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# Real-Time Systems

L. Sha et. al., "Priority Inheritance Protocols : An Approach to Real-Time Synchronization", IEEE Transactions on Computers, 39(9), 1990

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- Background and Motivation
  - Objectives
  - The Priority Inversion Problem
  - PIP (Priority Inheritance Protocol)
  - PCP (Priority Ceiling Protocol)
  - Schedulability Analysis
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# Background and Motivation

- A direct application of commonly used synchronization primitives such as semaphores, monitors, or the Ada rendezvous can lead to uncontrolled priority inversion.
    - a situation in which a higher priority job is blocked by lower priority jobs for an indefinite period of time
  - Priority inversion is a serious problem in real-time systems by adversely affecting both the schedulability and predictability of real-time systems.
    - Sources of priority inversion
      - non-preemptible regions of code
      - interrupts
      - non-unique priorities for some tasks (if there are not enough priority levels)
      - FIFO queues
      - synchronization and mutual exclusion
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# Objectives

- Investigate the synchronization problem in the context of priority-driven preemptive scheduling
  - Show that both protocols solve this uncontrolled priority inversion problem
    - PIP (Priority Inheritance Protocol)
    - PCP (Priority Ceiling Protocol)
    - To prevent unbounded priority inversion
  - Schedulability Test with RMS
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# Priority Inversion Problem

## ■ Terms

### □ Priority inversion

- Phenomenon where a higher priority job is blocked by lower priority jobs
- Common situation arises when two jobs attempt to access shared data.
  - To maintain consistency, the access must be serialized.

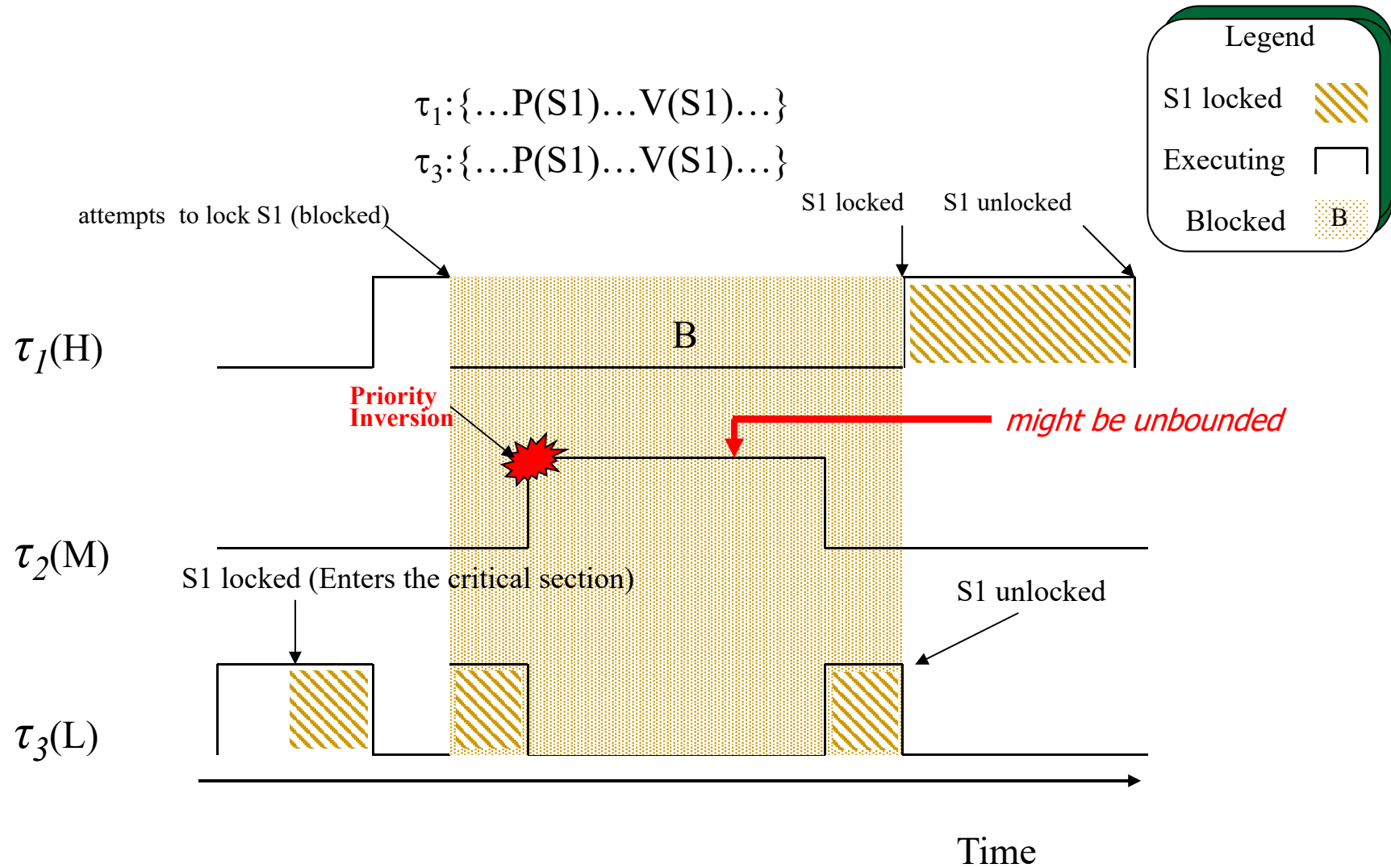
### □ Blocking

- A form of priority inversion where a higher priority job must wait for the processing of a lower priority job

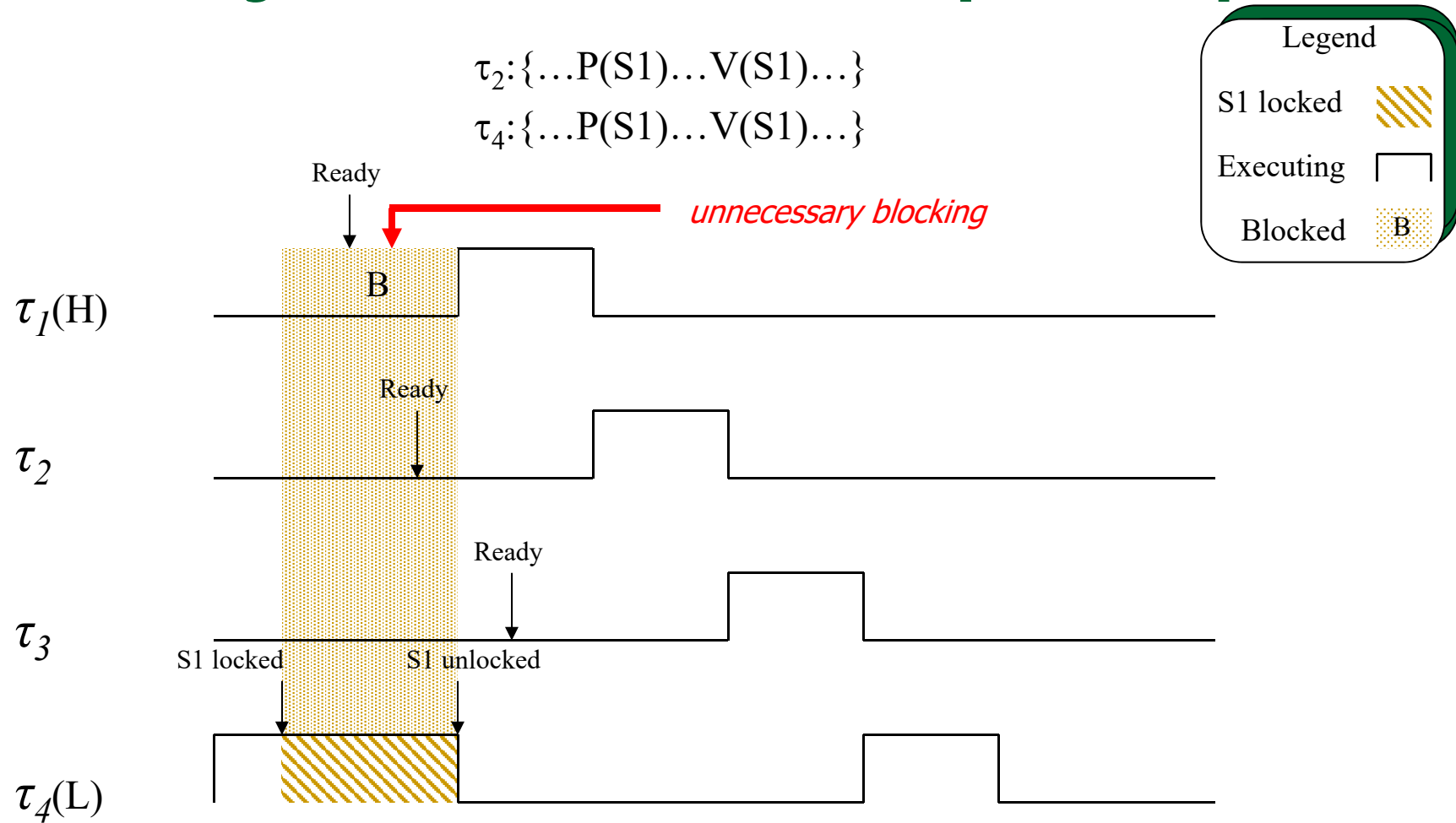
## ■ Critical sections are sections of code that use a resource, during which other tasks must not use that resource.

