Supplementary Material of Gaining-Sharing Knowledge Based Algorithm Assisted Differential Evolution

1. Algorithms

1.1. AMCDE

The pseudo-Code of AMCDE is given in Algorithm S1. In Algorithm S1, Algorithm S2 is called. The algorithms are followed by more details.

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r3,g}), \tag{1}$$

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r2,g}), \tag{2}$$

$$\vec{v}_{i,g} = F_i \cdot \vec{x}_{r1,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{r2,g}). \tag{3}$$

In the three equations, $r1 \in \{1, 2, ..., NP\}$, $r2 \in \{1, 2, ..., NP\}$, $r3 \in \{1, 2, ..., NP + |A|\}$, and $r1 \neq r2 \neq r3 \neq i$. In fact, A is used to store individuals eliminated from the population. $|A|_{min} = 0$, while $|A|_{max} = NP$. Moreover, $\vec{x}_{\phi,g}$ is among the best ϕ individuals in the gth generation. Here, F_i is F for the ith position of the population.

$$u_{j,i,g} = \begin{cases} \begin{cases} v_{j,i,g}, & if\ rand(0,1) \leq CR\ or\ j = randn(i), \\ x_{j,i,g}, & otherwise, \end{cases} & if\ rand(0.1) \leq p \\ \begin{cases} v_{j,i,g}, & for\ j = \langle l \rangle_D, \langle l+1 \rangle_D, ..., \langle l+L-1 \rangle_D \\ x_{j,i,g}, & for\ all\ other\ j \in [1, D], \end{cases} & otherwise, \end{cases} \tag{4}$$

where p is the probability of the binomial manner. That is, the exponential manner has the probability $1 - p$. Moreover, $l \in \{1, 2, ..., D\}$ is randomly decided during execution, while $L \in \{1, 2, ..., D\}$, as a parameter, requires to be set before execution.

Let $\overline{\Delta f}_k$ be the average improvement from target vector to trial vector among all the positions in the population controlled by the kth mutation strategy. The controlling ratio of the kth mutation

Algorithm 1 Pseudo-Code of AMCDE

Input: NP_{init}, NP_{min}, $MaxFES$, G_n, p_{bc1}, p_{bc2}, p_w, and p_r;

Parameter: g_n, g, FES;

Output: S

Obtain the initial generation of the population P_0
$FES = NP_{init}$, $g_n = 0$, $g = 0$
Evaluate individuals in the population
$FES = FES + NP_{init}$ and $g = g + 1$
Choose DE/current-to-ϕbest/1 with archive shown in Equation S1 as the monopolizer
while $FES < MaxFES$ **do**
  if $g_n < G_n$ **then**
    Execute the chosen mutation strategy
    Execute the binomial or exponential crossover in Equation S4 where $p = p_{bc1}$
    Evaluate offspring
    Execute Equation 2 for selection
    if the best fitness is not improved **then**
      $g_n = g_n + 1$
    else
      $g_n = 0$
    end if
  else
    Execute Algorithm S2 with parameters, (P_g, p_{bc2}, p_w, p_r), for a generation
    if the best fitness is improved **then**
      $g_n = 0$ and $G_n = G_n + 1$
      Choose the mutation strategy obtaining the new best fitness among Equations S1-S3 as the new monopolizer
    end if
  end if
  $FES = FES + NP$ and $g = g + 1$
  if $FES \geq 0.85 \cdot MaxFES$ **then**
    Apply the local search method based on sequential quadratic programming
    Update FES
  end if
  Decrease NP according to the linear population size reduction based on NP_{init} and NP_{min}
end while
Report the final solution S

Algorithm 2 Pseudo-Code of a generation with competition

Input: P_g, p_{bc2}, p_w, p_r and Cr;

