Diagnostic_Plotting

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NOTE: This document can be viewed in PDF or HTML (recommended) form. It can also be run as an interactive Jupyter notebook.

The HTML and PDF versions are located in Rayleigh/doc/Diagnostic_Plotting.{html,pdf} The Jupyter notebook is located in Rayleigh/etc/analysis/Diagnostic_Plotting.ipynb Standalone Python example scripts for each output type may also found in Rayleigh/etc/analysis/

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2 I. Running a Benchmark with Sample Output

Before you can plot data, you will need to generate data. The code samples in this notebook assume that you have run the model described by the input file found in:

rayleigh/etc/input_examples/benchmark_diagnostics_input

This input file instructs *Rayleigh* to run the Christensen et al. (2001) hydrodynamic (case 0) benchmark. Running this model with the prescribed outputs will generate approximately 70 MB of data.

To run this model: 1. Create a directory for your simulation (e.g., mkdir my_test_run) 2. Copy the input file: cp rayleigh/etc/input_examples/benchmark_diagnostics_input my_test_run/main_input 3. Copy or soft-link the rayleigh executable: cp Rayleigh/bin/rayleigh.opt my_test_run/. 4. Run the code: mpiexec -np N ./rayleigh -nprow n -npcol m (choose values of {N,n,m} such that n x m = N)

The code will run for 40,000 timesteps, or four viscous diffusion times. While it runs, *Rayleigh* will perform an in-situ analysis of the accuracy benchmark. Reports are written once every 1,000 time steps and are stored in the *Benchmark_Reports* subdirectory. Examine file 00030000 and ensure that you see similar results to those below. Your exact numbers may differ slightly, but all quantities should be under 1% difference.

Observable	Measured	Suggested	% Difference	Std. Dev.
Kinetic Energy	58.347827	58.348000	-0.000297	0.000000
Temperature	0.427424	0.428120	-0.162460	0.000101
Vphi	-10.119483	-10.157100	-0.370356	0.013835
Drift Frequency	0.183016	0.182400	0.337630	0.007295

If necessary, copy the data to the system on which you intend to conduct your analysis. Before you can plot, you will need to configure your Python environment.

3 II. Configuring Your Python Environment

Rayleigh comes packaged with a Python library (*rayleigh_diagnostics.py*) that provides data structures and methods associated with each type of diagnostic output in Rayleigh. This library relies on Numpy and is compatible with Python 3.x or 2.x (The *print* function is imported from the *future* module).

If you wish to follow along with the plotting examples described in this document, you will need to have the Numpy, Matplotlib, and (optionally) Basemap Python packages installed. The following versions of these packages were used when creating these examples: * Matplotlib v2.0.2 * Numpy v1.13.1 * Basemap v1.0.7

Unless you are experienced at installing and managing Python packages, I recommend setting up a virtual environment for Python using Conda. You may also install the required packages manually, but the advantage of this approach is that you maintain an entirely separate version of Python and related packages for this project. Below are directions for setting up a Python/Conda environment with Intel-optimized Python packages on a Linux system (Mac and Windows work similarly).

3.1 Conda Installation on Linux Systems

Step 1: Download the appropriate Miniconda installation script from https://conda.io/miniconda.html (choose Python 3.x)

Step 2: Make the shell script executable via: **chmod** +**x Miniconda3-latest-Linux-x86_64.sh** (or similar script name)

Step 3: Run the installation script: ./Miniconda3-latest-Linux-x86_64.sh

NOTE: The default installation directory is your home directory. This is also where Python packages for your Conda environments will be installed. Avoid installing to a disk with limited space (user home directories on HPC systems are often limited to a few GB).

NOTE: Unless you have a specific reason not to do so, answer "yes" to the question concerning prepending to PATH.

Step 5: Update your Conda: conda update conda

Step 6: Add the Intel Conda channel: conda config --add channels intel

Step 7: Create a virtual environment for Intel's Conda distribution: **conda create -n idp** intelpython3_full python=3

NOTE: In this case, *idp* will be your virtual environment name. You are free to pick an alternative when running conda create.

NOTE: A number of Python packages will be downloaded, including Numpy and Matplotlib. The process may appear to hang at the last step. Be patient.

Step 8: Activate your virtual environment: source activate idp

Step 9: Install the Basemap package: conda install -c intel basemap

Step 10: Verify your installation. Type **python** and then type the following commands at the prompt: 1. import numpy 2. import matplotlib 3. import mpl_tookits.basemap

If those commands worked without error, you may close Python (type exit()). You can revert to your native environment by typing source deactivate (or just close the terminal). Whenever you wish to access your newly-installed Python, type source activate idp first, before running python.

3.2 Preparing to Plot

All examples in this document rely on the rayleigh_diagnostics module. This module is located in Rayleigh/etc/analysis, along with several standalone scripts copies from the individual sections of this document. For example, the script **plot_G_Avgs.py** contains the code from section IV below. All python files you wish to use will need to reside in either your run directory (recommended) or a directory within your PYTHONPATH.

We suggest copying all python files to your my_test_run directory: 1. cp Rayleigh/etc/analysis/.py my_test_run/. 2. cp Rayleigh/etc/analysis/.ipynb my_test_run/.

3.3 The Jupyter Notebook

This document resides in three places: 1. Rayleigh/doc/Diagnostic_Plotting.pdf 2. Rayleigh/doc/Diagnostic_Plotting.html 3. Rayleigh/etc/analysis/Diagnostic_Plotting.ipynb

The third file is a Jupyter notebook file. This source code was used to generate the html and pdf documents. The notebook is designed to be run from within a Rayleigh simulation directory. If you wish to follow along interactively, copy the Jupyter notebook file from etc/analysis into your Rayleigh simulation directory (step 2 from *Preparing to Plot*). You can run the file in Jupyter via: 1. source activate idp 2. jupyter notebook (from within your my_test_run directory) 3. select Diagnostic_Plotting.ipynb in the file menu that presents itself.

When finished: 1. To close the notebook, type **ctrl+c** and enter "yes" when prompted to shut down the notebook server. 2. type **source deactivate**

4 III. Overview of Diagnostics in Rayleigh

Rayleigh's diagnostics package facilitates the in-situ analysis of a simulation using a variety of sampling methods. Each sampling method may be applied to a unique set of sampled quantities. Sampling methods are hereafter referred to as *output types* and sampled quantities as *output variables*.

Files of each output type are stored in a similarly-named subdirectory within the *Rayleigh* simulation directory. Output files are numbered by the time step of the final data record stored in the file. Output behavior for each simulation is controlled through the *main_input* file. For each

output type, the user specifies the output variables, cadence, records-per-file, and other properties by modifying the appropriate variables in the **output_namelist** section of *main_input*.

4.1 Basic Output Control

Each output type in *Rayleigh* has at least three namelist variables that govern its behavior:

{OutputType}_values: comma-separated list of menu codes corresponding to the desired output variables

{OutputType}_frequency: integer value that determines how often this type of output is performed

{OutputType}_nrec: integer value that determines how many records are stored in each output file.

All possible output variables and their associated menu codes are described in rayleigh/doc/rayleigh_output_variables.pdf You may find it useful to have that document open while following along with examples in this notebook.

As an example of how these variables work, suppose that we want to occasionally output equatorial cuts (output type) of temperature, kinetic energy density, and radial velocity (output variables). At the same time, we might wish to dump full-volume averages (output type) of kinetic and magnetic energy (output variables) with a higher cadence. In that case, something similar to the following would appear in main_input:

```
globalavg_values = 401, 1101
globalavg_frequency = 50
globalavg_nrec = 100
equatorial_values = 1, 401, 501
equatorial_frequency = 2500
equatorial_nrec = 2
```

This tells *Rayleigh* to output full-volume-averages of kinetic energy density (value code 401) and magnetic energy density (value code 1101) once every 50 time steps, with 100 records per file. Files are named based on the time step number of their final record. As a result, information from time steps 50, 100, 150, ..., 4950, 5000 will be stored in the file named $G_Avgs/00005000$. Time steps 5050 through 10,000 will stored in $G_Avgs/00010000$, and so on.

For the equatorial cuts, *Rayleigh* will output radial velocity (code 1), the kinetic energy density (code 401) and temperature (code 501) in the equatorial plane once every 2,500 time steps, storing two time steps per file. Data from time steps 2,500 and 5,000 will be stored in *Equatorial_Slices/00005000*. Data from time steps 7,500 and 10,000 will be stored in *Equatorial_Slices/00010000*, and so on.

This general organizational scheme for output was adapted from that developed by Thomas Clune for the ASH code.

4.2 Positional Output Control

Many of *Rayleigh's* output types allow the user to specify a set of gridpoints at which to sample the simulation. A user can, for example, output spherical surfaces sampled at arbitrary radii, or a meridional plane sampled at a specific longitude. This behavior is controlled through additional namelist variables; we refer to these variables as positional specifiers. In the sections that follow, positional specifiers associated with a given output type, if any, will be defined.

Positional specifiers are either indicial or normalized. In the main_input file, indicial specifiers

can be assigned a comma-separated list of grid indices on which to perform the output. For example,

shellslice_levels = 1, 32, 64, 128

instructs *Rayleigh* to output shell slices at { radius[1], radius[32], radius[64], radius[128]}. Note that radius[1] is the outer boundary.

While useful in some situations, specifying indices can lead to confusion if a simulations resolution needs to be changed at some point during a model's evolution. For example if the radial grid initially had 128 points, index 128 would correspond to the lower boundary. If the resolution were to double, index 128 would correspond to mid-shell.

For this reason, all positional specifiers may also be written in normalized form. Instead of integers, the normalized specifier is assigned a comma separated list of real values in the range [0,1]. The value of zero corresponds to the lowest-value grid coordinate (e.g., the inner radial boundary or theta=0 pole). The value 1 corresponds to the maximal coordinate (e.g., the outer radial boundary or theta=pi pole). A value of 0.5 corresponds to mid-domain. Normalized coordinates are indicated by adding *_nrm* to the indicial specifier's name. For example,

shellslice_levels_nrm= 0, 0.5, 0.95

instructs *Rayleigh* to output shell slices at the lower boundary, mid-shell, and slightly below the upper boundary. *Rayleigh* does not interpolate, but instead picks the grid coordinate closest to each specified normalized coordinate.

We recommend using normalized coordinates to avoid inconsistencies between restarts. They also overcome difficulties associated with the non-uniform nature of the radial and theta grids wherein grid points cluster near the boundaries.

