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Lab 3

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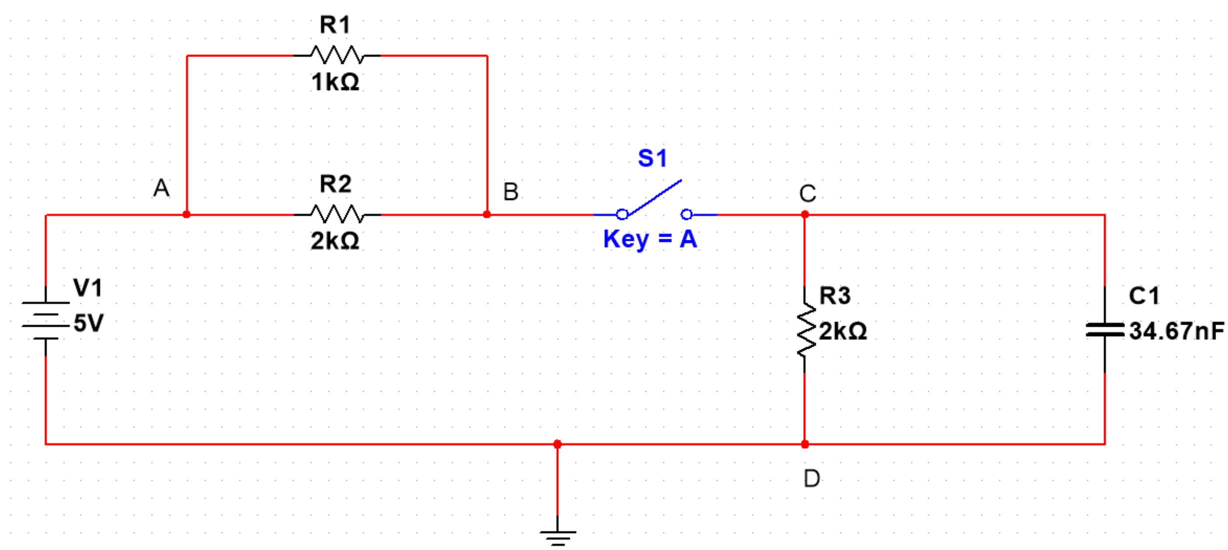
Introduction

In this lab, I am going to be solving the circuit below using an analytical, digital, and physical solution. First, I plan to derive a voltage equation across the capacitor (C1) for both charging and discharging states. This will be done by finding out the initial voltage, final voltage, and time constant and then plugging it into the first-order transient analysis equation. After that, I will use Multisim to build my circuit and use the Tektronix oscilloscope to confirm my time constants from my analysis for both charging and discharging. I will repeat the same procedure with a physical circuit, built on a breadboard. Moreover, I will also measure the scope impedance and solve for the energy stored in the capacitor.



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Analytical Solution

Charging (Analytical)

Charging the capacitor implies that before t=0, the switch is open and at t=0, the switch is closed. This will result in the following voltages across the capacitor:

$$V(0) = 0\ V$$

$$V_c = \frac{R_3}{R_3 + R_{par}} \cdot V_1$$

$$where\ R_{par} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{2000}{3} \Omega$$

$$\therefore V(\infty) = V_c = 3.75V$$

Because $V(0^-) = V(0^+) = 0$ thus the voltage across the capacitor at t=0 will be 0V. For finding out the voltage as time approaches infinity, I used the voltage divider equation (with R1 and R2 combined to form Rpar) to calculate the voltage at node C. Because the R3 is in parallel with the capacitor, the voltages across both the components will be the same resulting in the voltage at node C being equal to the voltage across the capacitor when time approaches infinity.

In addition, we need to find out the time constant. We will use the following equations to measure it. We shall consider the Thevenin Equivalent resistance seen by the capacitor as the effective resistance.

$$\tau = R_{th} C$$

$$where\ R_{th} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$$

By plugging the values in Maple (see appendix), we get:

$$R_{th} = 500.0\ \Omega$$

$$\tau = 17.335\ \mu s$$

Using the values of τ and voltage at t=0 and t=∞ , we can determine the voltage at time t.

$$V(t) = V(0)\, e^{-\frac{t}{\tau}} + V(\infty)\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$V(t) = 0 \cdot e^{-\frac{t}{17.335 \times 10^{-6}}} + 3.75 \cdot \left(1 - e^{-\frac{t}{17.335 \times 10^{-6}}}\right)$$

$$V(t) = 0 + 3.75 \cdot \left(1 - e^{-\frac{t}{17.335 \times 10^{-6}}}\right)$$

$$\mathbf{V(t) = 3.75 - 3.75}\boldsymbol{e}^{-\frac{t}{17.335\ \mu s}}$$

Discharging (Analytical)

For discharging it is implied that the switch is closed before t=0 and opened at t=0. By using the same logic as in charging the capacitor, we get:

$$V(0) = 3.75\ V$$

$$V(\infty) = 0\ V$$

The voltage at t=0 is because the capacitor is fully charged. When the switch is opened, the capacitor will act as the voltage source and will discharge causing the voltage to go to 0 when time approaches infinity.

