Anurag Parcha – 301558925

CMPT 732: Practices in Visual Computing I

Assignment-3: Part-1 – Epipolar Geometry

Fundamental Matrix and Normalized 8-point algorithm

Fundamental Matrix (F) is used to define a relationship between corresponding points in two different images that represent two different viewpoints (stereo images) of a scene in epipolar geometry. More specifically, the Fundamental Matrix applied to a point in image 1 gives the line along which the corresponding point lies on called the epipolar line.

$\mathbf{x}'^{\top}\mathbf{F}\mathbf{x} = 0.$ Fx describes the line corresponding to the point x' in the second image.

Fundamental matrix can be estimated with the normalized 8-point algorithm where 8 corresponding points are taken. The points are converted to homogenous coordinates and normalized by applying transformations (T, T') which centers the two sets of points' centroid at origin and scales the mean distance of all points to their centroid to $\sqrt{2}$. This changes the epipolar constraint to the below.

$$\begin{matrix}\bar{\mathbf{y}} = \mathbf{T}\,\mathbf{y} \\ \bar{\mathbf{y}}' = \mathbf{T}'\,\mathbf{y}'\end{matrix} \qquad 0 = (\bar{\mathbf{y}}')^T\,((\mathbf{T}')^T)^{-1}\,\mathbf{F}\,\mathbf{T}^{-1}\,\bar{\mathbf{y}} = (\bar{\mathbf{y}}')^T\,\bar{\mathbf{F}}\,\bar{\mathbf{y}}$$

This would mean that after computing the fundamental matrix, we would have to de-Normalize the Fundamental Matrix by applying the inverse Transforms to F'.

$$\mathbf{F} = (\mathbf{T}')^T\,\bar{\mathbf{F}}\,\mathbf{T}$$

While calculating the Fundamental matrix, we have to also enforce the rank 2 constraint. Fundamental Matrix has a rank of 2 since the epipoles of the epipolar line always satisfy the epipolar constraint making the Fundamental Matrix have a null vector other than the 0 vector. So after calculating the SVD of the F matrix, we have to set the last sigma value to 0 and recalculate the F matrix.

The below images show the results of the in-built openCV functions (mentions the "true" keyword) and manually computed estimations of the Fundamental Matrix for both the N estimation and RANSAC method.

