



기계 진동학

PART 1.

1-2 주차

진동학이란?

기계 시스템 내의 에너지를 다루는 것

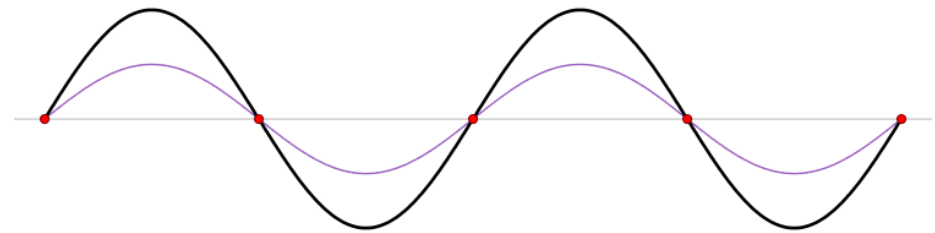
Wave란?

에너지의 흐름

Standing Wave란?

진폭이나 크기가 시간에 따라 변하지 않는 파동

*nodal point : 움직이지 않는 점



고유진동수(Natural frequency)란?

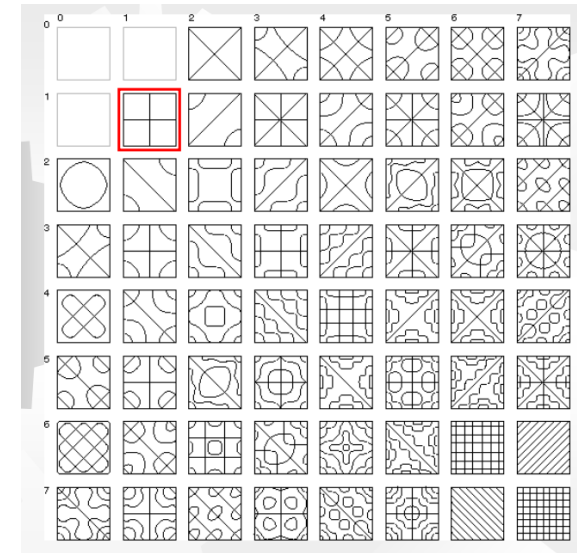
각 물체가 가지는 고유한 진동 특성 (재료, 조건 등 Boundary Condition이 이를 결정)

Mode shape이란?

고유진동수에 의해 생기는 Wave
수 없이 많지만 현실에서는 lower freq 만 다루므로 2개만 다룸

진동학 목적

공진(Resonance)를 없애는 것



관성이란?

움직임의 변화에 저항하는 힘

Kinetic(운동) Energy



Inertia (관성)

Potential(위치) Energy

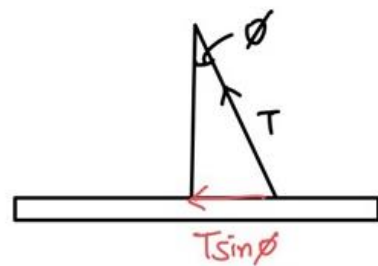


Spring (탄성)

Energy loss



Damping



$$\Sigma M = I\ddot{\theta} = -2TR\sin\phi$$

$$r\phi = l\theta \text{ 이므로}$$

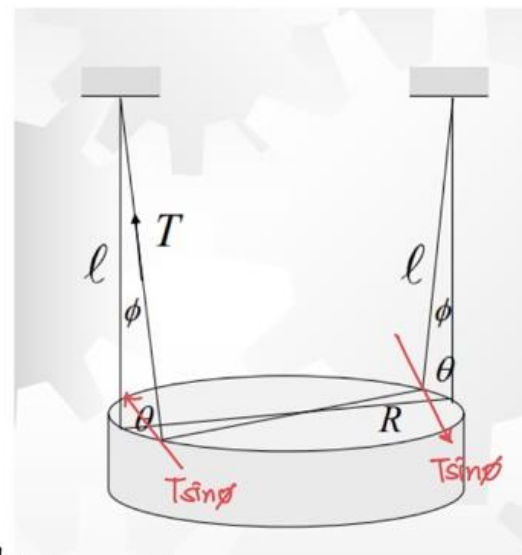
$$I\ddot{\theta} + 2TR \frac{R}{l} \theta = 0 \text{ 이다.}$$

$$\hookrightarrow \ddot{\theta} + \frac{mgR^2}{Il} \theta = 0$$

$$\theta = H \sin \omega t \quad \ddot{\theta} = -\omega^2 H \sin \omega t = 0$$

$$\left(\underbrace{-\omega^2 + \frac{mgR^2}{Il}}_{=0} \right) \underbrace{H \sin \omega t}_{=0} = 0$$

$$\omega^2 = \frac{mgR^2}{Il} \rightarrow \text{natural frequency}$$



등가 관성

Energy를 구하라

피니언이 지나간 거리 = \dot{x}

$$R\dot{\theta} = \dot{x}$$
$$\dot{\theta} = \frac{\dot{x}}{R}$$

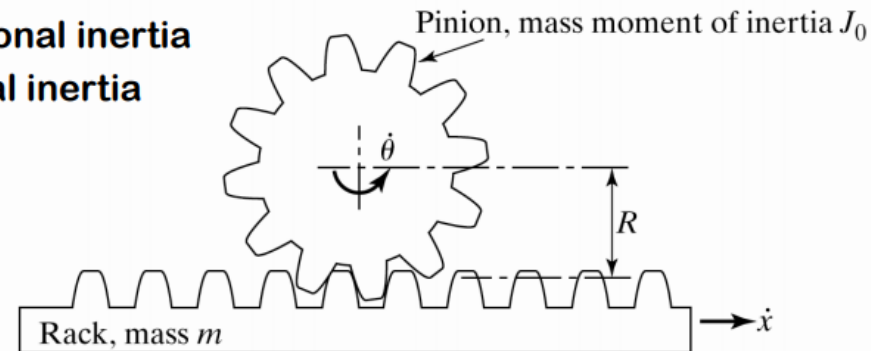
이를 아래 Energy Equation에 대입

$$KE = \frac{1}{2}mv^2, \quad \frac{1}{2}I\dot{\theta}$$

$$PE = \frac{1}{2}K\dot{\theta}, \quad \frac{1}{2}K_{\theta}\theta^2$$

Example 1 – Rack and Pinion Arrangement

- Translational inertia
- Rotational inertia



Total kinetic energy is given by

$$T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I_0\dot{\theta}^2$$
$$= \frac{1}{2}\left(m + \frac{I_0}{R^2}\right)\dot{x}^2 = \frac{1}{2}m_{eq}\dot{x}^2 \quad \text{where } m_{eq} = m + \frac{I_0}{R^2}$$
$$= \frac{1}{2}(mR^2 + I_0)\dot{\theta}^2 = \frac{1}{2}I_{eq}\dot{\theta}^2 \quad \text{where } I_{eq} = mR^2 + I_0$$

Use kinetic energy equation to evaluate the equivalent inertia from different types of inertia

$$\Delta = \frac{pl^3}{3EI}$$

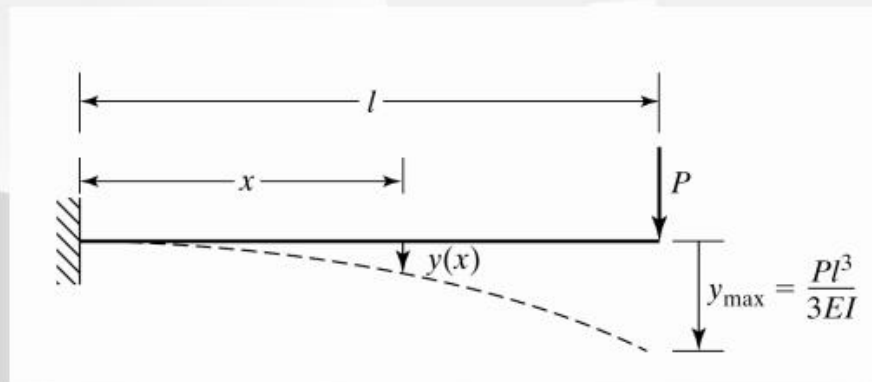
$$P = K_{eq} * \Delta$$

$$K_{eq} = \frac{3EI}{l^3}$$

$$T = \int_0^l \frac{1}{2} \dot{y}(x)^2 dm$$

$$dm = \frac{m}{l} dx$$

Example 2 – Cantilever Shaft



$$y = \frac{Px^2}{6EI}(3l - x)$$

$$= \frac{y_{\max}}{2l^3} x^2 (3l - x)$$

$$\text{where } y_{\max} = \frac{Pl^3}{3EI}$$

Kinetic Energy

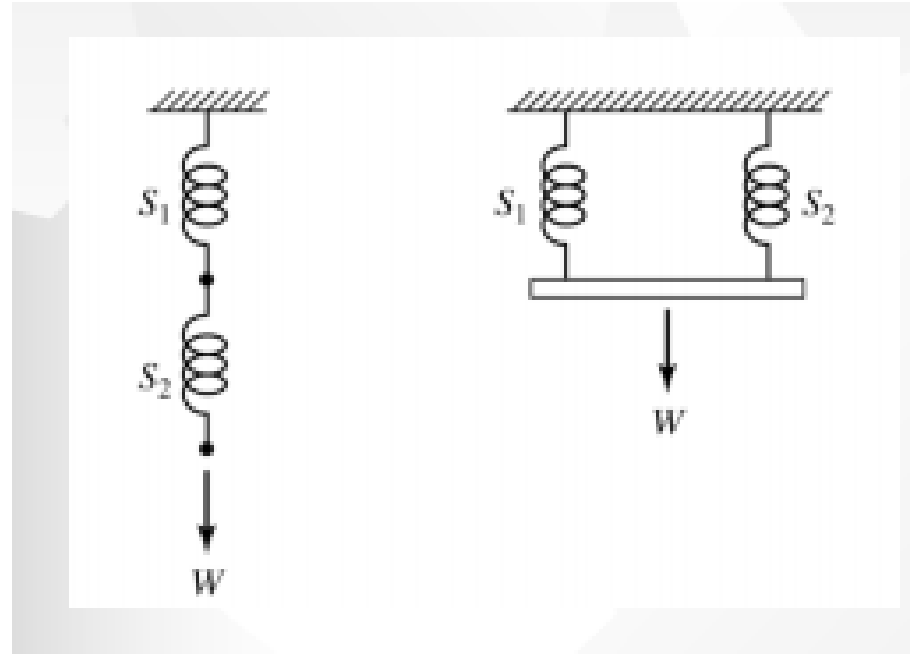
$$T = \int_0^l \frac{1}{2} \frac{m}{l} \dot{y}^2(x) dx \quad \leftarrow \dot{y}(x) = \frac{\dot{y}_{\max}}{2l^3} x^2 (3l - x)$$

$$T = \frac{m}{2l} \left(\frac{\dot{y}_{\max}}{2l^3} \right)^2 \int_0^l x^4 (3l - x)^2 dx = \frac{1}{2} \left(\frac{33}{140} m \right) \dot{y}_{\max}^2 = \frac{1}{2} m_{eq} \dot{y}_{\max}^2$$

$$\therefore m_{eq} = \frac{33}{144} m = 0.2357m$$

직렬 spring

스프링의 늘어난 길이가 다름



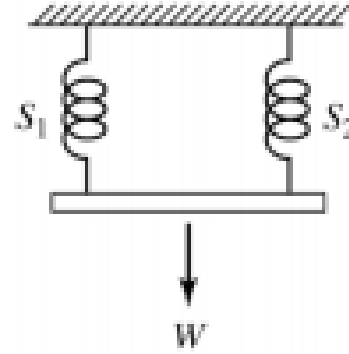
$$\Delta = \Delta_1 + \Delta_2$$

$$\frac{P}{K_{eq}} = \frac{P}{K_1} + \frac{P}{K_2}$$

$$K_{eq} = \frac{K_1 * K_2}{K_1 + K_2}$$

병렬 spring

스프링의 늘어난 길이가 같음



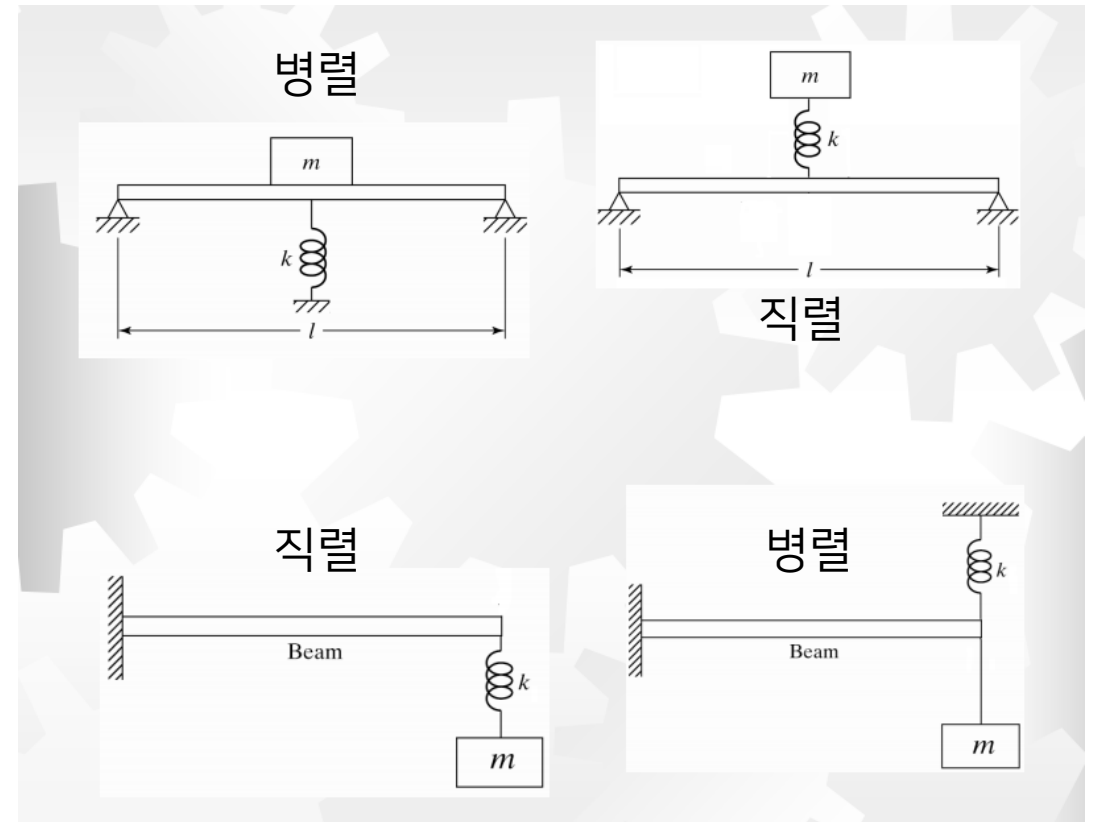
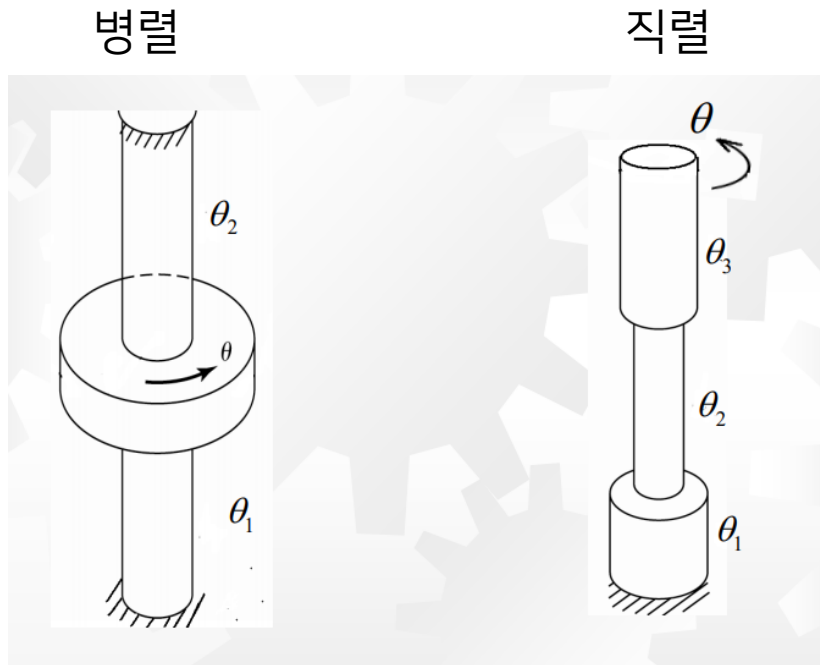
$$\Delta = \Delta_1 = \Delta_2$$

