上节回顾

1. 离散型随机变量的数学期望 $E(X)=\sum_{k=1}^{+\infty} x_k p_k.$

2. 连续型随机变量数学期望的定义 $E(X)=\int_{-\infty}^{+\infty} x f(x)\,dx.$

3. 随机变量函数的数学期望

1.一维离散随机变量函数的数学期望 $E(Y)=E[g(X)]=\sum_{i=1}^{\infty} g(x_i)p_i$

2.一维连续随机变量函数的数学期望 $E(Y)=E[g(X)]=\int_{-\infty}^{+\infty} g(x)f(x)dx$

3. 二维离散随机变量函数的数学期望 $E(Z)=E[g(X,Y)]=\sum_i\sum_j g(x_i,y_j)p_{ij}$

4. 二维连续随机变量函数的数学期望 $E(Z)=E[g(X,Y)]=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)f(x,y)dxdy$

上周回顾

4. 数学期望的性质

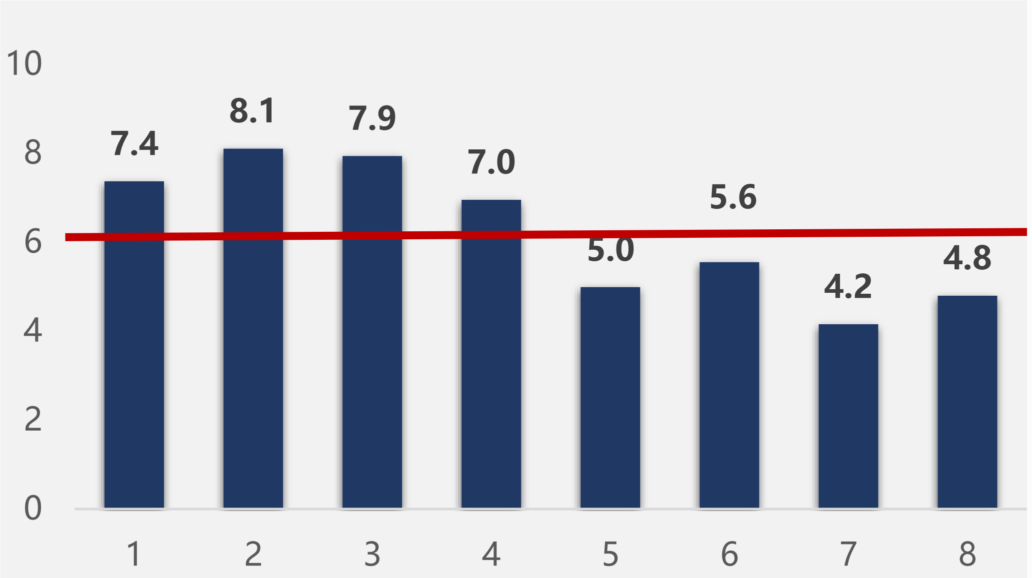
$$\begin{cases} 1^0 \quad E(C) = C; \\ 2^0 \quad E(CX) = CE(X); \\ 3^0 \quad E(X+Y) = E(X) + E(Y); \\ 4^0 \quad X, Y \text{ 独立} \Rightarrow E(XY) = E(X)E(Y). \end{cases}$$

Course Review

Definition The mathematical expectation (or simply expectation or expected value or mean) for a discrete random variable X is defined as

$$E(X) = \mu_x = \sum_k x_k \mathrm{P}(X = x_k)$$

given that the above series converges absolutely, where $\mathrm{P}(X = x_k)$ p.m.f. of X.