Positional Ranges Ranges of coordinates can be specified using shorthand, if desired. The inclusive coordinate range [X,Y] is indicated by a positive/negative number pair appearing in the indicial or normalized coordinate list. Multiple ranges can be specified within a list. For example,

shellslice_levels = 1,10,-15, 16, 20,-25, 128

would instruct *Rayleigh* to output shell slices at radial indices = { 1, 10, 11, 12, 13, 14, 15, 16, 20, 21, 22, 23, 24, 25, 128}

Similarly,

 $shellslice_levels_nrm = 0,-0.5, 1.0$

instructs *Rayleigh* to output shells at all radii in the lower half of the domain, and at the outer boundary.

4.3 IV. Global Averages

Summary: Full-volume averages of requested output variables over the full, spherical shell

Subdirectory: G_Avgs

main_input prefix: globalavg

Python Class: G_Avgs

Additional Namelist Variables:

None

Before proceeding, ensure that you have copied Rayleigh/etc/analysis/rayleigh_diagnostics.py to your simulation directory. This Python module is required for reading Rayleigh output into Python.

Examining the *main_input* file, we see that the following output values have been denoted for the Global Averages (see *rayleigh_output_variables.pdf* for the mathematical formulae):

Menu Code	Description
401	Full Kinetic Energy Density (KE)

Menu Code	Description
402	KE (radial motion)
403	KE (theta motion)
404	KE (phi motion)
405	Mean Kinetic Energy Density (MKE)
406	MKE (radial motion)
407	MKE (theta motion)
408	MKE (phi motion)
409	Fluctuating Kinetic Energy Density (FKE)
410	FKE (radial motion)
411	FKE (theta motion)
412	FKE (phi motion)

In the example that follows, we will plot the time-evolution of these different contributions to the kinetic energy budget. We begin with the following preamble:

```
In [1]: from rayleigh_diagnostics import G_Avgs, build_file_list
    import matplotlib.pyplot as plt
    import numpy
```

The preamble for each plotting example will look similar to that above. We import the numpy and matplotlib.pyplot modules, aliasing the latter to *plt*. We also import two items from *rayleigh_diagnostics*: a helper function *build_file_list* and the *GlobalAverage* class.

The G_Avgs class is the Python class that corresponds to the full-volume averages stored in the G_Avgs subdirectory of each Rayleigh run.

We will use the build_file_list function in many of the examples that follow. It's useful when processing a time series of data, as opposed to a single snapshot. This function accepts three parameters: a beginning time step, an ending time step, and a subdirectory (path). It returns a list of all files found in that directory that lie within the inclusive range [beginning time step, ending time step]. The file names are prepended with the subdirectory name, as shown below.

We can create an instance of the G_Avgs class by initializing it with a filename. The optional keyword parameter *path* is used to specify the directory. If *path* is not specified, its value will default to the subdirectory name associated with the datastructure (*G_Avgs* in this instance).

Each class was programmed with a **docstring** describing the class attributes. Once you created an instance of a rayleigh_diagnostics class, you can view its attributes using the help function as shown below.

```
In [3]: a = G_Avgs(filename=files[0],path='') # Here, files[0]='G_Avgs/00010000'
#a= G_Avgs(filename='00010000') would yield an equivalent result
help(a)
```

Help on G_Avgs in module rayleigh_diagnostics object:

```
class G_Avgs(builtins.object)
| Rayleigh GlobalAverage Structure
   -----
| self.niter
                            : number of time steps
| self.nq
                           : number of diagnostic quantities output
| self.qv[0:nq-1]
                           : quantity codes for the diagnostics output
self.vals[0:niter-1,0:nq-1] : The globally averaged diagnostics
| self.iters[0:niter-1]
                          : The time step numbers stored in this output file
 | self.time[0:niter-1] : The simulation time corresponding to each time step
 | self.version
                           : The version code for this particular output (internal use)
| self.lut
                           : Lookup table for the different diagnostics output
  Methods defined here:
   __init__(self, filename='none', path='G_Avgs/')
       filename : The reference state file to read.
              : The directory where the file is located (if full path not in filename
       path
   ______
  Data descriptors defined here:
   __dict__
       dictionary for instance variables (if defined)
   __weakref__
       list of weak references to the object (if defined)
```

Examining the docstring, we see a few important attributes that are common to the other outputs discussed in this document: 1. niter -- the number of time steps in the file 2. nq -- the number of output variables stored in the file 3. qv -- the menu codes for those variables 4. vals -- the actual data 5. time -- the simulation time corresponding to each output dump

The first step in plotting a time series is to collate the data.

```
In [4]: # Loop over all files and concatenate their data into a single array
    nfiles = len(files)
    for i,f in enumerate(files):
        a = G_Avgs(filename=f,path='')
        if (i == 0):
            nq = a.nq
            niter = a.niter
            gavgs = numpy.zeros((niter*nfiles,nq),dtype='float64')
            iters = numpy.zeros(niter*nfiles,dtype='int32')
            time = numpy.zeros(niter*nfiles,dtype='float64')
        i0 = i*niter
```

```
i1 = (i+1)*niter
gavgs[i0:i1,:] = a.vals
time[i0:i1] = a.time
iters[i0:i1] = a.iters
```

4.4 The Lookup Table (LUT)

The next step in the process is to identify where within the *gavgs* array our deisired output variables reside. Every Rayleigh file object possesses a lookup table (lut). The lookup table is a python list used to identify the index within the vals array where a particular menu code resides. For instance, the menu code for the theta component of the velocity is 2. The location of v_theta in the vals array is then stored in lut[2].

Note that you should never assume that output variables are stored in any particular order. Moreover, the lookup table is unique to each file and is likely to change during a run if you modify the output variables in between restarts. When running the benchmark, we kept a consistent set of outputs throughout the entirety of the run. This means that the lookup table did not change between outputs and that we can safely use the final file's lookup table (or any other file's table) to reference our data.

4.5 Plotting Kinetic Energy

Let's examine the different contributions to the kinetic energy density in our models. Before we can plot, we should use the lookup table to identify the location of each quantity we are interested in plotting.

```
In [5]: #The indices associated with our various outputs are stored in a lookup table #as part of the GlobalAverage data structure. We define several variables to #hold those indices here:
```

```
lut = a.lut
ke = lut[401] # Kinetic Energy (KE)
rke = lut[402] # KE associated with radial motion
tke = lut[403] # KE associated with theta motion
pke = lut[404] # KE associated with azimuthal motion
#We also grab some energies associated with the mean (m=0) motions
mke = lut[405]
mrke = lut[406] # KE associated with mean radial motion
mtke = lut[407] # KE associated with mean theta motion
mpke = lut[408] # KE associated with mean azimuthal motion
#We also output energies associated with the fluctuating/nonaxisymmetric
#motions (e.g., v-v_{m=0})
fke = lut[409]
frke = lut[410] # KE associated with mean radial motion
ftke = lut[411] #KE associated with mean theta motion
fpke = lut[412] # KE associated with mean azimuthal motion
```

To begin with, let's plot the total, mean, and fluctuating kinetic energy density during the initial transient phase, and then during the equilibrated phase.

```
In [6]: sizetuple=(10,3)
        fig, ax = plt.subplots(ncols=2, figsize=sizetuple)
        ax[0].plot(time, gavgs[:,ke], label='KE')
        ax[0].plot(time, gavgs[:,mke],label='MKE')
        ax[0].plot(time, gavgs[:,fke], label='FKE')
        ax[0].legend(loc='center right', shadow=True)
        ax[0].set_xlim([0,0.2])
        ax[0].set_title('Equilibration Phase')
        ax[0].set_xlabel('Time')
        ax[0].set_ylabel('Energy')
        ax[1].plot(time, gavgs[:,ke], label='KE')
        ax[1].plot(time, gavgs[:,mke], label = 'MKE')
        ax[1].plot(time,gavgs[:,fke],label='FKE')
        ax[1].legend(loc='center right', shadow=True)
        ax[1].set_title('Entire Time-Trace')
        ax[1].set_xlabel('Time')
        ax[1].set_ylabel('Energy')
        saveplot = False # Plots appear in the notebook and are not written to disk (set to True
        savefile = 'energy_trace.pdf' #Change .pdf to .png if pdf conversion gives issues
        plt.tight_layout()
        plt.show()
                   Equilibration Phase
                                                           Entire Time-Trace
      80
                                              80
      60
                                              60
                                       KE
                                                                               KE
      40
                                       MKE
                                              40
                                                                               MKE
                                       FKE
                                                                               FKE
      20
                                              20
```

0.0 0.5 1.0

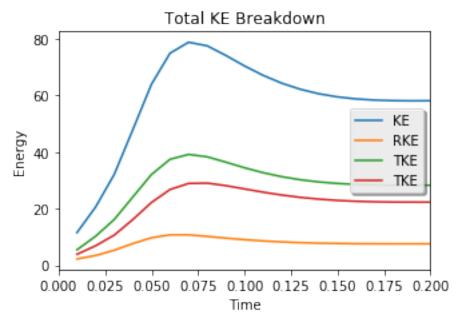
2.0

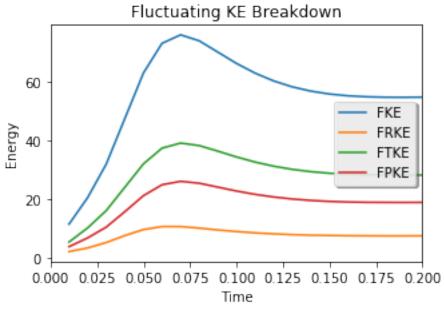
3.0

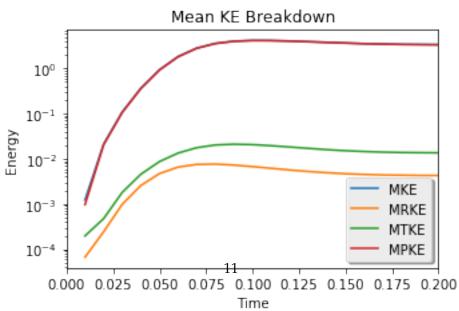
We can also look at the energy associated with each velocity component. Note that we log scale in the last plot. There is very little mean radial or theta kinetic energy; it is mostly phi energy.

