A Supplementary File for "An Easy-to-use Real-world Multi-objective Optimization Problem Suite"

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Abstract

This is a supplementary file for "An Easy-to-use Real-world Multi-objective Test Optimization Problem Suite".

S.1. Details of the RE problems

S.1.1. RE2-4-1: Four bar truss design problem

The RE2-4-1 problem is to minimize the structural volume (f_1) and the joint displacement (f_2) of the four bar truss [S.1]. The two objective functions of the RE2-4-1 problem are given:

$$f_1(\boldsymbol{x}) = L(2x_1 + \sqrt{2}x_2 + \sqrt{x_3} + x_4), \tag{S.1}$$

$$f_2(\boldsymbol{x}) = \frac{F\,L}{E}\left(\frac{2}{x_1} + \frac{2\sqrt{2}}{x_2} - \frac{2\sqrt{2}}{x_3} + \frac{2}{x_4}\right), \tag{S.2}$$

where $x_1, x_4 \in [a, 3a]$, $x_2, x_3 \in [\sqrt{2}a, 3a]$, and $a = F/\sigma$. The four variables determine the length of four bars, respectively. The parameters are given as follows: $F = 10$ kN, $E = 2 \times 10^5$kN/cm^2, $L = 200$ cm, $\sigma = 10$ kN/cm^2.

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S.1.2. RE2-3-2: Reinforced concrete beam design problem

The first objective of the RE2-3-2 problem is to minimize the total cost of concrete and reinforcing steel of the beam [S.2]:

$$f_1(\boldsymbol{x}) = 29.4x_1 + 0.6x_2x_3, \tag{S.3}$$

$$g_1(\boldsymbol{x}) = x_1x_3 - 7.735\frac{x_1^2}{x_2} - 180 \geq 0, \tag{S.4}$$

$$g_2(\boldsymbol{x}) = 4 - \frac{x_3}{x_2} \geq 0, \tag{S.5}$$

where $x_2 \in [0, 20]$ and $x_3 \in [0, 40]$. We determined the ranges of x_2 and x_3 according to initial solutions in [S.2]. x_1 has a pre-defined discrete value from 0.2 to 15[1]. The three variables (x_1, x_2, and x_3) represent the area of the reinforcement, the width of the beam, and the depth of the beam, respectively.

The second objective of RE2-3-2 is the sum of the two constraint violations:

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{2} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.6}$$

S.1.3. RE2-4-3: Pressure vessel design problem

The first objective of the RE2-4-3 problem is to minimize the total cost (including the cost of material and the cost of forming and welding) of a cylindrical pressure vessel [S.3]:

$$\begin{aligned} f_1(\boldsymbol{x}) =& 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 \\ &+ 3.1661x_1^2x_4 + 19.84x_1^2x_3, \end{aligned} \tag{S.7}$$

$$g_1(\boldsymbol{x}) = x_1 - 0.0193x_3 \geq 0, \tag{S.8}$$

$$g_2(\boldsymbol{x}) = x_2 - 0.00954x_3 \geq 0, \tag{S.9}$$

$$g_3(\boldsymbol{x}) = \pi x_3^2x_4 + \frac{4}{3}\pi x_3^3 - 1\,296\,000 \geq 0, \tag{S.10}$$

where $x_1, x_2 \in \{1, ..., 100\}$, $x_3 \in [10, 200]$, and $x_4 \in [10, 240]$. x_1 and x_2 are integer multiples of 0.0625. x_1, x_2, x_3, and x_4 represent the thicknesses of the shell, the head of a pressure vessel, the inner radius, and the length of the cylindrical section, respectively. We determined the ranges of x_2 and x_3 according to [S.3].

The second objective of RE2-4-3 is the sum of the three constraint violations:

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{3} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.11}$$

[1] For details, $x_1 \in \{0.2, 0.31, 0.4, 0.44, 0.6, 0.62, 0.79, 0.8, 0.88, 0.93, 1, 1.2, 1.24, 1.32, 1.4, 1.55, 1.58, 1.6, 1.76, 1.8, 1.86, 2, 2.17, 2.2, 2.37, 2.4, 2.48, 2.6, 2.64, 2.79, 2.8, 3, 3.08, 3, 1, 3.16, 3.41, 3.52, 3.6, 3.72, 3.95, 3.96, 4, 4.03, 4.2, 4.34, 4.4, 4.65, 4.74, 4.8, 4.84, 5, 5.28, 5.4, 5.53, 5.72, 6, 6.16, 6.32, 6.6, 7.11, 7.2, 7.8, 7.9, 8, 8.4, 8.69, 9, 9.48, 10.27, 11, 11.06, 11.85, 12, 13, 14, 15\}$.

S.1.4. RE2-2-4: Hatch cover design problem

The first objective of the RE2-2-4 problem is to minimize the weight of the hatch cover [S.2]:

$$f_1(\boldsymbol{x}) = x_1 + 120x_2, \tag{S.12}$$

$$g_1(\boldsymbol{x}) = 1.0 - \frac{\sigma_b}{\sigma_{b,\max}} \geq 0, \tag{S.13}$$

$$g_2(\boldsymbol{x}) = 1.0 - \frac{\tau}{\tau_{\max}} \geq 0, \tag{S.14}$$

$$g_3(\boldsymbol{x}) = 1.0 - \frac{\delta}{\delta_{\max}} \geq 0, \tag{S.15}$$

$$g_4(\boldsymbol{x}) = 1.0 - \frac{\sigma_b}{\sigma_k} \geq 0, \tag{S.16}$$

where $x_1 \in [0.5, 4]$ and $x_2 \in [4, 50]$. The ranges of the two variables were determined according to solutions reported in [S.2]. x_1 and x_2 represent the flange thickness and the beam height of the hatch cover, respectively. The parameters are given as follows: $\sigma_{b,\max} = 700\text{kg/cm}^2$, $\tau_{\max} = 450$ kg/cm, $\delta_{\max} = 1.5$cm, $\sigma_k = Ex_1^2/100$ kg/cm^2, $\sigma_b = 4\,500/(x_1 x_2)\text{kg/cm}^2$ $\tau = 1\,800/x_2\text{kg/cm}^2$, $\delta = 56.2 \times 10^4/(Ex_1x_2^2)$, and $E = 700\,000$ kg/cm^2.

The second objective of RE2-2-4 is the sum of the four constraint violations:

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{4} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.17}$$

S.1.5. RE2-3-5: Coil compression spring design problem

The first objective of the RE2-3-5 problem is to minimize the volume of spring steel wire which is used to manufacture the spring [S.4]:

$$
\begin{aligned}
f_1(\boldsymbol{x}) =& \frac{\pi^2 x_2 x_3^2 (x_1 + 2)}{4}, && \text{(S.18)}\\
g_1(\boldsymbol{x}) =& -\frac{8 C_f F_{\max} x_2}{\pi x_3^3} + S \geq 0, && \text{(S.19)}\\
g_2(\boldsymbol{x}) =& -l_f + l_{\max} \geq 0, && \text{(S.20)}\\
g_3(\boldsymbol{x}) =& -3 + \frac{x_2}{x_3} \geq 0, && \text{(S.21)}\\
g_4(\boldsymbol{x}) =& -\sigma_p + \sigma_{\mathrm{pm}} \geq 0, && \text{(S.22)}\\
g_5(\boldsymbol{x}) =& -\sigma_p - \frac{F_{\max} - F_p}{K} \\
& - 1.05(x_1 + 2)x_3 + l_f \geq 0, && \text{(S.23)}\\
g_6(\boldsymbol{x}) =& -\sigma_w + \frac{F_{\max} - F_p}{K} \geq 0, && \text{(S.24)}\\
C_f =& \frac{4(x_2/x_3) - 1}{4(x_2/x_3) - 4} + \frac{0.615 x_3}{x_2}, && \text{(S.25)}\\
K =& \frac{G x_3^4}{8 x_1 x_2^3}, && \text{(S.26)}\\
\sigma_p =& \frac{F_p}{K}, && \text{(S.27)}\\
l_F =& \frac{F_{\max}}{K} + 1.05(x_1 + 2)x_3, && \text{(S.28)}
\end{aligned}
$$

where $x_1 \in \{1, ..., 70\}$, $x_2 \in [0.6, 30]$, and x_3 has a predefined discrete value from 0.009 to 0.5.[2] x_1, x_2, and x_3 indicate the number of spring coils, the outside diameter of the spring, and the spring wire diameter, respectively. The parameters are given as follows: $F_{\max} = 1\,000$lb, $S = 189\,000$psi, $l_{\max} = 14$inch, $d_{\min} = 0.2$inch, $D_{\max} = 3$inch, $F_p = 300$lb, $\sigma_{pm} = 6$ inch, $\sigma_w = 1.25$inch, $G = 11.5 \times 10^6$.

