毕设进展报告2

Visual Tracking via Adaptive Spatially-Regularized Correlation Filters中的公式推导

由（4）到（5）的推导

$$E(H,w)=\frac{1}{2}\left\|y-\sum_{k=1}^{K}x_k*(P^Th_k)\right\|_2^2+\frac{\lambda_1}{2}\sum_{k=1}^{K}||w\odot h_k||_2^2+\frac{\lambda_2}{2}||w-w^r||_2^2 \tag{4}$$

其中，$P\in\mathbb{R}^{T\times T}$是一个二进制矩阵，训练集$X=[x_1,x_2,\cdots,x_K]\in\mathbb{R}^{T\times K}$，滤波器$H=[h_1,h_2,\cdots,h_K]\in\mathbb{R}^{T\times K}$,$k$为通道数,$w$为待优化的权重参数。

注意到(4)中有卷积运算，频域下卷积运算更加容易，于是试图将（4）变换到频域

令$g_k=P^Th_k$,对向量$\alpha\in\mathbb{R}^{T\times 1}$的傅里叶变换可以写作以下形式

$$\hat{\alpha}=\sqrt{T}F\alpha$$

其中F为正交的$T\times T$的傅里叶变换矩阵。

因此，g_k的傅里叶变换形式如下

$$\hat{g}=\sqrt{T}FP^Th_k$$

于是得到（5）

$$E(H,\hat{G},w)=\frac{1}{2}\left\|y-\sum_{k=1}^{K}\hat{x}_k\odot\hat{g}_k\right\|_2^2+\frac{\lambda_1}{2}\sum_{k=1}^{K}||w\odot h_k||_2^2+\frac{\lambda_2}{2}||w-w^r||_2^2 \tag{5}$$
$$s.t.\ \hat{g}_k=\sqrt{T}FP^Th_k,k=1,\cdots,K$$

将$\hat{g}_k$单独拿出来作为线性约束条件。

运用ADMM求解

将（5）写成增广拉格朗日形式（ALM），得到（6）

$$L(H,\hat{G},w,\hat{V})=E(H,\hat{G},w)+\frac{\mu}{2}\sum_{k=1}^{K}||\hat{g}_k-\sqrt{T}FP^Th_k||_2^2+\hat{v}_k^T\sum_{k=1}^{k}(\hat{g}_k-\sqrt{T}FP^Th_k) \tag{6}$$

其中$V=[v_1,v_2,\cdots,v_K]\in\mathbb{R}^{T\times K}$为拉格朗日乘子，它的傅里叶形式为$\hat{V}=[\hat{v}_1,\hat{v}_2,\cdots,\hat{v}_K]\in\mathbb{R}^{T\times K}$,引入$s_k=\frac{1}{\mu}v_k$,将（6）化简

$$
\begin{aligned}
L(H,\hat{G},w,\hat{V}) &= E(H,\hat{G},w)+\frac{\mu}{2}\sum_{k=1}^{K}(\hat{g}_k-\sqrt{T}FP^Th_k)^T(\hat{g}_k-\sqrt{T}FP^Th_k)+\hat{s}_k\mu\sum_{k=1}^{k}(\hat{g}_k-\sqrt{T}FP^Th_k) \\
&= E(H,\hat{G},w)+\frac{\mu}{2}\sum_{k=1}^{K}\big[(\hat{g}_k-\sqrt{T}FP^Th_k)^T(\hat{g}_k-\sqrt{T}FP^Th_k)+ \\
&\qquad 2\hat{s}_k(\hat{g}_k-\sqrt{T}FP^Th_k)+\hat{s}_k^2-\hat{s}_k^2\big] \\
&= E(H,\hat{G},w)+\frac{\mu}{2}\sum_{k=1}^{K}\big[(\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k)^T(\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k)-\hat{s}_k^2\big] \\
&= E(H,\hat{G},w)+\frac{\mu}{2}\sum_{k=1}^{K}\big|\big|\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k\big|\big|_2^2-\frac{\mu}{2}\sum_{k=1}^{K}\hat{s}_k^2
\end{aligned}
\tag{7}
$$

优化H

$$
h_k^*=arg\ \min_{h_k}\left\{\frac{\lambda_1}{2}||w\odot h_k||_2^2+\frac{\mu}{2}\big|\big|\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k\big|\big|_2^2\right\}
\tag{8}
$$

令$W=diag(w)$,可以将循环卷积换位矩阵相乘，得到$w\odot h_k=Wh_k$

$$
\begin{aligned}
&\frac{\partial}{\partial h_k}\left\{\frac{\lambda_1}{2}||w\odot h_k||_2^2+\frac{\mu}{2}\big|\big|\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k\big|\big|_2^2\right\} \\
&=\left\{\frac{\lambda_1}{2}||Wh_k||_2^2+\frac{\mu}{2}\big|\big|\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k\big|\big|_2^2\right\} \\
&=\lambda_1W^T(Wh_k)-\mu(\sqrt{F}TP^T)^T(\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k) \\
&=(\lambda_1W^TW+\mu TPP^T)h_k-\mu(\sqrt{T}FP^T)^T(\hat{g}_k+\hat{s}_k)
\end{aligned}
\tag{9}
$$