We can then calculate the time constant using Maple (see appendix):

$$\tau = R_3 C$$

$$\tau = 69.34\ \mu s$$

Now we shall substitute these values into the transient equation:

$$V(t) = V(0)\, e^{-\frac{t}{\tau}} + V(\infty)\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$V(t) = 3.75 e^{-\frac{t}{69.34 \times 10^{-6}}} + 0 \cdot \left(1 - e^{-\frac{t}{69.34 \times 10^{-6}}}\right)$$

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$$V(t) = 3.75e^{-\frac{t}{69.34\ \mu s}}$$

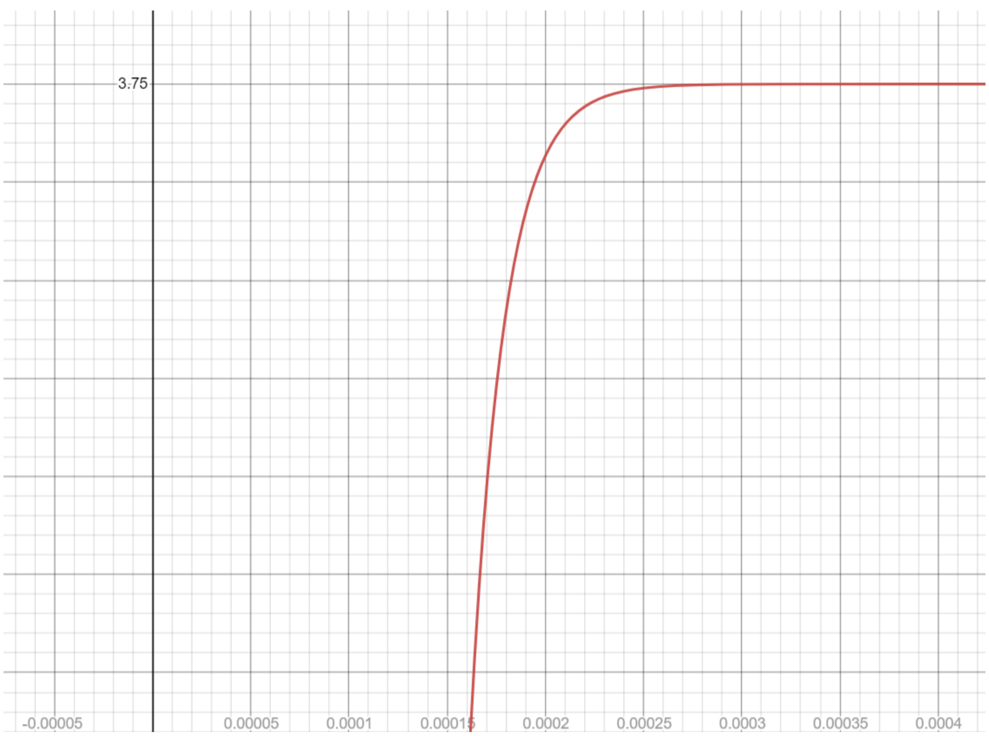
Analysis Solution (Charging)

Variable/Equation	**Values**
Voltage at t=0	0 V
Voltage at t=∞	3.75 V
Time constant (τ)	17.335 μs
Voltage equation across capacitor	$V(t) = 3.75 - 3.75e^{-\frac{t}{17.335\ \mu s}}$

Analysis Solution (Discharging)

Variable/Equation	**Values**
Voltage at t=0	3.75 V
Voltage at t=∞	0 V
Time constant (τ)	69.34 μs
Voltage equation across capacitor	$V(t) = 3.75e^{-\frac{t}{69.34\ \mu s}}$

Voltage versus Time graph for charging



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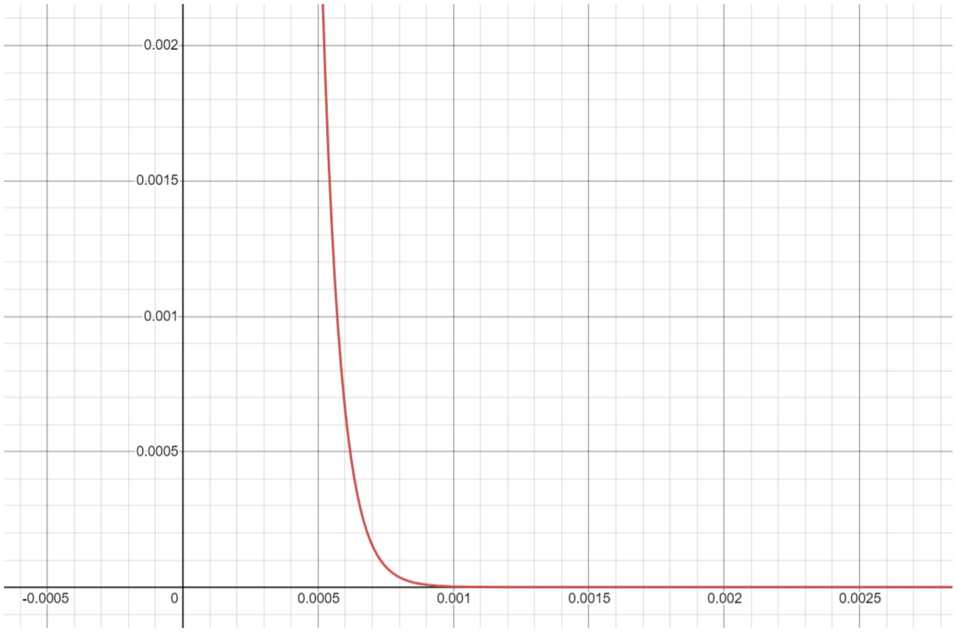
Voltage versus Time Graph for Discharging