- It is possible to guard critical sections by locking semaphores
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# Priority Inversion: Illustrated



# Priority Inversion: Non-preemptive



Disallow preemption during the execution of all critical sections!!

# Highest-Lockers Priority Protocol

$\tau_2: \{\dots P(S1) \dots V(S1) \dots\}$

$\tau_4: \{\dots P(S1) \dots V(S1) \dots\}$

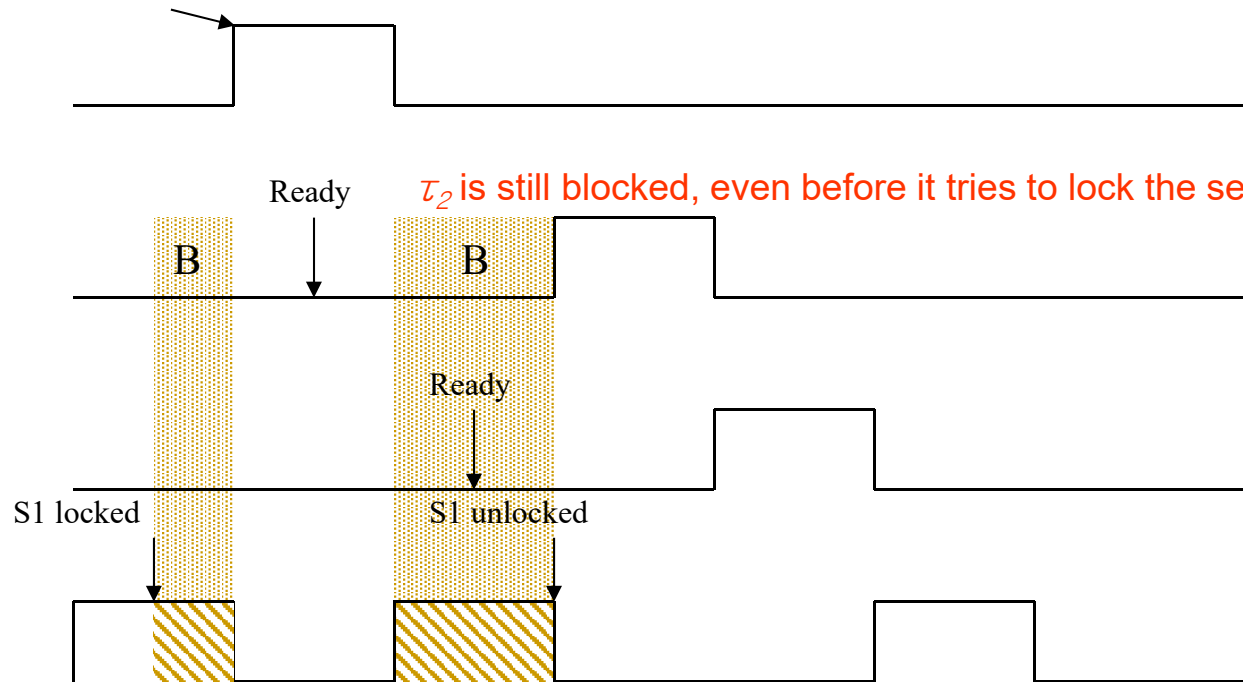
High-priority tasks that do not lock the semaphore are no longer blocked

$\tau_1(H)$

$\tau_2$

$\tau_3$

$\tau_4(L)$



Legend

S1 locked



Executing



Blocked



Execute  $\tau_4$  using the priority of the highest-priority task that may lock the semaphore ( $\tau_2$ )



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# Problem Statement

## ■ Notations

- $J_i$  : job
  - $z_{i,j}$  :  $j^{\text{th}}$  critical section of job  $J_i$
  - $S_{i,j}$  : semaphore guarding the critical section  $z_{i,j}$
  - $R_{i,j}$  : resource associated with  $z_{i,j}$
  - $z_{i,j} \subset z_{i,k} : z_{i,j}$  is entirely contained in  $z_{i,k}$
-

# Assumptions

- Jobs  $J_1, J_2, \dots, J_n$  are listed in **descending order of nominal priority**, with  $J_1$  having the highest nominal priority
- Jobs do not suspend themselves
- The **critical sections** used by any task **are properly nested**
  - that is, given any pair  $z_{i,j}$  and  $z_{i,k}$ , then either  $z_{i,j} \subset z_{i,k}$ ,  $z_{i,k} \subset z_{i,j}$ , or  $z_{i,j} \cap z_{i,k} = \emptyset$
- Critical sections are guarded by binary semaphores.
  - This means that **only one job at a time** can be within the critical section corresponding to a particular semaphore  $S_k$

# Basic Priority Inheritance Protocol (PIP)

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# Basic Inheritance Protocol (BIP)