Parameter: $S_{k,g}(k = 1, 2, 3)$, and $S_{win,g}$

Output: P_{g+1}

Equal number of positions are randomly allocated to the three mutation strategies and grouped into $S_{k,g}$
c_k is computed according to Equation S5
$win = 1$
for $x = 1$ to 3 **do**
  if $c_x > c_{win}$ **then**
    $win = x$
  end if
end for
if g is not the first generation in the current round of competition **then**
  for $x = 1$ to 3 **do**
    if $x \neq win$ **then**
      $S_{x,g} = S_{x,g} \backslash S_{win,g-1}$
    else
      $S_{x,g} = S_{x,g} \cup S_{win,g-1}$
    end if
  end for
end if
Execute mutation strategy of each position
Execute the binomial or exponential crossover in Equation S4 where $p = p_{bc2}$ and Cr is set based on Equation S6
Evaluate offspring
Execute the revised selection method where, at p_r, each of the worst p_w% target vectors are replaced by its worse trial vector
Compute c_k according to Equation S5
Find $win \in \{1, 2, 3\}$ to make $c_{win} = max(c_1, c_2, c_3)$

strategy c_k is computed as below,

$$c_k = \frac{\overline{\Delta f}_k}{\sum_{i=1}^{3} \overline{\Delta f}_i}, \tag{5}$$

where $k = \{1, 2, 3\}$. Then, c_k is adjusted to ensure $c_k \geq 0.1$ on the constant premise of $\sum_{i=1}^{3} c_k = 1$. In detail, if $c_m < 0.1$ ($m \in 1, 2, 3$), c_m is turned to 0.1 by reducing c_{max}.

$$Cr = \begin{cases} 0, & if\ FES < MaxFES \cdot 0.5 \\ (\frac{FES}{MaxFES} - 0.5) \cdot 2, & otherwise. \end{cases} \tag{6}$$

1.2. APGSK

The pseudo-Code of APGSK is given in Algorithm S3. The algorithm is followed by more details.

The process of adapting the control parameters begins by choosing a pool for the two parameters and probability parameter kw_p. The pool used for setting the parameters consists of the following two pairs (K_f, K_r): [(0.1, 0.2), (1.0, 0.1), (0.5, 0.9), and (1.0, 0.9)] which is applied during first 50% of $MaxFES$ while the another pairs: [(-0.15, 0.2), (-0.05, 0.1), (-0.05, 0.9), and (-0.15, 0.9)] will be activated after 50% of $MaxFES$ with probability less than 0.3 for enhancing the diversity of the population to ensure escaping from local optima and to reduce possibility of stagnation. The probability parameter kw_p includes a probability parameter p for each setting of the above-mentioned pool of settings. Therefore, every individual in the population will be assigned only one setting according to its probability parameter p.

The probability parameter adaptation kw_p will start after 10% of the function evaluations. The adaptation of the probability parameter will depend on the performance of each setting via the following formula:

$$\omega_{ps} = \sum_{i=1}^{n} [f(\vec{x}_{i,g}^{new}) - f(\vec{x}_{i,g})], \tag{7}$$

where ω_{ps} represents the sum of the differences between old fitness value and the new fitness value for every individual belonging to parameter setting ps, $f(\cdot)$ represents the fitness function, $\vec{x}_{i,g}^{new}$ is the new solution, $\vec{x}_{i,g}$ is the old solution, and n represents the number of solutions that belong to the parameter setting ps. After that, the improvement rate (Δps) could be calculated for each parameter setting by

$$\Delta_{ps} = max(0.05, \frac{\omega_{ps}}{\sum(omega_{ps})}),. \tag{8}$$

Algorithm 3 Pseudo-Code of APGSK

Input: NP_{init}, NP_{min}, $MaxFES$;

Parameter: NP, FES, k_f, k_r, and kw_p

Output: P_{g+1}

Obtain the initial generation of the population with individuals x_i $(i = 1, 2, ..., NP)$
$g = 0$
Initialize parameters: N, k_f ,k_r,K and P
Evaluate the individuals
while $FES < MaxFES$ **do**
  if $FES > 0.1 \cdot MaxFES$ **then**
    Update kw_p
  end if
  Compute The number of dimensions for junior action D_j and that for senior action D_s based on k_r according to Equations 4 and 5, respectively
  Junior action is executed at the probability k_r in the D_j dimensions according to Equations 6 or 7
  Senior action is executed at the probability k_r in the D_s dimensions according to Equations 8 or 9
  for i=1 to NP **do**
    Assign value of k_f and k_r for $\vec{x}_{i,g}$
    if $f(\vec{x}^{new}_{i,g}) < f(\vec{x}_{i,g})$ **then**
      $\vec{x}_{i,g} = \vec{x}^{new}_{i,g}$
    end if
  end for
end while

Here, 0.05 is used to express the minimum probability that could be assigned for each parameter setting in order to guarantee that all settings have a probability of being selected. The improvement rate (Δps) for each parameter setting is used for updating kw_p due to the following formula:

$$kw_{p,g+1} = (1-c)kw_{p,g} + c \cdot \Delta_{ps}, . \tag{9}$$

where c represents the learning rate. A constant learning rate c is used in order to make a benefit from the cumulative knowledge about each parameter setting's performance.

