TU: Linear Regression

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Introduction

Fit Linear Regression

Include path

```
addpath('../../Library')
```

Examples

Example 1: Fit with Linear Regression

Data Acquisition

- Feature: One-Dimension, p=1
- True value: ytrue = 2X+3

```
x = randn(100,1);
y = x*2 + 3+randn(100,1);
```

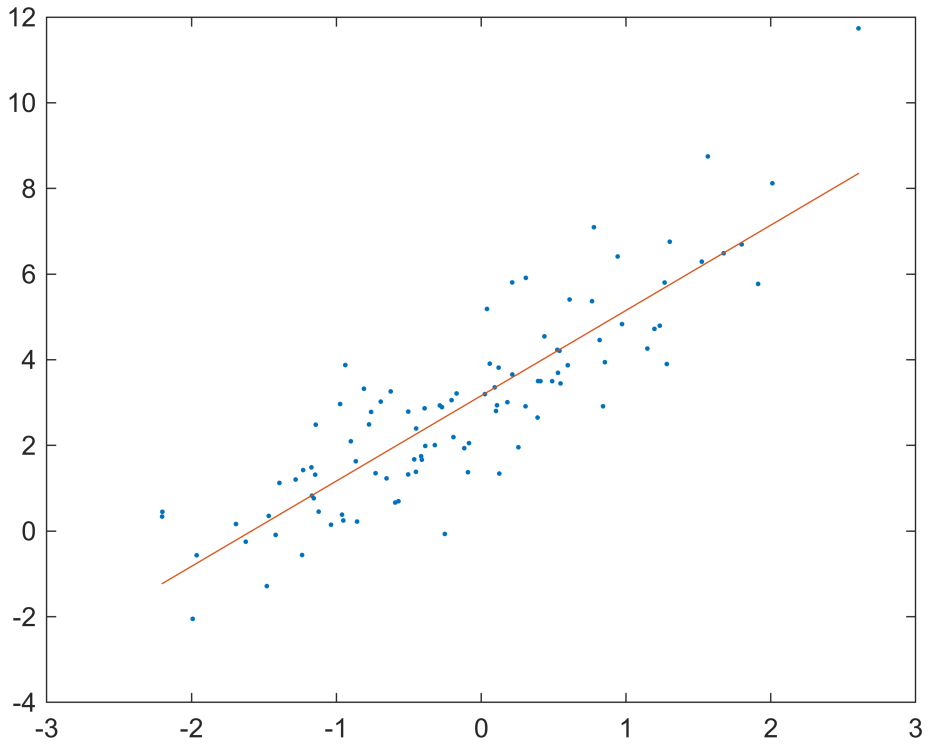
Fit Linear Regression

```
mdl = fitlm(x,y)
```

```
mdl =
선형 회귀 모델:
    y ~ 1 + x1

추정된 계수:
                   Estimate      SE       tStat       pValue
                   ________    _______    ______    __________

    (Intercept)     3.1607     0.11468    27.562     5.871e-48
    x1              1.9916     0.11442    17.406    9.6192e-32


관측값 개수: 100, 오차 자유도: 98
RMS 오차: 1.14
```

