CAE
LAB #4

Impact Hammer Test

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I. PreLAB

① **Summarize the investigation on Impact Hammer Test.**

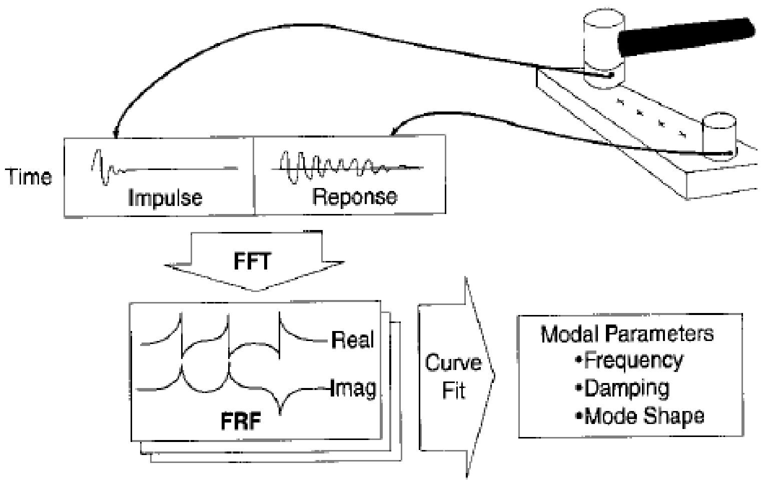


Figure 1: Illustration of a typical impact hammer modal test and signal processing

The "Hammer Impact Test," also known as "Modal Test," is a method for measuring how a structure responds to external forces.

A. Principle and Operation:

- The Impact Hammer consists of a small mass hammer that imparts an impact on the test subject.
- The impact is generated by the rapid movement of the hammer, delivering energy to the test subject.

B. Measurement of Response:

- The response of the test subject to the impact is measured using accelerometers or acceleration sensors.
- These sensors accurately measure the acceleration of the structure or system, recording changes in acceleration over time.

C. Time-Domain and Frequency-Domain Analysis:

- The measured acceleration data is transformed into time-domain and frequency-domain representations.
- In the time-domain, changes in acceleration over time are examined to evaluate the dynamic characteristics of the structure.
- In the frequency-domain, frequency spectra are analyzed to identify the structure's natural frequencies and response characteristics.

D. Data Interpretation:

- Analyzing the measured data reveals the structure's natural frequencies, frequency response spectra, damping ratio, and other relevant information.
- These results are used to assess the structure's integrity, strength, safety, and other performance aspects.

E. Calibration for Accuracy Enhancement:

- Calibration is performed before testing to ensure accurate knowledge of the properties and characteristics of the test subject.

② Examine the FFT of the following function and sketch the spectrum in the frequency domain.

FFT stands for Fast Fourier Transform, an algorithm that transforms signals from the time domain to the frequency domain efficiently by performing complex computations. This algorithm effectively reduces the computational workload.

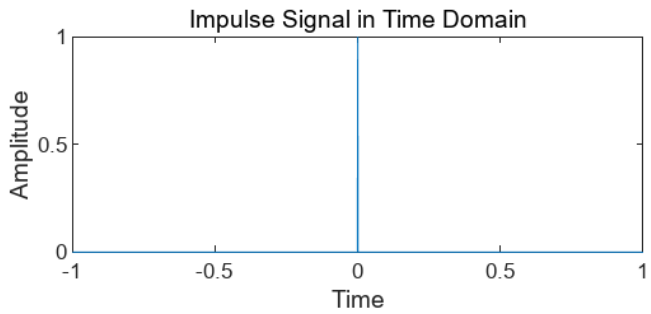
$$X(\omega) = \int_{-\infty}^{\infty} X(t) \cdot e^{-j\omega t}\, d\mathrm{t}$$

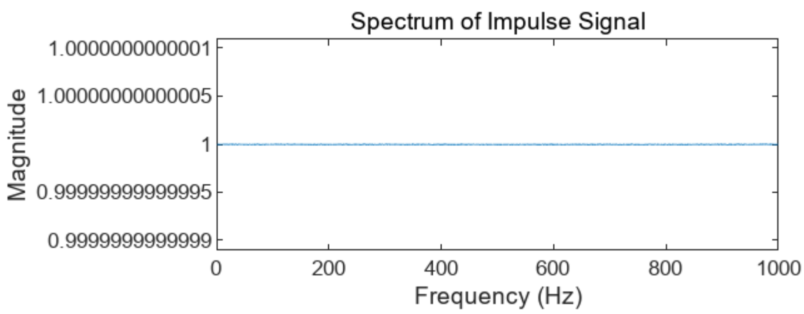
- **Impulse**

$$X(t) = \delta(t) = \begin{cases} \infty \ (t = 0) \\ 0 \ (t \neq 0) \end{cases}$$