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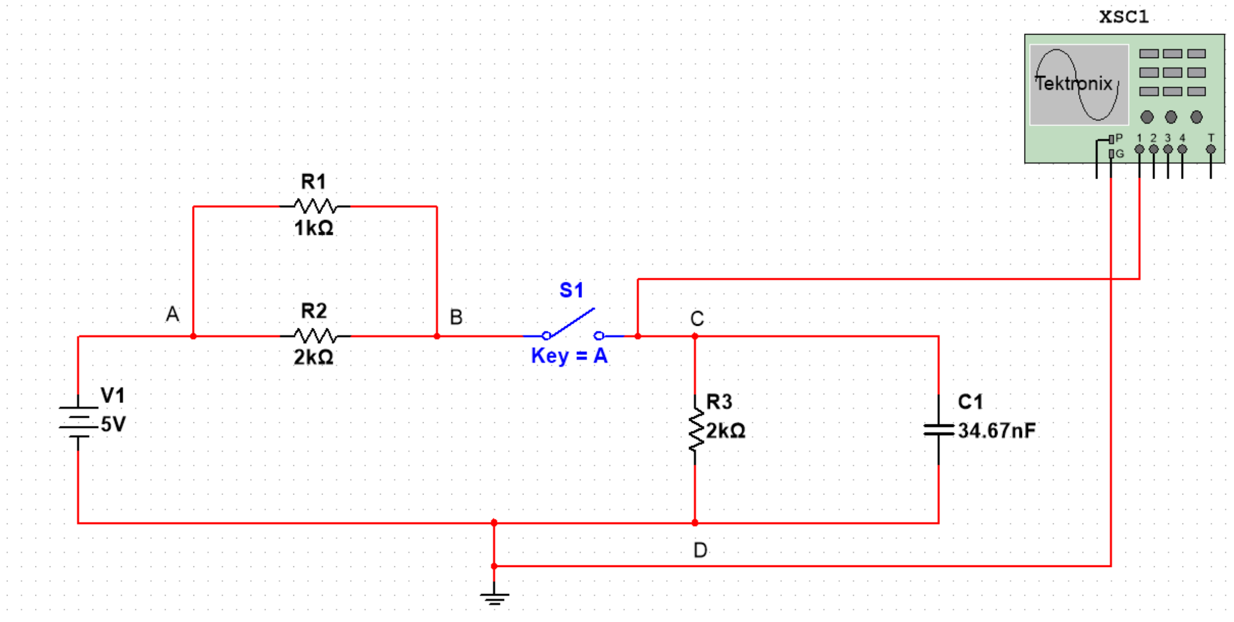
Digital Solution



For my digital solution, I used the Tektronix oscilloscope to measure the charging and discharging states. I connected the ground probe to the ground of the circuit and the channel 1 to node C (as seen in the circuit diagram above).

After running the simulation, I set the trigger and selected the rising option for the slope (as I was measuring the charging state). Moreover, I selected the coupling to be DC and the mode to be normal (ideally should be single mode but the answers did not differ) for being easily able to visualize the curve. Then I closed the switch which produced a curve as seen below.

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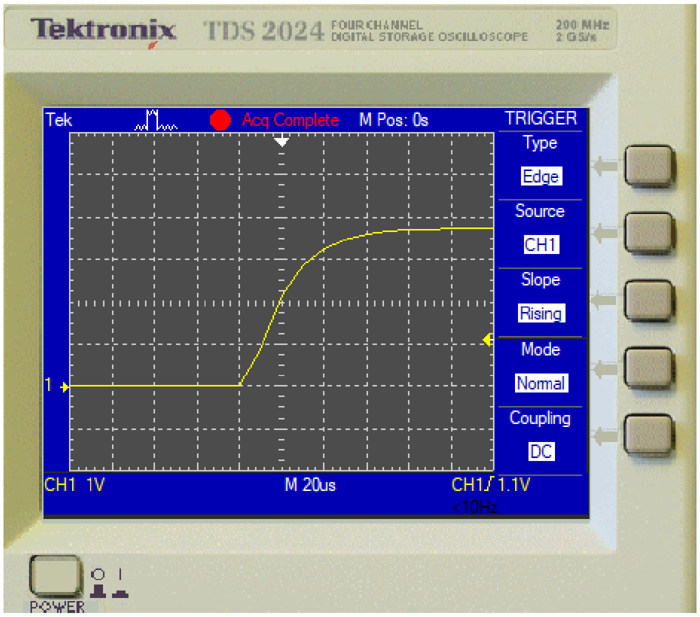
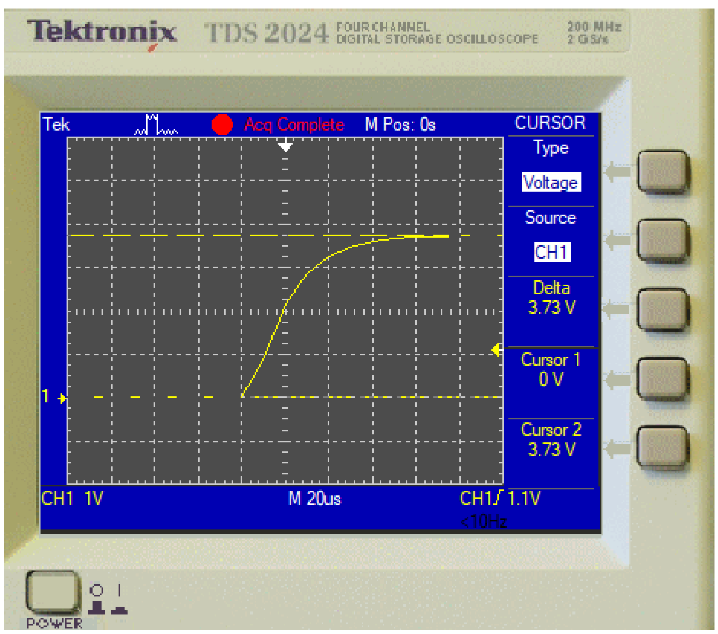