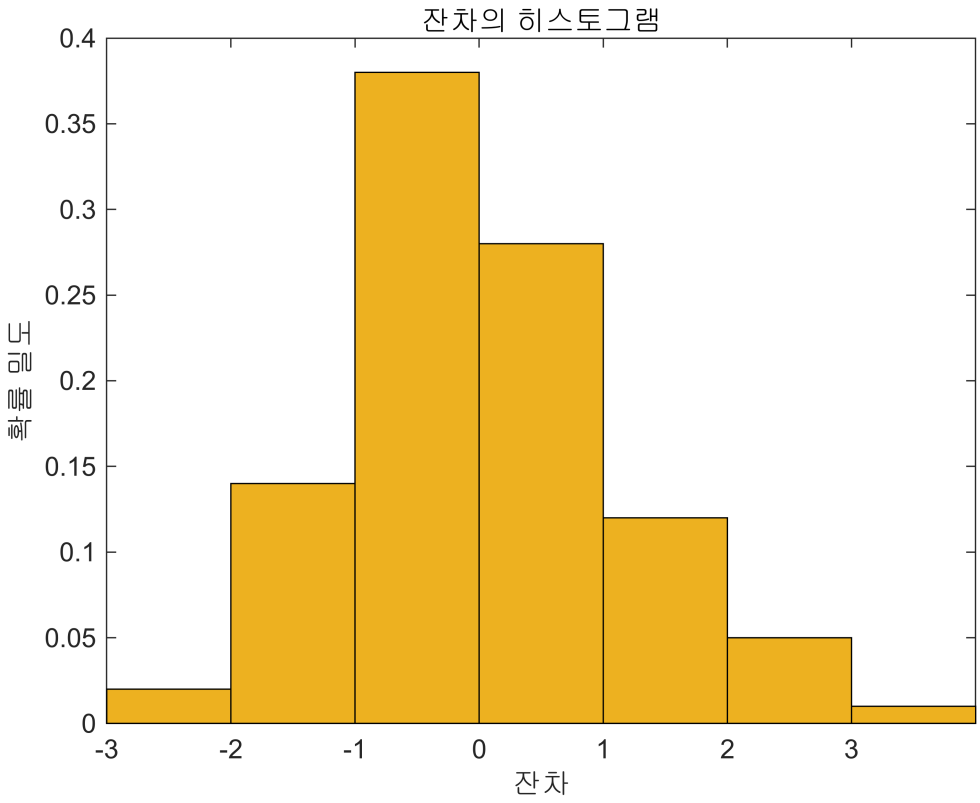
```
결정계수: 0.756, 수정된 결정계수: 0.753
상수 모델에 대한 F-통계량: 303, p-값 = 9.62e-32
```

```
% Plot prediction
plot(x,y,'.',x,mdl.Fitted, '-')
```



Analyze Linear Regression

```
% Plot residual histogram
plotResiduals(mdl)
```




```
% From remove outlier from histogram
outl = find(abs(mdl.Residuals.Raw) > 2);
mdl.Residuals.Raw(outl)
```

```
ans = 8×1
    2.2185
    3.3878
    2.4698
   -2.7272
    2.3842
    2.1376
    2.5840
   -2.0667
```

```
% Fit with outlier removed
mdl2 = fitlm(x,y,'Exclude',outl)
```

```
mdl2 =
선형 회귀 모델:
    y ~ 1 + x1

추정된 계수:
                   Estimate       SE       tStat       pValue
                   ________    ________    ______    __________

    (Intercept)     3.0119     0.094688    31.809    9.7326e-51
    x1              1.8087     0.096003     18.84    5.4685e-33
```

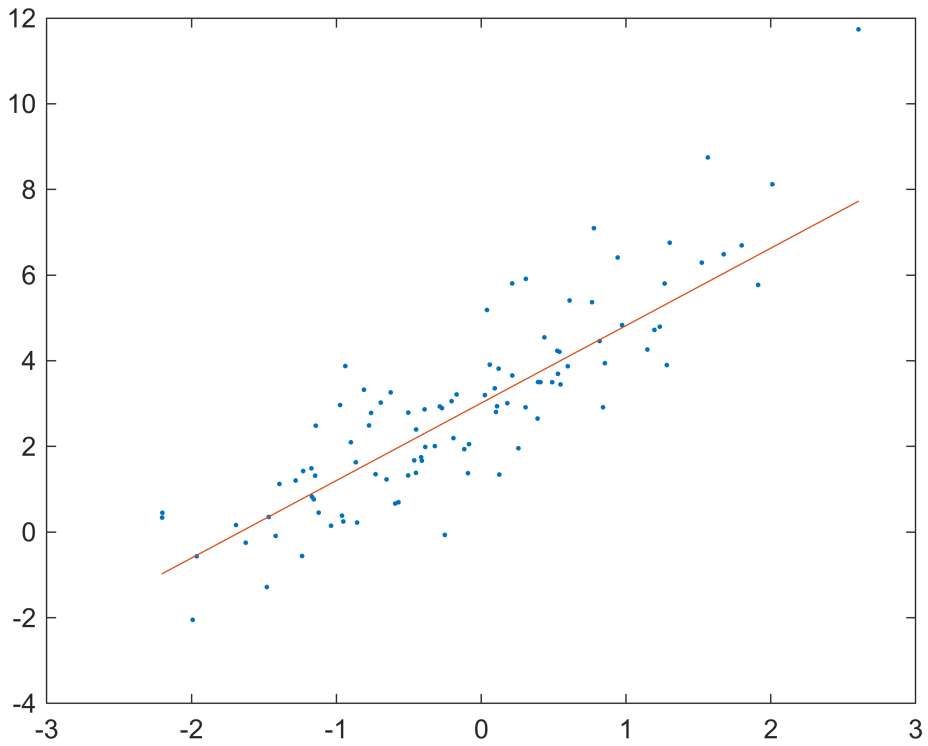
```
관측값 개수: 92, 오차 자유도: 90
RMS 오차: 0.89
결정계수: 0.798, 수정된 결정계수: 0.795
상수 모델에 대한 F-통계량: 355, p-값 = 5.47e-33
```