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t}\, d\mathrm{t}$$

$$F[\delta(t)] = \int_{-\infty}^{\infty} 1 \cdot e^{-j\omega t}\, d\mathrm{t} = 1$$





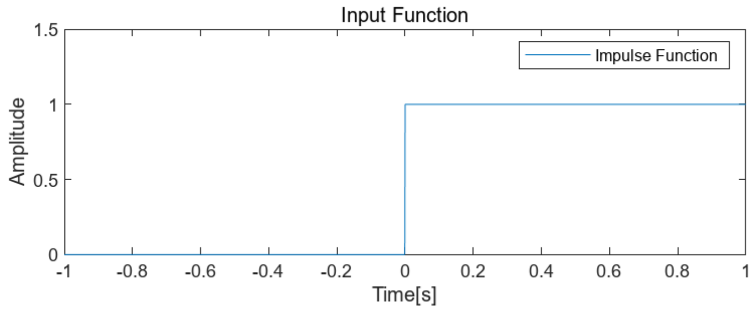
➢ **Unit Step Function**

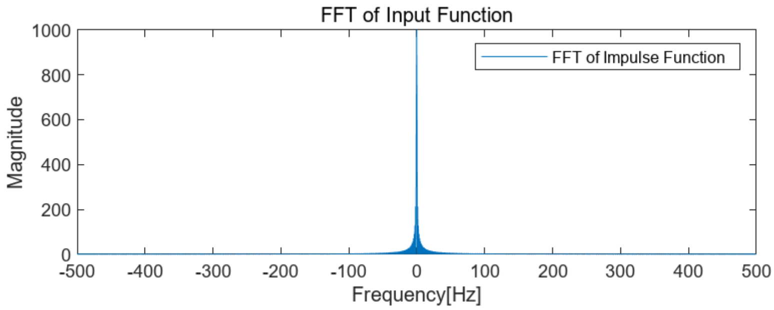
The unit step function, often denoted as u(t), is a mathematical function that takes on the value of 1 for all non-negative inputs and 0 for negative inputs. It is defined as follows:

$$X(t) = u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0) \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} u(t) \cdot e^{-j\omega t}\, \mathrm{dt}$$

$$F[u(t)] = \int_{0}^{\infty} e^{-j\omega t} dt = \lim_{A \to \infty} \left| -\frac{1}{j\omega} e^{-j\omega t} \right|_{0}^{A} = \frac{1}{j\omega}$$





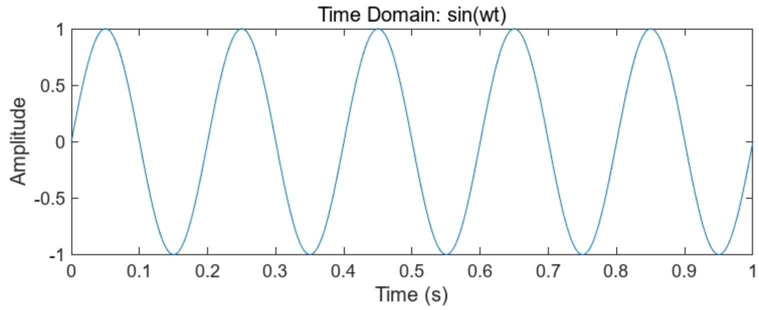
➢ **sin(ωt)**

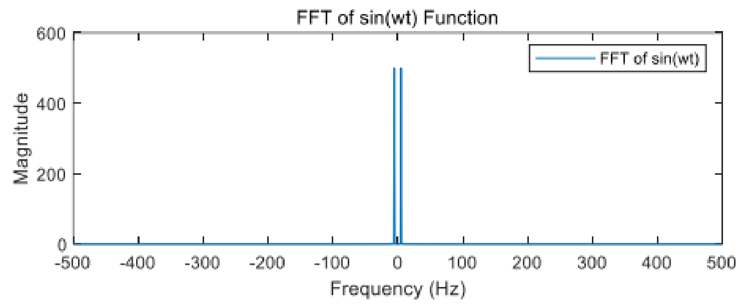
The function sin(wt) represents a sine wave with respect to time t. In this expression, w denotes the angular frequency, which is defined as 2π times the frequency f. Thus, $\omega = 2\pi f$. This sine function oscillates periodically, and the period T is given by $T = \frac{\omega}{2\pi}$. The period represents the time required for one complete cycle of the sine wave.