2. Tables

Table 1: Classification of the functions in the two CEC benchmark test suites

Type	CEC 2020	CEC 2022
Unimodal functions	F1	F1
Basic functions	F2-F4	F2-F5
Hybrid functions	F5-F7	F6-F8
Composition functions	F8-F10	F9-F12

Table 2: Setting of the CEC 2020 and CEC 2022 benchmark test suites

D	MaxFES	
	2020	2022
5	5.00E+04	-
10	1.00E+06	1.00E+06
15	3.00E+06	-
20	1.00E+07	1.00E+07

Table 3: The problems selected from the CEC 2011 suite

Problem number	Problem name	Dimensionality
P1	Parameter estimation for frequency modulated sound waves	6
P3	Bifunctional catalyst blend optimal control problem	1
P4	Optimal control of a non-linear stirred tank teacto	1
P8	Transmission network expansion planning problem	7
P10	Circular antenna array design problem	12
P11.3	Static economic load dispatch instance 1	6
P11.4	Static economic load dispatch instance 2	13
P11.5	Static economic load dispatch instance 3	15

Table 4: Value of $MaxFES$ for the problems selected from the CEC 2011 suite

D	$MaxFES$
$D \leq 5$	5.00E+04
$5 < D \leq 10$	1.00E+06
$10 < D \leq 15$	3.00E+06

Table 5: Settings of the involved Peers

Algorithm	Parameters
IMODE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$
APGSK	$NP_{max} = D \cdot 200$, $NP_{min} = 4$, $p = 0.05$, $r_{super} = 5\%$, $r_{medium} = 90\%$, $r_{inferior} = 5\%$, and $c = 0.05$
APGSK-IMODE	$NP_2^{max} = \frac{D \cdot 30}{4}$, $NP_2^{min} = 12$, $NP_1^{max} = D \cdot 30 - NP_2^{max}$, $NP_1^{min} = 4$, and $CS = 50$
NL-SHADE-RSP	$NP_{max} = 30D$, $M_{f,r} = 0.2$, $M_{CR,r} = 0.2$, and $n_A = 0.5$
MLS-L-SHADE	$NP_{max} = D \cdot 18$, $NP_{min} = 4$,$A_{rate} = 2.60$, and $H = D^2 \cdot 0.36$,
EA4eig	$NP_{max} = 100$ and $NP_{min} = 10$
AMCDE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$, $G_{n\_init} = 5$, $p_{bc1} = 0.4$, $p_{bc2} = 0.4$, $p_r = 0.01$, and $p_w = 0.2$

