CAE
LAB #2

Combined Stress

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I. PreLAB

① What is Generalized Hookes Law and present it in a formula.

When the material at a point is subjected to a state of triaxial stress, $\sigma_x, \sigma_y, \sigma_z$, then these stresses can be related to the normal strains $\epsilon_x, \epsilon_y, \epsilon_z$ by using the principle of superposition, Poisson's ratio, $\epsilon_{lat} = -v\epsilon_{long}$ and Hooke's law as it applies in the uniaxial direction, $\epsilon = \sigma/E$.

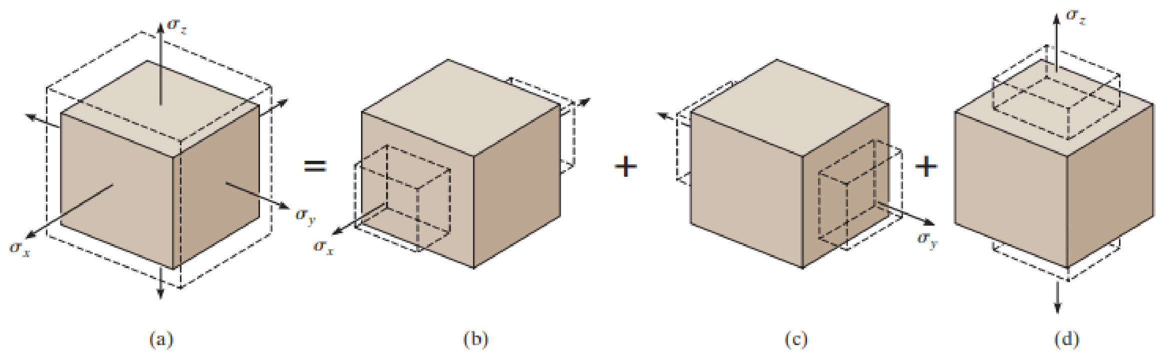

Figure 1. Bending stress in the Generalized Hooke's Law

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_x + \sigma_z\right)\right]$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$

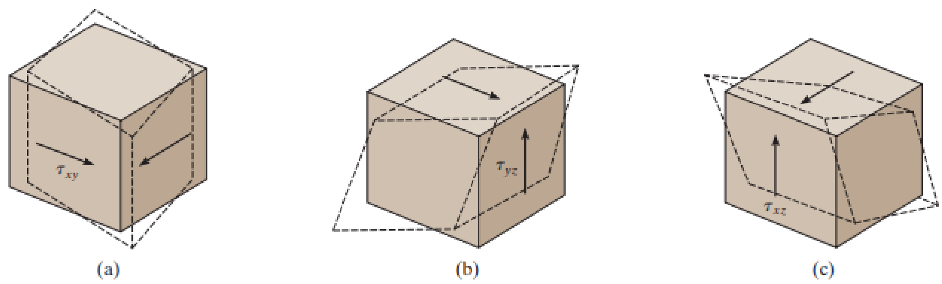

Figure 2. Shear stress in the Generalized Hooke's Law

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}, \quad \gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}$$

② **When a strain gauge rosette (a sensor with three strain gauges) is attached to the surface of an object to measure 2D strains $(\epsilon_x, \epsilon_y, \epsilon_z)$, it allows us to determine the 2D stress state of the surface $(\sigma_x, \sigma_y, \tau_{xy})$ Please provide the mathematical expressions for this process using a 45-degree rosette gauge.**

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \sin\theta\cos\theta$$

$$\begin{cases} for + 45°, & \varepsilon_a = \dfrac{\varepsilon_x}{2} + \dfrac{\varepsilon_y}{2} + \dfrac{\gamma_{xy}}{2} \\ for\ 0°, & \varepsilon_b = \varepsilon_x \\ for - 45°, & \varepsilon_c = \dfrac{\varepsilon_x}{2} + \dfrac{\varepsilon_y}{2} - \dfrac{\gamma_{xy}}{2} \end{cases}$$

$$\therefore \varepsilon_x = \varepsilon_b, \qquad \varepsilon_y = \varepsilon_a + \varepsilon_c - \varepsilon_b, \qquad \gamma_{xy} = \varepsilon_a - \varepsilon_c$$

$$\sigma_x = \frac{E(\varepsilon_x + v\varepsilon_y)}{(1 - v^2)}, \qquad \sigma_y = \frac{E(\varepsilon_y + v\varepsilon_x)}{(1 - v^2)}, \qquad \tau_{xy} = G\gamma_{xy} = \frac{E\gamma_{xy}}{2(1 + v)}$$