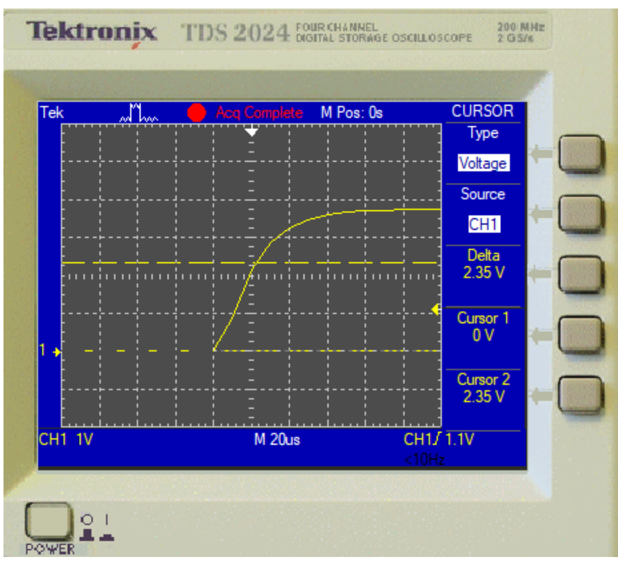
I then went to the cursor menu to select the voltage cursors and placed them to measure the voltage at the node C.

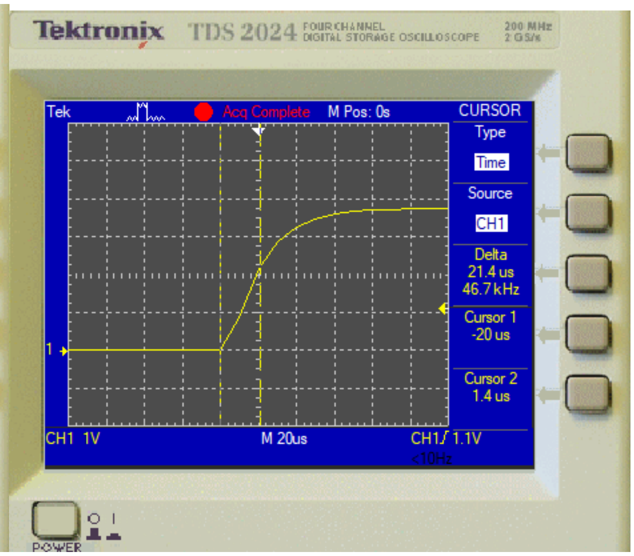


With this voltage, I am now able to calculate the voltage when the time is equal to the time constant.

$$V(0) = 0V$$
$$V(\infty) = 3.73V$$
$$V(\tau) = V(0)\, e^{-\frac{\tau}{\tau}} + V(\infty)\left(1 - e^{-\frac{\tau}{\tau}}\right)$$
$$V(\tau) = V(0)\, e^{-1} + V(\infty)\left(1 - e^{-1}\right)$$
$$V(\tau) = 0 \cdot e^{-1} + 3.73 \cdot \left(1 - e^{-1}\right)$$
$$V(\tau) = 2.3578\ V$$

With this value of voltage at t = τ, we can set our voltage probes so that the difference between them is around 2.35 V. By noting the position where the second probe cuts the graph, we can place our time cursors at that point and the point where the slope begins rising. This will give us the time difference which is the time constant.





This method shows that when t= τ, the voltage is 2.3578 V. This means that after 63.21% of the graph, the time is equal to τ. By calculating the voltage when t= τ, we can find the time it takes for the graph to reach that voltage which equals to the time constant.

Using the same probe placement and settings as for the charged state, we can now visualize the discharge curve. There would be only one change which is to set the slope as falling. After the switch is opened, we can then see the curve (shown below) on the oscilloscope.

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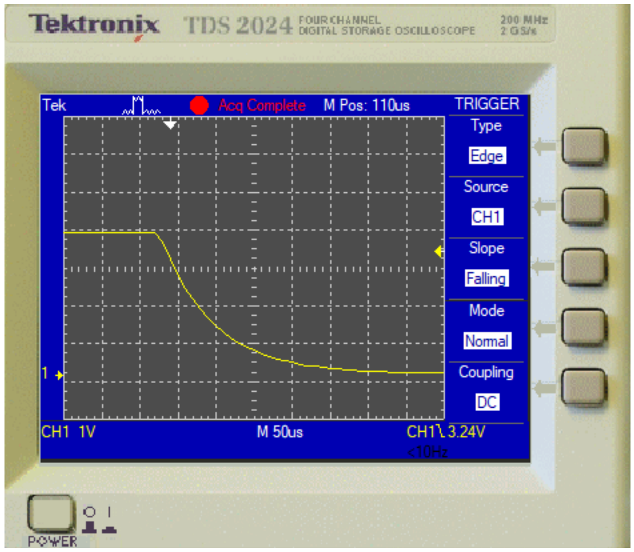
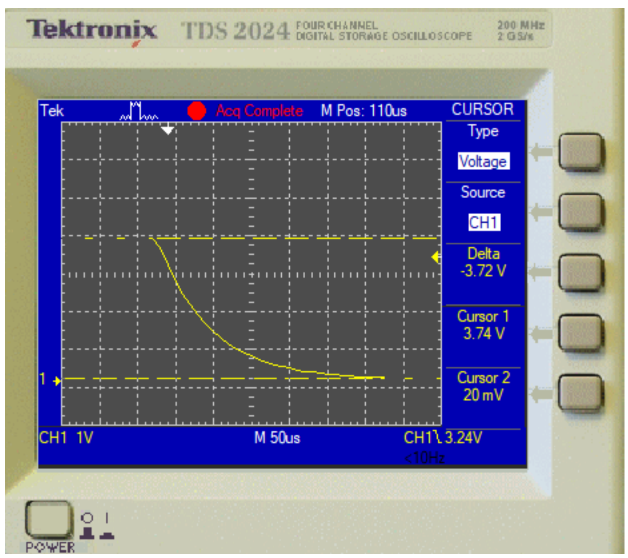


As like before, we will measure the voltage difference using the voltage probes.



