# EE338:: Filter Design assignment

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# Contents

Filter-1(Bandpass) Details	3
Un-normalized Discrete Time Filter Specifications	3
Normalized Digital Filter Specifications	3
Analog filter specifications for Band-pass analog filter using Bilinear Transformation	4
Frequency Transformation & Relevant Parameters	4
Frequency Transformed Lowpass Analog Filter Specifications	5
Analog Lowpass Transfer Function	5
Analog Bandpass Transfer Function:	6
Discrete Time Filter Transfer Function	7
FIR Filter Transfer Function using Kaiser Window	7
Filter-2(Bandstop) Details	8
Un-normalized Discrete Time Filter Specifications	8
Normalized Digital Filter Specifications	8
Analog filter specifications for Band-stop analog filter using Bilinear Transformation	9
Frequency Transformation & Relevant Parameters	9
Frequency Transformed Lowpass Analog Filter Specifications	10
Analog Lowpass Transfer Function	10
Analog Bandstop Transfer Function	12
Discrete Time Filter Transfer Function	12
FIR Filter Transfer Function using Kaiser Window	13
MATLAB PLOTS	15
Bandpass filter	15
IIR filter	15
FIR filter	17
Bandstop filter	19
IIR Filter	19
FIR Filter	21
Brief inferences/comparison of the FIR and IIR filters	22
Review of Report by Hitul Desai(18D070009)	22
Appendix and link to Programming codes used for producing the plots	23

# Filter-1(Bandpass) Details

#### Un-normalized Discrete Time Filter Specifications

Filter Number = 121

Since filter number is > 80, m = 121 - 80 = 41 and passband will be equiripple

$$q(m) = \lceil 0.1 * m \rceil - 1 = \lceil 4.1 \rceil - 1 = 4$$

$$r(m) = m - 10 * q(m) = 41 - 10 * 4 = 1$$

$$BL(m) = 25 + 1.7q(m) + 6.1r(m) = 25 + 1.7 * 4 + 6.1 * 1 = 37.9$$

$$BH(m) = BL(m) + 20 = 37.9 + 20 = 57.9$$

The first filter is given to be a **Band-Pass filter** with passband from BL(m) kHz to BH(m) kHz.

Therefore the specifications are :-

- Passband: 37.9 kHz to 57.9 kHz
- Transition band: 4 kHz on either side of passband
- Stopband: 0-33.9 kHz and 61.9 165 kHz (Since the sampling rate is 330 kHz)
- Tolerance: 0.15 in magnitude
- Passband Nature: Equiripple
- Stopband Nature: Monotonic

#### Normalized Digital Filter Specifications

Sampling Rate = 330 kHz.

In the normalized frequency axis, sampling rate corresponds to  $2\pi$ .

Thus, any frequency  $(\Omega)$  upto 165kHz  $\left(\frac{\text{SamplingRate}}{2}\right)$  can be represented on the normalized axis  $(\omega)$  as:-

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(\text{SamplingRate})}$$

Therefore the corresponding normalized discrete filter specifications are :-

- Passband:  $0.2297\pi$  to  $0.351\pi$
- Transition band:  $0.024\pi$  on either side of passband
- Stopband:  $0-0.205\pi$  and  $0.375\pi$   $\pi$
- Tolerance: 0.15 in magnitude
- Passband Nature: Equiripple
- Stopband Nature: Monotonic

# Analog filter specifications for Band-pass analog filter using Bilinear Transformation

The bilinear transformation is given as :-

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

ω	Ω
$0.2297\pi$	0.377
$0.351\pi$	0.615
$0.205\pi$	0.334
$0.375\pi$	0.668
0	0
$\pi$	$\infty$

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are:-

• **Passband:**  $0.377(\Omega_{P_1})$  to  $0.615(\Omega_{P_2})$ 

• Transition band: 0.334 to 0.377 & 0.615 to 0.668

• Stopband: 0-0.334 $(\Omega_{s_1})$  and 0.668 $(\Omega_{s_2})$  -  $\infty$ 

• Tolerance: 0.15 in magnitude for both passband and stopband.

