



MapReduce: Matrix Calculation

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MATRIX ADDITION



Matrix Addition: $A + B = C$

■ Input

- Two matrices, A and B
- Formatted: a line has a single element
- E.g.,
 - $A, 3, 2, 4.3 \rightarrow A_{32} = 4.3$

■ Map

- Input: $\langle [A|B], i, j, \text{value} \rangle$
- Output: $\langle \text{key}=\underline{\hspace{2cm}}, \text{value}=\underline{\hspace{2cm}} \rangle$

■ Reduce

- Input: $\langle \text{key}, \text{a list of values} \rangle$
- Output: $\langle \text{key}=\underline{\hspace{2cm}}, \text{value}=\underline{\hspace{2cm}} \rangle$

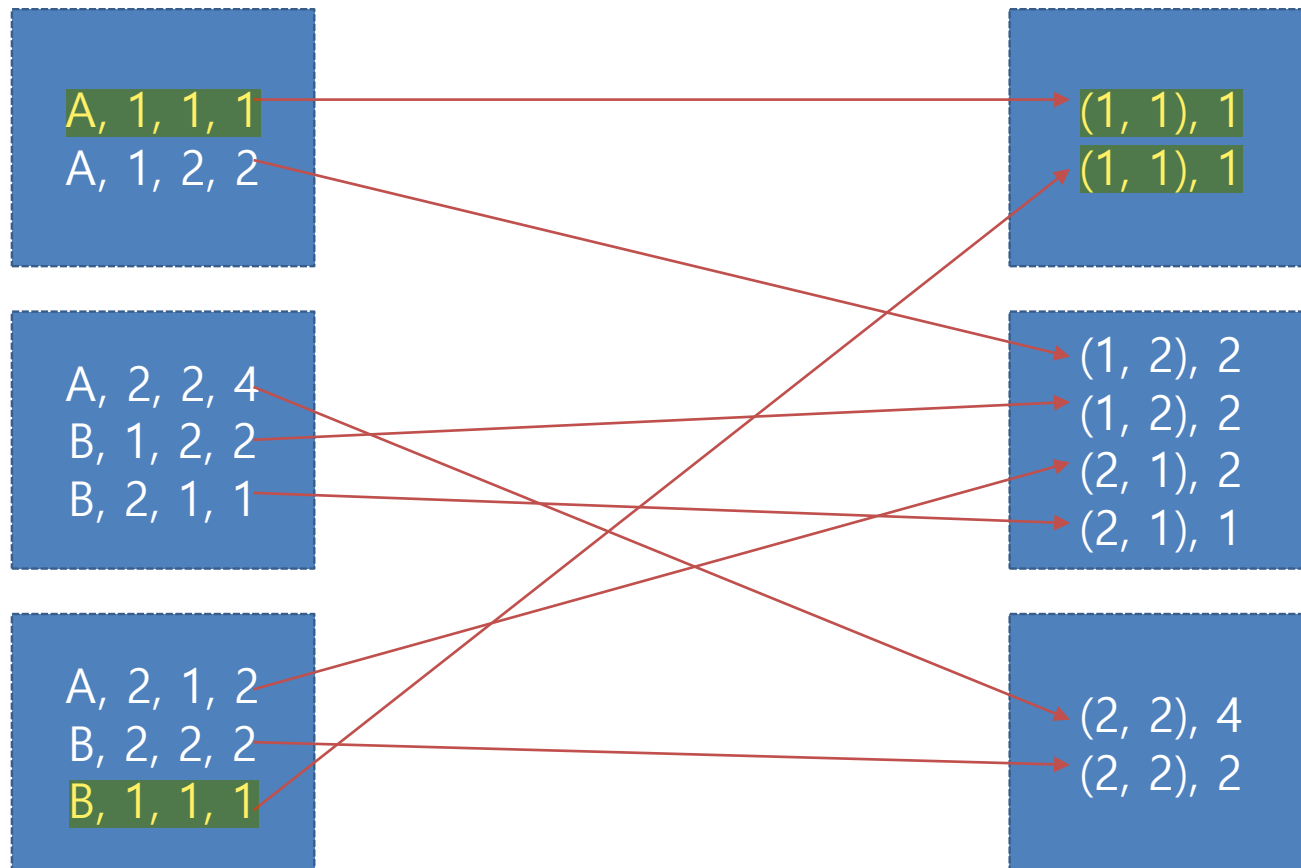


Matrix Addition: $A + B = C$

- Input
 - Two matrices, A and B
- Map
 - Input: $\langle [A|B], i, j, \text{value} \rangle$
 - Output: $\langle \text{key}=\{i,j\}, \text{value}=[A_{ij}|B_{ij}] \rangle$
- Reduce
 - Input: $\langle \text{key}, \text{a list of values}=\{A_{ij}, B_{ij}\} \rangle$
 - Output: $\langle \text{key}=\{i,j\}, \text{value}=A_{ij}+B_{ij} \rangle$

Matrix Addition: $A + B = C$

```
RDD.map(lambda t: ((t[1], t[2]), t[3]))
```



```
.reduceByKey(lambda a,b: a+b)
```



Thinking in MapReduce

1. 맵 함수의 입력데이터를 정의한다.
 - "A, 3, 2, 4.3"
2. 리듀스 함수의 키값을 정의한다.
 - Index (i, j) of output matrix C
