Budget Project: Roman Aqueduct to Tikal

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CC 340 - Water and The Roman City

By and large, **Route 2** appears to be the ideal aqueduct plan, although it would likely require both more surplus resources (i.e. money) and more advanced technology to construct. Essentially, it seems Route 1 should only ever be considered if *it is the only option available,* such as if the society simply lacks a sufficient labor force or technical ability, but even this is not that sensible as the difference in initial investment ends up being largely trivial.

Budget Calculations

Cost / Meter of Aqueduct Based on Type		
Type	**Initial Cost**	**Yearly Cost**
Cut-Cover	100	5 + 1
Bridge	500	8 + 1
Tunnel	1000	2 + 1

Maintenance overhead appended.
Unit for Initial: $Sestertii\ /\ Meter$
Unit for Recurring: $Sestertii\ /\ (Meter\ \cdot\ Year)$

Spring Value	
A	500
B	400
C	800
D	120
E	80

Abstractly represents source usefulness, related to volume.

We can imagine that a quinariae is similar to a liter, or some other absolute quantity of material / liquid, but it didn't feel sensible to consider this value as a constant quantity since this benefit is never 'used up' like the resources it took to build it, it is perpetually available after being built (but still not unlimited).

Specifically, the amount your society benefited from an aqueduct should be kind of proportional to how long that aqueduct has been flowing and being useful.

This sounded a lot like $Volume\ /\ Time$ **instead of just** $Volume$ **to me,** *so I decided to treat the quinariae assigned to each spring as like a yearly reward,* $Quinariae\ /\ Year$ **.**

Also, this formed a very tempting narrative in my head, as perhaps the $quinariae$ *mentioned in artifacts was actually referencing volumetric flow rate, as then it would be perfectly valid to describe values in terms of cross-sectional area alone (if you can assume water speed, which would've been pretty constant since speed depends only on altitude gradient, which was always minimized as much as could be). As a roman engineer, it wouldn't really be useful to discuss things in terms of absolute volume, since aqueducts are constantly flowing. Instead, I'd want describe how much a new source might increase the system's flow. (just a thought, I did not research this further)*

Route 1

Initial Costs					
Type	**Length**	**Initial Rate**	**Build Cost**	**Land Cost**	**Initial Cost**
tunnel	200	1000	200000	10000	210000
cutcover	2770	100	277000	138500	415500
bridge	140	500	70000	7000	77000
bridge	30	500	15000	1500	16500
tunnel	100	1000	100000	5000	105000
bridge	120	500	60000	6000	66000
tunnel	70	1000	70000	3500	73500
TOTALS:	**3430**	**4600**	**792000**	**171500**	**963500**

Recurring Cost Total at Year:			
Yearly Cost	**25**	**70**	**300**
600	15000	42000	180000
16620	415500	1163400	4986000
1260	31500	88200	378000
270	6750	18900	81000
300	7500	21000	90000
1080	27000	75600	324000
210	5250	14700	63000
20340	**508500**	**1423800**	**6102000**

Route 2

Initial Costs					
Type	**Length**	**Initial Rate**	**Build Cost**	**Land Cost**	**Initial Cost**
cutcover	5140	100	514000	257000	771000
tunnel	200	1000	200000	10000	210000
tunnel	200	1000	200000	10000	210000
bridge	70	500	35000	3500	38500
TOTALS:	**5610**	**2600**	**949000**	**280500**	**1229500**

Recurring Costs At Year X:			
Yearly Cost	**25**	**70**	**300**
30840	771000	2158800	9252000
600	15000	42000	180000
600	15000	42000	180000
630	15750	44100	189000
32670	**816750**	**2286900**	**9801000**

Route 1 Totals

Total Costs At Year X:		
25	**70**	**300**
225000	252000	390000
831000	1578900	5401500
108500	165200	455000
23250	35400	97500
112500	126000	195000
93000	141600	390000
78750	88200	136500
1472000	**2387300**	**7065500**

Cumulative Value of Aqueduct			
Value A+B=	**25**	**70**	**300**
900/year	22500	63000	270000

Efficiency of Investment At Year X:			
'Efficiency' = total value delivered by an aqueduct during lifetime, over the total cost spent on it during lifetime.	**25**	**70**	**300**
	1.53%	**2.64%**	**3.82%**
	Interpret as: for every 100 sestertii spent on this project, how much water did we get		

Alternate Efficiency 10000X			
Instead of calculating the cumulative value delivered by aqueduct over many years, we just use the raw *quinariae quantity as an absolute measure of delivered value.* *Multiplied 10000X* for clarity.	**25**	**70**	**300**
	6.1141	3.7699	1.2737
	Interpret as: for every 10,000 sestertii spent on this project, how much quinariae did we introduce		

Route 2 Totals

Total Costs At Year X:		
25	**70**	**300**
1542000	2929800	10023000
225000	252000	390000
225000	252000	390000
54250	82600	227500
2046250	**3516400**	**11030500**

Cumulative Value of Aqueduct			
Value A+C+D+E=	**25**	**70**	**300**
1500/year	37500	105000	450000

Efficiency of Investment At Year X:			
'Efficiency' = total value delivered by an aqueduct during lifetime, over the total cost spent on it during lifetime.	**25**	**70**	**300**
	1.83%	**2.99%**	**4.08%**
	Interpret as: for every 100 sestertii spent on this project, how much water did we get		

Alternate Efficiency 10000X			
Instead of calculating the cumulative value delivered by aqueduct over many years, we just use the raw *quinariae quantity as an absolute measure of delivered value.* *Multiplied 10000X* for clarity.	**25**	**70**	**300**
	7.3304	4.2657	1.3598
	Interpret as: for every 10,000 sestertii spent on this project, how much quinariae did we introduce		

Conclusions

The primary concern, and essentially the only concern, with Route 2 is the much shallower gradient that may be required as compared to Route 1. Both routes experience identical total changes in elevation, but Route 2's length may force the construction of impossibly shallow gradients, which require great technical precision, skill, and understanding to pull off. I can't tell if 6 meters over this distance is something that should raise any alarms.