③ **Investigate how to find the maximum shear stress, maximum tensile stress (principal stresses), and Von-Mises stress once you know the 2D stress state of the surface (σx, σy, τxy).**

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{VM} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2}$$

II. Main LAB

i. Introduction

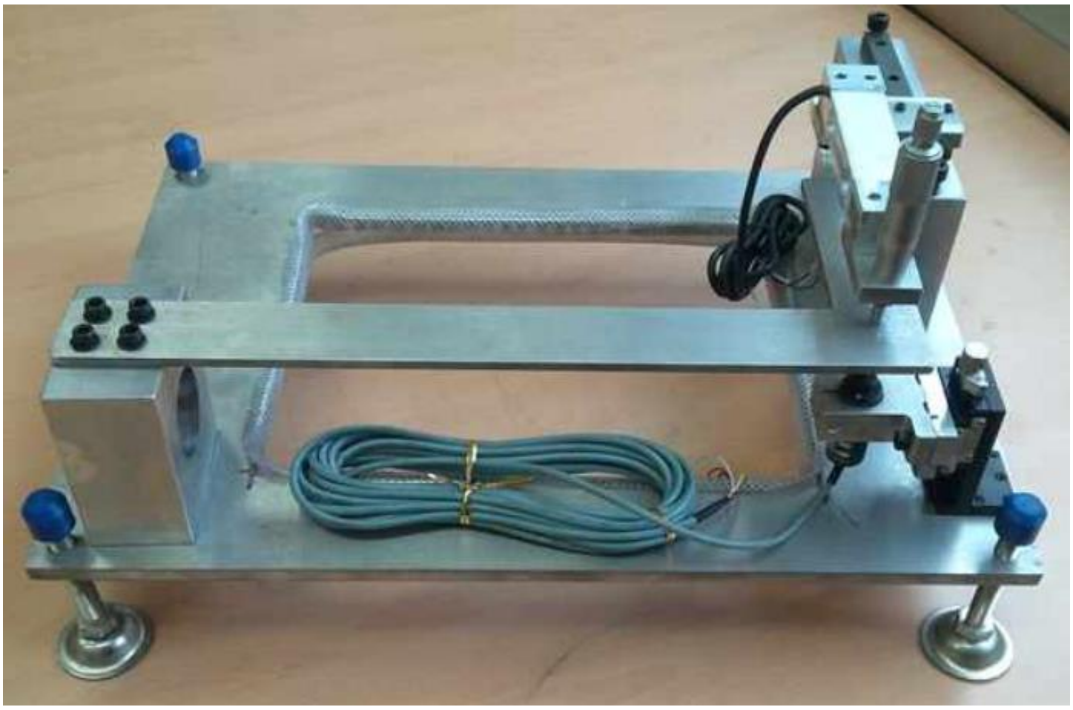

Figure 3. Composite Stress Testing Equipment

The composite stress testing equipment involves the use of a micrometer head to apply a vertical load to an L-shaped beam, with the aim of determining the stress at the strain gauge attachment point. The position of the micrometer head, which is mounted on LM guides, can be adjusted on the L-shaped beam. While this adjustment doesn't affect the bending stress, it does affect the torsional stress. Consequently, the composite stress varies depending on the micrometer's position.

Using NI-DAQ USB-6001 and LabVIEW coding, the voltage of the strain rosette is measured, and the strain state is determined by applying the strain rosette equation. From this information, the stress state is calculated, and through stress transformation, the principal stresses and maximum shear stress are determined.

ii. Experimental Results

① Checking the gain value

To verify the gain of the amplifier, press the calibration (CAL) button on each of the three strain gauge amplifiers in sequence and record the output voltage values in Table 2. For a gain of 1500, the value near 2.25V should be obtained.