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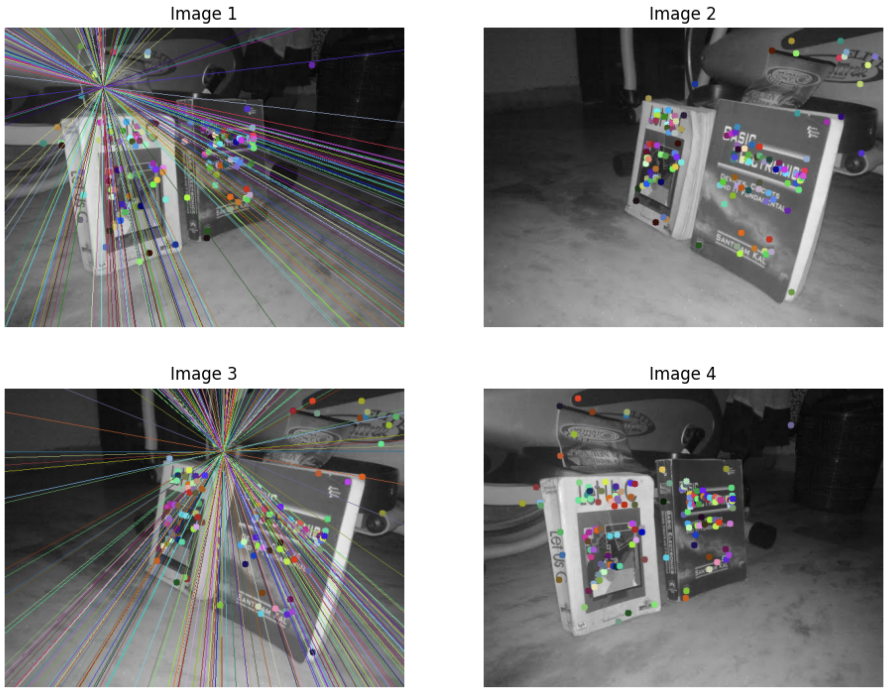
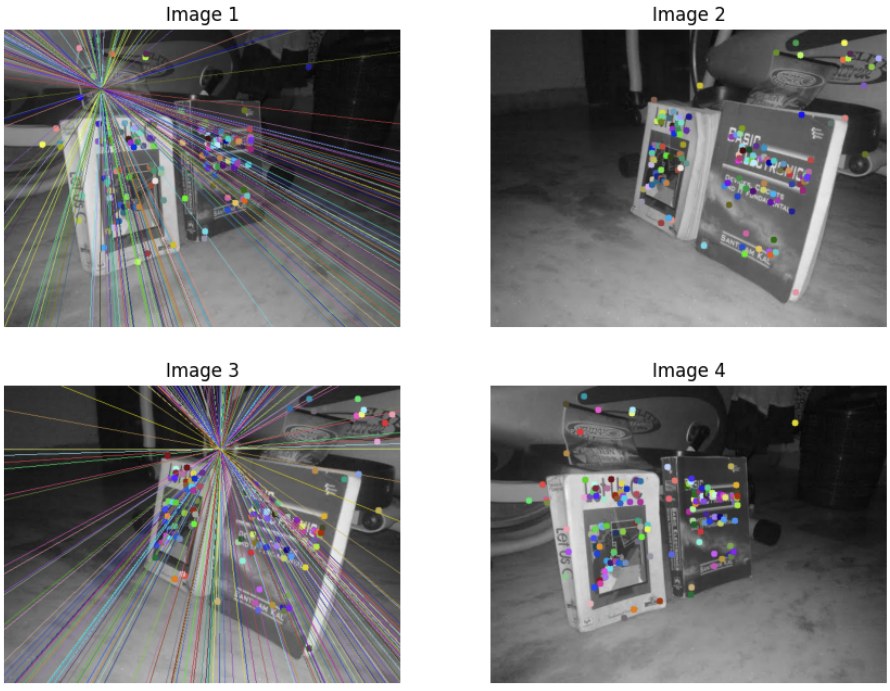
1. Dataset: myleft.jpg and myright.jpg

F_true: Fundamental Matrix computed using the OpenCV built-in function for 8-point Algorithm.



F: Manually Estimated Fundamental Matrix using 8-point Algorithm.

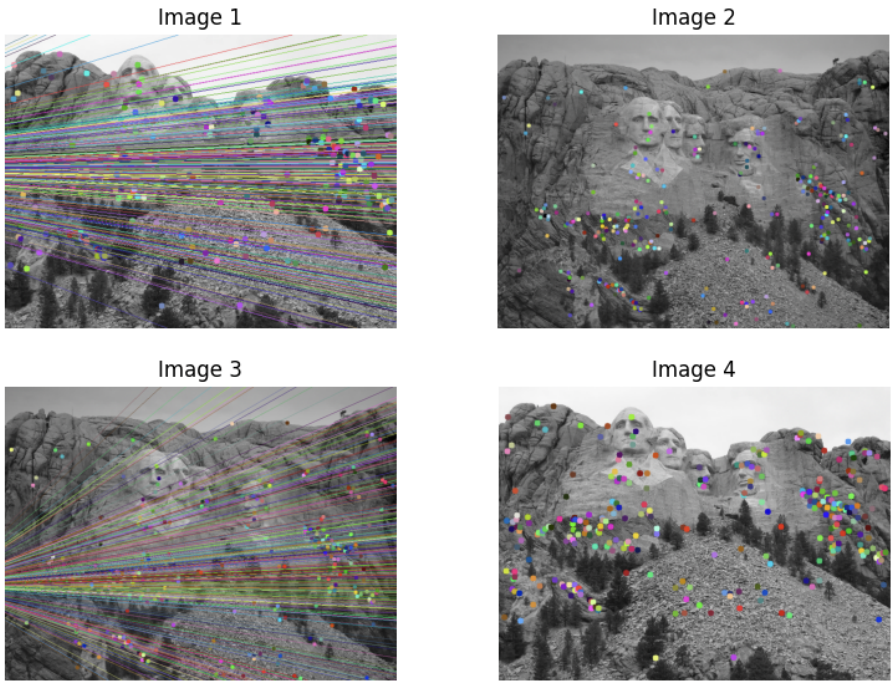


```
eight_point_fw
/Users/anuragparcha/Desktop/MPCS_FALL2022/VC/Lab/A3/A3-1/
CV2 Estimated Fundamental Matrix:
 [[ 1.30650775e-05  1.81420944e-05 -3.61564062e-03]
 [-2.57558327e-05  1.90938190e-05  2.13164449e-03]
 [-1.84707041e-03 -7.93245689e-03  1.00000000e+00]]
Manually Estimated Fundamental Matrix:
 [[ 1.30650775e-05  1.81420944e-05 -3.61564062e-03]
 [-2.57558327e-05  1.90938190e-05  2.13164449e-03]
 [-1.84707041e-03 -7.93245689e-03  1.00000000e+00]]
```

