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Q1)

a) $f(x, y, z) = x^2 + y^2 + 2z^2$

$$\frac{\partial f}{\partial x} = 2x, \quad \frac{\partial f}{\partial y} = 2y, \quad \frac{\partial f}{\partial z} = 4z$$

The gradient vector is:

$$\nabla f(x, y, z) = [2x, 2y, 4z]$$

Using Hessian matrix:

$$\nabla^2 f(x, y, z) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\ \frac{\partial^2 f}{\partial z \partial x} & \frac{\partial^2 f}{\partial z \partial y} & \frac{\partial^2 f}{\partial z^2} \end{bmatrix}$$

$$\nabla^2 f(x, y, z) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

b) $f(x_1, x_2) = \ln(e^{x_1} + e^{x_2})$

$$\frac{\partial f}{\partial x_1} = \frac{e^{x_1}}{e^{x_1}+e^{x_2}}$$

$$\frac{\partial f}{\partial x_2} = \frac{e^{x_2}}{e^{x_1}+e^{x_2}}$$

The gradient vector is :

$$\nabla f(x_1, x_2) = \left(\frac{e^{x_1}}{e^{x_1}+e^{x_2}}, \frac{e^{x_2}}{e^{x_1}+e^{x_2}}\right)$$

$$\nabla^2 f(x_1, x_2) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} \\ \frac{\partial^2 f}{\partial x_2 x_1} & \frac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$\xcancel{\nabla^2 f(x_1, x_2) = \begin{bmatrix} \frac{e^{x_1}(e^{x_2}-e^{x_1})}{(e^{x_1}+e^{x_2})^2} & -\frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2} \\ -\frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2} & \frac{e^{x_2}(e^{x_1}-e^{x_2})}{(e^{x_1}+e^{x_2})^2} \end{bmatrix}}$$

$$\frac{\partial^2 f}{\partial x_1^2} = \frac{e^{x_1}(e^{x_1}+e^{x_2}) - e^{x_1}e^{x_1}}{(e^{x_1}+e^{x_2})^2} = \frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2}$$

$$\frac{\partial^2 f}{\partial x_2^2} = \frac{e^{x_2}(e^{x_1}+e^{x_2}) - e^{x_2}e^{x_2}}{(e^{x_1}+e^{x_2})^2} = \frac{e^{x_2}e^{x_1}}{(e^{x_1}+e^{x_2})^2}$$

$$\nabla^2 f(x_1 x_2) = \begin{bmatrix} \frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2} & -\frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2} \\ -\frac{e^{x_1}e^{x_2}}{(e^{x_1}+e^{x_2})^2} & \frac{e^{x_2}e^{x_1}}{(e^{x_1}+e^{x_2})^2} \end{bmatrix}$$

Q2)

a) $f(x,y) = -9x + x^2 + 11y + 4y^2 - 2xy$

$$H(x,y) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

$$H(x,y) = \begin{bmatrix} 2 & -2 \\ -2 & 8 \end{bmatrix}$$

$\det(H(x,y)) = 2 \times 8 - (-2) \times (-2) = 16 - 4 = 12$

$\text{trace}(H(x,y)) = 2 + 8 = 10$

The discriminant is:

$D = \text{trace}^2 - 4 \times \det$

$D = 10^2 - 4 \times 12 = 52 > 0$ so the eigenvalues are real and positive, which means that the Hessian matrix is positive definite. As a result, the function $f(x,y)$ is convex

b) $\frac{\partial f}{\partial x} = 2x - 9 - 2y$

$\frac{\partial f}{\partial y} = 11 + 8y - 2x$

Using initial guesses $x = 0$, $y = 0$, $h = 0.$

$$x_{k+1} = x_k - h \frac{\partial f}{\partial x}(x_k, y_k)$$

$$y_{k+1} = y_k - h\frac{\partial f}{\partial y}(x_k, y_k)$$

Iteration 1:

$$x_1 = 0 - 0.1(2\times0 - 9 - 2\times0) = 0.9$$
$$y_1 = 0 - 0.1(11 + 8\times0 - 2\times0) = -1.1$$