0.000 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200

```
ax[0].plot(time, gavgs[:,pke], label='TKE')
ax[0].legend(loc='center right', shadow=True)
ax[0].set_xlim(xlims)
ax[0].set_title('Total KE Breakdown')
ax[0].set_xlabel('Time')
ax[0].set_ylabel('Energy')
ax[1].plot(time, gavgs[:,fke], label='FKE')
ax[1].plot(time, gavgs[:,frke], label='FRKE')
ax[1].plot(time, gavgs[:,ftke], label='FTKE')
ax[1].plot(time, gavgs[:,fpke], label='FPKE')
ax[1].legend(loc='center right', shadow=True)
ax[1].set_xlim(xlims)
ax[1].set_title('Fluctuating KE Breakdown')
ax[1].set_xlabel('Time')
ax[1].set_ylabel('Energy')
ax[2].plot(time, gavgs[:,mke], label='MKE')
ax[2].plot(time, gavgs[:,mrke], label='MRKE')
ax[2].plot(time, gavgs[:,mtke], label='MTKE')
ax[2].plot(time, gavgs[:,mpke], label='MPKE')
ax[2].legend(loc='lower right', shadow=True)
ax[2].set_xlim(xlims)
ax[2].set_title('Mean KE Breakdown')
ax[2].set_xlabel('Time')
ax[2].set_ylabel('Energy')
ax[2].set_yscale('log')
plt.tight_layout()
plt.show()
```







5 V. Shell Averages

Summary: Spherical averages of requested output variables. Each output variable is stored as a 1-D function of radius.

Subdirectory: Shell_Avgs main_input prefix: shellavg Python Class: Shell_Avgs Additional Namelist Variables:

None

The Shell-Averaged outputs are useful for examining how quantities vary as a function of radius. They are particularly useful for examining the distribution of energy as a function of radius, or the heat flux balance established by the system.

Examining the *main_input* file, we see that the following output values have been denoted for the Shell Averages (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity
501	Temperature Perturbation
1438	Radial Convective Heat Flux
1468	Radial Conductive Heat Flux

In the example that follows, we will plot the spherically-averaged velocity field as a function of radius, the mean temperature profile, and the radial heat flux. We begin with a preamble similar to that used for the Global Averages. Using the help function, we see that the Shell_Avgs data structure is similar to that of the G_Avgs. There are three important differences: * There is a radius attribute (necessary if we want to plot anything vs. radius) * The dimensionality of the values array has changed; radial index forms the first dimension. * The second dimension of the values array has a length of 4. In addition to the spherical mean, the 1st, 2nd and 3rd moments are stored in indices 0,1,2, and 3 respectively.

```
In [8]: from rayleigh_diagnostics import Shell_Avgs, build_file_list
    import matplotlib.pyplot as plt
    import numpy

# Build a list of all files ranging from iteration 0 million to 1 million
    files = build_file_list(0,1000000,path='Shell_Avgs')
    a = Shell_Avgs(filename=files[0], path='')
    help(a)

Help on Shell_Avgs in module rayleigh_diagnostics object:

class Shell_Avgs(builtins.object)
```

```
| Rayleigh Shell Average Structure
| self.niter
                                     : number of time steps
| self.nq
                                     : number of diagnostic quantities output
| self.nr
                                   : number of radial points
| self.qv[0:nq-1]
                                    : quantity codes for the diagnostics output
| self.radius[0:nr-1]
                                    : radial grid
| For version 1:
| self.vals[0:nr-1,0:nq-1,0:niter-1] : The spherically averaged diagnostics
| For version 2:
  self.vals[0:n-1,0:3,0:nq-1,0:niter-1] : The spherically averaged diagnostics
                                           0-3 refers to moments (index 0 is mean, index 3 is
| self.iters[0:niter-1]
                                     : The time step numbers stored in this output file
| self.time[0:niter-1]
                                    : The simulation time corresponding to each time step
                                     : The version code for this particular output (internal u
| self.version
| self.lut
                                     : Lookup table for the different diagnostics output
 Methods defined here:
 __init__(self, filename='none', path='Shell_Avgs/', ntheta=0)
      filename : The reference state file to read.
      path
                : The directory where the file is located (if full path not in filename
 Data descriptors defined here:
 __dict__
      dictionary for instance variables (if defined)
  __weakref__
      list of weak references to the object (if defined)
```

While it can be useful to look at instaneous snapshots of Shell Averages, it's often useful to examine these outputs in a time-averaged sense. Let's average of all 200 snapshots in the last file that was output. We could average over data from multiple files, but since the benchmark run achieves a nearly steady state, a single file will do in this case.

```
nmom = 4
niter = a.niter
radius = a.radius
savg=numpy.zeros((nr,nmom,nq),dtype='float64')
for i in range(niter):
    savg[:,:,:] += a.vals[:,:,:,i]
savg = savg*(1.0/niter)

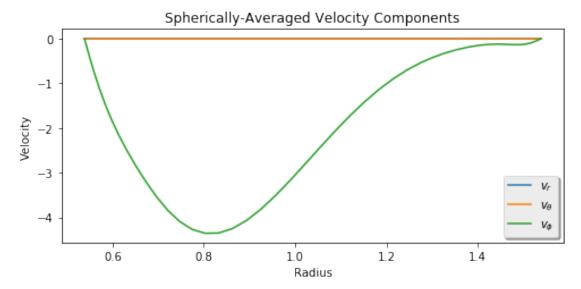
lut = a.lut
vr = lut[1]  # Radial Velocity
vtheta = lut[2]  # Theta Velocity
vphi = lut[3]  # Phi Velocity
thermal = lut[501]  # Temperature

eflux = lut[1438]  # Convective Heat Flux (radial)
cflux = lut[1468]  # Conductive Heat Flux (radial)
```

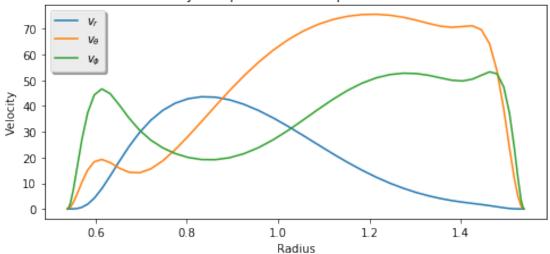
5.1 Velocity vs. Radius

Next, we plot the mean velocity field, and its first moment, as a function of radius. Notice that the radial and theta velocity components have a zero spherical mean. Since we are running an incompressible model, this is a good sign!

```
In [10]: sizetuple = (7,7)
         fig, ax = plt.subplots(nrows=2, ncols =1, figsize=sizetuple)
         ax[0].plot(radius,savg[:,0,vr],label=r'$v_r$')
         ax[0].plot(radius,savg[:,0,vtheta], label=r'$v_\theta$')
         ax[0].plot(radius,savg[:,0,vphi], label=r'$v_\phi$')
         ax[0].legend(shadow=True,loc='lower right')
         ax[0].set_xlabel('Radius')
         ax[0].set_ylabel('Velocity')
         ax[0].set_title('Spherically-Averaged Velocity Components')
         ax[1].plot(radius,savg[:,1,vr],label=r'$v_r$')
         ax[1].plot(radius,savg[:,1,vtheta], label=r'$v_\theta$')
         ax[1].plot(radius,savg[:,1,vphi], label=r'$v_\phi$')
         ax[1].legend(shadow=True,loc='upper left')
         ax[1].set_xlabel('Radius')
         ax[1].set_ylabel('Velocity')
         ax[1].set_title('Velocity Components: First Spherical Moment')
         plt.tight_layout()
         plt.show()
```







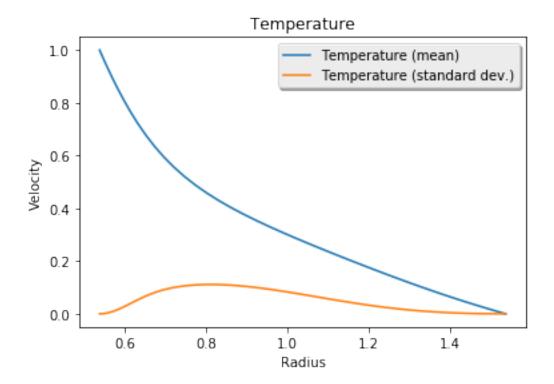
5.2 Radial Temperature Profile

We might also look at temperature ...

```
In [11]: fig, ax = plt.subplots()

ax.plot(radius,savg[:,0,thermal],label='Temperature (mean)')
ax.plot(radius,savg[:,1,thermal]*10, label='Temperature (standard dev.)')
ax.legend(shadow=True,loc='upper right')
ax.set_xlabel('Radius')
ax.set_ylabel('Velocity')
ax.set_title('Temperature')
```

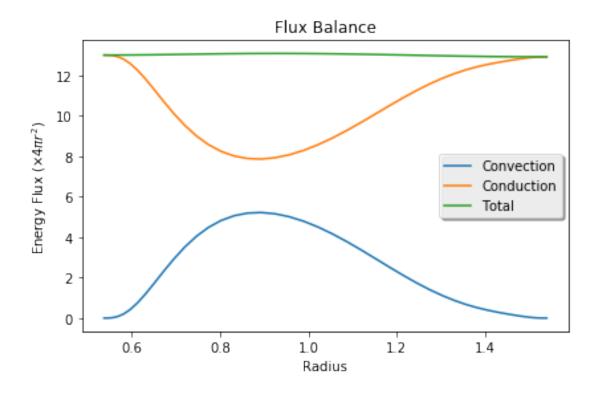
plt.show()



5.3 Heat Flux Contributions

We can also examine the balance between convective and conductive heat flux. In this case, before plotting these quantities as a function of radius, we normalize them by the surface area of the sphere to form a luminosity.

```
In [12]: fpr=4.0*numpy.pi*radius*radius
        elum = savg[:,0,eflux]*fpr
        clum = savg[:,0,cflux]*fpr
        tlum = elum+clum
        fig, ax = plt.subplots()
        ax.plot(radius,elum,label='Convection')
        ax.plot(radius,clum, label='Conduction')
        ax.plot(radius,tlum, label='Total')
        ax.set_title('Flux Balance')
        ax.set_ylabel(r'Energy Flux ($\times 4\pi r^2$)')
        ax.set_xlabel('Radius')
        ax.legend(shadow=True)
        plt.tight_layout()
        plt.show()
```



6 VI. Azimuthal Averages

Summary: Azimuthal averages of requested output variables. Each output variable is stored as a 2-D function of radius and latitude.

Subdirectory: AZ_Avgs **main_input prefix:** azavg **Python Class:** AZ_Avgs

Additional Namelist Variables:

None

Azimuthally-Averaged outputs are particularly useful for examining a system's mean flows (i.e., differential rotation and meridional circulation).

