CAE
LAB #2

Combined Stress

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I. PreLAB

① **For the system represented by the equation below, derive the dynamic equations, and obtain the response of the system's displacement, $x(t)$, to an external force, $F(t) = F_0 sin(\omega t)$, assuming that this system is underdamped. Explain the resonance phenomenon.**

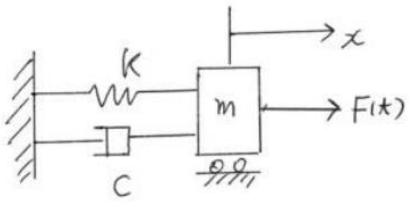

Figure 1. Harmonious Damped System

$$\Sigma F = m\ddot{x} = -kx - c\dot{x} + F(t)$$

$$m\ddot{x} + kx + c\dot{x} = F(t)$$

To understand the characteristic equation of the given system, we make the following assumptions:

$$F(t) = 0, \qquad x(t) = X_0 e^{st}$$

$$(ms^2 + cs + k)X_0 e^{st} = 0$$

$$s_1, s_2 = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

$$\zeta = \frac{c}{2\sqrt{mk}}, \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s_1, s_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

Given that the system is described as an underdamped system in the problem statement, it implies that $\zeta < 1$. Therefore,

$$s_1, s_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}, \quad \omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$x(t) = c_1 e^{(-\zeta\omega_n + j\omega_d)t} + c_2 e^{(-\zeta\omega_n - j\omega_d)t}$$

$$= e^{-\zeta\omega_n t}(X_1 cos\omega_d t + X_2 sin\omega_d t)$$

$$\dot{x}(t) = -\zeta\omega_n e^{-\zeta\omega_n t}(X_1 \cos\omega_d t + X_2 sin\omega_d t) + e^{-\zeta\omega_n t}\omega_d(-X_1 sin\omega_d t + X_2 cos\omega_d t)$$

$$\dot{x}(0) = \dot{x_0} = -\zeta\omega_n X_1 + \omega_d X_2$$

$$\omega_d X_2 = \dot{x}_0 + \zeta\omega_n X_0$$

$$X_2 = \frac{\dot{x_0} + \zeta\omega_n x_0}{\omega_d}$$

$$x(t) = e^{-\zeta\omega_n t}(\, x_0 cos\omega_d t + \frac{\dot{x_0} + \zeta\omega_n x_0}{\omega_d} sin\omega_d t\,)$$

When an object experiences an impact,

$$x_0 = 0, \qquad \dot{x_0} \neq 0$$

$$\therefore x(t) = \frac{\dot{x_0}}{\omega_d} \times e^{-\zeta\omega_n t} sin\omega_d t$$

$$x(t) = Xsin(\omega_d t - \emptyset)$$

Having derived the characteristic equation, we can now determine the displacement x(t) in response to an external force $F(t) = F_0 \sin\omega t$, where F_0 is the amplitude and ω is the angular frequency.

$$\ddot{x}(t) = -\omega_d^2\, Xsin(\omega_d t - \emptyset)$$

$$\dot{x}(t) = \omega_d\, Xcos(\omega_d t - \emptyset)$$

$$m\big(-\omega_d^2\, Xsin(\omega_d t - \emptyset)\big) + k\big(Xsin(\omega_d t - \emptyset)\big) + c\big(\omega_d\, Xcos(\omega_d t - \emptyset)\big) = F_0 \sin(\omega t)$$

$$X[(-\mathrm{m}\omega_{\mathrm{d}}^2 + \mathrm{k}) \sin(\omega_d t - \emptyset) + c\omega_d cos(\omega_d t - \emptyset) = F_0 \sin(\omega t)$$

$$(-\mathrm{m}\omega_{\mathrm{d}}^2 + \mathrm{k})\, \mathrm{X}\{\sin(\omega_d t) cos\emptyset - cos(\omega_d t) sin\emptyset\} + c\omega_d X\{cos(\omega_d t) cos\emptyset + \sin(\omega_d t)\, sin\emptyset\} = F_0 \sin(\omega t)$$