• Passband Nature: Equiripple

• Stopband Nature: Monotonic

#### Frequency Transformation & Relevant Parameters

We need to transform a Band-Pass analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandpass transformation which is given as:-

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

The two parameters in the above equation are B and  $\Omega_0$ . They can be determined using the specifications of the bandpass analog filter using the following relations:-

Considering  $\Omega_{Lp} = 1$  on the passband edge,

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.377 * 0.615} = 0.4815$$

$$B = \Omega_{P_2} - \Omega_{P_1} = 0.615 - 0.377 = 0.238$$

Ω	$\Omega_L$
0+	$-\infty$
$0.334  (\Omega_{S_1})$	$-1.5131 \left(\Omega_{L_{S_1}}\right)$
$0.377 \left(\Omega_{P_1}\right)$	$-1\left(\Omega_{L_{P_{1}}}\right)$
$0.4815(\Omega_0)$	0
$0.615 \left(\Omega_{P_2}\right)$	$1\left(\Omega_{L_{P_2}}\right)$
$0.668  (\Omega_{S_2})$	$1.3484 \left(\Omega_{L_{S_2}}\right)$
$\infty$	$\infty$

#### Frequency Transformed Lowpass Analog Filter Specifications

• Passband Edge:  $1 (\Omega_{L_P})$ 

• Stopband:  $\min \left(-\Omega_{L_{S_1}}, \Omega_{L_{S_2}}\right) = \min(1.5131, 1.3484) = 1.3484 (\Omega_{L_S})$ 

• Tolerance: 0.15 in magnitude for both Passband and Stopband

• Passband Nature: Equiripple

• Stopband Nature: Monotonic

#### **Analog Lowpass Transfer Function**

We need an Analog Filter which has an equiripple passband and a monotonic stopband. Therefore we need to design using the **Chebyshev** approximation. since the tolerance ( $\delta$ ) in both passband and stopband is 0.15, we define two new quantities in the following way:-

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$
$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now choosing the parameter  $\epsilon$  of the Chebyshev filter to be  $\sqrt{D_1}$ , we get the min value of N as:-

$$N_{\min} = \lceil \frac{\cosh^{-1}(\sqrt{\frac{D_2}{D_1}})}{\cosh^{-1}(\frac{\Omega_{L_S}}{\Omega_{L_P}})} \rceil$$
$$N_{\min} = \lceil 3.761 \rceil = 4$$

Now, the poles of the transfer function can be obtained by solving the equation:

$$1 + D_1 \cosh^2 \left( N_{\min} \cosh^{-1} \left( \frac{s}{j} \right) \right) = 1 + 0.3841 \cosh^2 \left( 4 \cosh^{-1} \left( \frac{s}{j} \right) \right) = 0$$

Solving for the roots(using Wolfram) we get: Note that the above figure shows the poles of the Magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function(The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

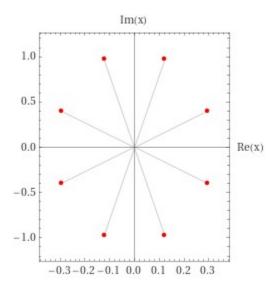


Figure 1: Roots of the denominator above

$$\begin{aligned} p_1 &= -0.12216 - 0.96981\iota \\ p_2 &= -0.12216 + 0.96981\iota \\ p_3 &= -0.29492 + 0.40171\iota \\ p_4 &= -0.29492 - 0.40171\iota \end{aligned}$$

Using the above poles which are in the left half plane and the fact that N is even we can write the Analog Lowpass Transfer Function as:-

$$H_{\text{analog},LPF}\left(s_{L}\right) = \frac{\left(-1\right)^{4} p_{1} p_{2} p_{3} p_{4}}{\sqrt{\left(1+D_{1}\right)\left(s_{L}-p_{1}\right)\left(s_{L}-p_{2}\right)\left(s_{L}-p_{3}\right)\left(s_{L}-p_{4}\right)}}$$

$$H_{\text{analog},\text{LPF}}\left(s_{L}\right) = \frac{0.198}{\left(s_{L}^{2}+0.24432 s_{L}+0.95545\right)\left(s_{L}^{2}+0.58984 s_{L}+0.24835\right)}$$

Note that since it is even order we take the DC Gain to be  $\frac{1}{\sqrt{1+\epsilon^2}}$ 

#### **Analog Bandpass Transfer Function:**

The transformation equation is given by :-

$$s_L = \frac{s^2 + \Omega_0^2}{Bs}$$

Substituting the values of the parameters B(0.238) and  $\Omega_0$ (0.4815), we get :-

$$s_L = \frac{s^2 + 0.2318}{0.238s}$$

Substituting this value into  $H_{\text{analog},LPF}(s_L)$  we get  $H_{\text{analog},BPF}(s)$  as:-

$$\frac{6.336*10^{-4}s^4}{\left(s^8+0.1977s^7+1.0034s^6+0.1458s^5+0.3585s^4+0.0338s^3+0.0539s^2+0.0025s^1+0.0029\right)}$$

#### Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as:-

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get  $H_{\text{discrete},BPF}(z)$  from  $H_{\text{analog},BPF}(s)$  as:-

$$\frac{0.00022 - 0.000907z^{-2} + 0.00136z^{-4} - 0.000907z^{-6} + 0.000226z^{-8}}{1.0000 + 4.7054z^{-1} + 11.9020z^{-2} + 19.3225z^{-3} + 22.0611z^{-4} + 17.8443z^{-5} + 10.1499z^{-6} + 3.7045z^{-7} + 0.7275z^{-8}}$$

#### FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15 Therefore  $\delta = 0.15$  and we get the minimum stopband attenuation to be:-

$$A = -20\log(0.15) = 16.4782dB$$

since A < 21, we get  $\beta$  to be 0 where  $\beta$  is the shape parameter of Kaiser window. Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \ge \frac{A - 7.95}{2.285 * \Delta\omega_T}$$

Here  $\Delta \omega_T$  is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{4kHz*2\pi}{330kHz} = 0.024\pi$$
$$N \ge 50$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB, it was found that a window length of 71 is required to satisfy the required constraints. Also, since  $\beta$  is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-pass impulse response samples were generated as the difference between two low-pass filters as done in class.

# Filter-2(Bandstop) Details

#### Un-normalized Discrete Time Filter Specifications

Filter Number = 121

Since filter number is > 80, m = 121 - 80 = 41 and passband will be monotonic

$$q(m) = \lceil 0.1 * m \rceil - 1 = \lceil 4.1 \rceil - 1 = 4$$

$$r(m) = m - 10 * q(m) = 41 - 10 * 4 = 1$$

$$BL(m) = 25 + 1.9q(m) + 4.1r(m) = 25 + 1.9 * 4 + 4.1 * 1 = 36.7$$

$$BH(m) = BL(m) + 20 = 36.7 + 20 = 56.7$$

The first filter is given to be a **Band-stop filter** with passband from BL(m) kHz to BH(m) kHz.

Therefore the specifications are :-

- **Stopband:** 36.7 kHz to 56.7 kHz
- Transition band: 4 kHz on either side of passband
- Passband: 0 32.7 kHz and 60.7 130 kHz (Since the sampling rate is 260 kHz)
- Tolerance: 0.15 in magnitude
- Passband Nature: Monotonic
- Stopband Nature: Monotonic

#### Normalized Digital Filter Specifications

Sampling Rate = 260 kHz.

In the normalized frequency axis, sampling rate corresponds to  $2\pi$ .

Thus, any frequency  $(\Omega)$  upto 130kHz  $\left(\frac{\text{SamplingRate}}{2}\right)$  can be represented on the normalized axis  $(\omega)$  as:-

$$\omega = \frac{\Omega * 2\pi}{\Omega_s(\text{SamplingRate})}$$

Therefore the corresponding normalized discrete filter specifications are :-

- Stopband:  $0.2823\pi$  to  $0.4362\pi$
- Transition band:  $0.0307\pi$  on either side of passband
- Passband:  $0-0.2515\pi$  and  $0.4669\pi$   $\pi$
- Tolerance: 0.15 in magnitude
- Passband Nature: Monotonic
- Stopband Nature: Monotonic

# Analog filter specifications for Band-stop analog filter using Bilinear Transformation

The bilinear transformation is given as:-

$$\Omega = \tan\left(\frac{\omega}{2}\right)$$

Applying the Bilinear transform to the frequencies at the band-edges, we get :-

$\omega$	Ω
$0.2823\pi$	0.4749
$0.4362\pi$	0.8173
$0.2515\pi$	0.4169
$0.4669\pi$	0.9011
0	0
$\pi$	$\infty$

Therefore the corresponding analog filter specifications for the same type of analog filter using the bilinear transformation are:-

• Stopband:  $0.4749(\Omega_{s_1})$  to  $0.8173(\Omega_{s_2})$ 

• Transition band: 0.4169 to 0.4749 & 0.8173 to 0.9011

• **Passband:** 0-0.4169( $\Omega_{p_1}$ ) and 0.9011( $\Omega_{p_2}$ ) -  $\infty$ 

• Tolerance: 0.15 in magnitude for both passband and stopband.

