Ngô Lê Thiên Ân — ITITDK 24030

Q1)

$$10x_1 + 2x_2 - x_3 = 27$$
$$-3x_1 - 6x_2 + 2x_3 = -61.5$$
$$x_1 + x_2 + 5x_3 = -21.5$$

a)
$$\begin{bmatrix} 10 & 2 & -1 & | & 27 \\ -3 & -6 & 2 & | & -61.5 \\ 1 & 1 & 5 & | & -21.5 \end{bmatrix}$$

Row1 = $\dfrac{\text{Row1}}{10}$
$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ -3 & -6 & 2 & | & -61.5 \\ 1 & 1 & 5 & | & -21.5 \end{bmatrix}$$

Row2 = Row2 + 3 × Row1
$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ 0 & -5.4 & 1.7 & | & -53.4 \\ 1 & 1 & 5 & | & -21.5 \end{bmatrix}$$

Row3 = Row3 - Row1
$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ 0 & -5.4 & 1.7 & | & -53.4 \\ 0 & 0.8 & 5.1 & | & -24.2 \end{bmatrix}$$

Row2 = $\dfrac{-1}{5.4}$ Row2
$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ 0 & 1 & -\dfrac{17}{54} & | & \dfrac{89}{9} \\ 0 & 0.8 & 5.1 & | & -24.2 \end{bmatrix}$$

Row 3 = Row 3 - 0.8 × Row 2

$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ 0 & 1 & -\frac{17}{54} & | & \frac{89}{9} \\ 0 & 0 & \frac{289}{54} & | & -\frac{289}{9} \end{bmatrix}$$

Row 3 = $\dfrac{1}{\frac{289}{54}}$ × Row 3

$$\begin{bmatrix} 1 & 0.2 & -0.1 & | & 2.7 \\ 0 & 1 & -\frac{17}{54} & | & \frac{89}{9} \\ 0 & 0 & 1 & | & -6 \end{bmatrix} \Rightarrow (x_1, x_2, x_3)$$
$$= (0.5, 8, -6)$$

Q2)

$$8x_1 + 2x_2 - 2x_3 = 8$$

a)
$$10x_1 + 2x_2 + 4x_3 = 16$$
$$12x_1 + 2x_2 + 2x_3 = 16$$

$$\begin{bmatrix} 8 & 2 & -2 & | & 8 \\ 10 & 2 & 4 & | & 16 \\ 12 & 2 & 2 & | & 16 \end{bmatrix}$$

Swap first rows and 3rd row

$$\begin{bmatrix} 12 & 2 & 2 & | & 16 \\ 10 & 2 & 4 & | & 16 \\ 8 & 2 & -2 & | & 8 \end{bmatrix}$$

$$Row\,1 = \frac{Row\,1}{12}$$

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{6} & | & \frac{4}{3} \\ 10 & 2 & 4 & | & 16 \\ 8 & 2 & -2 & | & 8 \end{bmatrix}$$

$$Row\,2 = Row\,2 - Row\,1 \times 10$$
$$Row\,3 = Row\,3 - Row\,1 \times 8$$

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{6} & | & \frac{4}{3} \\ 0 & \frac{1}{3} & \frac{7}{3} & | & \frac{8}{3} \\ 0 & \frac{2}{3} & -\frac{10}{3} & | & \frac{8}{3} \end{bmatrix}$$

Swap 2nd and 3rd row

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{6} & | & \frac{4}{3} \\ 0 & \frac{2}{3} & -\frac{10}{3} & | & -\frac{8}{3} \\ 0 & \frac{1}{3} & \frac{7}{3} & | & \frac{8}{3} \end{bmatrix}$$

Row 2 = $\dfrac{\text{Row 2}}{\frac{2}{3}}$

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{13}{6} & | & \frac{4}{3} \\ 0 & 1 & -5 & | & -4 \\ 0 & \frac{1}{3} & \frac{7}{3} & | & \frac{8}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{6} & | & \frac{4}{3} \\ 0 & 1 & -5 & | & -4 \\ 0 & 0 & 4 & | & 4 \end{bmatrix}$$

Row 3 = $\dfrac{\text{Row 3}}{4}$

$$\begin{bmatrix} 1 & \frac{1}{6} & \frac{1}{6} & | & \frac{4}{3} \\ 0 & 1 & -5 & | & -4 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

$X_3 = 1$

$X_2 = 1$

$X_1 = 1$

Q3)

a)

$2x_1 + x_2 - x_3 = 2$

$5x_1 + 2x_2 + 2x_3 = 9$

$3x_1 + x_2 + x_3 = 5$

$$\begin{bmatrix} 2 & 1 & -1 & | & 2 \\ 5 & 2 & 2 & | & 9 \\ 3 & 1 & 1 & | & 5 \end{bmatrix}$$

$R_3 = 2R_3 - 3R_1$

$R_2 = 2R_2 - 5R_1$

$$\begin{bmatrix} 2 & 1 & -1 & | & 2 \\ 0 & -1 & 9 & | & 8 \\ 0 & -1 & 5 & | & 4 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$R_3 = R_3 - R_2$

$$\begin{bmatrix} 2 & 0 & 8 & | & 10 \\ 0 & -1 & 9 & | & 8 \\ 0 & 0 & -4 & | & -4 \end{bmatrix}$$

$R_1 = 2R_3 + R_1$

$R_2 = 4R_2 + 9R_3$

$$\begin{bmatrix} 2 & 0 & 0 & | & 2 \\ 0 & -4 & 0 & | & -4 \\ 0 & 0 & -4 & | & -4 \end{bmatrix}$$

$$R_1 = \frac{R_1}{2}$$

$$R_2 = -\frac{R_2}{4}$$

$$R_3 = \frac{R_3}{4}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & 1 \\ 0 & 0 & 1 & | & 1 \end{bmatrix}$$

$$\Rightarrow (x_1, x_2, x_3) = (1, 1, 1)$$

Q4)

$$15c_1 - 3c_2 - c_3 = 3300$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2400$$

Rename $c_1 = x$, $c_2 = y$, $c_3 = z$

$$15x - 3y - z = 3300$$
$$-3x + 18y - 6z = 1200$$
$$-4\textcircled{0}x - y + 12z = 2400$$

$$x_{k+1} = \frac{1}{15}(3300 + 3y_k + z_k)$$

$$y_{k+1} = \frac{1}{18}(1200 + 3x_{k+1} + 6z_k)$$

$$z_{k+1} = \frac{1}{12}(2400 + 4x_{k+1} + y_{k+1})$$

Start with initial guess : $x = y = z = ?$

Relative error	Iteration	X	y	z
1	1	220	103.333	281.944
0.49	2	259.46	203.89	303.478
0.07	3	281.01	214.66	311.558
0.01	4	283.70	217.80	312.717