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Q1 $2x_1 - 6x_2 - x_3 = -38$
$-3x_1 - x_2 + 7x_3 = -34$
$-8x_1 + x_2 - 2x_3 = -20$

a) $\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ -3 & -1 & 7 & -34 \\ -8 & 1 & -2 & -20 \end{array}\right]$

Row2 = Row2 + $\frac{3}{2}$ x Row1
Row3 = Row3 + $\frac{8}{2}$ x Row1

$\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ 0 & -10 & 5.5 & -91 \\ 0 & -23 & -6 & -172 \end{array}\right]$

Swap Row2 and Row3

$\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ 0 & -23 & -6 & -172 \\ 0 & -10 & 5.5 & -91 \end{array}\right]$

Row3 = Row3 + $\frac{-10}{23}$ x Row2

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$$\left[\begin{array}{ccc|c} 2 & -6 & -1 & -38 \\ 0 & -23 & -6 & -172 \\ 0 & 0 & \frac{373}{46} & \frac{-373}{23} \end{array}\right]$$

$$\Rightarrow \begin{cases} x_3 = -2 \\ x_2 = 8 \\ x_1 = 4 \end{cases}$$

Q2)

a) $x_1 + x_2 - x_3 = -3$

$6x_1 + 2x_2 + 2x_3 = 2$

$-3x_1 + 4x_2 + x_3 = 1$

Using naive gauss elimination

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array}\right]$$

Row 2 = Row 2 − 6 x Row 1

Row 3 = Row 3 + 3 x Row 1

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 7 & -2 & -8 \end{array}\right]$$

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$$Row3 = Row3 + \frac{7}{4} \times Row2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 0 & 12 & 27 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_3 = 2.25 \\ x_2 = -0.5 \\ x_1 = -0.25 \end{cases}$$

b) Using Gauss elimination with partial pivoting

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array}\right]$$

We don't need to swap rows because the 1st row already has the largest absolute value

$$Row2 = Row2 - 6 \times Row1$$
$$Row3 = Row3 + 3 \times Row1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 7 & -2 & -8 \end{array}\right]$$

$$Row3 = Row3 + \frac{7}{4} \times Row2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 0 & 12 & 27 \end{array}\right] \Rightarrow \begin{cases} x_3 = 2.25 \\ x_2 = -0.5 \\ x_1 = -0.25 \end{cases}$$

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c) Using Gauss-Jordan without partial pivoting

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 6 & 2 & 2 & 2 \\ -3 & 4 & 1 & 1 \end{array}\right]$$

Row2 = Row2 − 6 × Row1

Row3 = Row3 + 3 × Row1

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 7 & -2 & -8 \end{array}\right]$$

Row3 = Row3 + $\frac{7}{4}$ × Row2

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & -4 & 8 & 20 \\ 0 & 0 & 12 & 27 \end{array}\right]$$

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$$Row2 = \frac{Row2}{-4}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 12 & 27 \end{array}\right]$$

$$Row1 = Row1 - Row2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 12 & 27 \end{array}\right]$$

$$Row3 = \frac{Row3}{12}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 2.25 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -2 & -5 \\ 0 & 0 & 1 & 2.25 \end{array}\right]$$

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Row 2 = Row 2 x $\frac{-1}{2}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2.5 \\ 0 & 0 & 1 & 2.25 \end{array}\right]$$

Row 2 = Row 2 − Row 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & 2.25 \end{array}\right]$$

Row 1 = Row 1 − Row 3

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -0.25 \\ 0 & 1 & 0 & -0.5 \\ 0 & 0 & 1 & 2.25 \end{array}\right]$$

$$\Rightarrow \begin{cases} x_1 = -0.25 \\ x_2 = -0.5 \\ x_3 = 2.25 \end{cases}$$

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(Q3)

a) $7x_1 + 2x_2 - 3x_3 = -12$
$2x_1 + 5x_2 - 3x_3 = -20$
$x_1 - x_2 - 6x_3 = -26$

$$\left[\begin{array}{ccc|c} 7 & 2 & -3 & -12 \\ 2 & 5 & -3 & -20 \\ 1 & -1 & -6 & -26 \end{array}\right]$$

$\text{Row2} = \text{Row2} - \frac{2}{7}\text{Row1}$

$\text{Row3} = \text{Row3} - \frac{1}{7}\text{Row1}$

$$\left[\begin{array}{ccc|c} 7 & 2 & -3 & -12 \\ 0 & \frac{31}{7} & -\frac{15}{7} & -\frac{116}{7} \\ 0 & -\frac{9}{7} & -\frac{39}{7} & -\frac{170}{7} \end{array}\right]$$