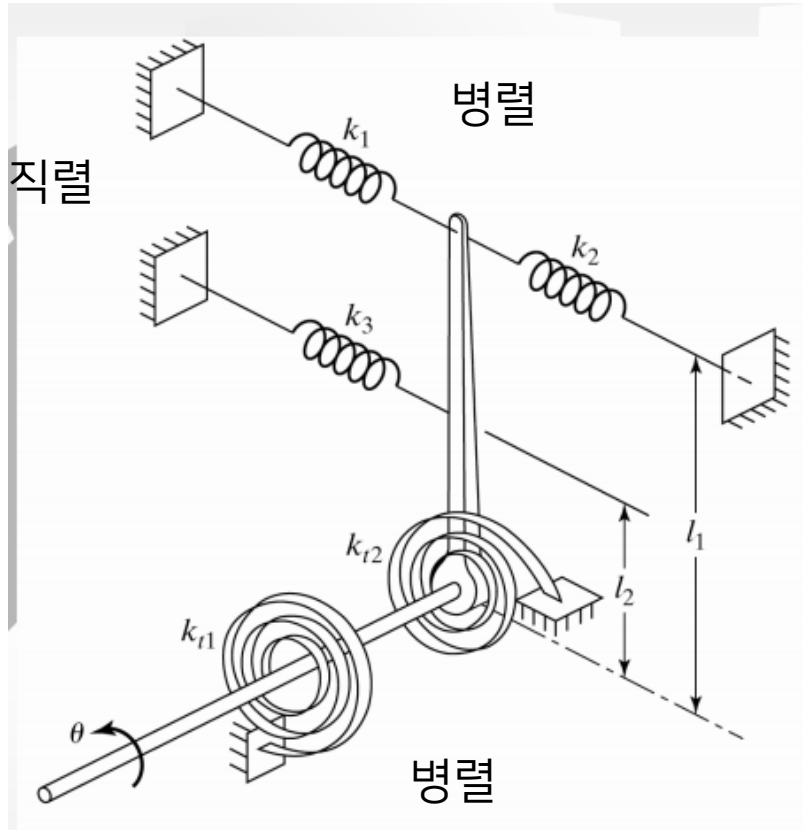
$$P = P_1 + P_2$$

$$P_1 = K_1 \Delta_1 + K_2 \Delta_2$$

$$K_{eq} = (K_1 + K_2)$$

직렬, 병렬 스프링 예제





$$k_{12} = k_1 + k_2$$

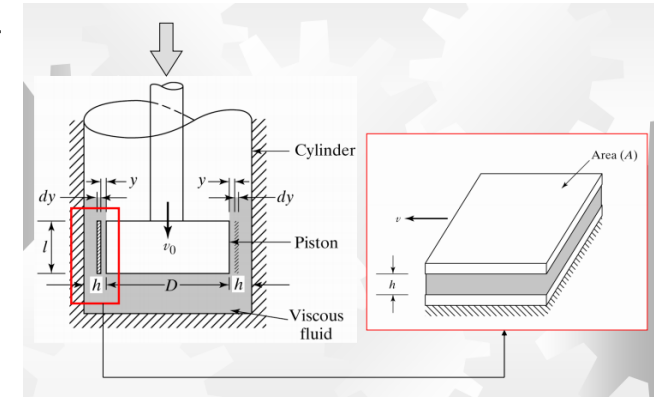
$$k_{t12} = k_{t1} + k_{t2}$$

$$\begin{aligned}
 U &= \frac{1}{2}(k_{t1} + k_{t2})\theta^2 \\
 &\quad + \frac{1}{2}k_3(\ell_2\theta)^2 + \frac{1}{2}k_{12}(\ell_1\theta)^2 \\
 &= \frac{1}{2}(k_{t1} + k_{t2} + k_3\ell_2^2 + k_{12}\ell_1^2)\theta^2 \\
 &= \frac{1}{2}k_{eq}\theta^2
 \end{aligned}$$

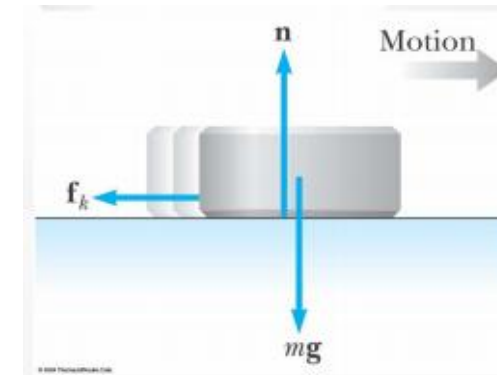
$$k_{eq} = k_{t1} + k_{t2} + k_3\ell_2^2 + (k_1 + k_2)\ell_1^2$$

Types of damping

1. Viscous Damping (점성 제동) : 주로, 점성 유체와 물체 표면 사이의 상대 운동에 의존
 - 속도에 비례하여 감소



2. Coulomb Damping (쿨롱 제동, 건 마찰 제동) : 마찰 표면에 의해 감소

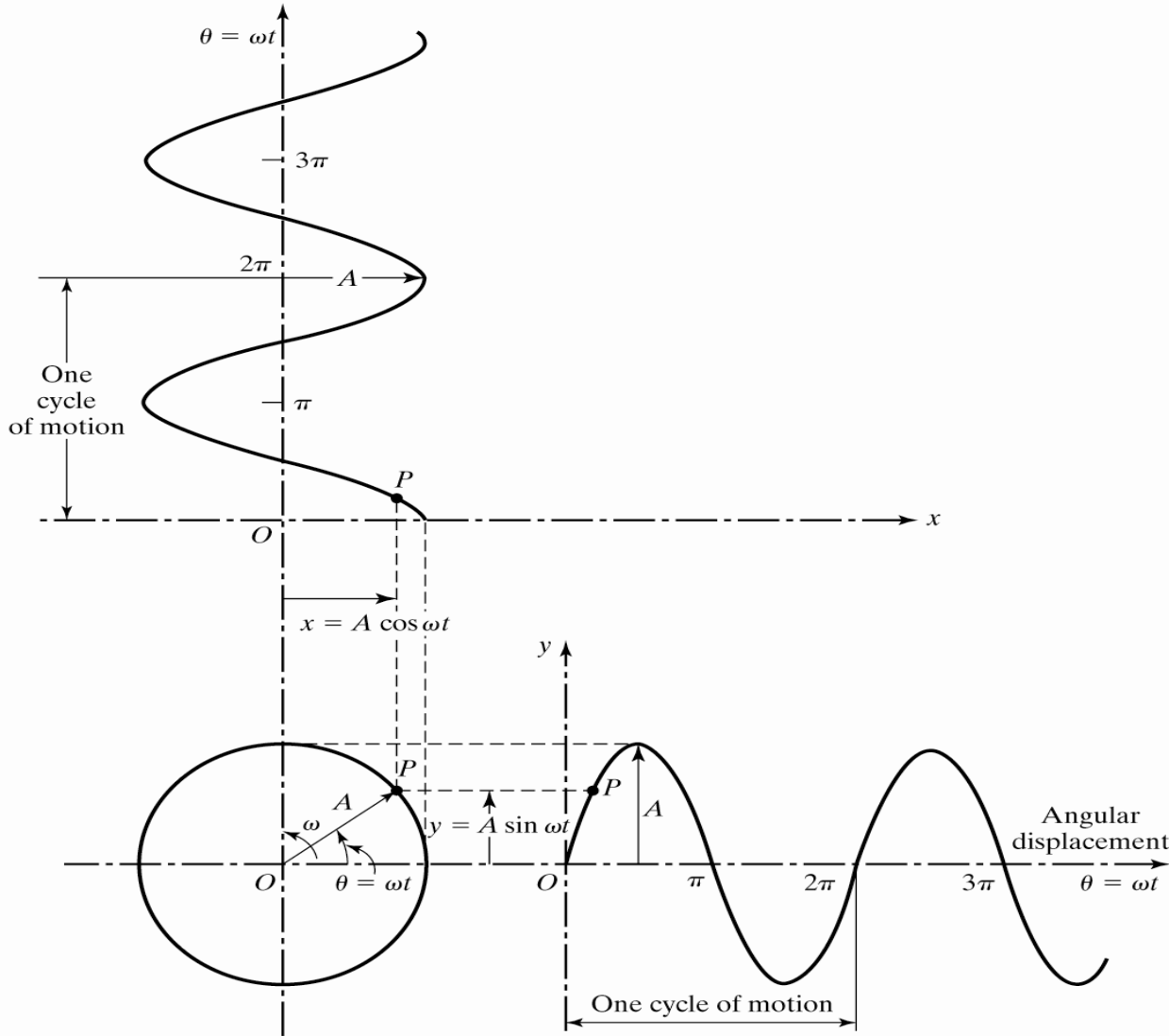


3. Material or Solid or Hystertic Damping : 물체 내부의 변형에 의한 물체 내부 면의 마찰력으로 인한 감소

PART 1.

3 주차

Harmonic Motion(조화 운동)



- 고유 주파수 0, 가장 단순한 형태의 진동
- 일정 주기로 반복
- 진동은 대부분 주기가 있다.
- EX) 용수철

$$f = \frac{\omega}{2\pi} (Hz) : \text{cycles per second}$$

$$x = A \cos \omega t$$

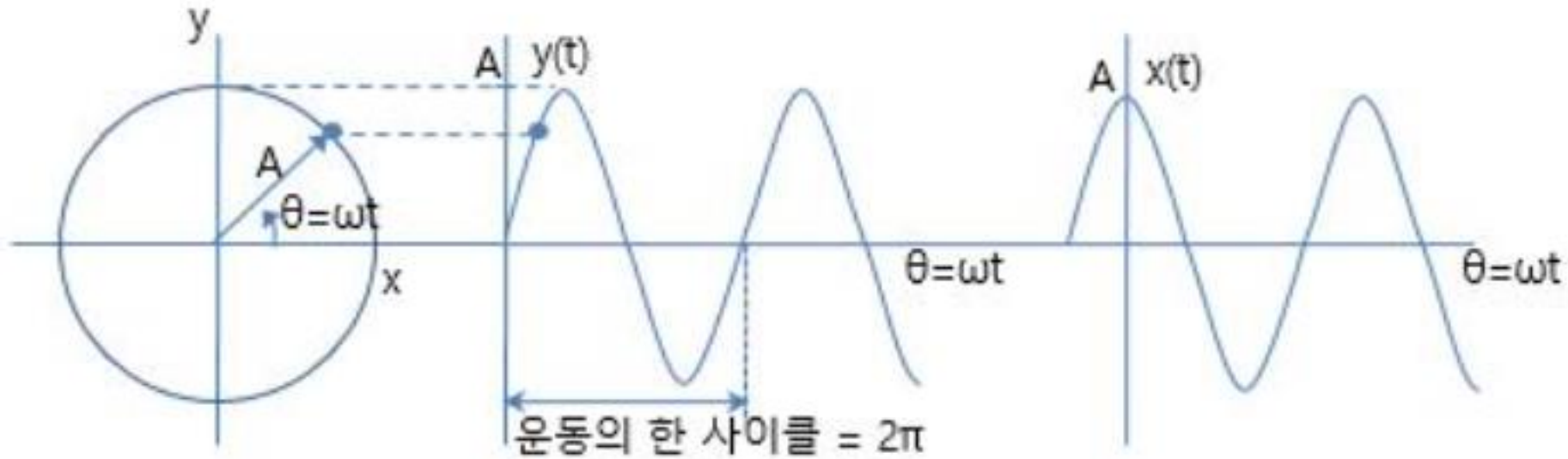
$$y = A \sin \omega t$$

$$z = x + jy = A(\cos \omega t + j \sin \omega t) = A e^{j\omega t}$$

조화운동의 미분방정식 :

$$\frac{d^2 x}{dt^2} + \omega_o^2 x = 0$$



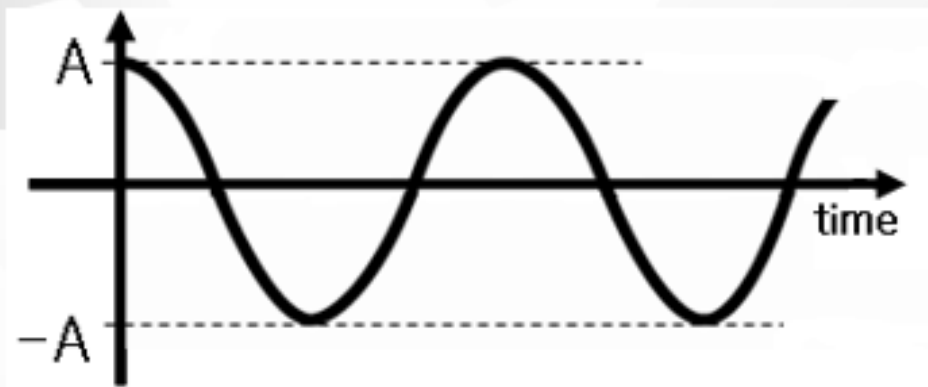


반지름 A → 진폭 A

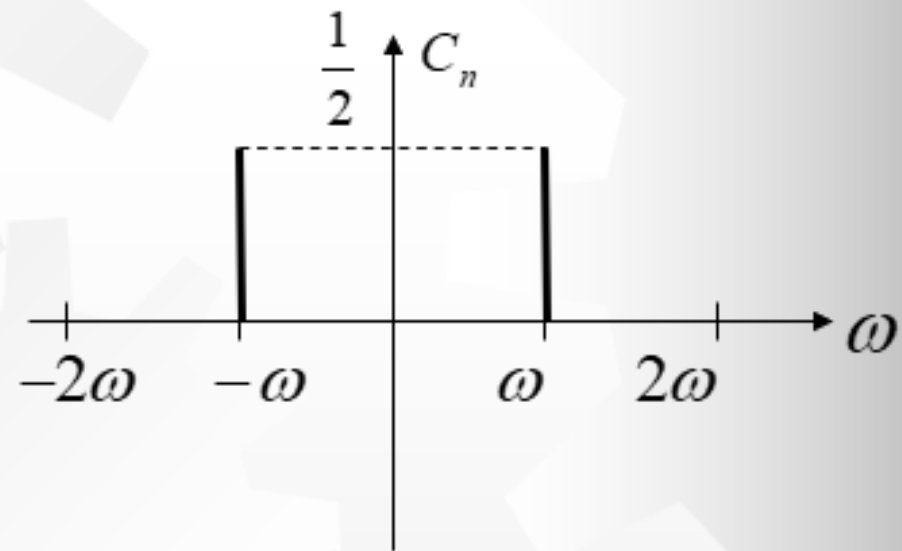
각 변위 $\theta = \omega t$ → 위상 변위 $\theta = \omega t$

각속도 $\omega = \frac{d\theta}{dt}$ → 각 진동수 $\omega = 2\pi f$

회전수 f → 진동수 $f = \frac{1}{T}$



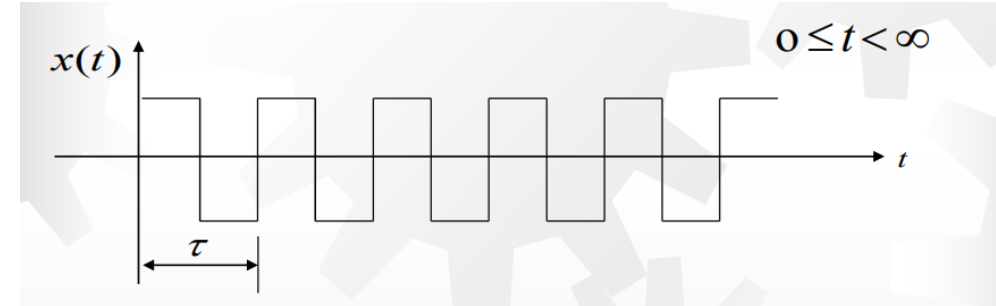
Time domain



Frequency domain

****특정 주파수를 관찰하기 용이하다****

Fourier Transform 유도 과정



Fourier Series로 부터 시작

$$X(t) = \frac{a_0}{2} + (a_1 \cos wt + a_2 \cos 2wt + \dots) + (b_1 \sin wt + b_2 \sin 2wt + \dots)$$

$$x(t) = \frac{a_0}{2} + \sum (a_n \cos nwt + b_n \sin nwt), \quad a_n = \frac{2}{\tau} \int_0^\tau x(t) \cos nwt \, dt, \quad b_n = \frac{2}{\tau} \int_0^\tau x(t) \sin nwt \, dt$$

$$C_n = \frac{1}{2}(a_n - jb_n) = \frac{1}{\tau} \int_0^\tau x(t) (\cos nwt - j \sin nwt) dt = \frac{1}{\tau} \int_0^\tau x(t) e^{-jnwt} dt$$

$$C_{-n} = \frac{1}{2}(a_{-n} - jb_{-n}) = \frac{1}{2}(a_n + jb_n)$$

$$a_n = c_n + c_{-n}$$

$$b_n = j(c_n - c_{-n})$$

$$\cos n\omega t = \frac{1}{2}(e^{jn\omega t} + e^{-jn\omega t})$$

$$\sin \omega t = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

$$\begin{aligned} x(t) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} (\underline{a_n \cos n\omega t + b_n \sin n\omega t}) \\ &= (C_n + C_{-n}) \frac{1}{2} (e^{jn\omega t} + e^{-jn\omega t}) \\ &\quad + \cancel{j} (C_n - C_{-n}) \frac{1}{2\cancel{j}} (e^{jn\omega t} - e^{-jn\omega t}) \\ &= C_n e^{jn\omega t} + C_{-n} e^{-jn\omega t} \end{aligned}$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x(t) \sin n\omega t \, dt$$

$$b_0 = 0$$

$$\begin{aligned} x(t) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \\ &= \left(\frac{a_0}{2} + j \frac{b_0}{2} \right) + \sum_{n=1}^{\infty} (C_n e^{jn\omega t} + C_{-n} e^{-jn\omega t}) \\ &= 0 \\ &= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \end{aligned}$$

$$\begin{aligned}
 C_n &= \frac{1}{2} (a_n - j b_n) \\
 &= \frac{1}{2} \cdot \frac{1}{\tau} \int_0^{\tau} x(t) (\cos n\omega t - j \sin n\omega t) dt \\
 &= \frac{1}{\tau} \int_0^{\tau} x(t) e^{-jn\omega t} dt
 \end{aligned}$$

$$\begin{aligned}
 x(t) &= \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t} \\
 \text{where } C_n &= \frac{1}{\tau} \int_0^{\tau} x(t) e^{-jn\omega t} dt
 \end{aligned}$$

Fourier Series

$$\begin{aligned}
 n\omega &= \omega_n \\
 \omega &= 2\pi f = \frac{2\pi}{T} \\
 n\omega &= \omega_n = \frac{2n\pi}{T} \\
 \Delta\omega &= \frac{2\pi}{\Delta T}
 \end{aligned}$$

