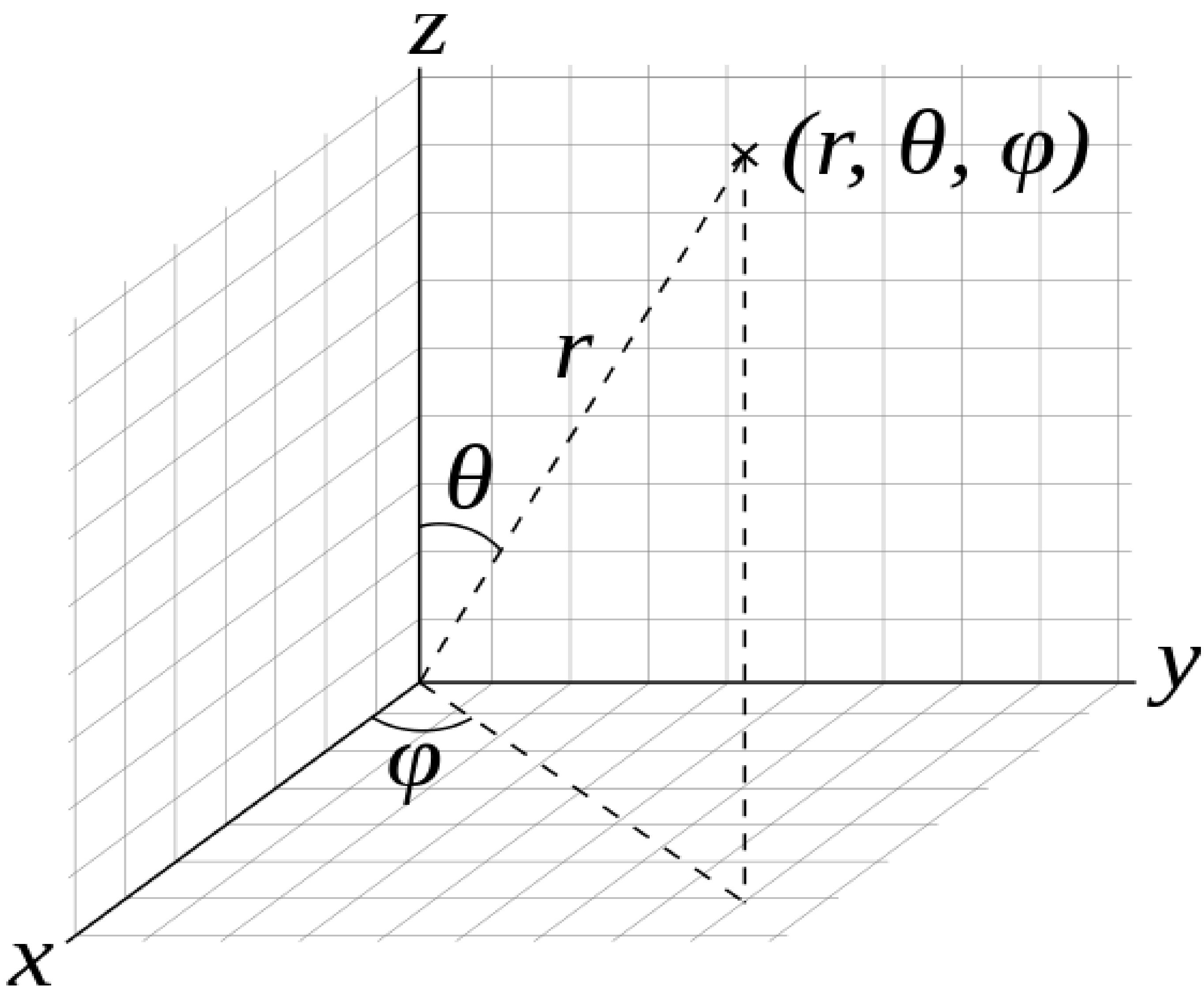
Derivations for *setArmPosition(r, θ, ϕ)* function



