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Ngày . . No.

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Q1)

| t | 2 | 2.1 | 2.2 | 2.7 | 3 | 3.4 |
|---|---|---|---|---|---|---|
| z | 6 | 7.752 | 30.256 | 36.576 | 66 | 423.168 |

a) Using Newton interpolating polynomials

$$f(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$$

$t = 2.5$ between 2.2 and 2.7

First order:

$$f(x) = f(2.2) + \frac{f(2.7) - f(2.2)}{2.7 - 2.2}(x - 2.2)$$

$$\Rightarrow 30.256 + \frac{36.576 - 30.256}{2.7 - 2.2}(x - 2.2)$$

$$f(x) = 30.256 + 12.64(x - 2.2)$$

$$f(2.5) = 34.048$$

Second order:

$$f(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) + \frac{f(x_3, x_2) - f(x_2,}{x_3 - x_1}$$

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Substitutes 2.1, 2.2, 2.7

$$f(x) = f(2.1) + \frac{f(2.2) - f(2.1)}{2.2 - 2.1}(x - 2.1)$$

$$+ \frac{f(x_3, x_2) - f(x_2, x_1)}{2.7 - 2.1}(x - 2.1)(x - 2.2)$$

with $f(x_3, x_2) = \frac{f(x_3) - f(x_2)}{x_3 - x_2} = \frac{f(2.7) - f(2.$
$x_3 - x_2$ $2.7 - 2.2$

$= 12.64$

2.2)

$f(x) = 7.752 + 285.04(x - 2.1) - 354(x - 2.$
$(x - 2.2)$

2.2)

$f(2.5) = 55.288$

$$\frac{f(x_3, x_2) - f(x_2, x_1)}{x_3 - x_1}(x - x_1)(x - x_2)$$

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c) $f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) +$

$\frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2) + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}$

$+ \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$

For $t = 2.5$

use $x_1 = 2.1, x_2 = 2.2, x_3 = 2.7, x_4 = 3$

$f(x) = \frac{(x-2.2)(x-2.7)(x-3)}{(2.1-2.2)(2.1-2.7)(2.1-3)} \times 7.752$

$+ \frac{(x-2.1)(x-2.7)(x-3)}{(2.2-2.1)(2.2-2.7)(2.2-3)} \times 30.256$

$+ \frac{(x-2.1)(x-2.2)(x-3)}{(2.7-2.1)(2.7-2.2)(2.7-3)} \times 36.576$

$+ \frac{(x-2.1)(x-2.2)(x-2.7)}{(3-2.1)(3-2.2)(3-2.7)} \times 66$

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at $t = 2.5$

$f(2.5) = 43$

$f(x_3)$ ~~d) The result of (a) and (c) is not same~~

Q3) $\frac{d^2y}{dx^2} + 0.5\frac{dy}{dx} + 7y = 0$

We reduce the 2nd to 1st order

Put $z = \frac{dy}{dx} \Rightarrow \frac{dz}{dx} = \frac{d^2y}{dx^2}$

$\frac{dz}{dx} + 0.5z + 7y = 0$

$\frac{dz}{dx} = -0.5z - 7y$

We have $\frac{dy}{dx} = z$ and $\frac{dz}{dx} = -0.5z - 7y$

$f(x, y, z) = z$

and $g(x, y, z) = -0.5z - 7y$

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We have: $K_1 = h f(x_0, y_0, z_0)$

$K_1 = 0.5 f(0, 4, 0) = 0$

$l_1 = h g(x_0, y_0, z_0)$

$l_1 = 0.5 \times (-28) = -14$

$K_2 = 0.5 f\left(0 + \frac{0.5}{2}, 4 + \frac{0}{2}, 0 + \frac{-14}{2}\right)$

$K_2 = 0.5 \times (-7) = -3.5$

$l_2 = 0.5 \times (-24.5) = 12.25$

$K_3 = 0.5 \times 6.125 = 3.0625$

$l_3 = 0.5 \times (-0.5 \times 6.125 - 7 \times 2.25) = 9.4$

$K_4 = 0.5 \times 9.4 = 4.703$

there is no requirement of $l_4$

Substitute $K_1, K_2, K_3, K_4$

$y(x) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

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$y(x) = \frac{x}{6} + \frac{1}{6}[0 + 2x(3.5) + 2x3.0625 + 4.703]$

~~$y(x) = 4.638$~~

$y(0) = 0.638$

$y(1) = 1.638$

$y(2) = 2.638$

$y(3) = 3.638$

$y(4) = 4.638$

$y(5) = 5.638$

KOKUYO

Q2)

a) Using Newton interpolating polynomials

$$f(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1)$$

$x = 3.5$ between 3 and 4.5

First order:

$$f(x) = f(3) + \frac{f(4.5) - f(3)}{4.5 - 3}(x - 3)$$

$$f(3.5) = f(3) + \frac{f(4.5) - f(3)}{4.5 - 3}(3.5 - 3)$$

$$f(3.5) = 3.125$$

Second order:

$$f(x) = f(x_1) + \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) + \frac{f(x_3 x_2) - f(x_2 x_1)}{x_3 - x_1}(x - x_1)(x - x_2)$$

$$f(x) = f(2.5) + \frac{f(3) - f(2.5)}{3 - 2.5}(x - 2.5) + \frac{-0.75 + 3.75}{4.5 - 2.5}(x - 2.5)(x - 3)$$

$f(3.5) = 2.375$

b) Using third order lagrange

$$f(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} f(x_1) + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} f(x_2) + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} f(x_3) + \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} f(x_4)$$

~~$f(3.5) =$~~ Use $x_1 = 2.5$

$x_2 = 3$

$x_3 = 4.5$

$x_4 = 5$

$f(3.5) = 2.375$