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Q1)

b) Using bisection method, with initial guesses of $x_\ell = 0.5$ and $x_u = 0.2$

Iteration 1:

$f(x_\ell) = \ln(0.5^4) - 0.7 = -3.4725$

$f(x_u) = \ln(2^4) - 0.7 = 2.0725$

$$x_1 = \frac{x_\ell + x_u}{2} = \frac{0.5+2}{2} = 1.25$$

$f(x_1) = \ln(1.25^4) - 0.7 = 0.1925$

Since $f(x_\ell) . f(x_1) < 0$, we replace x_u with x_1

$\Rightarrow x_\ell = 0.5,\ x_u = 1.25$

Iteration 2:

$f(x_\ell) = -3.4725$

$f(x_u) = \ln(1.25^4) - 0.7 = 0.1925$

$$x_2 = \frac{x_\ell + x_u}{2} = \frac{0.5+1.25}{2} = 0.875$$

$f(x_2) = \ln(0.875^4) - 0.7 = -1.234$

Since $f(x_\ell) . f(x_2) > 0$, we replace x_ℓ with x_2

$\Rightarrow x_\ell = 0.875,\ x_u = 1.25$

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Iteration 3:

$f(X_\ell) = -1.234$

$f(X_u) = 0.1925$

$$X_3 = \frac{X_\ell + X_u}{2} = \frac{0.875 + 1.25}{2} = 1.0625$$

$f(X_3) = \ln(1.0625^4) - 0.7 = -0.4575$

Since $f(X_\ell) \cdot f(X_3) > 0$, we replace X_ℓ with X_3

$\Rightarrow X_\ell = 1.0625$, $X_u = \cancel{0.4925}\ 1.25$

After 3 iterations, the approximate root is in the interval $[1.0625, 1.25]$

c) Using false position method

Iteration 1:

$f(X_\ell) = \ln(0.5^4) - 0.7 = -3.4725$

$f(X_u) = \ln(2^4) - 0.7 = 2.0725$

$$X_1 = X_u - \frac{f(X_u)(X_\ell - X_u)}{f(X_\ell) - f(X_u)}$$

$$X_1 = 2 - \frac{2.0725(0.5-2)}{-3.4725 - 2.0725} = 1.439$$

$f(X_1) = \ln(1.439^4) - 0.7 = 0.755$

Since $f(X_1) \cdot f(X_\ell) < 0$

$\Rightarrow X_\ell = 0.5$, $X_u = 1.439$

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Iteration 2:

$f(X_\ell) = -3.4725$

$f(X_u) = 0.755$

$$X_2 = 1.439 - \frac{0.755(0.5-1.439)}{-3.4725-0.755}$$

$X_2 = 1.271$

$f(X_2) = \ln(1.271^4) - 0.7 = 0.259$

Since $f(X_\ell) \cdot f(X_2) < 0$

$\Rightarrow X_\ell = 0.5, X_u = 1.271$

Iteration 3:

$f(X_\ell) = -3.4725$

$f(X_u) = 0.259$

$$X_3 = 1.271 - \frac{0.259(0.5-1.271)}{-3.4725-0.259}$$

$X_3 = 1.2174$

$f(X_3) = \ln(1.2174^4) - 0.7 = 0.086$

Since $f(X_\ell) \cdot f(X_3) < 0$

$\Rightarrow X_\ell = 0.5, X_u = 1.2174$

After 3 iterations, the interval is

$[0.5, 1.2174]$

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Q3) We have the exact value of $f(0.5)$

$$f(0.5) = 0.5^4 e^{-3\times(0.5)^2}$$
$$= 0.0625 e^{-0.75}$$

Derivative $f(x) = x^4 e^{-3x^2}$.