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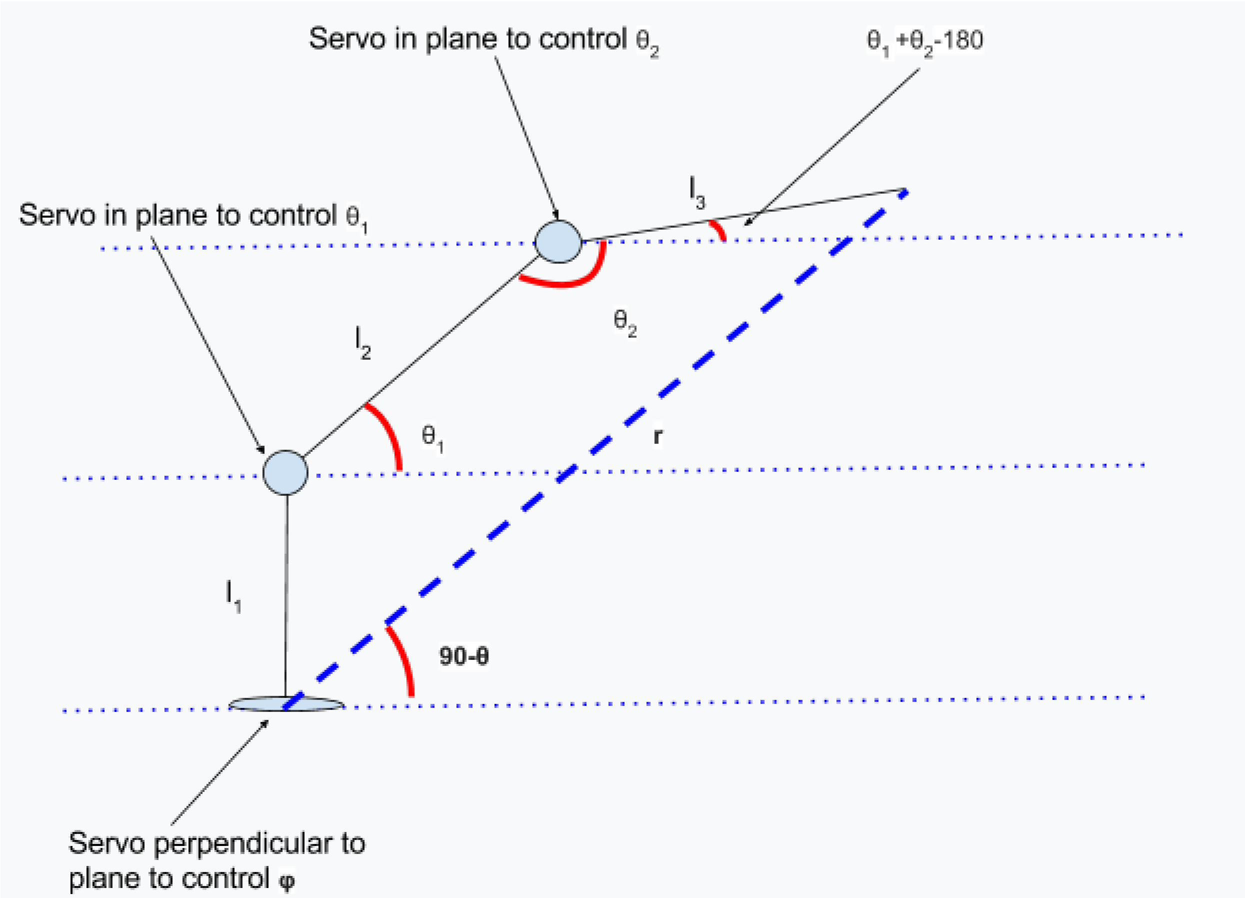
The Spherical Coordinate System : radial distance r, polar angle θ (theta), and azimuthal angle φ (phi).