Although Route 2 is substantially greater in length and meanders quite a bit, it manages to avoid running into most existing structures by remaining in the more rural southwestern part of the city. Route 1, though, must run directly under many structures in densely populated parts of the city, and seemingly even monuments and other potential sites of significance. Route 2's length is advantageous because it allows for the stratified population in the southern region access to flowing water in close proximity, whereas Route 1's proximity to the northern population would not be as useful because they are much closer to the aqueduct sources themselves.

Route 2 is more simplistic in design; almost the entire length is cut-cover construction, requiring just one very short bridge construction, and two critical tunnels that allow the aqueduct to draw from reservoirs. The greater number and variety of water sources adds redundancy to the system.

The cost of initially constructing Route 2 as well as maintaining Route 2 is quite a bit greater than Route 1, but when we compare numerical performance, other than the higher upfront cost, Route 2 demonstrates a better value proposition in every situation. Right at year 0, the water quantity Route 2 provides is 167% of Route 1, while only costing 127% of Route 1's cost. Looking at maintenance costs, Route 2 is more expensive to maintain at 160% of the recurring costs of Route 1, but since 160% is just a hair less than the 167% water volume output increase, the value of Route 2 in terms of maintenance costs is also superior. Since the initial investment and the recurring costs are both a better deal for Route 2 than for Route 1, we do not even need to assume any service life end date, at any given year past construction, Route 2 would have provided more water per unit cost.

The budget model can be simplified and represented as these relationships:

$$c_{total}\left(x_{years}, c_{initial}, c_{yearly}\right) = c_{initial} + c_{yearly} \cdot x_{years}$$

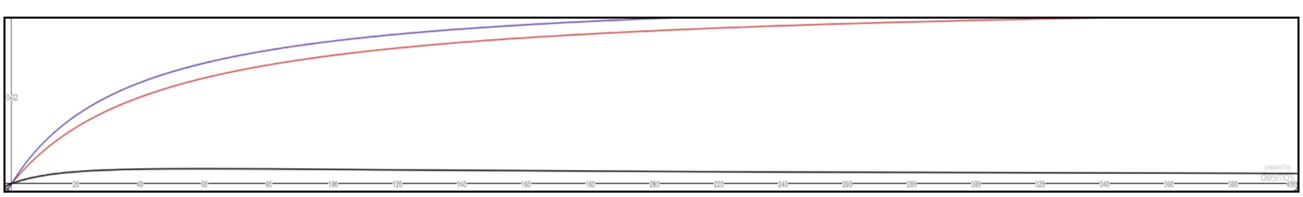
$$v_{total}\left(x_{years}, v_{yearly}\right) = v_{yearly} \cdot x_{years}$$

$$k_{efficiency}\left(x_{years}, c_{initial}, c_{yearly}, v_{yearly}\right) = \frac{v_{total}\left(x_{years}, v_{yearly}\right)}{c_{total}\left(x_{years}, c_{initial}, c_{yearly}\right)}$$

c_{total}: $cumulative\ cost\ of\ aqueduct$

v_{total}: $cumulative\ value\ provided\ by\ aqueduct$

$k_{efficiency}$: $value\ from\ aqueduct\ per\ unit\ cost\ spent\ at\ x_{years}\ since\ construction$



When the $k_{efficiency}$ of both routes is graphed over a few hundred years, it is clear that Route 2 **(purple)** will always remain more cost efficient than Route 1 **(red)**.

But accepting a lower value proposition may be reasonable, if, for example, both routes provide more than sufficient water supply needs, and we are free to reduce costs by employing the cheaper plan. In this situation, it is critical to note the immense cost of doing any kind of maintenance work on aqueducts, and within 30-40 years the entire cost of the initial investment will have been paid off again. As a result, the recurring costs are what dominate discussions of feasibility, and even large differences in upfront cost quickly become negligible within 1-2 decades. In our specific case, the recurring cost value proposition (value / yearly cost) is about equal for both Route 1 & 2, and this should not hold significant weight in our decision for Route.

Overall, it seems that Route 2 is truly the better route by every regard, and assuming it's construction would be possible by the Tikal at the time, it should always be chosen over Route 1. Regarding alterations & 3rd option, Route 2 also allows for optionally not tunneling to the reservoirs, allowing for an incredibly cost effective, almost pure cut-cover style construction which would actually have cheaper initial investment as compared to Route 1. Unlike Route 1, which must be built to completion to have any use at all, the tunnels and reservoirs in Route 2 can simply be integrated later, allowing the resources and capital investment to be spread out over time.

EC

I enjoyed this project, especially the analytical component - I was hoping for there to be some counterintuitive reveal in the math, but alas, Route 2 was simply always better :) .

This may be something you are saving for another assignment, but having students map out their own route solutions to minimize cost - deciding what kind of construction would go where - to a given set of terrain, and then comparing the student solution with what the Romans actually decided to construct would be quite interesting. Terrain maps are a bit hard to think about though - the concept is simple but it feels very easy to misread / make major mistakes. Allowing the use of 3D models would be much appreciated.