$$X(t) = \sin(\omega t) = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$$

$$F[X(t)] = \int_{-\infty}^{\infty} \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})e^{-j\omega t}dt = \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{j\omega_0 t - j\omega t}dt + \int_{-\infty}^{\infty} e^{-j\omega_0 t - j\omega t}dt\right]$$

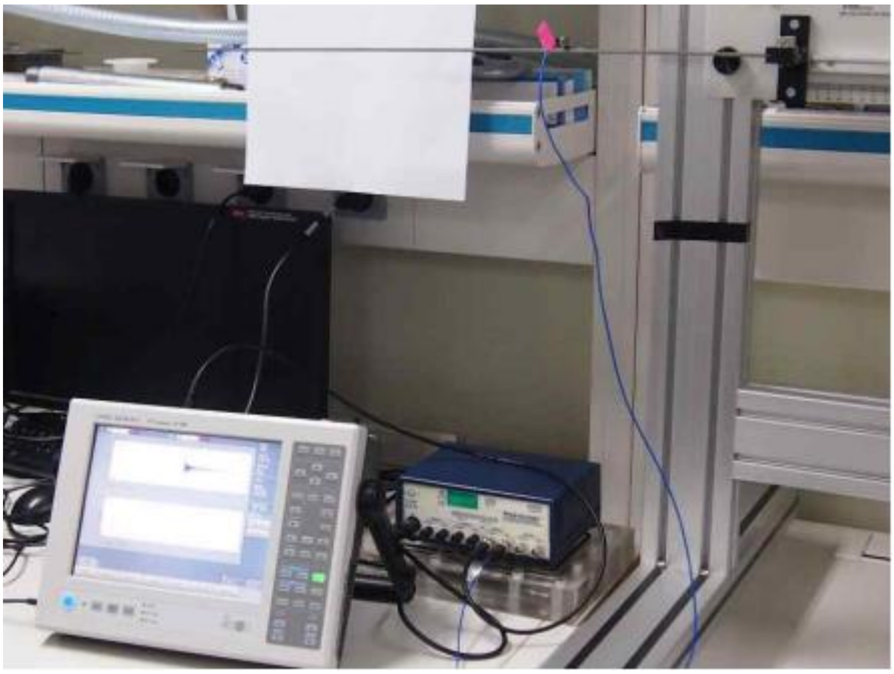
$$= \frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$





II. Main LAB

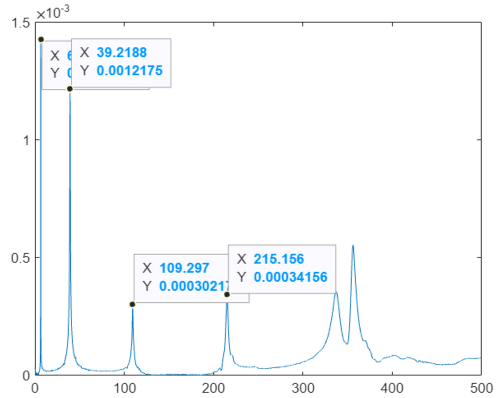
i. Introduction



In the experimental apparatus shown in Fig., an outrigger is attached, and an accelerometer is mounted at an appropriate position on the beam. When the end of the beam is lightly struck with a ball-peen hammer, the beam begins to vibrate, and the acceleration signal is amplified through a signal processor and appears on the FFT analyzer. The time-domain signal gradually decreases due to damping, and through frequency spectrum transformation, the 1st-order natural frequency of the system is displayed on the screen. By adjusting the sampling rate, it is possible to identify the desired number of natural frequencies.

ii. Results

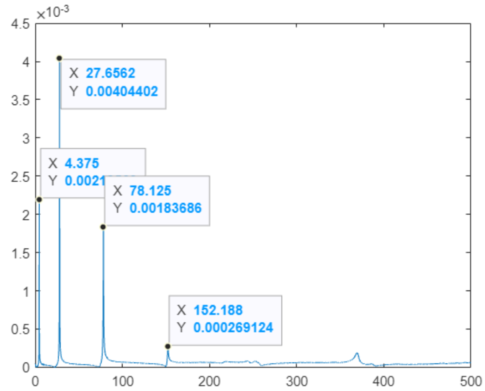
① Experimental Results.