## ■ Basic idea

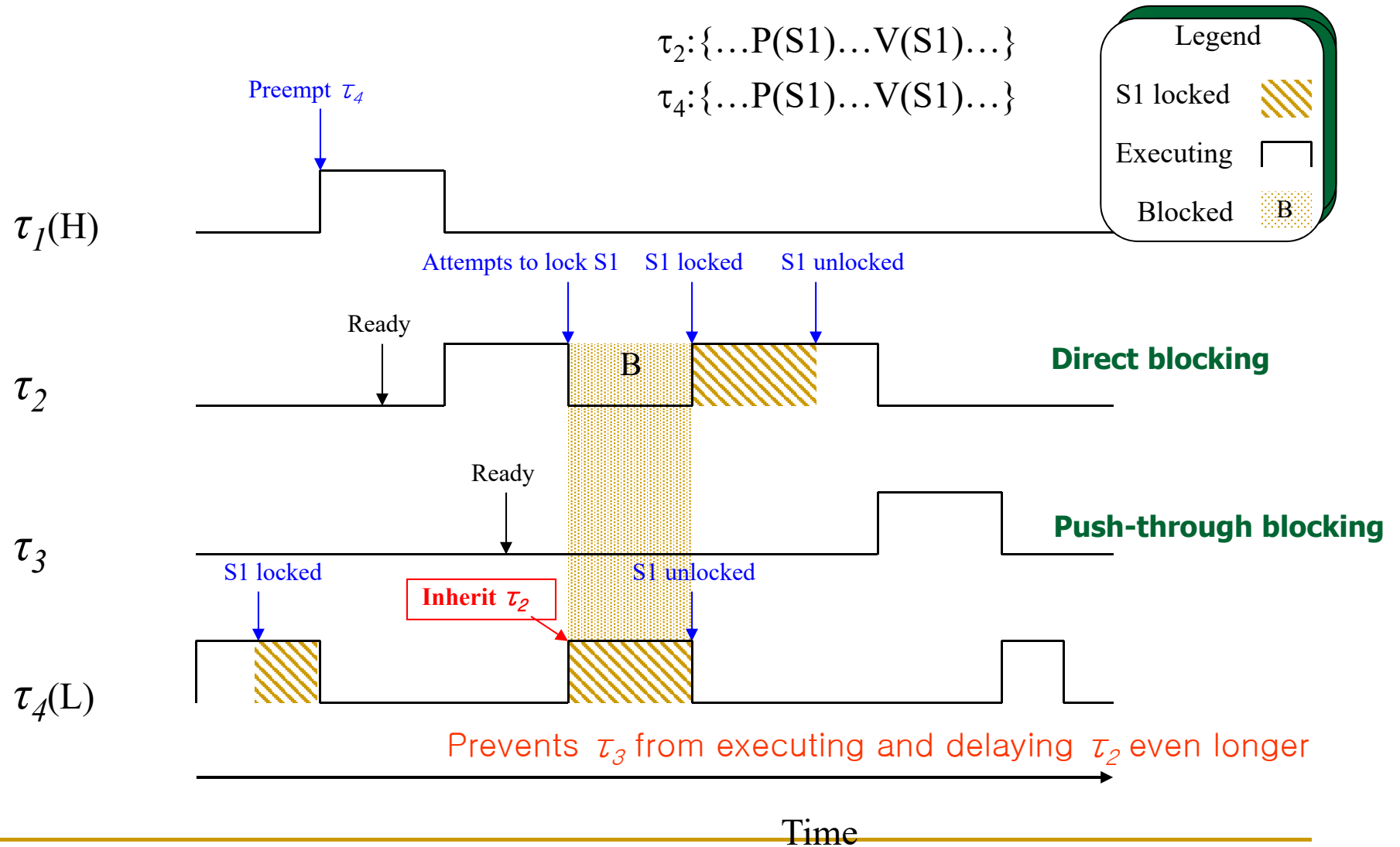
- When a job J blocks one or more higher priority jobs, it **ignores its original priority** assignment and **executes its critical section at the highest priority level of all the jobs it blocks**
  - Allows lower-priority tasks to temporarily inherit higher priorities: *if they are executing within a critical section* and *if they are blocking a higher-priority task*.
    - Prevents medium-priority tasks from preempting the low-priority task and prolonging the blocking duration experienced by a high-priority task.
  - After exiting its critical section, job J returns to its original priority level
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# Priority Inheritance Protocol (PIP)

- Definition of the basic protocol
    - 1) Job J, which has the highest priority among the jobs ready to run, is assigned the processor.
      - Job J will be blocked, if semaphore S has been already locked.
      - When Job J exits its critical section, critical section unlocked, and the highest priority job blocked by job J will be awakened
    - 2) If job J blocks higher priority jobs, J inherits the highest priority of the jobs blocked by J
      - When J exits a critical section, it resumes the priority it had at the point of entry into the critical section
    - 3) Priority inheritance is transitive.
      - Job  $J_3$  blocks  $J_2$ , and  $J_2$  blocks  $J_1$ ,  $J_3$  would inherit the priority of  $J_1$  via  $J_2$
    - 4) Job J can preempt another job  $J_L$  if job J is not blocked and its priority is higher than the priority, inherited or assigned, at which job  $J_L$  is executing
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# BIP Illustrated

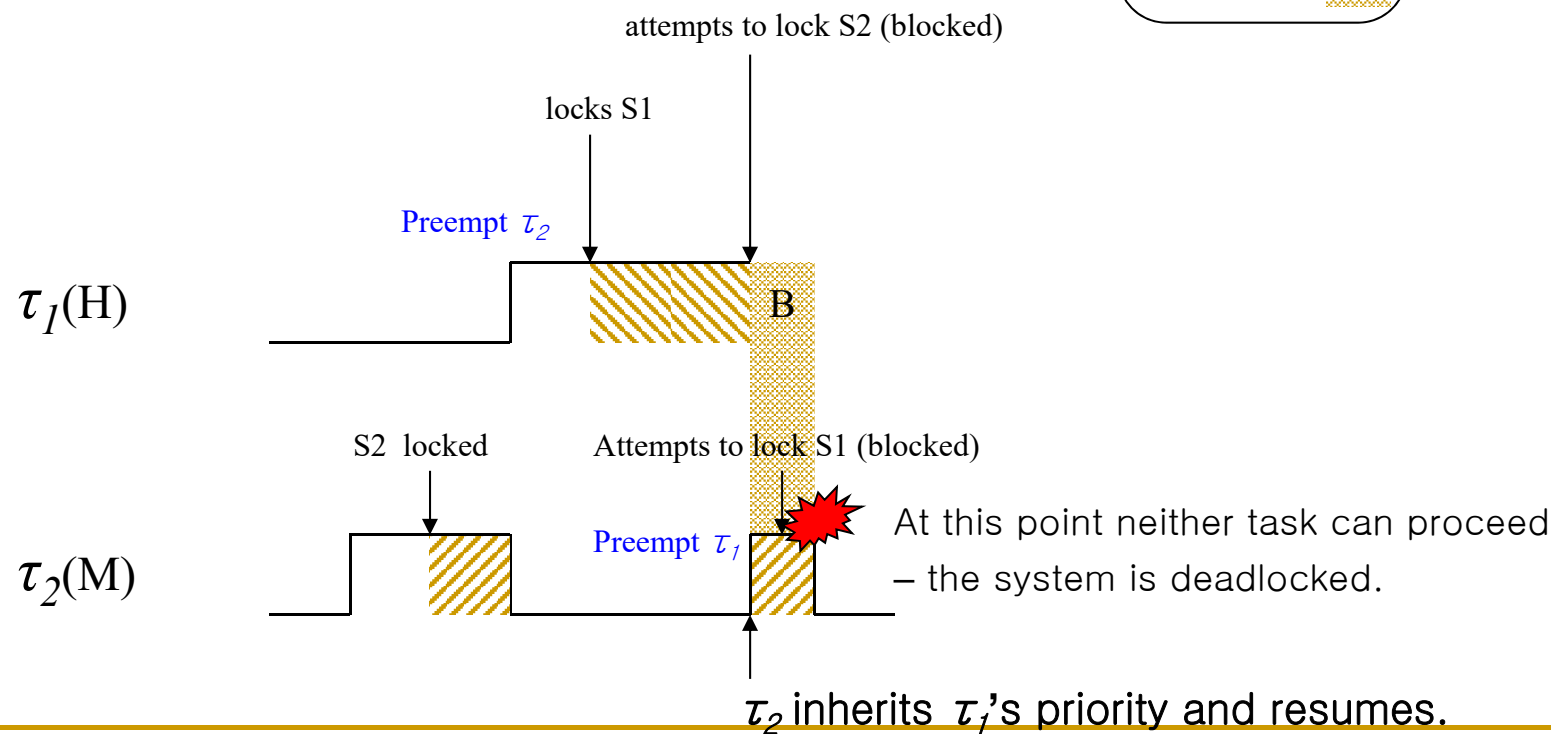
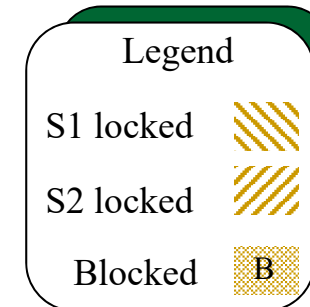