With a voltage of 3.72 V, we can plug this value into our equation and calculate the voltage when t= τ.

$$V(0) = 3.72V$$

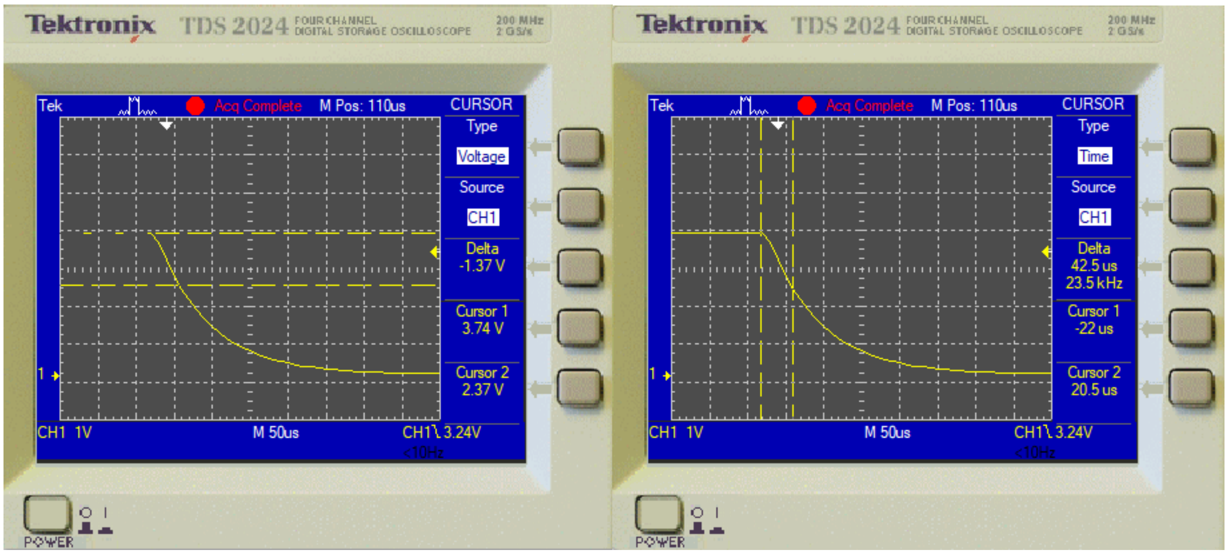
$$V(\infty) = 0V$$

$$V(\tau) = V(0)\, e^{-\frac{\tau}{\tau}} + V(\infty)\left(1 - e^{-\frac{\tau}{\tau}}\right)$$

$$V(\tau) = V(0)\, e^{-1} + V(\infty)\left(1 - e^{-1}\right)$$

$$V(\tau) = 3.72 \cdot e^{-1} + 0 \cdot \left(1 - e^{-1}\right)$$

$$V(\tau) = 1.369\ V$$



Using the voltage at τ, we measured the time constant using the time cursors. This shows that it takes time τ for the graph to fall by 63.21%.

Digital Solution (Charging)

Variable/Equation	**Values**
Voltage at t=0	0 V
Voltage at t=∞	3.73 V
Time constant (τ)	21.4 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.73 - 3.73e^{-\frac{\mathrm{t}}{21.4\ \mu\mathrm{s}}}$

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Digital Solution (Discharging)

Variable/Equation	**Values**
Voltage at t=0	3.72 V
Voltage at t=∞	0 V
Time constant (τ)	42.5 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.72e^{-\frac{\mathrm{t}}{42.5\ \mu\mathrm{s}}}$

As we can see from the above table, the results from my analysis and digital solution are very close to each other, thus confirming my solution.

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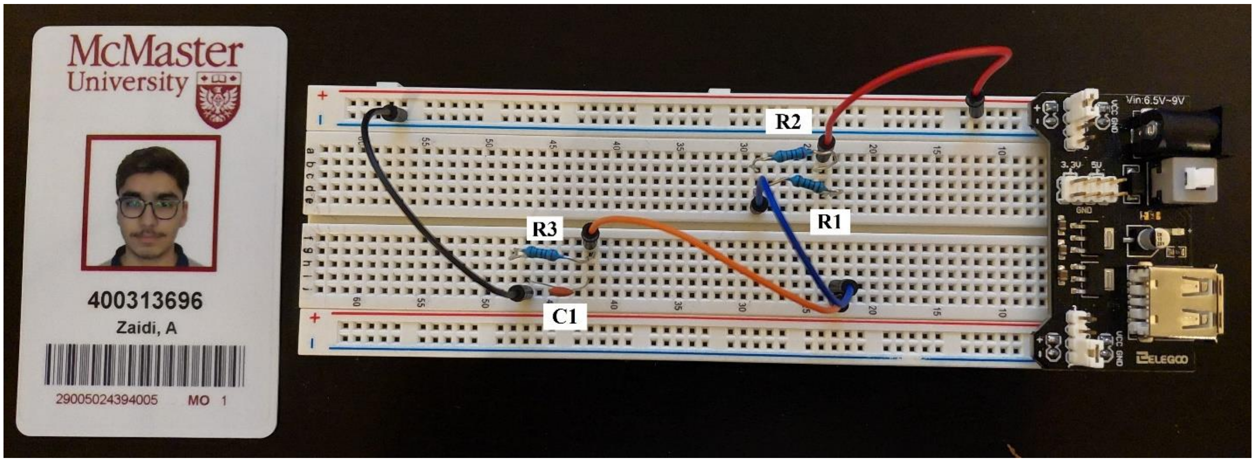
Physical Solution

I used the breadboard to replicate the circuit physically as seen below.