코드 ^

```
xlim("auto")
ylim("auto")
ax = gca;
chart = ax.Children(1);
datatip(chart,215.2,0.0003416);
datatip(chart,39.22,0.001218);
datatip(chart,109.3,0.0003022);
datatip(chart,6.328,0.001427);
```

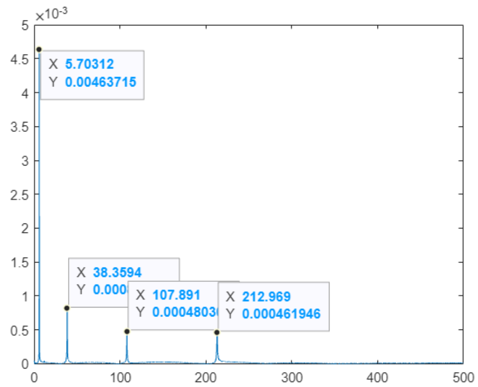
코드 업데이트 복사



코드 ^

```
xlim("auto")
ylim("auto")
ax = gca;
chart = ax.Children(1);
datatip(chart,152.2,0.0002691);
datatip(chart,78.12,0.001837);
datatip(chart,27.66,0.004044);
datatip(chart,4.375,0.002196);
```

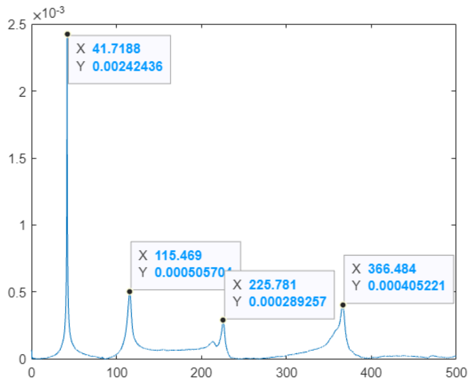
코드 업데이트 복사



코드 ^

```
xlim("auto")
ylim("auto")
ax = gca;
chart = ax.Children(1);
datatip(chart,213,0.0004619);
datatip(chart,107.9,0.0004803);
datatip(chart,38.36,0.0008158);
datatip(chart,5.703,0.004637);
```

코드 업데이트 복사



코드 ^

```
xlim("auto")
ylim("auto")
ax = gca;
chart = ax.Children(1);
datatip(chart,366.5,0.0004052);
datatip(chart,225.8,0.0002893);
datatip(chart,115.5,0.0005057);
datatip(chart,41.72,0.002424);
```

코드 업데이트 복사

② **Theory Results**

$$I = \frac{0.019 \times 0.003^3}{12} = 4.275 \times 10^{-11}[m^4]$$

$$A = 0.019 \times 0.003 = 5.7 \times 10^{-5}[m^2]$$

	E [Gpa]	$\boldsymbol{\rho}$ $[\boldsymbol{kg/m\text{^}3}]$
Steel	200	7850
Brass	115	8450
Aluminum	71	2770

$\boldsymbol{Fix - Free}$: $\beta_1 l = 1.875104,\quad \beta_2 l = 4.694091,\quad \beta_3 l = 7.854757,\quad \beta_4 l = 14.137165$

$\boldsymbol{Fix - Fix}$: $\beta_1 l = 4.730041,\quad \beta_2 l = 7.853205,\quad \beta_3 l = 10.995608,\quad \beta_4 l = 14.137165$

	$\boldsymbol{\beta_1}$	$\boldsymbol{\beta_2}$	$\boldsymbol{\beta_3}$	$\boldsymbol{\beta_4}$
Fix-Free	3	7.51	12.57	17.59
Fix-Fix	7.88	13.08	18.33	23.56