Table 6: The results for the CEC 2020 benchmark test functions when $D = 10$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)							
	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	3.93E+00 (3.41E+00)+	2.06E+01 (1.11E+01)−	6.04E+00 (4.80E+00)≈	7.03E-01 (2.06E+00)+	8.73E+00 (5.07E+00)−	4.42E+00 (5.10E+00)+	5.52E+00 (3.11E+00)≈	5.62E+00 (3.11E+00)
F3	1.24E+01 (7.58E-01)−	9.51E+00 (3.77E+00)≈	1.20E+01 (7.73E-01)≈	7.97E+00 (1.85E+01)+	1.02E+01 (3.15E+00)≈	1.07E+01 (1.22E+00)+	9.91E+00 (3.05E+00)+	1.08E+01 (2.56E+00)
F4	2.96E-03 (4.60E-03)−	4.09E-02 (2.24E-02)−	1.14E-01 (7.40E-02)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	3.08E-01 (7.35E-02)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F5	4.23E-01 (3.88E-01)≈	4.27E-01 (3.08E-01)≈	5.51E-01 (4.05E-01)−	2.47E-01 (7.64E-02)+	2.53E+00 (2.13E+00)−	5.55E-02 (9.36E-02)+	1.87E-01 (1.38E-01)+	3.51E-01 (2.86E-01)
F6	1.20E-01 (7.22E-02)−	7.43E-02 (6.92E-02)≈	7.30E-02 (9.29E-02)≈	6.90E-02 (5.83E-03)≈	3.94E-01 (2.13E-01)−	4.78E-02 (9.67E-02)+	3.97E-02 (2.77E-02)≈	6.05E-02 (6.21E-02)
F7	6.76E-04 (7.30E-04)≈	7.49E-02 (1.58E-01)−	8.52E-04 (1.53E-03)≈	1.10E-03 (1.37E-06)−	1.36E-01 (1.50E-01)−	2.09E-02 (7.96E-02)−	1.28E-04 (1.61E-04)+	6.93E-04 (1.73E-03)
F8	2.72E+00 (7.46E+00)≈	0.00E+00 (0.00E+00)≈	3.20E+01 (3.62E+01)−	3.29E+00 (5.70E+01)≈	1.28E+01 (1.95E+01)−	1.00E+02 (0.00E+00)−	2.45E+00 (7.27e+00)≈	1.15E+00 (3.51E+00)
F9	4.15E+01 (4.38E+01)≈	1.33E+01 (3.46E+01)+	8.85E+01 (3.11E+01)−	6.44E+01 (2.28E+03)≈	7.42E+01 (3.77E+01)−	2.56E+02 (9.65E+01)−	6.43E+01 (4.80E+01)≈	5.40E+01 (5.02E+01)
F10	3.98E+02 (3.42E-12)−	1.00E+02 (8.92E-05)+	3.58E+02 (1.03E+02)−	3.68E+02 (8.25E+03)−	3.68E+02 (8.92E+01)−	4.09E+02 (1.96E+01)−	3.68E+02 (9.08e+01)−	2.89E+02 (1.46E+02)
Wilcoxon −/+/≈	4/1/5	3/2/5	5/0/5	2/3/5	7/0/3	5/4/1	1/3/6	-/-/-
Friedman ranking	4.65	4.25	5.75	3.60	5.90	5.35	3.00	3.50

Table 7: The results for the CEC 2020 benchmark test functions when $D = 20$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)							
	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	3.46E-01 (5.66E-01)+	2.92E+00 (1.54E+00)−	2.63E-01 (5.13E-01)+	1.98E-02 (4.36E-04)+	1.27E+00 (9.83E-01)≈	3.53E+00 (2.30E+00)−	1.25E+00 (1.01E+00)≈	1.71E+00 (1.63E+00)
F3	2.05E+01 (1.24E-01)≈	2.04E+01 (1.73E-01)−	2.04E+01 (0.00E+00)+	2.04E+01 (0.00E+00)+	2.07E+01 (1.37E+00)−	2.22E+01 (8.19E-01)−	2.04E+01 (7.77E-02)≈	2.04E+01 (6.82E-02)
F4	5.00E-01 (8.08E-02)≈	2.07E-01 (7.64E-02)+	5.50E-01 (5.73E-02)≈	0.00E+00 (0.00E+00)+	0.00E+00 (0.00E+00)+	7.24E-01 (1.32E-01)−	4.74E-01 (8.27E-02)≈	5.18E-01 (1.07E-01)
F5	1.05E+01 (4.11E+00)−	3.51E+01 (4.89E+01)−	6.57E+00 (3.41E+00)−	1.37E+01 (9.28E+02)≈	5.05E+00 (6.98E+00)≈	1.26E+01 (2.97E+01)≈	4.24E+00 (2.77E+00)≈	4.13E+00 (2.69E+00)
F6	2.87E-01 (7.40E-02)−	1.79E-01 (5.50E-02)≈	2.03E-01 (6.80E-02)≈	1.38E-01 (1.83E-03)≈	3.39E-01 (7.83E-02)−	1.63E-01 (9.35E-02)≈	1.77E-01 (1.24E-01)≈	2.00E-01 (1.11E-01)
F7	5.10E-01 (1.78E-01)≈	7.64E-01 (7.97E-01)≈	1.43E+00 (2.88E+00)−	2.05E-01 (7.73E-03)+	6.28E-01 (1.51E-01)−	5.72E-01 (1.56E+00)−	4.85E-01 (1.33E-01)≈	5.43E-01 (1.46E-01)
F8	8.42E+01 (1.83E+01)≈	9.39E+01 (1.27E+01)≈	9.82E+01 (6.09E+00)−	8.80E+01 (3.29E+02)≈	9.31E+01 (1.93E+01)≈	1.00E+02 (2.12E-13)−	9.23E+01 (1.59E+01)≈	8.32E+01 (2.57E+01)
F9	9.67E+01 (1.83E+01)≈	8.67E+01 (3.46E+01)≈	1.32E+02 (9.98E+01)≈	1.00E+02 (3.43E-26)≈	1.00E+02 (1.06E-08)≈	4.06E+02 (3.24E+00)−	9.80E+01 (1.07E+01)≈	9.67E+01 (1.83E+01)
F10	4.00E+02 (4.44E-01)+	3.99E+02 (1.56E-02)+	4.05E+02 (5.66E+00)≈	3.99E+02 (1.19E-01)+	4.11E+02 (3.51E+00)−	4.06E+02 (1.53E-02)−	4.08E+02 (7.20E+00)−	4.02E+02 (5.66E+00)
Wilcoxon −/+/≈	2/2/6	3/2/5	3/2/5	0/5/5	4/1/5	7/0/3	1/0/9	-/-/-
Friedman ranking	4.10	4.50	5.35	2.90	5.35	6.35	3.75	3.70