• Passband Nature: Monotonic

• Stopband Nature: Monotonic

#### Frequency Transformation & Relevant Parameters

We need to transform a Band-Stop analog filter to a Lowpass analog filter. We require two parameters in such a case. We can make use of the Bandstop transformation which is given as:-

$$\Omega_L = \frac{B\Omega}{\Omega_0^2 - \Omega^2}$$

The two parameters in the above equation are B and  $\Omega_0$ . They can be determined using the specifications of the bandpass analog filter using the following relations:-

$$\Omega_0 = \sqrt{\Omega_{P_1}\Omega_{P_2}} = \sqrt{0.4169 * 0.9011} = 0.6129$$

$$B = \Omega_{P_2} - \Omega_{P_1} = 0.9011 - 0.4169 = 0.4842$$

Ω	$\Omega_L$
0+	0+
$0.4169 \left(\Omega_{P_1}\right)$	$+1\left(\Omega_{L_{P_{1}}}\right)$
$0.4749  (\Omega_{S_1})$	$1.5318 \left(\Omega_{L_{S_1}}\right)$
$0.6129 \left(\Omega_0^-\right)$	$\infty$
$0.6129 \left(\Omega_0^+\right)$	$-\infty$
$0.8173 \left(\Omega_{S_2}\right)$	$-1.3537 \left(\Omega_{L_{S_2}}\right)$
$0.9011 (\Omega_{P_2})$	$-1\left(\Omega_{L_{P_2}}\right)$
$\infty$	0-

#### Frequency Transformed Lowpass Analog Filter Specifications

• Passband Edge:  $1 (\Omega_{L_P})$ 

• Stopband:  $\min (\Omega_{L_{S_1}}, -\Omega_{L_{S_2}}) = \min(1.5318, 1.3537) = 1.3537 (\Omega_{L_S})$ 

• Tolerance: 0.15 in magnitude for both Passband and Stopband

• Passband Nature: Monotonic

• Stopband Nature: Monotonic

#### **Analog Lowpass Transfer Function**

We need an Analog Filter which has a monotonic passband and a monotonic stopband. Therefore we need to design using the Butterworth approximation. since the tolerance ( $\delta$ ) in both passband and stopband is 0.15, we define two new quantities in the following way:-

$$D_1 = \frac{1}{(1-\delta)^2} - 1 = \frac{1}{0.85^2} - 1 = 0.3841$$
$$D_2 = \frac{1}{\delta^2} - 1 = \frac{1}{0.15^2} - 1 = 43.44$$

Now using the inequality on the order N of the filter for the Butterworth Approximation we get :-

$$N_{\min} = \left\lceil \frac{\log \sqrt{\frac{D_2}{D_1}}}{\log \frac{\Omega_{L_S}}{\Omega_{L_P}}} \right
ceil$$

$$N_{\min} = \lceil 7.806 \rceil = 8$$

The cut-off frequency  $(\Omega_c)$  of the Analog LPF should satisfy the following constraint:

$$\frac{\Omega_{L_P}}{D_1^{\frac{1}{2N}}} \le \Omega_c \le \frac{\Omega_{L_S}}{D_2^{\frac{1}{2N}}}$$

$$1.0616 \le \Omega_c \le 1.0694$$

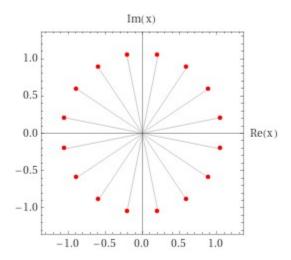


Figure 2: The roots calculated

Thus we can choose the value of  $\Omega_c$  to be 1.065 Now, the poles of the transfer function can be obtained by solving the equation:

$$1 + \left(\frac{s}{j\Omega_c}\right)^{2N} = 1 + \left(\frac{s}{j1.065}\right)^{16} = 0$$

Solving for the roots(using Wolfram) we get :-

Note that the above figure shows the poles of the magnitude Plot of the Transfer Function. In order to get a stable Analog LPF, we must include the poles lying in the Left Half Plane in the Transfer Function (The poles are symmetric about origin and we can pick one from each pair to be a part of our Transfer Function).

$$\begin{aligned} p_1 &= -(1.04454 + 0.20777\iota) \\ p_2 &= -(0.885515 + 0.591682\iota) \\ p_3 &= -(0.591682 + 0.885515\iota) \\ p_4 &= -(0.207771 + 1.04454\iota) \\ p_5 &= -(1.04454 - 0.20777\iota) \\ p_6 &= -(0.885515 - 0.591682\iota) \\ p_7 &= -(0.591682 - 0.885515\iota) \\ p_8 &= -(0.207771 - 1.04454\iota) \end{aligned}$$