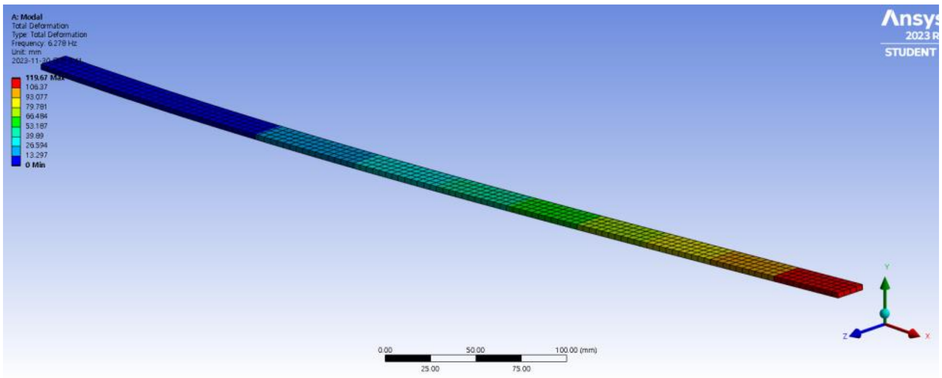
$$\beta^4 = \frac{\rho A \omega^2}{EI}$$

$$\omega = \sqrt{\frac{EI\beta^4}{\rho A}},\qquad f = \frac{2\pi}{\omega}$$

Fix-Free	$\boldsymbol{f_1}$	$\boldsymbol{f_2}$	$\boldsymbol{f_3}$	$\boldsymbol{f_4}$
Steel	6.2614	39.2384	109.9263	215.2596
Brass	4.5763	28.6782	80.3419	157.3269
Aluminum	6.2803	39.3568	110.2581	215.9095

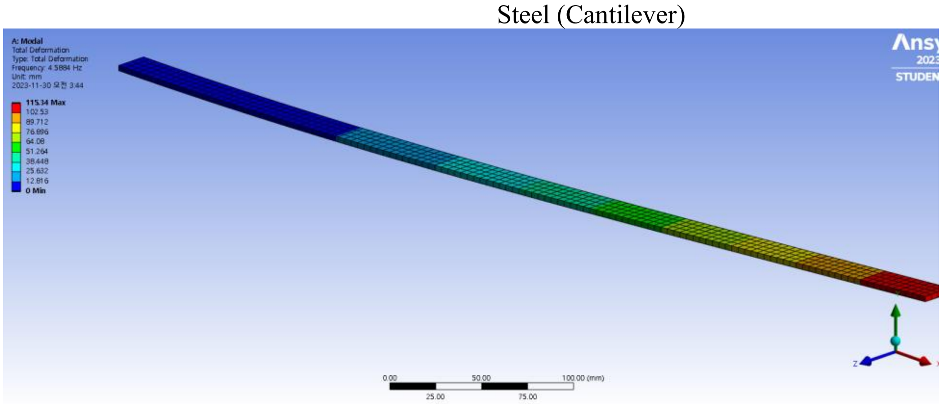
Free-Free	$\boldsymbol{f_1}$	$\boldsymbol{f_2}$	$\boldsymbol{f_3}$	$\boldsymbol{f_4}$
Steel	46.09	126.98	249.38	411.98

③ Simulation Results



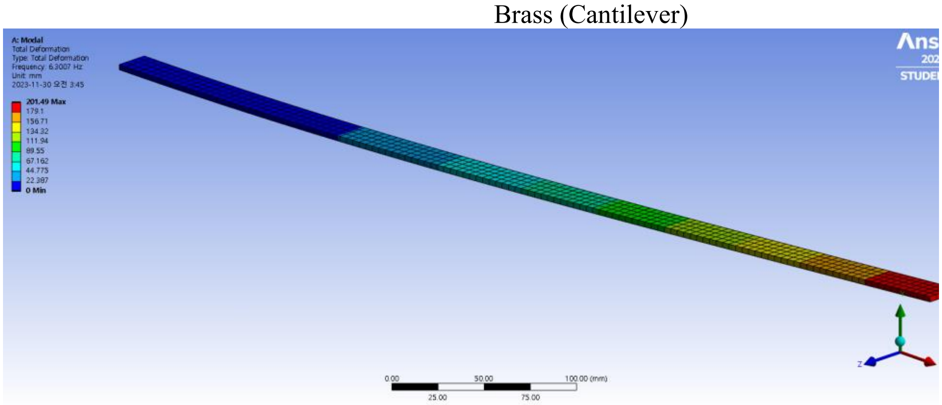
	Mode	Frequency [Hz]
1	1.	6.278
2	2.	39.339
3	3.	39.658
4	4.	110.14
5	5.	215.82
6	6.	247.47

