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Ngô Lê Thiên Ân - ITITDK21080

Q1)

a) The mean average:

$$Mean = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{28.65 + 26.55 + \ldots + 27.25}{28}$$

$$= \frac{784.7}{28} = 28.025$$

b) The standard deviation:

$$\text{Standard deviation} = \sqrt{\frac{\sum_{i=1}^{n} (X_i - mean)^2}{n}}$$

$$\sqrt{\frac{(28.65 - 28.025)^2 + (26.55 - 28.025)^2 + \ldots + (27.25 - 28.025)^2}{28}}$$

$SD = 1.085$

c) The variance:

$$\text{Variance} = SD^2 = 1.177$$

d) The coefficient of variation:

$$\text{Coefficient variation} = \left(\frac{\text{Standard deviation}}{\text{Mean}}\right) \times 100$$

$= 3.871$

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e) 90% confidence interval for the mean:

Confidence interval = Mean $\pm$ (90% $\times$ $\frac{\text{Standard deviation}}{\sqrt{n}}$)

$= 28.025 \pm \left(0.9 \times \frac{1.085}{\sqrt{28}}\right)$

$= (27.84, 28.20)$

-27.25

-$(27.25-28.025)^2$

g) Range that ~~68%~~ encompasses 68% of readings:

Lower range = Mean − Standard deviation
$= 28.025 - 1.085$
$= 26.94$

Upper range = Mean + Standard deviation
$= 28.025 + 1.085$
$= 29.11$

Q2)

tion) x100

a) $n = 9$

$\Sigma x_i = 45$

$\Sigma y_i = 47.5$

$\Sigma x_i y_i = (1\times1)+(2\times1.5)+\ldots+(9\times13) = 325$

$\Sigma x_i^2 = 1^2+2^2+3^2+\ldots+9^2 = 285$

~~Slope = n $\Sigma x_i y_i$~~

$\bar{y} = \frac{47.5}{9} = 5.27$

$\Sigma y_i^2 = 1^2+1.5^2+\ldots+13^2 = 387.25$

$\bar{x} = \frac{45}{9} = 5$

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Slope: $m = \frac{n(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{n(\Sigma x_i^2) - (\Sigma x_i)^2}$

$m = \frac{(9 \times 325) - (45 \times 47.5)}{9 \times 285 - 45^2} = 1.4583$

Intercept:

$b = \frac{(\Sigma y_i) - m(\Sigma x_i)}{n} = \frac{47.5 - 1.4583 \times 45}{9}$

$= \cancel{1.972} \; -2.0137$

Predict Value: $\hat{y} = mx + b$

Standard error of the estimate $\left(S_{y/x}\right)$:

$S_{y/x} = \sqrt{\frac{\Sigma(y - \hat{y})^2}{n-2}} = \sqrt{1.1523}$

Coefficient of determination:

$R^2 = 1 - \frac{\Sigma(y - \hat{y})^2}{\Sigma(y - \bar{y})^2} = 0.9143$

Correlation coefficient:

$r = \frac{n(\Sigma x_i y_i) - (\Sigma x_i)(\Sigma y_i)}{\sqrt{\left[n(\Sigma x_i^2) - (\Sigma x_i)^2\right]\left[n(\Sigma y_i^2) - (\Sigma y_i)^2\right]}}$

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$$r = \frac{9 \times 325 - (45 \times 47.5)}{\sqrt{(9 \times 285 - 45^2) \times (9 \times 387.25 - 47.5^2)}}$$

$r = 0.9666$

b) Polynomical regression

$n = 9$

$\sum x_i = 45$ $\sum y_i = 47.5$

$\sum x_i^2 = 285$ $\sum x_i y_i = 325$

$\sum x_i^3 = 2025$

$\sum x_i^4 = 15333$ $\sum x_i^2 y_i = 2438$

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

$$\begin{bmatrix} 9 & 45 & 285 \\ 45 & 285 & 2025 \\ 285 & 2025 & 15333 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 47.5 \\ 325 \\ 2438 \end{bmatrix}$$

$a_0 = 1.488$

$a_1 = -0.451$

$a_2 = 0.191$

$y = 0.191x^2 - 0.451x + 1.488$

Intercept is 1.488

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Q3) $n=9$

a) $\Sigma x_1 = 9$ $\quad \Sigma x_1 x_2 = 33$

$\Sigma x_2 = 27$ $\quad \Sigma x_1 y = 109$

$\Sigma y = 135$ $\quad \Sigma x_2 y = 449$

$\Sigma x_1^2 = 15$ $\quad \Sigma x_2^2 = 117$

Let equation for $y = a_0 + a_1 x_1 + a_2 x_2$

$$\begin{cases} \Sigma y = n a_0 + a_1 \Sigma x_1 + a_2 \Sigma x_2 \\ \Sigma x_1 y = a_0 \Sigma x_1 + a_1 \Sigma x_1^2 + a_2 \Sigma x_1 x_2 \\ \Sigma x_2 y = a_0 \Sigma x_2 + a_1 \Sigma x_1 x_2 + a_2 \Sigma x_2^2 \end{cases}$$

$135 = 9a_0 + 9a_1 + 27a_2$

$109 = 9a_0 + 15a_1 + 33a_2$

$449 = 27a_0 + 33a_1 + 117a_2$

$$\left[\begin{array}{ccc|c} 9 & 9 & 27 & 135 \\ 9 & 15 & 33 & 109 \\ 27 & 33 & 117 & 449 \end{array}\right]$$

$R_2 = R_2 - R_1$

$$\left[\begin{array}{ccc|c} 9 & 9 & 27 & 135 \\ 0 & 6 & 6 & -26 \\ 27 & 33 & 117 & 449 \end{array}\right]$$

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$R_3 = R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 9 & 9 & 27 & 135 \\ 0 & 6 & 6 & -26 \\ 0 & 6 & 36 & 44 \end{array}\right]$$

$R_3 = R_3 - R_2$

$$\left[\begin{array}{ccc|c} 9 & 9 & 27 & 135 \\ 0 & 6 & 6 & -26 \\ 0 & 0 & 30 & 70 \end{array}\right]$$

$30a_2 = 70 \Rightarrow a_2 = \frac{7}{3}$

$6a_1 + 6a_2 = -26$

$\Rightarrow a_1 = \frac{-20}{3}$

$9a_0 + 9a_1 + 27a_2 = 135$

$\Rightarrow a_0 = \frac{44}{3}$

Predicted value: $\hat{y} = \frac{44}{3} - \frac{20}{3}x_1 + \frac{7}{3}x_2$

$\hat{y} = 44 - 20x_1 + 7x_2$

Standard error:

$S_{y/x} = \sqrt{\frac{\Sigma(y-\hat{y})^2}{n-3}} = 1.4142$

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Coefficient of determination

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2} = 0.9583$$

Correlation coefficient:

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = 0.9789$$