2-D Representation of the Robotic Arm

For this project I am using the spherical coordinate system to denote points in 3D space.As φ can be set directly using a servo , an expression has to derived for θ_1 and θ_2 in terms of r and θ .

Derivation

Comparing vector components , we have :

$$rcos(90-\theta) = rsin(\theta) = l_2cos(\theta_1) + l_3cos(\theta_1+\theta_2-180)$$

$$\Rightarrow rsin(\theta) = l_2cos(\theta_1) - l_3cos(\theta_1+\theta_2) \quad \textbf{-(1)}$$

Similarly,

$$rcos(\theta) = l_1 + l_2sin(\theta_1) - l_3sin(\theta_1+\theta_2) \quad \textbf{-(2)}$$

Eliminating $cos(\theta_1+\theta_2)$, $sin(\theta_1+\theta_2)$,from (1) and (2):

$$\left(\frac{rcos(\theta)-l_1-l_2sin(\theta_1)}{-l_3}\right)^2 + \left(\frac{rsin(\theta)-l_2cos(\theta_1)}{-l_3}\right)^2 = 1$$

$$(rcos(\theta)-l_1-l_2sin(\theta_1))^2 + (rsin(\theta)-l_2cos(\theta_1))^2 = l_3^{\,2}$$

$$[r^2cos^2(\theta) + l_1^{\,2} + l_2^{\,2}sin^2(\theta_1) - 2rl_1cos(\theta) - 2rl_2cos(\theta)sin(\theta_1) + 2l_1l_2sin(\theta_1)] + [r^2sin^2(\theta)$$
$$+ l_2^{\,2}cos^2(\theta_1) - 2rl_2sin(\theta)cos(\theta_1)] = l_3^{\,2}$$

$$r^2 + l_1^{\,2} + l_2^{\,2} - l_3^{\,2} - 2rl_1cos(\theta) + (2l_1l_2 - 2rl_2cos(\theta))sin(\theta_1) + (-2rl_2sin(\theta))cos(\theta_1) = 0$$

$$-2l_2[(rcos(\theta)-l_1)sin(\theta_1) + rsin(\theta)cos(\theta_1)] = l_3^{\,2} - l_2^{\,2} - l_1^{\,2} - r^2 + 2rl_1cos(\theta)$$

$$(rcos(\theta)-l_1)sin(\theta_1) + rsin(\theta)cos(\theta_1) = \frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2}$$

$$\frac{rcos(\theta)-l_1}{\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}}sin(\theta_1) + \frac{rsin(\theta)}{\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}}cos(\theta_1) =$$
$$\frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}}$$

$$sin[\theta_1 + sin^{-1}(\frac{rsin(\theta)}{\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}})] = \frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}}$$

$$\theta_1 = sin^{-1}(\frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}}) - sin^{-1}(\frac{rsin(\theta)}{\sqrt{(rcos(\theta)-l_1)^2+(rsin(\theta))^2}})$$

$$\theta_1 = sin^{-1}(\frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}}) - sin^{-1}(\frac{rsin(\theta)}{\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}})$$

Similarly eliminating $cos(\theta_1)$, $sin(\theta_1)$

$$\theta_1 + \theta_2 = sin^{-1}(\frac{l_2^{\,2}-l_1^{\,2}-l_3^{\,2}-r^2+2rl_1cos(\theta)}{2l_3\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}}) - sin^{-1}(\frac{rsin(\theta)}{\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}})$$

$$\theta_2 = sin^{-1}(\frac{l_2^{\,2}-l_1^{\,2}-l_3^{\,2}-r^2+2rl_1cos(\theta)}{2l_3\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}}) - sin^{-1}(\frac{l_1^{\,2}+l_2^{\,2}-l_3^{\,2}+r^2-2rl_1cos(\theta)}{2l_2\sqrt{l_1^{\,2}+r^2-2rl_1cos(\theta)}})$$
