

第二节 方差

- 一、方差的定义
- 二、常见分布的方差
- 三、方差的性质
- 四、小结

4.2.1 方差的定义

1. 方差的定义

设 X 为随机变量,若 $E\{[X - E(X)]^2\}$ 存在,则称其为随机变量 X 的**方差**,记为 $D(X)$ 或 $\text{Var}(X)$,即

$$D(X) = \text{Var}(X) = E\{[X - E(X)]^2\}.$$

$\sqrt{D(X)}$ 是与随机变量 X 具有相同量纲的量,称为随机变量 X 的**标准差**或**均方差**,记为 $\sigma(X)$, 即 $\sigma(X) = \sqrt{D(X)}$.

随机变量 X 的方差和标准差描述了随机变量对数学期望的偏离程度. 它是衡量 X 取值分散程度的一个尺度.

方差与标准差越小, 随机变量的取值越集中, 反之, 随机变量的取值越分散.

4.2.1 方差的定义

2. 方差的计算方法

(1) 利用定义计算

由定义知道，方差实际上是随机变量 X 的函数 $g(X) = [X - E(X)]^2$ 的数学期望，因此根据定理4.1.1

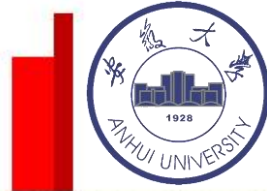
对于离散型随机变量 X ，若其分布律为 $p_k = P\{X=x_k\}$, $k=1,2,\dots$ ，则随机变量 X 的方差为

$$D(X) = \sum_k [x_k - E(X)]^2 p_k$$

对于连续型随机变量 X ，若其密度函数为 $f(x)$ ，则 X 的方差为

$$D(X) = \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x) dx$$

4.2.1 方差的定义



(2) 利用公式计算

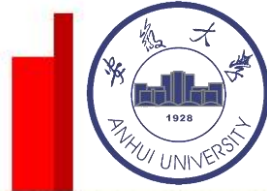
若随机变量 X 的方差存在, 则有

$$D(X) = E(X^2) - [E(X)]^2$$

证明

$$\begin{aligned} D(X) &= E\{[X - E(X)]^2\} \\ &= E\{X^2 - 2XE(X) + [E(X)]^2\} \\ &= E(X^2) - 2E(X)E(X) + [E(X)]^2 \\ &= E(X^2) - [E(X)]^2 \end{aligned}$$

4.2.1 方差的定义



例4.2.1 P109 一台设备由三大部件构成, 在设备运转中各部件需要调整的概率分别为0.10, 0.20, 0.30. 假设各部件的状态相互独立, 以 X 表示需要调整的部件数, 试求 X 的分布律, 数学期望 $E(X)$ 以及方差 $D(X)$.

解 设 $A_i = \{\text{部件}i\text{需要调整}\}$, $i = 1, 2, 3$, 则

$$P(A_1) = 0.10, P(A_2) = 0.20, P(A_3) = 0.30,$$

随机变量 X 的所有可能取值为0, 1, 2, 3, 由于 A_1, A_2, A_3 相互独立, 所以

$$P(X = 0) = P(\overline{A_1} \overline{A_2} \overline{A_3}) = 0.9 \times 0.8 \times 0.7 = 0.504$$

$$\begin{aligned} P(X = 1) &= P(A_1 \overline{A_2} \overline{A_3}) + P(\overline{A_1} A_2 \overline{A_3}) + P(\overline{A_1} \overline{A_2} A_3) \\ &= 0.1 \times 0.8 \times 0.7 + 0.9 \times 0.2 \times 0.7 + 0.9 \times 0.8 \times 0.3 = 0.398 \end{aligned}$$

$$\begin{aligned} P(X = 2) &= P(A_1 A_2 \overline{A_3}) + P(A_1 \overline{A_2} A_3) + P(\overline{A_1} A_2 A_3) \\ &= 0.1 \times 0.2 \times 0.7 + 0.1 \times 0.8 \times 0.3 + 0.9 \times 0.2 \times 0.3 = 0.092 \end{aligned}$$

$$P(X = 3) = P(A_1 A_2 A_3) = 0.1 \times 0.2 \times 0.3 = 0.006$$

4.2.1 方差的定义



故 X 的分布律为

X	0	1	2	3
P	0.504	0.398	0.092	0.006

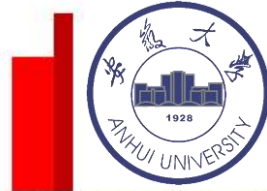
故数学期望 $E(X) = 1 \times 0.398 + 2 \times 0.092 + 3 \times 0.006$

$$= 0.6$$

故方差 $D(X) = E(X^2) - [E(X)]^2$

$$= 1^2 \times 0.398 + 2^2 \times 0.092 + 3^2 \times 0.006 - 0.6$$
$$= 0.46$$

4.2.1 方差的定义



例4.2.2 仍然考虑例4.1.9, 在长为 a 的线段上任取两个点 X 和 Y , 求此两点间距离的方差.

解 由例4.1.9, 已得到两点间距离的数学期望 $E|X - Y| = \frac{1}{3}a$, 所以

$$D|X - Y| = E(|X - Y|^2) - (E|X - Y|)^2$$

$$\begin{aligned} E(|X - Y|^2) &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (x - y)^2 f(x, y) dx dy \\ &= \int_0^a \int_0^a \frac{1}{a^2} (x - y)^2 dx dy \\ &= \frac{1}{a^2} \int_0^a \int_0^a (x^2 - 2xy + y^2) dx dy = \frac{1}{6} a^2 \end{aligned}$$

$$\text{故 } D|X - Y| = E(|X - Y|^2) - (E|X - Y|)^2 = \frac{1}{18} a^2$$

4.2.1 方差的定义

练习2 设X的密度函数为 $f(x) = \frac{1}{2}e^{-|x|}$, 求D(X)。

解 因为 $E(X) = \int_{-\infty}^{+\infty} xf(x)dx$

$$= \int_0^{+\infty} x \cdot \frac{1}{2}e^{-x}dx + \int_{-\infty}^0 x \cdot \frac{1}{2}e^x dx = 0,$$
$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)dx = 2 \int_0^{+\infty} x^2 \cdot \frac{1}{2}e^{-x}dx$$
$$= -x^2 e^{-x} \Big|_0^{+\infty} + \int_0^{+\infty} 2xe^{-x}dx$$
$$= -2xe^{-x} \Big|_0^{+\infty} + \int_0^{+\infty} 2e^{-x}dx = -2e^{-x} \Big|_0^{+\infty} = 2,$$

所以, $D(X) = E(X^2) - [E(X)]^2 = 2.$

4.2.2 常见分布的方差



1. 0-1分布/伯努利分布

已知随机变量 X 的分布律为

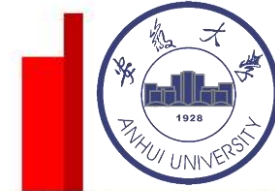
X	0	1
P	$1 - p$	p

$$\text{则 } E(X^2) = 0^2 \times (1-p) + 1^2 \times p = p$$

由于 $E(X) = p$, 故

$$\begin{aligned} D(X) &= E(X^2) - [E(X)]^2 \\ &= p(1 - p) = pq \end{aligned}$$

4.2.2 常见分布的方差



2. 二项分布

设随机变量 X 服从参数为 n, p 二项分布, 其分布律为

$$P\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, (k = 0, 1, 2, \dots, n),$$

$$E(X) = np$$

$$D(X) = np(1-p) = npq$$

4.2.2 常见分布的方差

3. 泊松分布

设随机变量 $X \sim P(\lambda)$, 其分布律为

$$P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, 2, \dots, \quad \lambda > 0.$$

已知 $E(X) = \lambda$, 而

$$\begin{aligned} E(X^2) &= \sum_{k=0}^{\infty} k^2 \cdot \frac{\lambda^k}{k!} e^{-\lambda} = \sum_{k=1}^{\infty} [k(k-1) + k] \cdot \frac{\lambda^k}{k!} e^{-\lambda} \\ &= \sum_{k=1}^{\infty} k(k-1) \cdot \frac{\lambda^k}{k!} e^{-\lambda} + \sum_{k=0}^{\infty} k \cdot \frac{\lambda^k}{k!} e^{-\lambda} \\ &= e^{-\lambda} \sum_{k=2}^{\infty} \frac{\lambda^k}{(k-2)!} + E(X) = \lambda^2 e^{-\lambda} \sum_{l=0}^{\infty} \frac{\lambda^l}{l!} + E(X) \\ &= \lambda^2 e^{-\lambda} \cdot e^{\lambda} + \lambda = \lambda^2 + \lambda \end{aligned}$$

所以方差 $D(X) = E(X^2) - [E(X)]^2 = \lambda$

泊松分布的数学期望和方差都等于参数 λ

4.2.2 常见分布的方差



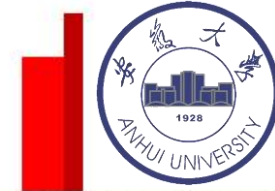
4. 均匀分布

设随机变量 $X \sim U(a, b)$, 则

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \int_a^b x^2 \frac{1}{b-a} dx = \frac{a^2 + ab + b^2}{3}$$

$$E(X) = \frac{a+b}{2}, \text{ 故 } D(X) = E(X^2) - [E(X)]^2 = \frac{(b-a)^2}{12}$$

4.2.2 常见分布的方差



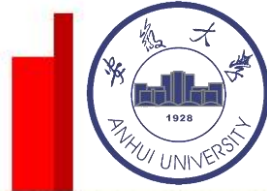
5. 指数分布

设随机变量 $X \sim E(\lambda)$, 则

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \int_0^{+\infty} x^2 \cdot \lambda e^{-\lambda x} dx = \frac{2}{\lambda^2}$$

$$E(X) = \frac{1}{\lambda}, \text{ 故 } D(X) = E(X^2) - [E(X)]^2 = \frac{1}{\lambda^2}$$

4.2.2 常见分布的方差



6. 正态分布

设 $X \sim N(\mu, \sigma^2)$, $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $\sigma > 0$, $-\infty < x < \infty$.

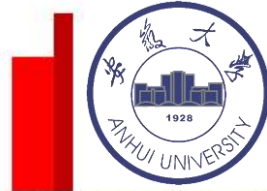
$$\begin{aligned} D(X) = E\{[X - E(X)]^2\} &= \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x) dx \\ &= \int_{-\infty}^{+\infty} (x - \mu)^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \end{aligned}$$

$$\text{令 } y = \frac{x-\mu}{\sigma}$$

正态分布的数学期望和方差分别为两个参数 μ 和 σ^2

$$\begin{aligned} &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \sigma^2 y^2 \cdot e^{-\frac{y^2}{2}} dy \\ &= \frac{\sigma^2}{\sqrt{2\pi}} (-ye^{-\frac{y^2}{2}} \Big|_{-\infty}^{+\infty} + \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2}} dy) \\ &= \sigma^2 \end{aligned}$$

4.2.2 常见分布的方差



7. Γ 分布

设随机变量 $X \sim \Gamma(r, \lambda)$, 则

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x) dx = \frac{(r+1)r}{\lambda^2}$$

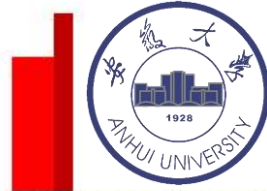
$$\text{因为 } E(X) = \frac{r}{\lambda}, \text{ 故 } D(X) = E(X^2) - [E(X)]^2 = \frac{r}{\lambda^2}$$

8. 对数正态分布

设随机变量 $X \sim LN(\mu, \sigma^2)$, 则随机变量 X 的方差为

$$D(X) = (e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$$

4.2.3 方差的性质



性质1 若 C 为常数, 则 $D(C) = 0$.

性质2 若 X 为随机变量, C 为常数, 则

$$D(CX) = C^2 D(X), \quad D(X + C) = D(X).$$

性质3 若 X 与 Y 是任意的两个随机变量, 则

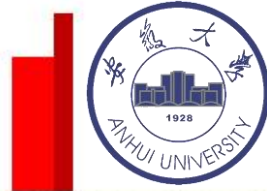
$$D(X + Y) = D(X) + D(Y) + 2E\{[X - E(X)][Y - E(Y)]\}$$

$$D(X - Y) = D(X) + D(Y) - 2E\{[X - E(X)][Y - E(Y)]\}$$

一般地, 对于 n 个随机变量 X_1, X_2, \dots, X_n , 有

$$D(X_1 + X_2 + \dots + X_n) = \sum_{i=1}^n D(X_i) + 2 \sum_{1 \leq i < j \leq n} E\{[X_i - E(X_i)][X_j - E(X_j)]\}$$

4.2.3 方差的性质



性质4 若随机变量 X 与 Y 相互独立, 则

$$D(X + Y) = D(X) + D(Y)$$

$$D(X - Y) = D(X) + D(Y)$$

一般地, 若 n 个随机变量 X_1, X_2, \dots, X_n 两两独立, 则

$$D(X_1 + X_2 + \dots + X_n) = D(X_1) + D(X_2) + \dots + D(X_n)$$

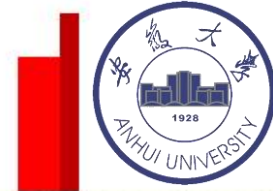
证明 $D(X + Y) = D(X) + D(Y) + 2E\{[X - E(X)][Y - E(Y)]\}$

$$\begin{aligned} E\{[X - E(X)][Y - E(Y)]\} &= E[XY - XE(Y) - YE(X) + E(X)E(Y)] \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

随机变量 X 与 Y 相互独立 $= 0$

所以 $D(X + Y) = D(X) + D(Y)$ 同理 $D(X - Y) = D(X) + D(Y)$

4.2.3 方差的性质



一般地，对于 n 个随机变量 X_1, X_2, \dots, X_n ，有

$$D(X_1 + X_2 + \dots + X_n) = \sum_{i=1}^n D(X_i) + 2 \sum_{1 \leq i < j \leq n} E\{[X_i - E(X_i)][X_j - E(X_j)]\}$$

若 n 个随机变量 X_1, X_2, \dots, X_n 两两独立，则

$$D(X_1 + X_2 + \dots + X_n) = D(X_1) + D(X_2) + \dots + D(X_n)$$

4.2.3 方差的性质

性质5 $D(X) = 0$ 的充要条件是:

随机变量 X 以概率1取常数 $E(X)$,即 $P(X = E(X)) = 1$

证明 充分性

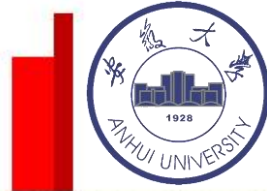
$$\text{设 } P(X = E(X)) = 1$$

$$\text{则 } P(X^2 = E(X^2)) = 1$$

$$\text{所以 } D(X) = E(X^2) - [E(X)]^2 = 0$$

必要性 需要用到切比雪夫不等式

4.2.3 方差的性质



切比雪夫不等式

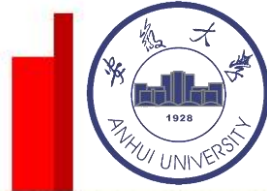
定理4.2.1 设随机变量 X 具有数学期望 $E(X) = \mu$ 和方差 $D(X) = \sigma^2$, 则对任意的正数 ε ,

$$P(|X - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$

$$P(|X - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}$$

上述结论表明, 在分布未知但数学期望和方差已知的情形下, 切比雪夫不等式可以用来估计 $P(|X - \mu| \geq \varepsilon)$ 的上界或者 $P(|X - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}$ 的下界。

4.2.3 方差的性质



例4.2.3 设 $U \sim U[-2, 2]$

$$X = \begin{cases} -1, & U \leq -1, \\ 1, & U > -1, \end{cases} \quad Y = \begin{cases} -1, & U \leq 1, \\ 1, & U > 1, \end{cases}$$

试求：1) (X, Y) 的联合分布律；2) $D(X + Y)$

解 1) 随机向量 (X, Y) 有四个可能取值 $(-1, -1), (-1, 1), (1, -1), (1, 1)$ 且

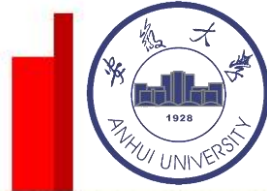
$$P(X = -1, Y = -1) = P(U \leq -1, U \leq 1) = \frac{1}{4}$$

$$P(X = -1, Y = 1) = P(U \leq -1, U > 1) = 0$$

$$P(X = 1, Y = -1) = P(U > -1, U \leq 1) = \frac{1}{2}$$

$$P(X = 1, Y = 1) = P(U > -1, U > 1) = \frac{1}{4}$$

4.2.3 方差的性质



由此可得随机向量 (X, Y) 的联合分布律为

X	Y	
	-1	1
-1	1/4	0
1	1/2	1/4

由 (X, Y) 的联合分布律, 可得 X 和 Y 的边缘分布律分别为

$$X \sim \begin{pmatrix} -1 & 1 \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix} \quad Y \sim \begin{pmatrix} -1 & 1 \\ \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$

从而, 可得

$$E(X) = (-1) \times \frac{1}{4} + 1 \times \frac{3}{4} = \frac{1}{2}, \quad E(X^2) = (-1)^2 \times \frac{1}{4} + 1^2 \times \frac{3}{4} = 1,$$

$$E(Y) = (-1) \times \frac{3}{4} + 1 \times \frac{1}{4} = -\frac{1}{2}, \quad E(Y^2) = (-1)^2 \times \frac{3}{4} + 1^2 \times \frac{1}{4} = 1,$$

4.2.3 方差的性质

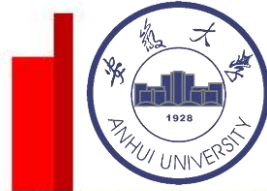


$$\text{故 } D(X) = E(X^2) - [E(X)]^2 = \frac{3}{4}, \quad D(Y) = E(Y^2) - [E(Y)]^2 = \frac{3}{4}$$

$$\text{因为 } D(X+Y) = D(X) + D(Y) + 2E\{[X - E(X)][Y - E(Y)]\}$$

$$\begin{aligned} E\{[X - E(X)][Y - E(Y)]\} &= E\left\{\left[X - \frac{1}{2}\right]\left[Y + \frac{1}{2}\right]\right\} \\ &= \left(-1 - \frac{1}{2}\right)\left(-1 + \frac{1}{2}\right) \times \frac{1}{4} + \left(-1 - \frac{1}{2}\right)\left(1 + \frac{1}{2}\right) \times 0 \\ &\quad + \left(1 - \frac{1}{2}\right)\left(-1 + \frac{1}{2}\right) \times \frac{1}{2} + \left(1 - \frac{1}{2}\right)\left(1 + \frac{1}{2}\right) \times \frac{1}{4} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{所以 } D(X+Y) &= D(X) + D(Y) + 2E\{[X - E(X)][Y - E(Y)]\} \\ &= \frac{3}{4} + \frac{3}{4} + 2 \times \frac{1}{4} = 2 \end{aligned}$$



1. 方差的意义

方差是一个常用来体现随机变量 X 取值分散程度的量. 如果 $D(X)$ 值大, 表示 X 取值分散程度大, $E(X)$ 的代表性差; 而如果 $D(X)$ 值小, 则表示 X 的取值比较集中, 以 $E(X)$ 作为随机变量的代表性好.

2. 方差的计算方法

(1) 利用定义计算

1) 对于离散型随机变量 X , 若其分布律为 $p_k = P\{X=x_k\}$, $k=1, 2, \dots$. 则随机变量 X 的方差为

$$D(X) = \sum_k [x_k - E(X)]^2 p_k$$

2) 对于连续型随机变量 X , 若其密度函数为 $f(x)$, 则 X 的方差为

$$D(X) = \int_{-\infty}^{+\infty} [x - E(X)]^2 f(x) dx$$

(2) 利用公式计算

若随机变量 X 的方差存在, 则有

$$D(X) = E(X^2) - [E(X)]^2$$



3. 方差的性质

性质1 若 C 为常数, 则 $D(C) = 0$.

性质2 若 X 为随机变量, C 为常数, 则

$$D(CX) = C^2 D(X), \quad D(X + C) = D(X).$$

性质3 若 X 与 Y 是任意的两个随机变量, 则

$$D(X + Y) = D(X) + D(Y) + 2E\{[X - E(X)][Y - E(Y)]\}$$

$$D(X - Y) = D(X) + D(Y) - 2E\{[X - E(X)][Y - E(Y)]\}$$

一般地, 对于 n 个随机变量 X_1, X_2, \dots, X_n , 有

$$D(X_1 + X_2 + \dots + X_n) = \sum_{i=1}^n D(X_i) + 2 \sum_{1 \leq i < j \leq n} E\{[X_i - E(X_i)][X_j - E(X_j)]\}$$

性质4 若随机变量 X 与 Y 相互独立, 则

$$D(X + Y) = D(X) + D(Y)$$

$$D(X - Y) = D(X) + D(Y)$$

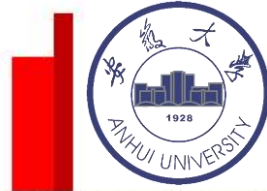
一般地, 若 n 个随机变量 X_1, X_2, \dots, X_n 两两独立, 则

$$D(X_1 + X_2 + \dots + X_n) = D(X_1) + D(X_2) + \dots + D(X_n)$$

性质5 $D(X) = 0$ 的充要条件是: 随机变量 X 以概率1取常数 $E(X)$, 即

$$P(X = E(X)) = 1$$

4.2.3 方差的性质



切比雪夫不等式

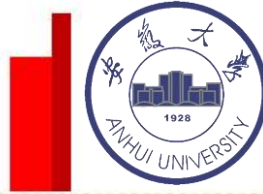
定理4.2.1 设随机变量 X 具有数学期望 $E(X) = \mu$ 和方差 $D(X) = \sigma^2$,
则对任意的正数 ε ,

$$P(|X - \mu| \geq \varepsilon) \leq \frac{\sigma^2}{\varepsilon^2}$$

$$P(|X - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}$$

上述结论表明, 在分布未知但数学期望和方差已知的情形下, 切比雪夫不等式可以用来估计 $P(|X - \mu| \geq \varepsilon)$ 的上界或者 $P(|X - \mu| < \varepsilon) \geq 1 - \frac{\sigma^2}{\varepsilon^2}$ 的下界。

Properties of the variance



Example 3.

It is known that the average number of white blood cells per milliliter in normal male adult blood is 7,300, and the mean square error is 700.

Try to use Chebyshev's inequality to estimate the probability that the number of white blood cells per milliliter is between 5200 and 9400.

Answer: Let X : the number of white blood cells per milliliter

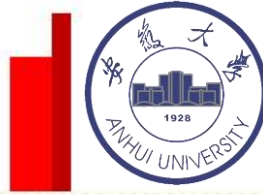
$$E(X) = 7300, \quad Var(X) = 700^2$$

Now request: $P(5200 \leq X \leq 9400) = ?$

$$\begin{aligned} &P(5200 - 7300 \leq X - 7300 \leq 9400 - 7300) \\ &= P(-2100 \leq X - E(X) \leq 2100) \\ &= P(|X - E(X)| \leq 2100) \end{aligned}$$

From Chebyshev's inequality: \Rightarrow

Properties of the variance



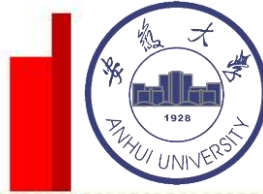
$$\begin{aligned} P\{|X - E(X)| \leq 2100\} &\geq 1 - \frac{\text{Var}(X)}{(2100)^2} \\ &= 1 - \left(\frac{700}{2100}\right)^2 = 1 - \frac{1}{9} = \frac{8}{9} \end{aligned}$$

That's means, the probability that the number of white blood cells per milliliter is between 5200 and 9400 is not less than 8/9.

Example 4. In each experiment, the probability of event A occurring is 0.75. Use Chebyshev's inequality to find: how big n needs to be so that in n independent repeated trials, the probability of event A occurring between 0.74 and 0.76 is at least 0.90?

Answer: Suppose X: the number of occurrences of event A in n trials.

Properties of the variance



$$X \sim B(n, 0.75)$$

$$E(X) = 0.75 n, \text{Var}(X) = 0.75 \times 0.25n = 0.1875n$$

It is necessary to find the minimum n that satisfying to condition:

$$P(0.74 < \frac{X}{n} < 0.76) \geq 0.90$$

$$P(0.74 < \frac{X}{n} < 0.76) = P(0.74n < X < 0.76n)$$

$$= P(0.74n - 0.75n < X - 0.75n < 0.76n - 0.75n)$$

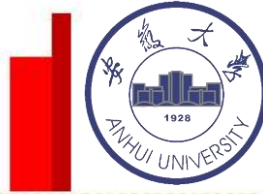
$$= P(-0.01n < X - 0.75n < 0.01n)$$

$$= P(|X - E(X)| \leq 0.01n)$$

The probability that the frequency of occurrence of event A is between 0.74 and 0.76 is at least 0.90

Take in Chebyshev's inequality $\varepsilon = 0.01 n$:

Properties of the variance



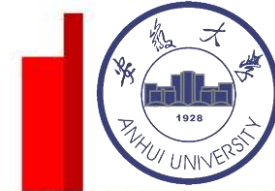
Then there are:
$$P(0.74 < \frac{X}{n} < 0.76) = P(|X - E(X)| \leq 0.01n)$$
$$\geq 1 - \frac{Var(X)}{(0.01n)^2} = 1 - \frac{0.1875n}{0.0001n^2} = 1 - \frac{1875}{n}$$

Let:
$$1 - \frac{1875}{n} \geq 0.9$$

$$n \geq \frac{1875}{1 - 0.9} = 18750$$

Conclusion: When n is 18750, the probability that event A occurs between 0.74 and 0.76 in n independent repeated trials can be at least 0.90.

Properties of the variance



Example 6. Assume the mathematical expectation of X is $E(X)$, and the variance is $Var(X)$. **Calculate:** The mathematical expectation and variance of random variable

$$Y = \frac{X - E(X)}{\sqrt{Var(X)}}$$

Answer:

$$\begin{aligned} E(Y) &= E\left(\frac{X - E(X)}{\sqrt{Var(X)}}\right) = \frac{1}{\sqrt{Var(X)}} E[X - E(X)] \\ &= \frac{1}{\sqrt{Var(X)}} [E(X) - E(X)] = 0 \end{aligned}$$

$$\begin{aligned} Var(Y) &= Var\left(\frac{X - E(X)}{\sqrt{Var(X)}}\right) = \frac{1}{Var(X)} Var[X - E(X)] \\ &= \frac{Var(X)}{Var(X)} = 1 \end{aligned}$$

Y is a standardized random variable