The second objective of RE2-3-5 is the sum of the six constraint violations:

$$f_2(\boldsymbol{x}) = \sum_{i=1}^{6} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.29}$$

[2] For details, $x_3 \in \{0.009, 0.0095, 0.0104, 0.0118, 0.0128, 0.0132, 0.014, 0.015, 0.0162, 0.0173, 0.018, 0.02, 0.023, 0.025, 0.028, 0.032, 0.035, 0.041, 0.047, 0.054, 0.063, 0.072, 0.08, 0.092, 0.105, 0.12, 0.135, 0.148, 0.162, 0.177, 0.192, 0.207, 0.225, 0.244, 0.263, 0.283, 0.307, 0.331, 0.362, 0.394, 0.4375, 0.5\}$.

S.1.6. RE3-3-1: Two bar truss design problem

The first and second objectives of the RE3-3-1 problem are to minimize the structural weight (f_1) and resultant displacement of join (f_2) of the two bar truss [S.5]:

$$f_1(\boldsymbol{x}) = x_1\sqrt{16 + x_3^2} + x_2\sqrt{1 + x_3^2}, \tag{S.30}$$

$$f_2(\boldsymbol{x}) = \frac{20\sqrt{16 + x_3^2}}{x_3 x_1}, \tag{S.31}$$

$$g_1(\boldsymbol{x}) = 0.1 - f_1(\boldsymbol{x}) \geq 0, \tag{S.32}$$

$$g_2(\boldsymbol{x}) = 10^5 - f_2(\boldsymbol{x}) \geq 0, \tag{S.33}$$

$$g_3(\boldsymbol{x}) = 10^5 - \frac{80\sqrt{1 + x_3^2}}{x_3 x_2} \geq 0, \tag{S.34}$$

where $x_1, x_2 \in [10^{-5}, 100]^3$ and $x_3 \in [1, 3]$. x_1 and x_2 indicate the length of the two bars. x_3 represents the vertical distance from the second bar.

The third objective of RE3-3-1 is the sum of the three constraint violations:

$$f_3(\boldsymbol{x}) = \sum_{i=1}^{3} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.35}$$

[3] The original range of x_1 and x_2 is as follows: $x_1, x_2 > 0$. Thus, x_1 and x_2 can be any non-negative values. To define this problem as a bound constrained problem, we set the upper bound and the lower bound.

S.1.7. RE3-4-2: Welded beam design problem

The first and second objectives of the RE3-4-2 problem are to minimize cost (f_1) and end deflection (f_2) of a welded beam [S.6]:

$$f_1(\boldsymbol{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2), \tag{S.36}$$

$$f_2(\boldsymbol{x}) = \frac{4PL^3}{Ex_4x_3^3}, \tag{S.37}$$

$$g_1(\boldsymbol{x}) = \tau_{\max} - \tau(\boldsymbol{x}) \geq 0, \tag{S.38}$$

$$g_2(\boldsymbol{x}) = \sigma_{\max} - \sigma(\boldsymbol{x}) \geq 0, \tag{S.39}$$

$$g_3(\boldsymbol{x}) = x_4 - x_1 \geq 0, \tag{S.40}$$

$$g_4(\boldsymbol{x}) = P_C(\boldsymbol{x}) - P \geq 0, \tag{S.41}$$

$$\tau(\boldsymbol{x}) = \sqrt{(\tau')^2 + \frac{2\tau'\tau''x_2}{2R} + (\tau'')^2}, \tag{S.42}$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2}, \tag{S.43}$$

$$\tau'' = \frac{MR}{J}, \tag{S.44}$$

$$M = P\left(L + \frac{x_2}{2}\right), \tag{S.45}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, \tag{S.46}$$

$$J = 2\left(\sqrt{2}x_1x_2\left(\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right)\right), \tag{S.47}$$

$$\sigma(\boldsymbol{x}) = \frac{6PL}{x_4x_3^2}, \tag{S.48}$$

$$P_C(\boldsymbol{x}) = \frac{4.013E\sqrt{x_3^2x_4^6/36}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right), \tag{S.49}$$

where $x_1, x_4 \in [0.125, 5]$ and $x_2, x_3 \in [0.1, 10]$. The four variables adjust the size of the beam. The parameters are given as follows: $P = 6\,000$lb, $L = 14$in, $E = 30 \times 10^6$psi, $G = 12 \times 10^6$psi, $\tau_{\max} = 13\,600$psi, and $\sigma_{\max} = 30\,000$psi.

The third objective of RE3-4-2 is the sum of the four constraint violations:

$$f_3(\boldsymbol{x}) = \sum_{i=1}^{4} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.50}$$

S.1.8. RE3-4-3: Disc brake design problem

The first and second objectives of the RE3-4-3 problem are to minimize the mass of the brake (f_1) and the minimum stopping time (f_2) of a disc brake [S.6]:

$$f_1(\boldsymbol{x}) = 4.9 \times 10^{-5}(x_2^2 - x_1^2)(x_4 - 1), \tag{S.51}$$

$$f_2(\boldsymbol{x}) = 9.82 \times 10^6 \left(\frac{x_2^2 - x_1^2}{x_3 x_4 (x_2^3 - x_1^3)} \right), \tag{S.52}$$

$$g_1(\boldsymbol{x}) = (x_2 - x_1) - 20 \geq 0, \tag{S.53}$$

$$g_2(\boldsymbol{x}) = 0.4 - \frac{x_3}{3.14(x_2^2 - x_1^2)} \geq 0, \tag{S.54}$$

$$g_3(\boldsymbol{x}) = 1 - \frac{2.22 \times 10^{-3} x_3 (x_2^3 - x_1^3)}{(x_2^2 - x_1^2)^2} \geq 0, \tag{S.55}$$

$$g_4(\boldsymbol{x}) = \frac{2.66 \times 10^{-2} x_3 x_4 (x_2^3 - x_1^3)}{(x_2^2 - x_1^2)} - 900 \geq 0, \tag{S.56}$$

where $x_1 \in [55, 80]$, $x_2 \in [75, 110]$, $x_3 \in [1000, 3000]$, $x_4 \in [11, 20]$[4]. The four variables (x_1, x_2, x_2, and x_4) represent the inner radius of the discs, the outer radius of the discs, the engaging force, and the number of friction surfaces, respectively.

The third objective of RE3-4-3 is the sum of the four constraint violations:

$$f_3(\boldsymbol{x}) = \sum_{i=1}^{4} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.57}$$

S.1.9. RE3-5-4: The vehicle crashworthiness design problem

The first, second, and third objectives of the RE3-5-4 problem are to minimize the weight (f_1), acceleration characteristics (f_2), and toe-board instruction (f_3) of the vehicle design [S.7]:

$$\begin{aligned} f_1(\boldsymbol{x}) =& 1640.2823 + 2.3573285x_1 + 2.3220035x_2 \\ &+ 4.5688768x_3 + 7.7213633x_4 \\ &+ 4.4559504x_5, \end{aligned} \tag{S.58}$$

$$\begin{aligned} f_2(\boldsymbol{x}) =& 6.5856 + 1.15x_1 - 1.0427x_2 + 0.9738x_3 \\ &+ 0.8364x_4 - 0.3695x_1x_4 + 0.0861x_1x_5 \\ &+ 0.3628x_2x_4 - 0.1106x_1^2 - 0.3437x_3^2 \\ &+ 0.1764x_4^2, \end{aligned} \tag{S.59}$$

[4]The original range for x_4 is $[2, 20]$, but x_4 has a constraint such that $x_4 \geq 11$.

$$
\begin{aligned}
f_3(\boldsymbol{x}) =& -0.0551 + 0.0181x_1 + 0.1024x_2 \\
&+ 0.0421x_3 - 0.0073x_1x_2 + 0.024x_2x_3 \\
&- 0.0118x_2x_4 - 0.0204x_3x_4 - 0.008x_3x_5 \\
&- 0.0241x_2^2 + 0.0109x_4^2, \qquad \text{(S.60)}
\end{aligned}
$$

where $x_i \in [1, 3]$ for each $i \in \{1, ..., 5\}$. The five variables specify the thickness of five reinforced members around the frontal structure of the vehicle. The parameters in (S.58)–(S.60) were obtained by the response surface method using data sampled from a simulation of a full-scale vehicle model.