$$F_0 = X[\big((-m\omega^2 + k)cos\emptyset + c\omega \sin\emptyset\big],$$

$$X[(m\omega^2 - k)sin\emptyset + c\omega cos\emptyset] = 0$$

$$X = \frac{F_0}{(k - m\omega^2)cos\emptyset + c\omega sin\emptyset}$$

$$= \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\emptyset = \tan^{-1}\frac{c\omega}{k - m\omega^2}$$

$$\therefore x(t) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} \sin{(\omega t - \emptyset)}$$

II. Main LAB

i. Introduction

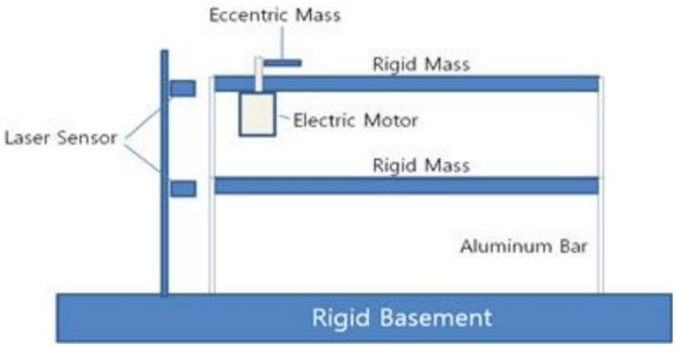

Figure 2. Experment Setting

In the vibration experiment apparatus shown in Fig. 1, the aluminum plate forming the side of the structure is thin, making it susceptible to bending deformation in the lateral direction. A motor is installed on the second floor of the structure, with an unbalanced mass attached to the motor shaft. As the motor rotates, centrifugal force is generated, causing the structure to oscillate in the direction of lower stiffness, namely the lateral direction. Acceleration sensors are attached to the first and second floors of the structure, or laser displacement gauges are installed on the side of the structure to measure the lateral accelerations of the structure. The rotation speed of the motor is gradually increased, and when it reaches the first natural frequency of the structure, first-mode resonance is observed. Further increasing the rotation speed, the vibration decreases until reaching the second natural frequency, at which point second-mode resonance is observed.

By observing the signal from the sensors in the time domain using a spectrum analyzer, the oscillation becomes pronounced. When the phases of the first and second floor sensors are the same, it corresponds to the first natural frequency mode, and when the phases are opposite, it corresponds to the second natural frequency mode. Analyzing the data from the spectrum analyzer transmitted to a PC allows for understanding the first and second natural frequencies and mode shapes in both the time and frequency domains.

ii. Results

① Experimental Results.