# Potential Drawback

## Deadlock

$\tau_1: \{ \dots P(S1) \dots P(S2) \dots V(S2) \dots V(S1) \dots \}$

$\tau_2: \{ \dots P(S2) \dots P(S1) \dots V(S1) \dots V(S2) \dots \}$



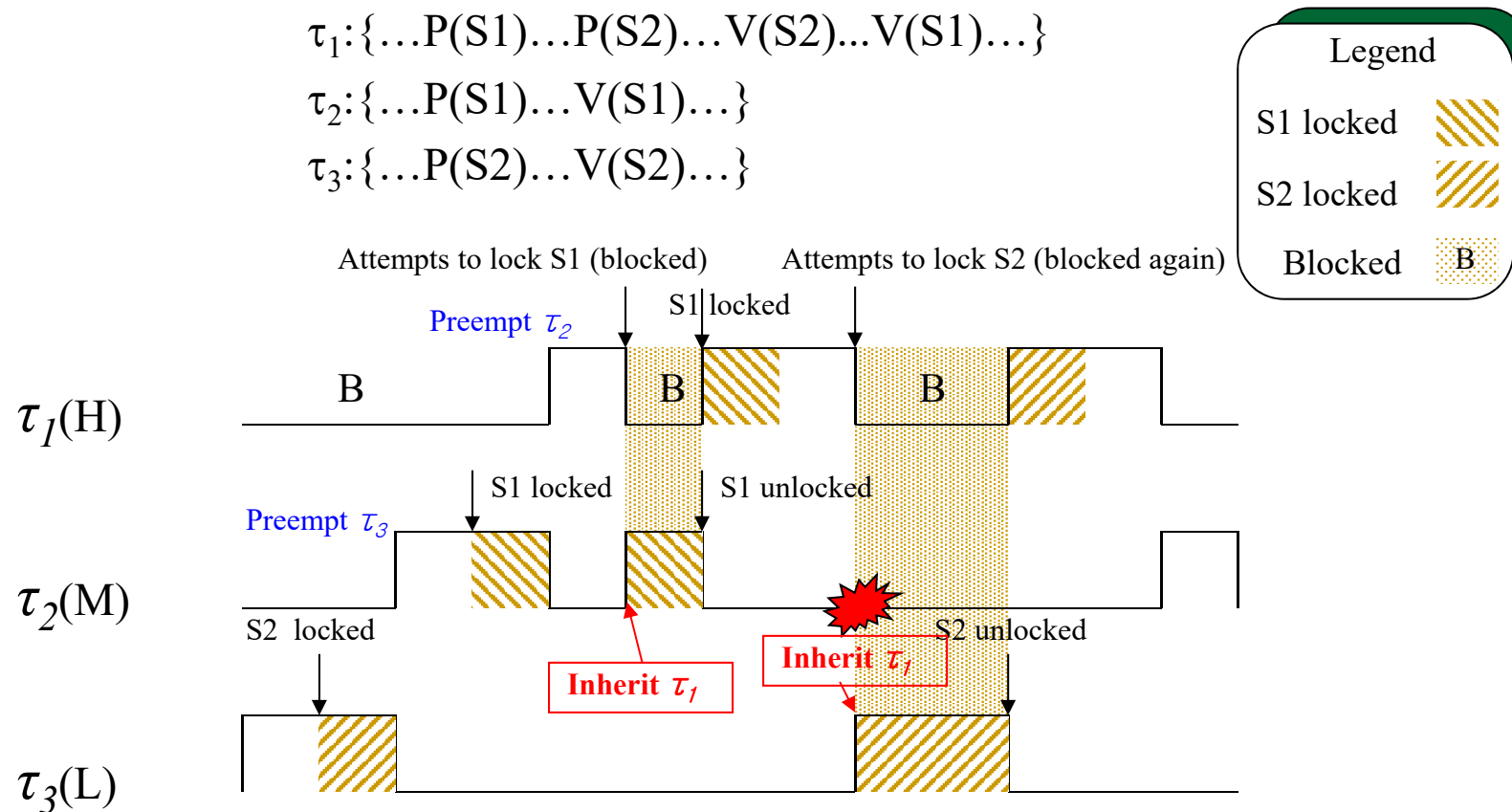
# Potential Drawback

## ■ Chained Blocking

$\tau_1: \{ \dots P(S1) \dots P(S2) \dots V(S2) \dots V(S1) \dots \}$

$\tau_2: \{ \dots P(S1) \dots V(S1) \dots \}$

$\tau_3: \{ \dots P(S2) \dots V(S2) \dots \}$



When a high-priority task shares more than one semaphore with lower-priority tasks, it may be blocked on each request to lock a semaphore → chained blocking.



# Priority Ceiling Protocol (PCP)

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# Priority Ceiling Protocol (PCP)

- Goal

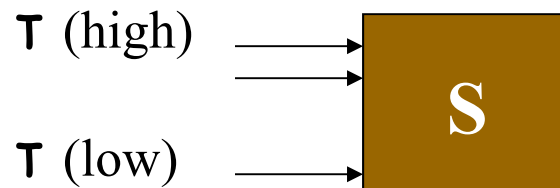
- To prevent the formation of deadlocks and of chained blocking

- Basic idea

- When a job J preempts the critical section of another job and executes its own critical section z, the priority of J should be guaranteed to be higher than the inherited priorities of all the preempted critical sections
    - If this condition cannot be satisfied, job J is denied entry into the critical section z and suspended, and the job that blocks J inherits J's priority
    - Allow a job J to start a new critical section only if J's priority is higher than all priority ceilings of all the semaphores locked by jobs other than J
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# Priority Ceiling

- Priority ceiling of a semaphore S
  - simply the priority of the highest priority task that may lock semaphore S
- System ceiling
  - the maximum ceiling of all semaphores **currently locked by other tasks**
- The idea behind PCP
  - To create a total priority ordering of executing and suspended critical sections



priority ceiling of semaphore S is the priority of  $\tau$  (high)

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## Priority Ceiling - Example

Critical section	Accessed by	Priority ceiling
$S_1$	$T_1, T_2$	$P(T_1)$
$S_2$	$T_1, T_2, T_3$	$P(T_1)$
$S_3$	$T_3$	$P(T_3)$
$S_4$	$T_2, T_3$	$P(T_2)$

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## PCP Rules (1)

- *Preemption*: A task with a higher execution priority always preempts tasks with lower execution priorities.
  - *Ceiling*: A task cannot enter its critical section unless its priority is higher than the **system ceiling**.
  - *Inheritance*: A lower priority task that blocks a higher priority task  $J_h$  inherits the priority of task  $J_h$ .  
[When there is only one semaphore, PCP works just like BIP.]
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## PCP Rules (2)