S.1.10. RE3-7-5: The speed reducer design problem

The first and second objectives of the RE3-7-5 problem are to minimize the volume (f_1) and the stress in one of the two gear shafts (f_2) of a speed reducer [S.8]:

$$
\begin{aligned}
f_1(\boldsymbol{x}) =& 0.7854x_1x_2^2\left(\frac{10x_3^2}{3} + 14.933x_3 - 43.0934\right) \\
&- 1.508x_1(x_6^2 + x_7^2) + 7.477(x_6^3 + x_7^3) \\
&+ 0.7854(x_4x_6^2 + x_5x_7^2), && \text{(S.61)} \\
f_2(\boldsymbol{x}) =& \frac{\sqrt{(745x_4/x_2x_3)^2 + 1.69 \times 10^7}}{0.1x_6^3}, && \text{(S.62)} \\
g_1(\boldsymbol{x}) =& \frac{1}{27} - \frac{1}{x_1x_2^2x_3} \geq 0, && \text{(S.63)} \\
g_2(\boldsymbol{x}) =& \frac{1}{397.5} - \frac{1}{x_1x_2^2x_3^2} \geq 0, && \text{(S.64)} \\
g_3(\boldsymbol{x}) =& \frac{1}{1.93} - \frac{x_4^3}{x_2x_3x_6^4} \geq 0, && \text{(S.65)} \\
g_4(\boldsymbol{x}) =& \frac{1}{1.93} - \frac{x_5^3}{x_2x_3x_7^4} \geq 0, && \text{(S.66)} \\
g_5(\boldsymbol{x}) =& 40 - x_2x_3 \geq 0, && \text{(S.67)} \\
g_6(\boldsymbol{x}) =& 12 - \frac{x_1}{x_2} \geq 0, && \text{(S.68)} \\
g_7(\boldsymbol{x}) =& -5 + \frac{x_1}{x_2} \geq 0, && \text{(S.69)} \\
g_8(\boldsymbol{x}) =& -1.9 + x_4 - 1.5x_6 \geq 0, && \text{(S.70)} \\
g_9(\boldsymbol{x}) =& -1.9 + x_5 - 1.1x_7 \geq 0, && \text{(S.71)} \\
g_{10}(\boldsymbol{x}) =& 1300 - f_2(\boldsymbol{x}) \geq 0, && \text{(S.72)} \\
g_{11}(\boldsymbol{x}) =& 1100 - \frac{\sqrt{(745x_5/x_2x_3)^2 + 1.575 \times 10^8}}{0.1x_7^3} \geq 0, && \text{(S.73)}
\end{aligned}
$$

where $x_1 \in [2.6, 3.6]$, $x_2 \in [0.7, 0.8]$, $x_3 \in \{17, ..., 28\}$, $x_4 \in [7.3, 8.3]$, $x_5 \in [7.3, 8.3]$, $x_6 \in [2.9, 3.9]$, and $x_7 \in [5, 5.5]$. x_3 is an integer variable. The seven variables represent the gear face width (x_1), the teeth module (x_2), the number of teeth of a pinion (x_3), the distance between bearings on the first shaft (x_4), the distance between bearings on the second shaft (x_5), the diameter of the first shaft (x_6), and the diameter of the second shaft (x_7).

The third objective of RE3-7-5 is the sum of the 11 constraint violations:

$$f_3(\boldsymbol{x}) = \sum_{i=1}^{11} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.74}$$

S.1.11. RE3-4-6: Gear train design problem

Th first and second objectives of this problem are to minimize the error between the realized gear ration and the given required gear ration (f_1) and the maximum size of the four gears (f_2) [S.9]:

$$f_1(\boldsymbol{x}) = \left| 6.931 - \frac{x_3}{x_1}\frac{x_4}{x_2} \right|, \tag{S.75}$$

$$f_2(\boldsymbol{x}) = \max\{x_1, x_2, x_3, x_4\}, \tag{S.76}$$

$$g_1(\boldsymbol{x}) = 0.5 - \frac{f_1(\boldsymbol{x})}{6.931} \geq 0, \tag{S.77}$$

where the four variables must be integer, and $x_1, x_2, x_3, x_4 \in \{12, ..., 60\}$. The four variables represent the number of teeth in each of the four gears.

The third objective of RE3-4-6 is the constraint violation of g_1:

$$f_3(\boldsymbol{x}) = \max\{g_1(\boldsymbol{x}), 0\}. \tag{S.78}$$

S.1.12. RE3-4-7: Rocket injector design problem

The original RE3-4-7 problem is a four-objective problem [S.10]. However, as in [S.11, 12], we handle RE3-4-7 as a three-objective problem. The first, second, and third objectives of the RE3-4-7 problem are to minimize the maximum temperature of the injector face (f_1), the distance from the inlet (f_2), and the maximum temperature on the post tip (f_3) [S.11]:

$$
\begin{aligned}
f_1(\boldsymbol{x}) =& 0.692 + (0.477x_1) - (0.687x_2) - (0.080x_3) - (0.0650x_4) - (0.167x_1x_1) \\
& - (0.0129x_2x_1) + (0.0796x_2x_2) - (0.0634x_3x_1) - (0.0257x_3x_2) \\
& + (0.0877x_3x_3) - (0.0521x_4x_1) + (0.00156x_4x_2) + (0.00198x_4x_3) \\
& + (0.0184x_4x_4) \qquad \text{(S.79)}
\end{aligned}
$$

$$
\begin{aligned}
f_2(\boldsymbol{x}) =& 0.153 - (0.322x_1) + (0.396x_2) + (0.424x_3) + (0.0226x_4) \\
& + (0.175x_1x_1) + (0.0185x_2x_1) - (0.0701x_2x_2) - (0.251x_3x_1) \\
& + (0.179x_3x_2) + (0.0150x_3x_3) + (0.0134x_4x_1) + (0.0296x_4x_2) \\
& + (0.0752x_4x_3) + (0.0192x_4x_4) \qquad \text{(S.80)}
\end{aligned}
$$

$$
\begin{aligned}
f_3(\boldsymbol{x}) =& 0.370 - (0.205x_1) + (0.0307x_2) + (0.108x_3) + (1.019x_4) \\
& - (0.135x_1x_1) + (0.0141x_2x_1) + (0.0998x_2x_2) + (0.208x_3x_1) \\
& - (0.0301x_3x_2) - (0.226x_3x_3) + (0.353x_4x_1) - (0.0497x_4x_3) \\
& - (0.423x_4x_4) + (0.202x_2x_1x_1) - (0.281x_3x_1x_1) - (0.342x_2x_2x_1) \\
& - (0.245x_2x_2x_3) + (0.281x_3x_3x_2) - (0.184x_4x_4x_1) - (0.281x_2x_1x_3) \\
& \qquad \text{(S.81)}
\end{aligned}
$$

where $x_i \in [0, 1]$ for each $i \in \{1, ..., 4\}$. x_1 is the hydrogen flow angle (α). x_2 is the hydrogen area (ΔHA). x_3 is the oxygen area (ΔOA). x_4 is the oxidizer post tip thickness (OPTT). The parameters in (S.79)–(S.81) were obtained by the response surface method using data obtained by a CFD-based simulation.

S.1.13. RE4-7-1: Car side impact design problem

The car side impact design problem, which was originally used in [S.13, 14], is the problem of minimizing the weight of the car (f_1), the pubic force experienced by a passenger (f_2), and the average velocity of the V-pillar responsible for withstanding the impact load (f_3) [S.15]. Unlike the original problem, four stochastic parameters are excluded in this version.