Examining the *main_input* file, we see that the following output values have been denoted for the Azimuthal Averages (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity
201	Radial Mass Flux
202	Theta Mass Flux
501	Temperature Perturbation

17

In the example that follows, we demonstrate how to plot azimuthal averages, including how to generate streamlines of mass flux. Note that since the benchmark is Boussinesq, our velocity and mass flux fields are identical. This is not the case when running an anelastic simulation.

We begin with the usual preamble and also import two helper routines used for displaying azimuthal averages.

Examining the data structure, we see that the vals array is dimensioned to account for latitudinal variation, and that we have new attributes costheta and sintheta used for referencing locations in the theta direction.

```
In [13]: from rayleigh_diagnostics import AZ_Avgs, build_file_list, plot_azav, streamfunction
        import matplotlib.pyplot as plt
        import pylab
        import numpy
        #from azavg_util import *
        files = build_file_list(30000,40000,path='AZ_Avgs')
        az = AZ_Avgs(files[0],path='')
        help(az)
Help on AZ_Avgs in module rayleigh_diagnostics object:
class AZ_Avgs(builtins.object)
 | Rayleigh AZ_Avgs Structure
 | -----
 | self.niter
                                              : number of time steps
 | self.nq
                                              : number of diagnostic quantities output
                                              : number of radial points
 | self.nr
                                              : number of theta points
 self.ntheta
 | self.qv[0:nq-1]
                                              : quantity codes for the diagnostics output
 | self.radius[0:nr-1]
                                              : radial grid
 | self.costheta[0:ntheta-1]
                                              : cos(theta grid)
 | self.sintheta[0:ntheta-1]
                                             : sin(theta grid)
 self.vals[0:ntheta-1,0:nr-1,0:nq-1,0:niter-1] : The phi-averaged diagnostics
 | self.iters[0:niter-1]
                                              : The time step numbers stored in this output
 | self.time[0:niter-1]
                                              : The simulation time corresponding to each ti
                                              : The version code for this particular output
 | self.version
 | self.lut
                                               : Lookup table for the different diagnostics of
 | Methods defined here:
   __init__(self, filename='none', path='AZ_Avgs/')
       filename : The reference state file to read.
       path : The directory where the file is located (if full path not in filename
   _____
  Data descriptors defined here:
   \_\_\mathtt{dict}\_\_
       dictionary for instance variables (if defined)
```

```
|
| __weakref__
| list of weak references to the object (if defined)
```

Before creating our plots, let's time-average over the last two files that were output (thus sampling the equilibrated phase).

```
In [14]: nfiles = len(files)
         tcount=0
         for i in range(nfiles):
             az=AZ_Avgs(files[i],path='')
             if (i == 0):
                 nr = az.nr
                 ntheta = az.ntheta
                 nq = az.nq
                 azavg=numpy.zeros((ntheta,nr,nq),dtype='float64')
             for j in range(az.niter):
                 azavg[:,:,:] += az.vals[:,:,:,j]
                 tcount+=1
         azavg = azavg*(1.0/tcount) # Time steps were uniform for this run, so a simple average
         lut = az.lut
         vr = azavg[:,:,lut[1]]
         vtheta = azavg[:,:,lut[2]]
         vphi = azavg[:,:,lut[3]]
         rhovr = azavg[:,:,lut[201]]
         rhovtheta = azavg[:,:,lut[202]]
         temperature = azavg[:,:,lut[501]]
         radius = az.radius
         costheta = az.costheta
         sintheta = az.sintheta
```

Before we render, we need to do some quick post-processing: 1. Remove the spherical mean temperature from the azimuthal average. 2. Convert v_phi into omega 3. Compute the magnitude of the mass flux vector 4. Compute stream function associated with the mass flux field

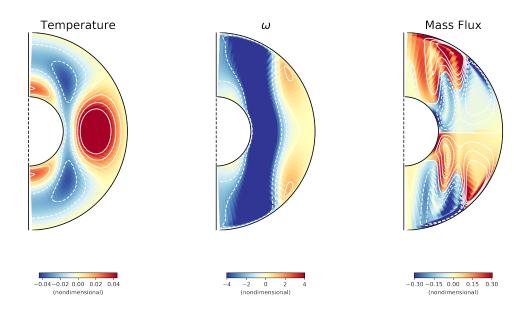
```
omega[:,i]=vphi[:,i]/(radius[i]*sintheta[:])

#Generate a streamfunction from rhov_r and rhov_theta
psi = streamfunction(rhovr,rhovtheta,radius,costheta,order=0)
#contours of mass flux are overplotted on the streamfunction PSI
rhovm = numpy.sqrt(rhovr**2+rhovtheta**2)*numpy.sign(psi)
```

Finally, we render the azimuthal averages.

NOTE: If you want to save any of these figures, you can mimic the saveplot logic at the bottom of this example.

```
In [16]: #
             We do a single row of 3 images
             Spacing is default spacing set up by subplot
         figdpi=300
         sizetuple=(5.5*3,3*3)
         tsize = 20
                      # title font size
         cbfsize = 10  # colorbar font size
         fig, ax = plt.subplots(ncols=3,figsize=sizetuple,dpi=figdpi)
         plt.rcParams.update({'font.size': 14})
         #temperature
         \#ax1 = f1.add\_subplot(1,3,1)
         units = '(nondimensional)'
         plot_azav(fig,ax[0],temperature,radius,costheta,sintheta,mycmap='RdYlBu_r',boundsfactor
                   boundstype='rms', units=units, fontsize = cbfsize)
         ax[0].set_title('Temperature',fontsize=tsize)
         #Differential Rotation
         \#ax1 = f1.add\_subplot(1,3,2)
         units = '(nondimensional)'
         plot_azav(fig,ax[1],omega,radius,costheta,sintheta,mycmap='RdYlBu_r',boundsfactor = 1.5
                   boundstype='rms', units=units, fontsize = cbfsize)
         ax[1].set_title(r'$\omega$',fontsize=tsize)
         #Mass Flux
         \#ax1 = f1.add\_subplot(1,3,3)
         units = '(nondimensional)'
         plot_azav(fig,ax[2],psi,radius,costheta,sintheta,mycmap='RdYlBu_r',boundsfactor = 1.5,
                   boundstype='rms', units=units, fontsize = cbfsize, underlay = rhovm)
         ax[2].set_title('Mass Flux',fontsize = tsize)
         saveplot=False
         if (saveplot):
             p.savefig(savefile)
         else:
             plt.show()
```



7 VII. Simulation Slices

7.1 VII.1 Equatorial Slices

Summary: 2-D profiles of selected output variables in the equatorial plane.

Subdirectory: Equatorial_Slices main_input prefix: equatorial Python Class: Equatorial_Slices Additional Namelist Variables:

None

The equatorial-slice output type allows us to examine how the fluid properties vary in longitude and radius.

Examining the *main_input* file, we see that the following output values have been denoted for the Equatorial Slices (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity
2	Theta Velocity

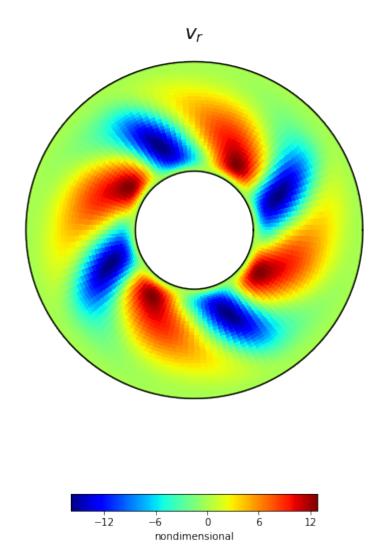
In the example that follows, we demonstrate how to create a 2-D plot of radial velocity in the equatorial plane (at a single time step).

We begin with the usual preamble. Examining the data structure, we see that the *vals* array is dimensioned to account for longitudinal variation, and that we have the new coordinate attribute *phi*.

```
import matplotlib.pyplot as plt
        from matplotlib import ticker, font_manager
        istring = '00040000'
        es = Equatorial_Slices(istring)
        tindex =1 # Grab second time index from this file
        help(es)
Help on Equatorial_Slices in module rayleigh_diagnostics object:
class Equatorial_Slices(builtins.object)
  Rayleigh Equatorial Slice Structure
| -----
 | self.niter
                                             : number of time steps
| self.nq
                                             : number of diagnostic quantities output
| self.nr
                                              : number of radial points
| self.nphi
                                             : number of phi points
| self.qv[0:nq-1]
                                             : quantity codes for the diagnostics output
| self.radius[0:nr-1]
                                             : radial grid
                                             : The equatorial_slices
 | self.vals[0:phi-1,0:nr-1,0:nq-1,0:niter-1]
| self.phi[0:nphi-1]
                                             : phi values (in radians)
| self.iters[0:niter-1]
                                             : The time step numbers stored in this output
 | self.time[0:niter-1]
                                             : The simulation time corresponding to each ti
 | self.version
                                             : The version code for this particular output
| self.lut
                                              : Lookup table for the different diagnostics of
| Methods defined here:
  __init__(self, filename='none', path='Equatorial_Slices/')
       filename : The reference state file to read.
                : The directory where the file is located (if full path not in filename
       path
   ______
   Data descriptors defined here:
   __dict__
       dictionary for instance variables (if defined)
   __weakref__
       list of weak references to the object (if defined)
# Equatorial Slice
        #Set up the grid
        remove_mean = True # Remove the m=0 mean
        nr = es.nr
```

```
nphi = es.nphi
r = es.radius/numpy.max(es.radius)
phi = numpy.zeros(nphi+1,dtype='float64')
phi[0:nphi] = es.phi
phi[nphi] = numpy.pi*2  # For display purposes, it is best to have a redunant data poin
#We need to generate a cartesian grid of x-y coordinates (both X & Y are 2-D)
radius_matrix, phi_matrix = numpy.meshgrid(r,phi)
X = radius_matrix * numpy.cos(phi_matrix)
Y = radius_matrix * numpy.sin(phi_matrix)
qindex = es.lut[1] # radial velocity
field = numpy.zeros((nphi+1,nr),dtype='float64')
field[0:nphi,:] =es.vals[:,:,qindex,tindex]
field[nphi,:] = field[0,:] #replicate phi=0 values at phi=2pi
#remove the mean if desired (usually a good idea, but not always)
if (remove_mean):
   for i in range(nr):
        the_mean = numpy.mean(field[:,i])
       field[:,i] = field[:,i]-the_mean
#Plot
sizetuple=(8,5)
fig, ax = plt.subplots(figsize=(8,8))
             # title font size
tsize = 20
cbfsize = 10  # colorbar font size
img = ax.pcolormesh(X,Y,field,cmap='jet')
ax.axis('equal') # Ensure that x & y axis ranges have a 1:1 aspect ratio
ax.axis('off')
                 # Plot bounding circles
ax.plot(r[nr-1]*numpy.cos(phi), r[nr-1]*numpy.sin(phi), color='black') # Inner circle
ax.plot(r[0]*numpy.cos(phi), r[0]*numpy.sin(phi), color='black') # Outer circle
ax.set_title(r'$v_r$', fontsize=20)
#colorbar ...
cbar = plt.colorbar(img,orientation='horizontal', shrink=0.5, aspect = 15, ax=ax)
cbar.set_label('nondimensional')
tick_locator = ticker.MaxNLocator(nbins=5)
cbar.locator = tick_locator
cbar.update_ticks()
cbar.ax.tick_params(labelsize=cbfsize) #font size for the ticks
t = cbar.ax.xaxis.label
t.set_fontsize(cbfsize) # font size for the axis title
```

plt.tight_layout()
plt.show()



7.2 VII.2 Meridional Slices

Summary: 2-D profiles of selected output variables sampled in meridional planes.