令（9）为零，可得

$$
\begin{aligned}
h_k^* &= \frac{\mu(\sqrt{T}FP^T)^T(\sqrt{T}Fg_k+\sqrt{T}Fs_k)}{\lambda_1W^TW+\mu TPP^T} \\
&= \frac{\mu(\sqrt{T}PF^T)\sqrt{T}F(g_k+s_k)}{\lambda_1W^TW+\mu TPP^T} \\
&= \frac{\mu TP(g_k+s_k)}{\lambda_1W^TW+\mu TPP^T}
\end{aligned}
\tag{10}
$$

令$\mathbf{p}=[P_{11},P_{22},\cdots,P_{TT}]^T$,$\mathbf{p}$为$P$的对角元素组成的列向量，且$PP^T=\mathbf{p}$，带入（10）得

$$
h_k^*=\frac{\mu T\mathbf{p}\odot(g_k+s_k)}{\lambda_1(\mathbf{w}\odot\mathbf{w})+\mu T\mathbf{p}}
\tag{11}
$$

优化$\hat{G}$

$$
\hat{G}^*=\arg\min_{\hat{G}}\left\{\frac{1}{2}\left|\left|\hat{y}-\sum_{k=1}^{K}\hat{x}_k\odot\hat{g}_k\right|\right|_2^2+\frac{\mu}{2}\sum_{k=1}^{K}\big|\big|\hat{g}_k-\sqrt{T}FP^Th_k+\hat{s}_k\big|\big|_2^2\right\}
\tag{12}
$$

然而（12）中$\sum_{k=1}^{K}\hat{x}_k\odot\hat{g}_k$在范式内，便不能用优化$H$（8）的方式优化$\hat{G}$

于是，考虑将$\hat{g}_k$中的每个分量单独提出来计算，记作$v_j^*(\hat{g}) \in \mathbb{R}^{K\times 1}$,代表滤波器$\hat{g}$的底$j$个分量在所有通道的值。

$$L(v_j(\hat{G})) = \frac{1}{2}\left|\left|\hat{y}_j - v_j(\hat{X})^T v_j(\hat{G})\right|\right|_2^2 + \frac{\mu}{2}\left|\left|v_j(\hat{G}) - v_j(\hat{M})\right|\right|_2^2 \\ v_j(\hat{M}) = v_j(\sqrt{T}FP^T H) - v_j(\hat{S}) \tag{13}$$

对（13）进行求导

$$\frac{\partial L(v_j(\hat{G}))}{\partial v_j(\hat{G})} = v_j(\hat{X})[\hat{y}_j - v_j(\hat{X})^T v_j(\hat{G})] + \mu[v_j(\hat{G}) - v_j(\hat{M})] \tag{14}$$

令（14）为零

$$\begin{aligned} 0 &= v_j(\hat{X})[\hat{y}_j - v_j(\hat{X})^T v_j(\hat{G})] + \mu[v_j(\hat{G}) - v_j(\hat{M})] \\ (v_j(\hat{X})v_j(\hat{X})^T + \mu)v_j(\hat{G}) &= v_j(\hat{X})\hat{y}_j + \mu v_j(\hat{M}) \\ v_j(\hat{G}) &= \frac{v_j(\hat{X})\hat{y}_j + \mu v_j(\hat{M})}{v_j(\hat{X})v_j(\hat{X})^T + \mu} \end{aligned} \tag{15}$$

直接求逆运算量巨大，考虑使用Sherman Morrsion公式：

$$(\mathbf{A} + \mathbf{u}\mathbf{v}^T)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{u}\mathbf{v}^T\mathbf{A}^{-1}}{1 + \mathbf{v}^T\mathbf{A}^{-1}\mathbf{u}} \tag{16}$$

令$A = \mu I, \mathbf{u} = v_j(\hat{X}), \mathbf{v} = v_j(\hat{X})^T$，带入（16）得

$$\begin{aligned} (v_j(\hat{X})v_j(\hat{X})^T + \mu)^{-1} &= \frac{1}{\mu}I - \frac{\frac{1}{\mu^2}v_j(\hat{X})v_j(\hat{X})^T}{1 + \frac{1}{\mu}v_j(\hat{X})^T v_j(\hat{X})} \\ &= \frac{1}{\mu}\left(I - \frac{v_j(\hat{X})v_j(\hat{X})^T}{\mu + v_j(\hat{X})^T v_j(\hat{X})}\right) \end{aligned} \tag{17}$$