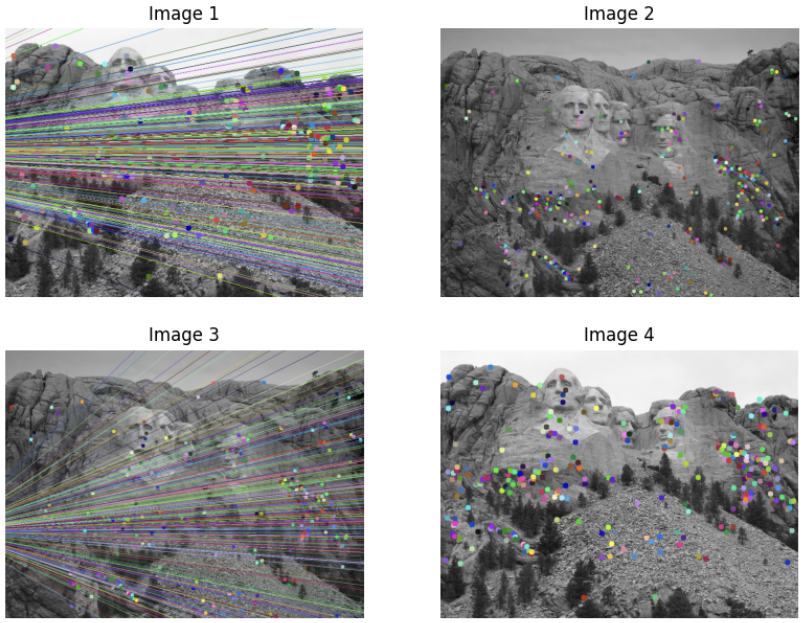
As we can see, the value for the Manually estimated Fundamental Matrix matches the OpenCV built-in function.

2. Dataset: mount_rushmore_1.jpg and mount_rushmore_2.jpg

F_true: Fundamental Matrix computed using the OpenCV built-in function for 8-point Algorithm.



F: Manually Estimated Fundamental Matrix using 8-point Algorithm.

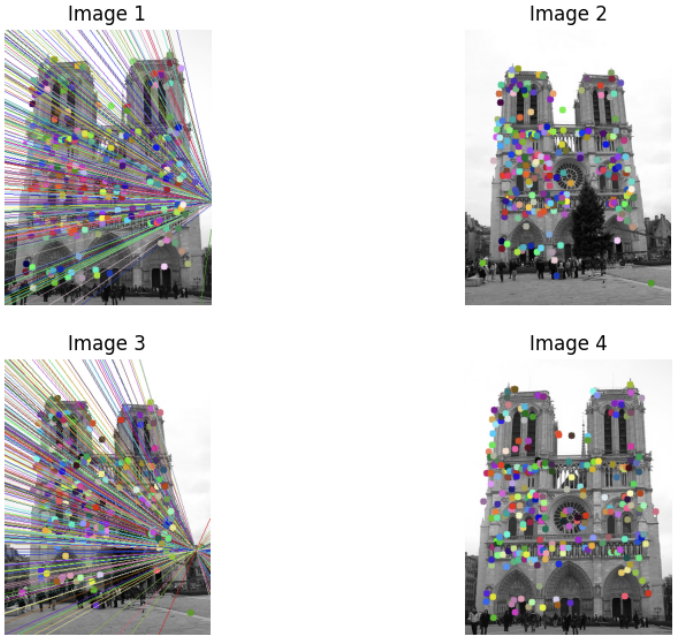



```
eight_point_fw
/Users/anuragparcha/Desktop/MPCS_FALL2022/VC/Lab/A3/A3-1/ve
CV2 Estimated Fundamental Matrix:
 [[-6.21019078e-07  1.94057859e-05 -4.61012984e-03]
 [-9.63274968e-06 -3.36322689e-06 -3.39544243e-03]
 [ 4.69881036e-03  4.57321966e-03  1.00000000e+00]]
Manually Estimated Fundamental Matrix:
 [[-6.21019078e-07  1.94057859e-05 -4.61012984e-03]
 [-9.63274968e-06 -3.36322689e-06 -3.39544243e-03]
 [ 4.69881036e-03  4.57321966e-03  1.00000000e+00]]
```