I used the 5V supply of the Soulbay power supply. The red wire represents current into the circuit and the black wire represents the earth connection (negative supply). The orange wire represents the switch which I used to generate the curves. Below are the measurements of all the components present on the circuit.

Note that the value shown for the capacitance is different because this is one of the three values, I used to calculate the average capacitance shown in the Multisim circuit previously.

For calculating the time constant, I used the same method as for my digital solution. The following diagram was generated for the charging state.

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The change in voltage was measured using the voltage probes which allowed me to do the following calculation for finding out the voltage when t= τ.

$$V(0) = 0V$$
$$V(\infty) = 3.64V$$
$$V(\tau) = V(0)\,e^{-\frac{\tau}{\tau}} + V(\infty)\left(1 - e^{-\frac{\tau}{\tau}}\right)$$
$$V(\tau) = V(0)\,e^{-1} + V(\infty)\left(1 - e^{-1}\right)$$
$$V(\tau) = 0 \cdot e^{-1} + 3.64 \cdot \left(1 - e^{-1}\right)$$
$$V(\tau) = 2.3009\ V$$

Using this value, I used the voltage probes to measure that point on the graph and then used the time probes to find the time difference which equals to the time constant.

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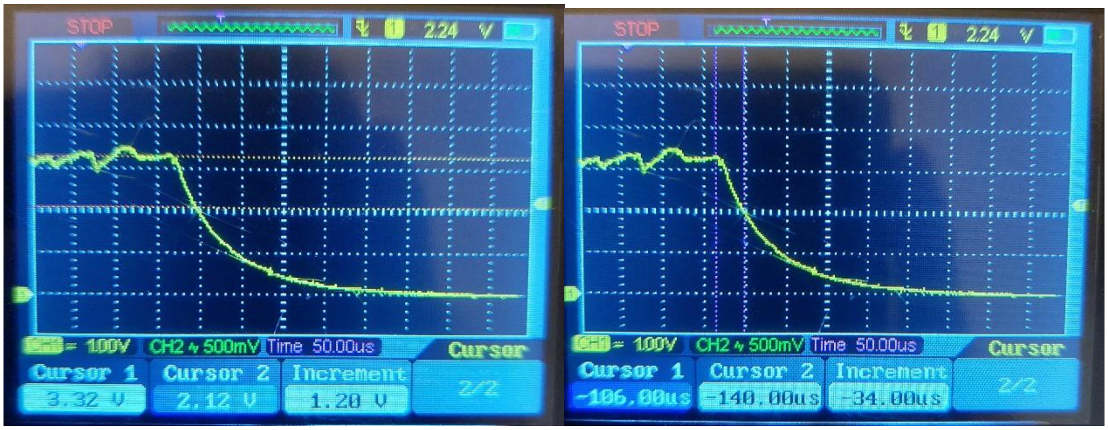
For discharging I used the same exact method as for charging. Here is the curve I obtained.

Using the value of voltage at the time constant, I performed the following calculation:

$$V(0) = 3.28V$$
$$V(\infty) = 0V$$
$$V(\tau) = V(0)\, e^{-\frac{\tau}{\tau}} + V(\infty)\left(1 - e^{-\frac{\tau}{\tau}}\right)$$
$$V(\tau) = V(0)\, e^{-1} + V(\infty)\left(1 - e^{-1}\right)$$
$$V(\tau) = 3.28 \cdot e^{-1} + 0 \cdot \left(1 - e^{-1}\right)$$
$$V(\tau) = 1.2066\ V$$

I then measured the time constant using the same method as for the charging state.



Physical Solution (Charging)

Variable/Equation	**Values**
Voltage at t=0	0 V
Voltage at t=∞	3.64 V
Time constant (τ)	17.6 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.64 - 3.64e^{-\frac{\mathrm{t}}{17.6\ \mu\mathrm{s}}}$

Physical Solution (Discharging)

Variable/Equation	**Values**
Voltage at t=0	3.28 V
Voltage at t=∞	0 V
Time constant (τ)	34.00 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.28e^{-\frac{\mathrm{t}}{34\ \mu\mathrm{s}}}$

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Table Summarizing Results

Charging

Variable/Equation	Analytical	Digital	Physical
Voltage at t=0	0 V	0 V	0 V
Voltage at t= ∞	3.75 V	3.73 V	3.64 V
Time constant (τ)	17.335 μs	21.4 μs	17.6 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.75 - 3.75e^{-\frac{\mathrm{t}}{17.335\ \mathrm{\mu s}}}$	$V(\mathrm{t}) = 3.73 - 3.73e^{-\frac{\mathrm{t}}{21.4\ \mathrm{\mu s}}}$	$V(\mathrm{t}) = 3.64 - 3.64e^{-\frac{\mathrm{t}}{17.6\ \mathrm{\mu s}}}$

Discharging

Variable/Equation	Analytical	Digital	Physical
Voltage at t=0	3.75 V	3.72 V	3.28 V
Voltage at t= ∞	0 V	0 V	0 V
Time constant (τ)	69.34 μs	42.5 μs	34.00 μs
Voltage equation across capacitor	$V(\mathrm{t}) = 3.75e^{-\frac{\mathrm{t}}{69.34\ \mu s}}$	$V(\mathrm{t}) = 3.72e^{-\frac{\mathrm{t}}{42.5\ \mathrm{\mu s}}}$	$V(\mathrm{t}) = 3.28e^{-\frac{\mathrm{t}}{34\ \mathrm{\mu s}}}$

