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Ngày . . No.

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Q1)

a) $y = x^3 + 3x - 15$ at $x = 0$, $h = 0.25$

Apply the first order central difference approximation of $O(h^4)$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

$$f'(0) = \frac{-f(2h) + 8f(h) - 8f(-h) + f(-2h)}{12h}$$

$)(x - x_3) f(x_4)$

$2)(x_4 - x_3)$

with $h = 0.25$

$$f'(0) = \frac{-f(0.5) + 8f(0.25) - 8f(-0.25) + f(-0.5)}{3}$$

Substitute values in $y = x^3 + 3x - 15$ we get

$$f'(0) = \frac{-(-13.375) + 8(-14.234) - 8(-15.765) + (-16.625)}{3}$$

$f'(0) \approx 3$

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b) $y = x^2 \cos x$ at $x = 0.5$, $h = 0.1$

Apply the first order central difference approximation

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

~~$f'(0) = -f(2h) + 8f(h)$~~

with $x = 0.5$, $h = 0.1$

$$f'(0.5) = \frac{-f(0.5 + 2\times 0.1) + 8f(0.5 + 0.1) - 8f(0.5 - 0.1) + f(0.5 - 2\times 0.1)}{1.2}$$

$$f'(0.5) = \frac{-f(0.7) + 8f(0.6) - 8f(0.4) + f(0.3)}{1.2}$$

Substitute values in $y = x^2 \cos x$

$$f'(0.5) = \frac{-(0.489) + 8(0.359) - 8(0.159) + 0.089}{1.2}$$

$f'(0.5) = 1$

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x−2h)

1)−

(0.3)

59)+ 0.089

c) $y = \tan(x/3)$ at $x=2$, $h=0.5$

Apply the first order central difference approximation

$$f'(x) = \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$$

with $x=2$, $h=0.5$

$$f'(2) = \frac{-f(3)+8f(2.5)-8f(1.5)+f(1)}{6}$$

Substitute values in $y = \tan\left(\frac{x}{3}\right)$

$$f'(2) = \frac{-\tan(1)+8\tan\left(\frac{2.5}{3}\right)-8\tan\left(\frac{1.5}{3}\right)+\tan\left(\frac{1}{3}\right)}{6}$$

$f'(2) \approx 0.54$

d) $y = \frac{\sin(0.5\sqrt{x})}{x}$ at $x=1$, $h=0.2$

$$f'(x) = \frac{-f(x+2h)+8f(x+h)-8f(x-h)+f(x-2h)}{12h}$$

with $x=1$, $h=0.2$

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$$f'(1) = \frac{-f(1.4) + 8f(1.2) - 8f(0.8) + f(0.6)}{2.4}$$

Substitute values in $y = \frac{\sin(0.5\sqrt{x})}{x}$

$f'(1) \approx -0.259$

e) $y = e^x + x$ at $x = 3$, $h = 0.2$

$$f'(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$$

with $h = 0.2$, $x = 3$

$$f'(3) = \frac{-f(3.4) + 8f(3.2) - 8f(2.8) + f(2.6)}{2.4}$$

Substitute values in $y = e^x + x$

$f'(3) \approx 21.084$

Q3) $\dfrac{dy}{dx} = (1+2t)\sqrt{x}$

a) $\int dy = \int (1+2t)\sqrt{x}\, dx$

$$y = \frac{2}{3}(1+2t)x^{\frac{3}{2}} + C$$

Substitute $y(0) = 1$

$$1 = \frac{2}{3}(1+2\times 0)\times 0^{\frac{3}{2}} + C$$

$$\Rightarrow C = 1$$

$$y = \frac{2}{3}(1+2t)x^{\frac{3}{2}} + 1$$

x	y
0	1
0.25	1.125
0.5	1.471
0.75	2.082
1	3

b) Using Euler's method:

$$y_{i+1} = y_i + h(1+2t_i)\sqrt{x_i}$$

$$y_0 = 0$$

$$y_1 = y_0 + 0.25(1+2\times 0)\sqrt{0}$$

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$$y_1 = 1 + 0 = 1$$

$$y_2 = y_1 + 0.25(1 + 2 \times 0.25) \times \sqrt{0.5}$$

$$y_2 = 1.265$$

$$y_3 = y_2 + 0.25(1 + 2 \times 0.5) \times \sqrt{0.75}$$

$$y_3 = 1.698$$

$$y_4 = y_3 + 0.25(1 + 2 \times 0.75) \times \sqrt{1}$$

$$y_4 = 2.323$$