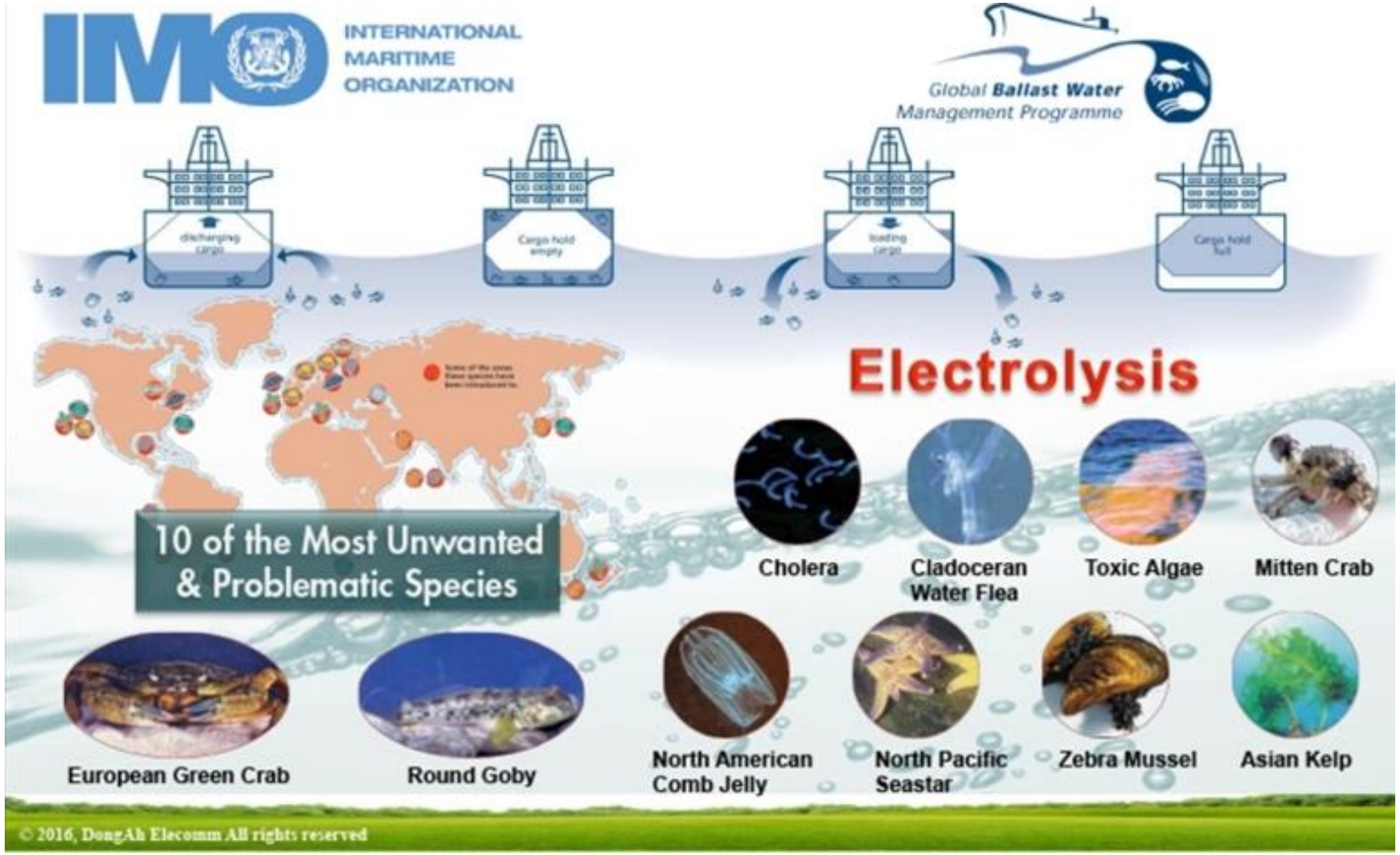
Course Review

Distribution	Distribution law	$E(X)$
$X \sim B(1,p)$	$P\{X=k\}=p^k(1-p)^{1-k}$ $\quad k=0,1$	p
$X \sim B(n,p)$	$P\{X=k\}=C_n^k p^k(1-p)^{n-k}$ $\quad k=0,1,2,\ldots,n$	np
$X \sim P(\lambda)$	$P\{X=k\}=\dfrac{\lambda^k}{k!}e^{-\lambda}$ $\quad k=0,1,2,\ldots$	λ
$X \sim G(p)$	$P\{X=k\}=(1-p)^{k-1}p$ $\quad k=1,2,\ldots$	$\dfrac{1}{p}$

Theorem 3.1.6: If a discrete r.v. X obeys Poisson distribution with parameter λ, then the expectation of X is $E(X)=\lambda$.

Problem Introduction

Ballast water is one of the largest non-native species transfer carriers in the world.

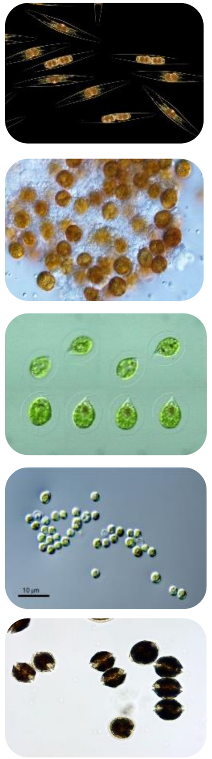


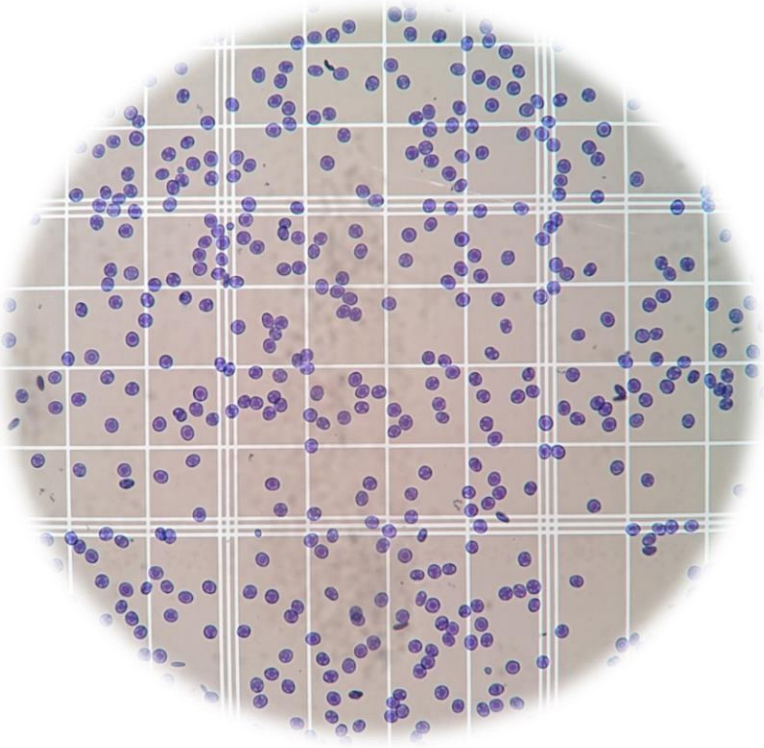





Problem Introduction

The mean cell density of living algal in water bodies is difficult to obtain directly.