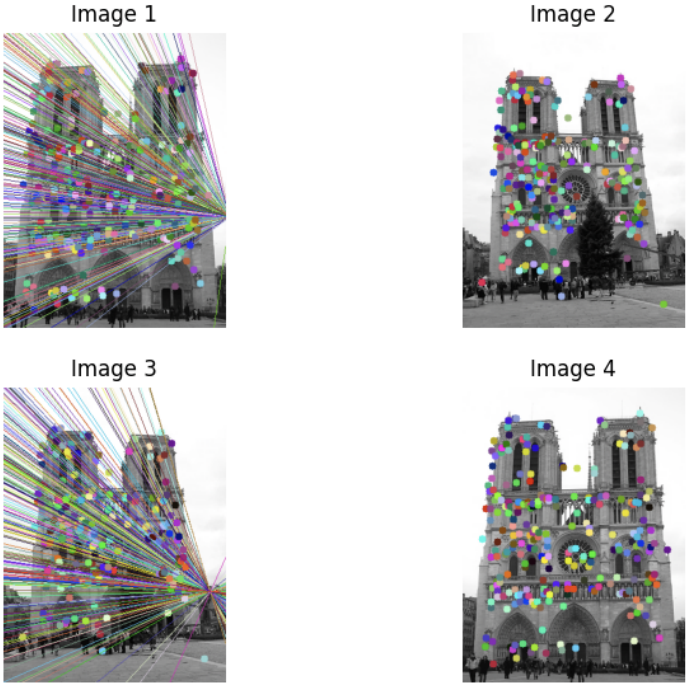
As we can see, the value for the Manually estimated Fundamental Matrix matches the OpenCV built-in function.

3. Dataset: notredam_1.jpg and notredam2.jpg

F_true: Fundamental Matrix computed using the OpenCV built-in function for 8-point Algorithm.



F: Manually Estimated Fundamental Matrix using 8-point Algorithm.



```
/Users/anuragparcha/Desktop/MPCS_FALL2022/VC/Lab/A3/A3-1/v
CV2 Estimated Fundamental Matrix:
 [[ 1.43824939e-04  5.82103337e-04 -1.94254472e-01]
 [-5.86388808e-04 -3.12812932e-05  1.91403923e-01]
 [ 1.24757087e-01 -1.56040682e-01  1.00000000e+00]]
Manually Estimated Fundamental Matrix:
 [[ 1.43824939e-04  5.82103337e-04 -1.94254472e-01]
 [-5.86388808e-04 -3.12812932e-05  1.91403923e-01]
 [ 1.24757087e-01 -1.56040682e-01  1.00000000e+00]]
```

As we can see, the value for the Manually estimated Fundamental Matrix matches the OpenCV built-in function.

Fundamental Matrix and Normalized 8-point algorithm using RANSAC

To get a more accurate fundamental matrix, we use the RANSAC or Random Sample Consensus Algorithm. In this, we run N iterations and in each iteration, we compute a Fundamental matrix with a random set of 8 points from the total set of n correspondences and calculate number of points that satisfy the Epipolar constraint with the that iteration's Fundamental Matrix. Then we calculate the number of these "inliers" and store it. After the N iterations, we take the best Fundamental Matrix with the maximum number of inliers. This gets rid of the inaccurate correspondences in the set of points of image 1 and image 2.