3. 맵 함수에서 출력할 키-값 쌍을 정의한다.
 - ((3, 2), 4.3)
4. 리듀스 함수를 정의한다.
 - Sum of list

MATRIX-VECTOR MULTIPLICATION



Matrix Multiplication: $Av=b$

- Multiply a matrix and a vector
 - $A * v = b$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}$$



Thinking in MapReduce

1. 맵 함수의 입력데이터를 정의한다.
 - "A, 3, 2, 4.3" or "v, 1, 1.1"
2. 리듀스 함수의 키값을 정의한다.
 - The row number i of output vector b
3. 맵 함수에서 출력할 키-값 쌍을 정의한다.
4. 리듀스 함수를 정의한다.



MapReduce Algorithm

■ Input

- A n-by-m matrices A ($\rightarrow A, [\text{row \#}], [\text{col \#}], [\text{val}]$)
- A m-dimensional vector v ($\rightarrow v, [\text{row \#}], [\text{val}]$)
- E.g.,
 - $A, 3, 2, 4.3 \rightarrow a_{32} = 4.3$
 - $v, 10, 3.7 \rightarrow v_{10} = 3.7$

■ Map

- Input: $\langle [A|v], i, [j]?, \text{value} \rangle$
- Output: $\langle \text{key} = ?, \text{value} = ? \rangle$

■ Reduce

- Input: $\langle \text{key}, \text{a list of values} = \{?\} \rangle$
- Output: $\langle \text{key} = \text{the row number } i, \text{value} = (Av)_i \rangle$



Thinking in MapReduce

1. 맵 함수의 입력데이터를 정의한다.
 - "A, 3, 2, 4.3" or "v, 1, 1.1"
2. 리듀스 함수의 키값을 정의한다.
 - The row number i of output vector b
3. 맵 함수에서 출력할 키-값 쌍을 정의한다.
 - $(i, \text{all data to compute } b[i] \text{ if it has})$
4. 리듀스 함수를 정의한다.
 - Compute $b[i]$