The first, second, and third objectives of the RE4-7-1 problem are given as follows:

$$\begin{aligned}
f_1(\boldsymbol{x}) =&1.98 + 4.9x_1 + 6.67x_2 + 6.98x_3 \\
&+ 4.01x_4 + 1.78x_5 + 10^{-5}x_6 \\
&+ 2.73x_7, && \text{(S.82)}\\
f_2(\boldsymbol{x}) =&4.72 - 0.5x_4 - 0.19x_2x_3, && \text{(S.83)}\\
f_3(\boldsymbol{x}) =&0.5(V_{\mathrm{MBP}}(\boldsymbol{x}) + V_{\mathrm{FD}}(\boldsymbol{x})), && \text{(S.84)}\\
g_1(\boldsymbol{x}) =&1 - 1.16 + 0.3717x_2x_4 \\
&+ 0.0092928x_3 \geq 0, && \text{(S.85)}\\
g_2(\boldsymbol{x}) =&0.32 - 0.261 + 0.0159x_1x_2 + 0.06486x_1 \\
&+ 0.019x_2x_7 - 0.0144x_3x_5 \\
&- 0.0154464x_6 \geq 0, && \text{(S.86)}\\
g_3(\boldsymbol{x}) =&0.32 - 0.214 - 0.00817x_5 + 0.045195x_1 \\
&+ 0.0135168x_1 - 0.03099x_2x_6 \\
&+ 0.018x_2x_7 - 0.007176x_3 \\
&- 0.023232x_3 + 0.00364x_5x_6 \\
&+ 0.018x_2^2 \geq 0, && \text{(S.87)}\\
g_4(\boldsymbol{x}) =&0.32 - 0.74 + 0.61x_2 + 0.031296x_3 \\
&+ 0.031872x_7 - 0.227x_2^2 \geq 0, && \text{(S.88)}\\
g_5(\boldsymbol{x}) =&32 - 28.98 - 3.818x_3 + 4.2x_1x_2 \\
&- 1.27296x_6 + 2.68065x_7 \geq 0, && \text{(S.89)}
\end{aligned}$$

$$\begin{aligned}
g_6(\boldsymbol{x}) =&32 - 33.86 - 2.95x_3 + 5.057x_1x_2 \\
&+ 3.795x_2 + 3.4431x_7 - 1.45728 \geq 0, && \text{(S.90)}\\
g_7(\boldsymbol{x}) =&32 - 46.36 + 9.9x_2 + 4.4505x_1 \geq 0, && \text{(S.91)}\\
g_8(\boldsymbol{x}) =&4 - f_2(\boldsymbol{x}) \geq 0, && \text{(S.92)}\\
g_9(\boldsymbol{x}) =&9.9 - V_{MBP}(\boldsymbol{x}) \geq 0, && \text{(S.93)}\\
g_{10}(\boldsymbol{x}) =&15.7 - V_{FD}(\boldsymbol{x}) \geq 0, && \text{(S.94)}\\
V_{\mathrm{MBP}}(\boldsymbol{x}) =&10.58 - 0.674x_1x_2 - 0.67275x_2, && \text{(S.95)}\\
V_{\mathrm{FD}}(\boldsymbol{x}) =&16.45 - 0.489x_3x_7 - 0.843x_5x_6, && \text{(S.96)}
\end{aligned}$$

where $x_1 \in [0.5, 1.5]$, $x_2 \in [0.45, 1.35]$, $x_3 \in [0.5, 1.5]$, $x_4 \in [0.5, 1.5]$, $x_5 \in [0.875, 2.625]$, $x_6 \in [0.4, 1.2]$, and $x_7 \in [0.4, 1.2]$. The parameters in (S.82)–(S.96) were obtained by the response surface method similar to the RE3-5-4 problem.

The fourth objective of RE4-7-1 is the sum of the 10 constraint violations:

$$f_4(\boldsymbol{x}) = \sum_{i=1}^{10} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.97}$$

S.1.14. RE4-6-2: Conceptual marine design problem

Th first, second, and third objectives of the RE4-6-2 problem are to minimize the transportation cost (f_1), the light ship weight (f_2), and the annual cargo transport capacity (f_3) [S.16]:

$$f_1(\boldsymbol{x}) = \frac{\text{annual\_costs}}{\text{annual\_cargo}}, \tag{S.98}$$

$$\begin{aligned} f_2(\boldsymbol{x}) =& \text{light\_ship\_weight} \\ =& W_s + W_o + W_m, \end{aligned} \tag{S.99}$$

$$\begin{aligned} f_3(\boldsymbol{x}) =& \text{annual\_cargo} \\ =& -\text{cargo\_DWT} \times \text{RTPA}, \end{aligned} \tag{S.100}$$

$$g_1(\boldsymbol{x}) = \frac{L}{B} - 6 \geq 0, \tag{S.101}$$

$$g_2(\boldsymbol{x}) = 15 - \frac{L}{D} \geq 0, \tag{S.102}$$

$$g_3(\boldsymbol{x}) = 19 - \frac{L}{T} \geq 0, \tag{S.103}$$

$$g_4(\boldsymbol{x}) = 0.45\text{DWT}^{0.31} - T \geq 0, \tag{S.104}$$

$$g_5(\boldsymbol{x}) = 0.7D + 0.7 - T \geq 0, \tag{S.105}$$

$$g_6(\boldsymbol{x}) = \text{DWT} - 3\,000 \geq 0, \tag{S.106}$$

$$g_7(\boldsymbol{x}) = 500\,000 - \text{DWT} \geq 0, \tag{S.107}$$

$$g_8(\boldsymbol{x}) = 0.32 - F_n \geq 0, \tag{S.108}$$

$$\begin{aligned} g_9(\boldsymbol{x}) =& KB + BM_T - KG \\ & - 0.07B \geq 0, \end{aligned} \tag{S.109}$$

$$\begin{aligned} \text{annual\_costs} =& \text{capital\_costs} \\ & + \text{running\_costs} \\ & + \text{voyage\_costs}, \end{aligned} \tag{S.110}$$

$$\text{capital\_costs} = 0.2\text{ship\_cost}, \tag{S.111}$$

$$\begin{aligned} \text{ship\_cost} =& 1.3(2\,000W_s^{0.85} + 3\,500W_o \\ & + 2\,400P^{0.8}), \end{aligned} \tag{S.112}$$

$$
\begin{aligned}
\text{steel\_weight} =& W_s \\
=& 0.034 L^{1.7} B^{0.7} D^{0.4} C_B^{0.5}, && \text{(S.113)} \\
\text{outfit\_weight} =& W_o \\
=& 1.0 L^{0.8} B^{0.6} D^{0.3} C_B^{0.1}, && \text{(S.114)} \\
\text{machinery\_weight} =& W_m = 0.17 P^{0.9}, && \text{(S.115)} \\
\text{displacement} =& 1.025 LBTC_B, && \text{(S.116)} \\
\text{power} =& P \\
=& \frac{\text{displacement}^{2/3} V_k^3}{a + bF_n}, && \text{(S.117)} \\
\text{Froudenumber} =& F_n = \frac{V}{(gL)^{0.5}}, && \text{(S.118)} \\
V =& 0.5144 V_k, && \text{(S.119)} \\
g =& 9.8065, && \text{(S.120)} \\
\text{AC} =& a + bF_n, && \text{(S.121)} \\
a =& 4\,977.06 C_B^2 \\
& - 8\,105.61 C_B \\
& + 4\,456.51, && \text{(S.122)} \\
b =& - 10\,847.2 C_B^2 \\
& + 12\,817 C_B \\
& - 6\,960.32, && \text{(S.123)} \\
\text{running\_costs} =& 40\,000 \text{DWT}^{0.3}, && \text{(S.124)} \\
\text{deadweight} =& \text{DWT} = \text{displacement} \\
& - \text{light\_ship}, && \text{(S.125)} \\
\text{voyage\_costs} =& (\text{fuel\_cost} \\
& + \text{port\_cost}) \text{RTPA}, && \text{(S.126)} \\
\text{fuel\_cost} =& 1.05\ \text{daily\_consumption} \\
& \times \text{sea\_days} \times \text{fuel\_price}, && \text{(S.127)} \\
\text{daily\_consumption} =& \frac{0.19 P \times 24}{1\,000} + 0.2, && \text{(S.128)} \\
\text{sea\_days} =& \frac{\text{round\_trip\_miles}}{24} V_k, && \text{(S.129)} \\
\text{round\_trip\_miles} =& 5\,000, && \text{(S.130)} \\
\text{fuel\_price} =& 100, && \text{(S.131)} \\
\text{port\_cost} =& 6.3 \text{DWT}^{0.8}, && \text{(S.132)} \\
\text{round\_trips\_per\_year} =& \text{RTPA} \\
=& \frac{350}{\text{sea\_days} + \text{port\_days}}, && \text{(S.133)}
\end{aligned}
$$