	$Gauge\_a\ (+45°)$	$Gauge\_b\ (0°)$	$Gauge\_c\ (-45°)$
$e_{out-CAL}\ [V\}$	2.20	2.23	2.30

Table 1. Calibration Voltage of Three Gauges

② Elasticity Coefficient Estimation Experiment

To obtain the elastic modulus through regression analysis, record the displacement values for loads of 1, 2, and 3 kg, as well as the voltage values of the strain gauge b (0° gauge) in a table. Pay attention to the hysteresis of the displacement gauge.

Load [kg]	Displacement [mm]	$e_{out}\ [V]\ for\ Gauge\_b$
1	2.67	0.34
2	5.31	0.69
3	8.40	1.08

Table 2. δ (displacement) and Output voltage of Gauge_b

③ Stress Measurement Experiment

Record three experimental values of the strain rosette voltage when a 3 kg load is applied in a table.

Count	$e_{out}\ for\ Gauge_a\ [V]$	$e_{out}\ for\ Gauge_b\ [V]$	$e_{out}\ for\ Gauge_c\ [V]$
1	+0.82	+0.95	-0.25
2	+0.84	+0.93	-0.32
3	+0.81	+0.93	-0.19
Average	+0.823	+0.937	-0.253

Table 3. Output, Average Voltage of Three Gauges

iii. Discussion

① **Compare and analyze the elastic modulus obtained through two methods: the Deflection Method and the Strain Method.**

- Strain Method

$$e_{out} = \frac{\Delta R}{4R} \times E \times G_{amp}; \qquad R = 120\Omega, \qquad E = 5V, \qquad G_{amp} = 1500$$

$$\epsilon = \frac{\Delta R}{R} \times \frac{1}{GF}; \qquad GF = 2.13$$

$$\sigma = \frac{Mc}{I} = \frac{(m \times 9.81) \times 150 \times 3}{\frac{40 \times 6^3}{12}}$$

$$\sigma = E\epsilon$$

Load [kg]	e_{out} [V] $for\ Gauge\_b$	ΔR [Ω]	ϵ	σ [Mpa]	E [Gpa]
1	0.34	0.02176	8.513×10^{-5}	6.13	72
2	0.69	0.04416	1.728×10^{-4}	12.26	70.9
3	1.08	0.06912	2.704×10^{-4}	18.39	68

Table 4. Elasticity obtained through the Strain Method

The elasticity value obtained above, when analyzed using linear regression in Matlab, results in the following.

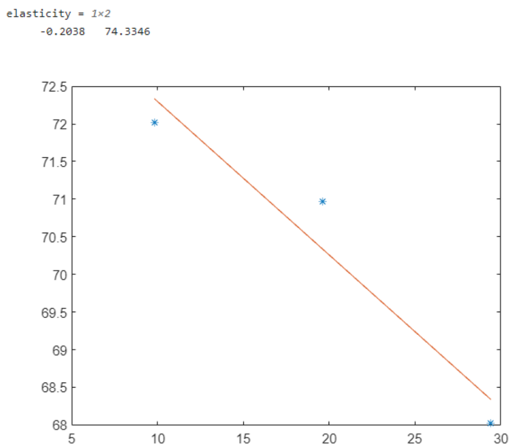

Figure 4. Linear regression graph (Strain Method)

E = 74.335 [Gpa]

- Deflection Method

$$\delta = \frac{PL^3}{3EI}, \qquad E = \frac{PL^3}{3\delta I} = \frac{(m \times 9.81) \times 335^3}{3 \times \delta \times \frac{40 \times 6^3}{12}}$$

Load [kg]	Displacement [mm]	E
1	2.67	63.95
2	5.31	64.31
3	8.40	60.98

Table 5. Elasticity obtained by Deflection Method

The elasticity value obtained above, when analyzed using linear regression in Matlab, results in the following.

