and has the most error. Because it is performed in a non-ideal world, it will have some deviance from the true value. The breadboard may cause current to "leak" which could result in a lower reading. Moreover, the supply wires may not be attached properly causing a lower reading. I used alligator clips to reduce this error but still some was present. I also observed a lot of noise in my channel 2 solution which the lab technician explained was normal for a voltage reading this small.

This lab proved that adding a capacitor in parallel to a load network (containing an inductor) will correct the power factor. It can also be shown that this can work the other way round; an inductor added in parallel to the load network (containing a capacitor) will also cause the power factor to go to 1. The significance of 1 as the value of the power factor is because this value occurs in loads which are purely resistive. We are aiming for this in order to minimize the apparent power which can only be done by decreasing the imaginary part of the complex power to 0.

As shown in lab 2, the inductor causes the voltage to lead and the current to lag. In my load network, without the capacitor added, the wave forms have phase difference of a positive value as the voltage is leading. When the capacitor is attached, it causes the current to lead and the voltage to lag thus eliminating the change created by the inductor. The challenge, however, was to find the exact capacitance that would lead to a zero-degree phase shift which was calculated in Maple.

Reflecting on Previous labs

The knowledge from lab 2 was essential over here. Without having the experience of dealing with AC supplies, this lab would have been impossible. More importantly, the conclusions from that lab were essential in deducing the working of the capacitor and inductor which formed the theoretical backbone of this lab. The third lab gave another outlook on how the capacitors and inductors change with time. In addition, the fourth lab steered us another way, showing us another application of these components, which will prove useful to us in future mechatronics courses.

Conclusion

Overall, this lab has strengthened my knowledge of the previous labs. Using the conclusions deduced in previous labs, which were now used in this one, was my favourite part of this lab. I learnt about how exactly the electric companies bill us and have found a solution that is used in industries today. Thus, it is a very modern solution to a problem which could end up saving the factory/company a lot of money.

Video Presentation

https://youtu.be/mtoVPjvtRc4