Steel (Cantilever)



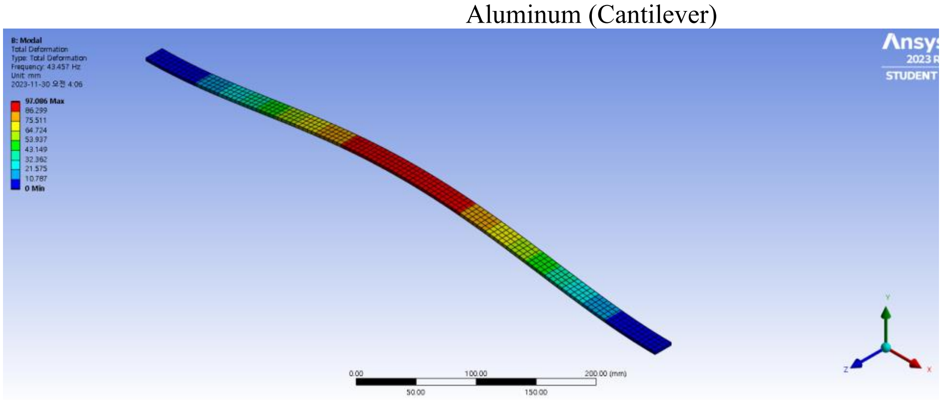
	Mode	Frequency [Hz]
1	1.	4.5884
2	2.	28.752
3	3.	28.985
4	4.	80.499
5	5.	157.73
6	6.	180.87

Brass (Cantilever)



	Mode	Frequency [Hz]
1	1.	6.3007
2	2.	39.481
3	3.	39.783
4	4.	110.54
5	5.	216.6
6	6.	248.23

Aluminum (Cantilever)



	Mode	Frequency [Hz]
1	1.	43.457
2	2.	119.77
3	3.	234.75
4	4.	272.37
5	5.	387.99
6	6.	579.5

Steel (Fixed-Fixed)

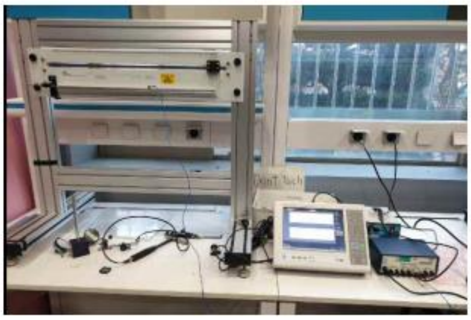
Steel (Cantilever)	$1st\ f_n$	$2nd\ f_n$	$3rd\ f_n$	$4th\ f_n$
Experimental	6.328	39.219	109.297	215.156
Theory	6.2614	39.2384	109.9263	215.2596
Simulation	6.278	39.339	110.14	215.82

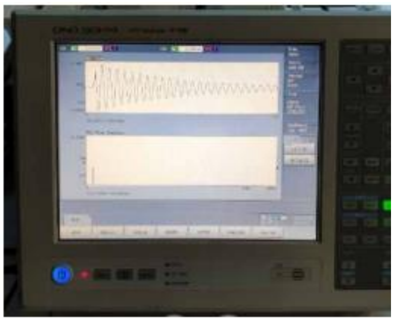
Brass (Cantilever)	$1st\ f_n$	$2nd\ f_n$	$3rd\ f_n$	$4th\ f_n$
Experimental	4.375	27.656	78.125	152.188
Theory	4.5763	28.6782	80.3419	157.3269
Simulation	4.5884	28.752	80.499	157.73

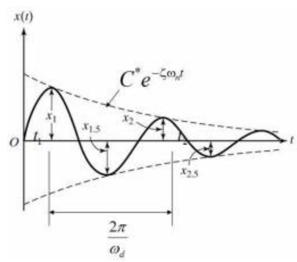
Al (Cantilever)	$1st\ f_n$	$2nd\ f_n$	$3rd\ f_n$	$4th\ f_n$
Experimental	5.7	38.359	107.891	212.969
Theory	6.2803	39.3568	110.2581	215.9095
Simulation	6.3007	39.481	110.54	216.6