将（17）带回（15）

$$v_j(\hat{G}) = \frac{1}{\mu}\left(I - \frac{v_j(\hat{X})v_j(\hat{X})^T}{\mu + v_j(\hat{X})^T v_j(\hat{X})}\right)\left(v_j(\hat{X})\hat{y}_j + \mu v_j(\sqrt{T}FP^T H) - \mu v_j(\hat{S})\right) \tag{18}$$

优化$\mathbf{w}$

$$\mathbf{w}^* = \arg\min_{\mathbf{w}}\left\{\frac{\lambda_1}{2}\sum_{k=1}^{K}||w \odot h_k||_2^2 + \frac{\lambda_2}{2}||w - w^r||_2^2\right\} \tag{19}$$

令$N_k = diag(h_k) \in \mathbb{R}^{T\times T}$，进而有

$$w \odot h_k = N_k w \tag{20}$$

(20)代入(19)

$$\mathbf{w}^* = \arg\min_{\mathbf{w}}\left\{\frac{\lambda_1}{2}\sum_{k=1}^{K}||N_k w||_2^2 + \frac{\lambda_2}{2}||w - w^r||_2^2\right\} \tag{21}$$

对目标函数求导

$$
\begin{aligned}
0 &= \frac{\partial}{\partial w}\left\{\frac{\lambda_1}{2}\sum_{k=1}^{K}(N_k w)^T(N_k w) + \frac{\lambda_2}{2}(w-w^r)^T(w-w^r)\right\} \\
&= \lambda_1 \sum_{k=1}^{K} N_k^T N_k w + \lambda_2 (w - w^r)
\end{aligned}
$$

移项得

$$
\lambda_2 w^r = \lambda_1 \sum_{k=1}^{K} N_k^T N_k w + \lambda_2 w = (\lambda_1 \sum_{k=1}^{K} N_k^T N_k + \lambda_2 I) w
$$

进而求得$\mathbf{w}^*$

$$
\begin{aligned}
\mathbf{w}^* &= \frac{\lambda_2 w^r}{\lambda_1 \sum_{k=1}^{K} N_k^T N_k + \lambda_2 I} \\
&= \frac{\lambda_2 w^r}{\lambda_1 \sum_{k=1}^{K} h_k \odot h_k + \lambda_2 I}
\end{aligned}
\tag{22}
$$

遇到的问题

- （6）到（7）的推导结果与论文中的不一致

  论文结果如下：

$$
\begin{aligned}
L(H,\hat{G},w,\hat{V}) = \frac{1}{2}\left\|y - \sum_{k=1}^{K}\hat{x}_k \odot \hat{g}_k\right\|_2^2 + \frac{\lambda_1}{2}\sum_{k=1}^{K}||w \odot h_k||_2^2 + \frac{\lambda_2}{2}||w - w^r||_2^2 \\
+ \frac{\mu}{2}\sum_{k=1}^{K}||\hat{g}_k - \sqrt{T}FP^T h_k + \hat{s}_k||_2^2
\end{aligned}
$$

而我的推导结果如下：

$$
\begin{aligned}
L(H,\hat{G},w,\hat{V}) = \frac{1}{2}\left\|y - \sum_{k=1}^{K}\hat{x}_k \odot \hat{g}_k\right\|_2^2 + \frac{\lambda_1}{2}\sum_{k=1}^{K}||w \odot h_k||_2^2 + \frac{\lambda_2}{2}||w - w^r||_2^2 \\
+ \frac{\mu}{2}\sum_{k=1}^{K}||\hat{g}_k - \sqrt{T}FP^T h_k + \hat{s}_k||_2^2 - \frac{\mu}{2}\sum_{k=1}^{K}\hat{s}_k^2
\end{aligned}
$$

解答

注意到，$\hat{s}_k^2$并不是待优化的主元，所以可以当作常数舍弃。

- 优化$\hat{G}$时，得到的结果（18），最后的结果少了个T

  论文结果如下：

$$
v_j(\hat{G}) = \frac{1}{\mu T}\left(I - \frac{v_j(\hat{X})v_j(\hat{X})^T}{\mu T + v_j(\hat{X})^T v_j(\hat{X})}\right)\left(v_j(\hat{X})\hat{y}_j + \mu v_j(\sqrt{T}FP^T H) - \mu v_j(\hat{S})\right)
$$

  而我的推导结果如下：

$$
v_j(\hat{G}) = \frac{1}{\mu}\left(I - \frac{v_j(\hat{X})v_j(\hat{X})^T}{\mu + v_j(\hat{X})^T v_j(\hat{X})}\right)\left(v_j(\hat{X})\hat{y}_j + \mu v_j(\sqrt{T}FP^T H) - \mu v_j(\hat{S})\right)
$$

接下来的工作

进一步理解测试代码，争取找到可改进的地方。