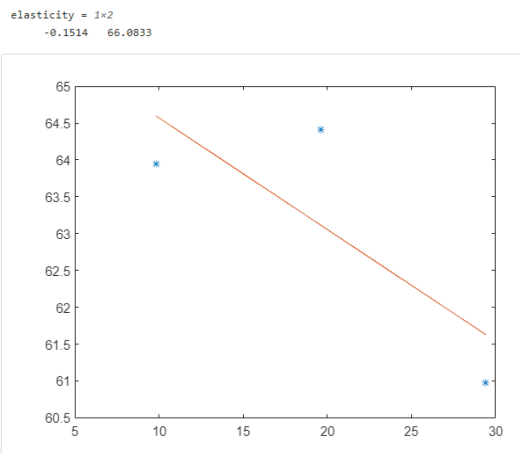

Figure 5. Linear regression graph (Deflection Method)

E = 66.083 [Gpa]

② **Compare and analyze the experimental and theoretical values of bending stress (σ_x) and shear stress (τ_{xy}).**

- First, we determined the change in resistance using the same method as in discussion 1 based on the output voltage obtained from the experiment. Then, we calculated the rate of change and used the generalized Hooke's law to obtain bending and shear stresses. The Young's modulus used in this calculation was the value obtained using the strain method in discussion 1 (74.335), and the Poisson's ratio (v) was assumed as 0.33 for AL-6061-T6.

	$Gauge_a$	$Gauge_b$	$Gauge_c$
e_{out}	0.823	0.937	-0.253
ΔR	0.053	0.059	-0.016
ϵ	2.0735E-4	2.308E-4	-6.259E-5

Table 6. The resistance change and strain for the three strain gauges

$$\varepsilon_x = \varepsilon_b = 2.308 \times 10^{-4}, \ \ \varepsilon_y = \varepsilon_a + \varepsilon_c - \varepsilon_b = -8.604 \times 10^{-5}$$

$$\sigma_x = \frac{E(\varepsilon_x + v\varepsilon_y)}{1 - v^2} = \mathbf{16.88\ [Mpa]}$$

$$\sigma_y = \frac{E(\varepsilon_y + v\varepsilon_x)}{1 - v^2} = -0.824\ [Mpa]$$

By Generalized Hooke's Law, experimental value of shear stress,

$$\gamma_{xy} = \varepsilon_a - \varepsilon_c = 2.6995 \times 10^{-4}$$

$$\tau_{xy} = G\gamma_{xy} = \frac{E\gamma_{xy}}{2(1 + v)} = \mathbf{7.54\ [Mpa]}$$

- Theoretical value of bending stress,

$$\sigma_x = \frac{Mc}{I} = \frac{(3 \times 9.81) \times 150 \times 3}{\frac{40 \times 6^3}{12}} = \mathbf{18.39\ [Mpa]}$$

Theoretical value of shear stress,

$$\tau_{max} = \frac{T}{cbh^2}, \ \ c = \frac{1}{3}\left(1 - \frac{0.63h}{b}\right)$$

$$\tau_{max} = \tau_{xy} = \frac{3 \times 9.81 \times 0.1}{\frac{1}{3} \times \left(1 - \frac{0.63 \times 0.006}{0.04}\right) \times 0.04 \times 0.006^2} = \mathbf{6.77\ [Mpa]}$$

The maximum shear stress acting on a square cross-section subjected to torsion occurs at the midpoint of the section. Therefore, in the experiment, strain gauges were attached to the center of the surface for measurement, and as a result, the calculated maximum value matches the theoretical value for the xy coordinates.

	$\sigma_x\ [Mpa]$	$\tau_{xy}\ [Mpa]$
Experimental Value	16.88	7.54
Theorical Value	18.39	6.77

Table 7. Comparison of experimental and theoretical bending and shear stress

③ **Compare and analyze the experimental, theoretical, and simulation values for principal stress ($\boldsymbol{\sigma_1}$), maximum shear stress ($\boldsymbol{\tau_{max}}$), and Von-Mises stress ($\boldsymbol{\sigma_{vm}}$)**

- In experimental,

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \mathbf{19.656}, -3.59\ [Mpa]$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \mathbf{11.628\ [Mpa]}$$

$$\sigma_{vm} = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = \mathbf{21.68\ [Mpa]}$$

- In theory,

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \mathbf{20.613}, -2.22\ [Mpa]$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \mathbf{11.89\ [Mpa]}$$

$$\sigma_{vm} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \mathbf{21.81\ [Mpa]}$$

- In simulation,

To observe the point at 150mm, we positioned the probe at 130mm, taking into account the coordinate system's origin, and verified the results.