$$\begin{aligned}
\text{port_days} =& 2\left(\frac{\text{cargo_DWT}}{\text{handling_rate}} + 0.5\right), && \text{(S.134)}\\
\text{cargo_deadweight} =& \text{cargo_DWT}\\
=& \text{DWT} - \text{fuel_carried}\\
& - \text{miscellaneous_DWT}, && \text{(S.135)}\\
\text{fuel_carried} =& \text{daily_consumption}\\
& \times (\text{sea_days} + 5), && \text{(S.136)}\\
\text{miscellaneous_DWT} =& 2\text{DWT}^{0.5}, && \text{(S.137)}
\end{aligned}$$

$$\begin{aligned}
\text{handling_rate} =& 8\,000, && \text{(S.138)}\\
\text{vertical_center_of_buoyancy} =& KB = 0.53T, && \text{(S.139)}\\
\text{metacentric_radius} =& BM_T\\
=& \frac{(0.085C_B - 0.002)B^2}{TC_B}, && \text{(S.140)}\\
\text{vertical_center_of_gravity} =& K\text{G} = 1 + 0.52D, && \text{(S.141)}
\end{aligned}$$

where L, B, D, T, V_k, and C_B correspond to six decision variables (x_1, x_2, x_3, x_4, x_5, and x_6), respectively. The range of the six variables are as follows: $L \in [150, 274.32]$, $B \in [20, 32.31]$, $D \in [13, 25]$, $T \in [10, 11.71]$, $V_k \in [14, 18]$, and $C_B \in [0.63, 0.75]$. We selected the range of the six variables according to [S.17]. L, B, D, T, V_k, and C_B indicate the values of length, beam, depth, draft, speed, and block coefficients, respectively. Although f_3 is to be maximized in the original definition, it is modified for a minimization problem.

The fourth objective of RE4-6-2 is the sum of the nine constraint violations:

$$f_4(\boldsymbol{x}) = \sum_{i=1}^{9} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.142}$$

S.1.15. RE6-3-1: Water resource planning problem

The five objectives of the RE6-3-1 problem are to minimize the drainage network cost (f_1), the storage facility cost (f_2), the treatment facility cost (f_3), the expected flood damage cost (f_4), and the expected economic loss due to flood (f_5) of a water resource planning [S.18]:

$$f_1(\boldsymbol{x}) = 106780.37(x_2 + x_3) + 61704.67, \tag{S.143}$$
$$f_2(\boldsymbol{x}) = 3000x_1, \tag{S.144}$$
$$f_3(\boldsymbol{x}) = \frac{305700 \times 2289x_2}{(0.06 \times 2289)^{0.65}}, \tag{S.145}$$
$$f_4(\boldsymbol{x}) = 250 \times 2289 \exp(-39.75x_2 + 9.9x_3 + 2.74), \tag{S.146}$$
$$f_5(\boldsymbol{x}) = 25\left(\frac{1.39}{x_1x_2} + 4940x_3 - 80\right), \tag{S.147}$$
$$g_1(\boldsymbol{x}) = 1 - \frac{0.00139}{x_1x_2} - 4.94x_3 + 0.08 \geq 0, \tag{S.148}$$
$$g_2(\boldsymbol{x}) = 1 - \frac{0.000306}{x_1x_2} - 1.082x_3 + 0.0986 \geq 0, \tag{S.149}$$
$$g_3(\boldsymbol{x}) = 50000 - \frac{12.307}{x_1x_2} - 49408.24x_3 - 4051.02 \geq 0, \tag{S.150}$$
$$g_4(\boldsymbol{x}) = 16000 - \frac{2.098}{x_1x_2} - 8046.33x_3 + 696.71 \geq 0, \tag{S.151}$$
$$g_5(\boldsymbol{x}) = 10000 - \frac{2.138}{x_1x_2} - 7883.39x_3 + 705.04 \geq 0, \tag{S.152}$$

$$g_6(\boldsymbol{x}) = 2000 - 0.417(x_1x_2) - 1721.26x_3 + 136.54 \geq 0, \tag{S.153}$$
$$g_7(\boldsymbol{x}) = 550 - \frac{0.164}{x_1x_2} - 631.13x_3 + 54.48 \geq 0, \tag{S.154}$$

where $x_1 \in [0.01, 0.45]$, $x_2 \in [0.01, 0.1]$, and $x_3 \in [0.01, 0.1]$. x_1, x_2, and x_3 represent the local detention storage capacity, the maximum treatment rate, and the maximum allowable overflow rate, respectively.

The sixth objective of RE6-3-1 is the sum of the seven constraint violations:

$$f_6(\boldsymbol{x}) = \sum_{i=1}^{7} \max\{g_i(\boldsymbol{x}), 0\}. \tag{S.155}$$

S.1.16. RE9-7-1: The car cab design

This is an alternative version of the RE4-7-1 problem [S.19]. The RE9-7-1 problem is closer than RE4-7-1 to the original problem [S.13]. The first objective of RE9-7-1 is to minimize the weight of the car (f_1) as follows (in the following formulations, three constraint functions are combined in a similar manner to [S.20]):

$$\begin{aligned} f_1(\boldsymbol{x}) =& 1.98 + 4.9x_1 + 6.67x_2 + 6.98x_3 \\ & + 4.01x_4 + 1.78x_5 + 0.00001x_6 \\ & + 2.73x_7, \end{aligned} \tag{S.156}$$

$$\begin{aligned} g_1(\boldsymbol{x}) =& 1 - (1.16 - 0.3717x_2x_4 \\ & - 0.00931x_2x_{10} - 0.484x_3x_9 \\ & + 0.01343x_6x_{10}) \geq 0, \end{aligned} \tag{S.157}$$

$$\begin{aligned} g_2(\boldsymbol{x}) =& 0.32 - (0.261 - 0.0159x_1x_2 \\ & - 0.188x_1x_8 - 0.019x_2x_7 \\ & + 0.0144x_3x_5 + 0.87570001x_5x_{10} \\ & + 0.08045x_6x_9 + 0.00139x_8x_{11} \\ & + 0.00001575x_{10}x_{11}) \geq 0, \end{aligned} \tag{S.158}$$

$$\begin{aligned} g_3(\boldsymbol{x}) =& 0.32 - (0.214 + 0.00817x_5 \\ & - 0.131x_1x_8 - 0.0704x_1x_9 \\ & + 0.03099x_2x_6 - 0.018x_2x_7 \\ & + 0.0208x_3x_8 + 0.121x_3x_9 \\ & - 0.00364x_5x_6 + 0.0007715x_5x_{10} \\ & - 0.0005354x_6x_{10} + 0.00121x_8x_{11} \\ & + 0.00184x_9x_{10} - 0.018x_2x_2) \geq 0, \end{aligned} \tag{S.159}$$

$$\begin{aligned} g_4(\boldsymbol{x}) =& 0.32 - (0.74 - 0.61x_2 - 0.163x_3x_8 \\ & + 0.001232x_3x_{10} - 0.166x_7x_9 \\ & + 0.227x_2x_2) \geq 0, \end{aligned} \tag{S.160}$$

$$g_5(\boldsymbol{x}) = 32 - \left(\frac{\mathrm{URD} \times \mathrm{MRD} \times \mathrm{LRD}}{3} \right) \geq 0, \tag{S.161}$$