Steel (Cantilever)	$1st\ f_n$	$2nd\ f_n$	$3rd\ f_n$	$4th\ f_n$
Experimental	41.719	115.469	225.781	366.484
Theory	43.2000	119.0273	233.7523	386.1726
Simulation	43.457	119.77	234.75	387.99

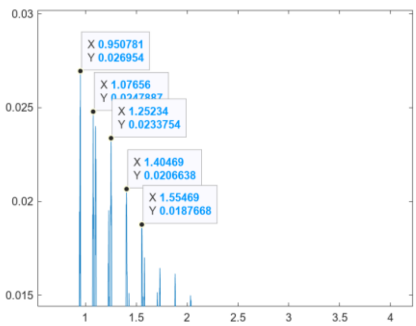
④ **Steel with Damping ratio test**







$$\omega_d = \omega_n\sqrt{1-\varsigma^2}, \qquad \delta = \frac{1}{n}\ln\frac{x_k}{x_{k+n}}, \qquad \varsigma = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}}$$



The ratio of the amplitude for 5 cycles (δ)	Damping ratio (ζ)
0.0724	0.0115

iii. Discussion

① Compare and discuss the differences between experimental, theoretical, and simulation results for each material.

The experimental, theoretical, and simulation values yielded nearly identical results. The discrepancy observed between the experimental and theoretical values can be attributed to the fact that the experimental environment is not theoretically perfect.

② Influence of Beam Material and Support Conditions

i. Discuss the impact of beam material comparison.

The natural frequencies varied based on the elastic modulus and density of the materials. The natural frequency is proportional to the square root of the elastic modulus and inversely proportional to the square root of the density. Therefore, materials like steel and aluminum, which have similar ratios of elastic modulus to density, exhibit similar natural frequencies. On the other hand, materials like brass, with a relatively smaller ratio of elastic modulus to density, have smaller natural frequencies.

ii. Compare and discuss the effect of support conditions on a steel rod.

As the support becomes more rigidly fixed, the value of β increases. Since the natural frequency is proportional to the square of β, the natural frequency increases with the strengthening of the support conditions. Therefore, even for the same material, the natural frequency becomes larger as the support conditions are enhanced.

③ Discuss the differences in the FFT results between the vibrations measured when the end of the beam was pressed by hand and suddenly released, and when the beam was lightly struck with a hammer.

When using a hammer, a forced impact is provided, inducing an immediate response from the system or structure. Additionally, the signals generated by the impact represent rapidly changing dynamic responses, facilitating the measurement and analysis of quick responses from structures or equipment. Moreover, signals generated through impact cover a broad frequency range, making it convenient to test and analyze responses within specific frequency bands. Lastly, in complex mode shape measurements of structures or mechanical systems, impact tests help rapidly activate and measure the mode shapes.

In contrast, when pressing with the hand, static force is applied, measuring the static response of the structure or system. This is useful for evaluating the static characteristics of structures or equipment. Furthermore, the force applied by hand provides relatively low energy compared to an impact, inducing small changes in the system or structure and making it useful for measuring sensitive responses. Additionally, the hand-applied force is applied smoothly compared to an impact, contributing to the gradual variation of mode shapes in the system or structure. Lastly, applying force by hand allows the measurement of the effects of static force on structures or equipment, aiding in understanding properties such as stiffness or deformation characteristics of structures.

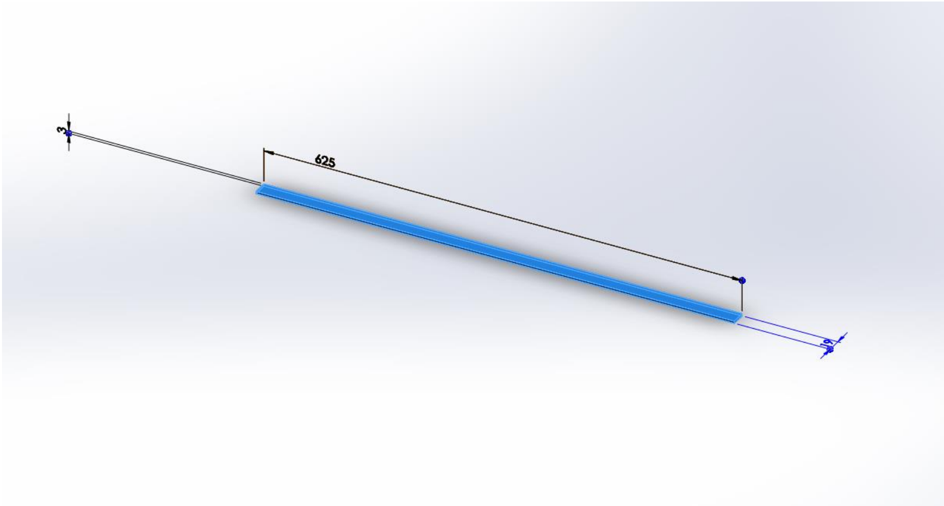
④ **Describe the impact of the attachment position of the accelerometer on the experimental results, and discuss precautions to be taken regarding the attachment location of the accelerometer during experiments.**