Using the above poles which are in the left half plane we can write the Analog Lowpass Transfer Function as:-

$$H_{\text{analog},LPF}\left(s_{L}\right) = \frac{\left(\Omega_{c}\right)^{N}}{\left(s_{L} - p_{1}\right)\left(s_{L} - p_{2}\right)\left(s_{L} - p_{3}\right)\left(s_{L} - p_{4}\right)\left(s_{L} - p_{5}\right)\left(s_{L} - p_{6}\right)\left(s_{L} - p_{7}\right)\left(s_{L} - p_{8}\right)}$$

$$= \frac{1.655}{\left(s_{L}^{2} + 2.08908s_{L} + 1.1342\right)\left(s_{L}^{2} + 1.77103s_{L} + 1.1342\right)\left(s_{L}^{2} + 1.18336s_{L} + 1.1342\right)\left(s_{L}^{2} + 0.41542s_{L} + 1.1342\right)}$$

Note that the scaling of the numerator is done in order to obtain a DC gain of 1.

#### **Analog Bandstop Transfer Function**

The transformation equation is given by:-

$$s_L = \frac{Bs}{\Omega_0^2 + s^2}$$

Substituting the values of the parameters B(0.4842) and  $\Omega_0$ (0.6129), we get :-

$$s_L = \frac{0.4842s}{0.3756 + s^2}$$

Substituting this value into  $H_{\text{analog},LPF}(s_L)$  we get  $H_{\text{analog},BSF}(s)$ . It can be written in the form N(s)/D(s) where the coefficients of the polynomials N(s) and D(s) are given as:

		Degr	ee	$s^{16}$	$s^{15}$			$s^{14}$		$s^{13}$		
		Coef	ficient	$t \mid 1 (a_1)$	$1(a_{16})$ $2.3304(a_{15})$		$a_{15})   5$	$5.7203(a_{14})$		8.1803		
		Degree $s^1$		$s^{12}$	s <sup>11</sup>			$s^{10}$		$s^9$		
		Coefficient 11		11.1673	$.1673(a_{12})$ 11.18		$42(a_{11})$ $10.4786$		4786 (a	(10)	$7.717 (a_{5}$	9)
		Degree		$s^8$	$s^8$		$s^7$		$s^6$			
		Coefficient		5.288	$5.2889(a_8)$		$85 (a_7)$	) 1.4782 (a		0.59	$26(a_5)$	
D	Degree $s^4$			$s^3$		$s^2$		$s^1$		$s^0$		
Coefficient 0		0.22	$23 (a_4)$	0.061	$1(a_3)$	0.016	$1(a_2)$	0.002	$\overline{5(a_1)}$	0.0003	$39(a_0)$	

Table 1: Coefficients of D(s)

And that of N(s) is given as:

	Degree		$s^{16}$ $s^{14}$			$s^{12}$	$s^{10}$				
	Coefficient		$1(b_1)$	$(a_6)  3.0048  (b_1)$		$a_{14})  3.9501  (b_{12})$		$(b_{12})$	$(2.9673 (b_{10}))$		
Degre	e	$s^8$		$s^6$		$s^4$		$s^2$		$s^0$	
Coeffi	Coefficient 1.393		$(b_8)$	0.4	$4186 (b_6)$	0.0	$786 (b_4)$	0.00	$84 (b_2)$	0.000	$39 (b_0)$

Table 2: Coefficients of N(s)

#### Discrete Time Filter Transfer Function

To transform the analog domain transfer function into the discrete domain, we need to make use of the Bilinear Transformation which is given as:-

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

Using above equation we get  $H_{\text{discrete},BSF}(z)$  from  $H_{\text{analog},BSF}(s)$ . It can be written in the form N(z)/D(z) where the coefficients of the polynomials N(z) and D(z) are given as:

	Degree	Degree			$z^{-15}$		$z^{-14}$		$z^{-13}$		
	Coeffici	Coefficient 0.1876		<sub>-16</sub> )	1.3625 (b_	-15)	5.8303 (b	$o_{-14}) \   \ 1$	7.3986	$(b_{-13})$	
	Degree	Degree $z^{-12}$			$z^{-11}$		$z^{-10}$		$z^{-9}$		
	Coefficien	Coefficient $40.1497 (b_{-})$		12)	$74.3955(b_{-11})$		$114.0685 (b_{-10})$		$146.4941 (b_{-9})$		
	Degree	Degree $z^{-8}$		$z^{-7}$			$z^{-6}$		$z^{-5}$		
	Coefficier	Coefficient 159.2094 ( <i>b</i>		-8)	$_{-8})$ 146.4941 ( $b_{-}$		) $114.0685(b_{-6})$		$74.3955 (b_{-5})$		
D	Degree	$z^{-4}$	:	$z^{-3}$		$z^{-2}$		$z^{-1}$		$z^0$	
С	Coefficient 4		$1497 (b_{-4})$	17.5	$3986 (b_{-3})$	5.83	$03(b_{-2})$	$1.3625(b_{-1})$		0.1876 (	$b_0)$

Table 3 : Coefficients of N(z)

Similarly, for D(z) the coefficients can be written as:-

	Degree	$z^{-16}$		$z^{-15}$		$z^{-14}$		$z^{-13}$	
	Coefficien	t $0.03519 (a_{-})$	-16)	$0.3078 (a_{-})$	15)	1.5808 (a_	14)	$5.6825(a_{-}$	-13)
	Degree	$z^{-12}$	,	$z^{-11}$		$z^{-10}$		$z^{-9}$	
	Coefficient	15.8346 (a <sub>-12</sub>	2)	$35.5724 (a_{-1})$	1)	66.3731 (a_	10)	104.2023	$(a_{-9})$
	Degree	$z^{-8}$	2	z <sup>-7</sup>		$z^{-6}$		$z^{-5}$	
	Coefficient	$139.0501 (a_{-8})$	3)	$157.8972 (a_{-}$	7)	152.4757 (a	_6)	124.0205	$(a_{-5})$
	Degree	$z^{-4}$	$z^{-}$	3	$z^{-}$	2	$z^{-1}$	L	$z^0$
C	Coefficient	$83.8940(a_{-4})$	45.	$.8394(a_{-3})$	19	$.4381 (a_{-2})$	5.7	$796(a_{-1})$	$1(a_0)$

Table 4 : Coefficients of D(z)

#### FIR Filter Transfer Function using Kaiser Window

The tolerance in both the stopband and passband is given to be 0.15 . Therefore  $\delta=0.15$  and we get the minimum stopband attenuation to be :-

$$A = -20\log(0.15) = 16.4782dB$$

since A < 21, we get  $\beta$  to be 0 where  $\beta$  is the shape parameter of Kaiser window. Now to estimate the window length required, we use the empirical formula for the lower bound on the window length.

$$N \ge \frac{A - 8}{2.285 * \Delta \omega_T}$$

Here  $\Delta \omega_T$  is the minimum transition width. In our case, the transition width is the same on either side of the passband.

$$\Delta\omega_T = \frac{4kHz*2\pi}{260kHz} = 0.031\pi$$

$$N > 39$$

The above equation gives a loose bound on the window length when the tolerance is not very stringent. On successive trials in MATLAB, it was found that a window length of 56 is required to satisfy the required constraints. Also, since  $\beta$  is 0, the window is actually a rectangular window.

The time domain coefficients were obtained by first generating the ideal impulse response samples for the same length as that of the window. The Kaiser Window was generated using the MATLAB function and applied on the ideal impulse response samples. For generating the ideal impulse response a separate function was made to generate the impulse response of Low-Pass filter. It took the cutoff value and the number of samples as input argument. The band-stop impulse response samples were generated as the difference between three low-pass filters (all-pass - bandpass ) as done in class.

# MATLAB PLOTS

#### Bandpass filter

#### IIR filter

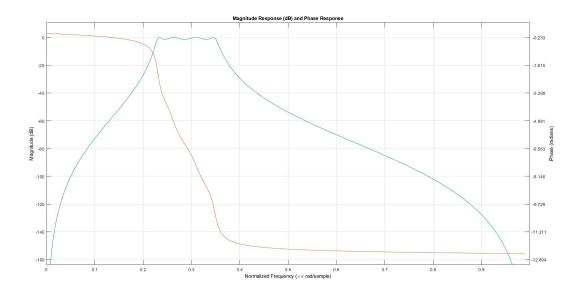


Figure 3: Chebyshev filter Magnitude and phase response

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the phase response is not linear.

