Dado $\operatorname{sen}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$, $\forall x \in \mathbb{R}$.

$$f(x) = \frac{\operatorname{sen}(2x)}{x}$$

$$\frac{1}{x} \cdot \operatorname{sen}(2x)$$

$$\operatorname{sen}(2x) = \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2x)^{2n+1}}{(2n+1)!}$$

$$\frac{\operatorname{sen}(2x)}{x} = \frac{1}{x} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2x)^{2n+1}}{(2n+1)!}$$

$$= \frac{1}{\cancel{x}} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n+1} \cdot x^{\cancel{2n+1}}}{(2n+1)!}$$

$$\therefore \sum_{n=0}^{\infty} \frac{(-1)^n \cdot 2^{2n+1} \cdot x^{2n}}{(2n+1)!}$$

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1) b)

$$\int_0^{\pi/2} f(x)dx \qquad f(x) = \text{sen}(2x)/x$$

menor a 0.001.

$$\Rightarrow \int_0^{\pi/2} \frac{\text{sen}(2x)}{x} \, dx$$

$$\sum_{n=0}^{\infty} = \frac{(-1)^n \cdot 2^{n+1} \cdot x^{2n}}{(2n+1)!} =$$

$$\sum_{n=0}^{\infty} \left(\frac{2^{2n+1} (-1)^n}{(2n+1)!} \int_0^{\pi/2} x^{2n} dx \right)$$

$$= \sum_{n=0}^{\infty} \left[\frac{2^{2n+1} (-1)^n}{(2n+1)!} \left(\frac{x^{2n+1}}{2n+1} \right) \Bigg|_0^{\pi/2} \right)$$

$$\lim_{n \to \infty} \frac{\pi^{2n+1}}{(2n+1)!} = 0 \Rightarrow \lim_{n \to \infty} \frac{\pi^{2n+1}}{(2n+1)! \, (2n+1)}$$

$$a_n < 0.001$$

$$\frac{\pi^{2n+1}}{(2n+1)!(2n+1)} < \frac{1}{1000}$$

$$1000 \leq \frac{(2n+1)! \, (2n+1)}{\pi^{2n+1}}$$

2.)

$$P(x) = 3x^3 - 11x^2 + 11x + 5$$
$$P(2+i) = 0$$

$x = -\frac{1}{3}$ $\quad x = 2+i$ $\quad x = 2-i$

3	-11	11	5	$-1/3$
↓	-1	4	-5	
3	-12	15	0	

$$3x^2 - 12x + 15$$

$$\Delta = b^2 - 4 \cdot a \cdot c$$
$$\Delta = (-12)^2 - 4 \cdot 3 \cdot 15 = -36$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 3 \cdot 15}}{2 \cdot 3}$$

$$= \frac{12 \pm \sqrt{-36}}{6}$$

$$= \frac{12 \pm \sqrt{36}\, i}{6}$$

$$= \frac{12}{6} \pm \frac{\sqrt{36}}{6} i$$

$$= 2 \pm i$$

3) $z = e^{\frac{-\pi}{2}i}$

$$z^z = \left(e^{\frac{-\pi}{2}i}\right)^{e^{\frac{-\pi}{2}i}}$$

$$z^z = e^{z\ln(z)} = e^{e^{\frac{-\pi}{2}i} \cdot \frac{-\pi}{2}i} = e^{-i \cdot \frac{\pi}{2}i} = e^{-\frac{\pi}{2}}$$

$$\ln(z) = \ln(e^{\frac{-\pi}{2}i}) = \frac{-\pi}{2}i$$

$$e^{\frac{-\pi}{2}i} = \text{cis}\left(\frac{-\pi}{2}\right) = \cos\left(\frac{-\pi}{2}\right) + i \cdot \text{sen}\left(\frac{-\pi}{2}\right)$$

$z = -i$

$z = \text{cis}\left(\frac{-\pi}{2}\right)$