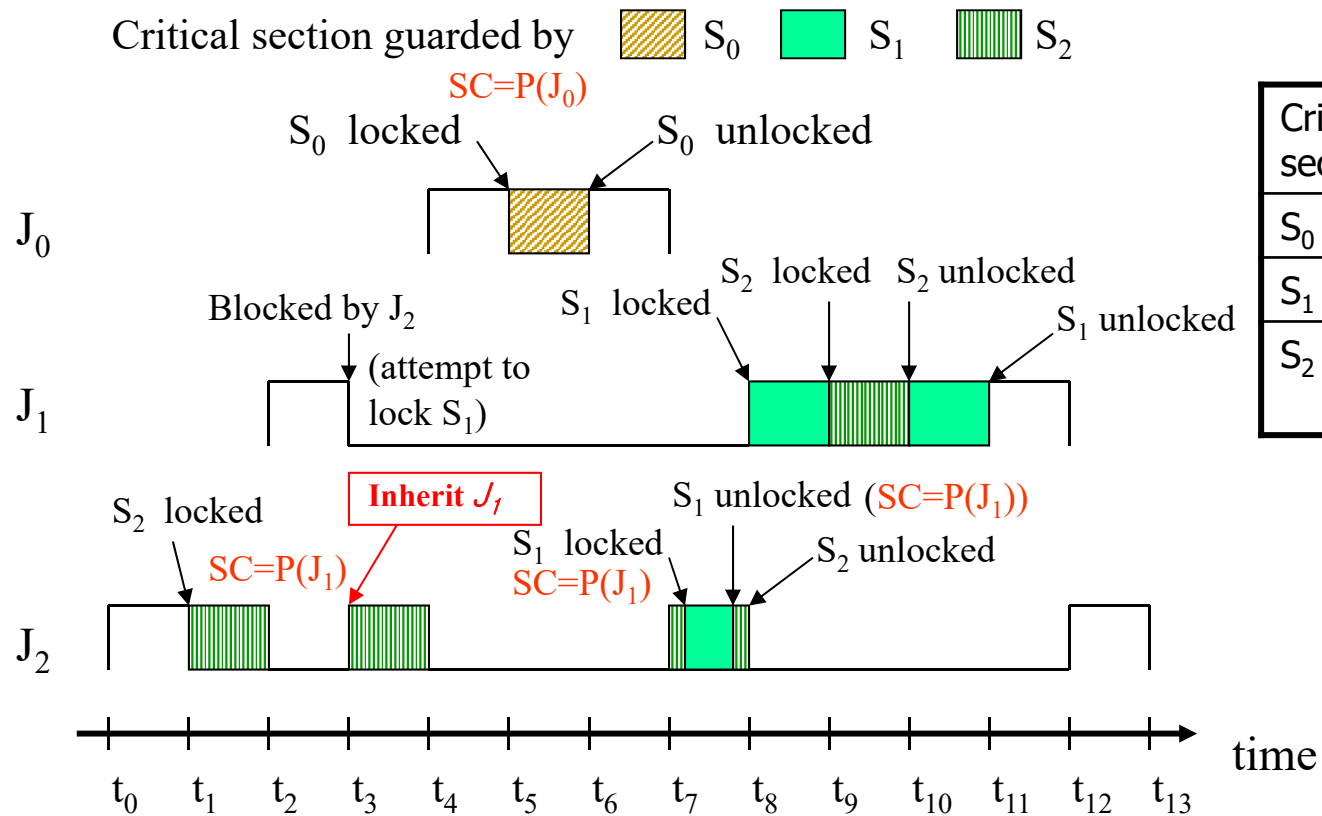
1. When J wants to enter the c. s.
    - its priority must be higher than system ceiling
    - when exiting, wake up the highest priority task among the blocked
  2. While running inside the c. s.
    - always inherits highest priority among the blocked
  3. *When J does not want to enter the c. s.*
    - can preempt a lower priority task
  4. When J completes execution normally
    - if there are many tasks of same priority ready for execution, schedule the task blocking other tasks
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# Example 1

$J_0 = \{ \dots, P(S_0), \dots, V(S_0), \dots \}$

$J_1 = \{ \dots, P(S_1), \dots, P(S_2), \dots, V(S_2), \dots, V(S_1), \dots \}$

$J_2 = \{ \dots, P(S_2), \dots, P(S_1), \dots, V(S_1), \dots, V(S_2), \dots \}$



Critical section	Accessed by	Priority ceiling
$S_0$	$J_0$	$P(J_0)$
$S_1$	$J_1, J_2$	$P(J_1)$
$S_2$	$J_1, J_2$	$P(J_1)$

## Example 2

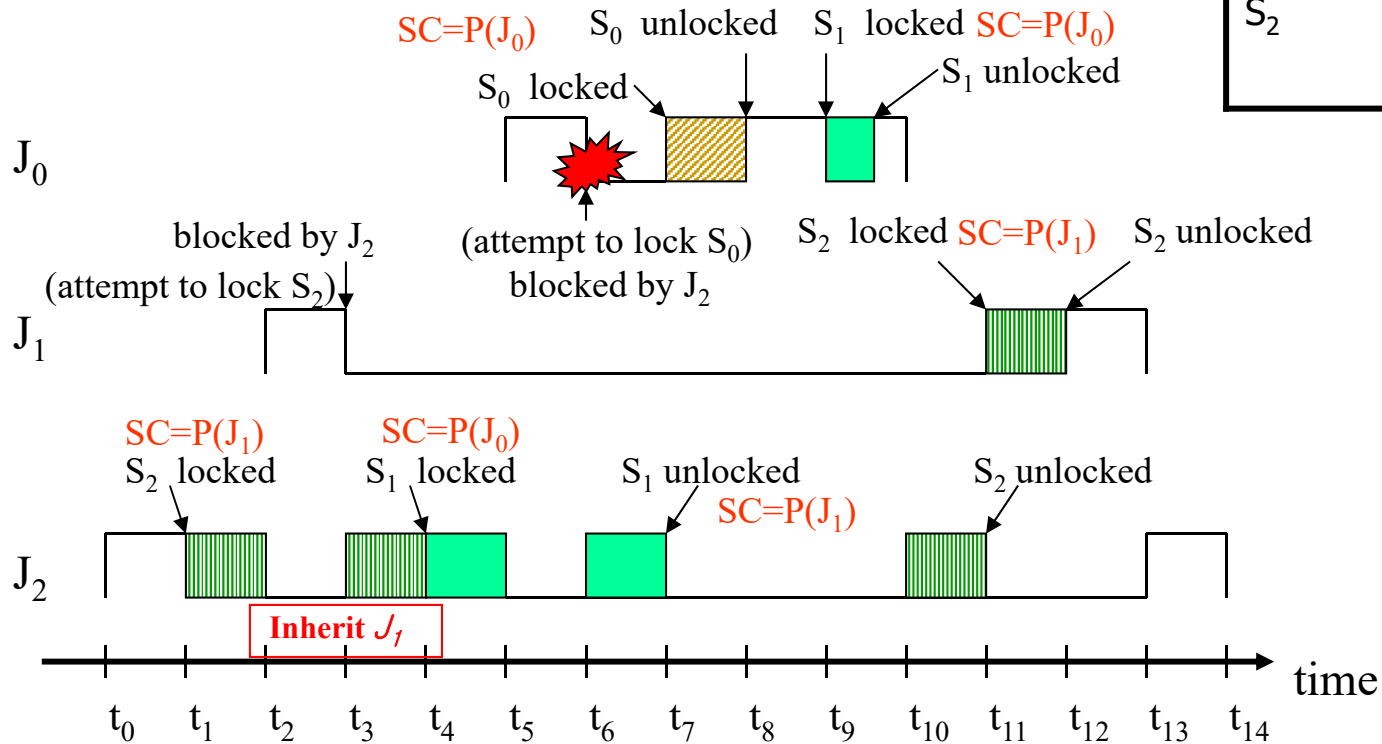
$J_0 = \{ \dots, P(S_0), \dots, V(S_0), \dots, P(S_1), \dots, V(S_1), \dots \}$