$$\begin{aligned} g_6(\boldsymbol{x}) =& 32 - (4.72 - 0.5x_4 - 0.19x_2x_3 \\ & - 0.0122x_4x_{10} + 0.009325x_6x_{10} \\ & + 0.000191x_{11}x_{11}) \geq 0, \end{aligned} \tag{S.162}$$

$$\begin{aligned} g_7(\boldsymbol{x}) =& 4 - (10.58 - 0.674x_1x_2 - 1.95x_2x_8 \\ & + 0.02054x_3x_{10} - 0.0198x_4x_{10} \\ & + 0.028x_6x_{10}) \geq 0, \end{aligned} \tag{S.163}$$

$$\begin{aligned} g_8(\boldsymbol{x}) =& 9.9 - (16.45 - 0.489x_3x_7 - 0.843x_5x_6 \\ & + 0.0432x_9x_{10} - 0.0556x_9x_{11} \\ & - 0.000786x_{11}x_{11}) \geq 0, \end{aligned} \tag{S.164}$$

$$
\begin{aligned}
\text{upper\_rib\_deflection} =& \text{URD} \\
=& 28.98 + 3.818x_3 - 4.2x_1x_2 \\
& + 0.0207x_5x_{10} + 6.63x_6x_9 \\
& - 7.77x_7x_8 + 0.32x_9x_{10}, && \text{(S.165)} \\
\text{middle\_rib\_deflection} =& \text{MRD} \\
=& 33.86 + 2.95x_3 \\
& + 0.1792x_{10} - 5.057x_1x_2 \\
& - 11x_2x_8 - 0.0215x_5x_{10} \\
& - 9.98x_7x_8 + 22x_8x_9, && \text{(S.166)} \\
\text{lower\_rib\_deflection} =& \text{LRD} \\
=& 46.36 - 9.9x_2 - 12.9x_1x_8 \\
& + 0.1107x_3x_{10}, && \text{(S.167)}
\end{aligned}
$$

where $x_1 \in [0.5, 1.5]$, $x_2 \in [0.45, 1.35]$, $x_3 \in [0.5, 1.5]$, $x_4 \in [0.5, 1.5]$, $x_5 \in [0.875, 2.625]$, $x_6 \in [0.4, 1.2]$, $x_7 \in [0.4, 1.2]$. The seven variables indicate the thickness of the following parts: B-Pillar inner (x_1), B-Pillar reinforcement (x_2), floor side inner (x_3), cross members (x_4), door beam (x_5), the door beltline reinforcement (x_6), and roof rail (x_7). The remaining four variables (x_8, x_9, x_{10}, and x_{11}) are stochastic parameters, which represent the material of B-Pillar inner (x_8), the material of floor side inner (x_9), the barrier height (x_{10}), and the barrier hitting position (x_{11}).

Each of the other eight objectives (f_2, ..., f_9) of RE9-7-1 corresponds to each of the eight constraint violations:

$$
\begin{aligned}
f_2(\boldsymbol{x}) &= \max\{g_2(\boldsymbol{x}), 0\}, && \text{(S.168)} \\
f_3(\boldsymbol{x}) &= \max\{g_3(\boldsymbol{x}), 0\}, && \text{(S.169)} \\
f_4(\boldsymbol{x}) &= \max\{g_4(\boldsymbol{x}), 0\}, && \text{(S.170)} \\
f_5(\boldsymbol{x}) &= \max\{g_5(\boldsymbol{x}), 0\}, && \text{(S.171)} \\
f_6(\boldsymbol{x}) &= \max\{g_6(\boldsymbol{x}), 0\}, && \text{(S.172)} \\
f_7(\boldsymbol{x}) &= \max\{g_7(\boldsymbol{x}), 0\}, && \text{(S.173)} \\
f_8(\boldsymbol{x}) &= \max\{g_8(\boldsymbol{x}), 0\}, && \text{(S.174)} \\
f_9(\boldsymbol{x}) &= \max\{g_9(\boldsymbol{x}), 0\}. && \text{(S.175)}
\end{aligned}
$$

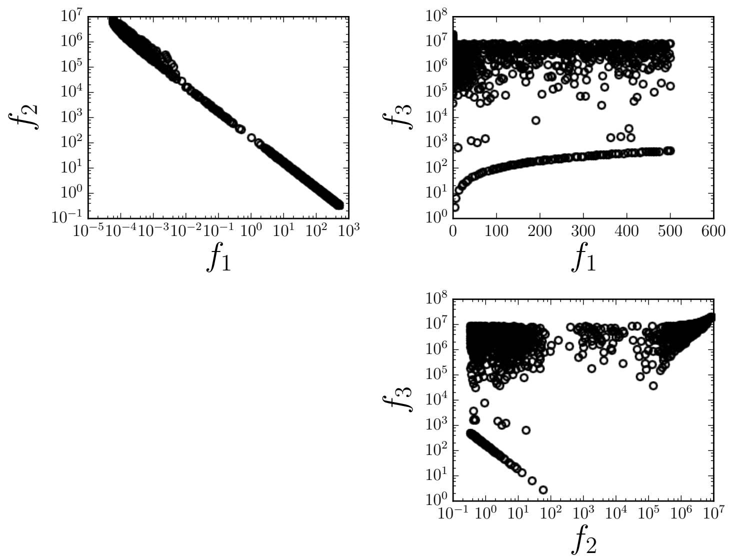

Figure S.1: Scatter matrix of approximated Pareto fronts of the RE3-3-1 problem.

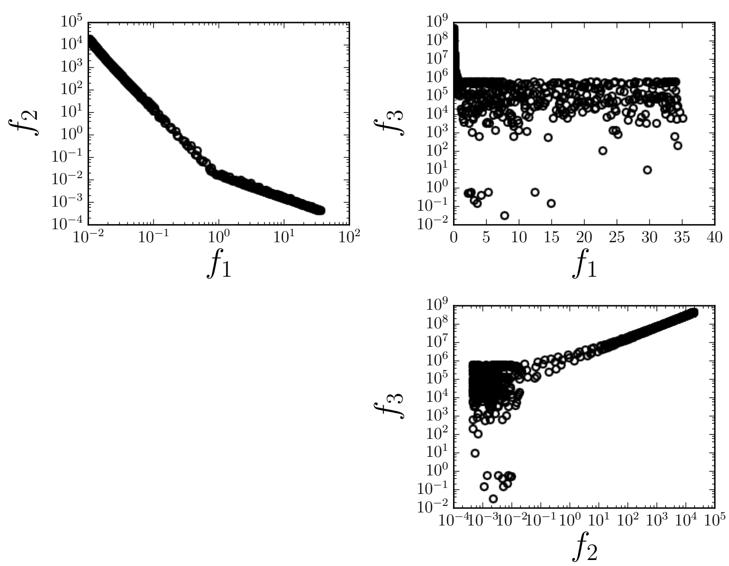

Figure S.2: Scatter matrix of approximated Pareto fronts of the RE3-4-2 problem.

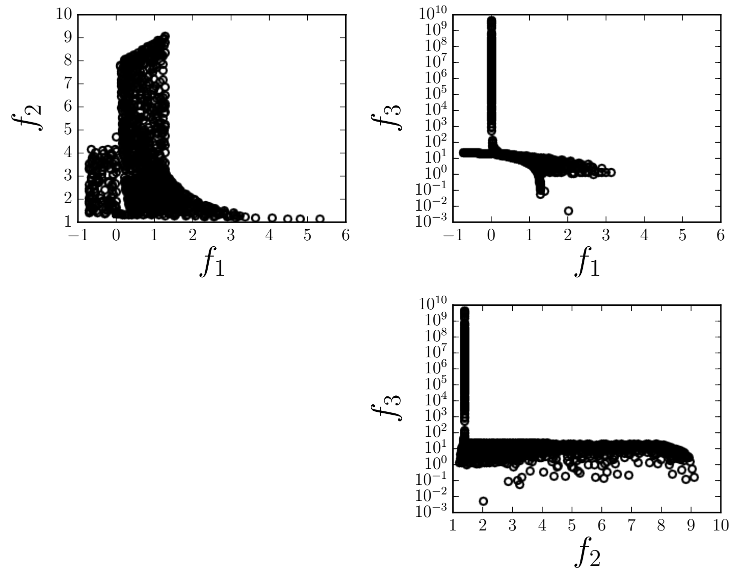

Figure S.3: Scatter matrix of approximated Pareto fronts of the RE3-4-3 problem.