Predict with Test data

```
% Ypredict and confidence interval
Xnew=2;
[Ynew,YnewI]=predict(mdl2,Xnew)
```

```
Ynew = 6.6294
YnewI = 1×2
    6.1717    7.0870
```

```
% Plot prediction
plot(x,y,'.',x,mdl2.Fitted, '-')
```



Example 2: Predict Car MPG

Find linear relationship of MPG(연비) with Weight & Displacement

Then, Predict MPG for a car with Weight=3000 , Displacement=300

Data Acquisition

- Dependet Variable: MPG
- Independet Variables: Weight, Displacement

```
clear
load carsmall

tbl = table(MPG,Weight, Displacement);  % table type
```

Fit Linear Regression

```
mdl = fitlm(tbl,'MPG~Weight+Displacement')
```

```
mdl =
선형 회귀 모델:
    MPG ~ 1 + Weight + Displacement

추정된 계수:
                    Estimate        SE         tStat       pValue
                   _________    _________    _______    __________

    (Intercept)         46.925       2.0858     22.497    6.0509e-39
    Weight          -0.0068422    0.0011337    -6.0353    3.3838e-08
    Displacement     -0.014593    0.0082695    -1.7647      0.080968


관측값 개수: 94, 오차 자유도: 91
RMS 오차: 4.09
결정계수: 0.747, 수정된 결정계수: 0.741
상수 모델에 대한 F-통계량: 134, p-값 = 7.22e-28
```

```
mdl.CoefficientNames
```

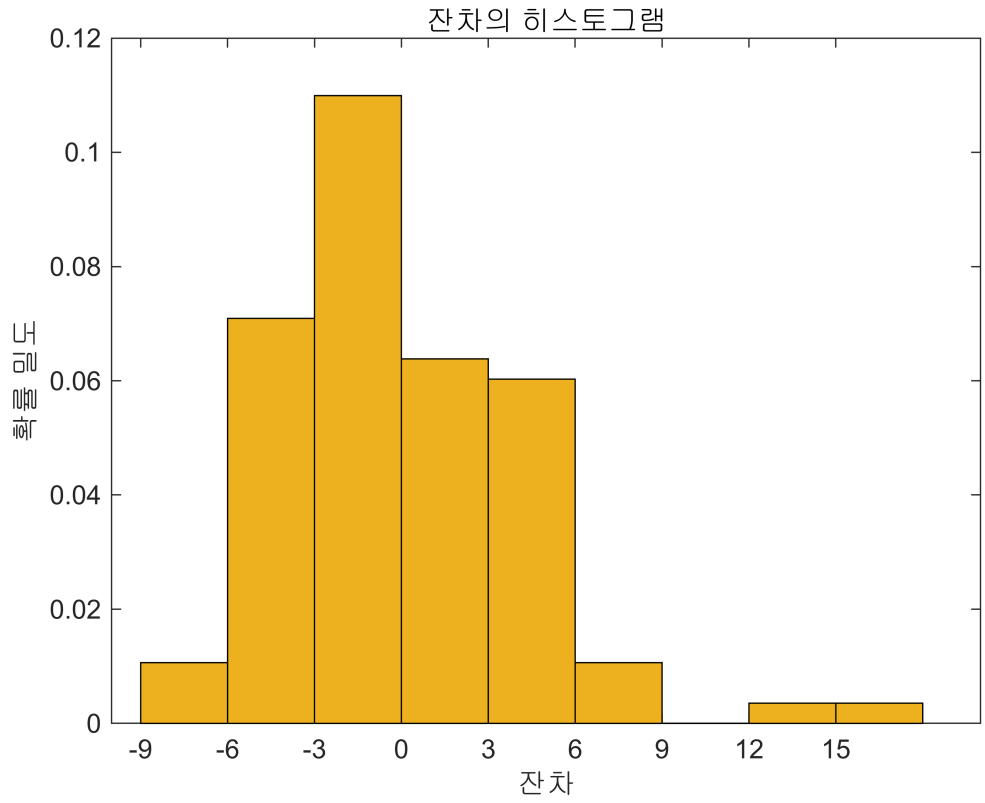
```
ans = 1×3 cell
'(Intercept)''Weight'     'Displacement'
```

```
mdl.Coefficients.Estimate
```

```
ans = 3×1
   46.9247
   -0.0068
   -0.0146
```