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} =$$

$\langle 1, (A, 1, a_{11}) \rangle$

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

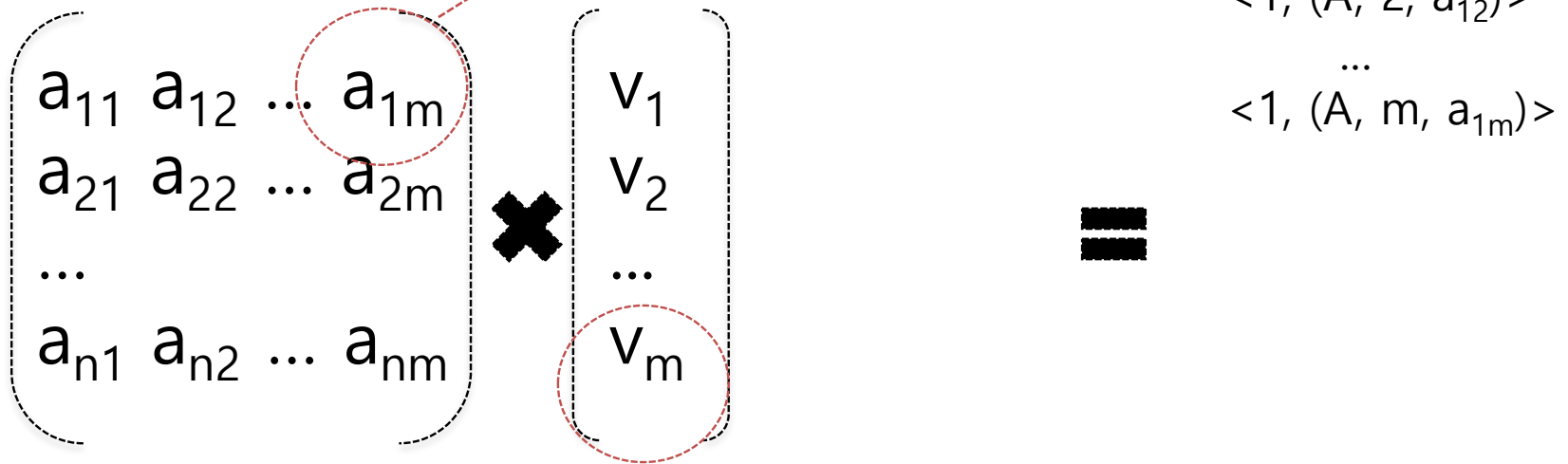
$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \dots \\ \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \end{pmatrix}$$



Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$


$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, m, a_{1m}) \rangle \end{pmatrix}$$

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, m, a_{1m}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \dots \\ \langle 2, (v, 1, v_1) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \end{pmatrix}$$

A red dashed arrow points from the circled v_1 in the vector to the first row of the resulting vector.

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, m, a_{1m}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \end{pmatrix}$$

A red dashed circle highlights the element v_2 in the vector v . A red dashed arrow points from this circle to the second element of the first row of the resulting vector, $\langle 1, (A, 2, a_{12}) \rangle$.

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, m, a_{1m}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle 1, (v, m, v_m) \rangle \\ \langle 2, (v, 1, v_1) \rangle \\ \langle 2, (v, 2, v_2) \rangle \\ \dots \\ \langle 2, (v, m, v_m) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, m, v_m) \rangle \end{pmatrix}$$

The diagram illustrates the multiplication of a matrix A (with elements a_{ij}) and a vector v (with elements v_i). The result is a vector of inner products. A red dashed arrow highlights the calculation of the first row's inner product, showing the dot product of the first row of the matrix with the vector v .

Matrix Multiplication

- Multiply a matrix and a vector

— $A * v$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix} \times \begin{pmatrix} v_1 \\ v_2 \\ \dots \\ v_m \end{pmatrix} = \begin{pmatrix} \langle 1, (A, 1, a_{11}) \rangle \\ \langle 1, (A, 2, a_{12}) \rangle \\ \dots \\ \langle 1, (A, m, a_{1m}) \rangle \\ \langle 1, (v, 1, v_1) \rangle \\ \langle 1, (v, 2, v_2) \rangle \\ \dots \\ \langle 1, (v, m, v_m) \rangle \\ \langle 2, (v, 1, v_1) \rangle \\ \langle 2, (v, 2, v_2) \rangle \\ \dots \\ \langle 2, (v, m, v_m) \rangle \\ \dots \\ \langle n, (v, 1, v_1) \rangle \\ \langle n, (v, 2, v_2) \rangle \\ \dots \\ \langle n, (v, m, v_m) \rangle \end{pmatrix}$$