Table 8: The results for the CEC 2022 benchmark test functions when $D = 10$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)							
	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.51E-01 (7.19E-01)≈	1.20E+00 (1.86E+00)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F3	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F4	6.70E+00 (1.54E+00)−	4.64E+00 (1.46E+00)−	3.48E+00 (1.19E+00)+	9.06E+00 (5.05E+00)−	1.86E+00 (7.59E-01)+	3.32E-01 (6.58E-01)+	2.26E+00 (1.17E+00)+	3.15E+00 (1.72E+00)
F5	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F6	1.22E-01 (9.16E-02)≈	4.29E-02 (3.07E-02)+	6.29E-02 (6.18E-02)≈	3.48E-02 (5.61E-04)+	1.05E-01 (6.71E-02)≈	8.10E-03 (3.65E-02)+	6.66E-02 (6.93E-02)≈	9.16E-02 (8.58E-02)
F7	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.83E-03 (2.16E-03)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F8	8.53E-03 (1.06E-02)≈	1.00E-01 (8.50E-02)−	5.66E-03 (6.90E-03)≈	2.23E-02 (5.84E-04)−	1.82E+00 (1.64E+00)−	2.57E-03 (4.74E-03)+	8.12E-03 (7.59E-03)≈	6.94E-03 (9.33E-03)
F9	2.22E+02 (4.19E+01)≈	1.48E+02 (1.06E+02)+	2.22E+02 (4.19E+01)≈	2.22E+02 (1.75E+03)≈	2.29E+02 (6.17E-13)≈	1.86E+02 (0.00E+00)+	2.14E+02 (5.82E+01)≈	2.06E+02 (7.00E+01)
F10	1.70E+01 (3.35E+01)−	2.14E+01 (1.12E+01)−	9.46E+01 (1.72E+01)−	0.00E+00 (0.00E+00)+	2.22E-01 (6.22E-01)≈	1.00E+02 (4.02E-02)−	9.50E-01 (1.52E+00)≈	7.04E-01 (1.63E+00)
F11	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F12	1.59E+02 (7.05E-01)≈	1.53E+02 (2.90E+01)+	1.59E+02 (7.65E-01)≈	1.60E+02 (1.33E+00)−	1.59E+02 (2.55e-01)≈	1.46E+02 (2.45E+00)+	1.60E+02 (1.13E+00)−	1.59E+02 (9.16E-01)
Wilcoxon −/+/≈	2/0/10	3/3/6	1/1/10	3/2/7	2/1/9	2/5/5	1/1/10	-/-/-
Friedman ranking	5.08	4.21	4.50	4.67	5.38	3.67	4.42	4.08