Analyze Linear Regression

```
plotResiduals(mdl)
```




```
% Remove outlier
outl = find((mdl.Residuals.Raw) > 9);

% Fit with outlier removed
mdl2 = fitlm(tbl,'MPG~Weight+Displacement','Exclude',outl)
```

```
mdl2 =
선형 회귀 모델:
    MPG ~ 1 + Weight + Displacement

추정된 계수:
                    Estimate         SE         tStat       pValue
                   __________    __________    _______    __________

    (Intercept)        45.548        1.8056     25.226    2.5559e-42
    Weight          -0.0062503    0.00097845    -6.3879    7.4767e-09
    Displacement     -0.018035     0.0071414    -2.5255      0.013324


관측값 개수: 92, 오차 자유도: 89
RMS 오차: 3.51
결정계수: 0.796, 수정된 결정계수: 0.792
상수 모델에 대한 F-통계량: 174, p-값 = 1.78e-31
```

```
mdl2.Coefficients.Estimate
```

```
ans = 3×1
   45.5477
   -0.0063
   -0.0180
```

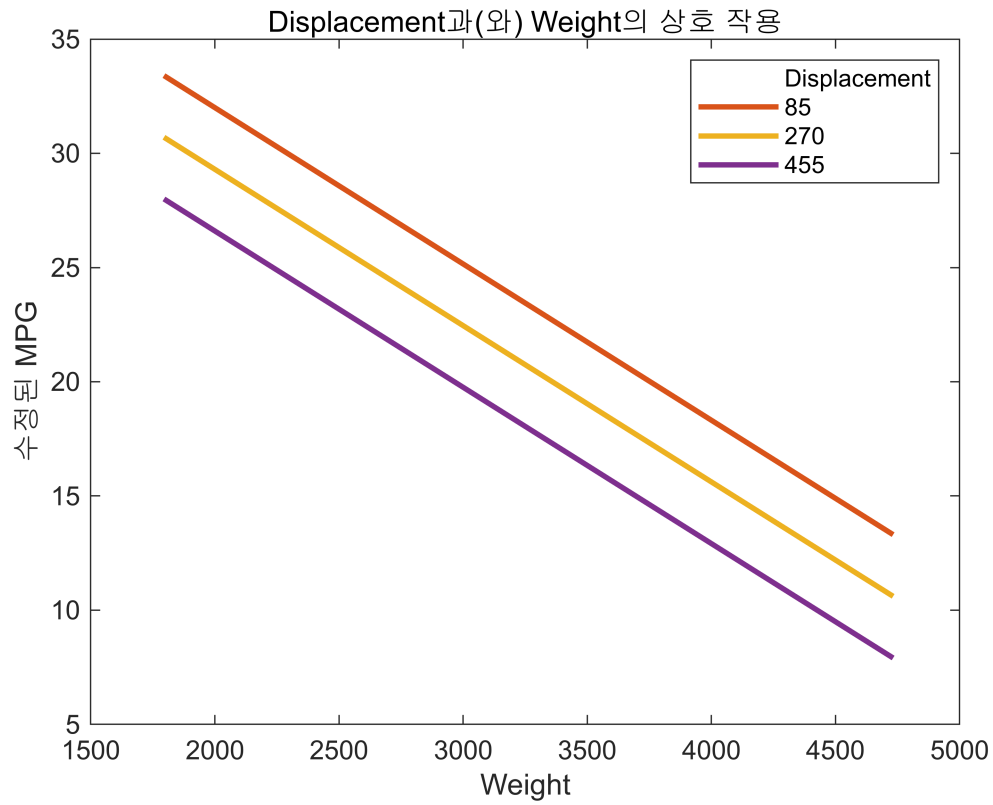
Predict

Detailed look at the interactions

```
% Ypredict and confidence interval
Xnew=[3000 300];
[Ynew,YnewI]=predict(mdl2,Xnew)
```

```
Ynew = 21.3863
YnewI = 1×2
   19.8515   22.9211
```

```
plotInteraction(mdl,'Displacement','Weight','predictions')
```



Exercise

Exercise 1 : Gradient Descent

Linear Regression Using Gradient Descent

Hypothesis.