$T \rightarrow \infty$ Fourier Transform

If $\int_{-\infty}^{\infty} |x(t)| dt$ exist,

$$\begin{aligned}
 x(t) &= \sum_{n=-\infty}^{\infty} \left[\frac{1}{\tau} \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t) e^{-jn\omega t} dt \right] e^{jn\omega t} \quad \left(\frac{1}{\Delta T} = \frac{\Delta\omega}{2\pi} \right) \\
 &= \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \left[\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} x(t) e^{j\omega_n t} dt \right] e^{j\omega_n t} d\omega
 \end{aligned}$$

As $\tau \rightarrow \infty$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right] e^{j\omega t} d\omega \quad S = \sigma + j\omega \quad \text{Laplace}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \quad \text{where } X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Fourier Transform

Fourier, Laplace

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

조화 운동으로부터 도출된 위의 식을 통해 푸리에 유도

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \quad \text{where } X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$s = \sigma + j\omega \quad \begin{array}{l} \xrightarrow{\sigma = 0} \\ \xrightarrow{\sigma \neq 0} \end{array}$$

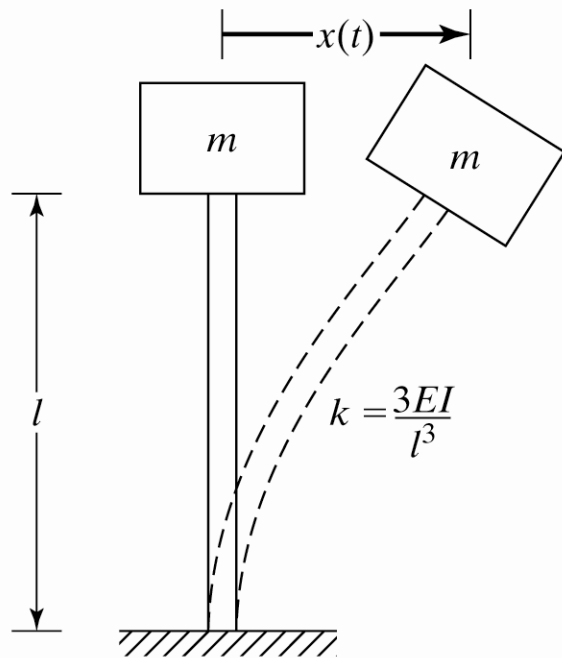
미분방정식 문제 풀이

→ Laplace (+, -, /, x)

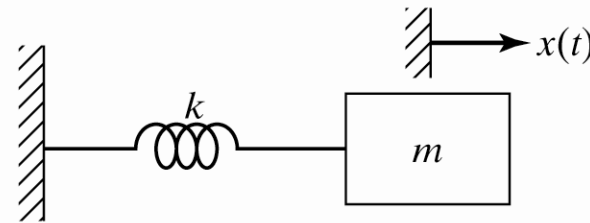
→ Inverse Laplace

Multi DOF

Multi DOF = single DOF + single DOF



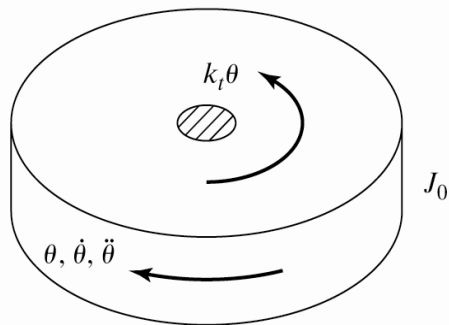
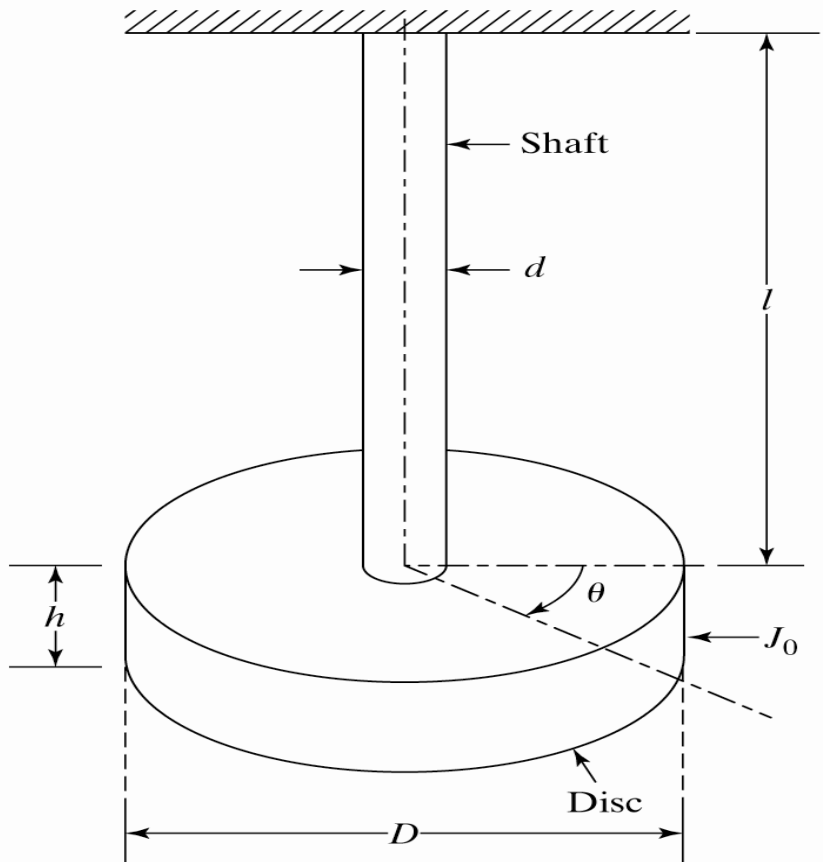
(a) Idealization of the tall structure



(b) Equivalent spring-mass system

Newtonian Mechanism : $F = ma$
Lagrangian Mechanism : Energy

모델링 : m , c , k 값 구하는 과정



$$I\ddot{\theta} = -k_t\theta \quad \leftarrow k_t = \frac{GJ}{\ell}$$

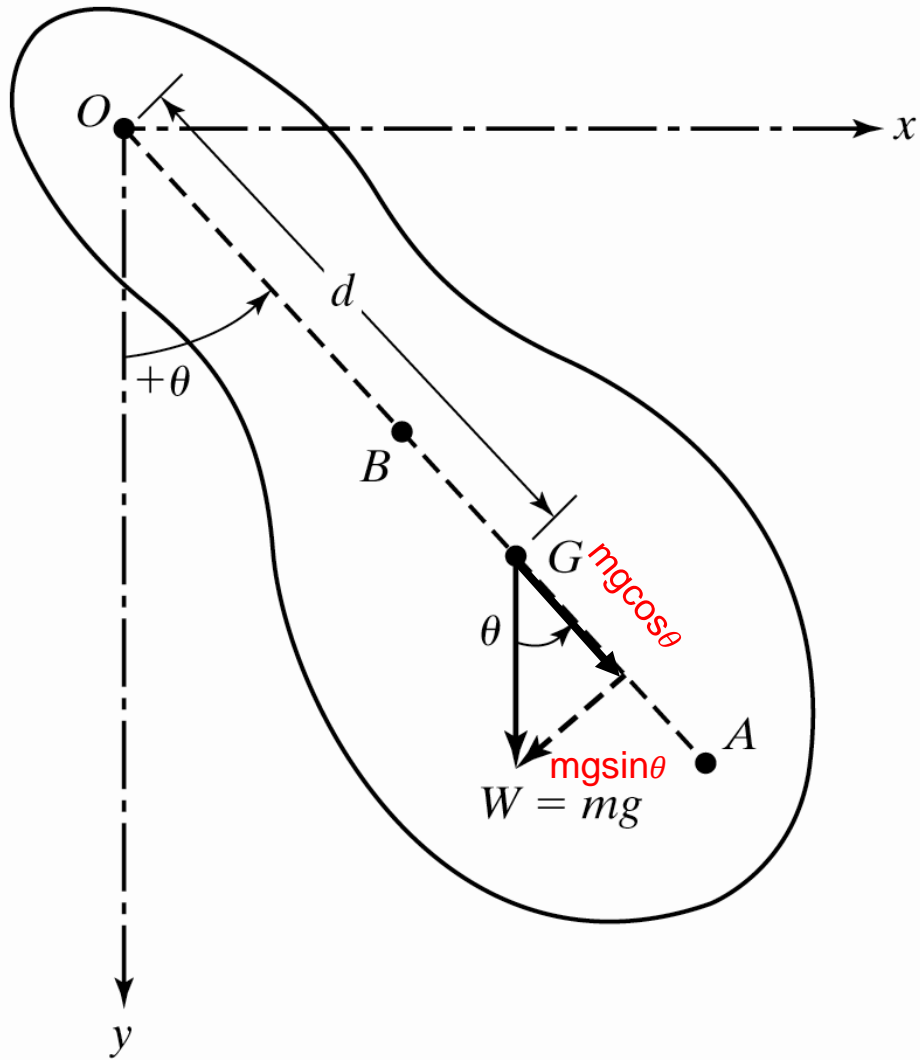
$$\phi = \frac{TL}{GJ}, T = \frac{GJ}{L}\phi$$

$$1. I = \frac{1}{2}mR^2 = \frac{1}{2}(\rho\pi R^2h)R^2 = \frac{1}{2}\rho\pi R^4h \quad (R = \frac{D}{2})$$

$$2. I\ddot{\theta} + k_t\theta = 0$$

$$3. \ddot{\theta} + \frac{k_t}{I}\theta = \ddot{\theta} + \omega_n^2\theta \quad (\omega_n^2 = \frac{GJ}{IL})$$

$$\rightarrow \theta = A\cos\omega_nt + B\sin\omega_nt$$

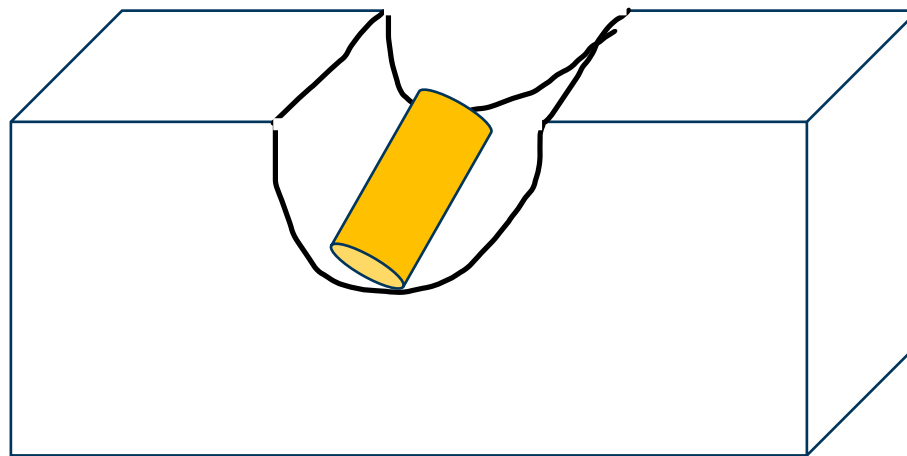
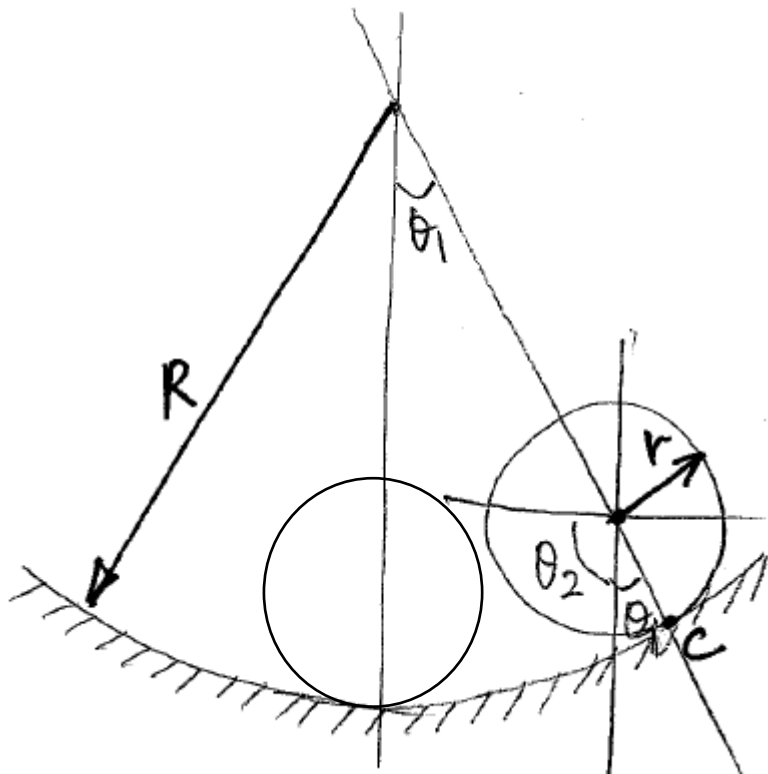


$$\sum M_0 = I_0 \ddot{\theta} = -mgd \sin \theta \cong -\theta$$

$$I \ddot{\theta} + mgd \theta = 0$$

$$\ddot{\theta} + \omega_n^2 \theta = 0$$

$$\omega_n^2 = \frac{mgd}{I}$$

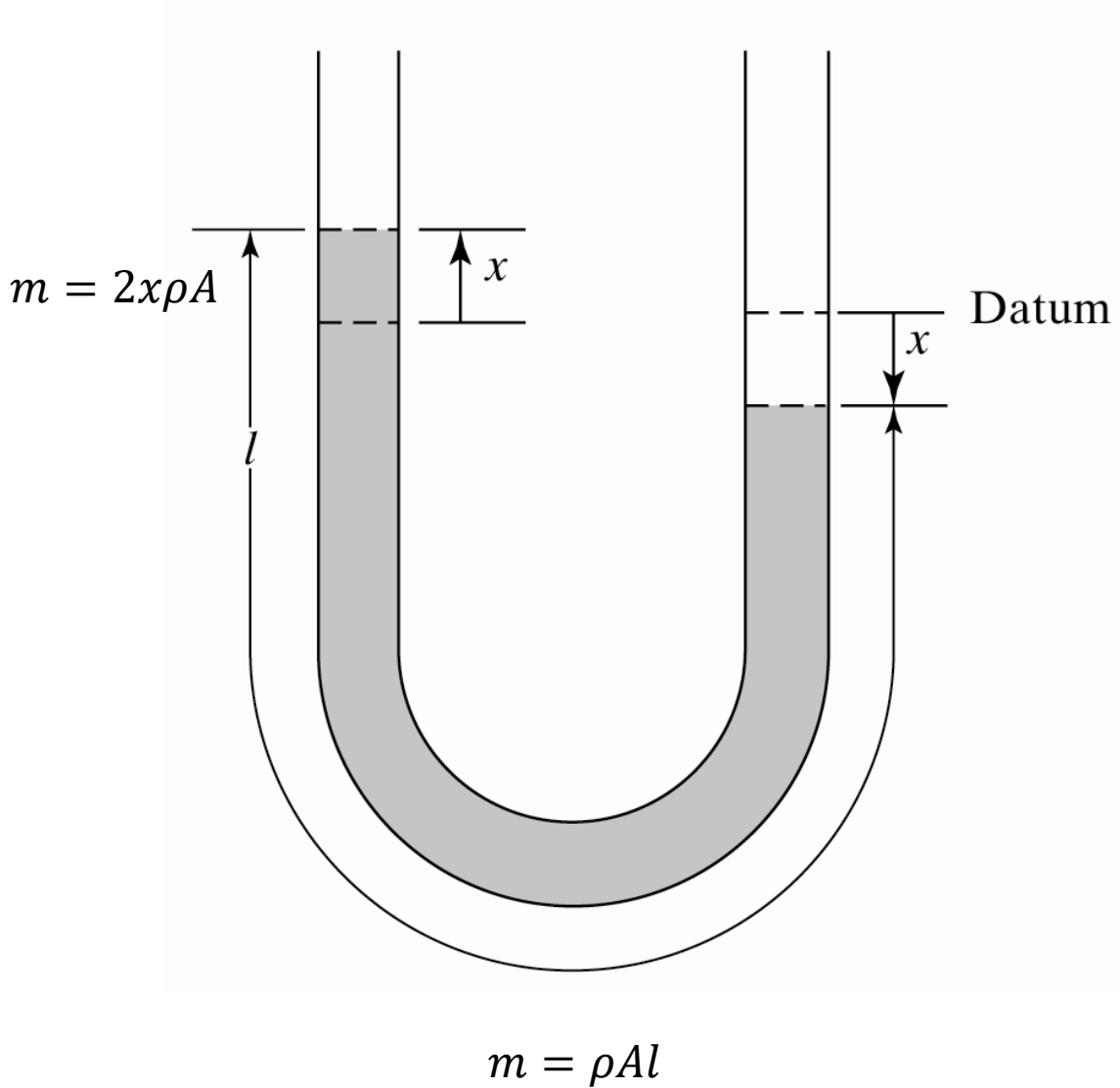


$$(\theta_1 + \theta_2)r = R\theta_1, \quad \theta_1(R - r) = \theta_2 r \rightarrow \theta_1 = \frac{r}{R-r}\theta_2$$

$$I_C \ddot{\theta}_2 = -mgr \sin \theta_1 \quad (T = I\ddot{\theta}) \quad \leftarrow I_C = \frac{3}{2}mr^2, \quad \sin \theta_1 \cong \theta_1$$

$$\therefore \frac{3}{2}mr^2 \cdot \frac{R-r}{r} \ddot{\theta}_1 = -mgr \theta_1$$

$$\therefore \ddot{\theta}_1 + \frac{2g}{3(R-r)} \theta_1 = \ddot{\theta} + \omega_n^2 \theta = 0$$



$$\sum F = m\ddot{x} = -2\rho Agx$$

$$m\ddot{x} + 2\rho Agx = 0$$

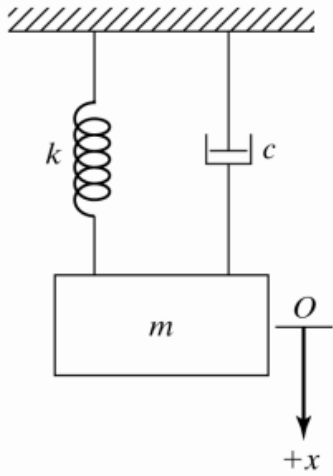
$$\rho Al\ddot{x} + 2\rho Agx = 0$$

$$\ddot{x} + \frac{2g}{l}x = 0$$

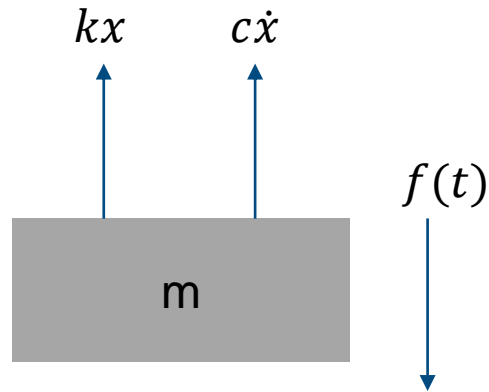
$$\ddot{x} + \omega_n^2 x = 0$$

PART 1.