Theoretically, for an accuracy of 99% we need at least 1177 iterations to ensure that the 8 randomly picked points are all inliers in a set of points with an outlier ratio of 50%. Threshold is another hyper parameter that determines what point is considered an inlier. Specifically, the calculation of the Epipolar constraint ($x'Fx$) should be below the threshold for an inlier.

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1. Dataset: myleft.jpg and myright.jpg

Threshold = 0.01

N = 1200

F_ransac_true: Fundamental Matrix computed using the OpenCV built-in RANSAC function.

Image 1

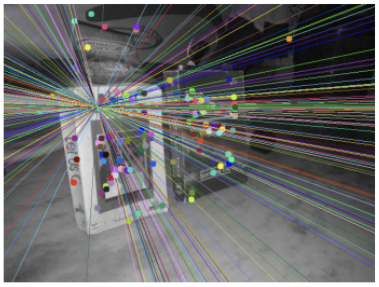


Image 2

Image 3

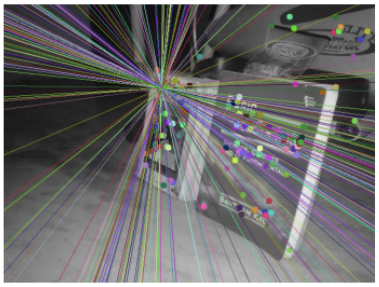


Image 4

F_ransac: Manually Estimated Fundamental Matrix using 8-point Algorithm and RANSAC.

Image 1

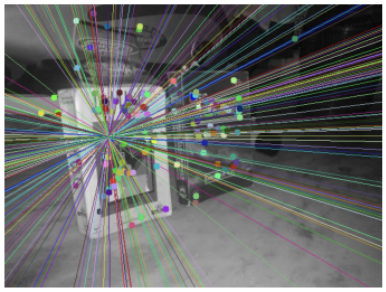


Image 2

Image 3

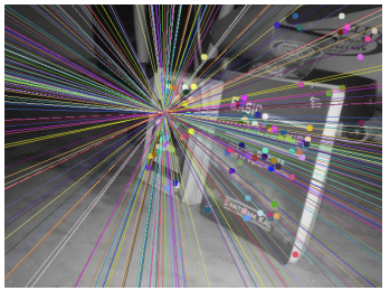


Image 4

Fundamental Matrices:

```
eight_point_fw
/Users/anuragparcha/Desktop/MPCS_FALL2022/VC/Lab/A3/A3-1/
CV2 Estimated Fundamental Matrix:
 [[ 1.30650775e-05  1.81420944e-05 -3.61564062e-03]
 [-2.57558327e-05  1.90938190e-05  2.13164449e-03]
 [-1.84707041e-03 -7.93245689e-03  1.00000000e+00]]
Manually Estimated Fundamental Matrix:
 [[ 1.30650775e-05  1.81420944e-05 -3.61564062e-03]
 [-2.57558327e-05  1.90938190e-05  2.13164449e-03]
 [-1.84707041e-03 -7.93245689e-03  1.00000000e+00]]
CV2 Estimated Fundamental Matrix with RANSAC:
 [[ 6.07617187e-06  2.00017214e-05 -4.32365787e-03]
 [-1.56906556e-05  8.90563230e-06  9.03109517e-04]
 [ 5.82083788e-04 -6.42914141e-03  1.00000000e+00]]
Manually Estimated Fundamental Matrix with RANSAC:
 [[ 6.73128341e-06  1.63982405e-05 -4.66938828e-03]
 [-1.21156790e-05  4.01961626e-06  1.17515032e-03]
 [ 4.00560640e-04 -4.95186115e-03  1.00000000e+00]]

Process finished with exit code 0
```

2. Dataset: mount_rushmore_1.jpg and mount_rushmore_1.jpg

Threshold = 0.1

N = 1200

F_ransac_true: Fundamental Matrix computed using the OpenCV built-in RANSAC function.