$\text{Row3} = \text{Row3} + \frac{9}{31}\text{Row2}$

$$\left[\begin{array}{ccc|c} 7 & 2 & -3 & -12 \\ 0 & \frac{31}{7} & -\frac{15}{7} & -\frac{116}{7} \\ 0 & 0 & -\frac{192}{31} & -\frac{902}{31} \end{array}\right]$$

This is the upper triangular matrix [U]

And the lower triangular matrix [L] is

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{7} & 1 & 0 \\ \frac{1}{7} & \frac{-9}{31} & 1 \end{bmatrix}$$

$$[L] \times [U] = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{7} & 1 & 0 \\ \frac{1}{7} & \frac{-9}{31} & 1 \end{bmatrix} \times \begin{bmatrix} 7 & 2 & -3 \\ 0 & \frac{31}{7} & \frac{-15}{7} \\ 0 & 0 & \frac{-192}{31} \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & -3 \\ 2 & 5 & -3 \\ 1 & -1 & -6 \end{bmatrix} = [A]$$

b) We assume: $y = [y_1, y_2, y_3]$

$$[L \mid Y] = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{7} & 1 & 0 \\ \frac{1}{7} & \frac{-9}{31} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -12 \\ -20 \\ -26 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y_1 = -12 \\ y_2 = \frac{-116}{7} \\ y_3 = \frac{-902}{31} \end{cases}$$

Similar: $x = [x_1, x_2, x_3]$

$$[U \mid X] = \begin{bmatrix} 7 & 2 & -3 \\ 0 & \frac{31}{7} & \frac{-15}{7} \\ 0 & 0 & \frac{-192}{31} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -12 \\ \\ -26 \end{bmatrix}$$

$$\begin{bmatrix} -12 \\ \frac{-116}{7} \\ \frac{-902}{31} \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 = \frac{23}{32} \\ x_2 = \frac{-47}{32} \\ x_3 = \frac{451}{96} \end{cases}$$

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c) We have $b^T = [12, 18, -6]$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{7} & 1 & 0 \\ \frac{1}{7} & \frac{-9}{31} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \\ -6 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y_1 = 12 \\ y_2 = \frac{102}{7} \\ y_3 = \frac{-108}{31} \end{cases}$$

$$\begin{bmatrix} 7 & 2 & -3 \\ 0 & \frac{31}{7} & \frac{-15}{7} \\ 0 & 0 & \frac{-192}{31} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ \frac{102}{7} \\ \frac{-108}{31} \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 = \frac{15}{16} \\ x_2 = \frac{57}{16} \\ x_3 = \frac{9}{16} \end{cases}$$

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Q4) $3x_1 - 2x_2 + x_3 = -10$
$2x_1 + 6x_2 - 4x_3 = 44$
$-x_1 - 2x_2 + 5x_3 = -26$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 2 & 6 & -4 & 44 \\ -1 & -2 & 5 & -26 \end{array}\right]$$

$\text{Row2} = \text{Row2} - \frac{2}{3}\text{Row1}$
$\text{Row3} = \text{Row3} + \frac{1}{3}\text{Row1}$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 0 & \frac{22}{3} & \frac{-14}{3} & \frac{152}{3} \\ 0 & \frac{-8}{3} & \frac{16}{3} & \frac{-88}{3} \end{array}\right]$$

$\text{Row3} = \text{Row3} + \frac{8}{22}\text{Row2}$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -10 \\ 0 & \frac{22}{3} & \frac{-14}{3} & \frac{152}{3} \\ 0 & 0 & \frac{40}{11} & \frac{-120}{11} \end{array}\right]$$

This is the upper triangular matrix

[a]

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And the lower triangular matrix [L] is

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ -\frac{1}{3} & -\frac{8}{22} & 1 \end{bmatrix}$$

$$[L|Y] = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ -\frac{1}{3} & -\frac{8}{22} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 44 \\ -26 \end{bmatrix}$$

$$\Rightarrow \begin{cases} y_1 = -10 \\ y_2 = \frac{152}{3} \\ y_3 = -\frac{420}{11} \end{cases}$$

$$[U|X] = \begin{bmatrix} 3 & -2 & 1 \\ 0 & \frac{22}{3} & -14 \\ 0 & 0 & -\frac{420}{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -10 \\ \frac{152}{3} \\ -\frac{420}{11} \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 = 1 \\ x_2 = 5 \\ x_3 = -3 \end{cases}$$

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