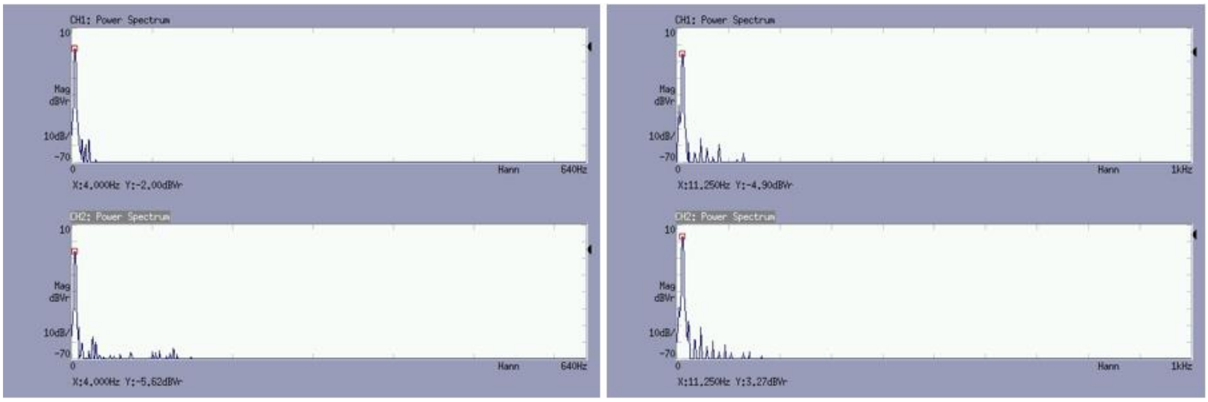

Figure 3. Freqency domain results

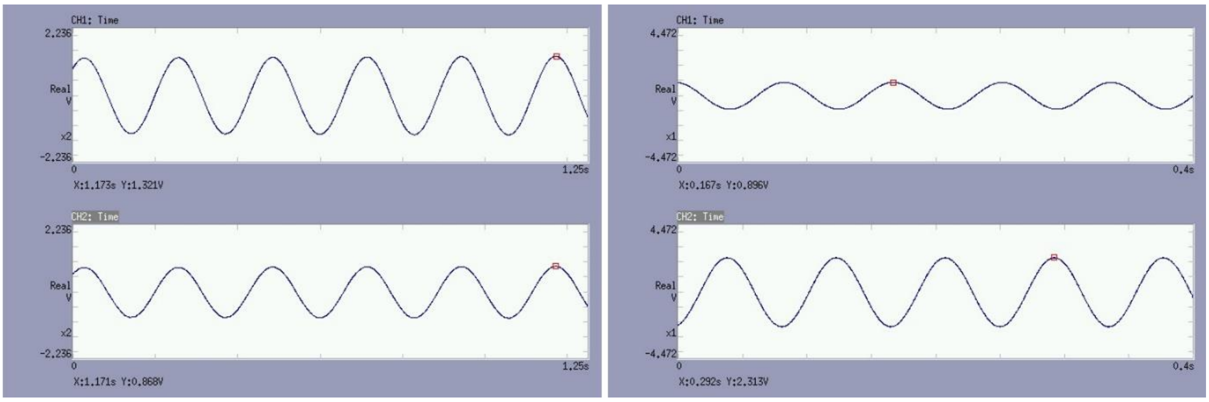

Figure 4. Time domain results

$$First\ f_n = 4\ [Hz]$$

$$Second\ f_n = 11.25\ [Hz]$$

$$First\ Mode\ Shape = \frac{1.321}{0.868} = 1.52$$

$$Second\ Mode\ Shape = -\frac{0.896}{2.313} = 0.39$$

② **Theory Results**

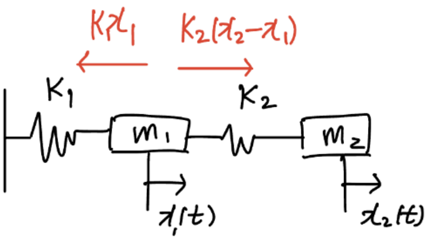

Figure 5. FBD

Equation of motion:

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = 0$$

$$m_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0$$

$$x(t) = Xcos(\omega t + \emptyset)$$

$$\ddot{x} = -\omega^2 Xcos(\omega t + \emptyset)$$

For harmonic motion $x_i(t) = X_i \cos(\omega t + \emptyset)\,;\ i = 1,2$

$$\begin{bmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 \\ -k_2 & -\omega^2 m_2 + k_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Frequency equation is:

$$\omega^2 = \frac{m_2 k_1 + m_2 k_2 + m_1 k_2 \pm \sqrt{(m_2 k_1 + m_2 k_2 + m_1 k_2)^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2}$$

$$I = \frac{bh^3}{12} = \frac{0.05 \times 0.0015^3}{12}$$

$$l = 0.205m,\ \ E = 70Gpa$$

Due to a steel plate between the first and second floors restricting the moment, the spring constant is as follows:

$$k' = \frac{12EI}{l^3}, \qquad k_1 = k_2 = 2 \times k = \frac{24EI}{l^3}$$

$$m_1 = 1.12\ kg,\ \ m_2 = 1.37\ kg$$

$$\omega_1 = 28.358\ [rad/s],\ \ \omega_2 = 78.067\ \ [rad/s]$$

First and second natural frequency is:

$$1st\ mode\ f_n = 4.5133\ [Hz], \qquad 2nd\ mode\ f_n = 12.4247\ [Hz]$$

Mode shapes,

$$r_1 = \frac{(-\omega_1^2 m_1 + k_1 + k_2)}{k_2} = 1.6716$$

$$r_2 = \frac{(-\omega_2^2 m_1 + k_1 + k_2)}{k_2} = -0.4891$$

$$\overrightarrow{x^{(1)}} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(2)} \end{pmatrix} = \begin{pmatrix} 1.0 \\ 1.6716 \end{pmatrix} x_1^{(1)}$$

$$\overrightarrow{x^{(2)}} = \begin{pmatrix} x_1^{(1)} \\ x_2^{(2)} \end{pmatrix} = \begin{pmatrix} 1.0 \\ -0.4891 \end{pmatrix} x_1^{(2)}$$

$$\therefore 1st\ mode\ shape = 1.6716,$$

$$2nd\ mode\ shpae = -0.4891$$

③ **Simulation Results**

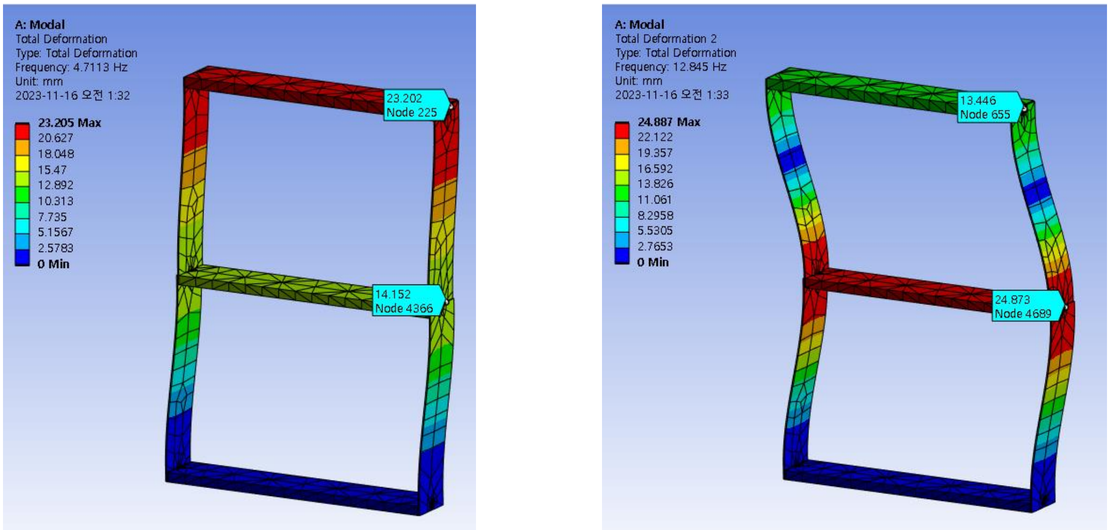

Figure 6. Simulation Results

	Mode	Frequency [Hz]
1	1.	4.7113
2	2.	12.845

Figure 7. Natural frequency results

$$First\ f_n = 4.711$$

$$Second\ f_n = 12.845$$

$$1st\ mode\ shape = \frac{23.202}{14.152} = 1.638$$

$$2nd\ mode\ shape = -\frac{13.446}{24.873} = -0.54$$

iii. Discussion

① Create the table and discuss the differences between experimental, theoretical, and simulation results.