Image 1

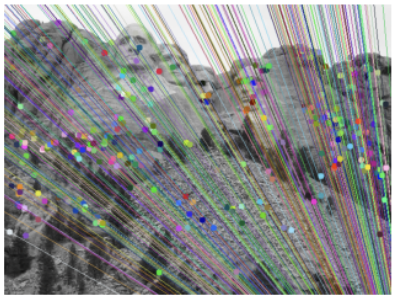


Image 2

Image 3

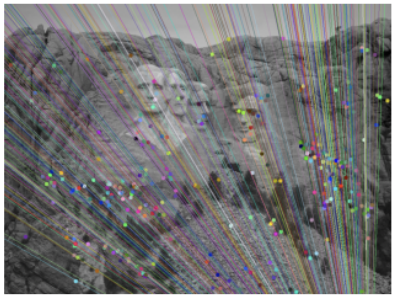


Image 4

F_ransac: Manually Estimated Fundamental Matrix using 8-point Algorithm and RANSAC.

Image 1

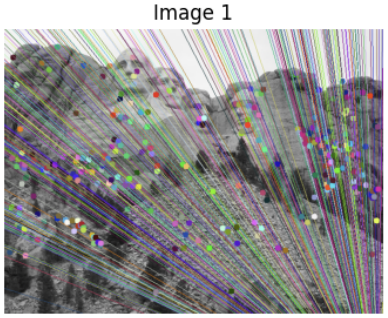


Image 2

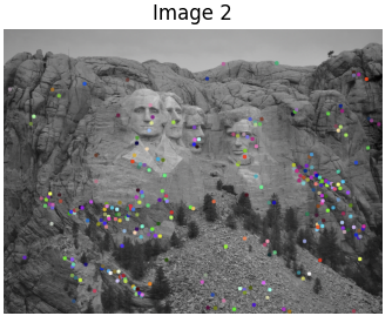


Image 3

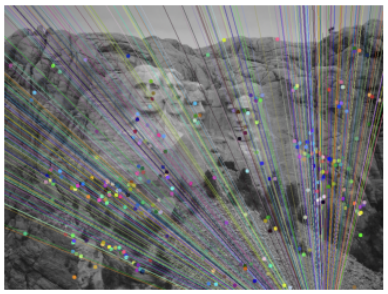


Image 4

Fundamental Matrices:

eight_point_fw

```
/Users/anuragparcha/Desktop/MPCS_FALL2022/VC/Lab/A3/A3-1/ve
CV2 Estimated Fundamental Matrix:
 [[-6.21019078e-07  1.94057859e-05 -4.61012984e-03]
 [-9.63274968e-06 -3.36322689e-06 -3.39544243e-03]
 [ 4.69881036e-03  4.57321966e-03  1.00000000e+00]]
Manually Estimated Fundamental Matrix:
 [[-6.21019078e-07  1.94057859e-05 -4.61012984e-03]
 [-9.63274968e-06 -3.36322689e-06 -3.39544243e-03]
 [ 4.69881036e-03  4.57321966e-03  1.00000000e+00]]
CV2 Estimated Fundamental Matrix with RANSAC:
 [[-1.53776350e-06 -5.97157200e-05  4.76960336e-02]
 [ 6.62637085e-05 -9.02240539e-06 -3.97449906e-02]
 [-7.35273978e-02  6.52321527e-02  1.00000000e+00]]
Manually Estimated Fundamental Matrix with RANSAC:
 [[-1.03388662e-06 -1.49920537e-05  1.04845708e-02]
 [ 2.04853295e-05 -3.74276660e-06 -1.07546737e-02]
 [-1.75707505e-02  1.57689968e-02  1.00000000e+00]]

Process finished with exit code 0
```

3. Dataset: notredam_1.jpg and notredam2.jpg

Threshold = 0.001

N = 1200

F_ransac_true: Fundamental Matrix computed using the OpenCV built-in RANSAC function.

Image 1

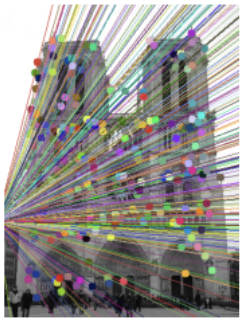


Image 2

Image 3

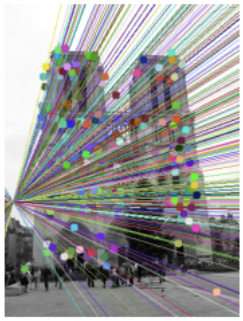


Image 4

F_ransac: Manually Estimated Fundamental Matrix using 8-point Algorithm and RANSAC.