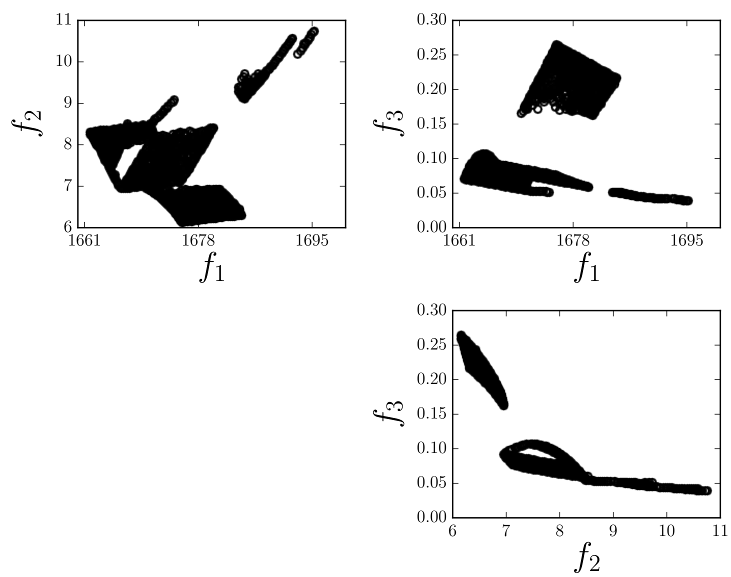

Figure S.4: Scatter matrix of approximated Pareto fronts of the RE3-5-4 problem.

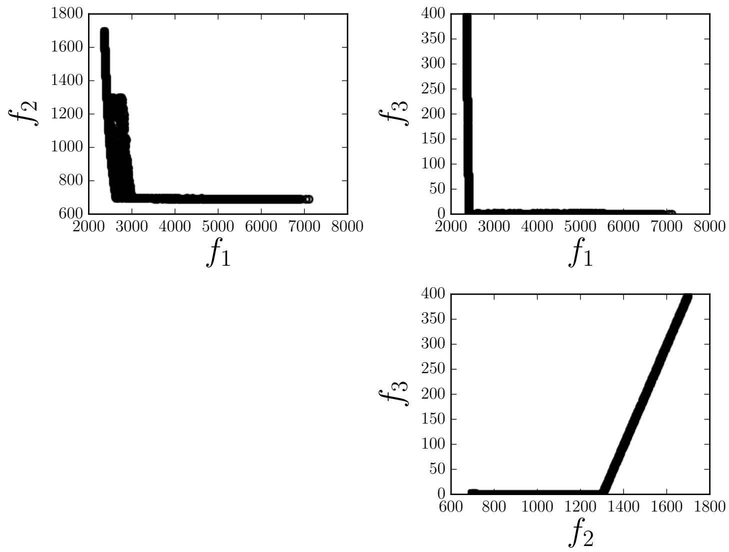

Figure S.5: Scatter matrix of approximated Pareto fronts of the RE3-7-5 problem.

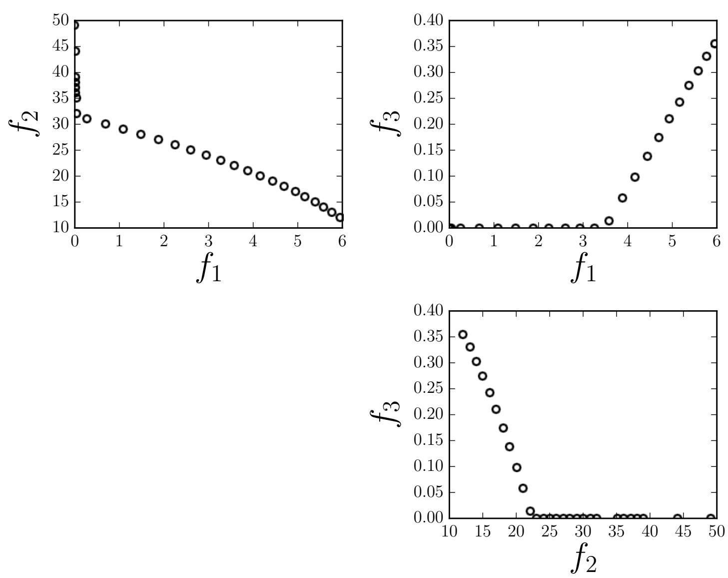

Figure S.6: Scatter matrix of approximated Pareto fronts of the RE3-4-6 problem.

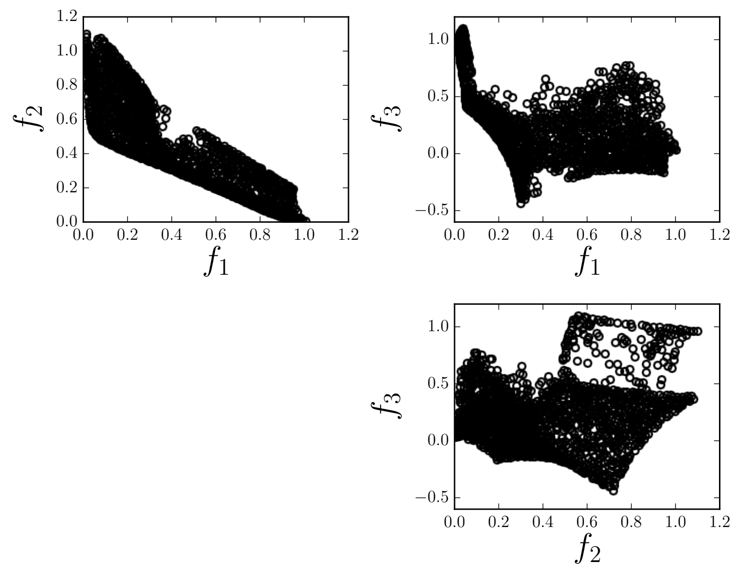

Figure S.7: Scatter matrix of approximated Pareto fronts of the RE3-4-7 problem.

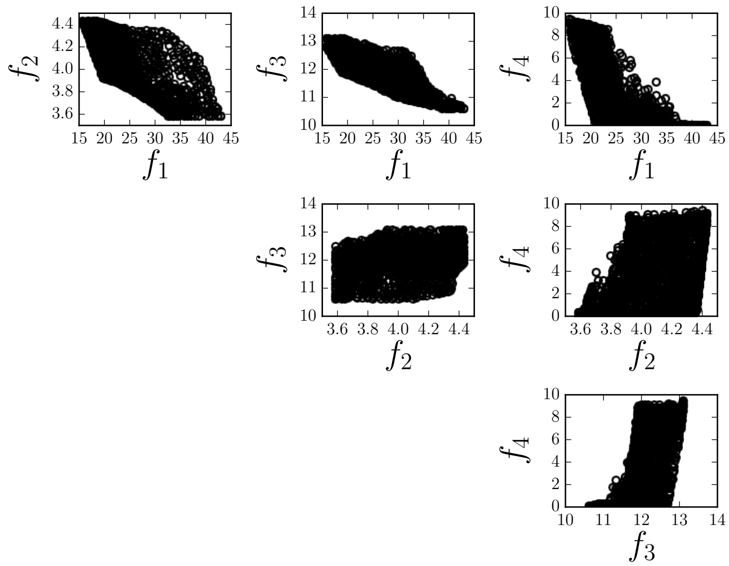

Figure S.8: Scatter matrix of approximated Pareto fronts of the RE4-7-1 problem.