MapReduce Algorithm

■ Map

- Input: $\langle [A|v], i, [j]?, [a_{ij} \mid v_i] \rangle$
- Output:
 - If "A": Output $\langle \text{key}=i, \text{value}= ("A", j, a_{ij}) \rangle$
 - Else if "v":
 - ◆ For $k=1$ to n :
 - » Output $\langle \text{key}=k, \text{value}= ("v", i, v_i) \rangle$

■ Reduce

- Input: $\langle \text{key}=i,$
a list of values $= \{(A, j, a_{ij}) \mid (v, j, v_j)\}_{j=1, \dots, m} \rangle$
- Output: $\langle \text{key}=i, \text{value} = \sum_{j=1}^m a_{ij} \cdot v_j \rangle$



Thinking in MapReduce

1. 맵 함수의 입력데이터를 정의한다.
 - "A, 3, 2, 4.3" or "v, 1, 1.1"
2. 리듀스 함수의 키값을 정의한다.
 - The row number i of output vector b
3. 맵 함수에서 출력할 키-값 쌍을 정의한다.
 - map
4. 리듀스 함수를 정의한다.
 - reduce

MATRIX-MATRIX MULTIPLICATION



Matrix Multiplication: $A \times B = C$

Map

— Input

- Two matrices, A ($n \times \ell$) and B ($\ell \times m$)
- E.g.,
 - ♦ $A, 2, 3, 3.2 \rightarrow a_{23} = 3.2$
 - ♦ $B, 2, 3, 3.2 \rightarrow b_{23} = 3.2$

— Output

- For (A, i, j, a_{ij}) : ?
- For (B, i, j, b_{ij}) : ?

Reduce

— Input

- $\langle \text{key}=?, ? \rangle$

— Output

- $\langle \text{key}=?, ? \rangle$



Matrix Multiplication: $A \times B = C$

Map

— Input

- Two matrices, A ($n \times \ell$) and B ($\ell \times m$)
- E.g.,
 - ♦ $A, 2, 3, 3.2 \rightarrow a_{23} = 3.2$
 - ♦ $B, 2, 3, 3.2 \rightarrow b_{23} = 3.2$

— Output

- For (A, i, j, a_{ij}) : ?
- For (B, i, j, b_{ij}) : ?

Reduce

— Input

- $\langle \text{key}=\{i,j\}, \{?\} \rangle$

— Output

- $\langle \text{key}=\{i,j\}, ? \rangle$



Matrix Multiplication

- A matrix: row 1 \rightarrow C matrix: row 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- A matrix: row 1 -> C matrix: row 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

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Matrix Multiplication

- A matrix: row 2 \rightarrow C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- A matrix: row 2 -> C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- A matrix: row 2 -> C matrix: row 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- A matrix i-th row \rightarrow C matrix i-th row
— key (i, 1), (i, 2), ... (i, m)

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 1 \rightarrow C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 1 \rightarrow C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 1 \rightarrow C matrix: col 1

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 2 -> C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 2 \rightarrow C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix: col 2 \rightarrow C matrix: col 2

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Matrix Multiplication

- B matrix j-th col \rightarrow C matrix j-th col
 - key $(1, j), (2, j), \dots (n, j)$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & & & \\ a_{n1} & a_{n2} & \dots & a_{nk} \end{pmatrix} \times \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & & & \\ b_{k1} & b_{k2} & \dots & b_{km} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1m} \\ c_{21} & c_{22} & \dots & c_{2m} \\ \dots & & & \\ c_{n1} & c_{n2} & \dots & c_{nm} \end{pmatrix}$$



Solution

■ Map

— Input

- Two matrices, A ($n \times l$) and B ($l \times m$)

— Output

- For A_{ij} : $\langle \{i, 1\}, \{j, A_{ij}\} \rangle, \dots, \langle \{i, m\}, \{j, A_{ij}\} \rangle$
- For B_{ij} : $\langle \{1, j\}, \{i, B_{ij}\} \rangle, \dots, \langle \{n, j\}, \{i, B_{ij}\} \rangle$

■ Reduce

— Input

- $\langle \{i, j\}, \{?\} \rangle$

— Output

- $\langle \{i, j\}, ? \rangle$



Solution

■ Map

— Input

- Two matrices, A ($n \times l$) and B ($l \times m$)

— Output

- For A_{ij} : $\langle \{i, 1\}, \{j, A_{ij}\} \rangle, \dots, \langle \{i, m\}, \{j, A_{ij}\} \rangle$
- For B_{ij} : $\langle \{1, j\}, \{i, B_{ij}\} \rangle, \dots, \langle \{n, j\}, \{i, B_{ij}\} \rangle$

■ Reduce

— Input

- $\langle \{i, j\}, \{\{1, A_{i1}\}, \{1, B_{1j}\}, \dots, \{l, A_{il}\}, \{l, B_{lj}\}\} \rangle$

— Output

- $\langle \{i, j\}, A_{i1} * B_{1j} + A_{i2} * B_{2j} + \dots + A_{il} * B_{lj} \rangle$

Matrix Multiplication: Example

- a_{ik}
 - key $(i, 1), (i, 2), \dots (i, m)$
 - value (k, a_{ik})

A 1 1 3

key (1,1) value (1,3)

key (1,2) value (1,3)

A 1 2 -5

A 2 1 6

A 2 2 12

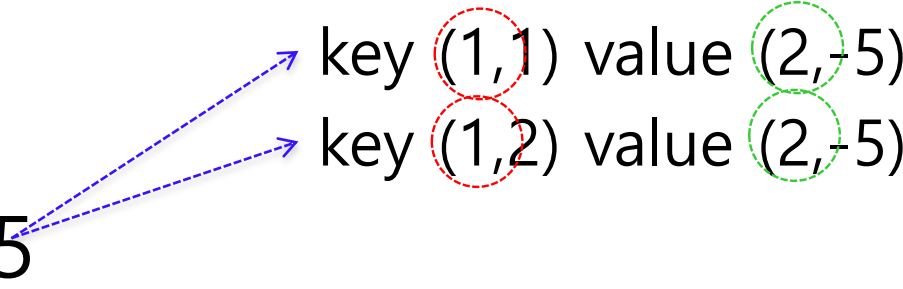
$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- a_{ik}
 - key $(i, 1), (i, 2), \dots (i, m)$
 - value (k, a_{ik})

A 1 1 3
A 1 2 -5
A 2 1 6
A 2 2 12

key (1,1) value (2,-5)
key (1,2) value (2,-5)



$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- a_{ik}
 - key $(i, 1), (i, 2), \dots (i, m)$
 - value (k, a_{ik})

A 1 1 3

A 1 2 -5

A 2 1 6

A 2 2 12

key (2,1) value (1,6)

key (2,2) value (1,6)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- a_{ik}
 - key $(i, 1), (i, 2), \dots (i, m)$
 - value (k, a_{ik})

A 1 1 3

A 1 2 -5

A 2 1 6

A 2 2 12

key (2,1) value (2,12)

key (2,2) value (2,12)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- b_{kj}
 - key $(1, j), (2, j), \dots (n, j)$
 - value (k, b_{kj})

B $\begin{matrix} \textcircled{1} & \textcircled{1} & 2 \end{matrix}$ $\begin{matrix} \nearrow \text{key } (1, 1) \text{ value } \textcircled{(1, 2)} \\ \searrow \text{key } (2, 1) \text{ value } \textcircled{(1, 2)} \end{matrix}$

B 1 2 11

B 2 1 1

B 2 2 -7

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- b_{kj}
 - key $(1, j), (2, j), \dots (n, j)$
 - value (k, b_{kj})

B 1 1 2
B 1 2 1 1
B 2 1 1
B 2 2 -7

key (1,2) value (1,11)
key (2,2) value (1,11)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- b_{kj}
 - key $(1, j), (2, j), \dots (n, j)$
 - value (k, b_{kj})

B 1 1 2
B 1 2 11
B 2 1 1
B 2 2 -7

key (1, 1) value (2, 1)
key (2, 1) value (2, 1)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

- b_{kj}
 - key $(1, j), (2, j), \dots (n, j)$
 - value (k, b_{kj})

B 1 1 2

B 1 2 11

B 2 1 1

B 2 2 -7

key (1, 2) value (2, -7)

key (2, 2) value (2, -7)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$



Matrix Multiplication

A emitted
key value

(1,1) (1,3)
(1,2) (1,3)
(1,1) (2,-5)
(1,2) (2,-5)
(2,1) (1,6)
(2,2) (1,6)
(2,1) (2,12)
(2,2) (2,12)

B emitted
key value

(1,1) (1,2)
(2,1) (1,2)
(1,2) (1,11)
(2,2) (1,11)
(1,1) (2,1)
(2,1) (2,1)
(1,2) (2,-7)
(2,2) (2,-7)

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$

Matrix Multiplication

A emitted
key value

(1,1) (1,3)
(1,2) (1,3)
(1,1) (2,-5)
(1,2) (2,-5)
(2,1) (1,6)
(2,2) (1,6)
(2,1) (2,12)
(2,2) (2,12)

B emitted
key value

(1,1) (1,2)
(2,1) (1,2)
(1,2) (1,11)
(2,2) (1,11)
(1,1) (2,1)
(2,1) (2,1)
(1,2) (2,-7)
(2,2) (2,-7)

key

(1,1)

<(1,3) , (2,-5), (1, 2), (2, 1)>

(1,2)

<(1,3), (2,-5), (1, 11), 2,-7)>

(2,1)

<(1,6) , (2,12), (1, 2), (2, 1)>

(2,2)

<(1,6) , (2,12), (1, 11), (2, -7)>

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} =$$



Matrix Multiplication

A emitted
key value

(1,1) (1,3)
(1,2) (1,3)
(1,1) (2,-5)
(1,2) (2,-5)
(2,1) (1,6)
(2,2) (1,6)
(2,1) (2,12)
(2,2) (2,12)

B emitted
key value

(1,1) (1,2)
(2,1) (1,2)
(1,2) (1,11)
(2,2) (1,11)
(1,1) (2,1)
(2,1) (2,1)
(1,2) (2,-7)
(2,2) (2,-7)

key

value list

(1,1) <(1,3), (2,-5), (1,2), (2,1)>

$$6 - 5 = 1$$

(1,2) <(1,3), (2,-5), (1,11), (2,-7)>

$$33 + 35 = 68$$

(2,1) <(1,6), (2,12), (1,2), (2,1)>

$$12 + 12 = 24$$

(2,2) <(1,6), (2,12), (1,11), (2,-7)>

$$66 - 84 = -18$$

$$\begin{pmatrix} 3 & -5 \\ 6 & 12 \end{pmatrix} \times \begin{pmatrix} 2 & 11 \\ 1 & -7 \end{pmatrix} = \begin{pmatrix} 1 & 68 \\ 24 & -18 \end{pmatrix}$$



Thinking in MapReduce

1. 맵 함수의 입력데이터를 정의한다.
2. 리듀스 함수의 키값을 정의한다.
3. 맵 함수에서 출력할 키-값 쌍을 정의한다.
4. 리듀스 함수를 정의한다.