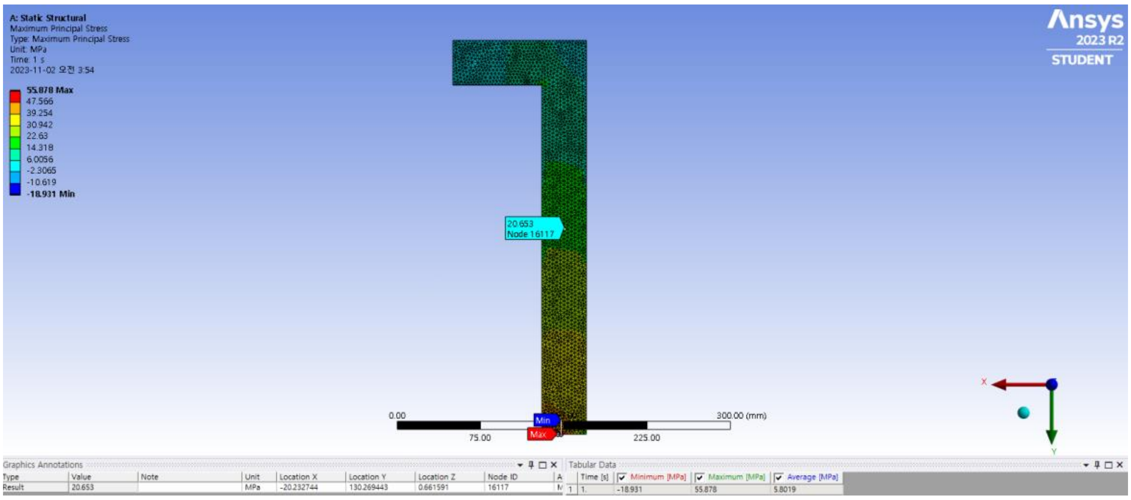

Figure 6. Maximum principal stress in ANSYS simulation

$$\sigma_1 = 20.653\ \ [Mpa]$$

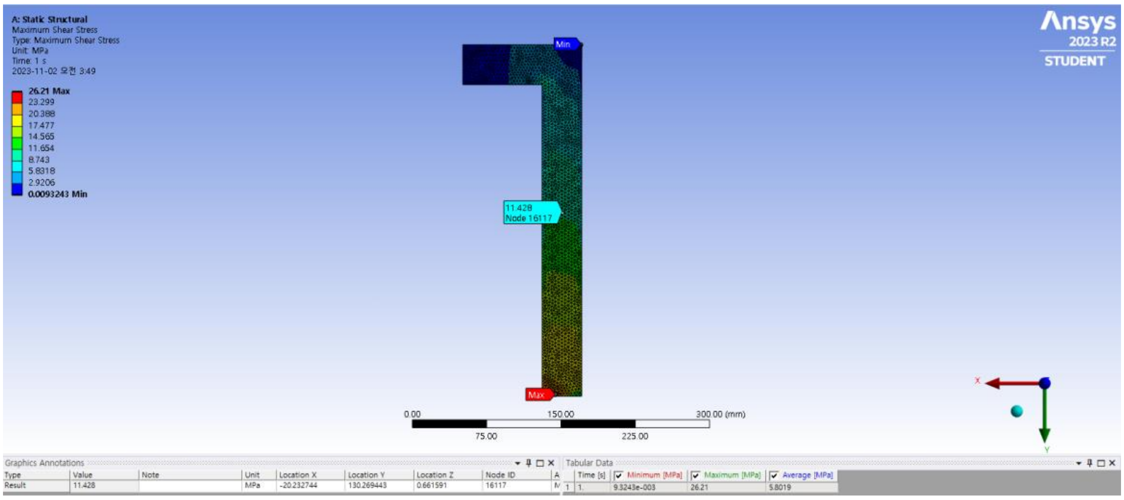

Figure 7. Maximum shear stress in ANSYS simulation

$$\tau_{max} = 11.428\ [Mpa]$$

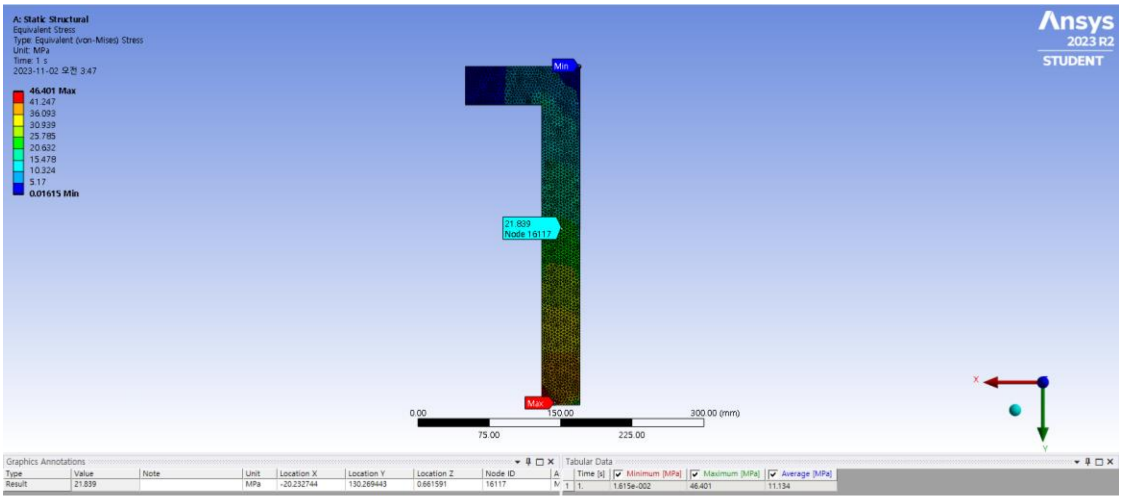

Figure 8. Von-mises stress in ANSYS simulation

$$\sigma_{vm} = 21.839\ [Mpa]$$

	$\boldsymbol{\sigma_1}$	$\boldsymbol{\tau_{max}}$	$\boldsymbol{\sigma_{vm}}$
Experiment	19.656	11.628	21.68
Theory	20.613	11.89	11.428
Simulation	20.653	11.428	21.839

④ **To ensure a safety factor of 3 and continue with this experiment next year, what is the maximum load that can be applied to the L-shaped beam?**

$$SF = 3 = \frac{\sigma_y}{\sigma_{vm\_max}}$$

$$\sigma_{vm\_max} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} = \sqrt{\left(\frac{Mc}{I}\right)^2 + 3 \times \left(\frac{T}{cbh^2}\right)^2}$$

$$= \sqrt{\left(\frac{m \times 9.81 \times 0.335 \times 0.003}{\frac{0.04 \times 0.006^3}{12}}\right)^2 + 3 \times \left(\frac{m \times 9.81 \times 0.1}{\frac{1}{3} \times \left(1 - \frac{0.63 \times 0.006}{0.04}\right) \times 0.04 \times 0.006^2}\right)^2}$$

$$= \sqrt{1.875 \times 10^{14} \times m^2 + 1.528 \times 10^{13} \times m^2} = \sqrt{2.0278 \times 10^{14}} \times m$$

$$m = \frac{280 \times 10^6}{3 \times \sqrt{2.0278 \times 10^{14}}} = 6.554\ [kg]$$

⑤ **Please discuss any observations and lessons learned from the experiment, in addition to the points mentioned above.**

✓ The area where stress is concentrated when a composite force is applied to the L-beam.