4 주차



System
(a)



FBD

$$\Sigma F = m\ddot{x} = -kx - c\dot{x} + f(t)$$

$$m\ddot{x} + kx + c\dot{x} = f(t)$$

$f(t) = 0$ 일때 : external force와 상관없이 물체의 고유 진동수를 아는 것

$$x = X_0 e^{st}$$

$$(ms^2 + cs + k)X_0 e^{st} = 0$$

$$ms^2 + cs + k = 0$$

$$s_1, s_2 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$\text{Critical Damping : } C_c^2 = 2\sqrt{mk}$$

$$\text{Damping Ratio : } \zeta = \frac{C}{C_c}$$

$$\text{Nature freq : } W_n = \sqrt{\frac{k}{m}}$$

$$s^2 + 2\zeta W_n s + W_n^2 = 0$$

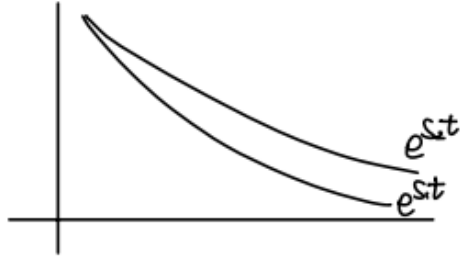
$$s_1, s_2 = -\zeta W_n \pm W_n \sqrt{\zeta^2 - 1}$$

$$s_1, s_2 = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1}$$

0.001 0.1
steel, 고무 등
대부분의 재료는 1보다 작다.

① $\zeta > 1$ 인 경우

$$x(t) = c_1 e^{-s_1 t} + c_2 e^{-s_2 t} \quad (s_1 < 0, s_2 < 0)$$



→ 진동 (oscillation) 이 없다.

$x(0) = X_0 = X_1$ 일 때

$$\dot{x}(t) = -\zeta \omega_n e^{-\zeta \omega_n t} (X_1 \cos \omega_d t + X_2 \sin \omega_d t) + e^{-\zeta \omega_n t} \omega_d (-X_1 \sin \omega_d t + X_2 \cos \omega_d t)$$

$$\dot{x}(0) = -\zeta \omega_n X_1 + \omega_d X_2 = \dot{x}_0$$

$$\omega_d X_2 = \dot{x}_0 + \zeta \omega_n X_0$$

$$X_2 = \frac{\dot{x}_0 + \zeta \omega_n X_0}{\omega_d}$$

$$x(t) = e^{-\zeta \omega_n t} \left(X_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta \omega_n X_0}{\omega_d} \sin \omega_d t \right)$$

② $\zeta = 1$ 인 경우

$$s_1, s_2 = -\zeta \omega_n$$

$$x(t) = (c_1 + c_2 t) e^{-\zeta \omega_n t}$$

$$x(0) = x_0 = c_1$$

$$x'(t) = c_2 e^{-\zeta \omega_n t} + (-\zeta \omega_n)(c_1 + c_2 t) e^{-\zeta \omega_n t}$$

$$c_2 = \dot{x}_0 - \zeta \omega_n x_0$$

$$x(t) = (x_0 + (\dot{x}_0 - \zeta \omega_n x_0) t) e^{-\zeta \omega_n t}$$

③ $\zeta < 1$ 인 경우

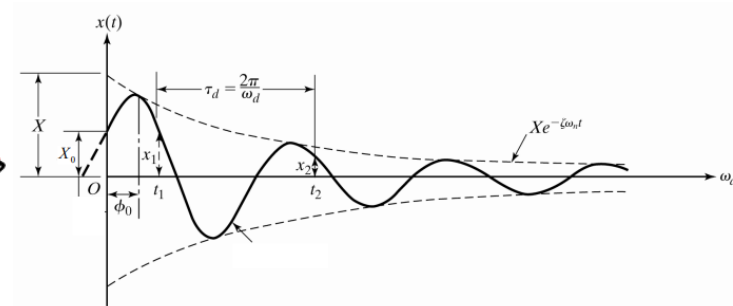
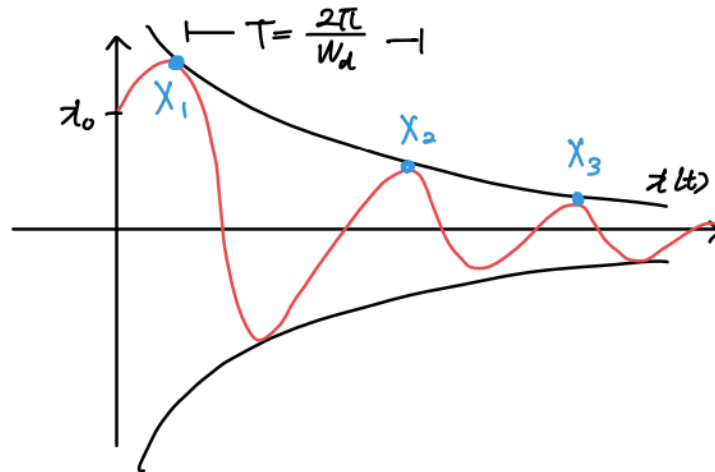
$$s_1, s_2 = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2} = -\zeta \omega_n \pm j \omega_d \quad \omega_d = \omega_n \sqrt{1 - \zeta^2}$$

→ damped natural freq

$$x(t) = c_1 e^{(-\zeta \omega_n + j \omega_d)t} + c_2 e^{(-\zeta \omega_n - j \omega_d)t}$$

$$= e^{-\zeta \omega_n t} (c_1 e^{j \omega_d t} + c_2 e^{-j \omega_d t})$$

$$= e^{-\zeta \omega_n t} (X_1 \cos \omega_d t + X_2 \sin \omega_d t)$$



$$x(t) = e^{-\zeta \omega_n t} \left(x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta \omega_n x_0}{\omega_d} \sin \omega_d t \right)$$

When $x_0 = 0$, $\dot{x} \neq 0$: **Impact**

$$x(t) = \frac{\dot{x}_0}{\omega_d} e^{-\zeta \omega_n t} \sin \omega_d t$$

$$t = KT, \quad x_K = \frac{\dot{x}_0}{\omega_d} e^{-\zeta \omega_n KT}$$

$$t = (K+1)T, \quad x_{K+1} = \frac{\dot{x}_0}{\omega_d} e^{-\zeta \omega_n (K+1)T}$$

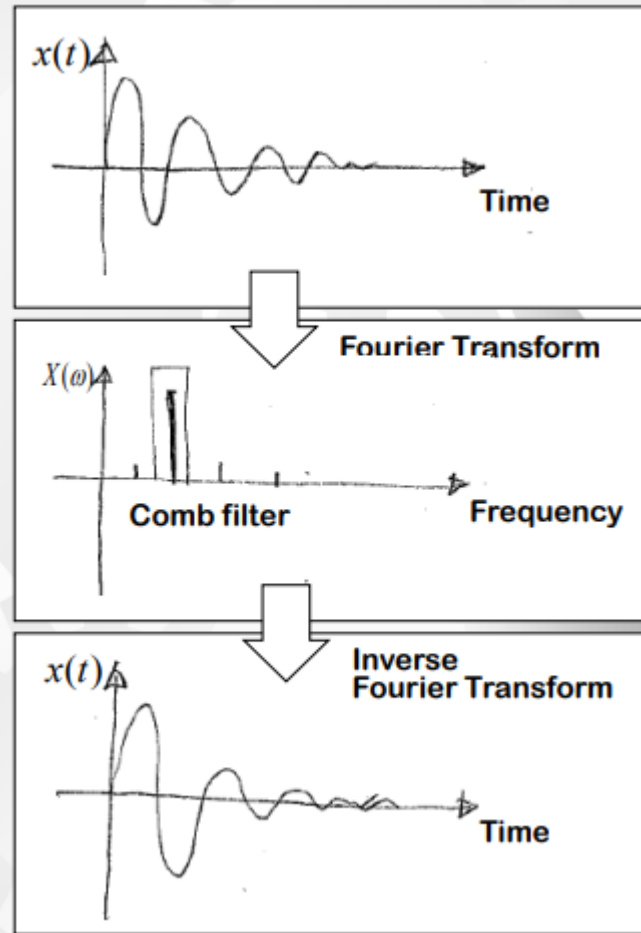
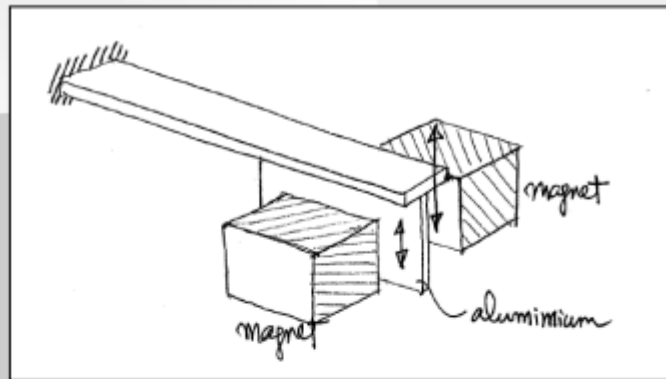
$$\frac{X_1}{X_2} = \frac{X_K}{X_{K+1}} = e^{\zeta \omega_n T}$$

대수 감소율(logarithmic decrement) : 진폭이 감쇠하는 정도

$$\begin{aligned} \delta &= \ln \frac{X_K}{X_{K+1}} = \zeta \omega_n T = \zeta \omega_n \frac{2\pi}{\omega_d} \\ &= \zeta \omega_n \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}} \\ &= \frac{2\pi \zeta}{\sqrt{1-\zeta^2}} \approx 2\pi \zeta \end{aligned}$$

$$\therefore \zeta = \frac{\delta}{2\pi}$$

$$\therefore \delta = \frac{2\pi c}{m\omega_d}$$



FFT 이후

특정 주파수를 찾는 다음

Inverse FFT로

Time Domain으로 가져오기

Stability

$$s_1, s_2 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$s_1, s_2 = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

$c < 0$ 일 경우 : dynamic unstable

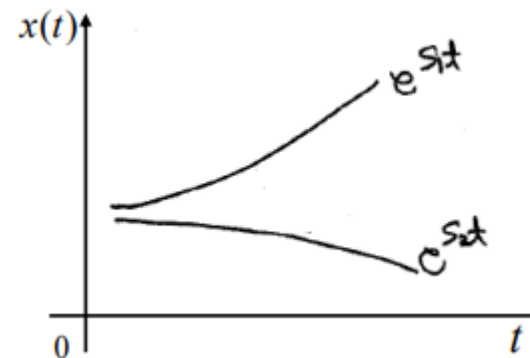
$k < 0$ 일 경우 : Statically unstable

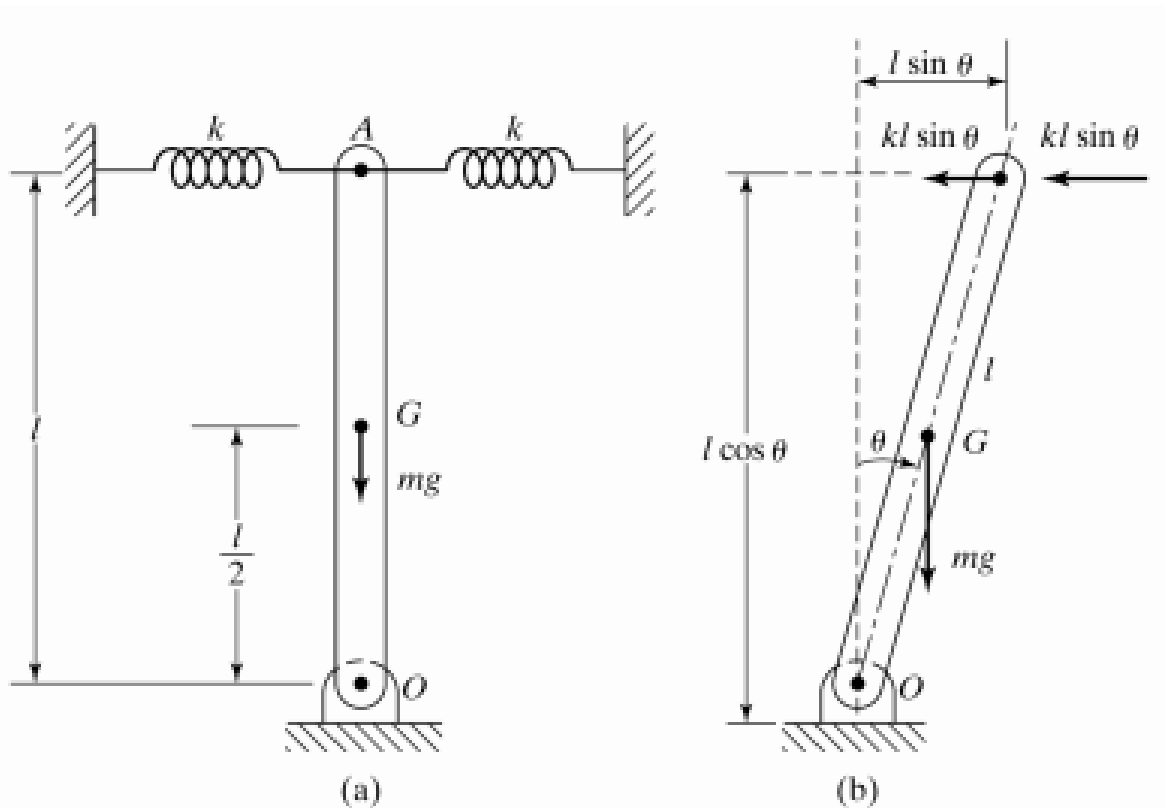
$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$ 이므로

$k < 0$ 인 경우 s_1 또는 s_2 가 양수이므로 $x(t)$ 가 발산하게 됨

$c < 0$ 인 경우 s_1, s_2 이 양수이므로 $x(t)$ 가 발산하게 됨

마찰 계수 c 를 바꿔주는 것은 쉽지 않으므로 k 로 진동을 통제해야 함





$$* F = kx$$

$$\Sigma M_O = I\ddot{\theta} = -2k(l \sin \theta) l \cos \theta + \frac{1}{2} mgl \sin \theta$$

$$\sin \theta = \epsilon, \quad \cos \theta = 1, \quad \sin \theta = \theta$$

$$I\ddot{\theta} + 2kl^2\theta - \frac{1}{2}mgl\theta = 0$$

$$I\ddot{\theta} + (2kl^2 - \frac{1}{2}mgl)\theta = 0$$

$$k_{eq} = 2kl^2 - \frac{1}{2}mgl, \quad I\ddot{\theta} + k_{eq}\theta = 0$$

k가 매우 작으면 $\frac{1}{2}mgl$ 가 더 크기 때문에 k_{eq} 가 마이너스가 되면서 숙 넘어가 버림

k가 크면 반복 왕복 운동하며 진동이 생김

가스터빈 EXHAUST 케이싱 베어링 AIR SEAL에서

오일 탄화물 축적에 의한 이상 진동

Abnormal High Vibration by the Accumulated Oil Carbide at the Exhaust
Casing Bearing Air Seal of a Gas Turbine

김동관*·박상호*·구재량**

Dong Kwan Kim, Sangho Park and Koo Jae Raeyang

압축된 공기와 연료가 혼합되어
연소 되면서 고온 고압의 기체가
팽창하고 이 힘으로 터빈을 구동함.
이 때 압축한 공기가 새어 나가지
않도록 Seal을 붙이는데
이 때문에 마찰(C)로 인한 진동 문제가 생김
($c < 0$) negative c

2.3 진동원인 판단

수 있는 기계적인 사항으로는, 베어링 진동센서 가
까운 부위에서 회전부와 고정부의 rubbing으로 인한
마찰 발생을 가정하여 볼 수 있다. 터빈 exhaust 베
어링에 있는 air seal 또는 oil seal 부위와 축이 미
세하게 접촉되면 Fig. 6과 같은 진폭변화가 있을 수
있다. 축과 고정체의 rubbing에 의한 마찰은 축 표
면을 뜨겁게 만들어 마찰방향으로 축을 휘게 만든
다. 휘어진 축의 질량 불평형은 기존의 불평형과는
다른 새로운 방향에 질량 불평형을 만들어 축을 휘
게하므로⁴⁾ 불평형에 의한 1X 성분의 진동이 상승하
게 되며 위상이 변화된다. 이러한 축의 굽힘 현상이

이상 진동의 진폭이 운전 시간의 경과와 함께 점
차로 커지고 지속시간이 순간적(수초~수분)인 것으
로 보아 다음과 같이 판단하였다. 가스터빈 exhaust
베어링 air seal 이나 oil seal 부위는 530℃ 배기가
스가 통과하는 통로 근처에 있으므로 오일 탄화물
쉽게 발생할 수 있는 환경으로써, 탄화물이 적층되
면 축의 회전에 의하여 탄화물 적층부위가 마찰하면
서 축의 열변형을 발생시키므로 순간적인 진동을 증
가시키고, 계속되는 마찰로 인하여 탄화물이 탈락하
는 현상이 반복되는 것이다.