Subdirectory: Meridional_Slices main_input prefix: meridional Python Class: Meridional_Slices Additional Namelist Variables:

• meridional_indices (indicial): indices along longitudinal grid at which to output meridional planes.

• meridional_indices_nrm (normalized) : normalized longitudinal grid coordinates at which to output

The meridional-slice output type allows us to examine how the fluid properties vary in latitude and radius.

Examining the *main_input* file, we see that the following output values have been denoted for the Meridional Slices (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity

In the example that follows, we demonstrate how to create a 2-D plot of radial velocity in a meridional plane. The procedure is similar to that used to plot an azimuthal average.

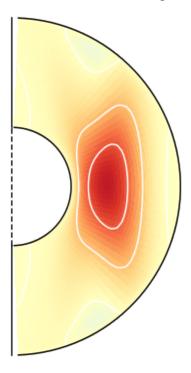
We begin with the usual preamble and import the *plot_azav* helper function. Examining the data structure, we see that it is similar to the AZ_Avgs data structure. The *vals* array possesses an extra dimension relative to its AZ_Avgs counterpart to account for the multiple longitudes that may be output, we see attributes *phi* and *phi_indices* have been added to reference the longitudinal grid.

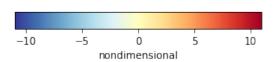
Help on Meridional_Slices in module rayleigh_diagnostics object:

```
class Meridional_Slices(builtins.object)
 | Rayleigh Meridional Slice Structure
 | self.niter
                                                 : number of time steps
                                                 : number of diagnostic quantities output
 | self.nq
 | self.nr
                                                 : number of radial points
 | self.ntheta
                                                 : number of theta points
                                                 : number of phi points sampled
 | self.nphi
 | self.qv[0:nq-1]
                                                 : quantity codes for the diagnostics output
 | self.radius[0:nr-1]
                                                 : radial grid
 | self.costheta[0:ntheta-1]
                                                 : cos(theta grid)
 | self.sintheta[0:ntheta-1]
                                                 : sin(theta grid)
```

```
| self.phi[0:nphi-1]
                                                 : phi values (radians)
 | self.phi_indices[0:nphi-1]
                                                : phi indices (from 1 to nphi)
 | self.vals[0:nphi-1,0:ntheta-1,0:nr-1,0:nq-1,0:niter-1] : The meridional slices
 | self.iters[0:niter-1]
                                                 : The time step numbers stored in this output
 | self.time[0:niter-1]
                                                 : The simulation time corresponding to each ti
 | self.version
                                                 : The version code for this particular output
 | self.lut
                                                 : Lookup table for the different diagnostics of
 | Methods defined here:
 __init__(self, filename='none', path='Meridional_Slices/')
       filename : The reference state file to read.
             : The directory where the file is located (if full path not in filename
 | Data descriptors defined here:
  __dict__
       dictionary for instance variables (if defined)
   __weakref__
       list of weak references to the object (if defined)
In [20]: radius = ms.radius
        costheta = ms.costheta
         sintheta = ms.sintheta
        phi_index = 0  # We only output one Meridional Slice
        vr_ms = ms.vals[phi_index,:,:,ms.lut[1],tindex]
        units = 'nondimensional'
         # Plot
        sizetuple=(8,5)
        fig, ax = plt.subplots(figsize=(8,8))
        tsize = 20  # title font size
         cbfsize = 10  # colorbar font size
         ax.axis('equal') # Ensure that x & y axis ranges have a 1:1 aspect ratio
         ax.axis('off') # Do not plot x & y axes
         plot_azav(fig,ax,vr_ms,radius,costheta,sintheta,mycmap='RdYlBu_r',boundsfactor = 4.5,
                  boundstype='rms', units=units, fontsize = cbfsize)
         ax.set_title('Radial Velocity',fontsize=tsize)
        plt.tight_layout()
        plt.show()
```

Radial Velocity





7.3 VII.3 Shell Slices

Summary: 2-D, spherical profiles of selected output variables sampled in at discrete radii.

Subdirectory: Shell_Slices main_input prefix: shellslice Python Class: Shell_Slices Additional Namelist Variables:

- shellslice_levels (indicial): indices along radial grid at which to output spherical surfaces.
- shellslice_levels_nrm (normalized): normalized radial grid coordinates at which to output spherical surfaces.

The shell-slice output type allows us to examine how the fluid properties vary on spherical surfaces.

Examining the *main_input* file, we see that the following output values have been denoted for the Shell Slices (see *rayleigh_output_variables.pdf* for mathematical formulae):

Description
Radial Velocity
Theta Velocity
Phi Velocity

In the examples that follow, we demonstrate how to create a 2-D plot of radial velocity: * on a Cartesian, lat-lon grid * projected onto a spherical surface using Basemap (MUST set use Basemap=True below)

Plotting on a lat-lon grid is straightforward and illustrated below. The shell-slice data structure is also displayed via the help() function in the example below and contains information needed to define the spherical grid for plotting purposes.

It is worth noting the *slice_spec* keyword (described in the docstring) that can be passed to the init method. When reading large shell slices, a user can save time and memory during the read process by specifying the slice they want to read.

```
# Shell Slice
        from rayleigh_diagnostics import Shell_Slices
        import numpy
        import matplotlib.pyplot as plt
        from matplotlib import ticker, font_manager
        # Read the data
        istring = '00040000'
        ss = Shell_Slices(istring)
        help(ss)
        ntheta = ss.ntheta
        nphi = ss.nphi
        costheta = ss.costheta
        theta = numpy.arccos(costheta)
        #help(ss)
        tindex =1 # All example quantities were output with same cadence. Grab second time-ind
        rindex = 0 # only output one radius
        sizetuple=(8,8)
        vr = ss.vals[:,:,rindex,ss.lut[1],tindex]
        fig, ax = plt.subplots(figsize=sizetuple)
        img = plt.imshow(numpy.transpose(vr), extent=[0,360,-90,90])
```

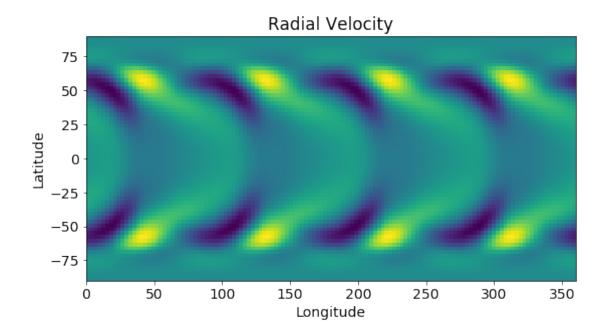
ax.set_xlabel('Longitude')
ax.set_ylabel('Latitude')

ax.set_title('Radial Velocity')

```
Help on Shell_Slices in module rayleigh_diagnostics object:
class Shell_Slices(builtins.object)
 | Rayleigh Shell Slice Structure
 _____
 | self.niter
                                               : number of time steps
 | self.nq
                                               : number of diagnostic quantities output
 | self.nr
                                               : number of shell slices output
 | self.ntheta
                                               : number of theta points
                                               : number of phi points
 | self.nphi
 | self.qv[0:nq-1]
                                               : quantity codes for the diagnostics output
 | self.radius[0:nr-1]
                                              : radii of the shell slices output
 | self.inds[0:nr-1]
                                              : radial indices of the shell slices output
 | self.costheta[0:ntheta-1]
                                              : cos(theta grid)
 self.sintheta[0:ntheta-1]
                                               : sin(theta grid)
 self.vals[0:nphi-1,0:ntheta-1,0:nr-1,0:nq-1,0:niter-1]
                                              : The shell slices
 | self.iters[0:niter-1]
                                              : The time step numbers stored in this output
 | self.time[0:niter-1]
                                              : The simulation time corresponding to each ti
 | self.version
                                               : The version code for this particular output
                                               : Lookup table for the different diagnostics of
 | self.lut
 | Methods defined here:
   __init__(self, filename='none', path='Shell_Slices/', slice_spec=[], rec0=False)
       filename : The reference state file to read.
                : The directory where the file is located (if full path not in filename
       path
       slice_spec : Optional list of [time index, quantity code, radial index]. If
                   specified, only a single shell is read. time indexing and radial
                   indexing start at 0
       rec0
                : Set to true to read the first timestep's data only.
   print_info(self, print_costheta=False)
       Prints all metadata associated with the shell-slice object.
   ______
 | Data descriptors defined here:
   __dict__
       dictionary for instance variables (if defined)
   __weakref__
       list of weak references to the object (if defined)
```

plt.tight_layout()

plt.show()



Plotting with Basemap If you wish to project your data onto a spherical grid, we recommend using the Basemap package. If you have not installed this package yet, recall that you can do so by activating your conda environment and then running:

conda install -c intel basemap

The example that follows illustrates how to use Basemap. So that this notebook will run for those without Basemap installed, we have nested the plotting code in a try/except statement. If Basemap fails to import, the code will report an error message instead of crashing.

```
In [22]: try:
             from mpl_toolkits.basemap import Basemap, addcyclic
             import numpy as np
             import matplotlib.pyplot as plt
             from matplotlib import ticker
             from rayleigh_diagnostics import Shell_Slices
             tsize = 20
                            # title font size
             cbfsize = 10  # colorbar font size
             #Specify which data to plot
             istring='00040000'
             rec_spec = [1,1,0] # grab time index 1, quantity code 1, and radial index 0
             ss = Shell_Slices(istring,slice_spec = rec_spec)
             nphi = ss.nphi
             ntheta = ss.ntheta
             save_figure = False # Set to true to save to a figure_file (below)
```