Table 9: The results for the CEC 2022 benchmark test functions when $D = 20$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)							
	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	4.27E+01 (1.46E+01)−	0.00E+00 (0.00E+00)+	3.86E+01 (1.64E+01)−	3.62E+01 (3.39E+02)−	1.34E-01 (7.23e-01)≈	5.32E-01 (1.38E+00)≈	3.46E+01 (1.94E+01)−	6.72E-01 (1.53E+00)
F3	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.89E-14 (4.31E-14)−	3.79E-15 (4.31E-28)≈	0.00E+00 (0.00E+00)≈	7.58E-15 (2.88E-14)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F4	4.87E+01 (6.49E+00)−	3.58E+01 (5.70E+00)−	2.26E+01 (3.64E+00)−	5.19E+01 (7.50E+01)−	5.04E+00 (8.10E-01)≈	8.59E+00 (3.32E+00)−	8.86E+00 (1.35E+01)≈	8.16E+00 (8.82E+00)
F5	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F6	2.88E-01 (7.98E-02)≈	3.14E-01 (9.27E-02)≈	4.72E-01 (4.77E-01)≈	4.44E-01 (1.56E-01)≈	1.88E-01 (6.83E-02)+	1.23E-01 (1.55E-01)+	3.19E-01 (8.74E-02)−	2.65E-01 (1.04E-01)
F7	1.96E+00 (7.58E-01)−	2.64E+00 (2.42E+00)−	1.91E+00 (1.12E+00)≈	5.40E-02 (5.93E-02)+	1.95E+00 (1.14E+00)≈	2.40E+00 (3.56E+00)≈	1.79E+00 (9.29E-01)≈	1.53E+00 (9.04E-01)
F8	1.62E+01 (4.19E+00)≈	1.72E+01 (4.65E+00)≈	1.80E+01 (5.85E+00)−	1.69E+01 (3.56E+01)≈	1.61E+01 (4.46E+00)≈	1.63E+01 (7.56E+00)−	1.35E+01 (6.70E+00)≈	1.57E+01 (4.84E+00)
F9	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (7.52E-27)≈	1.81E+02 (8.53E-14)≈	1.65E+02 (0.00E+00)+	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)
F10	0.00E+00 (0.00E+00)≈	9.44E+01 (1.65E+01)−	1.00E+02 (3.70E-02)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.08E+02 (2.93e+01)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F11	3.00E+02 (0.00E+00)−	0.00E+00 (0.00E+00)≈	2.40E+02 (1.22E+02)−	2.90E+02 (3.00E+03)−	0.00E+00 (0.00E+00)≈	3.23E+02 (4.30e+01)−	3.00E+02 (0.00E+00)−	0.00E+00 (0.00E+00)
F12	2.34E+02 (1.98E+00)−	2.34E+02 (1.53E+00)−	2.31E+02 (1.80E+00)≈	2.38E+02 (1.22E+01)−	2.32E+02 (8.63E-01)−	2.00E+02 (3.32e-04)+	2.32E+02 (1.68E+00)≈	2.32E+02 (2.22E+00)
Wilcoxon −/+/≈	5/0/7	4/1/5	6/0/6	4/1/7	1/1/10	4/3/5	3/0/9	-/-/-
Friedman ranking	5.17	4.88	5.58	5.33	3.25	4.42	4.13	3.25

Table 10: The results of the selected CEC 2011 real-world optimization problems with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)			
	NL-SHADE-RSP	EA4ig	AMCDE	APGSK-A-AMCDE
P1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	2.81E-01 (1.54E+00)≈	1.68E-13 (6.44E-13)
P3	1.15E-05 (2.18E-19)≈	1.03E-05 (3.65E-06)+	1.15E-05 (1.90E-19)≈	1.15E-05 (2.05E-19)
P4	1.62E+01 (3.16E+00)−	1.46E+01 (1.78E+00)−	1.65E+01 (3.13E+00)−	1.41E+01 (2.81E-01)
P8	2.20E+02 (0.00E+00)≈	2.20E+02 (0.00E+00)≈	2.20E+02 (0.00E+00)≈	2.20E+02 (0.00E+00)
P10	-2.17E+01 (9.76E-02)≈	-2.20E+01 (5.35E-01)+	-2.16E+01 (1.48E-03)−	-2.17E+01 (3.61E-02)
P11.3	1.54E+04 (1.16E+00)≈	1.54E+04 (3.53E-04)≈	3.08E+04 (2.69E+04)−	1.54E+04 (2.13E-07)
P11.4	1.88E+04 (1.02E+02)−	1.81E+04 (2.18E+01)≈	1.82E+04 (1.16E+02)−	1.81E+04 (6.09E+00)
P11.5	3.30E+04 (3.51E+01)−	3.27E+04 (4.59E-05)+	2.47E+05 (1.97E+05)−	3.29E+04 (4.27E+01)
Wilcoxon $-/+/\approx$	3/0/5	1/3/4	5/0/3	-/-/-