3.2 탄화물 적층 원인

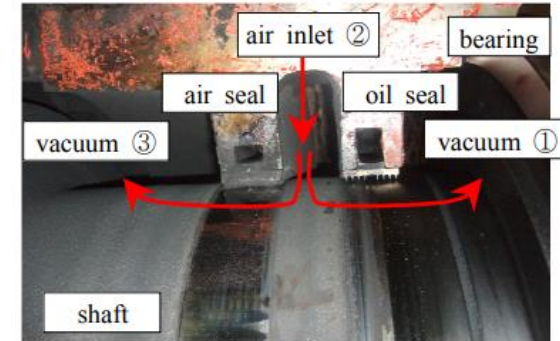
즉, 탄화물 발생 원인으로는 외부에서 유입된 이
물질(보온재)에 더하여, Fig. 8 과 같은 구조로 되어
있는 시스템에서 상호간 압력 불균형으로 인하여
exhaust 케이싱 베어링에 공급되는 오일의 비산
(mist) 증기가 메인 오일 탱크로 회수되지 못하고,
② 또는 ③ 으로 유입되어 가스터빈의 연소가스
와 함께 배출되었다는 것을 의미한다.

4. 결 론

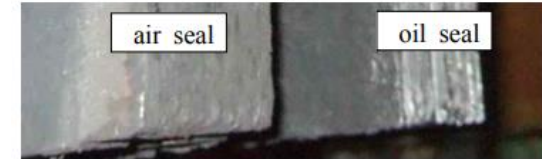
① 가스터빈의 exhaust 케이싱에 위치한 베어링의
오일 비산 증기(mist) 계통의 압력 불균형은, 탄화물
적층에 의하여 기동시에 고진동을 유발한다.

② 이를 방지하기 위하여, 터빈 운전시에는 메인
오일 탱크에 적절한 진공을 유지하고, 베어링 오일
을 공급하기 전에 반드시 오일 드레인 탱크의 부압
을 형성하여야 하며,

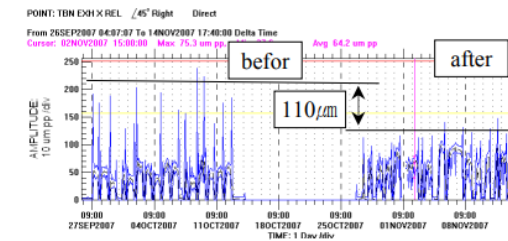
③ 정비시에는 베어링 오일의 비산 증기(mist)를
대기로 벤트하는 튜브와, 비산 증기를 메인 오일 탱
크로 회수하는 배관을 청결히 하여야 한다.



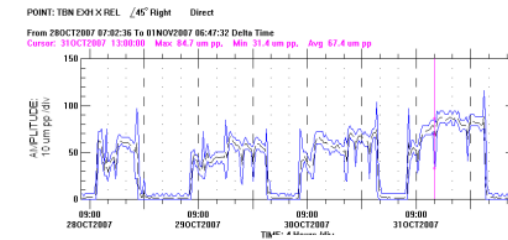
(a)



(b)



(a)



(b)

Fig. 9 Shaft vibration trend of the #1 bearing compared before with after maintenance

Free Vibration

$$\zeta = \frac{\delta}{2\pi}, \quad c = \zeta * 2 (mk)^{\frac{1}{2}}$$

ζ값을 구해도 m,k 값을 구해야 하기 때문에 c값을 구하기 어렵다.

$$\Sigma F = mx'' = -kx - cx'$$

$$\Delta E = \int f dx = \int (kx + cx') dx$$

Spring 의 Energy loss는 없다고 가정 했을 때

$$\int kx dx = 0$$

$$\int C \dot{x} dx = \int C \dot{x} \frac{dx}{dt} dt$$

$$= \int C \dot{x}^2 dt$$

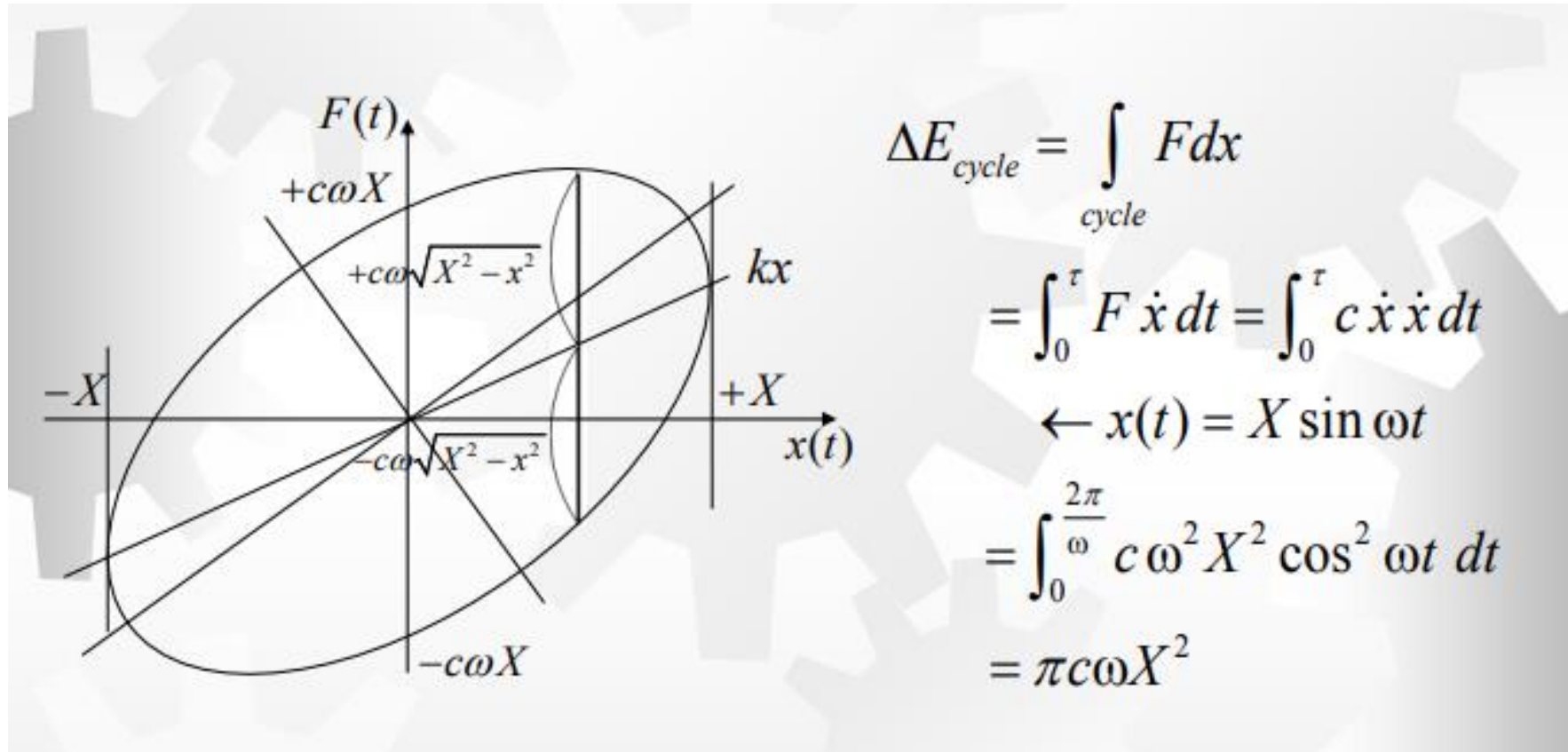
$x = X \sin \omega t$ 라 할 때 (진동의 sinusoidal 특성)

$$= \int_0^T (X\omega \cos \omega t)^2 dt, \quad T = \frac{2\pi}{\omega}$$

$$= X^2 \omega^2 c \int_0^T \cos^2 \omega t dt$$

$$= X^2 \omega^2 c \int_0^T \frac{1 + \cos 2\omega t}{2} dt$$

$$= c\omega X^2 \pi$$



K값과 C값을 구할 수 있음

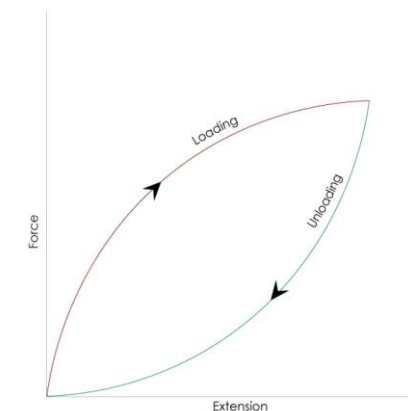
또한 X 값의 변화에 따라 K값과 C값이 변화함

$$f(x) = kX \sin \omega t + cX\omega \cos \omega t$$

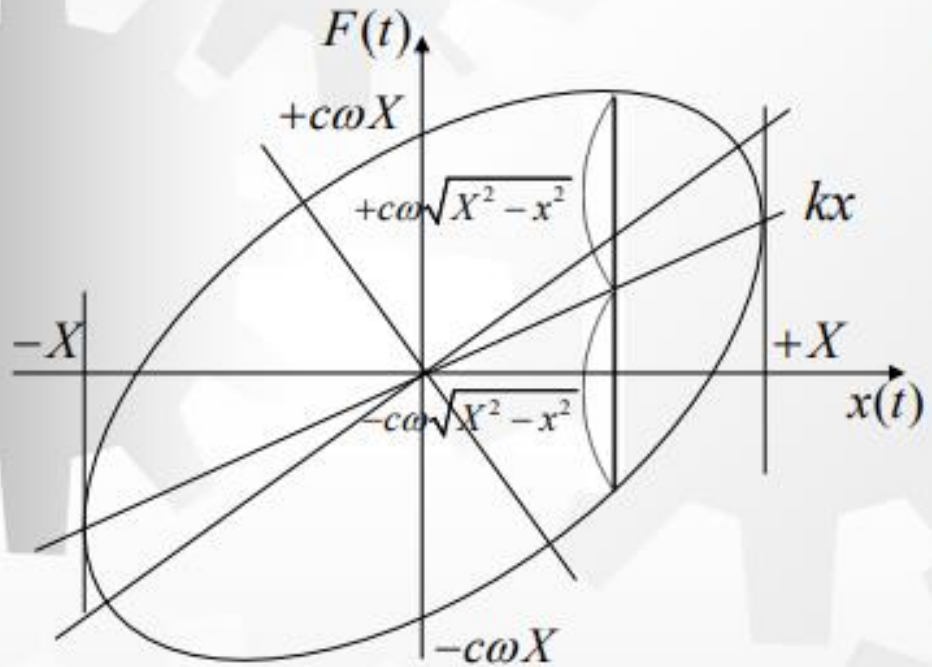
PART 1.

5 주차

Viscous damping	Coulomb damping	Structural damping
진동의 파형 곡선이 지수(Exponential)하게 감쇄	진동의 파형 곡선이 직선으로 감쇄	구조적인 원자의 마찰로 인한 댐핑
Damping Force \propto Velocity	Damping Force \propto 마찰계수	Damping Force $\propto Amplitude^2$
감쇄하는 진동이 0에 수렴하기까지 이론적으로 ∞ 시간이 걸림	Rest in equilibrium or displaced position depending upon the initial amplitude and amount of friction present	Stress-Strain Curve에서 인장/압축을 가한뒤에 발생하는 이력현상으로 인한 에너지 손실



Viscous Damping

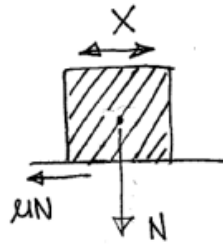
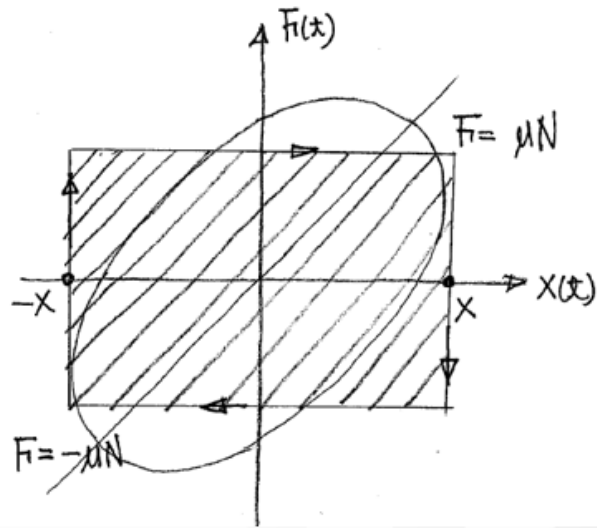


$$\begin{aligned}
 \Delta E_{\text{cycle}} &= \int_{\text{cycle}} F dx \\
 &= \int_0^{\tau} F \dot{x} dt = \int_0^{\tau} c \dot{x} \dot{x} dt \\
 &\leftarrow x(t) = X \sin \omega t \\
 &= \int_0^{\frac{2\pi}{\omega}} c \omega^2 X^2 \cos^2 \omega t dt \\
 &= \pi c \omega X^2
 \end{aligned}$$

$$\dot{x}(t) = \frac{d}{dt} X \sin(\omega t) = X \omega \cos(\omega t)$$

$$\cos^2(\omega t) = \frac{1 + \cos(2\omega t)}{2}$$

Coulomb(Dry friction) Damping



$$m\ddot{x} + \mu mg \operatorname{sgn}(\dot{x}) + kx = f(t)$$

선형 미분방정식으로 나타낼 수 있는 Viscous damping의 Energy loss 식으로 나타내보자.

$$\Delta E_{cycle} = 4\mu NX = c_{eq}\pi\omega X^2$$

$$c_{eq} = \frac{4\mu N}{\pi\omega X}$$

Structural (or Material) Damping

$$\Delta E_{cycle} \propto X^2 \text{ (Empirical)} \quad \longrightarrow \quad \Delta E_{cycle} = \alpha X^2$$

선형 미분방정식으로 나타낼 수 있는 Viscous damping의 Energy loss 식으로 나타내보자.

$$\alpha X^2 = c_{eq} \pi \omega X^2 \quad \longrightarrow \quad c_{eq} = \frac{\alpha}{\pi \omega}$$

$$\therefore m\ddot{x} + \frac{\alpha}{\pi \omega} \dot{x} + kx = f(t) \quad \leftarrow f(t) = F_0 e^{j\omega t}, \quad x(t) = X e^{j(\omega t - \phi)}$$

$$\therefore m\ddot{x} + \left(k + j \frac{\alpha}{\pi} \right) x = F_0 e^{j\omega t} \quad \leftarrow \left(k + j \frac{\alpha}{\pi} \right) : \text{complex stiffness}$$

$$\frac{\alpha}{\pi} : \text{structural damping constant}$$

Forced Vibration of single DOF systems (Magnification Factor)

input / output

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

Let $f(t) = F_0 e^{j\omega t}$ and $x(t) = X_0 e^{j\omega t}$ $\xrightarrow{\frac{d}{dt}}$ $\dot{x} = j\omega X_0 e^{j\omega t}$ $\xrightarrow{\frac{d}{dt}}$ $\ddot{x} = -\omega^2 X_0 e^{j\omega t}$

then $F_0 e^{j\omega t} = X_0 [(k - m\omega^2) + j\omega c] e^{j\omega t}$

Hook's law
 $kx = F_0$
 $x = \frac{F_0}{k} = \delta_{st}$ static deflection
 $\frac{m}{k} = \frac{1}{\omega_n^2}$
 $\frac{c}{k} = \frac{2\zeta \sqrt{mk}}{k}$
 $= 2\zeta \sqrt{\frac{m}{k}} = 2\zeta \frac{1}{\omega_n}$
 $r = \frac{\omega}{\omega_n}$

$$X_0 = \frac{F_0}{(k - m\omega^2) + j\omega c}$$

$$= \frac{F_0/k}{(1 - \frac{m}{k}\omega^2) + j\omega \frac{c}{k}}$$

$$= \frac{\delta_{st}}{(1 - \frac{\omega^2}{\omega_n^2}) + j(2\zeta \frac{\omega}{\omega_n})}$$

$$= \frac{\delta_{st}}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (2\zeta \frac{\omega}{\omega_n})^2}} e^{j \tan^{-1}(\frac{2\zeta \frac{\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2})}}$$

$(1 - \frac{\omega^2}{\omega_n^2}) + j(2\zeta \frac{\omega}{\omega_n}) \frac{0}{r} \rightarrow r e^{j\theta}$ $\frac{r}{2} \angle \theta$ 표현

$$\frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \doteq MF \text{ magnification factor}$$

$$X_0 = \delta_{st} \cdot MF$$

$$X = \delta_{st} \cdot MF e^{-j\phi} \quad \text{where } \phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$$

$$x(t) = \delta_{st} \cdot MF e^{j(\omega t - \phi)}$$

2-2-8 Quadratic Poles and Zeros

Now consider the second-order transfer function

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{1}{1 + (2\zeta/\omega_n)s + (1/\omega_n^2)s^2} \quad (2-83)$$

We are interested only in the case when $\zeta \leq 1$, because otherwise $G(s)$ would have two unequal real poles, and the Bode plot can be obtained by considering $G(s)$ as the product of two transfer functions with simple poles.

By letting $s = j\omega$, Eq. (2-83) becomes

$$G(j\omega) = \frac{1}{\left[1 - (\omega/\omega_n)^2\right] + j2\zeta(\omega/\omega_n)} \quad (2-84)$$

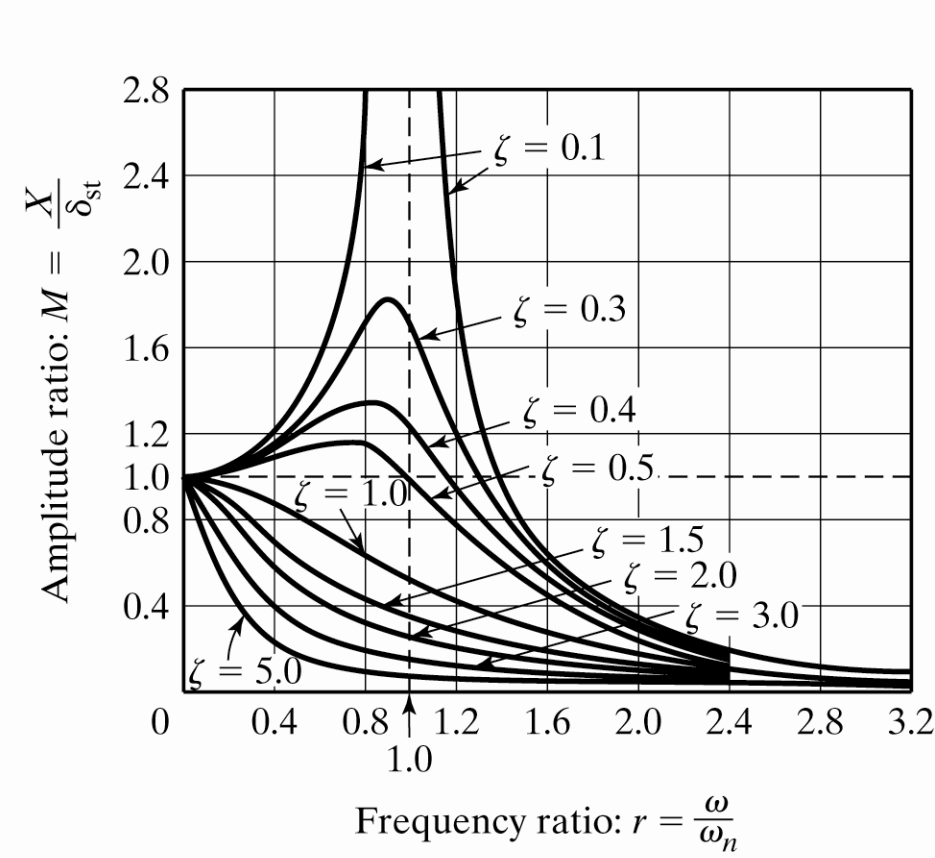
The magnitude of $G(j\omega)$ in dB is

$$|G(j\omega)| = 20 \log_{10} |G(j\omega)| = -20 \log_{10} \sqrt{\left[1 - (\omega/\omega_n)^2\right]^2 + 4\zeta^2(\omega/\omega_n)^2} \quad (2-85)$$

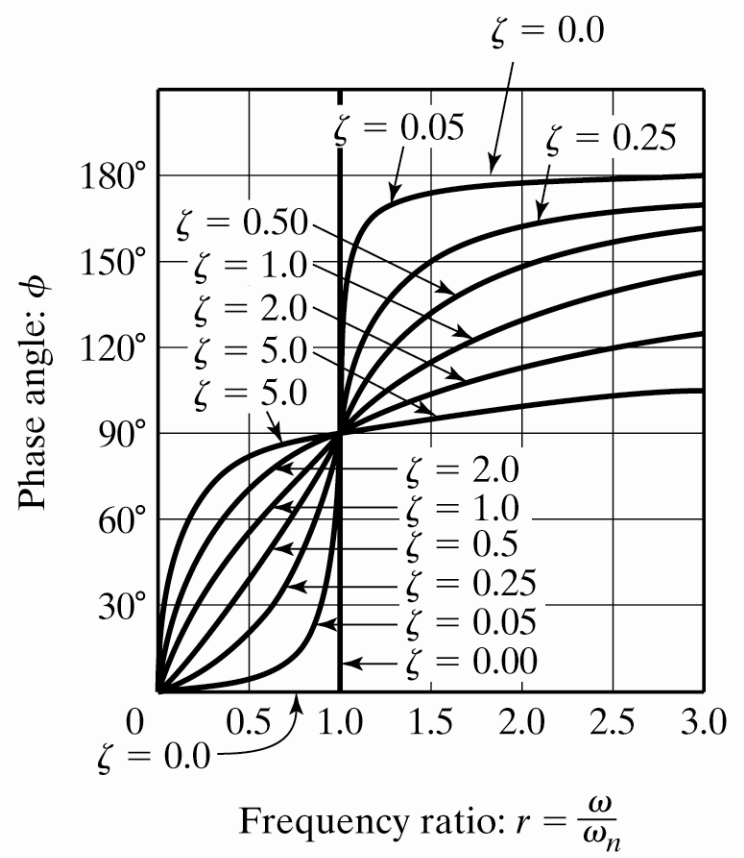
At very low frequencies, $\omega/\omega_n \ll 1$, Eq. (2-85) can be approximated as

$$|G(j\omega)|_{\text{dB}} = 20 \log_{10} |G(j\omega)| \cong -20 \log_{10} 1 = 0 \quad \text{dB} \quad (2-86)$$

Forced Vibration of single DOF systems (Magnification Factor)



(a)



(b)

Frequency ratio 비율에 따른 Amplitude 비율과 위상 차이

Forced Vibration of single DOF systems (Resonant frequency)

$$MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

MF의 극대점을 찾는 법 → MF를 r에 대해 미분하여 0이 되는 지점을 찾는다.

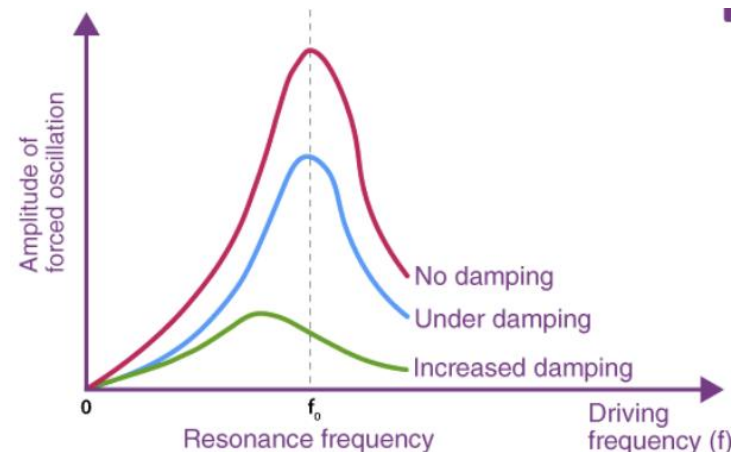
$$\frac{d}{dr}MF = \frac{-4r(1-r^2)+8\zeta^2r}{MF^{1.5}} = 0$$

분자가 0을 만족하면 미분값이 0이 된다. 따라서
 $-4r(1-r^2) + 8\zeta^2r = 0$

$$1 - r^2 = 2\zeta^2$$

$r = \frac{\omega}{\omega_n}$ r의 정의는 시스템의 입력주파수와 고유 주파수의 비

$\frac{\omega^2}{\omega_n^2} = 1 - 2\zeta^2 \rightarrow \omega = \omega_n \sqrt{1 - 2\zeta^2}$ 일 때 MF는 극대 값을 갖게 되고
이 때의 주파수를 Resonant frequency (ω_r)라고 부름



Forced Vibration of single DOF systems (Resonant frequency)

Maximum magnitude of $|H(\omega)|$ occurs at $\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$

$$\omega_n > \omega_d > \omega_r$$

Natural Frequency : ω_n

Damped Natural Frequency : $\omega_d = \omega_n \sqrt{1 - \zeta^2}$

Resonant Natural Frequency : $\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$

When $\omega_r = 0$, i.e., $1 - 2\zeta^2 = 0$ or $\zeta = \frac{1}{\sqrt{2}}$

Forced Vibration of single DOF systems (Resonant frequency)

① **when** $0 \leq \zeta < \frac{1}{\sqrt{2}} = 0.707$

At $\omega = \omega_r$

$$r = \frac{\omega_r}{\omega_n} = \sqrt{1 - 2\zeta^2} \quad \therefore r^2 = 1 - 2\zeta^2$$

$$\therefore (1 - r^2)^2 + (2\zeta r)^2 = (2\zeta^2)^2 + (2\zeta)^2(1 - 2\zeta^2) = 4\zeta^2(1 - \zeta^2)$$

$$\therefore \text{MF} = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

$$\therefore \left(\frac{X_0}{\delta_{st}} \right) \bigg|_{\omega=\omega_r} = \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

② **when** $\zeta \geq \frac{1}{\sqrt{2}}$, $\max(|H(\omega)|) = \delta_{st}$

when $\zeta = \frac{1}{\sqrt{2}}$ and $\omega = \omega_r$

$$1 - \zeta^2 \bigg|_{\zeta=\frac{1}{\sqrt{2}}} = \frac{1}{2}$$

$$\therefore \left(\frac{X}{\delta_{st}} \right)_{\omega=\omega_r, \zeta=\frac{1}{\sqrt{2}}} = 1$$

Forced Vibration of single DOF systems (Total response)

3) Total Response

Total response is given by

$$x(t) = e^{-\zeta\omega_n t} (C_1 \cos \omega_d t + C_2 \sin \omega_d t) + X_0 \cos(\omega t - \phi)$$

When the initial conditions are applied,

$$x(0) = C_1 + X_0 \cos \phi = x_0$$

$$\dot{x}(0) = -\zeta\omega_n C_1 + \omega_d C_2 + \omega X_0 \sin \phi = \dot{x}_0$$

Then

$$\begin{aligned} x(t) = & X_0 \cos(\omega t - \phi) \\ & + e^{-\zeta\omega_n t} \left[(x_0 \cos \omega_d t + \frac{\dot{x}_0 + \zeta\omega_n x_0}{\omega_d} \sin \omega_d t) \right. \\ & \left. - X_0 \cos \phi \cos \omega_d t - \frac{X_0}{\omega_d} (\zeta\omega_n \cos \phi + \omega \sin \phi) \sin \omega_d t \right] \end{aligned}$$

System's natural response

$$e^{-\zeta\omega_n t} (C_1 \cos \omega_d t + C_2 \sin \omega_d t)$$

Forced response

$$X_0 \cos(\omega t - \phi)$$

Forced Vibration of single DOF systems (Q-factor)

$$\text{Amplitude ratio} = \frac{X_0}{\delta_{st}} = MF = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \Big|_{r=1} = \frac{1}{2\zeta}$$

$\frac{1}{2\zeta} = Q$ -factor라고 함. 하지만 공진주파수부근 MF가 1일 때로부터 zeta값을 찾는 것은 calibration 문제로 힘들다!

Damping ratio가 충분히 작다고 판단될 때($\zeta < 0.05$) $\omega_r \cong \omega_n$

Power가 $\frac{1}{2}$ 이 되는 지점 \rightarrow 진폭이 $\frac{1}{\sqrt{2}}$ 가 되는 지점

$$r_1^2, r_2^2 = (1 - 2\zeta^2) \pm \sqrt{(1 - 2\zeta^2)^2 - (1 - 8\zeta^2)} \quad \leftarrow \text{Half-power point 를 찾아보자!}$$

$$r_2^2 - r_1^2 = 4\zeta\sqrt{1 + \zeta^2} \cong 4\zeta$$

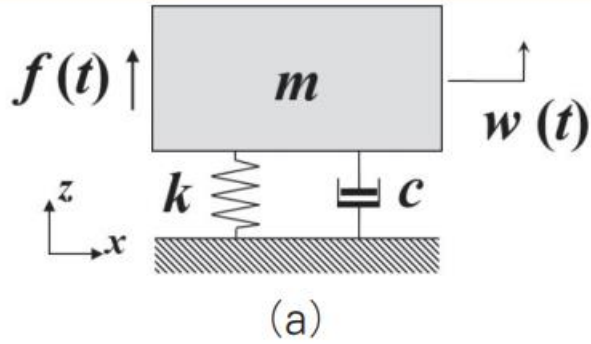
$$(r_2 - r_1)(r_2 + r_1) = 4\zeta$$

$$r_1 + r_2 = \frac{\omega_1 + \omega_2}{\omega_n} = 2$$

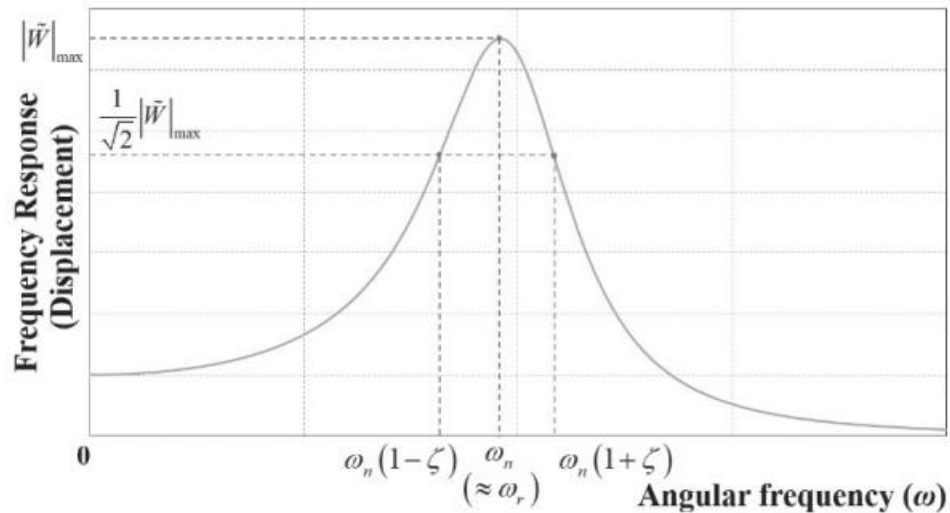
$$r_2 - r_1 = \frac{\omega_2 - \omega_1}{\omega_n} = 2\zeta$$

$$\leftarrow \zeta = \frac{\omega_2 - \omega_1}{2\omega_n}$$

Forced Vibration of single DOF systems (Q-factor)



Q -factor: 공진 주파수 부근의 곡선이 얼마나 예리한지 (Sharpness)에 대해 보여준다.



$$Q = \frac{\omega_n}{\Delta\omega} = \frac{\omega_n}{(\omega_2 - \omega_1)} = \frac{1}{2\zeta}$$

5주차 복습

Magnification Factor

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$x(t) = X_o \operatorname{Re}(e^{j\omega t})$$

$$f(t) = F_o \operatorname{Re}(e^{j\omega t})$$

$$[(k - m\omega^2) + jc\omega]X_o e^{j\omega t} = F_o e^{j\omega t}$$

$$X_o = \frac{F_o}{(k - m\omega^2) + jc\omega}$$

$$= \frac{\frac{F_o}{k}}{\left(1 - \frac{m}{k}\omega^2\right) + j\frac{c}{k}\omega}$$

$$\frac{m}{k} = \frac{1}{\omega_n^2}, \quad \frac{c}{k} = \frac{\zeta 2\sqrt{mk}}{k} = 2\zeta \frac{1}{\omega_n}, \quad kx = F_o = \delta_{st}$$

$$X_o = \frac{\delta_{st}}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j2\zeta \frac{\omega}{\omega_n}}, \quad r = \frac{\omega}{\omega_n}$$

$$= \frac{\delta_{st}}{(1 - r^2) + j2\zeta r}$$

$$= \frac{\delta_{st} e^{-j\phi}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$MF = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$X_o = \delta_{st} MF e^{-j\phi}$$

ϕ : phase delay

$$x(t) = \delta_{st} MF e^{j(\omega t - \phi)}$$

δ_{st} : static deflection

$$F(t) = F_o e^{j\omega t}$$

MF: static deflection이 얼마나 증가하였는지?

MF를 통해 공진주파수를 찾기

$$\frac{d}{dr}(MF) = \frac{2(1-r^2)(-2r) + 2(2\zeta r)(2\zeta)}{((1-r^2)^2 + (2\zeta r)^2)^{\frac{3}{2}}} = 0$$

$$r^2 = 1 - 2\zeta^2 = \left(\frac{w}{w_n}\right)^2$$

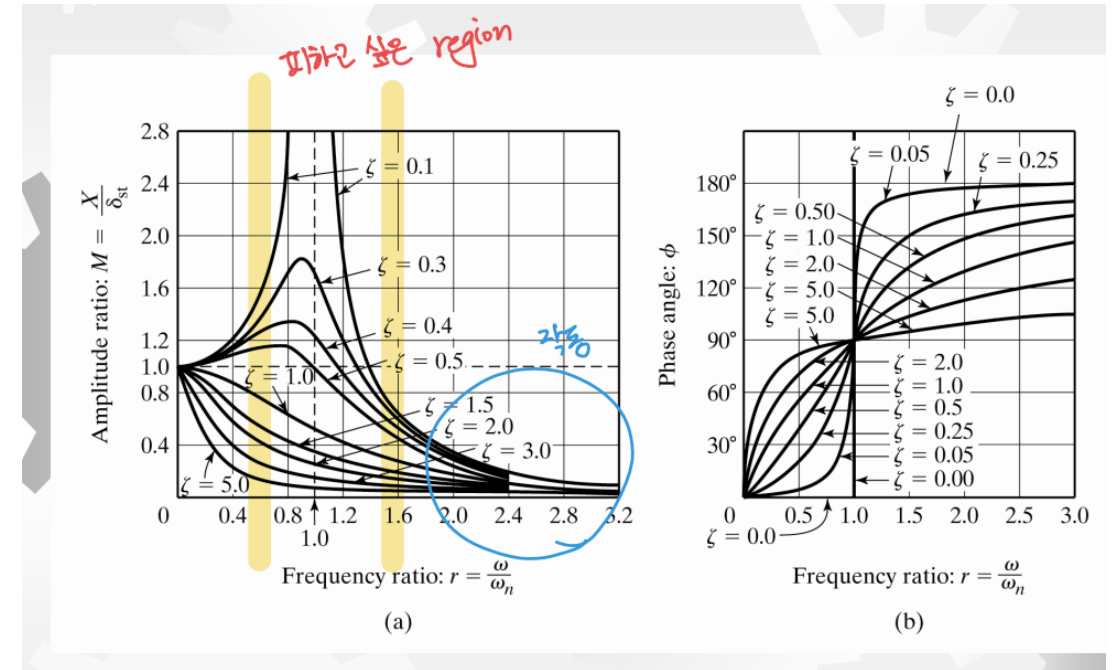
$$w^2 = w_n^2(1 - 2\zeta^2)$$

$$w = w_n\sqrt{1 - 2\zeta^2}$$

$$w_n = \sqrt{\frac{k}{m}}, \quad w_d = w_n\sqrt{1 - \zeta^2}, : \text{Damped nature freq}, \quad w_r = w_n\sqrt{1 - 2\zeta^2} : \text{Resonant freq}$$

$r = 1$ 일 때 $\zeta = 0$ 이므로 감쇠비가 0이다. $\rightarrow MF\left(\frac{x}{\delta_{st}}\right)$ 는 무한대

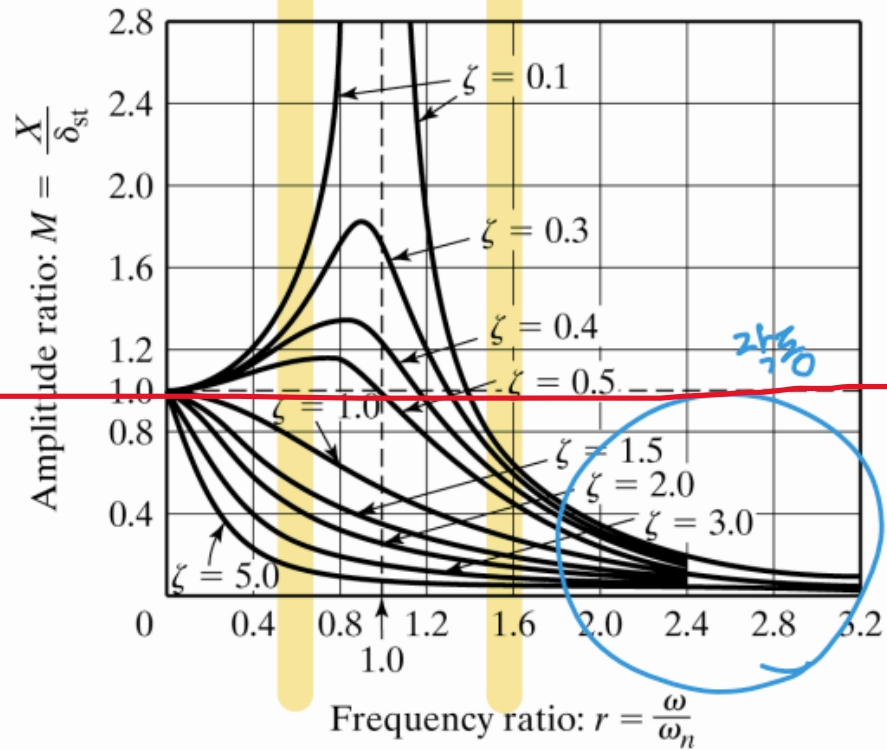
설계 조건 : $r > 2$, $\zeta = 0.7$?



$$\theta = \tan^{-1} \frac{2\zeta r}{1 - r^2}$$

$r = 1$ 일때 위상 지연이 $\frac{\pi}{2}$ 만큼 일어난다.

피하인 싶은 region

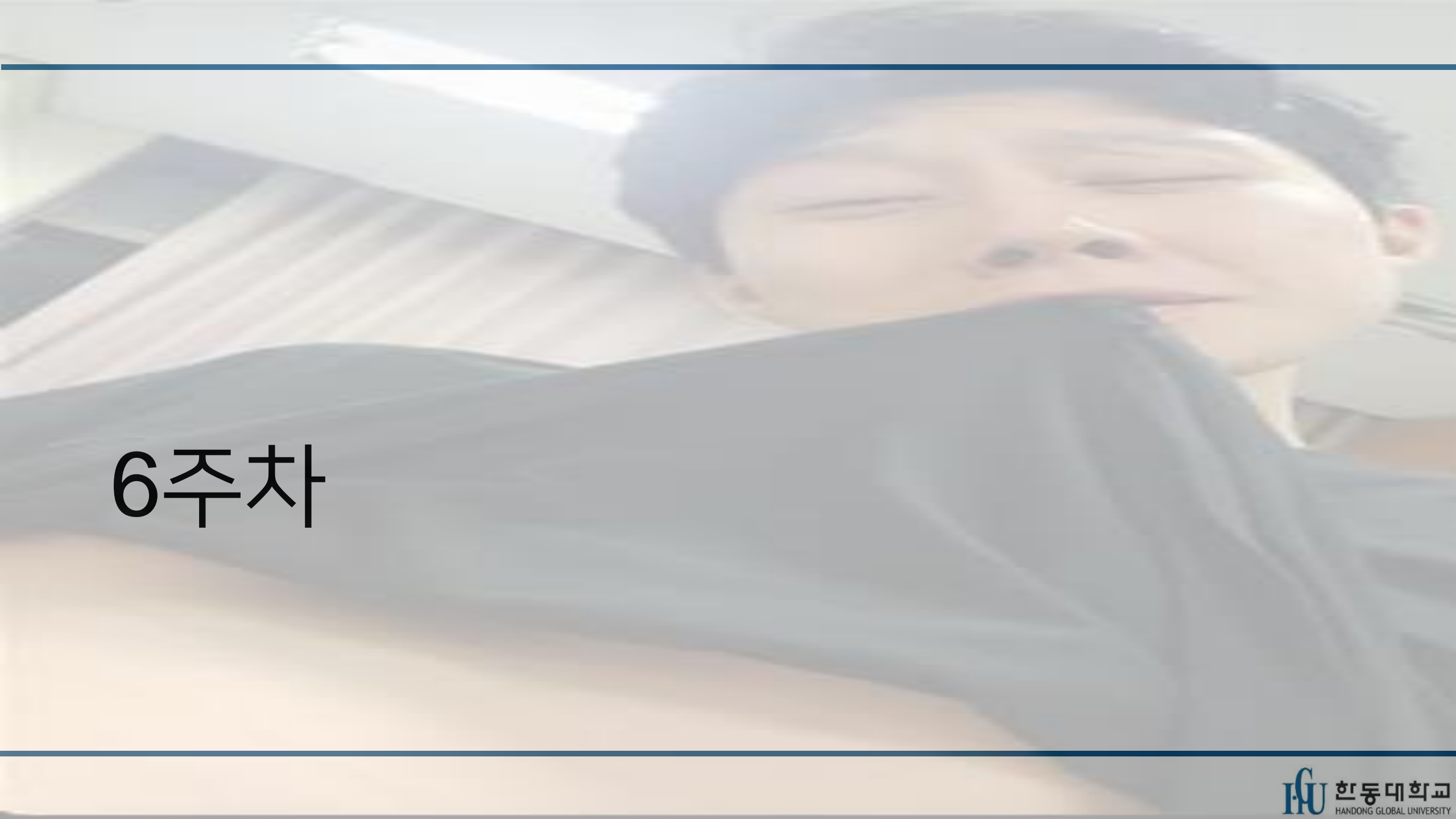


(a)

진동을 줄이려면 MF가 1보다 낮은 r 을 설계 해야함

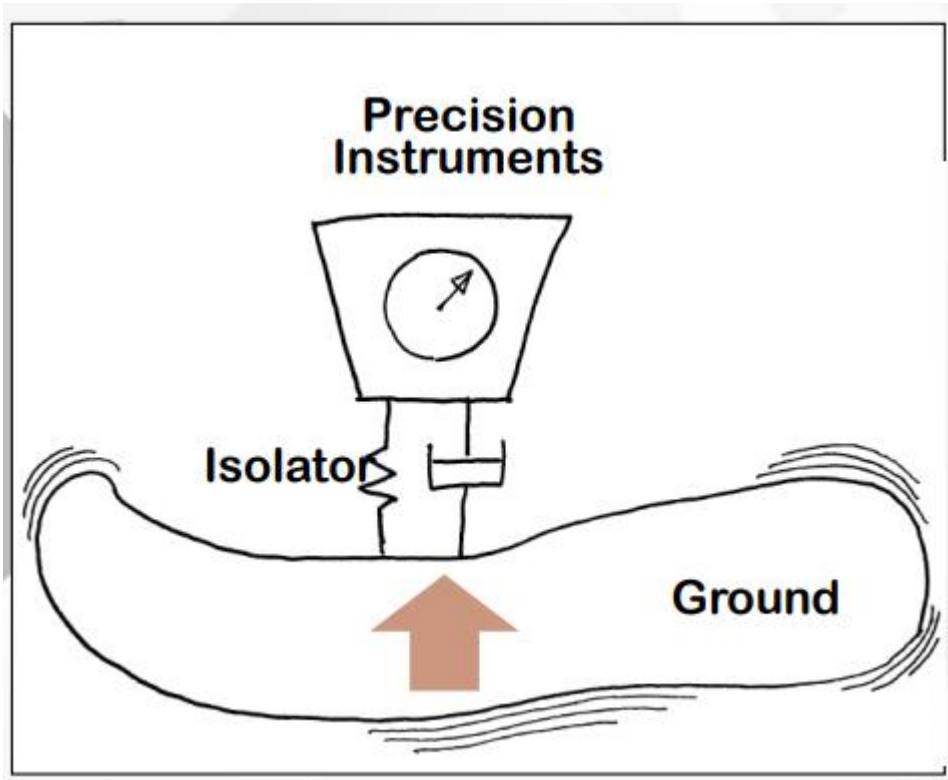
→ $\frac{w}{w_n}$ 중 w_n 이 작아야 한다. (w 는 외부의 진동)

→ K 가 작아야 한다 ($w_n = \sqrt{\frac{k}{m}}$)

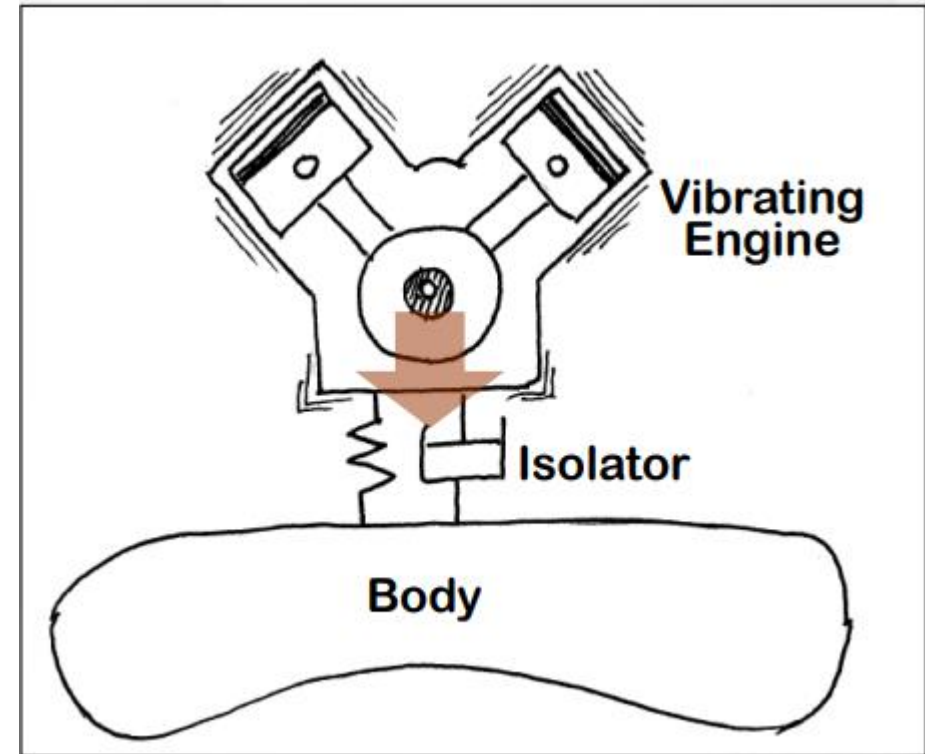
A person is lying in a hospital bed, appearing to be asleep. They are wearing a dark blue hospital gown. A clear nasal cannula is visible, inserted into their nostril. The background shows a hospital room with a window and some medical equipment.

6주차

Vibration Isolation

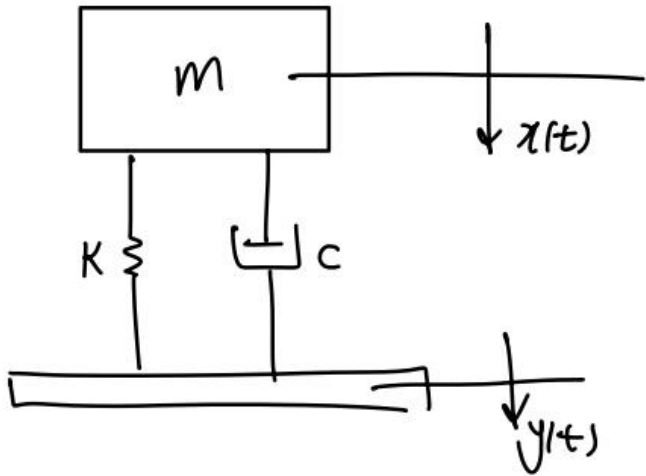


(1) 외부에서 일어난 진동을 장치로부터 격리시키는 것



(2) 장치로부터 일어난 진동을 본체와 격리시키는 것

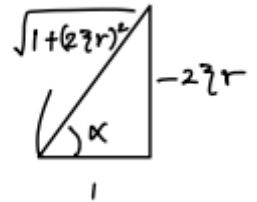
(1) 외부에서 일어난 진동을 장치로부터 격리시키는 것



$$x = \delta_{st} M F e^{j(\omega t - \phi)}$$

$$x = \frac{k + jcw}{k} Y_0 M F e^{j(\omega t - \phi)}$$

$$\frac{cw}{k} = \frac{2\zeta \sqrt{mk} * \omega}{k} = 2\zeta \frac{\omega}{\omega_n} = 2\zeta r$$



$$x = (1 + j2\zeta r) Y_0 \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} e^{j(\omega t - \phi)}$$

$$x(t) = Y_0 \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} e^{j(\omega t - \phi - j\alpha)}$$

$$\sum F = m\ddot{x} = -c(\dot{x} - \dot{y}) - k(x - y)$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$\text{if } y = Y_0 e^{j\omega t}$$

$$m\ddot{x} + c\dot{x} + kx = j(k + jc\omega) Y_0 e^{j\omega t}$$

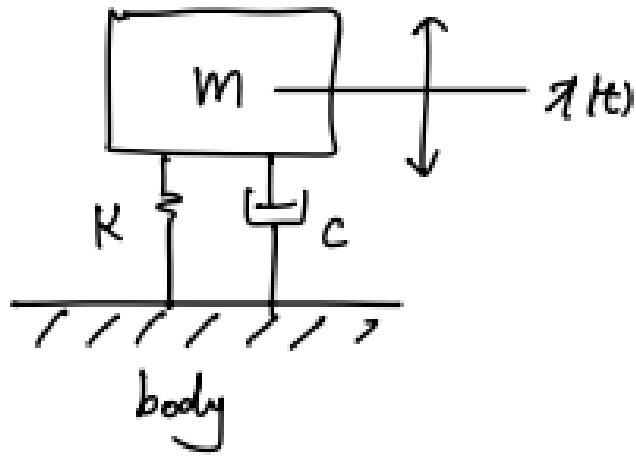
$$\frac{F_0}{k} = \frac{j(k + jc\omega)}{k} = \delta_{st}$$

$$\text{TM} : \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$x(t) = Y_0 T M e^{j(\omega t - \phi - j\alpha)}$$

Transmissibility : 전달성

(2) 장치로부터 일어난 진동을 본체와 격리시키는 것



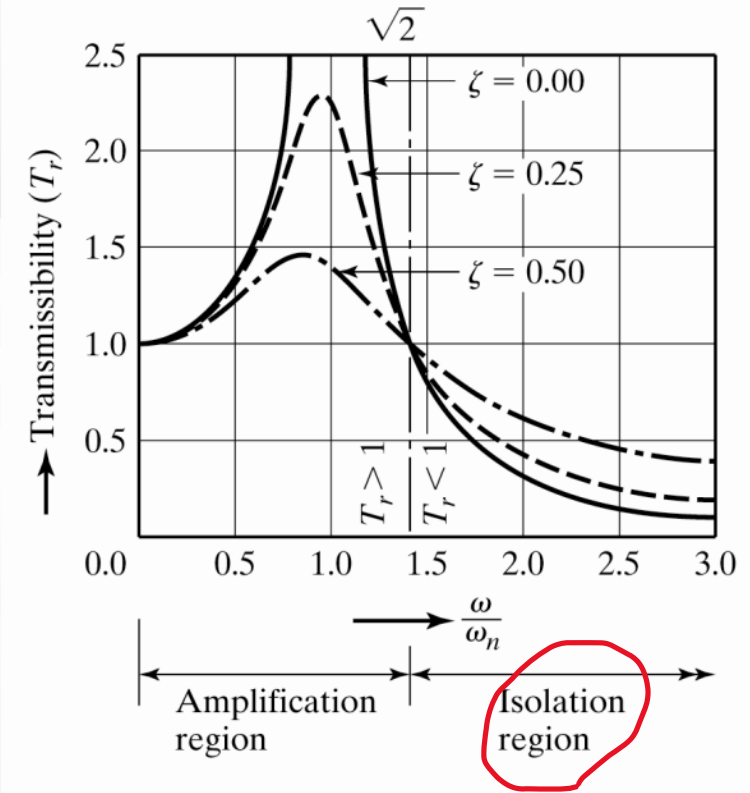
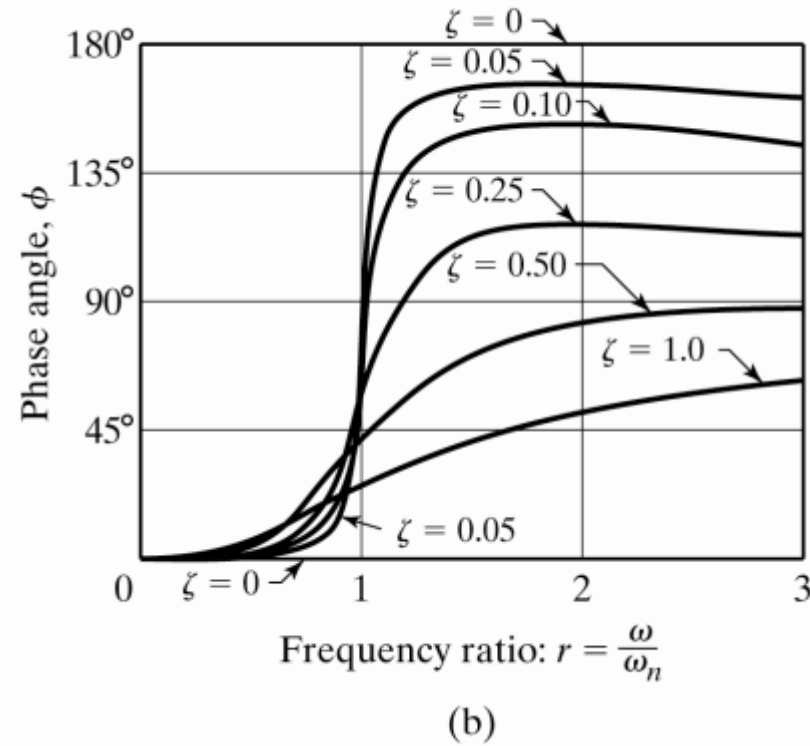
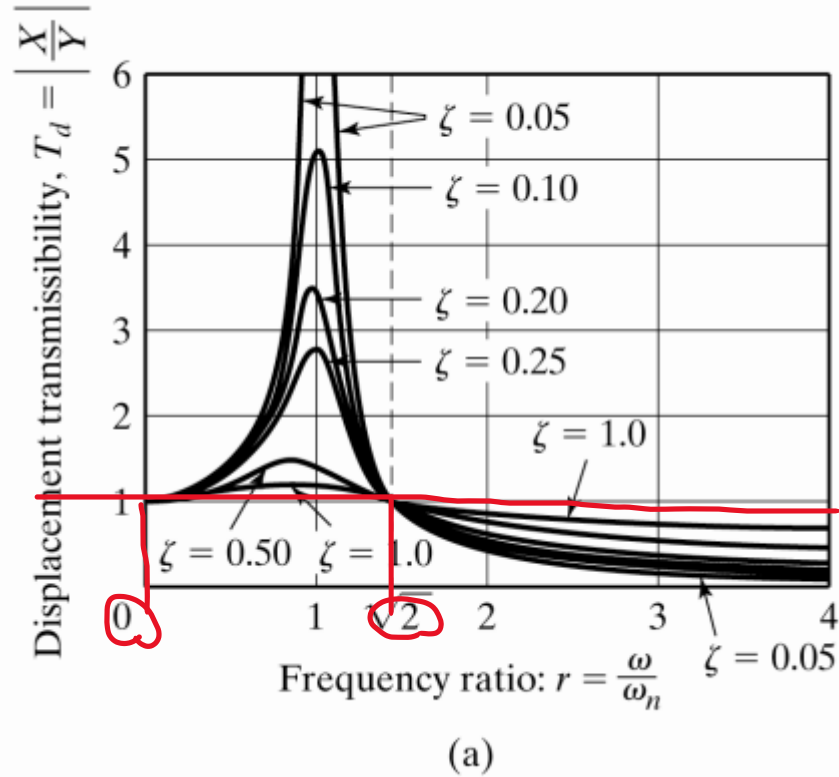
$$F(t) = c\dot{x} + kx$$

$$x = \delta_{st} M F e^{j(\omega t - \phi)}$$

$$= (k + j\omega c) \frac{F_0}{k} * \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} e^{j(\omega t - \phi)}$$

$$= F_0 T M e^{j(\omega t - \phi - \alpha)}$$

r 값에 따른 TM, Phase 변화



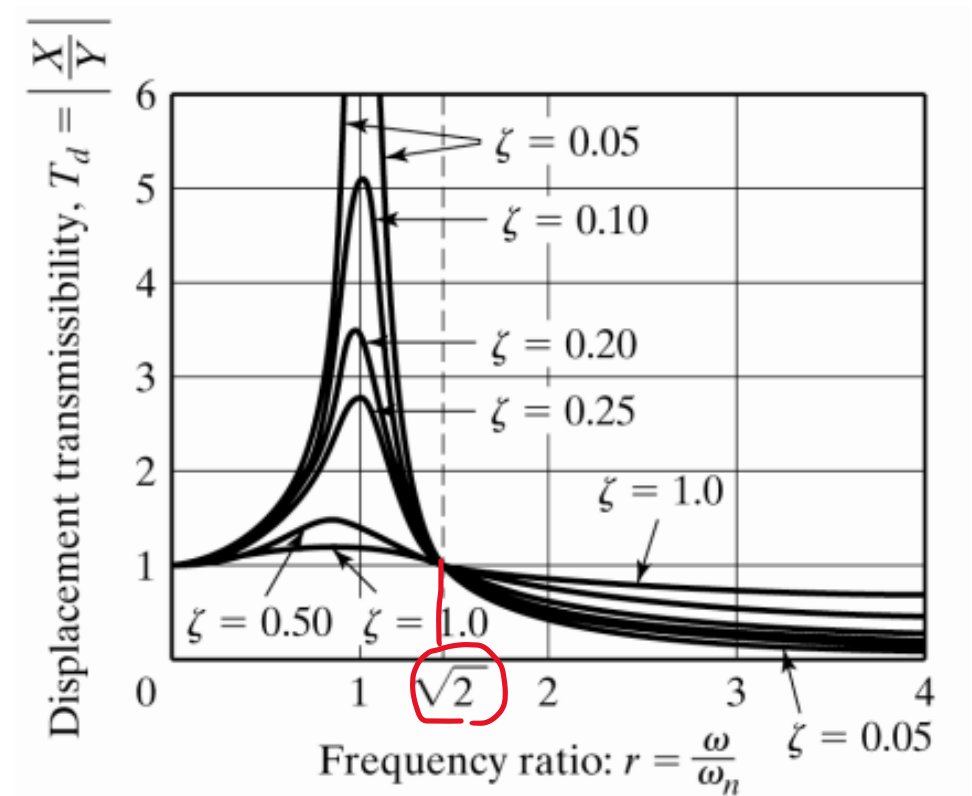
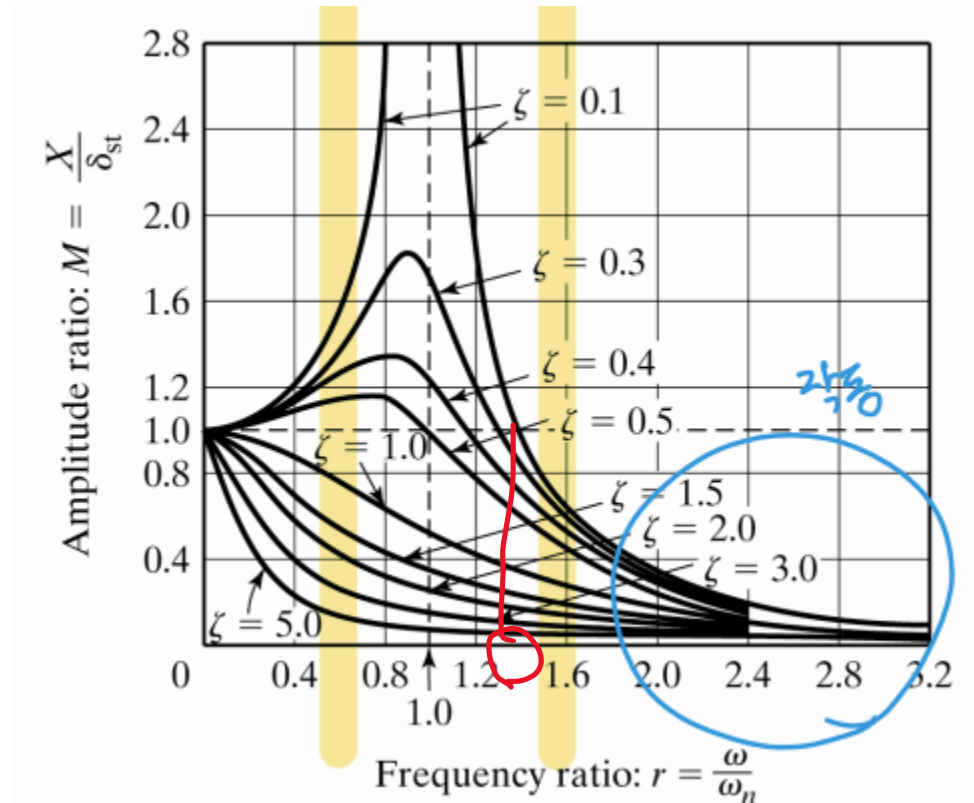
TM값이 1일 경우 $\rightarrow r = 0, r = \sqrt{2}$

따라서 Transmissibility가 1보다 작은 $r > \sqrt{2}$ 인 위치에서 설계해야 함

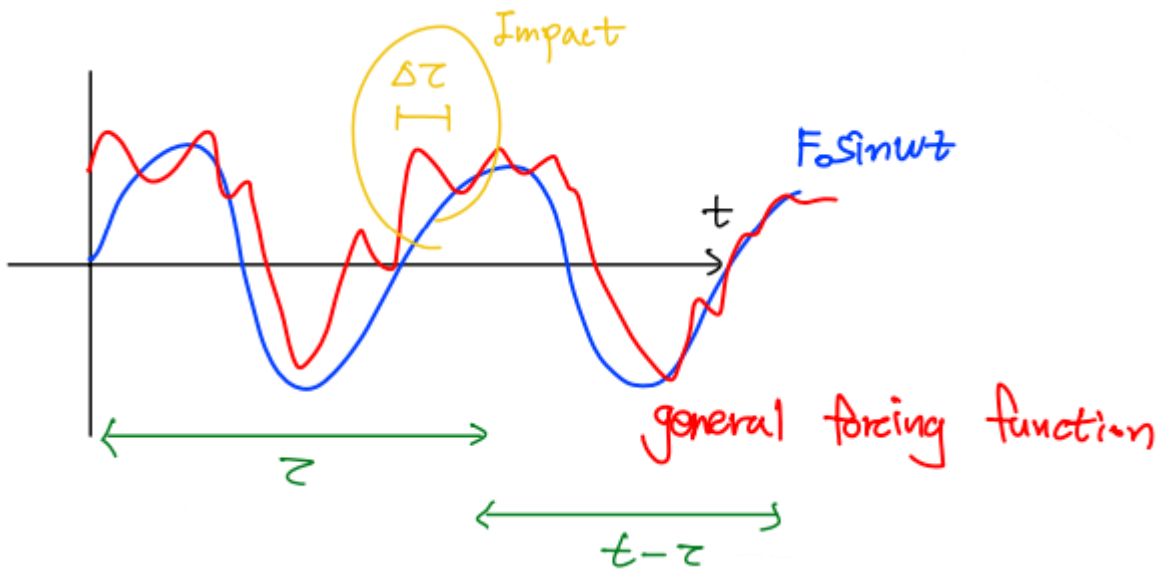
Forced Vibration, Vibration Isolation 정리

구분	System structure	$f(t) \rightarrow system \rightarrow x(t)$	Magnitude
Forced Vibration	동적 시스템에 외란을 가하여 생기는 진동	$x(t) = \delta_{st} M F e^{j(\omega t - \phi)}$	$M F = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$
Vibration Isolation	외부 진동으로부터 분리	$x(t) = Y_0 T M e^{j(\omega t - \phi - j\alpha)}$	$T M = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$
	내부 진동으로부터 본체 분리	$x(t) = F_0 T M e^{j(\omega t - \phi - \alpha)}$	

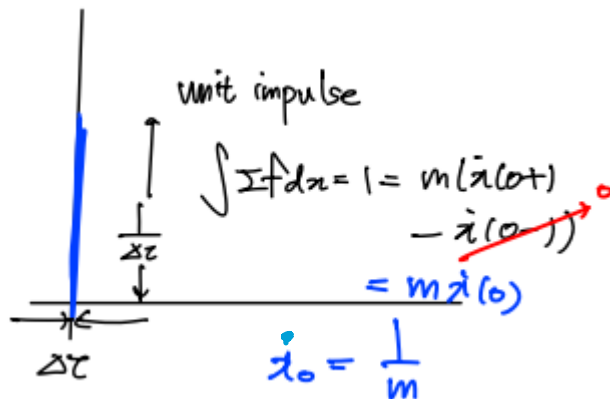
MF과 TM의 r에 따른 감쇠비 차이



Single Harmonic Forcing Function



General Forcing function: 일반 강제진동
 → 어떤 시스템이 갑작스런 힘을 받았을 때의 상황을 다루는 것 (정지 된 시스템에 갑작스런 충격)



$$\int \Sigma f(t) dt = m \int d\dot{x}$$

Linear impulse = Linear momentum

$$\dot{x}_0 = \frac{1}{m}$$

General Impulse

$$x(t) = e^{-\zeta \omega_n t} \left(x_0 \cos \omega_d t + \frac{x_0 + \dot{\zeta} \omega_n x_0}{\omega_d} \sin \omega_d t \right)$$

unit impulse의 경우 ($\dot{x} = \frac{1}{m}, x_0 = 0$)

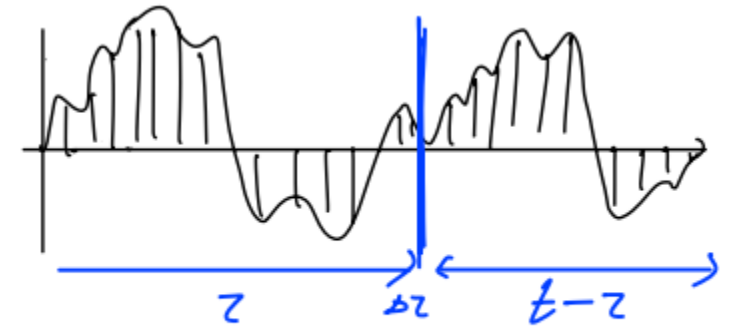
$$x(t) = \frac{1}{m \omega_d} e^{-\zeta \omega_n t} \sin \omega_d t : \text{unit impulse}$$

$$F_0 \Delta \tau = \frac{1}{m}$$

$$F_0 \Delta \tau * \frac{1}{m \omega_d} e^{-\zeta \omega_n t} \sin \omega_d t : \text{General Impulse}(t=0 \text{인 시점})$$

$$x(t) = F(\tau) \Delta \tau \frac{1}{m \omega_d} e^{-\zeta \omega_n (t-\tau)} \sin \omega_d (t-\tau) : \text{General Impulse}(t=\tau \text{인 시점})$$

$$h(t) = \frac{1}{m \omega_d} e^{-\zeta \omega_n t} \sin \omega_d t \text{ 라고 하면 } x(t) = F(\tau) \Delta \tau h(t-\tau)$$



impulse의 함으로 본다.

$$x(t) = \int_0^t F(\tau) h(t-\tau) d\tau$$

Convolution Integral

Convolution Integral

$$x(t) = \int_0^t F(z) h(t-z) dz$$

$$\textcircled{1} x(t) = \int_0^t F(z) h(t-z) dz = \int_0^t F(t-z) h(z) dz$$

$$\textcircled{2} X(s) = \mathcal{L}[x(t)] = F(s) H(s)$$

Convolution Integral의
2가지 특성

$$H(s) = \frac{H(s)}{F(s)} = \frac{\text{Output}}{\text{Input}} \quad (\text{system transformation})$$

unit impulse response

$$\mathcal{L}[h(t)] = \mathcal{L}\left[\frac{1}{n\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t\right]$$

unit impulse

1번 특성 증명

① 증명

$$\lambda(t) = \int_0^t f(z) h(t-z) dz$$

$$\begin{aligned} t-z &= t^* \\ -dz &= dt^* \\ z &= t-t^* \end{aligned} \quad \begin{cases} z=0, & t=t^* \\ z=t, & t^*=0 \end{cases}$$

$$\lambda(t) = \int_t^0 f(t-t^*) h(t^*) (-dt^*)$$

$$= \int_0^t f(t-t^*) h(t^*) dt^* \rightarrow \int_0^t f(t-z) h(z) dz$$