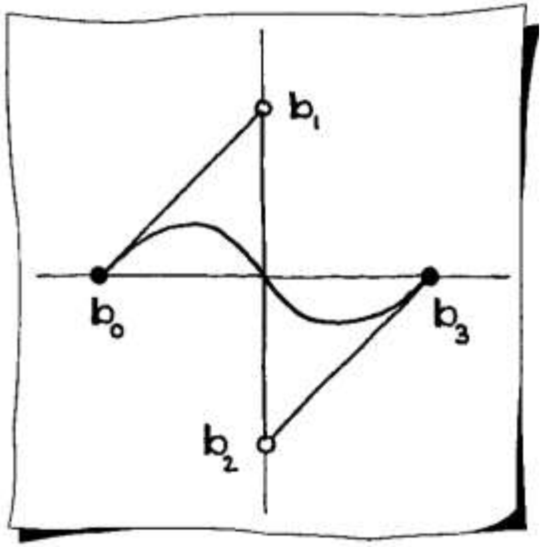
Cubic Bezier Polynomial (Added)

$$\boldsymbol{x}(t) = (1-t)^3\boldsymbol{b_0} + 3(1-t)^2 t\boldsymbol{b_1} + 3(1-t)t^2\boldsymbol{b_2} + t^3\boldsymbol{b_3}$$

Example $\mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -(1-t)^3 + t^3 \\ 3(1-t)^2 t - 3(1-t)t^2 \end{bmatrix}$



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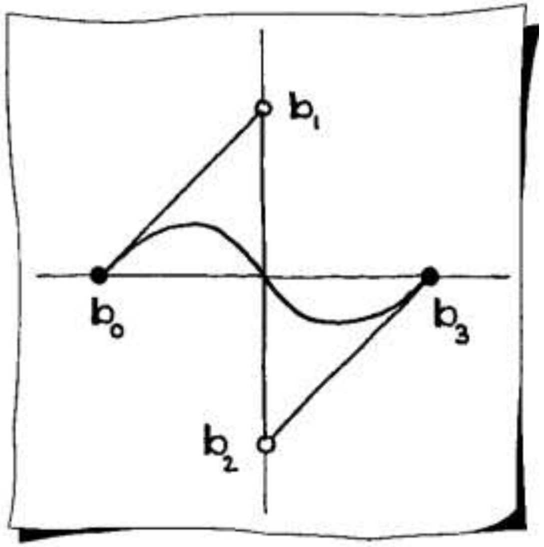
$$\Rightarrow \mathbf{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = (1-t)^3\begin{bmatrix} -1 \\ 0 \end{bmatrix} + 3(1-t)^2t\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 3(1-t)t^2\begin{bmatrix} 0 \\ -1 \end{bmatrix} + t^3\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$



Derivative of Cubic Bezier Polynomial (Added)

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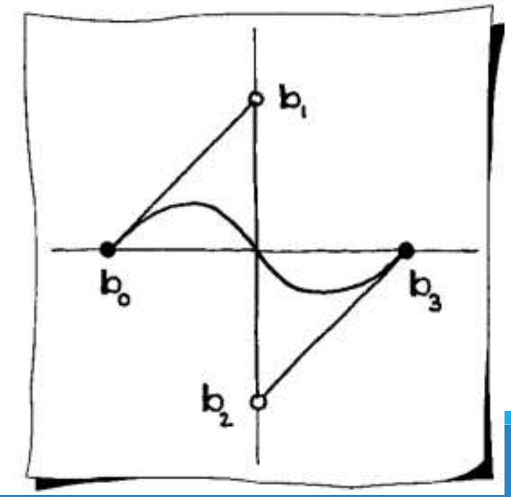

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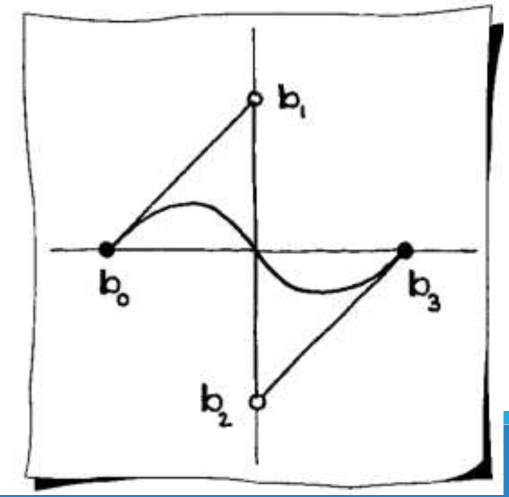
- Find the first derivative of $\mathbf{x}(t)$

$$\mathbf{x}'^{(t)} = \begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} (1-t)^2 + 6 \begin{bmatrix} 0 \\ -2 \end{bmatrix} (1-t)t + 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} t^2$$



Uniform Quadratic B-Spline (Added)

9. (15pt)

$$\mathbf{p_0} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad \mathbf{p_1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{p_2} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \mathbf{p_3} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{p_4} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

를 control points로 가지는 2차원 uniform 2차 B-spline 곡선 $\mathbf{f}(t)$는 다음과 같이 쓸 수 있다.

$$\mathbf{f}(t) = \sum_{i=0}^{4} \mathbf{p_i} N_i^2(t), \text{ where } N_i^2(t) = \begin{cases} \frac{1}{2}r^2, & r = t - \tau_i, \text{ if } t \in [\tau_i, \tau_{i+1}) \\ -r^2 + r + \frac{1}{2}, & r = t - \tau_{i+1}, \text{ if } t \in [\tau_{i+1}, \tau_{i+2}) \\ \frac{1}{2}(1-r)^2, & r = t - \tau_{i+2}, \text{ if } t \in [\tau_{i+2}, \tau_{i+3}) \\ 0, & \text{otherwise.} \end{cases}$$

(a) (4pt) $\tau_0 = 0$, $\tau_1 = 1$일 때, $\mathbf{f}(t)$의 knot sequence를 적으시오.

(b) (3pt) (a)의 knot sequence를 이용할 때, $\mathbf{f}(3.5)$의 값의 계산에 영향을 미치는 (즉, 0이 아닌) $N_i^2(t)$를 모두 쓰시오.

(c) (8pt) (a)의 knot sequence를 이용하여, $\mathbf{f}(3.5)$를 계산하시오.