$J_1 = \{ \dots, P(S_2), \dots, V(S_2), \dots \}$

$J_2 = \{ \dots, P(S_2), \dots, P(S_1), \dots, V(S_1), \dots, V(S_2), \dots \}$

Critical section guarded by   $S_0$    $S_1$    $S_2$

Critical section	Accessed by	Priority ceiling
$S_0$	$J_0$	$P(J_0)$
$S_1$	$J_0, J_2$	$P(J_0)$
$S_2$	$J_1, J_2$	$P(J_1)$

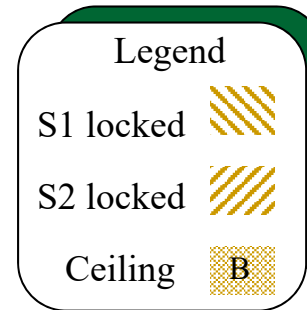




# Deadlock Avoidance

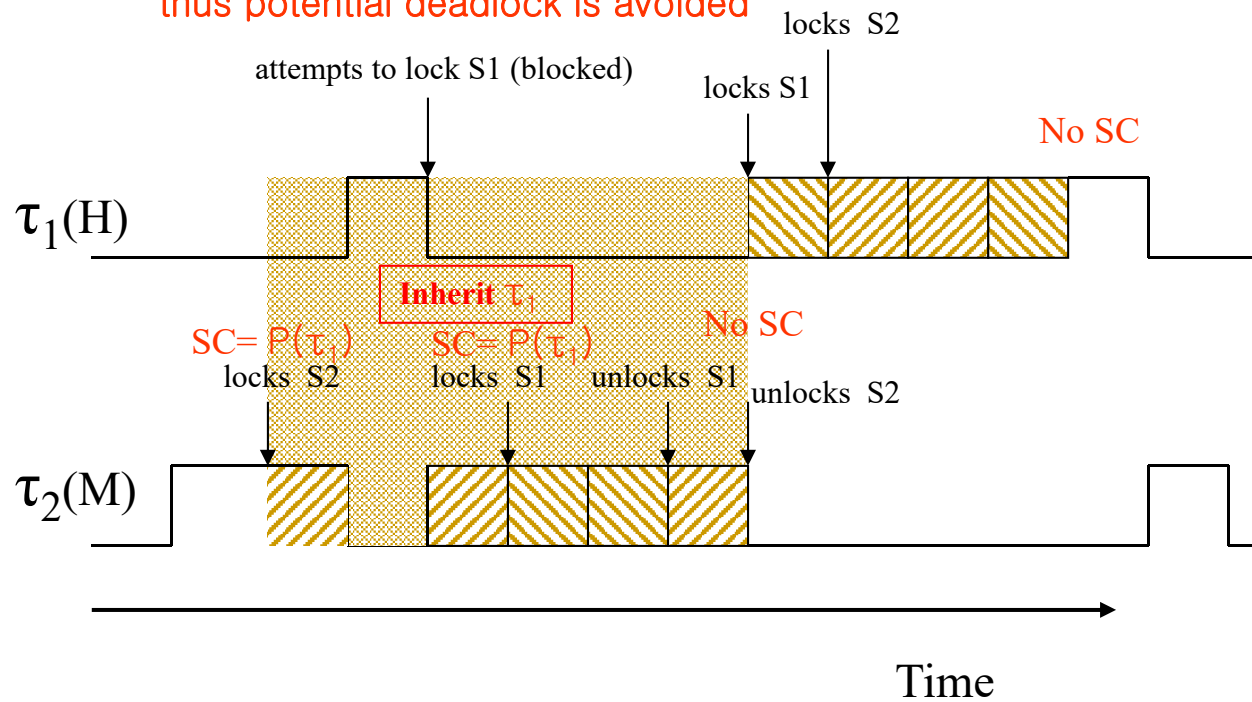
$\tau_1: \{ \dots P(S1) \dots P(S2) \dots V(S2) \dots V(S1) \dots \}$

$\tau_2: \{ \dots P(S2) \dots P(S1) \dots V(S1) \dots V(S2) \dots \}$



Critical section	Accessed by	Priority ceiling
$S_1$	$\tau_1 \tau_2$	$P(\tau_1)$
$S_2$	$\tau_1 \tau_2$	$P(\tau_1)$

$\tau_1$  is denied to lock S1 and thus potential deadlock is avoided

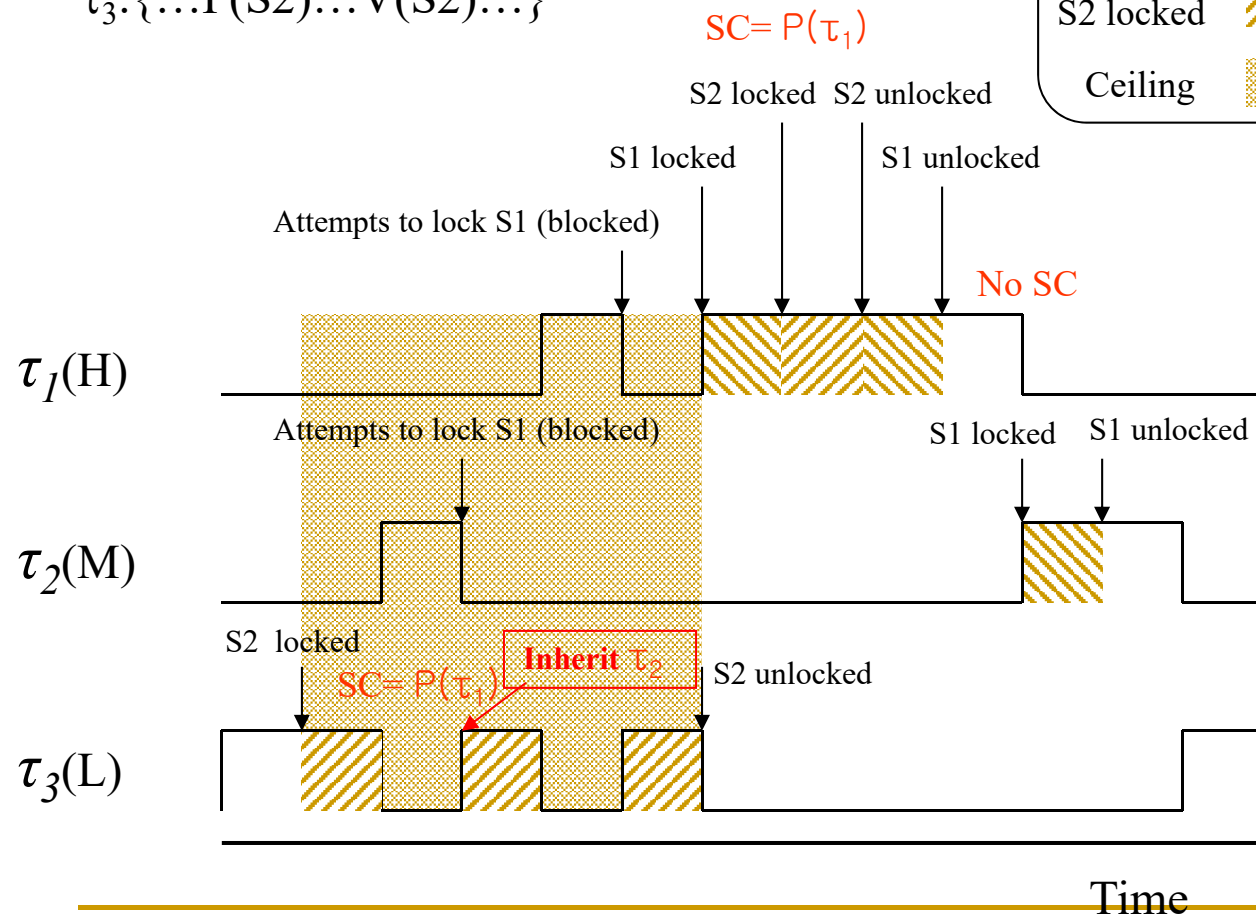
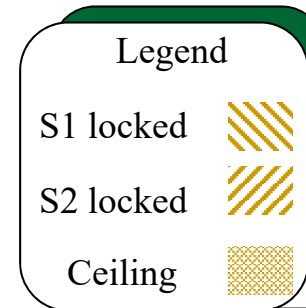