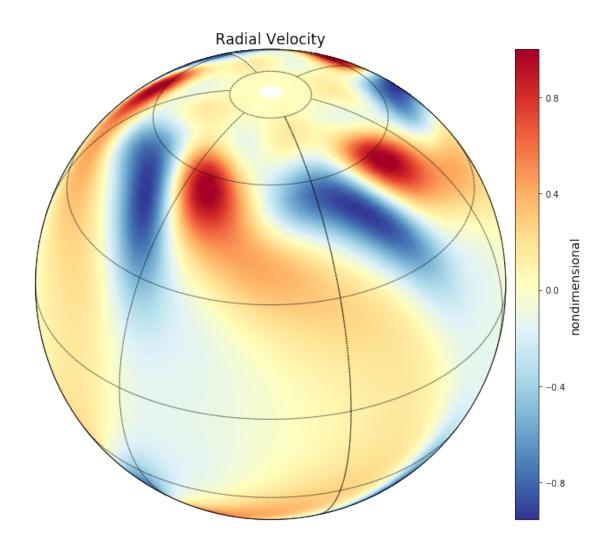
```
figure_file = 'shell_slice_basemap.png'
#The resolution of the png file or the view window in pixels
xpixels = 1024
ypixels = 1024
# Initialize the projection.
# lon_0, lat_0 are the center point of the projection.
# resolution = 'l' means use low resolution coastlines.
m = Basemap(projection='ortho',lon_0=-20,lat_0=35,resolution='l')
#Read in the data
data = ss.vals[:,:,0,0,0].reshape(nphi,ntheta)
data = np.transpose(data)
maxabs = 3*np.std(data) # saturate data at +/- 3 sigma
dlon = 360.0/(nphi)
dlat = 180.0/ntheta
#Scale the data
for i in range(ntheta):
    for j in range(nphi):
        if (data[i,j] > maxabs):
            data[i,j] = maxabs
        if (data[i,j] < -maxabs):</pre>
            data[i,j] = -maxabs
        data[i,j] = data[i,j]/maxabs
#Generate 1-D grids of latitude and longitude
lons = np.zeros(nphi)
for i in range(nphi):
    lons[i] = dlon*i-180.0
lats = np.zeros(ntheta)
for i in range(ntheta):
    lats[i] = 90.0-np.arccos(ss.costheta[i])*180.0/np.pi
    lats[i] = i*dlat-90
# Convert to 2-D grids
llons, llats = np.meshgrid(lons, lats)
# Get x-y projection points on the plane
x, y = m(llons, llats)
# Do some interpolation
```

```
nx = int((m.xmax-m.xmin)/2000.)+1; ny = int((m.ymax-m.ymin)/2000.)+1
             nx = 1024
             ny = 1024
             print('interpolating...')
             topodat,x,y =\
             m.transform_scalar(data,lons,lats,nx,ny,returnxy=True,masked=True,order=1)
             print ('...complete...')
             #Initialize the window
             plt.figure(figsize=(xpixels/100.0, ypixels/100.0))
             #View the data
             my_cmap = plt.cm.RdYlBu_r
             img=m.pcolormesh(x,y,topodat,cmap=my_cmap)
             # draw parallels and meridians.
             m.drawparallels(np.arange(-90.,120.,30.))
             m.drawmeridians(np.arange(0.,420.,60.))
             m.drawmapboundary(fill_color='white')
             #colorbar ...
             cbar = m.colorbar(img) # ,shrink=0.5, aspect = 15)
             cbar.set_label('nondimensional')
             tick_locator = ticker.MaxNLocator(nbins=5)
             cbar.locator = tick_locator
             cbar.update_ticks()
             cbar.ax.tick_params(labelsize=cbfsize) #font size for the ticks
             t = cbar.ax.xaxis.label
             t.set_fontsize(cbfsize) # font size for the axis title
             plt.title('Radial Velocity')
             if (save_figure):
                 plt.savefig(figure_file, fontsize=tsize)
             else:
                 plt.show()
             print('Basemap module is not installed.')
interpolating...
...complete...
/custom/software/miniconda3/envs/idp3/lib/python3.6/site-packages/mpl_toolkits/basemap/__init__.
  b = ax.ishold()
```

This is necessary for moderately-sized and large shell slices (ell_max >= 255)

```
/custom/software/miniconda3/envs/idp3/lib/python3.6/site-packages/mpl_toolkits/basemap/__init__.
See the API Changes document (http://matplotlib.org/api/api_changes.html)
for more details.
ax.hold(b)
```

/custom/software/miniconda3/envs/idp3/lib/python3.6/site-packages/mpl_toolkits/basemap/__init__.
fill_color = ax.get_axis_bgcolor()



8 VIII. Spherical Harmonic Spectra

Summary: Spherical Harmonic Spectra sampled at discrete radii.

Subdirectory: Shell_Spectra main_input prefix: shellspectra

Python Classes:

• Shell_Spectra : Complete data structure associated with Shell_Spectra outputs.

• PowerSpectrum : Reduced data structure -- contains power spectrum of velocity and/or magnetic fields only.

Additional Namelist Variables:

- shellspectra_levels (indicial): indices along radial grid at which to output spectra.
- shellspectra_levels_nrm (normalized): normalized radial grid coordinates at which to output spectra.

The shell-spectra output type allows us to examine the spherical harmonic decomposition of output variables at discrete radii.

Examining the *main_input* file, we see that the following output values have been denoted for the Shell Spectra (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity

Spherical harmonic spectra can be read into Python using either the **Shell_Spectra** or **Power-Spectrum** classes.

The **Shell_Spectra** class provides the full complex spectra, as a function of degree ell and azimuthal order m, for each specified output variable. It possesses an attribute named *lpower* that contains the associated power for each variable, along with its m=0 contributions separated and removed.

The **Power_Spectrum** class can be used to read a Shell_Spectra file and quickly generate a velocity or magnetic-field power spectrum. For this class to work correctly, your file must contain all three components of either the velocity or magnetic field. Other variables are ignored (use Shell_Spectrum's lpower for those).

We illustrate how to use these two classes below. As usual, we call the help() function to display the docstrings that describe the different data structures embodied by each class.

```
In [23]: import matplotlib.pyplot as plt
    from matplotlib import ticker
    import numpy
    from rayleigh_diagnostics import Shell_Spectra, Power_Spectrum
    istring = '00040000'

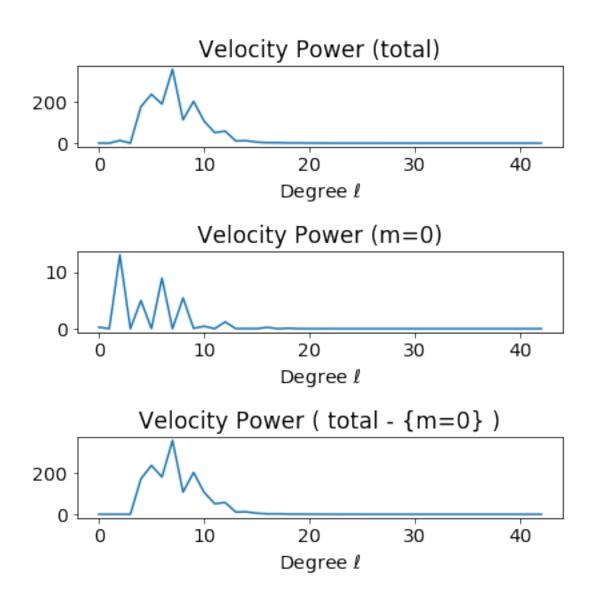
    tind = 0
    rind = 0
    #help(ss)

    vpower = Power_Spectrum(istring)
    help(vpower)
    power = vpower.power
```

```
fig, ax = plt.subplots(nrows=3, figsize=(6,6))
ax[0].plot(power[:,rind,tind,0])
ax[0].set_xlabel(r'Degree $\ell$')
ax[0].set_title('Velocity Power (total)')
ax[1].plot(power[:,rind,tind,1])
ax[1].set_xlabel(r'Degree $\ell$')
ax[1].set_title('Velocity Power (m=0)')
ax[2].plot(power[:,rind,tind,2])
ax[2].set_xlabel(r'Degree $\ell$')
ax[2].set_title('Velocity Power ( total - {m=0} )')
plt.tight_layout()
plt.show()
fig, ax = plt.subplots()
ss = Shell_Spectra(istring)
help(ss)
mmax = ss.mmax
lmax = ss.lmax
power_spectrum = numpy.zeros((lmax+1,mmax+1),dtype='float64')
for i in range(1,4): # i takes on values 1,2,3
    qind=ss.lut[i]
    complex_spectrum = ss.vals[:,:,rind,qind,tind]
    power_spectrum = power_spectrum+numpy.real(complex_spectrum)**2 + numpy.imag(complex_spectrum)
power_spectrum = numpy.transpose(power_spectrum)
tiny = 1e-6
img=ax.imshow(numpy.log10(power_spectrum+tiny), origin='lower')
ax.set_ylabel('Azimuthal Wavenumber m')
ax.set_xlabel(r'Degree $\ell$')
ax.set_title('Velocity Power Spectrum')
#colorbar ...
cbar = plt.colorbar(img) # ,shrink=0.5, aspect = 15)
cbar.set_label('Log Power')
tick_locator = ticker.MaxNLocator(nbins=5)
cbar.locator = tick_locator
cbar.update_ticks()
cbar.ax.tick_params() #font size for the ticks
plt.show()
```

Help on Power_Spectrum in module rayleigh_diagnostics object:

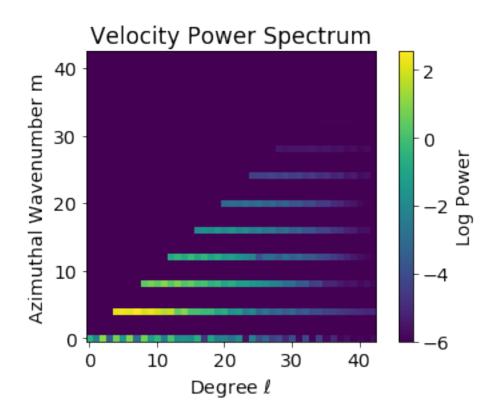
```
class Power_Spectrum(builtins.object)
| Rayleigh Power Spectrum Structure
| -----
| self.niter
                                              : number of time steps
| self.nr
                                              : number of radii at which power spectra are a
 | self.lmax
                                              : maximum spherical harmonic degree 1
| self.radius[0:nr-1]
                                              : radii of the shell slices output
| self.inds[0:nr-1]
                                              : radial indices of the shell slices output
 self.power[0:lmax,0:nr-1,0:niter-1,0:2]
                                              : the velocity power spectrum. The third
                                              : index indicates (0:total,1:m=0, 2:total-m=0
| self.mpower[0:lmax,0:nr-1,0:niter-1,0:2]
                                             : the magnetic power spectrum
 | self.iters[0:niter-1]
                                              : The time step numbers stored in this output
   self.time[0:niter-1]
                                              : The simulation time corresponding to each ti
 | self.magnetic
                                              : True if mpower exists
  Methods defined here:
   __init__(self, infile, dims=[], power_file=False, magnetic=False, path='Shell_Spectra')
       Initialize self. See help(type(self)) for accurate signature.
  blank_init(self, dims)
  power_file_init(self, pfile)
| set_pars(self, iters, time, inds, radius)
   spectra_file_init(self, sfile)
| write_power(self, ofile)
   ______
| Data descriptors defined here:
   __dict__
       dictionary for instance variables (if defined)
   __weakref__
       list of weak references to the object (if defined)
```



Help on Shell_Spectra in module rayleigh_diagnostics object:

```
class Shell_Spectra(builtins.object)
   Rayleigh Shell Spectrum Structure
   self.niter
                                                   : number of time steps
   self.nq
                                                   : number of diagnostic quantities output
   self.nr
                                                   : number of shell slices output
   self.nell
                                                   : number of ell values
   self.nm
                                                   : number of m values
   self.lmax
                                                   : maximum spherical harmonic degree 1
    self.mmax
                                                   : maximum spherical harmonic degree m
    self.qv[0:nq-1]
                                                   : quantity codes for the diagnostics output
```

```
| self.radius[0:nr-1]
                                                 : radii of the shell slices output
| self.inds[0:nr-1]
                                                 : radial indices of the shell slices output
| self.vals[0:lmax,0:mmax,0:nr-1,0:nq-1,0:niter-1]
                                                 : The complex spectra of the shells output
| self.lpower[0:lmax,0:nr-1,0:nq-1,0:niter-1,3] : The power as a function of ell, integrate
                                                    : index indicates (0:total,1:m=0, 2:total-
                                                 : The time step numbers stored in this output
| self.iters[0:niter-1]
  self.time[0:niter-1]
                                                 : The simulation time corresponding to each ti
| self.version
                                                 : The version code for this particular output
| self.lut
                                                 : Lookup table for the different diagnostics of
| Methods defined here:
  __init__(self, filename='none', path='Shell_Spectra/')
      filename : The reference state file to read.
      path
               : The directory where the file is located (if full path not in filename
  print_info(self)
      Prints all metadata associated with the shell-spectra object.
 Data descriptors defined here:
  __dict__
      dictionary for instance variables (if defined)
  __weakref__
      list of weak references to the object (if defined)
```



9 IX. Point Probes

Summary: Point-wise sampling of desired output variables.

Subdirectory: Point_Probes main_input prefix: point_probe Python Class: Point_Probes Additional Namelist Variables:

- point_probe_r : radial indices for point-probe output
- point_probe_theta : theta indices for point-probe output
- point_probe_phi : phi indices for point-probe output
- point_probe_r_nrm : normalized radial coordinates for point-probe output
- point_probe_theta_nrm : normalized theta coordinates for point-probe output
- point_probe_phi_nrm : normalized phi coordinates for point-probe output
- point_probe_cache_size: number of time-samples to save before accessing the disk

Point-probes allow us to sample a simulation at an arbitrary set of points. This output type serves two purposes: 1. It provides an analog to laboratory measurements where slicing and

averaging are difficult, but taking high-time-cadence using (for example) thermistors is common-practice. 2. It provides an alternative method of slicing a model (for when equatorial, meridional, or shell slices do yield the desired result).

9.1 IX.1 Specifying Point-Probe Locations

Point-probe locations are indicated by specifying a grid. The user does not supply a set of ordered coordinates (r,theta,phi). Instead, the user specifies nodes on the grid using the namelist variables described above. Examples follow.

Example 1: 4-point Coarse Grid

```
point_probe_r_nrm = 0.25, 0.5
point_probe_theta_nrm = 0.5
point_probe_phi_nrm = 0.2, 0.8
```

This example would produce point probes at the four coordinates { (0.25, 0.5, 0.2), (0.25, 0.5, 0.8), (0.5, 0.5, 0.2), (0.5, 0.5, 0.8) } (r,theta,phi; normalized coordinates).

```
Example 2: "Ring" in Phi
```

```
point_probe_r_nrm = 0.5
point_probe_theta_nrm = 0.5
point_probe_phi_nrm = 0.0, -1.0
```

This example describes a ring in longitude, sampled at mid-shell, in the equatorial plane. We have made use of the positional range feature here by indicating normalized phi coordinates of 0.0, -1.0. Rayleigh interprets this as an instruction to sample all phi coordinates.

```
** Example 3: 2-D Surface in (r,phi) **
point_probe_r_nrm = 0, -1.0
point_probe_theta_nrm = 0.25
point_probe_phi_nrm = 0, -1.0
```

This example uses the positional range feature along with normalized coordinates to generate a 2-D slice in r-phi at theta = 45 degrees (theta_nrm = 0.25). Using the syntax 0,-1.0 instructs *Rayleigh* to grab all r and phi coordinates.

```
** Example 4: 3-D Meridional "Wedges" **
point_probe_r_nrm = 0.0, -1.0
point_probe_theta_nrm = 0.0, -1.0
point_probe_phi_nrm = 0.20, -0.30, 0.7, -0.8
```

This example generates two 3-D wedges described by all r,theta points and all longitudes in the ranges [72 deg, 108 deg] and [252 deg, 288 deg].

9.2 IX.2 Point-Probe Caching

When performing sparse spatial sampling using point-probes, it may be desireable to output with a high-time cadence. As this may cause disk-access patterns characterized by frequent, small writes, the point-probes are programmed with a caching feature. This feature is activated by specifing the **point_probe_cache_size** variable in the output namelist.

This variable determines how many time-samples are saved in memory before a write is performed. Its default value is 1, which means that the disk is accessed with a frequency of **point_probe_frequency**. If the cache size is set to 10 (say), then samples are still performed at **point_probe_frequency** but they are only written to disk after 10 have been collected in memory.

NOTE: Be sure that **point_probe_cache_size** divides evenly into **point_probe_nrec**.

9.3 IX.3 Example: Force-Balance with Point Probes

Our example input file specifies a coarse, six-point grid. Examining the *main_input* file, we see that all variables necessary to examine the force balance in each direction have been specified. (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity
1201	Radial Advection (v dot grad v)
1202	Theta Advection
1203	Phi Advection
1216	Buoyancy Force (ell=0 component subtracted)
1219	Radial Coriolis Force
1220	Theta Coriolis Force
1221	Phi Coriolis Force
1228	Radial Viscous Force
1229	Theta Viscous Force
1230	Phi Viscous Force

Note that the pressure force appears to be missing. This is not an oversight. The diagnostic nature of the Pressure equation in incompressible/anelastic models, coupled with the second-order Crank-Nicolson time-stepping scheme, means that the pressure field can exhibit an even/odd sawtoothing in time. The *effective* pressure force (as implemented through the Crank-Nicolson scheme) is always a weighted average over two time steps **and is always well-resolved in time**.

When sampling at regular intervals as we have here, if we directly sample the pressure force, we will sample either the high or low end of the sawtooth envelope, and the force balance will be off by a large factor. The easiest fix is to output the velocity field and compute its time derivative. This, in tandem with the sum of all other forces, can be used to calculate the effective pressure as a post-processing step. The (undesireable) alternative is to output once every time step and compute the effective pressure using the Crank-Nicolson weighting.

