geographical analysis

Geographical Analysis (2013) **45**, 238–258

A Spatially Disaggregated Areal Interpolation Model Using Light Detection and Ranging-Derived Building Volumes

Harini Sridharan[1], Fang Qiu[2]

[1]Geographic Information Science & Technology, Oak Ridge National Laboratory, Oak Ridge, TN, USA, [2]Geospatial Sciences, University of Texas at Dallas, Richardson, TX, USA

Dasymetric areal interpolation is the process by which data are transferred from a spatial unit system for which they are available (source units) to another system for which they are required (target units) with the aid of ancillary information (control units). We propose a spatially disaggregated areal interpolation model for population data using light detection and ranging (LiDAR)-derived building volumes as an ancillary variable. Innovative methods are proposed for model initialization, iterative regression and adjustment, and stopping criteria to deal effectively with control units of unequal size. The model is derived and applied at the control unit level to minimize the modifiable areal unit problem, and an iterative adjustment process is utilized to overcome the spatial heterogeneity problem encountered in earlier approaches. The use of building volume to disaggregate the population into finer scales ensures maximum correspondence with the unit at which the original population data were collected and models not only the horizontal but also the vertical population distribution. A case study for Round Rock, Texas, demonstrates that the proposed spatially disaggregated model using LiDAR-derived building volumes outperforms earlier areal interpolation models using traditional area- and length-based ancillary variables.

Introduction

Owing to privacy issues and administrative convenience, population figures provided by most national census organizations are aggregated and presented at predefined spatial units, such as the census tract, block group, and block in the United States (Langford and Harvey 2001). Unfortunately, such predefined areas are not appropriate for all studies, particularly those that require specific areal units (Openshaw 1984, p. 3). Consequently, population data may be required at user-defined units that are different from those provided by the census. For example, socioeconomic studies may require population for special areas such as free trade

Correspondence: Fang Qiu, Geospatial Sciences, The University of Texas at Dallas, 800 W. Campbell Road, Richardson, TX 75080
e-mail: ffqiu@utdallas.edu

Submitted: June 7, 2012. Revised version accepted: December 10, 2012.

doi: 10.1111/gean.12010

zones, facility service areas, transportation survey zones, or urban enterprise corridors, whereas environmental studies may need demographic data for a drainage basin or for soil or vegetation zones. These zones seldom match census geography. Thus, demographic data are not readily available. Furthermore, census organizations often revise the boundaries of the spatial units used for reporting in order to accommodate shifts in the underlying population distribution over time. Consequently, modeling change over time often requires the comparison of data provided at different spatial units (Flowerdew and Green 1991). Finding suitable methods to reallocate population counts and other data from the census units for which they are available to the units for which they are needed is critical. This usually is accomplished by areal interpolation.

Areal interpolation is the process of transferring a data attribute (population in this case) from the spatial units for which it is measured to a different set of spatial units (Flowerdew and Openshaw 1987; Flowerdew, Green, and Kehris 1991; Langford 2006). In other words, when incompatibility exists among spatial units, areal interpolation estimates the values for the desired units based on values from the units for which they are available (Mrozinski and Cromley 1999). The spatial units for which the attribute data are available (census units in our case) are termed *source units*, whereas those to which the data are transferred are termed *target units*. *Simple* areal interpolation methods rely solely on the population data for the redistribution process, whereas *intelligent* methods incorporate ancillary variables often derived from remote sensing imagery, such as land use. Intelligent methods use the ancillary variables to disaggregate population data from the source units into *control units* and then subsequently reaggregate these data from the control units to the target units. Because density measures are often used as part of this process, some of the methods are referred to as *dasymetric*, a term relating to density (Wright 1936). Existing methodologies suffer from a variety of problems, which may include the failure of ancillary variables (such as land use) to capture adequately the vertical distribution of population, the classic *modifiable areal unit problem* (*MAUP*) when the relationship between population and ancillary variables quantified at the source units is assumed to apply to a different set of spatial units (such as the target units), and the failure of the modeling process to compensate for the effects of spatial heterogeneity caused by autocorrelation (the tendency of values in one spatial unit to be similar to and correlate with those of its neighbor) and nonstationarity (the tendency of the relationship between two variables to vary over space).

This article proposes a spatially disaggregated areal interpolation model (SDAIM) that uses light detection and ranging (LiDAR)-derived residential building volumes to achieve a dasymetric estimation of population at a fine spatial scale. Using buildings as the control unit provides a fine spatial scale, which should maximize accuracy because of its correspondence with the size of the unit at which the original population data were collected. Using volume rather than area as the ancillary variable captures the vertical as well as the horizontal distribution of population. The spatially disaggregated model should minimize MAUPs because the model is both derived from and applied to a single set of control units. The areal interpolation modeling technique also is specifically designed to address spatial autocorrelation and nonstationarity issues. The proposed SDAIM is compared with earlier methods in a case study of Round Rock, Texas, and is evaluated based on a variety of error measures, including root mean square error (RMSE), adjusted-RMSE (adj-RMSE), mean absolute percentage error (MAPE), median absolute percentage error (MedAPE), and population-weighted mean absolute error (PWMAE) (Qiu, Sridharan, and Chun 2010).

Background

Virtually all techniques that have been proposed for areal interpolation of population counts rely on specific assumptions about an underlying population distribution. One of the most widely used methods is area weighting interpolation (Goodchild and Lam 1980). In this method, the source units and target units are overlaid to obtain intersections, and the population count for each intersecting unit is obtained by multiplying the population density of its corresponding source unit by the area of the intersection (Flowerdew and Green 1991; Reibel and Bufalino 2005). Next, the intersections falling into each target unit are aggregated to estimate population for that target unit. Therefore, the population of the target unit is actually a weighted sum of the source unit densities, with the area of the intersections being the weights; hence, its name. Area weighting is classified as a simple method because it does not require any additional data besides the source and targets units. The implicit assumption of the method is that the population is homogeneously distributed in each source unit, which is rarely true, especially for large source units. It is also a volume-preserving method: when the population count of a source unit is disaggregated and summed back, the total equals that of the original data.

As an alternative, Tobler (1979) proposed a pycnophylactic approach that redistributes the population from the source units into a continuously changing density surface and then uses this surface to summarize the population for the target units. This density surface provides a gradient of population distribution and avoids area weighting's abrupt change in population density at the boundaries of the source units (Holt, Lo, and Hodler 2004; Kyriakidis 2004). To preserve volume, or the pycnophylactic property, an iterative step is employed to ensure that the original value of the source units is preserved in the transformation to the target units. As with areal weighting interpolation, smooth pycnophylactic interpolation also is categorized as a simple areal interpolation approach because population is transferred from the source units to the target units without the use of any ancillary data.

Because population distribution often is related to other spatial variables that are easy to measure, the so-called intelligent areal interpolation methods use ancillary variables to guide the redistribution of population (Flowerdew and Green 1991). Control units based on the ancillary data are used for transferring population from source units to target units. These methods disaggregate population from the source units into the control units, and then subsequently reaggregate it from the control units to the target units. Dasymetric methods are the most popular "intelligent" areal interpolation techniques. According to Wright (1936), the term dasymetric means "density measuring." Thus, dasymetric methods usually involve using the size of a control unit to estimate its population density, with the implicit assumption that population density is homogeneous within the control units (Eicher and Brewer 2001).

Assorted variables related to the spatial distribution of the underlying population have been used as ancillary data; control units extracted from remote sensing products are extensively employed. The most common control units include urbanized area (Ogrosky 1975; Lo and Welch 1977; Tobler 1979; Lisaka and Hegedus 1982; Weber 1994), residential land use (Langford and Unwin 1994; Fisher and Langford 1995; Mennis 2003; Holt, Lo, and Hodler 2004), buildings (Lo 1995; Wu, Wang, and Qiu 2008; Lwin and Murayama 2010), and road network length (Xie 1995; Reibel and Bufalino 2005). When selecting ancillary data, the size of the control units relative to that of the source and target units, as well as that of the unit at which the data were originally collected, is particularly important. The ideal control units are of a size corresponding to that of the units for which the population information was originally collected. For example, demographic data in the United States are collected from every household before being aggregated into census units. Control units much bigger than a housing unit, such as residential land use, usually include parts of a landscape where people do not reside, such as roads, sidewalks, and lawns. Therefore, population is not necessarily uniformly distributed within a control unit, a deviation from the implicit assumption of the dasymetric method. Furthermore, if the control units are much larger than the target units, additional spatial splitting of the control units via intersection overlap is needed before transferring population from the control units to the target units. In contrast, when the size of control units is much smaller than that of housing units, as in the case of using pixels from high spatial resolution imagery, only a small fraction of household population is transferred into each control unit. When these fractions of population are reaggregated into target zones, more accuracy is not necessarily achieved, but the computational expense is usually higher.

The degree to which ancillary variables are able to reflect an underlying residential settlement pattern is a critical factor affecting the performance of areal interpolation. Two major types of ancillary variables are available: geometric measurements of the control units themselves, such as their area or length (assumed to be directly related to the size of the residential settlement), or an attribute of the control units that is indirectly associated with the population distribution, such as the spectral value or the fraction of pixels indicating an impervious surface. Because dasymetric methods, by definition, imply density measurement, strictly speaking, only the first type of ancillary variable, which allows population density per geometric size to be obtained, is dasymetric. The most commonly used ancillary variables based on geometric size are either one-dimensional (1-D) length measures or two-dimensional (2-D) areal measures. Although these 1-D and 2-D measures may provide good estimates of the horizontal distribution of population, they are not able to capture its vertical dimension because population may reside in structures ranging from single-story, single-family houses, to multistory, multifamily complexes, to high-rise buildings that vary greatly in their number of stories. The lack of vertical information in these 1-D and 2-D ancillary variables makes them less than ideal as population indicators, which often results in underestimation of population in areas with high-rise buildings and overestimation in areas with low-rise buildings (Green 1956; Harvey 2002b).

Various approaches to implementing dasymetric interpolation have been proposed, the simplest being the binary method (Fisher and Langford 1995; Eicher and Brewer 2001; Holt, Lo, and Hodler 2004). Land use is classified into two categories: residential and nonresidential. Instead of assuming that population is homogeneous within the source units, the binary land use method allows population counts to be distributed only to residential land use in the source units, whereas nonresidential land use is considered unpopulated. However, the binary dasymetric approach still has an implicit assumption that the population density is uniformly distributed across the residential land use class, which again is seldom true (Mennis and Hultgren 2006; Maantay, Maroko, and Herrmann 2007). For this reason, the binary dasymetric method was extended to the multiclass dasymetric approach by classifying the control units into more than one residential class (Eicher and Brewer 2001), where these residential classes have different population densities. To obtain these density measures, Mennis (2003) suggests a heuristic method based on empirical sampling of population densities for three land use classes (low-density urban, high-density urban, and nonurban). When empirical samples are not available, a multivariate linear regression equation estimated with ordinary least squares has been utilized to statistically predict the population densities for the different residential classes (Flowerdew, Green, and Kehris 1991; Langford and Harvey 2001; Flowerdew and Green 1992, 1994;

Goodchild, Anselin, and Deichmann 1993; Yuan, Smith, and Limp 1997; Bielecka 2005; Langford 2006). However, this linear regression still assumes that the relationship between population counts and the geometric size of control units of the same type is constant across all source units in the entire study area. Population density for the same single-family class, for example, is likely to differ from place to place in a study area. Failure to capture this spatial heterogeneity in modeling the relationship between population counts and the geometric size of residential settlements may lead to overestimation in one place and underestimation in another, with positive or negative estimation errors often clustering in nearby neighborhoods.

Several researchers have addressed this problem by exploiting the spatial autocorrelation property of population using geostatistical and spatial statistical models. Because population density follows the first law of geography (Tobler 1979), it tends to be more similar in communities that are closer to each other. Kyriakidis (2004) suggests a cokriging-based theoretical framework to establish a spatial cross-correlation between areal and point variables to perform an area-to-point interpolation. Kyriakidis and Yoo (2005) demonstrate the workability of this framework by applying it to synthetic data and further compare it with Tobler's pycnophylactic interpolation (Yoo, Kyriakidis, and Tobler 2010). Liu, Kyriakidis, and Goodchild (2008) instantiate this conceptual framework by cokriging population density with built-up area compositions obtained from satellite imagery. Wu and Murray (2005) independently develop a cokriging-based areal interpolation by modeling population spatial autocorrelation and cross-correlation between population and impervious surface fractions extracted from Landsat 7 Enhanced Thematic Mapper Plus imagery. In a related context, Qiu, Sridharan, and Chun (2010) use spatial autoregressive (SAR) statistical models to account for the presence of spatial autocorrelation for population estimations based on building volumes at the census block level. Overall, these studies suggest that incorporating spatial autocorrelation into a small area population estimation model can generate results superior to those from standard linear regression for dasymetric areal interpolation.

Both geostatistical and SAR models are global, where a single regression function is derived to describe the relationship between the population and control variables with the same set of regression parameters throughout an entire study area. Spatial heterogeneity is addressed by integrating a spatially weighted average of the population of neighboring units into such a model, based on either distance or topological adjacency. Another group of recent studies addresses the spatial heterogeneity problem for dasymetric areal interpolation from a different perspective: spatial nonstationarity. Because both population and control variables usually are geographically varying phenomena, the relationship between them also can vary across space, resulting in spatial nonstationarity of the functional relationship between population and the control variables. For example, Lo (2008) demonstrates that the relationship between population density and land cover/land use is not constant across space, suggesting that a single regression function may not be able to model this spatially varying relationship adequately. To address this issue, he uses a geographically weighted regression (GWR) model for population estimation. Unlike the global models, GWR is a local model that allows the regression coefficient parameters to vary across space to capture spatial variation in the relationship between population and the ancillary variables. Lin, Cromley, and Zhang (2011) extend the GWR approach to areal interpolation to capture the relationship between population and multiclass land cover derived from Landsat Thematic Mapper (TM) imagery. When used to solve areal interpolation problems for both misaligned and nested areal units, the GWR approach performs better for the former than for the latter. As pointed out by Lo (2008), the spatial processes determining a population distribution are likely to vary more significantly at coarser scales than at finer scales, which tend to be more stationary. This change in scale may account for the poorer performance of GWR at finer spatial scales. Perhaps, Qiu, Sridharan, and Chun (2010) demonstrate that a SAR approach is effective in modeling a population distribution at finer spatial scales because spatial autocorrelation remains more persistent at these scales, whereas spatial nonstationarity does not.

The preceding regression-based approaches, whether linear or spatial, global or local, all attempt to model the statistical relationship between population and ancillary variables at the aggregate level of the source units. For example, with land use ancillary data, a regression analysis predicts the population counts for each source unit from the aggregated area of each land use class within that source unit. Then, the resulting parameter estimates of the regression function calculated at the source unit level are applied to the target units to obtain the estimates of population based on the aggregate area of each of the land use classes within the target units. The application of a spatially aggregated model developed for one areal unit system to a different areal unit system may lead to the well-known MAUP, caused by the scale effect and zoning effect (Openshaw 1984). The scale effect is the variation in outcomes that may be obtained when the same data are aggregated into areal units with different spatial scales (i.e., the number of areal units changes) (Openshaw and Taylor 1979). Clark and Avery (1976) demonstrate the scale effect in bivariate regression by investigating the impact of aggregation on correlation coefficient and regression parameter estimates. They show that as the level of data aggregation increases the correlation coefficients between the variables increase. Therefore, in the context of areal interpolation, if the scale of the target units is very different from that of the source units (as with nested areal units), then applying statistical models developed at source units (such as census tracts) to target units (such as census blocks) could be problematic. The zoning effect refers to the inconsistency in statistical results obtained from different partitioning of the same surface (i.e., the number of areal units may remain the same) (Openshaw 1984; Wong 1996). As a result, the model derived at the source unit level may not perform well when applied to target units with a different partitioning even if they are of similar spatial scale (as in the case of misaligned areal units). An alternative areal interpolation methodology is desirable to overcome these MAUP issues.

A possible strategy is suggested by Harvey (1999, 2002a, b), who introduces an areal interpolation model at a level finer than the source units by iteratively regressing population counts with the pixel values of TM imagery based on a least-squares approximation to the expectation-maximization (EM) algorithm. EM is a statistical approach for data sets with missing information (Dempster, Laird, and Rubin 1977), consisting of two iterative procedures: the expectation (E) step and the maximization (M) step. In the E step, a conditional E of the missing or unobserved data is estimated based upon complete covariate information. In the M step, a maximum likelihood estimation of the regression parameters is conducted as though no data were missing. These two steps are repeated iteratively until convergence to a stable set of parameters is achieved. The general EM algorithm has been elaborated by Martin (1984) and was further extended to a regression context by Griffith, Bennett, and Haining (1989). Griffith (2010) presents a dramatic simplification of this algorithm for the type of problem discussed here. Flowerdew and Green (1991) were the first to use EM procedures to interpolate the population from source units to target units by disaggregating the population to the intersection units. The areal interpolation problem is treated as a two-way table with the source unit populations as the rows and the target unit populations as the columns. The row sums equal the populations of each source unit, which are known values. However, the values of the cells in the table, representing the population of the intersection units, are unknown and essentially missing data. EM procedures are used to estimate the missing population of the intersection units, which are subsequently aggregated to the target units to interpolate their populations. Flowerdew and Green's (1991) approach is not a true dasymetric model because no control unit is employed. Harvey (2002a, b) extends EM-based areal interpolation by using residential pixels as control units and develops an iterative regression model predicting population counts from the spectral values at the pixel level rather than at the source or target unit level. Wu and Murray (2007) apply this least-squares approximation of EM using impervious surface fraction as the ancillary variable.

Results from these studies demonstrate that pixel-level (control unit) regression models can produce superior results for areal interpolation in comparison to the aggregate-level (source unit) models. This superiority is probably attributable to the population being spatially disaggregated and iteratively regressed at a finer scale (in this case, at the pixel level), which avoids the MAUP encountered by the regression models at the aggregate level. However, the size of the pixels in low spatial resolution TM imagery can be larger than that of a housing unit, and the associated mixed pixel problem may affect interpolation accuracy because nonresidential features may fall into the same pixel. Additionally, the pixel-level spectral value or impervious surface fraction is an attribute only indirectly associated with population counts, unlike the geometric size used in dasymetric models, which has a direct relationship. Like other 1-D and 2-D ancillary variables, these pixel-level variables also lack the height information necessary to capture the vertical distribution of population.

In summary, the performance of areal interpolation models is strongly influenced by (1) the size of the control units, (2) the relevance of the ancillary variables, and (3) the suitability of the areal interpolation modeling process. In the model proposed in this study, the fine-scale control unit, which is residential buildings, was chosen to maximize the correspondence with the size of unit at which the original population data were collected (the household). The ancillary variable, building volume derived from LiDAR data, was selected to account for the vertical distribution of population in addition to its horizontal distribution. The modeling techniques utilized were designed to minimize the MAUP scale and zoning effects and to address the spatial heterogeneity issue.

Methodology

The SDAIM developed in this study uses residential buildings as control units and their volumes as the ancillary variable. The census population is first disaggregated from the source units to the individual residential buildings based on volume and then is aggregated to estimate the population of the target units. Residential buildings are chosen as control units because of their close correspondence to both the pattern of human settlement and the level at which the data originally were collected. Compared with more widely used road network and land use data, individual residential buildings are the most detailed physical units characterizing where members of a population reside and do not contain any nonresidential components. Because the ancillary variable, volume, is a measure of geometric size, a population density per unit volume can be calculated, thus creating a dasymetric model. Among all variables derivable from remote sensing data, volume probably has the most direct relevance for population counts due to its three-dimensional (3-D) nature. In general, the larger the building volume, the more housing units it can accommodate regardless of the vertical structure of the building, be it single story or high rise, and therefore the more residents it can house. Furthermore, because they are the fundamental base unit for population, buildings can be easily reaggregated back to any size or type of target unit.

For fine-scale population estimation, the areas of residential buildings have long been extracted from remotely sensed imagery or from geographic information system (GIS)-formatted building footprint files. However, to extract the volumes of the buildings, 3-D height information also is required. LiDAR, although a relatively new technology, already has been used extensively for the accurate extraction of building heights (Maas and Vosselman 1999; Barnsley, Steel, and Barr 2003) and for the reconstruction of 3-D building structures, including their footprint areas (Hongjian and Shiqiang 2006; Gurram et al. 2007; Sampath and Shan 2007). Building volumes derived from the height measured by LiDAR multiplied by the area from LiDAR or other sources also have been employed for small-area population estimation by Wu, Wang, and Qiu (2008), Qiu, Sridharan, and Chun (2010), and Dong, Ramesh, and Nepali (2010). However, LiDAR-derived building volumes have not yet been utilized for areal interpolation, with the exception of Xie (2006), who proposes a theoretical framework for areal interpolation of population using high-resolution digital orthophoto quarter quad imagery and LiDAR data. However, no actual population areal interpolation or detailed accuracy assessment has been conducted.

The proposed SDAIM is built upon enhancements to the pixel-based areal interpolation method used by Harvey (1999, 2000, 2002a, b). Instead of using pixel values as the ancillary variable, the source unit population is spatially disaggregated to the individual residential buildings using LiDAR-derived volume. To this end, a regression model is formed at the residential building level rather than at the aggregated level of source units, with the population of each individual building being the dependent variable and its volume being the independent variable. Because the population of each building is missing information while its volume is a known value, an iterative procedure based on the least-squares approximation of the EM algorithm is used to specify the regression model. Considering a set of source units indexed by s, the goal is to disaggregate the source unit population (P_s) into each individual building (p_{si}) within that source unit using the building volume (v_{si}) as the ancillary variable. The disaggregation of population is subject to the constraint that the sum of the disaggregated building population must equal the population of its respective source unit:

$$P_s = \sum_i p_{si}, \tag{1}$$

where p_{si} is the population of building i belonging to source zone s. In other words, the SDAIM maintains the pycnophylactic property.

The model itself has three key components: (1) an initialization of population estimates for each residential building; (2) an iterative regression and adjustments at the residential building level; and (3) a stopping criterion to terminate the iterations upon convergence. Methods aimed at improving areal interpolation accuracy and efficiency are proposed for each component, specifically tailored to the chosen control unit (residential buildings) and its ancillary variable (building volume).

Initialization

The initialization procedure provides a rough estimate of the population for each building, which then is used as the dependent variable for a subsequent iterative regression at the building level. In his pixel-based areal interpolation approach, Harvey (1999, 2000) simply distributes the population of a source unit equally to all the control units within it. With pixels as control units, the equal initialization strategy uniformly assigns an initial value to all pixels in a source unit equal to the amount of the ratio of its population to the number of pixels. Albeit suitable for control units of equal size, this initialization is far less appropriate for varying-sized control units such as buildings. Consequently, we propose a new initialization approach that assigns a source unit population to its buildings in proportion to their sizes (in this case volumes):

$$P_{si(init)} = \left(\frac{P_s}{V_s}\right) * \upsilon_{si},\ i = 1, 2, \ldots n_s, \tag{2}$$

where $p_{si(init)}$ is the initial population assigned to building i in source unit s with volume υ_{si}, and V_s is the total volume of all the buildings in that source unit. This initialization method is equivalent to applying a binary dasymetric areal interpolation, except that the ancillary variable is building volume rather than area. The initial values may not be accurate but do serve as an appropriate starting point for further refinement. Therefore, the initial values obtained provide an expectation (E) of the populations for the individual buildings, which originally are missing data.

Iterative regression and adjustments

Following the initialization, a two-step iterative procedure is carried out to fine-tune estimation of the building-level population. In the first step, a regression model at the control unit level (individual building) is estimated using the expected population of the buildings as the response variable and their volumes as the covariate:

$$\hat{P}_{si} = \beta_o + \beta_1 \upsilon_{si}, \tag{3}$$

where $\hat{p}_{si}$ is the estimated building-level population, and β_0 and β_1 are the estimated regression parameters. Next, the building-level population values are reestimated using these newly estimated regression parameters.

Because the reestimated building-level populations may no longer maintain the pycnophylactic property, their values need to be refined to ensure that the sum of the building-level population estimates equals the census populations of their respective source units [equation (1)]. Subsequently, an iterative adjustment is utilized to redistribute the errors of estimation into individual buildings to minimize discrepancies between the estimated and the actual populations of the source units. The estimated population of a source unit $(\hat{p}_s)$ is obtained by aggregating the estimated building-level populations $\hat{p}_{si}$ of the source unit:

$$\hat{P}_s = \sum_{i=1}^{n_s} \hat{P}_{si}, \tag{4}$$

where n_s is the number of buildings in a source unit.

For the estimated population of a source unit $(\hat{p}_s)$ to equal its true population, an adjustment needs to be computed and applied to each building based on the discrepancy between its true and its estimated source unit populations. There are different ways to achieve this pycnophylactic property, two of which are widely adopted: ratio-based adjustment (Flowerdew and Green 1991; Bloom, Pedler, and Wragg 1996) and equal adjustment (Harvey 1999, 2000, 2002a). With ratio-based adjustment, the building-level population estimates are viewed as a gain and adjusted by using the ratio between the true and the estimated populations of the source unit as a scaling term:

$$\hat{P}_{si(adj)} = \hat{P}_{si}\left(\frac{P_s}{\hat{P}_s}\right), \tag{5}$$

where $\hat{p}_{si(adj)}$ is the adjusted population estimation for building i in source zone s.

With equal adjustment, as used in Harvey's (1999) pixel-based approach, the discrepancy is regarded as an offset and corrected by dividing the discrepancy between the true and the estimated populations by the number of control units (pixels) in a source unit, and then by adding this amount as a constant to the population estimates of all the control units within a source unit:

$$\hat{P}_{si(adj)} = \hat{P}_{si} + \frac{\left(P_s - \hat{P}_s\right)}{n_s}, \tag{6}$$

where $\hat{p}_{si(adj)}$ is the adjusted population estimate for building i in source zone s. Harvey (1999) used Lagrange multipliers to show that this error redistribution strategy minimizes the sum of squared residuals around its regression line while constraining the sum of the control units to the source unit population. However, this equal adjustment method, although suitable for pixels that are equal-sized control units, may not be applicable to control units of varying size, such as residential buildings.

We propose a size-based adjustment for varying-sized control units. In our method, the source unit-level population discrepancy also is treated as an offset, but redistributed to the control units in proportion to their sizes, rather than as a constant. In the case of building volume as the ancillary variable, the adjustment term added to a building equals the discrepancy first divided by the total volume of all buildings in unit s (V_s), and then multiplied by the volume of that building:

$$\hat{P}_{si(adj)} = \hat{P}_{si} + \left(\frac{\left(P_s - \hat{P}_s\right)}{v_s}\right)\upsilon_{si}, \tag{7}$$

where $V_s = \sum_{i=1}^{n_s} v_{si}$. Then, the adjusted estimates of building populations are used as the new expected values for the next iteration of building-level regression and adjustment to fine-tune iteratively the small-area estimation.

Stopping criterion

The stopping criterion for the iterative regression and adjustment method requires an indicator of model convergence at a desirable accuracy within a reasonable number of iterations (Bloom, Pedler, and Wragg 1996). Two commonly used stopping criteria are used. One is based on the change in the iterative regression parameters between two consecutive iterations (Flowerdew and Green 1991), which basically is related to the change in population estimates for the control units (Bloom, Pedler, and Wragg 1996). At the end of each iterative regression, differences in the regression parameters from the previous iteration are compared against predefined threshold values, and the iteration is stopped if the differences are less than the threshold values. The other stopping criterion, suggested by Harvey (1999, 2000, 2002a), uses the change in the R^2 of the iterative regression between two iterations. Because R^2 is determined by the values predicted from the iterative regression, which in turn is a function of the regression parameters, these two stopping criteria are, from a practical perspective, essentially identical and generally take the same number of iterations to achieve convergence.

Table 1 Various Possibilities for the Starting, Adjustment, and Stopping Factors

Factor	Possibilities
Initialization	1. Equal initialization
	2. Size-based initialization
Iterative adjustment	1. Equal adjustment
	2. Ratio-based adjustment
	3. Size-based adjustment
Stopping criterion	1. Parameter based
	2. RMSE based

We propose a new stopping criterion based on the change in the deviation from the pycnophylactic property between two iterations. At the end of each iterative regression, the estimated population of each source unit $(\hat{p}_s)$ is obtained by aggregating its estimated building-level populations using equation (4) and then comparing this value with its true population P_s. The deviation of the model from the pycnophylactic property is obtained by computing the RMSE:

$$RMSE = \sqrt{\frac{\sum_{1}^{n}\left(\hat{P}_s - P_s\right)^2}{n}}, \tag{8}$$

where n is the number of source units in a study area. The change in RMSE at iteration r is defined as

$$\Delta^r RMSE = RMSE^r - RMSE^{r-1}. \tag{9}$$

If $\Delta^r RMSE$ is less than a specified threshold, then the iterative process is stopped. Usually, a very low threshold value (0.000001) is used. The use of $\Delta^r RMSE$ as the stopping criterion is appropriate because achieving the pycnophylactic property is a key constraint of the modeling process. Additionally, computation of the RMSE includes the calculation of the discrepancy of population estimated at the source unit level (i.e., $\hat{P}_s - P_s$), which also is a fundamental process for the size-based iterative adjustment.

Upon convergence, the final step of the model is to obtain the target population (P_T). This is achieved by summing the final adjusted population estimates for the individual buildings that fall within each target zone:

$$P_T = \sum_i p_{Ti}, \tag{10}$$

where p_{Ti} is the population of the ith building in target zone T.

The accuracy and efficiency of the proposed model is affected by decisions with respect to the initialization, adjustment, and stopping criterion stages. To obtain the most accurate and efficient areal interpolation, various algorithm designs for these three steps need to be evaluated. Table 1 summarizes possible options to be considered, and Table 2 sums up the error measures to be used in the evaluation. These error measures are RMSE, adj-RMSE, MAPE, MedAPE, and PWMAE. The equations for their calculations also are provided and a detailed description is available in Qiu, Sridharan, and Chun (2010).

Table 2 Error Measures Based on the Difference between the Interpolated Population (P_t') and Observed Population (P_t) for the N_t Target Zones

Error measure	Equation
RMSE	$\sqrt{\frac{\sum_{t=1}^{N_t}(P_t'-P_t)^2}{N_t}}$
Adj-RMSE	$\sqrt{\frac{\sum_{t=1}^{N_t}\left(\frac{P_t'-P_t}{P_t}\right)^2}{N_t}}$
MAPE	$100\times\frac{\sum_{t=1}^{N_t}\frac{\lvert P_t'-P_t\rvert}{P_t}}{N_t}$
MedAPE	$100\times Median\frac{\lvert P_t'-P_t\rvert}{P_t}$
PWMAE	$100\times\frac{\sum_{t=1}^{N_t}P_t'\times\frac{\lvert P_t'-P_t\rvert}{P_t}}{\sum_{t=1}^{N_t}P_t'}$

Once the most appropriate algorithm design is identified, the spatially disaggregated model proposed here must be assessed relative to alternative interpolation modeling techniques, control units, and ancillary variables. The models used for this benchmarking are simple area weighting interpolation and dasymetric interpolation using 2-D residential building area, land use area ancillary data, and 1-D road network data. Additionally, a spatially disaggregated areal interpolation based on building area (rather than volume) is included to assess the degree to which volume data, which are generally more onerous to obtain, produce superior areal interpolation results over area data.

Case study

Round Rock, Texas, a rapidly expanding suburban city located 15 mi north of Austin in the Austin–Round Rock metropolitan area, is the focus for the study, primarily because of the ready availability of all the required data. It was the second fastest growing city in the United States between 2000 and 2006, increasing almost 41%, from 63,136 people in 2000 to an estimated 86,175 people in 2006. The study area covers 36 block groups comprising 1,147 blocks defined by the U.S. Census Bureau that had a total population of 88,948 in the 2000 census; individual block group populations range from 579 to 9,610, and the block groups contain 27,769 residential buildings (Fig. 1).

The city of Round Rock's GIS information center provided all the ancillary data including building footprints, land use parcels, and road networks. The building footprints were derived from high-resolution aerial photographs through a combined process of digital processing and manual interpretation. Because population is expected to reside only in residential buildings, the building layers were overlaid on the land use data, and those belonging to nonresidential parcels were eliminated. The land use and building footprints were available only for 2006 and hence had

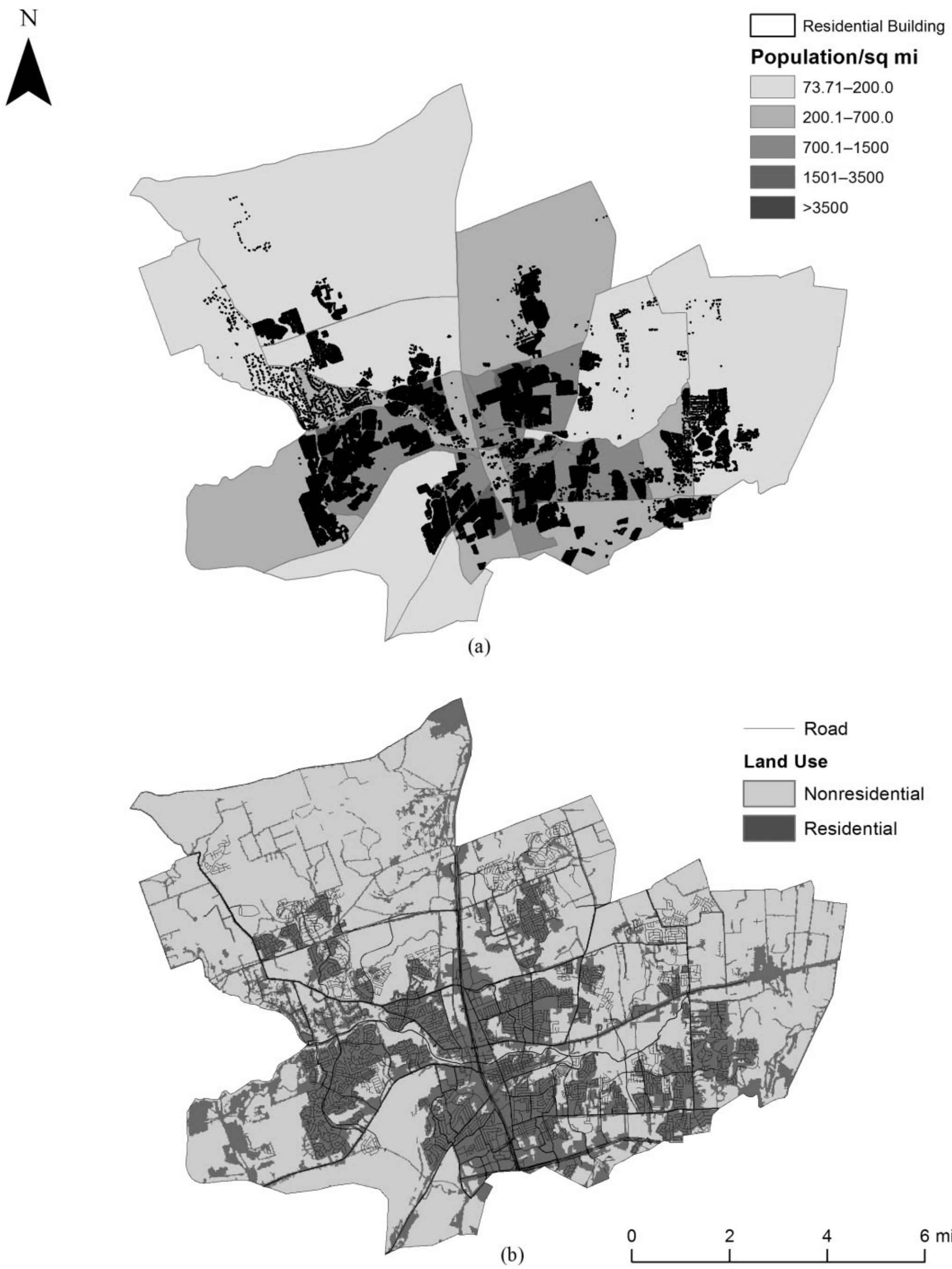

Figure 1. Study area, Round Rock, Texas. (a) The population density for the census block groups (source units) and the residential buildings (control units). (b) Residential land use zones and the road network.

Figure 2. Residential building volume-based areal interpolation.

to be corrected to match the census population data from 2000. This correction was achieved by comparing 2000 and 2006 aerial photographs, and then eliminating buildings that had been added during this time period (none had been demolished). The volumes of the residential buildings were obtained from LiDAR data collected in 2006, with an average point spacing of 1.4 m and a vertical accuracy of 10 cm. The relative heights of buildings from the ground were extracted from the LiDAR point cloud using a simple buffer-based approach (Qiu, Sridharan, and Chun 2010). A 1-ft (30-cm) two-way buffer was created along the perimeter of each building footprint, which served as an exclusion zone eliminating the LiDAR points reflected off the walls of the building. A 2-ft one-way outward buffer was drawn along the perimeter of the exclusion zone. This outer buffer contains LiDAR points bounced off the ground, and the median elevation of the points in this buffer provides the elevation of the ground's location. Similarly, a 2-ft one-way inward buffer was drawn inside a building to obtain the elevation of the building. By subtracting these two elevations, the above-ground height of a building was obtained, which then was combined with the building footprint data to compute residential building volumes.

The block groups were used as the source units, and the residential buildings were used as the control units (Fig. 2). Census blocks were used as target units in order to assess the accuracy of the proposed model for the more challenging case of finer resolution nesting of target units within source units. The study area's 1,147 blocks have at least 50 blocks nested within each block group.

Evaluation of settings strategies

Table 3 summarizes the efficiency and accuracy of the volume-based SDAIM with various combinations of initialization, adjustment, and stopping strategies. Overall, the model using a size-based adjustment with size-based initialization and a RMSE-based stopping criterion has the lowest error measures and the greatest efficiency in terms of the smallest number of iterations for convergence. For example, compared with the equal initialization, equal adjustment, and parameter-based stopping criterion combination, which is generally used by pixel-based areal interpolations, this model reduces the MedAPE from 25.95 to 18.92, and the number of iterations for convergence from 445 to 18.

To evaluate the impact of each algorithm design on model performance, we can keep one setting constant (such as the adjustment setting, which is used to organize the rows of Table 3) while changing the others. For every combination of initialization and stopping criterion, the size-based adjustment consistently produced the highest accuracy for all error measures. This size-based adjustment also converged faster, having fewer iterations than equal and ratio-based

Table 3 Efficiency and Accuracy Measures for the Volume-based SDAIM Using Different Algorithm Settings

Initialization	Adjustment	Stopping criterion	No. of iterations	RMSE	Adj-RMSE	MAPE	MedAPE	PWMAE	r between observed and predicted
Equal	Equal	Parameter	445	127.2	1.737	61.685	25.96	82.945	0.736
Size	Equal	Parameter	339	127.2	1.737	61.685	25.96	82.945	0.736
Equal	Equal	RMSE	231	127.1	1.737	61.67	25.93	82.880	0.736
Size	Equal	RMSE	195	127.1	1.737	61.67	25.93	82.880	0.736
Equal	Ratio	Parameter	117	128.6	1.779	58.526	20.00	83.523	0.75
Size	Ratio	Parameter	104	128.6	1.779	58.526	20.00	83.523	0.75
Equal	Ratio	RMSE	50	128.1	1.778	58.162	20.00	83.273	0.75
Size	Ratio	RMSE	37	128.1	1.778	58.162	20.00	83.273	0.75
Equal	Size	Parameter	44	117.2	1.731	55.625	19.14	74.87	0.77
Size	Size	Parameter	29	117.2	1.731	55.625	19.14	74.87	0.77
Equal	Size	RMSE	19	116.9	1.72	55.44	18.92	74.6	0.765
Size	Size	RMSE	18	116.9	1.72	55.44	18.92	74.6	0.765

adjustments, irrespective of initialization and stopping criterion settings. The ratio-based adjustment produced the largest interpolation errors for three error measures (RMSE, adj-RMSE, PWMAE), with slightly smaller values than the equal adjustment method for the MAPE and MedAPE measures. The ratio-based adjustment method also consistently resulted in faster convergence than the equal adjustment method. Overall, adjustment based on the size of control units, such as the volume of individual buildings, performed better than adjustment based on population of the source units only, such as the ratio-based setting.

As with size-based adjustment, equal adjustment, although not suitable for control units of unequal size (such as buildings), is based on information about the control units (in this case, the count of individual buildings within each source zone). Consequently, equal adjustment performs slightly better than ratio-based adjustment, which does not utilize any control unit-level information. Equal adjustment can be considered to be special case of size-based adjustment, which is applicable only to control units of equal size (e.g., pixels).

The primary impact of the initialization setting is on the computational efficiency of the spatially disaggregated model. With the same combination of adjustment and stopping criterion, the size-based initialization converges faster than the equal initialization. For example, with the same equal adjustment and parameter-based stopping criterion, the size-based initialization took 339 iterations to converge, whereas the equal-based initialization used 445 iterations for the Round Rock case study. However, the initialization had no impact on the accuracy achieved at convergence. Both initialization strategies resulted in exactly the same accuracy based on all error measures for the same combinations of the other two settings.

For the stopping criterion, the RMSE-based strategy consistently converged much faster than the parameter-based stopping criterion. For example, with equal initialization and size-based adjustment, the parameter-based stopping criterion took 44 iterations to converge, whereas the RMSE-based approach took less than half this number (19). The RMSE-based stopping criterion also produced a slight but consistently higher accuracy. This improvement is because parameter-based stopping primarily relies on a comparison between the estimates from two consecutive iterations, whereas RMSE depends on a comparison between each estimate and its true source unit value. Thus, the goal of areal interpolation to minimize prediction discrepancy is embedded in the model.

Evaluation relative to other models

Table 4 presents the error measures from benchmarking the volume-based SDAIM against four alternatives: simple area weighting interpolation, dasymetric interpolation using 2-D residential land use area ancillary data, dasymetric interpolation using 1-D road network data, and the spatially disaggregated model using building area rather than volume. Both spatially disaggregated models use the preceding optimum setting designs, namely the size-based initialization, the size-based iterative adjustment, and the RMSE-based stopping criterion. The simple area-weighted procedure produces the least accurate results, with much larger values for all error measures. For example, this procedure's PWMAE measure is 1,543.05, which is 6–20 times greater than all others. The area-weighted procedure assumes a uniform distribution of population within the source units, even when population resides only in concentrated subareas. Consequently, it performs poorly when compared with those utilizing ancillary data to provide information about the actual distribution of human settlement.

The two binary models using land use (residential) area and road network length as ancillary variables result in less accurate areal interpolations than the two spatially disaggregated models,

Table 4 Accuracy Measures for Different Areal Interpolation Techniques

Methodology	Ancillary variable	RMSE	Adj-RMSE	MAPE	MedAPE	PWMAE
Area weighting	N/A	164.29	19.31	331.97	69.63	1543.05
Binary dasymetric (area based)	Land use area	122.00	4.84	119.79	67.65	251.79
Binary dasymetric (length based)	Road length	130.70	8.83	102.49	60.39	101.50
Spatially disaggregated (area based)	Building area	134.15	1.56	62.35	29.95	84.81
Spatially disaggregated (volume based)	Building volume	116.97	1.72	55.44	18.92	74.60

N/A, not applicable.

with the road-based model slightly superior to the land use model (except in terms of RMSE and adj-RMSE). This outcome is understandable because residential land use also includes spaces such as parking lots, gardens, and yards, areas where people do not actually reside. The dasymetric model using road network length is based on the premise that human settlements always occur along roads, and therefore road length within residential land use can indicate where the population is located better than the residential land use area itself can. This is consistent with the results obtained by others (Hawley and Moellering 2005; Reibel and Bufalino 2005; Zhang and Qiu 2011). Nevertheless, a road network also may include lengths pertaining to nonresidential uses, such as parking lots and major thoroughfares, which may explain its underperformance relative to the spatially disaggregated models.

The spatially disaggregated model using residential building area as the control variable produces a better result than the land use area and road length models in terms of almost all measures. The area of individual residential buildings provides a significantly more precise depiction of population distribution than more generalized land use and road measures. Nevertheless, the use of residential building area is unable to capture the vertical (single story to high rise) dimension of settlement because it measures only building footprints. Consequently, for almost all of the error measures, the spatially disaggregated model using building area is inferior to that using residential building volume. However, the vertical dimension of population distribution is effectively captured by indexing the total volume of dwelling units in a building, a size variable that is directly related to the number of households, the basis on which census data are collected. Consequently, the spatially disaggregated model using residential building volumes achieves the highest areal interpolation accuracy among all models tested.

In general, building volumes tend to indicate more residents, which is modeled by the iterative regression steps of the spatially disaggregated areal interpolation procedure. However, other factors also can influence the volume/resident relationship. Higher income neighborhoods are likely to have higher building volumes per resident than lower income areas. The classic urban trade-off of less space for more accessibility (or other amenities) likewise tends to affect this relationship, as do "empty-nesters" (couples whose children have left home) and single seniors (e.g., people whose spouses have died) still living in the family home. Therefore, a regression model that does not incorporate these factors may result in overestimation in high-income neighborhoods, less accessible locations, and older residential areas, and in underestimation in other areas. However, this spatial heterogeneity of the relationship between building volume and the number of residents is captured by the SDAIM. Although the model is still global, a local constraint on the model is exerted through the iterative adjustment, which effectively offsets the under- or overestimation of the control units within a source unit. Additionally, because the iterative regression and adjustment are constructed at the building control unit level, the spatially disaggregated model does not involve the application of a model derived at the source unit level to the target units, which may differ significantly in size. Therefore, unlike spatially aggregated models, the performance of the spatially disaggregated model is independent of the size of target units relative to that of the affiliated source units.

Summary and future studies

In this study, we propose a SDAIM using residential building volume as the control variable. The model is built upon improvements to pixel-based approaches using a least-squares approximation to the EM algorithm, where an areal interpolation is treated as missing data. New designs are proposed for the initialization, the iterative adjustment, and the stopping criterion for the model iterations, combinations that are more appropriate for the varying-sized control units (residential buildings) used here rather than for the equal-sized pixel control units used in earlier EM applications. Our case study compares the performance of various designs and then evaluates the best performing design with earlier areal interpolation models in terms of accuracy and efficiency. Significantly better results are obtained, which is attributed both to the use of residential buildings as control units and building volumes as the ancillary variable, and to the improved spatially disaggregated modeling techniques. Residential buildings are far closer to the spatial resolution of human settlement than residential land use and road networks used hitherto. The LiDAR-derived building volume used as an ancillary variable accounts for the vertical distribution of population, which 1-D length and 2-D area information fails to capture. The spatially disaggregated model is built and used at the control unit level and therefore avoids the MAUP encountered in more traditional models constructed at the source unit level. By using an iterative regression and adjustment procedure, the proposed model is able to deal with spatial heterogeneity problems associated with the ancillary variable, irrespective of whether its cause is spatial autocorrelation or nonstationarity. In this study, the target units are census blocks, the smallest census units with population information. Because the model formulation aggregates into target units the population transferred to buildings from source units, its performance is independent of the resolution of the target units. Therefore, the model should work equally well with larger target units. Finally, the building volumes used in this study were obtained from LiDAR data. However, the model also is applicable to areas for which LiDAR data are unavailable but for which stereoscopic aerial photographs are available to provide building volume extraction through traditional photogrammetry.

The SDAIM using LiDAR-derived building volumes demonstrates significant improvement over existing conventional methods in the case study. However, the superiority of this approach needs to be further tested. In particular, it should be compared with geostatistical-, SAR-, and GWR-based models for addressing spatial heterogeneity issues. In this study, the source unit is the census block group, whereas the target unit is the block. This contribution enabled investigation of the more challenging issue of areal interpolation of a finer resolution areal unit nesting within a coarser resolution unit. However, the ideal source unit most likely is the census block because the smaller the source unit, the less the spatial heterogeneity within it. Additionally, the division of residence type into single-, duplex-, and multifamily units could be used to develop a multiclass spatially disaggregated model so that variation in the relationship between population and building volumes among different classes can be directly addressed. Furthermore, the impact of data inaccuracies such as missing buildings or mislabeling of land use types also should be carefully investigated. For instance, when a large apartment building is mislabeled as commercial in a given source zone, the areal interpolation process tries to redistribute population into the remaining buildings, possibly distorting the relationship between the building-level population and its volume. Data accuracy is crucial for any successful small area population areal interpolation.

Acknowledgements

This article was prepared for Oak Ridge National Laboratory, P.O. Box 2008, Oak Ridge, TN 37831-6285, managed by UT-Battelle, LLC, for the United States Department of Energy under Contract No. DEAC05-00OR22725.

References

Barnsley, M. J., A. M. Steel, and S. L. Barr. (2003). "Determining Urban Land Use through An Analysis of the Spatial Composition of Buildings Identified in LIDAR and Multispectral Image Data." In *Remotely Sensed Cities*, 83–108, edited by V. Mesev. London: Taylor and Francis.

Bielecka, E. (2005). "A Dasymetric Population Density Map of Poland." *Proceedings of the International Cartographic Conference*, 9–15 July 2005, A Coruña, Spain (CD).

Bloom, M. L., P. J. Pedler, and G. E. Wragg. (1996). "Implementation of Enhanced Areal Interpolation Using Mapinfo." *Computers & Geoscience* 22, 459–66.

Clark, W. A. V., and K. L. Avery. (1976). "The Effects of Data Aggregation in Statistical Analysis." *Geographical Analysis* 8, 428–38.

Dempster, A., N. Laird, and D. Rubin. (1977). "Maximum Likelihood from Incomplete Data Via the EM Algorithm." *Journal of the Royal Statistical Society, Series B* 39(1), 1–38.

Dong, P., S. Ramesh, and A. Nepali. (2010). "Evaluation of Small Area Population Estimation Using LiDAR, Landsat TM and Parcel Data." *International Journal of Remote Sensing* 31(21), 5571–86.

Eicher, C., and C. A. Brewer. (2001). "Dasymetric Mapping and Areal Interpolation: Implementation and Evaluation." *Cartography and Geographic Information Science* 28, 125–38.

Fisher, P. F., and M. Langford. (1995). "Modelling the Errors in Areal Interpolation between Zonal Systems by Monte Carlo Simulation." *Environment and Planning A* 27, 11–24.

Flowerdew, R., and M. Green. (1991). "Data Integration: Statistical Methods for Transferring Data between Zonal Systems." In *Handling Geographic Information: Methodology and Potential Applications*, 38–53, edited by I. Masser and M. B. Blakemore. London: Longman Scientific and Technical.

Flowerdew, R., and M. Green. (1992). "Developments in Areal Interpolation Methods and GIS." *Annals of Regional Science* 26, 67–78.

Flowerdew, R., and M. Green. (1994). "Areal Interpolation and Types of Data." In *Spatial Analysis and GIS*, 121–45, edited by S. Fotheringham and P. Rogerson. London: Taylor and Francis.

Flowerdew, R., and S. Openshaw. (1987). A Review of the Problem of Transferring Data from One Set of Areal Units to Another Incompatible Set." RR4, Northern Regional Research Laboratory, University of Lancaster and Newcastle upon Tyne.

Flowerdew, R., M. Green, and E. Kehris. (1991). "Using Areal Interpolation Methods in Geographic Information Systems." *Papers in Regional Science* 70, 303–15.

Goodchild, M., L. Anselin, and U. Deichmann. (1993). "A Framework for the Areal Interpolation of Socioeconomic Data." *Environment and Planning A* 25, 383–97.

Goodchild, M. F., and N. S.-N. Lam. (1980). "Areal Interpolation: A Variant of the Traditional Spatial Problem." *Geo–Processing* 1, 297–312.

Green, N. E. (1956). "Aerial Photographic Analysis of Residential Neighborhoods: An Evaluation of Data Accuracy." *Social Forces* 35, 142–47.

Griffith, D. A. (2010). "Some Simplifications for the Expectation-Maximization (EM) Algorithm: The Linear Regression Model Case." *InterStat 23*. Available at interstat.statjournals.net/YEAR/2010/articles/1003002.pdf.

Griffith, D. A., R. J. Bennett, and R. Haining. (1989). "Statistical Analysis of Spatial Data in the Presence of Missing Observations: A Methodological Guide and An Application to Urban Census Data." *Environment and Planning A* 21(11), 1511–23.

Gurram, P., H. Rhody, J. Kerekes, S. Lach, and E. Saber. (2007). "3D Scene Reconstruction through A Fusion of Passive Video and LIDAR Imagery." *Applied Imagery Pattern Recognition Workshop*, 36th IEEE, 133–38.

Harvey, J. T. (1999). Estimation of Population Using Satellite Imagery." PhD dissertation, University of Ballarat, Australia.

Harvey, J. T. (2000). "Small Area Population Estimation Using Satellite Imagery." *Statistics in Transition* 4(4), 611–33.

Harvey, J. T. (2002a). "Population Estimation Models Based on Individual TM Pixels." *Photogrammetric Engineering and Remote Sensing* 68(11), 1181–92.

Harvey, J. T. (2002b). "Estimating Census District Populations from Satellite Imagery: Some Approaches and Limitations." *International Journal of Remote Sensing* 23(10), 2071–95.

Hawley, K., and H. Moellering. (2005). "A Comparative Analysis of Areal Interpolation Methods." *Cartography and Geographic Information Science* 32, 411–23.

Holt, J. B., C. P. Lo, and T. W. Hodler. (2004). "Dasymetric Estimation of Population Density and Areal Interpolation of Census Data." *Cartography and Geographic Information Science* 31, 103–21.

Hongjian, Y., and Z. Shiqiang. (2006). "3D Building Reconstruction from Aerial CCD Image and Sparse Laser Sample Data." *Optics and Lasers in Engineering* 4(6), 555–66.

Kyriakidis, P. C. (2004). "A Geostatistical Framework for Area-to-Point Spatial Interpolation." *Geographical Analysis* 36(3), 259–89.

Kyriakidis, P. C., and E.-H. Yoo. (2005). "Geostatistical Prediction and Simulation of Point Values from Areal Data." *Geographical Analysis* 37(2), 124–51.

Langford, M. (2006). "Obtaining Population Estimates in Non-Census Reporting Zones: An Evaluation of the 3-Class Dasymetric Method." *Computers, Environment and Urban Systems* 30, 161–80.

Langford, M., and J. Harvey. (2001). "The Use of Remotely Sensed Data for Spatial Disaggregation of Published Census Population Counts." *Proceedings IEEE/ISPRS Joint Workshop on Remote Sensing and Data Fusion over Urban Areas*. Spazio Congressi, Rome, 260–64.

Langford, M., and D. J. Unwin. (1994). "Generating and Mapping Population Density Surfaces within A Geographical Information System." *The Cartographic Journal* 31, 21–6.

Lin, J., R. Cromley, and C. Zhang. (2011). "Using Geographically Weighted Regression to Solve the Areal Interpolation Problem." *Annals of GIS* 17(1), 1–14.

Lisaka, J., and E. Hegedus. (1982). "Population Estimation from Landsat Imagery." *Remote Sensing of Environment* 12(4), 259–72.

Liu, X. H., P. C. Kyriakidis, and M. F. Goodchild. (2008). "Population-Density Estimation Using Regression and Area-to-Point Residual Kriging." *International Journal of Geographical Information Science* 22(4), 431–44.

Lo, C. P. (1995). "Automated Population and Dwelling Unit Estimation from High-Resolution Satellite Images: A GIS Approach." *International Journal of Remote Sensing* 16, 17–34.

Lo, C. P. (2008). "Population Estimation Using Geographically Weighted Regression." *GIScience & Remote Sensing* 45(2), 131–48.

Lo, C. P., and R. Welch. (1977). "Chinese Urban Population Estimates." *Annals of the Association of American Geographer* 67(2), 246–53.

Lwin, K. K., and Y. Murayama. (2010). "Development of GIS Tool for Dasymetric Mapping." *International Journal of Geoinformatics* 6, 11–8.

Maantay, J. A., A. R. Maroko, and C. Herrmann. (2007). "Mapping Population Distribution in the Urban Environment: The Cadastral-Based Expert Dasymetric System (CEDS)." *Cartography and Geographic Information Science* 34, 77–102.

Maas, H.-G., and G. Vosselman. (1999). “Two Algorithms for Extracting Building Models from Raw Laser Altimetry Data.” *ISPRS Journal of Photogrammetry and Remote Sensing* 54, 153–63.

Martin, R. J. (1984). “Exact Maximum Likelihood for Incomplete Data from A Correlated Gaussian Process.” *Communications in Statistics: Theory and Methods* 13, 1275–88.

Mennis, J. (2003). “Generating Surface Models of Population Using Dasymetric Mapping.” *The Professional Geographer* 55, 31–42.

Mennis, J., and T. Hultgren. (2006). “Intelligent Dasymetric Mapping and Its Application to Areal Interpolation.” *Cartography and Geographic Information Science* 33, 179–94.

Mrozinski, R. D., Jr, and R. G. Cromley. (1999). “Singly- and Doubly-Constrained Methods of Areal Interpolation for Vector-Based GIS.” *Transactions in GIS* 3, 285–301.

Ogrosky, C. E. (1975). “Population Estimates from Satellite Imagery.” *Photogrammetric Engineering and Remote Sensing* 41, 707–12.

Openshaw, S. (1984). *The Modifiable Areal Unit Problem*. Norwich: Geo Books.

Openshaw, S., and P. Taylor. (1979). “A Million Or So Correlation Coefficients: Three Experiments on the Modifiable Area Unit Problem.” In *Statistical Applications in the Spatial Sciences*, 127–44, edited by N. Wrigley. London: Pion.

Qiu, F., H. Sridharan, and Y. Chun. (2010). “Spatial Autoregressive Models for Population Estimation at the Block Level Using LIDAR Derived Volume Information.” *Cartography and Geographic Information Science* 37(3), 239–57.

Reibel, M., and M. E. Bufalino. (2005). “Streetweighted Interpolation Techniques for Demographic Count Estimation in Incompatible Zone Systems.” *Environment and Planning A* 37, 127–39.

Sampath, A., and J. Shan. (2007). “Building Boundary Tracing and Regularization from Airborne LIDAR Point Clouds.” *Photogrammetric Engineering and Remote Sensing* 73(7), 805–12.

Tobler, W. (1979). “Smooth Pycnophylactic Interpolation for Geographical Regions.” *Journal of the American Statistical Association* 74, 519–30.

Weber, C. (1994). “Per-Zone Classification of Urban Land Use Cover for Urban Population Estimation.” In *Environmental Remote Sensing from Regional to Global Scales*, 142–48, edited by G. M. Foody and P. J. Curran. Chichester and New York: Wiley.

Wong, D. W. S. (1996). “Aggregation Effects in Geo-Referenced Data.” In *Practical Handbook of Spatial Statistics*, 83–106, edited by D. Griffith. Boca Raton, FL: CRC Press.

Wright, J. K. (1936). “A Method of Mapping Densities of Population.” *The Geographical Review* 26, 103–10.

Wu, C., and A. Murray. (2005). “A Cokriging Method for Estimating Population Density in Urban Areas.” *Computers Environment and Urban Systems* 29(5), 558–79.

Wu, C., and A. Murray. (2007). “Population Estimation Using Landsat Enhanced Thematic Mapper Imagery.” *Geographical Analysis* 39, 26–43.

Wu, S., L. Wang, and X. Qiu. (2008). “Incorporating GIS Building Data and Census Housing Statistics for Subblock-Level Population Estimation.” *The Professional Geographer* 60(1), 21–35.

Xie, Y. (1995). “The Overlaid Network Algorithms for Areal Interpolation Problem.” *Computers, Environment and Urban Systems* 19, 287–306.

Xie, Z. (2006). “A Framework for Interpolating the Population Surface at the Residential-Housing-Unit Level.” *GIScience & Remote Sensing* 43(3), 233–51.

Yoo, E.-H., P. C. Kyriakidis, and W. Tobler. (2010). “Reconstructing Population Density Surfaces from Areal Data: A Comparison of Tobler’s Pycnophylactic Interpolation Method and Area-to-Point Kriging.” *Geographical Analysis* 42(1), 78–98.

Yuan, Y., R. M. Smith, and W. F. Limp. (1997). “Remodeling Census Population with Spatial Information from Landsat TM Imagery.” *Computers, Environment and Urban Systems* 21, 245–58.

Zhang, C., and F. Qiu. (2011). “A Point-Based Intelligent Approach to Areal Interpolation.” *The Professional Geographer* 63(2), 262–76.