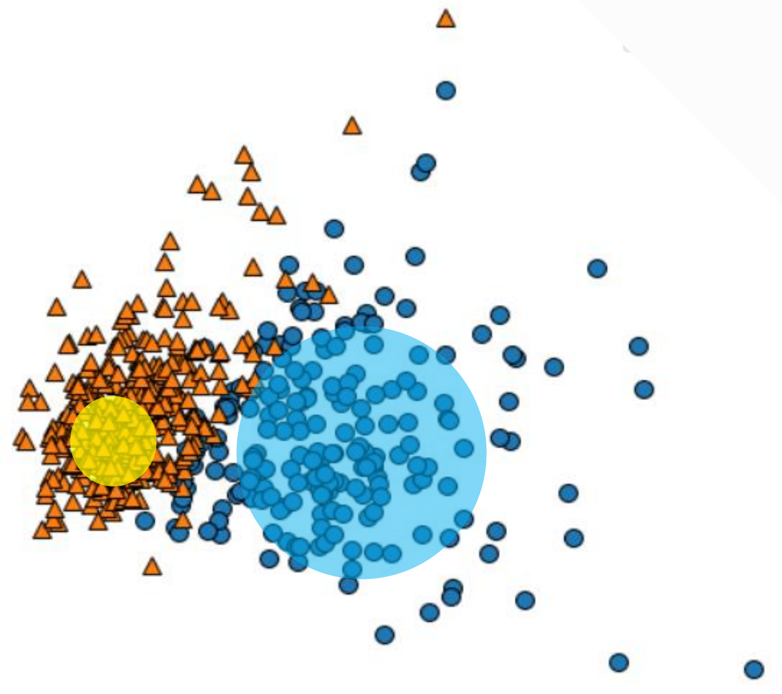


The cell density of algal in sample water obeys Poisson distribution.

The Variance of Poisson distribution

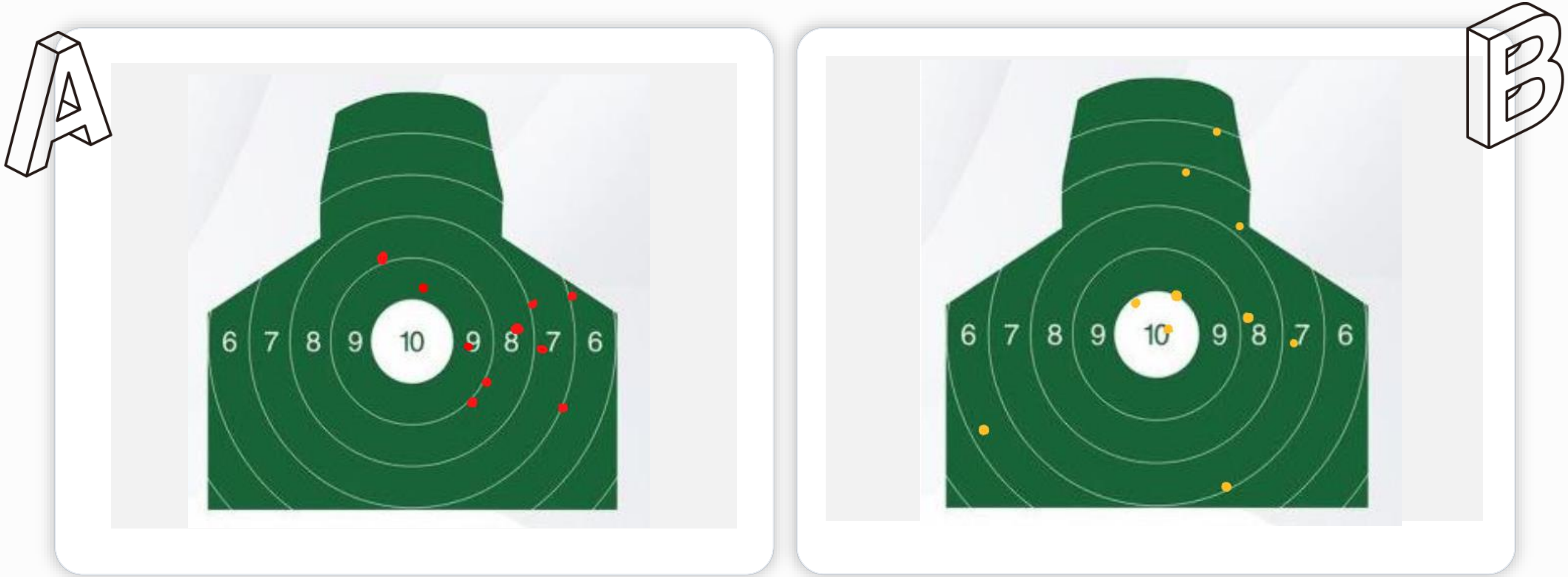
Variance and its properties

The variance measures the spread of the distribution around the mean. An r.v. with a zero variance is constant. The larger the variance, the more uncertain the r.v. is, in the mean square sense.



Section 2 Variance

Example 2.1: Assuming two shooters A and B have a competition, they shoot 10 times at each target, as shown in the figure. Which shooter's shooting is more stable?



Section 2 Variance

Example 2.1: Assuming two shooters A and B have a competition, they shoot 10 times at each target, as shown in the table. Which shooter's shooting is more stable?

Shooter A		**Shooter B**	
Number of rings	***Probability p***	***Number of rings***	***Probability p***
5	0.2	4	0.3
7	0.6	7	0.4
9	0.2	10	0.3

Section 2 Variance

Solution: (1) Find the mathematical expectation:

$$E(X) = \sum_k x_k P(X = x_k)$$

Shooter A		Shooter B	
Number of rings	*Probability p*	*Number of rings*	*Probability p*
5	0.2	4	0.3
7	0.6	7	0.4
9	0.2	10	0.3

$$\boldsymbol{E(X_A)} = 5 * 0.2 + 7 * 0.6 + 9 * 0.2 = \mathbf{7}$$

$$\boldsymbol{E(X_B)} = 4 * 0.3 + 7 * 0.4 + 10 * 0.3 = \mathbf{7}$$

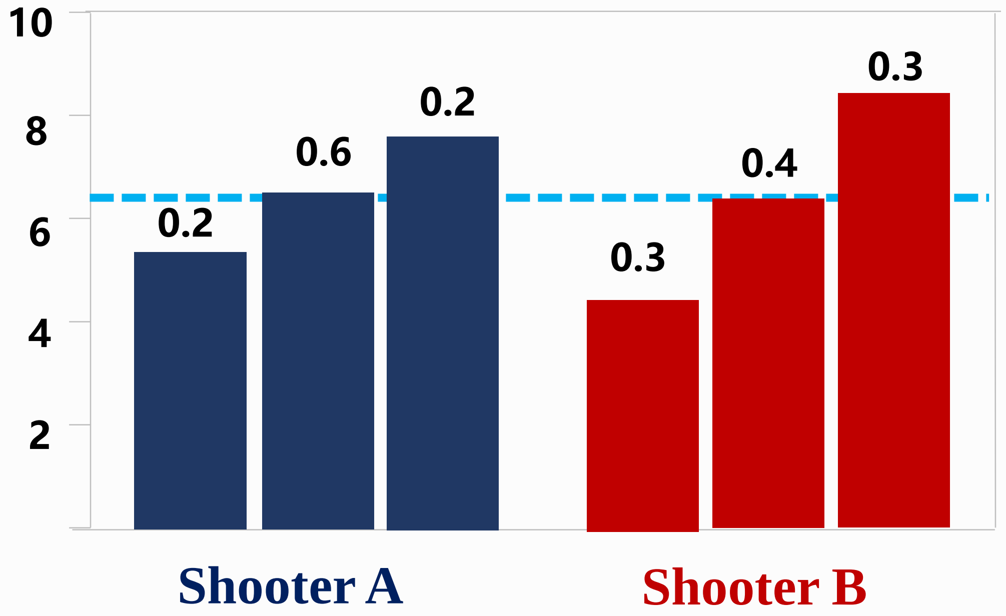


Fig1. The expectation of two shooters

Section 2 Variance

Solution: (2) Find the expectation of the deviation from its expectation:

$$E[X - E(X)]$$

Shooter A		Shooter B	
Number of rings	*Probability p*	*Number of rings*	*Probability p*
5	0.2	4	0.3
7	0.6	7	0.4
9	0.2	10	0.3
$E(X_A)=7$		$E(X_B)=7$	

$E[X_A - E(X_A)]$

$= (5-7)*0.2 + (7-7)*0.6 + (9-7)*0.2 = 0$

$E[X_B - E(X_B)]$

$=(4-7)*0.3 + (7-7)*0.4 \ + (10-7)*0.3 = 0$

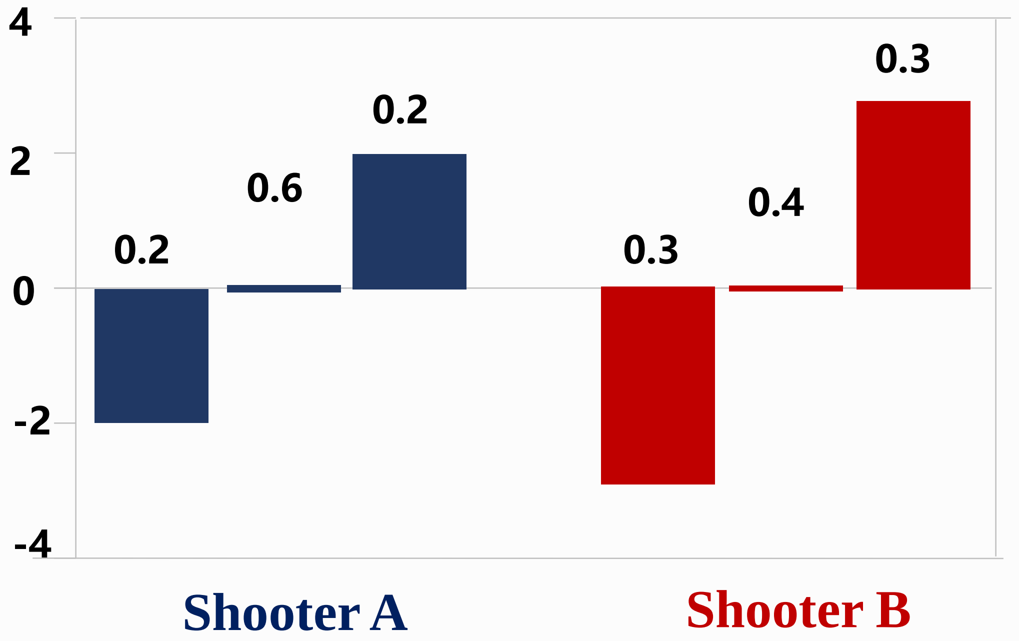


Fig2. The expectation of the two shooter's deviation

Section 2 Variance

Solution: Find the expectation of the square of the deviation between X and its expectation :

$$E[X - E(X)]^2$$

Shooter A		Shooter B	
Number of rings	*Probability p*	*Number of rings*	*Probability p*
5	0.2	4	0.3
7	0.6	7	0.4
9	0.2	10	0.3
$E(X_A)=7$		$E(X_B)=7$	

$$E[X_A - E(X_A)]^2$$

$$= (5-7)^2 * 0.2 + (7-7)^2 * 0.6 + (9-7)^2 * 0.2 = 1.6$$

$$E[X_B - E(X_B)]^2$$

$$=(4-7)^2 * 0.3 + (7-7)^2 * 0.4 + (10-7)^2 * 0.3 = 5.4$$

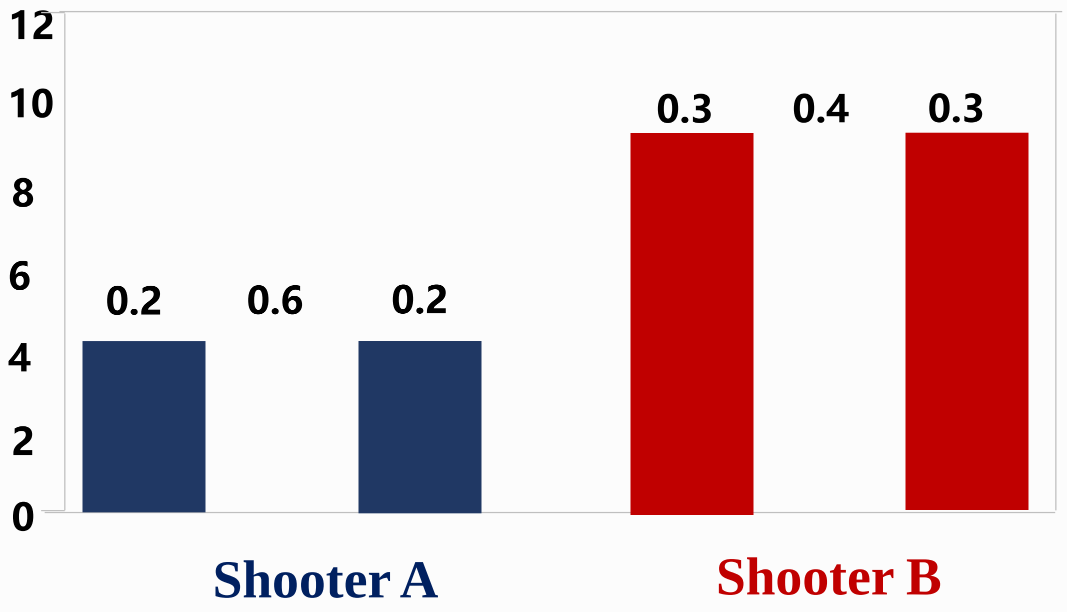


Fig3. The expectation of square of the two shooter's deviation

Definition of variance

Definition of the variance If X is an r.v.with $E(X) = \mu_x$, then $Var(X)$ or $D(X)$, called the variance of X, is defined by

$$Var(X) = D(X) = E[(X - \mu_x)^2]$$

and the Standard Deviation (SD) σ_x of X is

$$\sigma_X = \sqrt{Var(X)} = \sqrt{D(X)}$$

Definition of variance

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Improved Analytical Formula for the SAR Doppler Centroid Estimation Standard Deviation for a Dynamic Sea Surface

by Siqi Qiao, Baochang Liu * and Yijun He

School of

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Clinical Neurology and Neurosurgery

Volume 224, January 2023, 107553

Standard deviations of MR signal intensities
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Standard Deviation Effect of Average Structure Descriptor on Grain Boundary Energy Prediction

by Ruoqi Dang and Wenshan Yu *

State Key Laboratory for Strength and Vibration of Mechanical Structures, Shaanxi Engineering Laboratory for Vibration Control of Aerospace Structures, School of Aerospace Engineering, Xi'an Jiaotong University, Xi'an 710049, C

* Author t

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Information Sciences

Volume 626, May 2023, Pages 370-389

Group AHP framework based on geometric
standard deviation and interval group pairwise
comparisons

Petra Grošelj, Gregor Dolinar

Open Access Article

Improved Estimation of O-B Bias and Standard Deviation by an RFI Restoration Method for AMSR-2 C-Band Observations over North America

by Wangbin Shen [1], Zhaohui Lin [2,3], Zhengkun Qin [1,3,*] and Xuesong Bai [1]

> J Cataract Refract Surg. 2023 Feb 1;49(2):19

Corneal ectasia detectio
standard deviation from OCT

Marcella Q Salomão [1], Ana Luisa Hofling-Lima, Louise Pellegrino Gomes Esporcatte, Fernando Faria Correa, Erica Ferreira Meneses, Yan Li, David Huang, Bernardo Lopes, Nelson Sena Jr, Aydano P Machado, Renato Ambrósio Jr

Single-Option Question

Q: Why do we need a concept of standard deviation (SD) ?

A Cause SD of X has the same unit with r.v. X.

B Cause SD value of X is more smaller than r.v. X.

C Cause SD value of X is more easier to calculate.

D It make no difference with $Var(X)$.

设置

Q: Why do we need a concept of standard deviation (SD) ?

A

A Cause SD of X has the same unit with r.v. X.

B

B Cause SD value of X is more smaller than r.v. X.

C

C Cause SD value of X is more easier to calculate.

D

It make no difference with $Var(X)$.

提交

Variance calculating

Method 1# *Calculate according to its definition.*

$$Var(X) = D(X) = E[(X - \mu_x)^2]$$

Method 2# *A convenient way for calculating variance.*

$$Var(X) = E(X^2) - [E(X)]^2$$

Variance of Poisson distribution

Suppose that r.v. $X \sim P(\lambda)$, $P(X=k)=\frac{\lambda^{k}e^{-\lambda}}{k!}$, $k=0,1,2,$ then,

$$E(X^2)=E[X(X-1)+X]$$

$$=\sum_{k=0}^{\infty} k(k-1)\frac{\lambda^k}{k!}e^{-\lambda}+E(X)$$

$$=\lambda^2\cdot e^{-\lambda}\cdot\sum_{k=2}^{\infty}\frac{\lambda^{k-2}}{(k-2)!}+\lambda=\lambda^2\cdot \mathrm{e}^{-\lambda}\cdot \mathrm{e}^{\lambda}+\lambda=\lambda^2+\lambda$$

Thus, $Var(X)=E(X^2)-E(X)^2=\lambda=E(X)$

Theorem 3.2.6 : If r.v. X obeys Poisson distribution with parameter λ, $X \sim P(\lambda)$, then $Var(X)=\lambda$.

Solve the problem

- The mean cell density of living algal in water bodies is difficult to obtain directly.
- The cell density of algal in sample water obeys Poisson distribution.
- The expectation value of Poisson distribution equals to its variance !

We try to get the variance of the algal cell density in water!

Solve the problem

- Variable fluorescence is a byproduct of photosynthesis.
- The fluorescence intensity is positively correlated with the density of algae cells.

Solve the problem

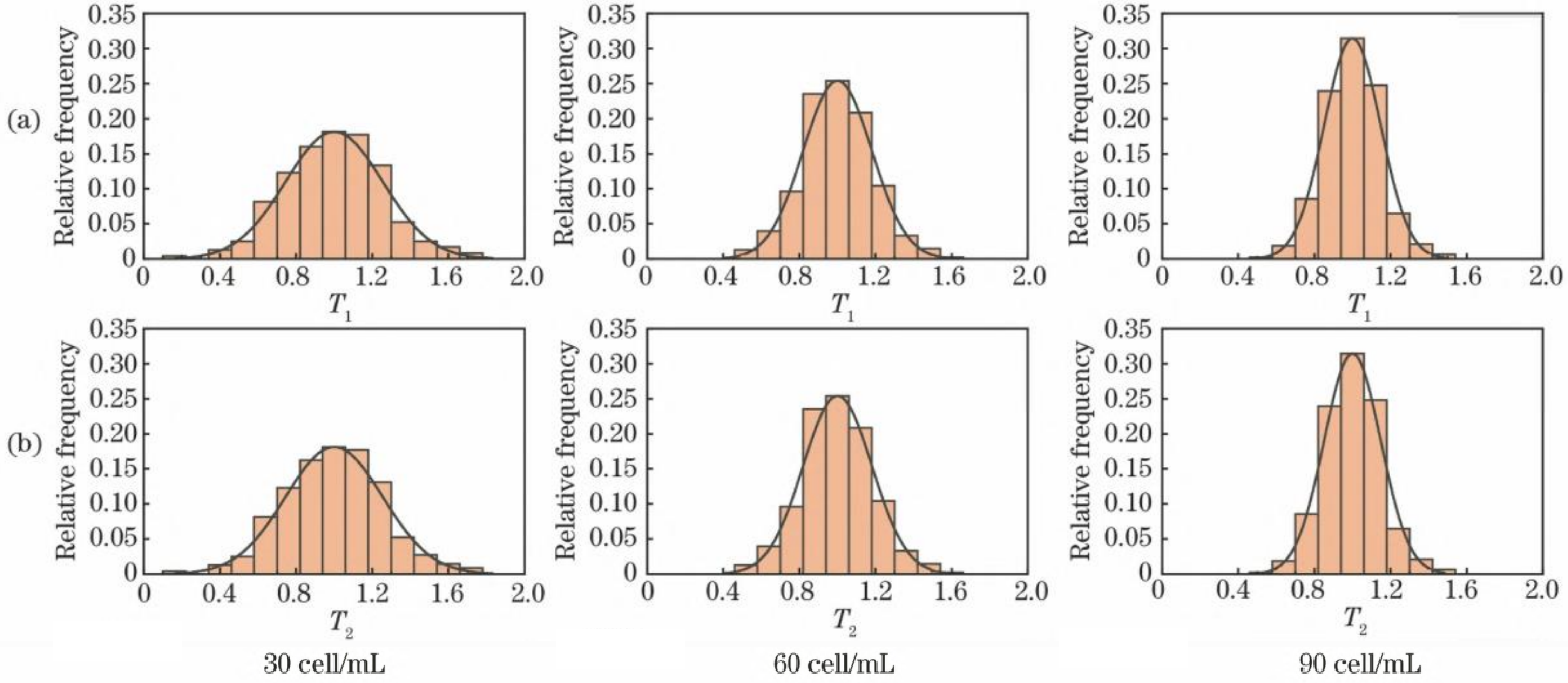

Fig.5 Distribution shape of cell number and Fv value under different cell density. (a)Distribution shape of cell number; (b)Distribution shape of Fv value.

Solve the problem

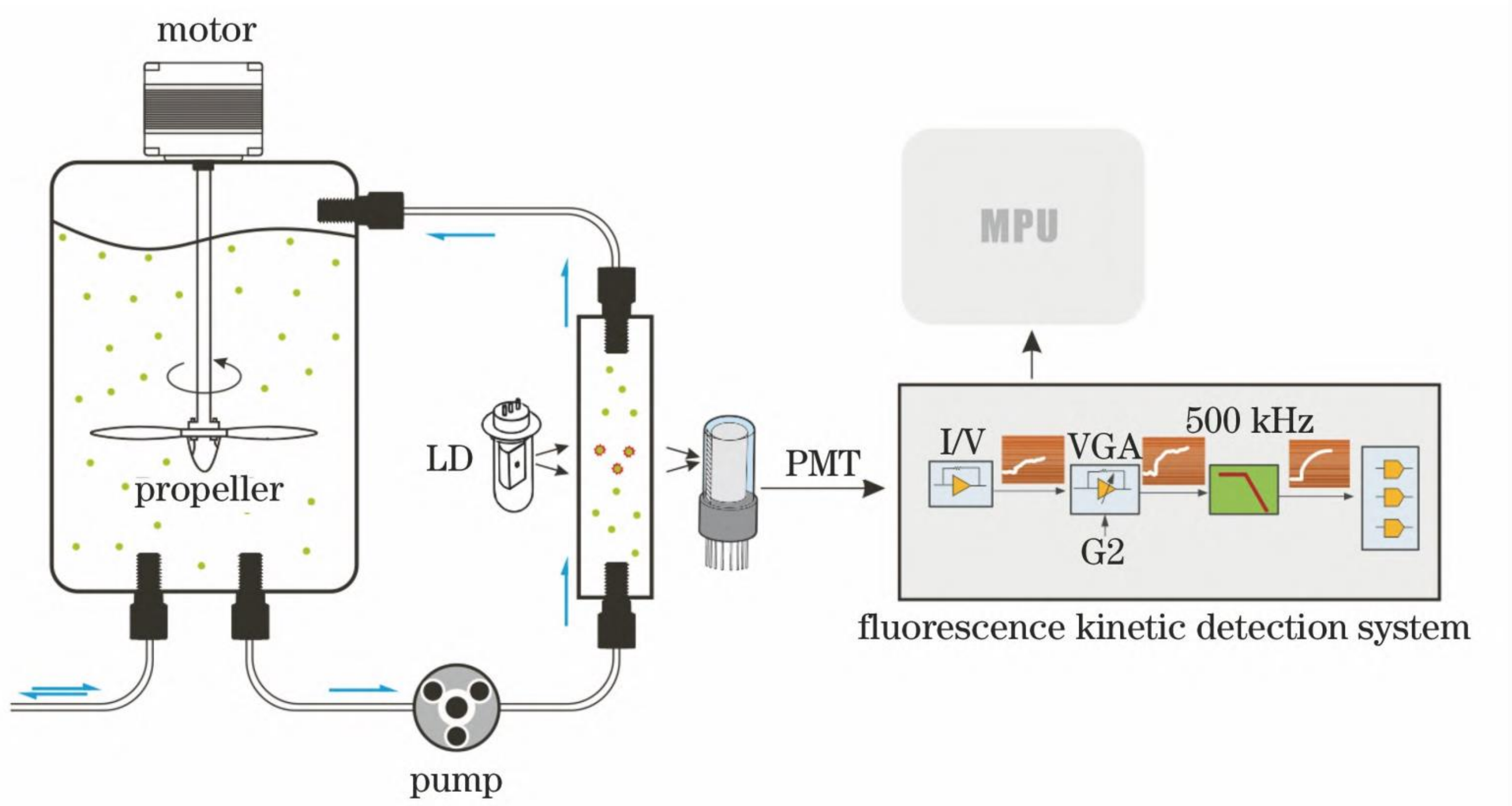

Fig4. Schematic diagram of the experimental device for variable fluorescence measurement of viable algae cells in water

Problem solved

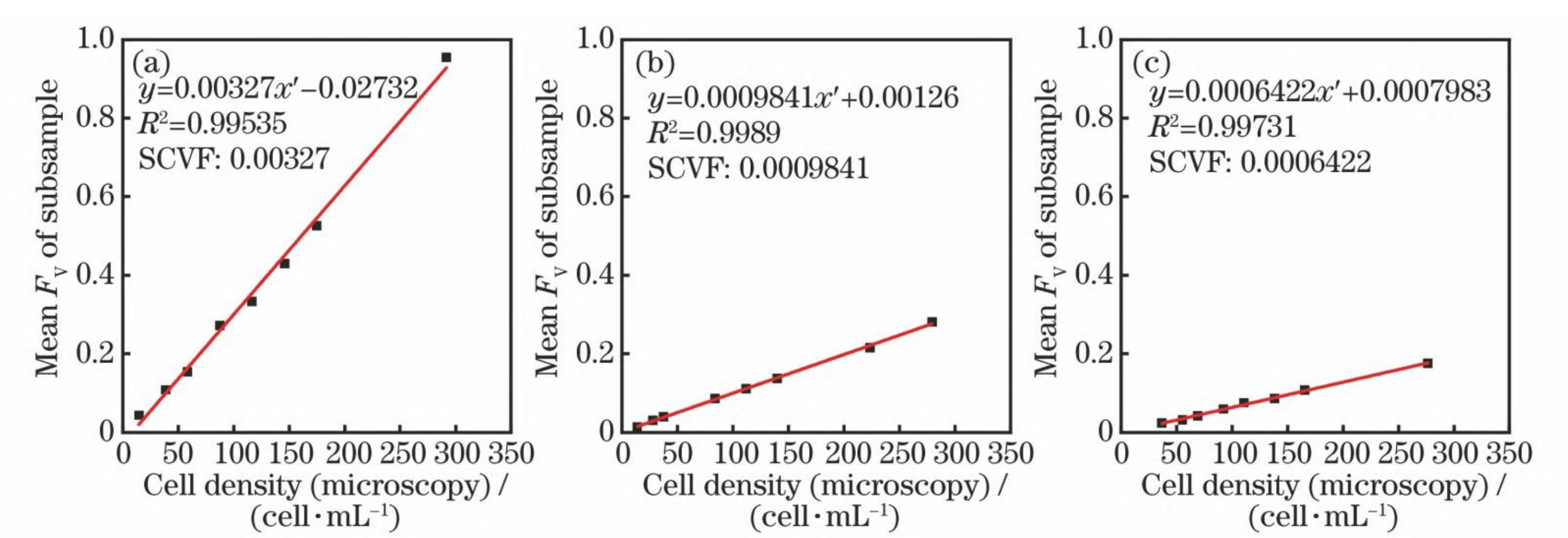

Fig.6 Comparison of viable algal cell density between microscopic examination and statistical analysis method. (a)Peridinium umbonatum var. inaequale; (b)Dunaliella salina;(c)Thalassiosira weissflogii;

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Verification

Discovery

Be cautions

Analysis

Innovation

- http://www.vipzhuanli.com/patent/202111081462.7/
- https://kns.cnki.net/kcms2/article/abstract?v=3uoqIhG8C44YLTlOAiTRKibYlV5Vjs7iJTKGjg9uTdeTsOI_ra5_XcqK_FrGDXbbb4_RP7CEmRWdAd6zewYXjYR_c14XZPb5&uniplatform=NZKPT

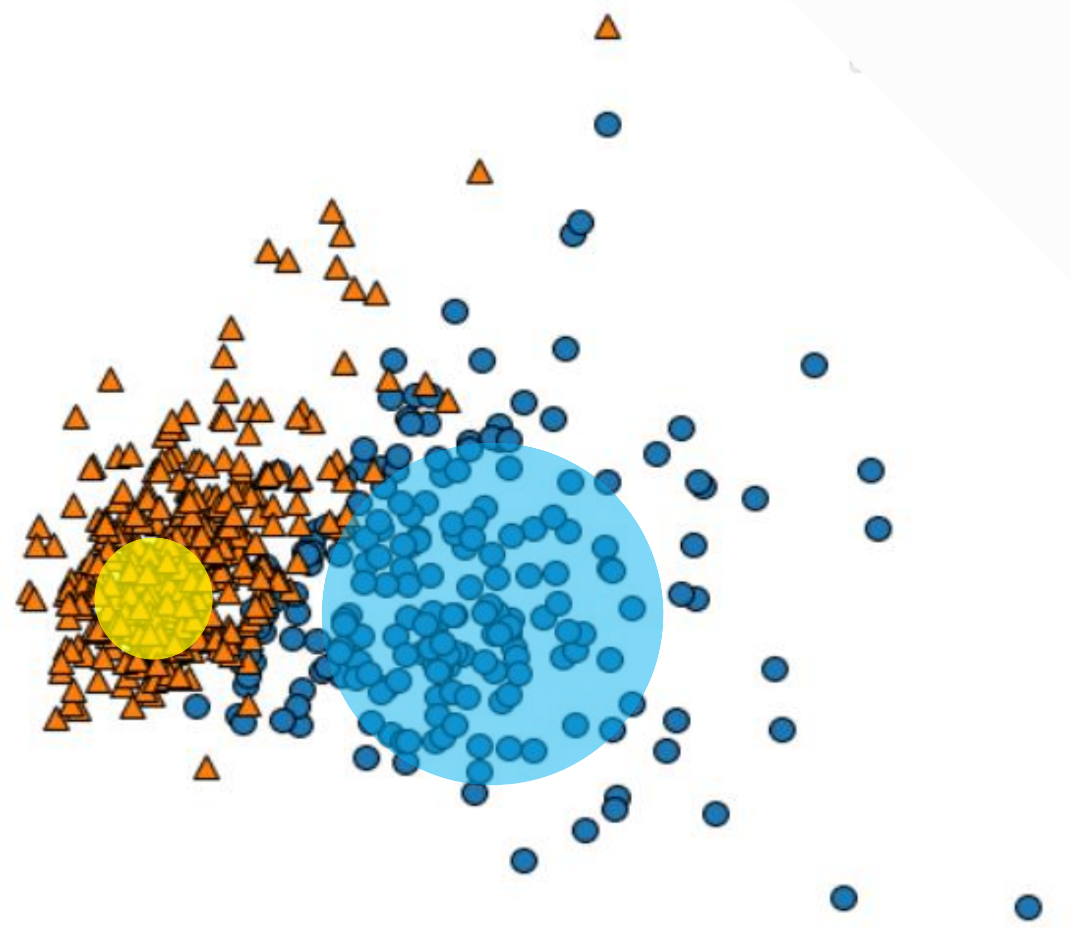
Summary & Homeworks

□ Summary:

$$Var(X) = D(X) = E[(X - \mu_x)^2]$$

$$\sigma_X = \sqrt{Var(X)} = \sqrt{D(X)}$$

$$Var(X) = E(X^2) - [E(X)]^2$$



□ Homeworks