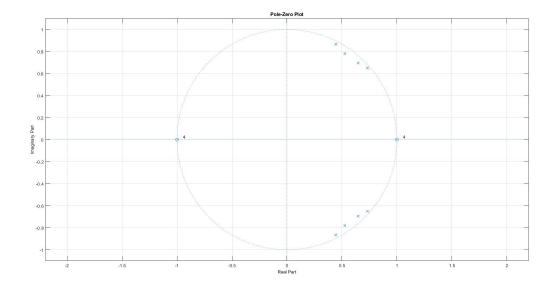


Figure 4: The transfer function, as expected is stable(All poles within unit circle)

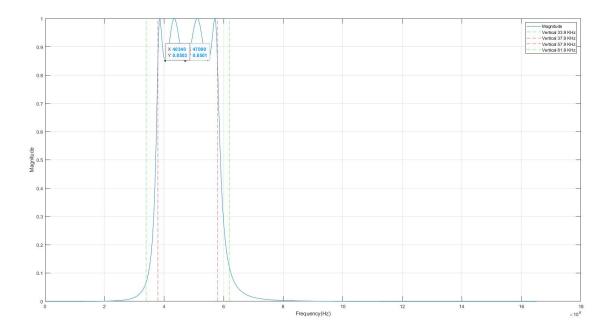


Figure 5: Caption

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

#### FIR filter

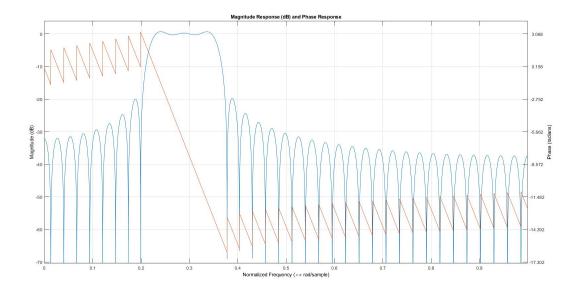


Figure 6: Phase And Magnitude Response for the FIR Kaiser filter

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter is indeed giving us a Linear Phase response which is desired.

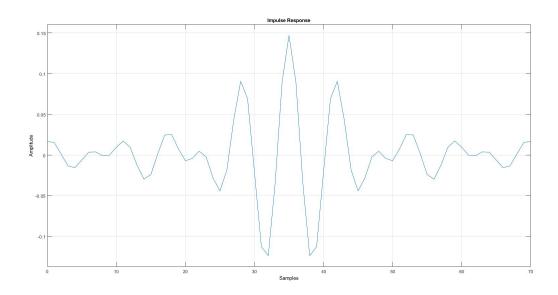


Figure 7: Impulse Response of the filter

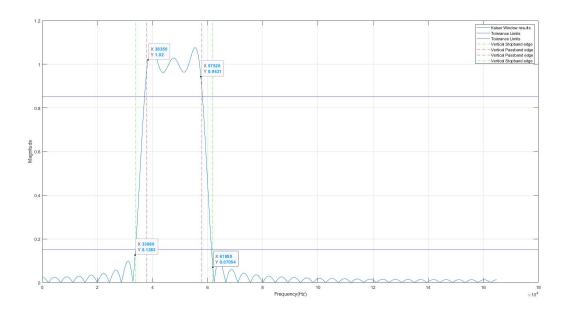


Figure 8: Frequency Magnitude Response plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

### Bandstop filter

#### IIR Filter

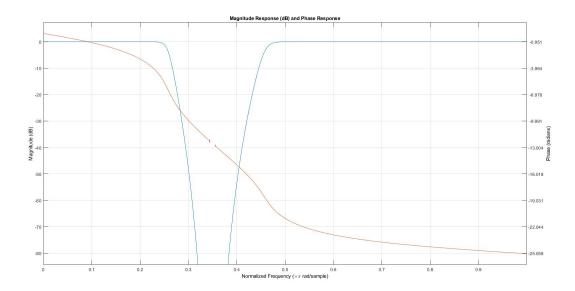


Figure 9: Frequency and Phase response of the filter

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the phase response is not linear.