# Blocked At Most Once

$\tau_1: \{ \dots P(S1) \dots P(S2) \dots V(S2) \dots V(S1) \dots \}$

$\tau_2: \{ \dots P(S1) \dots V(S1) \dots \}$

$\tau_3: \{ \dots P(S2) \dots V(S2) \dots \}$



Critical section	Accessed by	Priority ceiling
$S_1$	$\tau_1 \tau_2$	$P(\tau_1)$
$S_2$	$\tau_1 \tau_3$	$P(\tau_1)$

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# Summary of synchronization protocols

- No preemption
    - do not allow preemption during execution of critical sections
  - Highest locker's priority
    - execute critical sections with the priority of the highest priority task that may lock the semaphore
  - Priority inheritance
    - when a lower priority task blocks the execution of a higher priority task, it inherits the priority of the task it blocks
  - Priority ceiling
    - priority inheritance plus priority ceiling rule for locking semaphores
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# Summary of synchronization protocols

Protocol	Bounded priority inversion	Blocked at most once	Deadlock avoidance
Nonpreemptible critical sections	Yes	Yes <sup>1</sup>	Yes <sup>1</sup>
Highest locker's priority	Yes	Yes <sup>1</sup>	Yes <sup>1</sup>
Basic inheritance	Yes	No	No
Priority ceiling	Yes	Yes <sup>2</sup>	Yes

1 Only if tasks do not suspend within critical sections

2 PCP is not affected if tasks suspend within critical sections

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# Real-Time Systems

## Scheduling Algorithm

### – EDF (Earliest Deadline First)

**[Liu73] C. L. Liu and J.W. Layland, “Scheduling algorithms for multiprogramming in a hard real-time environment”, Journal of the ACM, 1973**

**[Baruah90] S. K. Baruah, L.E.Rosier, and R.R. Howell, “Algorithms and complexity concerning the preemptive scheduling of periodic, real-time tasks on one processor”, Journal of Real-time Systems, 1990**

**[Jeffay93] K. Jeffay and D.L. Stone, “Accounting for interrupt handling costs in dynamic priority task systems”, RTSS, 1993**

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# Contents

- EDF (Earliest Deadline First) Scheduling
- Schedulability Analysis based on Utilization Bound



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# EDF (Earliest Deadline First)

- Assumptions

- Same as RM (Rate Monotonic)

- Priority Assignment

- Priorities are assigned to tasks according to the deadlines of their current requests.
- Dynamic priority scheduling

- Run-time activity of scheduler

- The ready task with the highest priority, i. e., the one with the nearest deadline, is executed.
- Sort priorities such that tasks with closer deadlines get higher priorities

- [Liu73] (Refer to RM paper, section 7)

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# Schedulability Analysis

## Theorem

*For a given set of  $n$  tasks, the EDF scheduling algorithm is feasible if and only if*

$$(C_1/T_1) + (C_2/T_2) + \dots + (C_n/T_n) = U \leq 1$$

Proof:

(1) *Only if (necessary condition)*

Let  $T = T_1 T_2 \dots T_n$ ,

The total demand of computation time in  $[0, T]$  is

$$\sum_{i=1}^n \frac{C_i}{T_i} T = UT$$

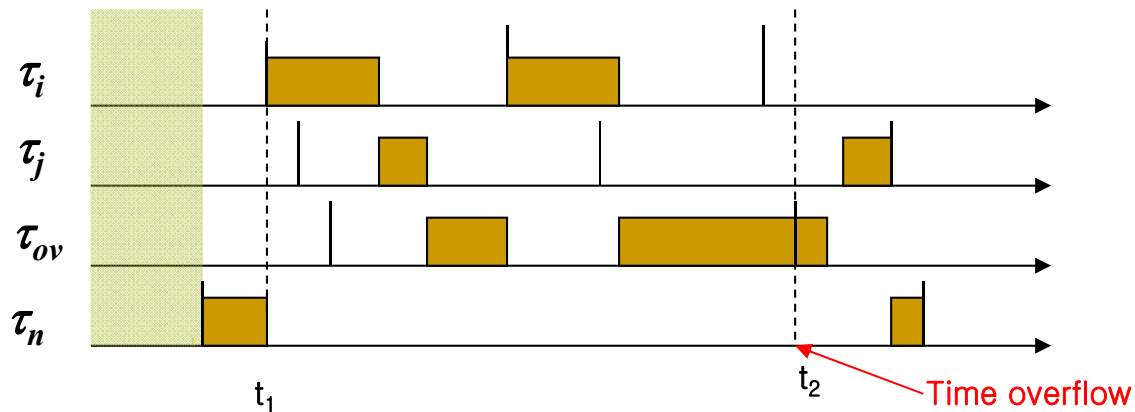
If  $U > 1$ , there is clearly no feasible schedule for the task set since the processor demand  $UT$  exceeds the available time  $T$ .



# Schedulability Analysis (cont'd)

Proof (cont'd):

(2) if (sufficient condition)



Assume that the condition  $U < 1$  is satisfied and yet the task set is not schedulable

$$C_p(t_1, t_2) = \sum_{r_k \geq t_1, d_k \leq t_2} C_k = \sum_{i=1}^n \left\lfloor \frac{t_2 - t_1}{T_i} \right\rfloor C_i$$

$$C_p(t_1, t_2) = \sum_{i=1}^n \left\lfloor \frac{t_2 - t_1}{T_i} \right\rfloor C_i \leq \sum_{i=1}^n \frac{t_2 - t_1}{T_i} C_i = (t_2 - t_1)U$$