Table 1. Experimental, Theory, Simulation's f_n and mode shapes

	$First\ f_n\ [Hz]$	$Second\ f_n\ [Hz]$	$1_{st}\ mode\ shape$	$2_{nd}\ mode\ shape$
Experiment	4	11.25	1.522	-0.39
Theory	4.51	12.42	1.672	-0.49
Simulation	4.71	12.845	1.639	-0.54

The reason for the inevitable discrepancies, although the results in experimentation, theory, and simulation exhibited similarities, lies in the mismatch between the experimental environment and the ideal theoretical conditions. Additionally, the joining configuration of the multilayered structure used in the experiment fell short of meeting the ideal experimental conditions, leading to discrepancies with the theoretical and simulation results.

② Discussing the vibrational characteristics of a multi-degree-of-freedom structure in comparison to a single-degree-of-freedom structure.

- **Natural Frequencies:**

Single DOF: Has a single natural frequency determined by its mass, stiffness, and damping characteristics.

Multi DOF: Exhibits multiple natural frequencies, each associated with a specific mode of vibration. The natural frequencies are influenced by the structure's mode shapes and modal masses.

- **Mode Shapes:**

Single DOF: One mode shape defines the amplitude distribution of displacement throughout the structure during vibration.

Multi DOF: Multiple mode shapes exist, each representing a unique pattern of deformation associated with a specific natural frequency. These mode shapes illustrate the relative motion between different parts of the structure.

- **Response to Excitation:**

Single DOF: Reacts to external forces with a straightforward, single-mode response.

Multi DOF: Responds with a combination of modes, and the interaction between modes becomes significant. The structure can exhibit more complex dynamic behavior due to coupling effects between degrees of freedom..

- **Modal Damping:**

Single DOF: Typically represented by a single damping ratio affecting the entire system.

Multi DOF: Modal damping considers damping associated with each mode, allowing for a more detailed representation of the structure's dynamic behavior.

③ When a large cooling fan is installed on the rooftop of a building, causing resonance in the building, consider methods to prevent the resonance.

- **Use of Vibration Isolation Devices:**

Install vibration isolation devices between the building and the cooling fan to minimize the transmission of vibrations between the two structures.

- **Application of Vibration Damping Devices:**

Install vibration damping devices on the cooling fan or the building structure to absorb the energy of vibrations and dissipate it.

- **Use of Vibration Isolation Pads:**

Install vibration isolation pads between the building and the cooling fan to absorb vibrations and disrupt their transmission.

- **Adjustment of Fan Location:**

Modify the location of the cooling fan to minimize its interaction with the building's primary resonance frequencies.

- **Change in Fan Operating Frequencies:**

Adjust the operating frequencies of the cooling fan to avoid conflicts with the building's resonance frequencies.

- **Structural Reinforcement and Optimization:**

Strengthen and optimize the building structure to prevent resonance at specific frequencies.

- **Compliance with Noise and Vibration Standards:**

Adhere to relevant regulations and standards to manage and minimize noise and vibration levels.

iv. Appendix

① Geometry Design (DesignModeler)

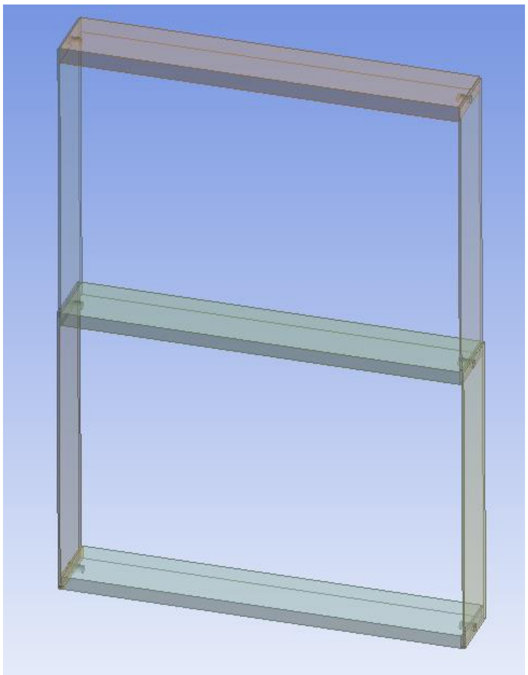

Figure 8. Geometry of multi-level building

② Material (AL-6061-T6)