Image 1

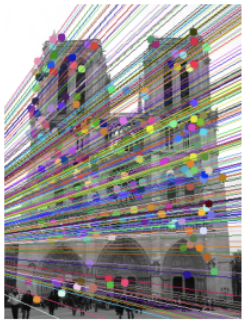


Image 2

Image 3

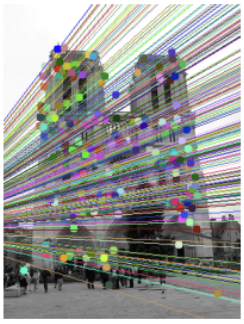
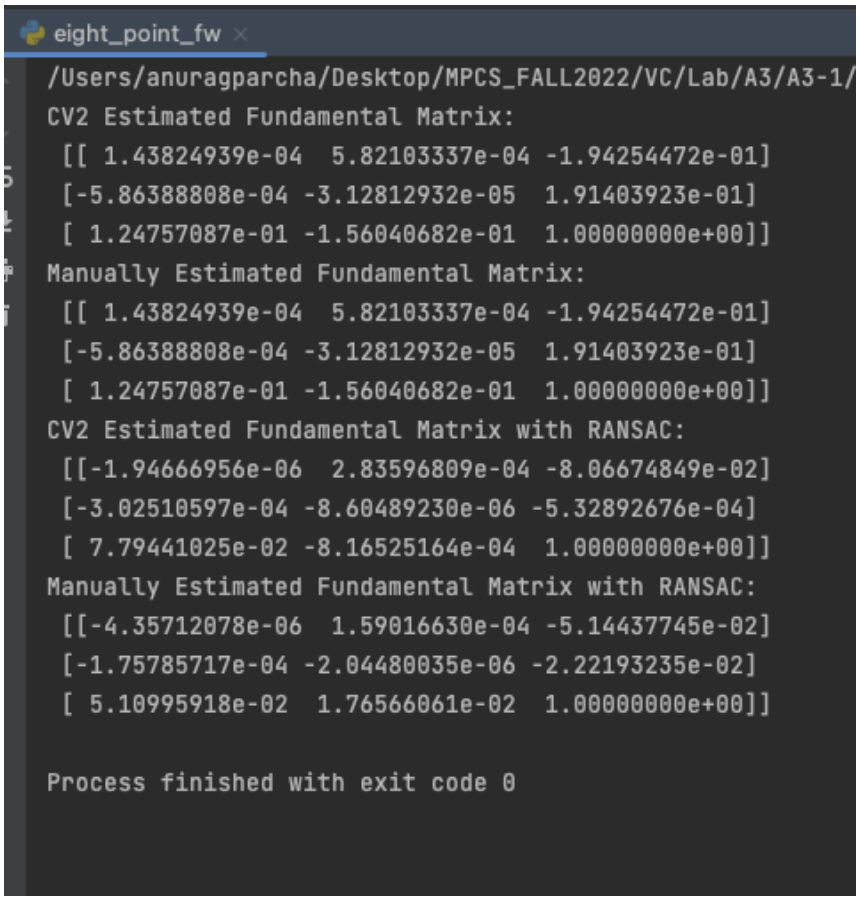


Image 4

Fundamental Matrices:



INFERENCE:

The output images show the geometric correspondences between the points in left and right images through epipolar lines, this is also known as the Epipolar constraint. In “image 1”s and “image 3”s, we plot the epipolar lines computed by the points in “image 2”s and “image 4”s with the Fundamental matrices. Along these epipolar lines is where that point’s corresponding point lies. This reduces the search for the exact point correspondence along that line instead os searching the whole image plane. These lines converge at a point which may or may not be in the image, this point of convergence is called the epipole. The epipole should correspond to the same point in image plane 1 and image plane 2 of left and right images respectively. Ultimately, this helps us gather the 3D information of the objects in the left and right images.

In the case of RANSAC, the output should be more accurate since it helps us eliminate the point correspondences which are actually outliers. This is done by taking the best Fundamental matrix generated by all 8 points which are inliers. And this is ensured by increasing the number of trials to match the outlier ratio. With an outlier ratio of 50%, we need atleast 1177 trials so that there is a 99% chance that all 8 points will be inliers giving us the best matrix.