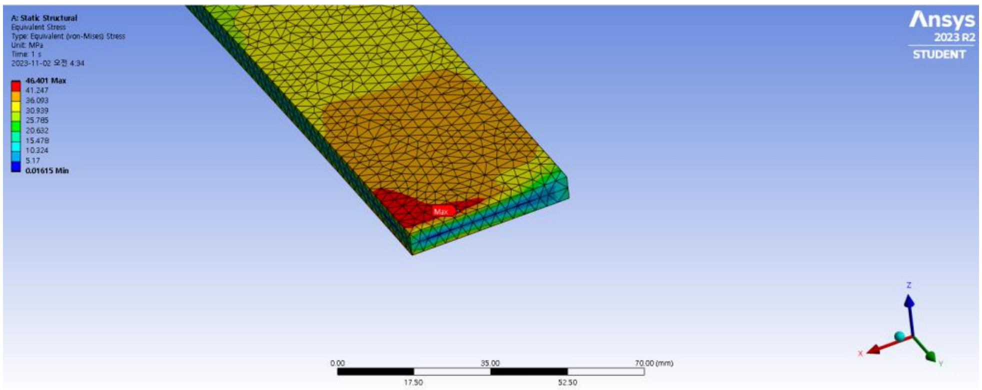

Figure 9. Von-Misses Stress

When torque was applied to the L-beam of a cantilever, it was observed that stress is concentrated closer to the center, indicating that this is due to the presence of stress in the y-direction when complex stresses occur. This was demonstrated through simulation.

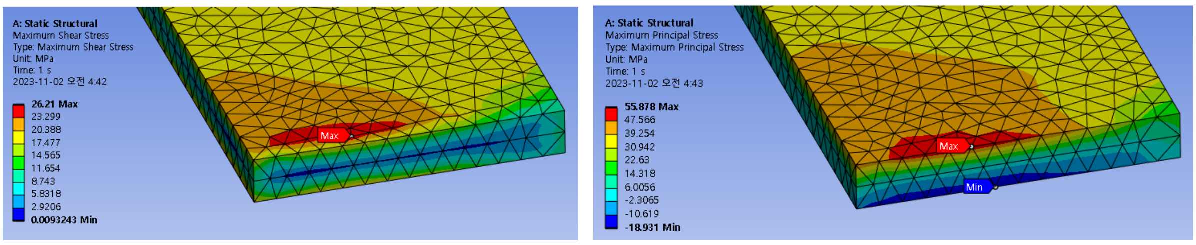

Figure 10. Maximum Shear and Principal Stress

This applies similarly to maximum shear stress and maximum principal stress. Through this, it can be inferred that the y-direction stress had an influence when calculating the three types of stress based on experimental data in discussion 3.

✓ Distribution of shear stress

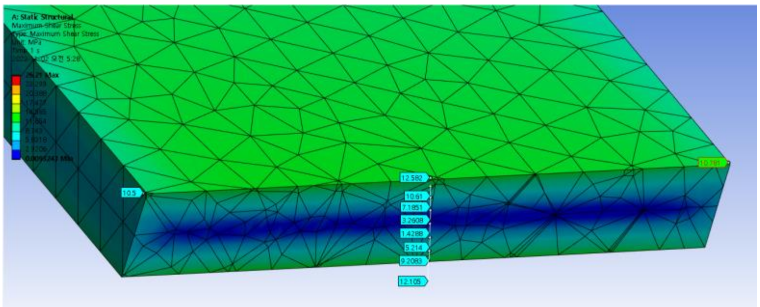

Figure 11. Shear stress distribution

When torque is applied to a square cross-section, it was confirmed through simulation that the central portion of the surface of the square cross-section has the highest shear stress, as indicated by the formula in discussion 2.

iv. Appendix

① Geometry Design (DesignModeler)

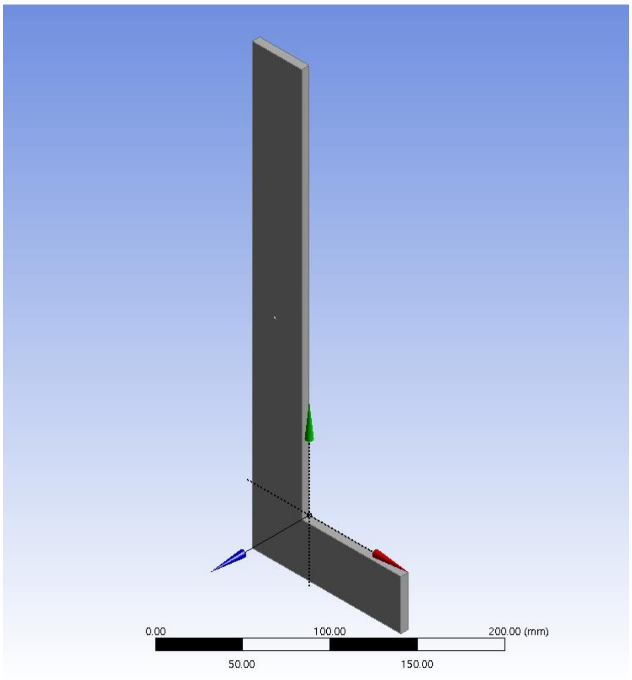

Figure 12. Geometry of L-Beam

② Material (AL-6061-T6)