Table 2. Material conditions

	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	2770	kg m^-3		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	2.3E-05	C^-1		
6	Isotropic Elasticity				
7	Derive from	Young's ...			
8	Young's Modulus	7.1E+10	Pa		
9	Poisson's Ratio	0.33			
10	Bulk Modulus	6.9608E+10	Pa		
11	Shear Modulus	2.6692E+10	Pa		
12	S-N Curve	Tabular			
13	Interpolation	Semi-Log			
14	Scale	1			
15	Offset	0	Pa		
16	Tensile Yield Strength	2.8E+08	Pa		
17	Compressive Yield Strength	2.8E+08	Pa		
18	Tensile Ultimate Strength	3.1E+08	Pa		
19	Compressive Ultimate Strength	0	Pa		

③ Experimental Condition

Table 3. Experimental conditions

Used Material	AL-6061-T6, Structural Steel
Mesh	default
Method	Modal Analysis
Solution	Total Deformation 1
	Total Deformation 2

④ Boundary Conditions

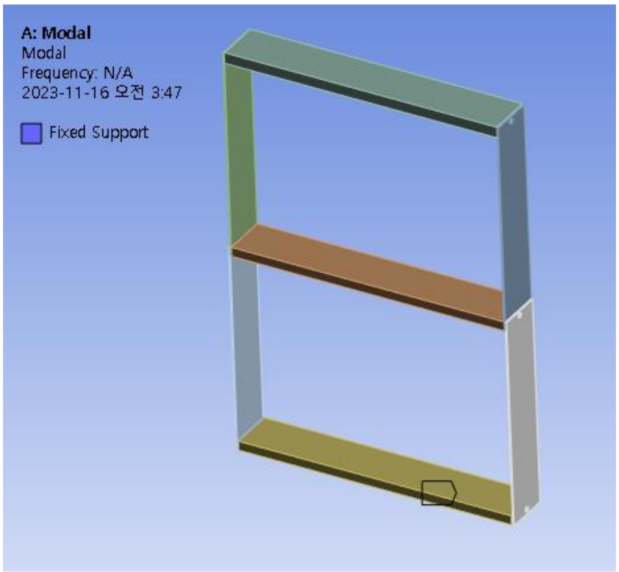

Figure 9. Fixed support at bottom face

⑤ Matlab Code

```
clc; clear; close all;

b = 0.05;
h = 0.0015;
I = b*h^3 / 12 ;
l = 0.205;
E = 70E09;


k1_ = 12*E*I/(l^3);
k2_ = 12*E*I/(l^3);

k1 = 2 * k1_;
k2 = 2 * k2_;

m1 = 1.12;
m2 = 1.37;

w1 = sqrt((m2*k1+m2*k2+m1*k2 - sqrt((m2*k1 + m2*k2 + m1*k2)^2 - 4*m1*m2*k1*k2)) / (2*m1*m2));
w2 = sqrt((m2*k1+m2*k2+m1*k2 + sqrt((m2*k1 + m2*k2 + m1*k2)^2 - 4*m1*m2*k1*k2)) / (2*m1*m2));

fn_1 = w1 / (2*pi)
fn_2 = w2 / (2*pi)

r1 = (-w1^2*m1 + k1 + k2) / k2
r2 = (-w2^2*m1 + k1 + k2) / k2
```

```
fn_1 = 4.5133
fn_2 = 12.4247

r1 = 1.6716
r2 = -0.4891
```

Figure 10. Code for calculating theory results

v. References

(1) R.C. Hibbeller, “Mechanics of Materials”, 8th Ed, Pearson, 2012

(2) Professor CS.Lee’s CAE textbook (2023) from HGU Mechanical Control Engineering