Iteration 2:

$$x_2 = 0.9 - 0.1(2\times0.9 - 9 - 2\times(-1.1)) = 1.4$$
$$y_2 = -1.1 - 0.1[8\times(-1.1) - 2\times(0.9) + 11] = -1.14$$

Iteration 3:

$$x_3 = 1.4 - 0.1(2\times1.4 - 9 - 2\times-1.14) = 1.792$$
$$y_3 = -1.14 - 0.1(11 + 8\times-1.14 - 2\times1.4) = -1.048$$

c) Using initial guesses $x_0 = 0$, $y = 0$ and $\alpha = 0.1$

$$x_{k+1} = x_k - \alpha\frac{\partial f}{\partial x}(x_k, y_k)$$
$$y_{k+1} = y_k - \alpha\frac{\partial f}{\partial y}(x_k, y_k)$$

Iteration 1:

$$x_1 = 0 - 0.1(2\times0 - 9 - 2\times0) = 0.9$$
$$y_1 = 0 - 0.1(11 + 8\times0 - 2\times0) = -1.1$$

After one iteration of the steepest descent method with the given initial guesses and step size, the updated valuss are $x_1 = 0.9$ and $y = -1.1$. These value are the approximate minimum of the function

d) Using Newton method:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = \begin{bmatrix} x_k \\ y_k \end{bmatrix} - \alpha \cdot H(x_k, y_k)^{-1} \cdot \nabla f(x_k, y_k)$$

Iteration 1: starting with initial guesses $x_0 = 0$, $y_0 = 0$

$$\nabla f(0,0) = \left(\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial x}(0,0) \right) = (-9, 11)$$

$$H(0,0) = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2}(0,0) & \frac{\partial^2 f}{\partial x \partial y}(0,0) \\ \frac{\partial^2 f}{\partial x \partial y}(0,0) & \frac{\partial^2 f}{\partial y^2}(0,0) \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 8 \end{bmatrix}$$

$$H(0,0)^{-1} = \begin{bmatrix} 2 & -2 \\ -2 & 8 \end{bmatrix}^{-1}$$

$$= \frac{1}{2 \times 8 - (-2) \times (-2)} \begin{bmatrix} 8 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= \frac{1}{12} \begin{bmatrix} 8 & 2 \\ 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \alpha \frac{1}{12} \begin{bmatrix} 8 & 2 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} -9 \\ 11 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \alpha \frac{1}{12} \begin{bmatrix} 8 \times -9 + 2 \times 11 \\ 2 \times -9 + 2 \times 11 \end{bmatrix}$$

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$x_1 = 0 - \alpha\frac{1}{12}(8\cancel{x}-9+2\times 11)$

$y_1 = 0 - \alpha\frac{1}{12}(2\times -9+2\times 11)$

If $\alpha = 0.1$

$x_1 = \frac{5}{12}$

$y_1 = \frac{-1}{30}$

Q3)

a) Let x be the number of units of product A $(x \geq 0)$

Let y be the number of units of product B $(y \geq 0)$

Product A requires 20 kg

Product B requires 5 kg

The total available raw material is 9500 kg

$20x + 5y \leq 9500$

Product A requires 0.04 hours

B 0.12 hours

The total available work hours in a week is 40 hours

$0.04x + 0.12y \leq 40$

The company can only store 550 kg of total product per week

$x + y \leq 550$

~~And quantities of produ~~

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b) $20x + 5y \le 9500 \Rightarrow y \le 1900 - 4x$

$0.04x + 0.12y \le 40 \Rightarrow y \le \dfrac{1000}{3} - \dfrac{1}{3}x$

$x + y \le 550 \Rightarrow y \le -x + 550$