The placement of the accelerometer can impact the resonant frequency, resonant characteristics of the structure, and the captured mode shapes, depending on the attachment position. Additionally, the attachment position can influence the amplitude and magnitude of the measured vibrations, with certain positions providing higher or lower amplitudes based on the structure's response.

Key considerations for accelerometer attachment include ensuring secure and consistent attachment for safety and reliability. Insecure or inconsistent attachment can introduce noise and inaccuracies into the data. Using multiple accelerometers at various locations allows for a comprehensive understanding of the structure's response, especially in complex structures with multiple modes. Regular calibration of accelerometers is essential for maintaining accuracy, particularly when changing attachment positions.

iv. Appendix

Geometry Design (SolidWorks)



Matlab Code

```
clear; clc;

I = 0.019*0.003^3/12;
A = 0.019*0.003;
E_s = 200E09;
E_b = 115E09;
E_a = 71E09;
d_s = 7850;
d_b = 8450;
d_a = 2770;

b1 = [3, 7.51, 12.57, 17.59];
b2 = [7.88, 13.08,  18.33,  23.56]

for i = 1:4
    f_s(i) = sqrt((E_s*I*b1(i)^4)/(d_s*A))/(2*pi);
end

f_s

for i = 1:4
    f_b(i) = sqrt((E_b*I*b1(i)^4)/(d_b*A))/(2*pi);
end

f_b

for i = 1:4
    f_a(i) = sqrt((E_a*I*b1(i)^4)/(d_a*A))/(2*pi);
end

f_a

for i = 1:4
    f_fix(i) = sqrt((E_s*I*b2(i)^4)/(d_s*A))/(2*pi);
end

f_fix
```

```
b2 = 1×4
      7.8800   13.0800   18.3300   23.5600

f_s = 1×4
      6.2614   39.2384  109.9263  215.2596

f_b = 1×4
      4.5763   28.6782   80.3419  157.3269

f_a = 1×4
      6.2803   39.3568  110.2581  215.9095

f_fix = 1×4
     43.2000  119.0273  233.7523  386.1726
```

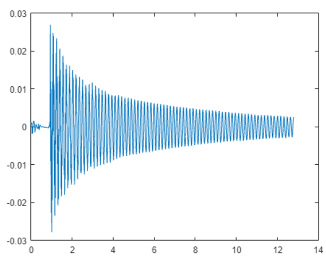
```
clc; clear all; close all;

% Excel 파일 경로 설정
% excelFilePath = 'steel_cantilever_fft.xls
% excelFilePath = 'brass_cantilever_fft.xls
% excelFilePath = 'aluminium_cantilever_fft
excelFilePath = 'steel_damping_ratio_time.x

% 데이터 읽기
data = xlsread(excelFilePath);
% data2 = xlsread(excelFilePath2);
% data3 = xlsread(excelFilePath3);
% data4 = xlsread(excelFilePath4);

% num에는 숫자 데이터, txt에는 텍스트 데이
freq = data(:, 1);
amplitude = data(:, 2);
% freq1 = data(:, 1);
% amplitude1 = data(:, 2);
% freq2 = data(:, 1);
% amplitude2 = data(:, 2);
% freq3 = data(:, 1);
% amplitude3 = data(:, 2);
% freq4 = data(:, 1);
% amplitude4 = data(:, 2);

plot(freq, amplitude);
hold on;
% plot(freq2, amplitude2);
% hold on;
% plot(freq3, amplitude3);
% hold on;
% plot(freq4, amplitude);
% hold on;
```



v. References

(1) Professor CS.Lee's CAE textbook (2023) from HGU Mechanical Control Engineering