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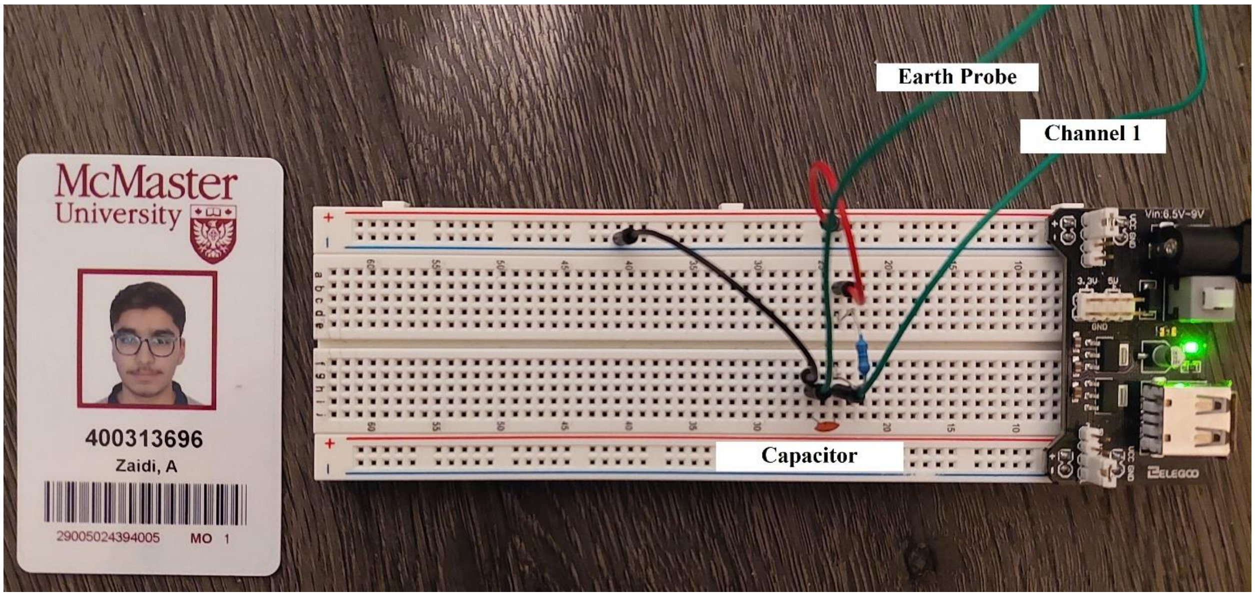
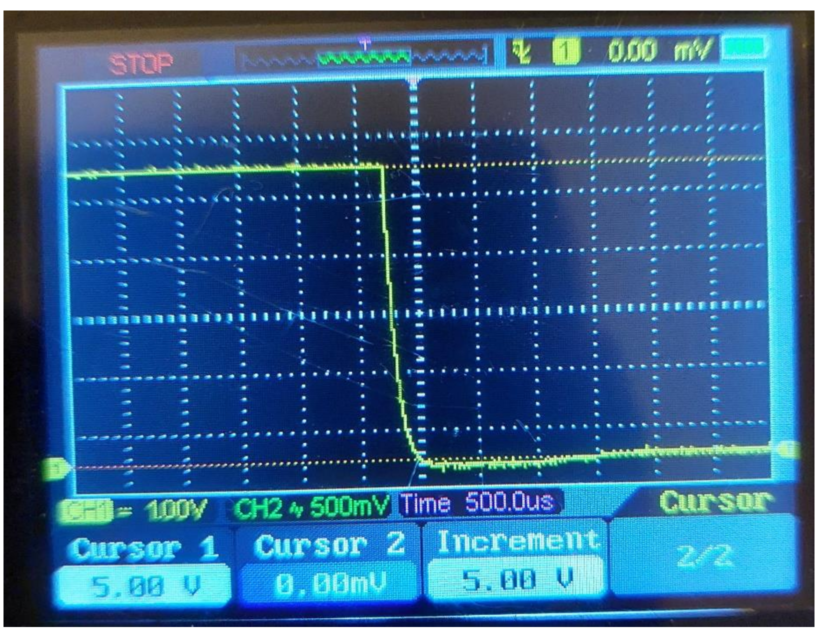
Scope Probe Impedance

I constructed the following circuit to measure the scope probe impedance. Because the probes are connected across the capacitor, the resistor could have any value and it will not affect the results. I connected the resistor mainly for safety and to prevent the capacitor from blowing up.



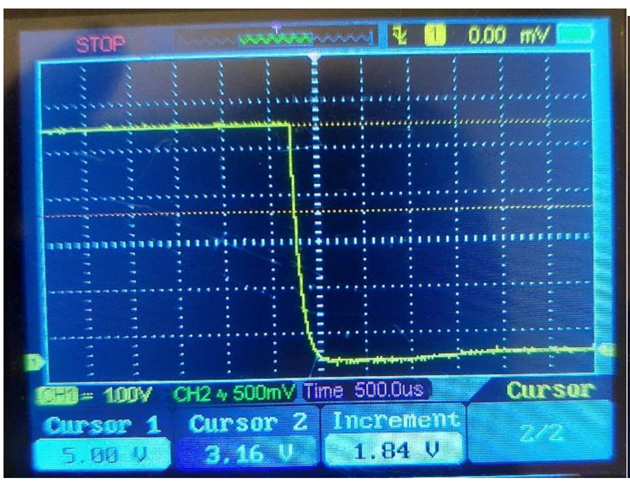
The following curve was obtained from the oscilloscope.