We demonstrate how to compute the effective pressure force via post-processing in the example below.

```
In [24]: from rayleigh_diagnostics import Point_Probes, build_file_list
    import numpy
    from matplotlib import pyplot as plt

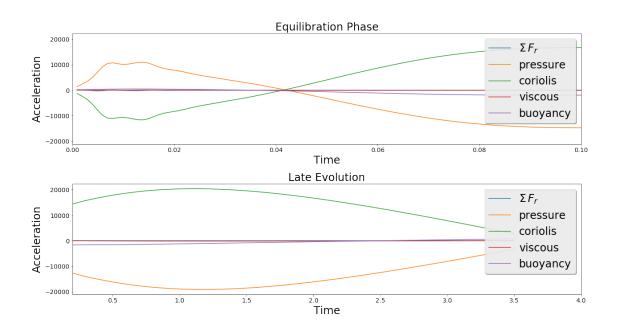
#Decide which direction you want to look at (set direction = {radial, theta, or phi})
    #This is used to determine the correct quantity codes below
    radial = 0
    theta = 1
    phi = 2
    direction=radial
    # Build a list of all files ranging from iteration 0 million to 1 million
    files = build_file_list(0,1000000,path='Point_Probes')
```

```
nfiles = len(files)-1
for i in range(nfiles):
   pp = Point_Probes(files[i],path='')
   if (i == 0):
      nphi = pp.nphi
      ntheta = pp.ntheta
      nr = pp.nr
      nq = pp.nq
      niter = pp.niter
      vals=numpy.zeros( (nphi,ntheta,nr,nq,niter*nfiles),dtype='float64')
      time=numpy.zeros(niter*nfiles,dtype='float64')
   vals[:,:,:,: i*niter:(i+1)*niter] = pp.vals
   time[i*niter:(i+1)*niter]=pp.time
istring='00040000' # iteration to examine
help(pp)
# We choose the coordinate indices **within**
# the Point-Probe array that we want to examine
# These indices start at zero and run to n_i-1
# where n_i is the number of points sampled in
# the ith direction
# Use help(pp) after loading the Point-Probe file
# to see the Point-Probe class structure
pind = 0
              # phi-index to examine
rind = 0
              # r-index to examine
tind = 0
                # theta-index to examine
   = Point_Probes(istring)
рp
lut = pp.lut
nt
    = pp.niter
# Grab velocity from the point probe data
u = vals[pind,0,rind,pp.lut[1+direction],:]
dt=time[1]-time[0]
# Use numpy to compute time-derivative of u
# (necessary to compute a smooth effective pressure without outputing every timestep)
```

```
#Depending on Numpy version, gradient function takes either time (array) or dt (scalar)
try:
   dudt = numpy.gradient(u,time)
except:
   dt = time[1]-time[0] # Assumed to be constant...
   dudt = numpy.gradient(u,dt)
Forces (modulo pressure)
# Note the minus sign for advection. Advective terms are output as u dot grad u, not -
advec = -vals[ pind, tind, rind, lut[1201 + direction], :]
   = vals[ pind, tind, rind, lut[1219 + direction], :]
visc = vals[ pind, tind, rind, lut[1228 + direction], :]
forces = visc+cor+advec
if (direction == radial):
   buoy = vals[ pind, tind, rind, lut[1216], :]
   forces = forces+buoy
# Construct effective pressure force
pres = dudt-forces
forces = forces+pres
# Set up the plot
yfsize='xx-large' # size of y-axis label
ustrings = [r'u_r', r'u_\theta', r'u_\phi']
ustring=ustrings[direction]
dstring = r'$\frac{\partial '+ustring+'}{\partial t}$'
fstrings = [r'$\Sigma\,F_r$' , r'$\Sigma\,F_\theta$' , r'$\Sigma\,F_\phi$' ]
fstring = fstrings[direction]
diff_string = dstring+' - '+fstring
pstring = 'pressure'
cstring = 'coriolis'
vstring = 'viscous'
bstring = 'buoyancy'
fig, axes = plt.subplots(nrows=2, figsize=(7*2.54, 9.6))
ax0 = axes[0]
ax1 = axes[1]
# Upper: dur/dt and F_total
#mpl.rc('xtick', labelsize=20) --- still trying to understand xtick label size etc.
```

```
#mpl.rc('ytick', labelsize=20)
        ax0.plot(time,forces, label = fstring)
        ax0.plot(time,pres,label=pstring)
        ax0.plot(time,cor,label=cstring)
        ax0.plot(time, visc, label=vstring)
        if (direction == radial):
            ax0.plot(time,buoy,label=bstring)
        ax0.set_xlabel('Time', size=yfsize)
        ax0.set_ylabel('Acceleration', size=yfsize)
        ax0.set_title('Equilibration Phase',size=yfsize)
        ax0.set_xlim([0,0.1])
        leg0 = ax0.legend(loc='upper right', shadow=True, ncol = 1, fontsize=yfsize)
        # Lower: Numpy Gradient Approach
        ax1.plot(time,forces,label=fstring)
        ax1.plot(time,pres,label=pstring)
        ax1.plot(time,cor,label=cstring)
        ax1.plot(time, visc, label=vstring)
        if (direction == radial):
            ax1.plot(time,buoy,label=bstring)
        ax1.set_title('Late Evolution',size=yfsize)
        ax1.set_xlabel('Time',size=yfsize)
        ax1.set_ylabel('Acceleration', size =yfsize)
        ax1.set_xlim([0.2,4])
        leg1 = ax1.legend(loc='upper right', shadow=True, ncol = 1, fontsize=yfsize)
        plt.tight_layout()
        plt.show()
Help on Point_Probes in module rayleigh_diagnostics object:
class Point_Probes(builtins.object)
 | Rayleigh Point Probes Structure
 _____
 | self.niter
                                                : number of time steps
 | self.nq
                                                : number of diagnostic quantities output
 | self.nr
                                                : number of radial points
 | self.ntheta
                                                : number of theta points
 | self.nphi
                                                : number of phi points sampled
 | self.qv[0:nq-1]
                                                : quantity codes for the diagnostics output
 | self.radius[0:nr-1]
                                                : radial grid
 | self.costheta[0:ntheta-1]
                                               : cos(theta grid)
 | self.sintheta[0:ntheta-1]
                                                : sin(theta grid)
 | self.phi[0:nphi-1]
                                                : phi values (radians)
```

```
self.phi_indices[0:nphi-1]
                                               : phi indices (from 1 to nphi)
self.vals[0:nphi-1,0:ntheta-1,0:nr-1,0:nq-1,0:niter-1] : The meridional slices
self.iters[0:niter-1]
                                               : The time step numbers stored in this output
self.time[0:niter-1]
                                               : The simulation time corresponding to each ti
                                               : The version code for this particular output
self.version
self.lut
                                               : Lookup table for the different diagnostics of
Methods defined here:
__init__(self, filename='none', path='Point_Probes/')
             : The reference state file to read.
              : The directory where the file is located (if full path not in filename
Data descriptors defined here:
__dict__
    dictionary for instance variables (if defined)
__weakref__
    list of weak references to the object (if defined)
```



10 X. Modal Outputs

Summary: Spherical Harmonic Spectral Coefficients sampled at discrete radii and degree ell.

Subdirectory: SPH_Modes main_input prefix: sph_mode Python Classes: SPH_Modes Additional Namelist Variables:

- sph_mode_levels (indicial): indices along radial grid at which to output spectral coefficients.
- sph_mode_levels_nrm (normalized): normalized radial grid coordinates at which to output spectral coefficients.
- sph_mode_ell: Comma-separated list of spherical harmonic degree ell to output.

The Modal output type allows us to output a restricted set of complex spherical harmonic coefficients at discrete radii. For each specified ell-value, all associated azimuthal wavenumbers are output.

This output can be useful for storing high-time-cadence spectral data for a few select modes. In the example below, we illustrate how to read in this output type, and we plot the temporal variation of the real and complex components of radial velocity for mode ell = 4, m = 4.

Examining the *main_input* file, we see that the following output values have been denoted for the Shell Spectra (see *rayleigh_output_variables.pdf* for mathematical formulae):

Menu Code	Description
1	Radial Velocity
2	Theta Velocity
3	Phi Velocity

We also see that ell=2,4,8 have been selected in the *main_input* file, leading to power at the following modes:

ell-value	m-values
2	0,1,2
4	0,1,2,3,4
8	0,1,2,3,4,5,6,7,8

```
In [25]: from rayleigh_diagnostics import SPH_Modes, build_file_list
    import matplotlib.pyplot as plt
    import numpy

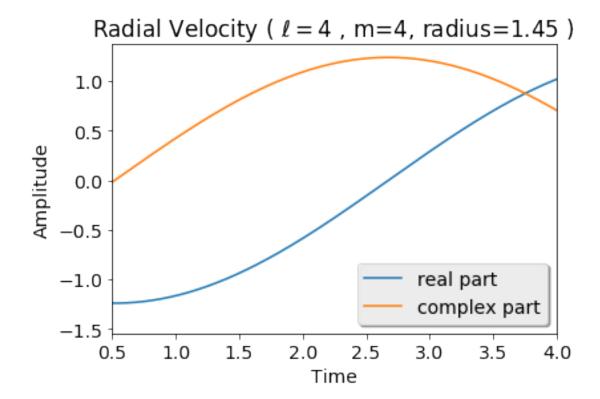
qind = 1  # Radial velocity
    rind = 0  # First radius stored in file

files = build_file_list(0,1000000,path='SPH_Modes')
    nfiles = len(files)
    for i in range(nfiles):
        spm = SPH_Modes(files[i],path='')
        if (i == 0):
```

```
nell = spm.nell
               nr = spm.nr
               nq = spm.nq
               niter = spm.niter
               lvals = spm.lvals
               max_ell = numpy.max(lvals)
               nt = niter*nfiles
               vr = spm.lut[qind]
               vals=numpy.zeros( (max_ell+1,nell,nr,nq,nt),dtype='complex64')
               time=numpy.zeros(nt,dtype='float64')
            vals[:,:,:,:, i*niter:(i+1)*niter] = spm.vals
            time[i*niter:(i+1)*niter]=spm.time
        help(spm)
        # Print some information regarding the bookkeeping
        print('....')
        print(' Contents')
        print(' nr = ', nr)
        print(' nq = ', nq)
        print(' nt = ', nt)
        for i in range(nell):
           lstring=str(lvals[i])
           estring = 'Ell='+lstring+' Complex Amplitude : vals[0:'+lstring+','+str(i)+',0:nr-1
            print(estring)
        print(' First dimension is m-value.')
        print('....')
        # Create a plot of the ell=4, m=4 real and imaginary amplitudes
        radius = spm.radius[rind]
        lfour_mfour = vals[4,1,rind,vr,:]
        fig, ax = plt.subplots()
        ax.plot(time,numpy.real(lfour_mfour), label='real part')
        ax.plot(time,numpy.imag(lfour_mfour), label='complex part')
        ax.set_xlabel('Time')
        ax.set_ylabel('Amplitude')
        rstring = "{0:4.2f}".format(radius)
        ax.set_title(r'Radial Velocity ( $\ell=4$ , m=4, radius='+rstring+' ) ')
        ax.legend(shadow=True)
        ax.set_xlim([0.5,4.0])
        plt.show()
Help on SPH_Modes in module rayleigh_diagnostics object:
class SPH_Modes(builtins.object)
| Rayleigh Shell Spectrum Structure
| -----
| self.niter
                                              : number of time steps
```

```
| self.nq
                                                  : number of diagnostic quantities output
 | self.nr
                                                  : number of shell slices output
 | self.nell
                                                  : number of ell values
 | self.qv[0:nq-1]
                                                  : quantity codes for the diagnostics output
 | self.radius[0:nr-1]
                                                 : radii of the shell slices output
 | self.inds[0:nr-1]
                                                  : radial indices of the shell slices output
 | self.lvals[0:nell-1]
                                                  : ell-values output
   self.vals[0:lmax,0:nell-1,0:nr-1,0:nq-1,0:niter-1]
                                                  : The complex spectra of the SPH modes output
                                                  : (here lmax denotes the maximum 1-value outp
  self.iters[0:niter-1]
                                                  : The time step numbers stored in this output
   self.time[0:niter-1]
                                                 : The simulation time corresponding to each ti
   self.version
                                                 : The version code for this particular output
                                                  : Lookup table for the different diagnostics of
   self.lut
 | Methods defined here:
   __init__(self, filename='none', path='SPH_Modes/')
        filename : The reference state file to read.
        path
              : The directory where the file is located (if full path not in filename
  Data descriptors defined here:
  __dict__
       dictionary for instance variables (if defined)
   __weakref__
        list of weak references to the object (if defined)
Contents
 nr = 3
 nq = 3
 nt = 400
Ell=2 Complex Amplitude : vals[0:2,0,0:nr-1,0:nq-1,0:nt-1]
Ell=4 Complex Amplitude : vals[0:4,1,0:nr-1,0:nq-1,0:nt-1]
Ell=8 Complex Amplitude : vals[0:8,2,0:nr-1,0:nq-1,0:nt-1]
First dimension is m-value.
```

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In []: