

# Full Bayesian Models to Handle Missing Data in Health Economic Evaluations

**A thesis submitted in partial fulfillment of the requirement for the  
Degree of M. Phil in Statistics**

Andrea Gabrio

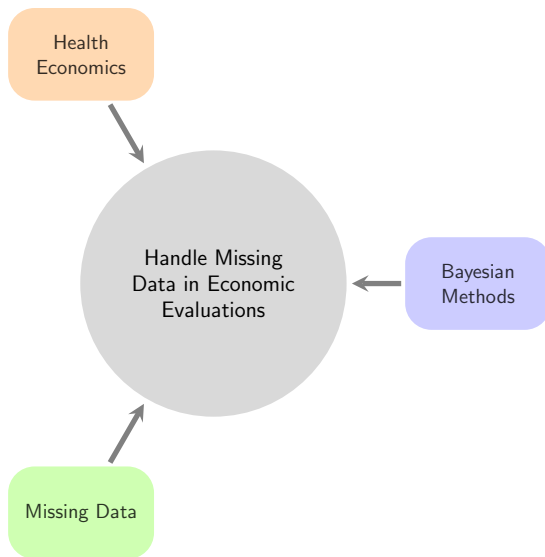
Primary Supervisor: Gianluca Baio

Secondary Supervisors: Alexina J. Mason, Rachael Hunter

DEPARTMENT OF STATISTICAL SCIENCE  
UNIVERSITY COLLEGE LONDON

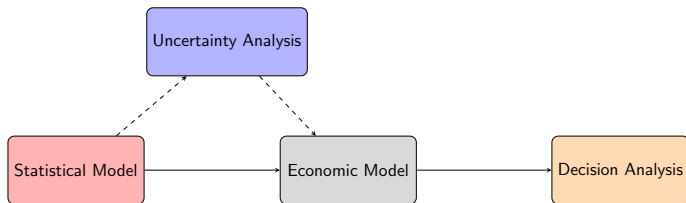


# Research Question



# Health Economics

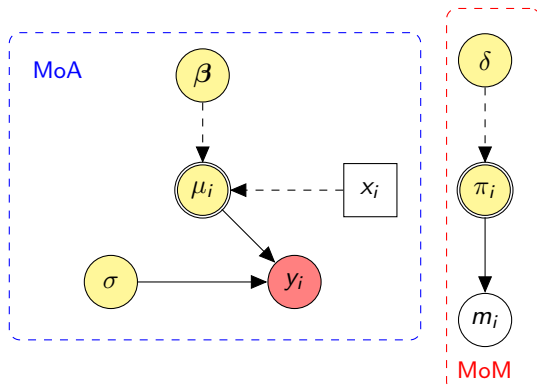
- Health Economic Evaluation involves the application of economic theory to *health* and *health care*
- The main objective is the comparison of alternative options in terms of their *costs* and *benefits* (e.g. QALYs)
- Provides decision-makers with information that can help resource allocation decisions



# Missing Data

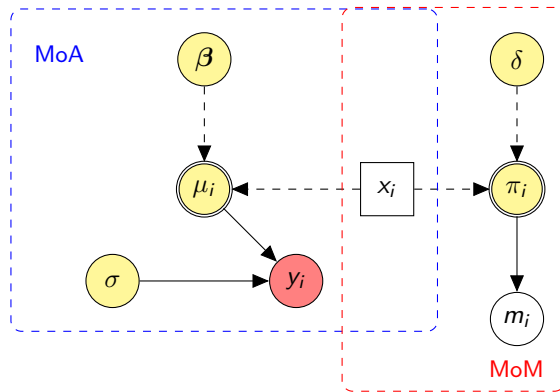
- In CEAs missing data handling is particularly challenging:
  - Missingness may occur in both benefits/costs
  - Quantify impact of uncertainty on the output of the decision process
- Assumptions cannot be tested from the data but need to be formulated based on the available state of knowledge
- This formally translates into an assumed *missing data mechanism* (Rubin, 1987) that is linked to the data generating process
  - Missing Completely At Random (MCAR)
  - Missing At Random (MAR)
  - Missing Not At Random (MNAR)

# Missing Data Mechanism: MCAR



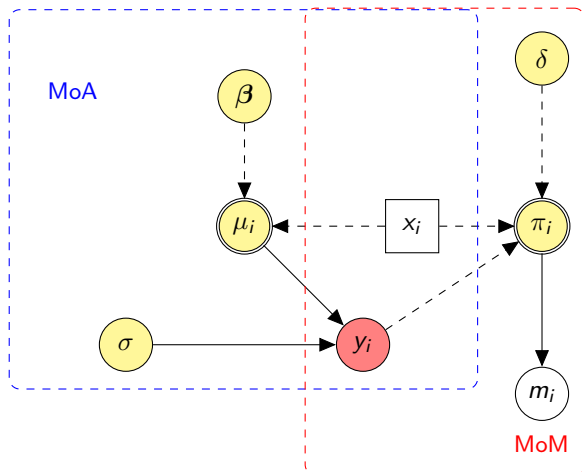
**Figure:** MoA=Model of Analysis, MoM=Model of Missingness

# Missing Data Mechanism: MAR



**Figure:** MoA=Model of Analysis, MoM=Model of Missingness

# Missing Data Mechanism: MNAR



**Figure:** MoA=Model of Analysis, MoM=Model of Missingness

# Missing Data Methods

- Complete Case Analysis
  - Elimination of partially observed cases
  - Simple but reduce efficiency and possibly bias parameter estimates
- Single Imputation
  - Imputation of missing data with a single value (mean, median, LVCF)
  - Does not account for the uncertainty in the imputation process
- Multiple Imputation (Rubin, 1987)
  - Missing data imputed  $H$  times to obtain  $H$  different imputed datasets
  - Each dataset is analysed and  $H$  sets of estimates are derived
  - Parameter estimates are combined into a single quantity
  - The uncertainty due to imputation is incorporated but the validity relies on the correct specification of the imputation model



# Full Bayesian Models

- Parameters are given probability distributions that describe the uncertainty before (*prior*) and after (*posterior*) observing the data

$$p(\boldsymbol{\theta} \mid y) \propto p(y \mid \boldsymbol{\theta})p(\boldsymbol{\theta})$$

- Incorporate both individual and parameter (missing data) uncertainty
- Naturally encode alternative missingness assumptions through the priors and assess the robustness of the results (Sensitivity Analysis)
- Often not analytically tractable and iterative approximation methods, e.g. MCMC (Brooks et al., 2011), are required

# Nonignorable Missingness: Selection Models

- *Selection Models* factor the joint distribution  $(y, m)$  as:

$$p(y, m \mid x, \theta^{MoA}, \theta^{MoM}) = p(y \mid x, \theta^{MoA})p(m \mid y, x, \theta^{MoM})$$

- Typically,  $m \sim \text{Bern}(\pi)$  with:  $\text{logit}(\pi) = \gamma_0 + \gamma_1 x + \delta y$ 
  - $\delta$  represents the impact on  $\pi$  of the missing values (MNAR parameter)
- Possible SAs (Mason et al., 2012) are:
  - **Assumption Sensitivity:** Vary MoA and/or MoM form
  - **Parameter Sensitivity:** Vary MoM assumptions (priors)
- If results are not robust, more information may need to be gathered

# Summary

- Missing data can bias inferences and mislead the decision output
- Most of the methods limit their validity to MAR, which may not hold and, more importantly, can never be verified from the data
- Full Bayesian models handle missingness by assessing and quantifying different types of uncertainty
- Selection Models are a possible choice to handle nonignorable missingness but SA is necessary to assess the robustness of the results to alternative assumptions

# Literature Review

Two-fold purpose:

- 1 Provide some guidelines on the reporting and analysis of missingness (Quality Evaluation Scheme)
  - 2 Review of the missing data methods in CEAs (2003-2015), updating the work of Noble et al. (2012)
- Original review focused only on missing costs in within-trial CEA studies
    - 88 articles between 2003-2009
  - Include missing effects and update the review
    - 81 studies between 2009-2015

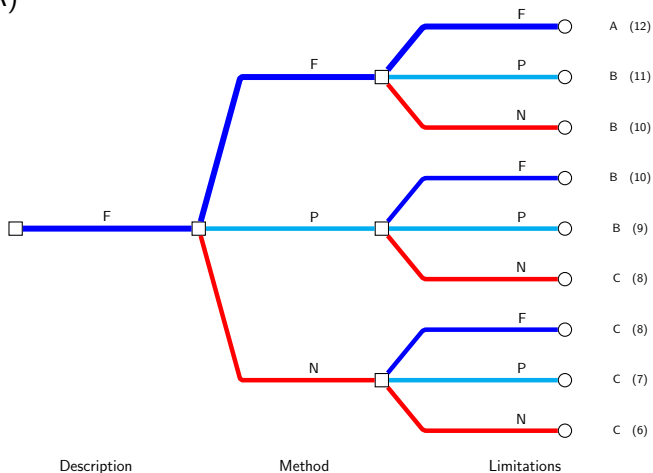
# Literature Review: Quality Evaluation Scheme

- Provide guidelines on how to report information on missingness:
  - **Description:** Missingness pattern and assumption
  - **Method:** Choose base-case and alternative methods
  - **Limitations:** Possible issues in assumptions
- Assign a grade to the articles based on the information in each component
- Match the articles to a score (0-12) based on the overall information provided, weighted by each component

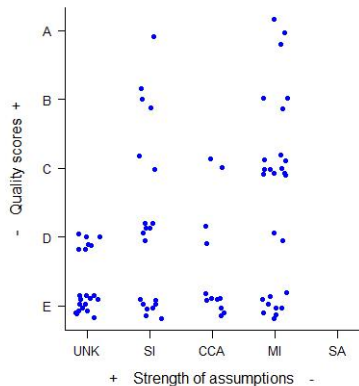
Score Content	Description	Method	Limitations
Full (F)	6	4	2
Partial (P)	3	2	1
Null (N)	0	0	0

# Literature Review: Quality Evaluation Scheme

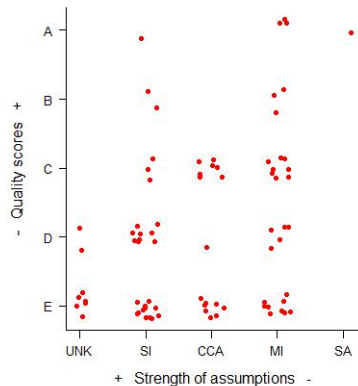
- Based on the grading, we group the articles into ordered categories (E-A)



# Literature Review: Quality Evaluation Review



**(a) Missing Cost Analyses (2009-2015)**



**(b) Missing Effect Analyses (2009-2015)**

# Literature Review: Conclusions

- High missing data proportions in within-trial CEAs may lead to imprecise economic evidences
- The review shows a movement towards more flexible methods in terms of missingness assumptions but:
  - Many studies do not provide transparent missing data information
  - Almost no study performs a sensitivity analysis
- QES could represent a valuable tool to improve missing data handling
  - Explicitly define assumptions and assess their impact on the conclusions.



# Case Study: The MenSS Trial

- The MenSS pilot RCT (Bailey et al., 2016) evaluates the cost-effectiveness of a new digital intervention to reduce the incidence of STI in young men with respect to the SOC
  - QALYs calculated from utilities (EQ-5D 3L)
  - Total costs calculated from different components (no baseline)

	Control ( $n_1=75$ )		Intervention ( $n_2=84$ )	
Time	observed	missing(%)	observed	missing(%)
Baseline	72	4%	72	14%
3 months	34	55%	23	73%
6 months	35	53%	23	73%
12 months	43	43%	36	57%
<b>complete cases</b>	27	64%	19	77%

# Case Study: Modelling strategy

model	MoA ( $e, c$ )	MoM ( $e$ )	MoM ( $c$ )
<i>Base-Case</i>	Independent Normal	MAR	MAR
$MAR(e, c)$	Joint Normal	MAR	MAR
$MNAR(e)$	Joint Normal	MNAR	MAR
$MNAR(c)$	Joint Normal	MAR	MNAR

MNAR-MoM:  $\text{logit}(\pi) = \gamma_0 + \delta y$  (Selection Model for  $y = e, c$ )

- MNAR( $e$ ):  $e^{mis} \approx (5 - 10\%)$  lower than  $e^{obs} \rightarrow \delta^e \sim N(-2, 1)$
- MNAR( $c$ ):  $c^{mis} \approx (60 - 70\%)$  higher than  $c^{obs} \rightarrow \delta^c \sim N(0, 1)$

## Case Study: Results

Parameter	Base-Case			MAR ( $e, c$ )			MNAR ( $e$ )			MNAR ( $c$ )		
	Mean	95% CI		Mean	95% CI		Mean	95% CI		Mean	95% CI	
<b>Control (<math>t = 1</math>)</b>												
mean QALY ( $\mu_1^e$ )	0.886			0.874	0.840	0.907	0.855	0.807	0.893	0.863	0.826	0.899
mean cost ( $\mu_1^c$ )	214			207.770	115.363	302.901	207.912	113.226	301.081	290.324	126.971	452.932
sd QALY ( $\sigma_1^e$ )				0.081	0.061	0.110	0.081	0.064	0.103	0.081	0.064	0.103
sd cost ( $\sigma_1^c$ )				257.964	197.201	341.123	259.517	191.160	344.420	267.924	197.633	356.626
<b>Intervention (<math>t = 2</math>)</b>												
mean QALY ( $\mu_2^e$ )	0.918			0.913	0.868	0.956	0.847	0.715	0.929	0.912	0.860	0.967
mean cost ( $\mu_2^c$ )	189			189.170	110.778	267.963	188.497	108.829	267.280	316.032	106.835	516.946
sd QALY ( $\sigma_2^e$ )				0.092	0.066	0.130	0.094	0.070	0.124	0.092	0.069	0.122
sd cost ( $\sigma_2^c$ )				174.082	124.350	252.623	176.378	121.666	249.735	190.872	128.897	275.189
<b>Incremental</b>												
mean QALY increment ( $\Delta^e$ )	0.032	-0.02	0.08	0.039	-0.016	0.095	-0.008	-0.122	0.072	0.049	-0.011	0.114
mean cost increment ( $\Delta^c$ )	-25	-145	97	-18.600	-141.081	102.463	-19.415	-140.283	104.196	25.708	-121.593	194.326

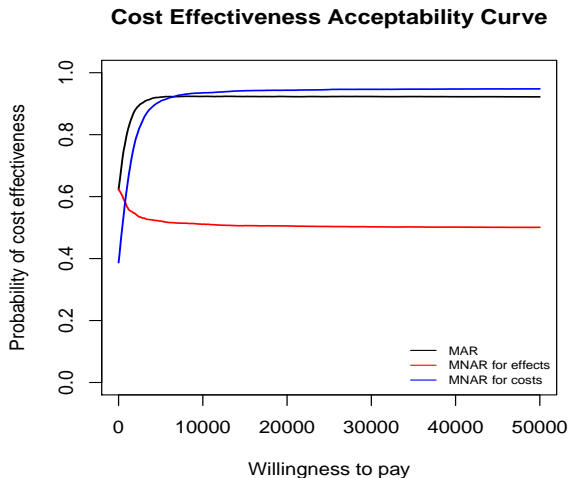
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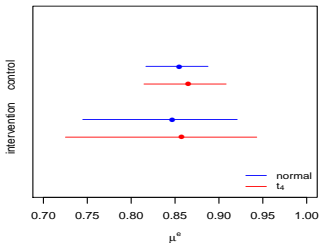
# Case Study: Economic Evaluation



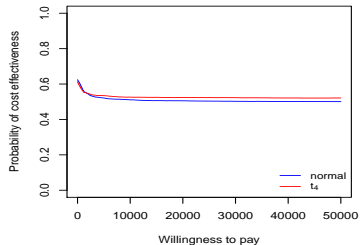
- Under  $MNAR(e)$  the assessment radically changes, with the new interventions being not cost-effective compared with the control

# Case Study: Sensitivity Analysis (AS)

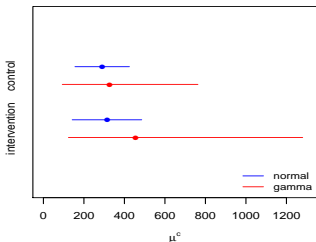
Estimated effect means



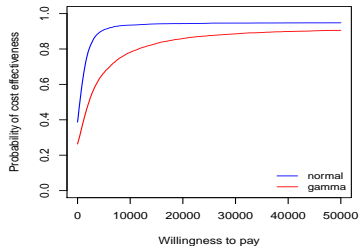
Cost Effectiveness Acceptability Curve



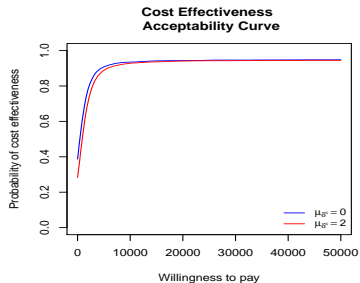
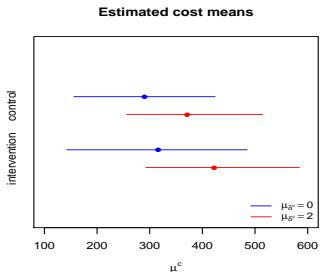
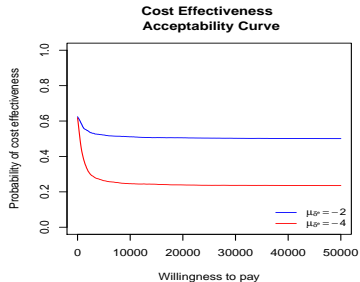
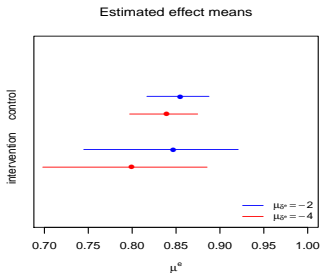
Estimated cost means



Cost Effectiveness Acceptability Curve



# Case Study: Sensitivity Analysis (PS)





# Case Study: Conclusions

- MAR is not likely to hold and the original study conclusions may overestimate the cost-effectiveness of the reference intervention
- The MNAR departures explored show how a relatively small variation in  $\text{MoM}(e)$  may substantially alter the decision output
- Lack of information about missingness may severely impair the analysis and force unrealistic assumptions

# Future Works

- Potential for Bayesian methods to handle nonignorable missingness in economic evaluations
- However, other extensions deserve further considerations
- Model Improvements
  - Explore other MoA distributions (e.g. truncated, skewed, mixture)
  - Consider alternative types of nonignorable models (e.g. PMM)
  - Model outcomes at disaggregated level (longitudinal models)
- Joint modelling of MNAR mechanisms
  - Explicitly include both MoMs and perform SA
  - Account for correlations between the two MoMs

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