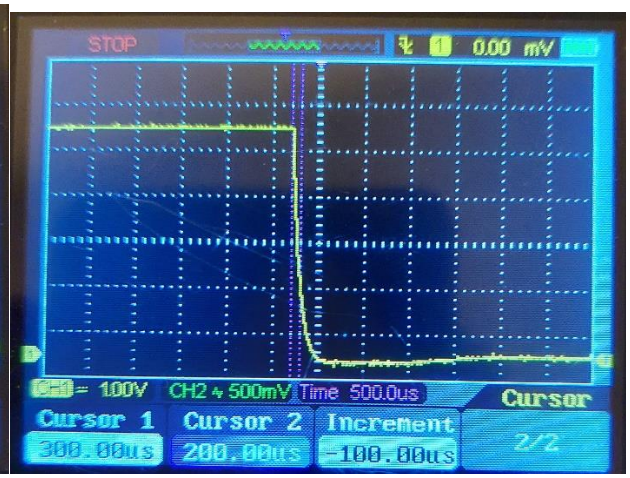


The voltage across it is 5V (same as power supply) meaning that the curve is correct. I will now perform the following calculation to calculate the voltage at the time constant.

$$V(0) = 5V$$
$$V(\infty) = 0V$$
$$V(\tau) = 5e^{-\frac{\tau}{\tau}} + V(\infty)\left(1 - e^{-\frac{\tau}{\tau}}\right)$$
$$V(\tau) = 5e^{-1} + V(\infty)\left(1 - e^{-1}\right)$$
$$V(\tau) = 5 \cdot e^{-1} + 0 \cdot \left(1 - e^{-1}\right)$$
$$V(\tau) = 1.839\ V$$

With this voltage, I can now use the voltage probes to have an increment of 1.84V and then measure the time difference (τ).





Now that I have found out the time constant, I can use the equation $\tau = RC$ to calculate the resistance which is going to be the scope probe impedance.

$$\tau = RC$$
$$R = \frac{\tau}{C}$$
$$R = \frac{100 \times 10^{-6}}{34.67 \times 10^{-9}}$$
$$R = 2884\ \Omega$$

Thus, the scope probe impedance is 2884 ohms.

Energy Stored in Capacitor

The energy stored in the capacitor will be the energy it will have before discharge. This means that before t=0, the capacitor will have the following amount of energy before reaching to 0 volts.

$$E=\frac{1}{2}\cdot CV^2$$

$$E=\frac{1}{2}\cdot\left(34.67\times 10^{-9}\right)\cdot(5)^2$$

$$E=4.33\times 10^{-7}J$$

Thus, the energy stored across the capacitor was 4.33×10^{-7} J.

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Analysis

The results from this lab were close to each other. There was some error involved which led to a range of values for discharging but overall, with regards to charging, the results matched with near perfection.

One of the main sources of error had to do with how the physical circuit was set up. With a wire acting as a switch, there was some noise involved with the signal produced. This was no problem for charging curves as for that we only needed to close the switch by plugging in the wire (which can even be done by touching the wire on the edge of the hole of the breadboard). However, when measuring the discharge, I had to remove the wire from the hole which resulted in noise hence a variation in results. A solution to this could be using a switch.

There was also a problem of reduced accuracy in digital and physical solutions. The voltage at the time constant calculated was more than three significant figures. This was not that much of a problem for the digital solution but for the physical solution, it led to a lot of systematic error. For example, for the charging solution for the physical part, my value for the voltage at the time constant was 2.30V. When I moved the cursors, however, I could only have a difference of either 2.28V or 2.32V which was not the correct solution.

Overall, the results show that for charging the initial voltage is always 0V because the capacitor has no current passing through it. When the switch opens, then the voltage increases to a stable value asymptotically which highlights the fact that the capacitor plates are fully charged as time approaches infinity and no current can pass though. On the other hand, when the capacitor discharges, it already has reached its stable value thus initially the capacitor voltage will have a non-zero value. Then when the switch opens, the capacitor provides energy (calculated in the last part) to the resistor (R3) until it reaches 0V.

Reflection

This lab not only strengthened my previous knowledge on first order transient analysis (from first year) but also strengthened by skills with regards to the oscilloscope. I got to see changes in real-time which increased my eagerness for the later contents of this course. The lab proved to be a great review of the material previously learnt and applying the theory we learnt to produce three solutions was very useful.

The lab would be useful in later chapters as it reinforced my knowledge on capacitors. Now I feel more confident in not only analytically solving but also physically solving a problem which involves capacitors. Being a very important part of everyday electronics, the knowledge of capacitors is necessary, and I think that I have built a strong foundation for the labs and projects in the future that involves capacitors.

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Appendix

```
> restart;
R1:=1e3: R2:=2e3: R3:=2e3: V1:=5: C1:=34.67e-9:
Rpar:=(1/R1 + 1/R2)^(-1);
Vc:=(R3/(R3+Rpar))*V1;
Rth:=((1/R1)+(1/R2)+(1/R3))^(-1);
Tau:=Rth*C1
```

$$Rpar := 666.6666667$$

$$Vc := 3.750000000$$

$$Rth := 500.0000000$$

$$\mathrm{T} := 0.00001733500000$$

```
> restart;
R1:=1e3: R2:=2e3: R3:=2e3: V1:=5: C1:=34.67e-9:
Rpar:=(1/R1 + 1/R2)^(-1);
Vc:=(R3/(R3+Rpar))*V1;
Tau:= R3*C1
```

$$Rpar := 666.6666667$$

$$Vc := 3.750000000$$

$$\mathrm{T} := 0.00006934$$