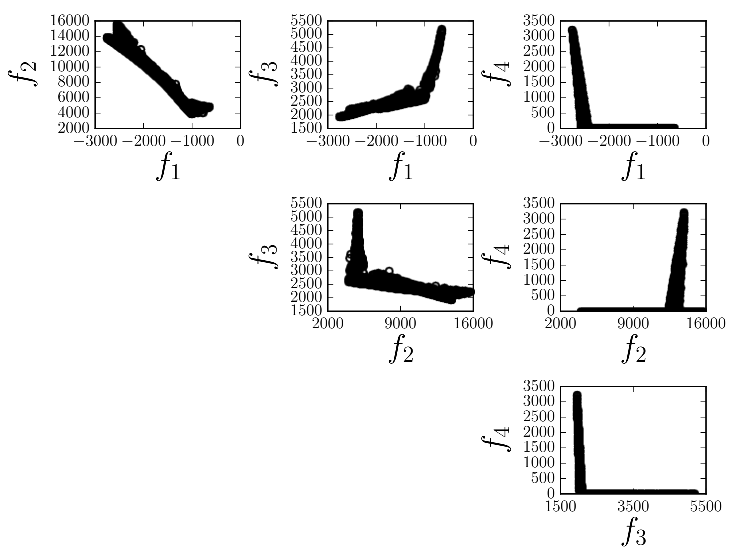

Figure S.9: Scatter matrix of approximated Pareto fronts of the RE4-6-2 problem.

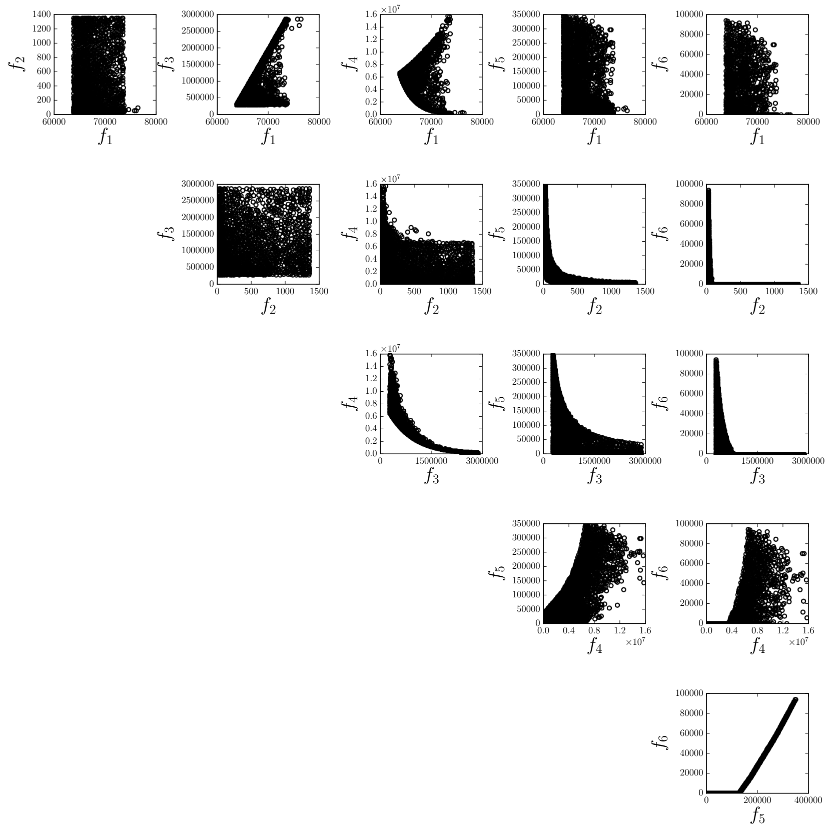

Figure S.10: Scatter matrix of approximated Pareto fronts of the RE6-3-1 problem.

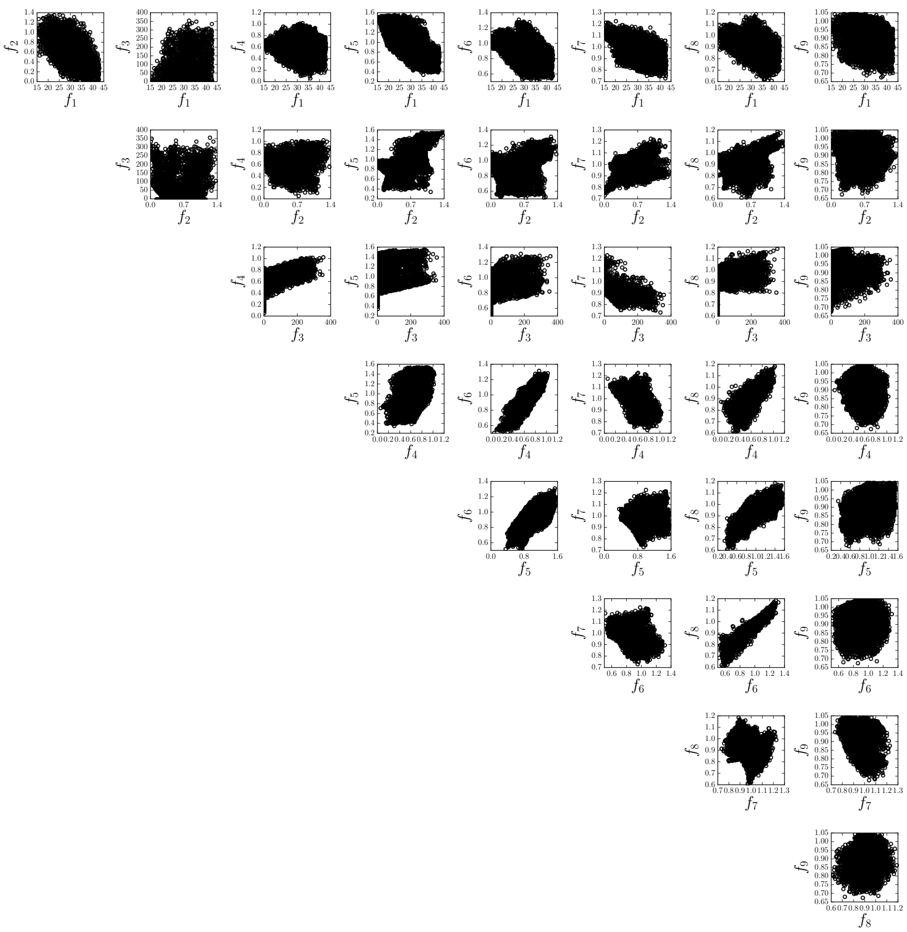

Figure S.11: Scatter matrix of approximated Pareto fronts of the RE9-7-1 problem.

Algorithm S.1: The distance-based selection method

```
   input  : A set of μ non-dominated solutions A
   output : A set of n = 500 × M non-dominated solutions A^approx
1  A^approx ← ∅;
2  for k ∈ {1, ..., μ} do
3      V_k ← false, D_k ← ∞;
4  for i ∈ {1, ..., M} do
5      j ← argmin_{k∈{1,...,μ} | V_k=false} f_i(x_k);
6      A^approx ← A^approx ∪ {x_j}, V_j ← true;
7      for k ∈ {1, ..., μ} do
8          if V_k = false then
9              D_k ← min(distance(x_k, x_j), D_k);
10 while |A^approx| < n do
11     j ← argmax_{k∈{1,...,μ} | V_k=false} D_k;
12     A^approx ← A^approx ∪ {x_j}, V_j ← true;
13     for k ∈ {1, ..., μ} do
14         if V_k = false then
15             D_k ← min(distance(x_k, x_j), D_k);
```

S.2. The distance-based selection method

We used the distance-based selection method [S.21] to select well-distributed solutions in the objective space from a large number of non-dominated solutions $\boldsymbol{A}$ found by various optimizers (see Subsection 3.1). It should be noted that the selection method uses the Euclidean distance in the objective space, not the solution space.

Algorithm S.1 shows the selection method presented in [S.21]. The function distance$(\boldsymbol{x}_1, \boldsymbol{x}_2)$ in Algorithm S.1 returns the Euclidean distance between two objective vectors $\boldsymbol{f}(\boldsymbol{x}_1)$ and $\boldsymbol{f}(\boldsymbol{x}_2)$. For each $i \in \{1, ..., M\}$, the i-th extreme solution that has the minimum objective value $f_i(\boldsymbol{x})$ is selected (lines 4–6). The selected M solutions are added to $\boldsymbol{A}^{\mathrm{approx}}$. Then, a solution that has the maximum distance to solutions in $\boldsymbol{A}^{\mathrm{approx}}$ in the objective space is selected (lines 10–12). This procedure is repeatedly performed until $|\boldsymbol{A}^{\mathrm{approx}}| = n(= 500 \times M)$.

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