	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	2770	kg m^-3		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	2.3E-05	C^-1		
6	Isotropic Elasticity				
7	Derive from	Young's ...			
8	Young's Modulus	7.1E+10	Pa		
9	Poisson's Ratio	0.33			
10	Bulk Modulus	6.9608E+10	Pa		
11	Shear Modulus	2.6692E+10	Pa		
12	S-N Curve	Tabular			
13	Interpolation	Semi-Log			
14	Scale	1			
15	Offset	0	Pa		
16	Tensile Yield Strength	2.8E+08	Pa		
17	Compressive Yield Strength	2.8E+08	Pa		
18	Tensile Ultimate Strength	3.1E+08	Pa		
19	Compressive Ultimate Strength	0	Pa		

Table 8. Material properties (AL-6061-T6)

③ Experimental Condition

Used Material	AL-6061-T6
Mesh	3mm
Force	-29.43N (3kg)
Solution	Total Deformation
	Equivalent Stress
	Directional Deformation
	Maximum Shear Stress
	Maximum Principal Stress

Table 9. Conditions

④ Boundary Conditions & Load Conditions

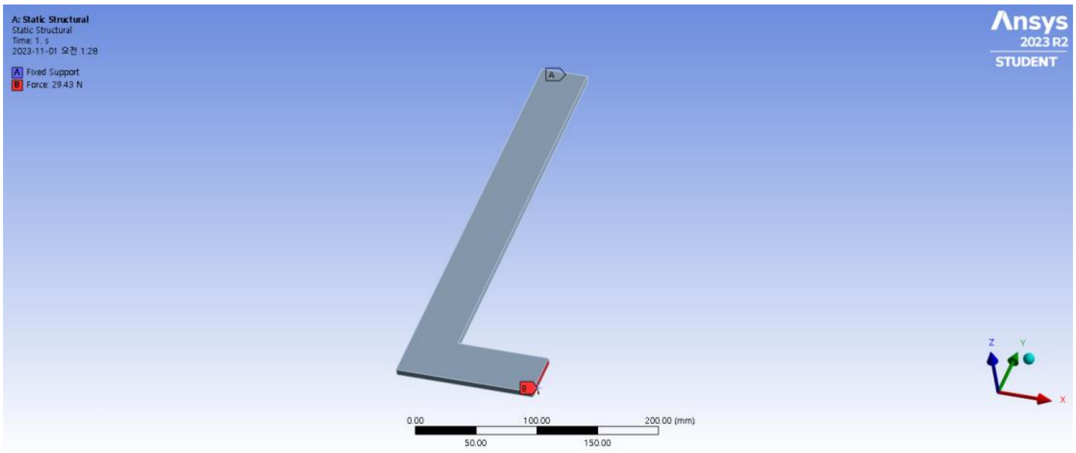

Table 10. BC & LC

⑤ Linear regression function using Matlab

```
clc; close all; clear;
% Strain Method
P = 9.81 * [1, 2, 3];
L = 150;
I = 40 * 6^3 / 12;
sigma = P*L*3/I * 10E6
epsilon = [8.513E-5, 1.728E-4, 2.704E-4];
E = sigma ./ epsilon / 10E9

plot(P, E, "*");

elasticity = polyfit(P,E,1)
curve_lin = elasticity(1).*P + elasticity(2);
hold on
plot(P, curve_lin)

% Deflection Method
P = 9.81 * [1, 2, 3];
E = [63.95, 64.41, 60.98];
plot(P, E, "*");
elasticity = polyfit(P,E,1)
curve_lin = elasticity(1).*P + elasticity(2);
hold on
plot(P, curve_lin)
```

Figure 13. Linear regression Code

v. References

(1) R.C. Hibbeller, “Mechanics of Materials”, 8th Ed, Pearson, 2012

(2) Professor CS.Lee’s CAE textbook (2023) from HGU Mechanical Control Engineering