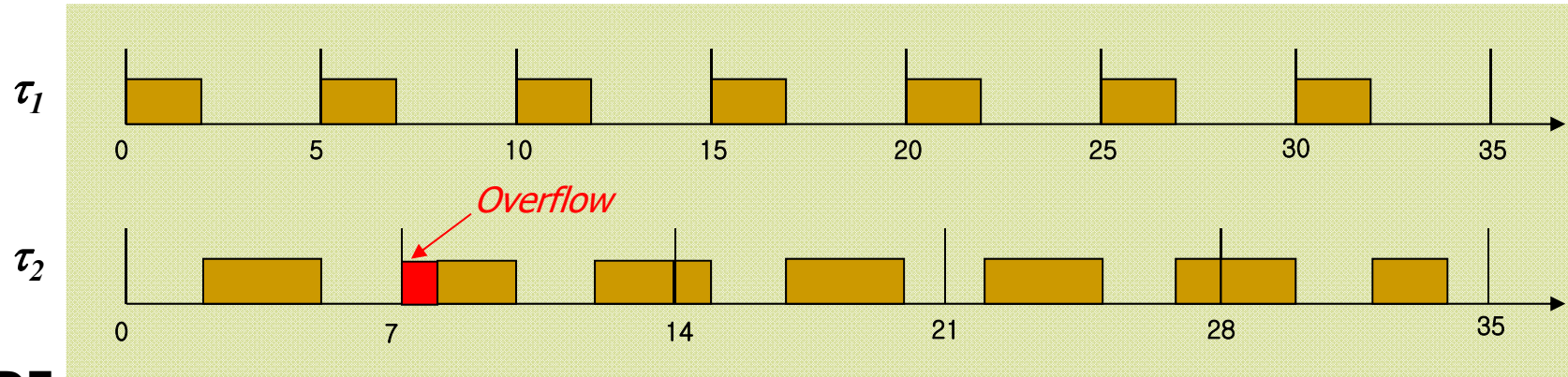
$$(t_2 - t_1) < C_p(t_1, t_2) \leq (t_2 - t_1)U$$

That is  $U > 1$ , which is a contradiction.

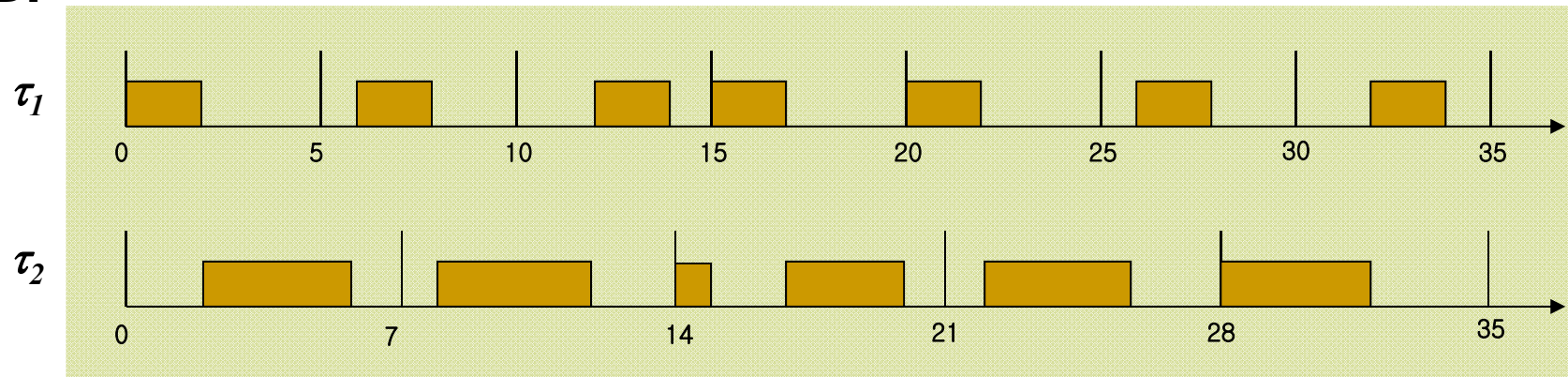
# Example

- Task set  $\tau = \{\tau_1, \tau_2\}$ ,  $\tau_1 = (2, 5, 5)$  and  $\tau_2 = (4, 7, 7)$

## RM



## EDF



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# **[Least/Minimum] [Slack Time/Laxity] First**

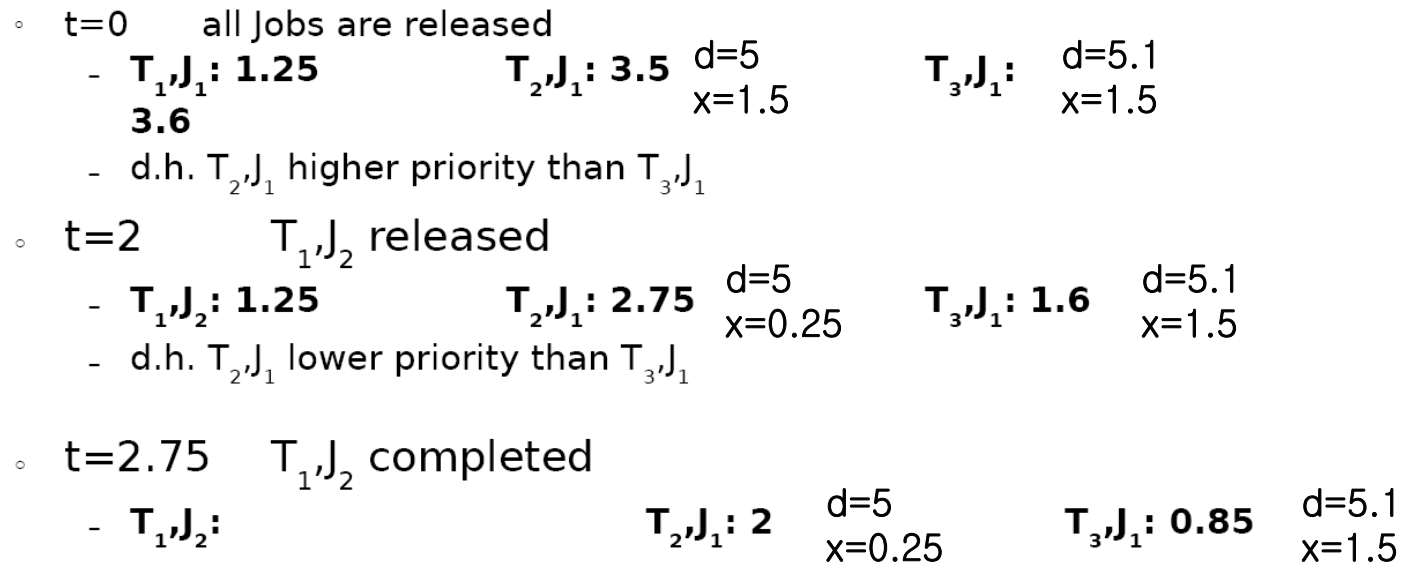
- **Select the task whose slack is the lowest**
  - **Slack Time = Laxity:**  
**(time to deadline - remaining execution time required to reach deadline)**
  - **slack time:  $d-x-t$** 
    - **$x$  remaining execution time of a job**
    - **$d$  absolute deadline**
    - **$t$  current time**
    - **dynamic per job, dynamic at task level**
      - **also optimal (analog EDF definition)**
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# MLF (LSF) Scheduling

- two versions:
    - Strict: slacks are computed at all times
      - ***Each instruction (prohibitively slow)***
      - ***Each timer "tick"***
    - Non-strict: slacks computed only at events (release and completion)
  - Scheduler checks slacks of all ready jobs and reorders queue
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- $T_1: (0.75, 2), T_2: (1.5, 5), T_3: (1.5, 5.1)$



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# Reading Assignment

- **T.P. Baker and Alan Show, "The Cyclic Executive Model and Ada", Real-Time Systems Journal, 1989**
  - **J. P. Lehoczky, L. Sha, and J. K. Strosnider, "Enhanced Aperiodic Responsiveness in Hard Real-Time Environments", IEEE RTSS, 1987**
  - **B. Sprunt, L. Sha, and J. Lehoczky, "Aperiodic Task Scheduling for Hard-Real-Time Systems", Journal of Real-Time Systems, 1989**
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