$u = x^4$ $\quad u' = 4x^3$

$v = e^{-3x^2}$ $\quad v' = -6x e^{-3x^2}$

$$f'(x) = 4x^3 e^{-3x^2} - 6x^5 e^{-3x^2}$$

Evaluate the first order Taylor series at $x = 0.5$ where $x_0 = 1$

$$T_1(0.5) = f(1) + f'(1)(0.5 - 1)$$

$$T_1(0.5) = 1^4 e^{-3} + (4e^{-3} - 6e^{-3})(-0.5)$$
$$= \frac{1}{e^3} + e^{-3}$$

Exact value: $f(0.5) = 0.0625 e^{-0.75}$

Approximation: $T_1(0.5) = \frac{1}{e^3} + e^{-3}$

Comparison: -0.07

As you can see, there is a significant difference between the exact value and the approximation. Taylor series may not be very accurate for this specific function.

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Q4)

a) Analytical solution:

$$x^{3.5} = 80$$

$$x = \sqrt[3.5]{80} = 3.497$$

b) Initial guesses: $x_1 = 2$, $x_2 = 5$

Iteration 1: $f(x_1) = 2^{3.5} - 80 = -68.68$

$$f(x_2) = 5^{3.5} - 80 = 199.5$$

Using false position:

$$x_{new} = 5 - \frac{199.5 \times (5-2)}{199.5 + 68.68} = 2.768$$

$$f(x_{new}) = -44.71$$

Relative error:

$$E = \frac{|2.768 - 5|}{2.768} = 0.806$$

Since $E > 0.025$, we proceed to next iteration

Iteration 2:

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$x_{new} = 3.176$

$f(x_{new}) = -22.89$

Relative error : $E = \frac{|3.176 - 2.768|}{3.176} = 0.128$

Since $E > 0.025$, we proceed to next iteration

Iteration 3:

$x_{new} = 3.36$

$f(x_{new}) = -10.467$

Relative error: $E = \frac{|3.36 - 3.176|}{3.36} = 0.054$

Since $E > 0.025$, we proceed to next iteratio

Iteration 4:

$x_{new} = 3.44$

$f(x_{new}) = -4.49$

Relative error : $E = \frac{|3.44 - 3.36|}{3.44} = 0.023$

So $E < 0.025$, the approximation valu of the root is 3.44 with $E = 0.023$.

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Q2)

$f(x) = -12x^5 - 6.4x^3 + 12$

$f(x) = 0$

$\Rightarrow 12x^5 + 6.4x^3 - 12 = 0$

Let $x=0$: $f(0) = -12 < 0$

Let $x=1$: $f(1) = 6.4 > 0$

$f(0).f(1) < 0 \Rightarrow 0 < \text{Root} < 1$

First approximation root

$$x_1 = \frac{0+1}{2} = 0.5$$

$f(0.5) = -10.825$

$f(0.5).f(1) < 0 \Rightarrow 0.5 < \text{root} < 1$

Second approximation root

$$x_2 = \frac{0.5+1}{2} = 0.75$$

Relative error: $E = \left|\frac{0.75 - 0.5}{0.5}\right| \times 100\%$

$= 50\%$

$f(0.75) = -6.45$

$f(0.75).f(1) < 0 \Rightarrow 0.75 < \text{root} < 1$

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Third approximation:

$$X_3 = \frac{0.75+1}{2} = 0.875$$

Relative error: $E = \left|\frac{0.875-0.75}{0.75}\right| \times 100\%$

$= 16.6\%$

$f(0.875) = 1.55$

$f(0.875).f(1) < 0 \Rightarrow 0.875 < root < 1$

Fourth approximation:

$$X_4 = \frac{0.875+1}{2} = 0.937$$

$$E = \left|\frac{0.937-0.875}{0.875}\right| \times 100\% = 7.08\%$$

$f(0.937) = 1.96$

$f(0.937).f(0.875) < 0 \Rightarrow 0.875 < root < 0.937$

Fifth approximation:

$$X_5 = \frac{0.875+0.937}{2} = 0.906$$

$$E = \left|\frac{0.937-0.906}{0.906}\right| \times 100\% = 3.42\%$$

$\Rightarrow E < 5\%$