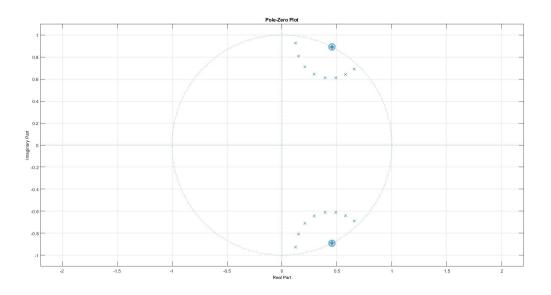


Figure 10: Pole-zero plot showing the stability of the filter

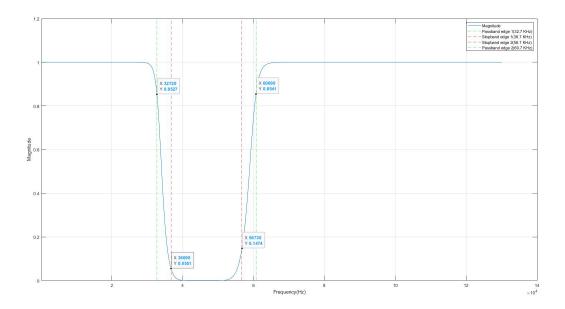


Figure 11: Final frequency response plot

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

#### FIR Filter

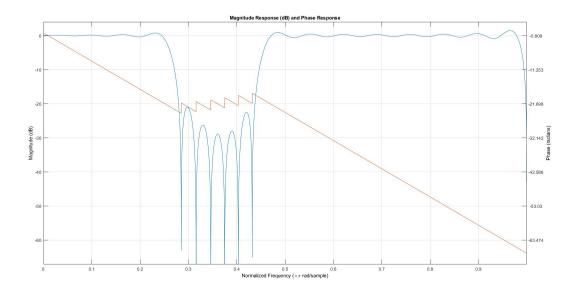


Figure 12: Phase-magnitude response for FIR filters

From the above plot, I have verified that the passband tolerance and stopband attenuation have been satisfied. It can be seen that the FIR Filter indeed gives us a Linear Phase response which is desired.

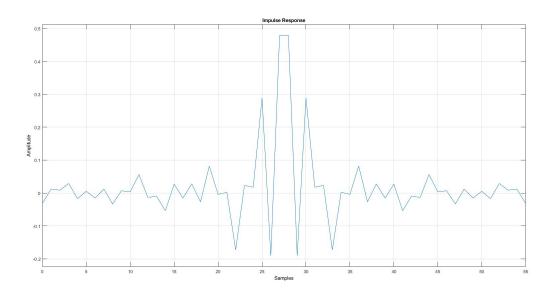


Figure 13: Time domain sequence values for the impulse response

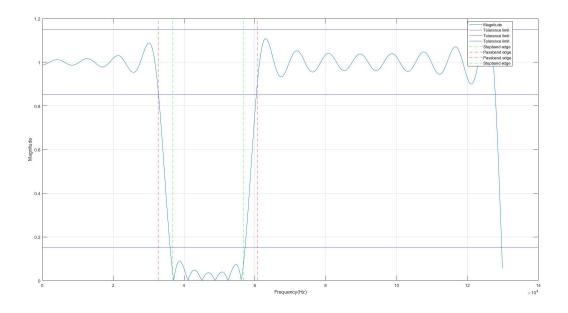


Figure 14: Frequency Response for the kaiser windowed filter

In the above plot, the band edge frequencies have been marked. From the magnitude at these frequencies it can be seen that the specifications required in the passband and the stopband have been met.

## Brief inferences/comparison of the FIR and IIR filters

The phase response is clearly a lot worse in case of an IIR filter. In the FIR filter the phase response is a lot better in the passband. However, the magnitude response of the IIR Filter is a lot better than that of a FIR filter and the order of the FIR filter used is higher, which indicates that more resources like summers, delay buffers and multipliers have to be used to realise this filter. More importantly, the FIR filter is non-causal albeit with a finite support hence it can be made causal using delay buffer which adds on to the existing demand of resources that are made by the FIR filter.

# Review of Report by Hitul Desai(18D070009)

- 1. All specifications of filters and the calculations have been verified to be correct.
- 2. Designer has met the frequency response specifications and this is clearly visible in the plots(Frequency response vs normalised angular frequency and Magnitude and phase plots) which are very well annotated with critical points and margins.
- 3. All the questions have been answered in great detail and a lot of attention to detail has been paid. Appreciate the designer's critical thinking around this design.

# Appendix and link to Programming codes used for producing the plots

The Plots of filter response was mainly produced in MATLAB. Besides them, I have used Wolfram Alpha to solve for complex roots. Also I had initially tried in python to calculate the coefficients of the polynomials involved.

The code is displayed in the following git repository:

https://github.com/AnDa-creator/EE338-Filter-Design-Assignment-2020