$$h_\theta(x) = \theta_1 x + \theta_0$$

To find the parameters, repeat until convergence

$$\theta_k = \theta_k - \alpha \frac{\partial}{\partial \theta_k} J(\theta_1, \theta_0)$$

where, cost(error) function

$$J(\theta_1, \theta_0) = \frac{1}{2n} E = \frac{1}{2n} \sum_{i=1}^{n} \left(y_i - h(x_i) \right)^2$$

and

$$\frac{\partial J}{\partial \theta_1} = -\frac{1}{n} \sum_{i=1}^{n} x_i \left(y_i - (\theta_1 x_i + \theta_0) \right)$$

$$\frac{\partial J}{\partial \theta_0} = -\frac{1}{n} \sum_{i=1}^{n} \left(y_i - (\theta_1 x_i + \theta_0) \right)$$

Data Acquisition

Feature: One-Dimension, p=1

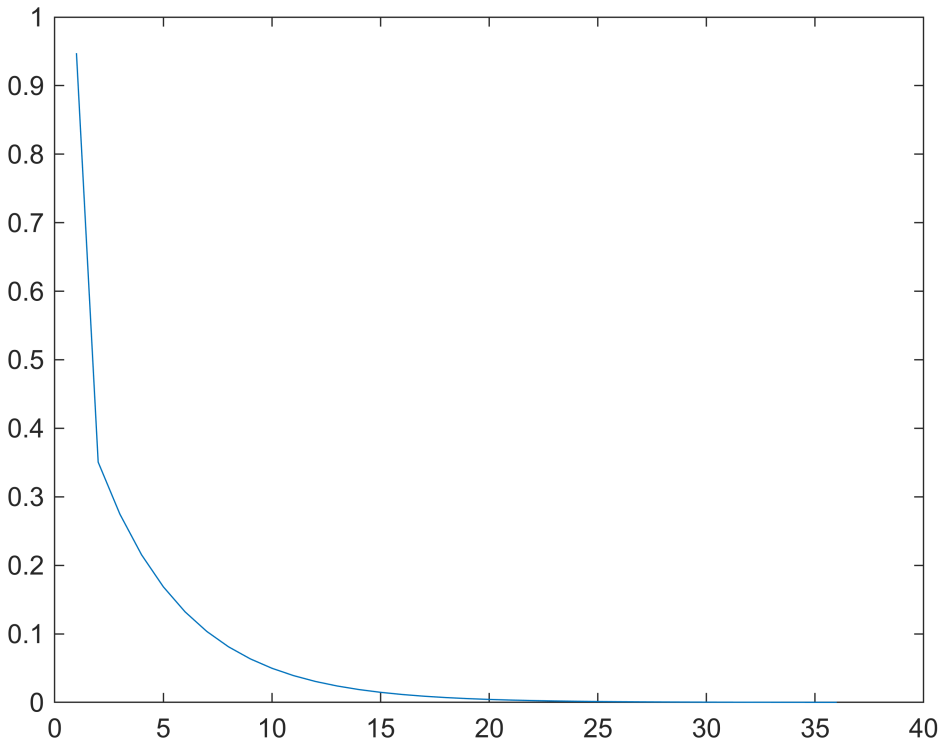
True: y = 2X+3

```
N=100;
X = randn(N,1);
Y = X*2 + 3+randn(N,1);
```

Fit Linear Regression: Gradient Descent

```
% dJ/dx
lamda=0.1;  % learning rate

% Initialization for t0, t1
t0=0.5;
t1=0.5;
loss=1;
itrN=1000;
k=1;
```

```
t1_prev=0;
t0_prev=0;

%% ADD your code here
while(loss>0.0001 && k<itrN)

    % Define function h(x)
    h=t1*X+t0;

    % Define gradient w.r.t theta_1 and theta_0
    dJt1 = -sum(X.*(Y-(t1*X+t0))) / N;
    dJt0 = -sum(Y-(t1*X+t0)) / N;

    % Update theta1, theta0
    t1=t1-lamda*(dJt1 + dJt0);
    t0=t0-lamda*(dJt1 + dJt0);

    % Cost Function v1
    loss=sum((Y-h).^2)/(2*N);

    % Cost Function v2
    loss= (norm(t1-t1_prev)+norm(t0-t0_prev))/2;
    t1_prev=t1;
    t0_prev=t0;

    loss_hist(k) = loss;
    k=k+1;
end


% Plot loss vs iteration
figure
plot(loss_hist)
```




```
% Predicted hypothesis y(x)
ypred=t1_prev*X + t0_prev; %%% ADD your code here

disp('optimal paraterms are')
```

```
optimal paraterms are
```

```
t1
```

```
t1 = 2.5670
```

```
t0
```

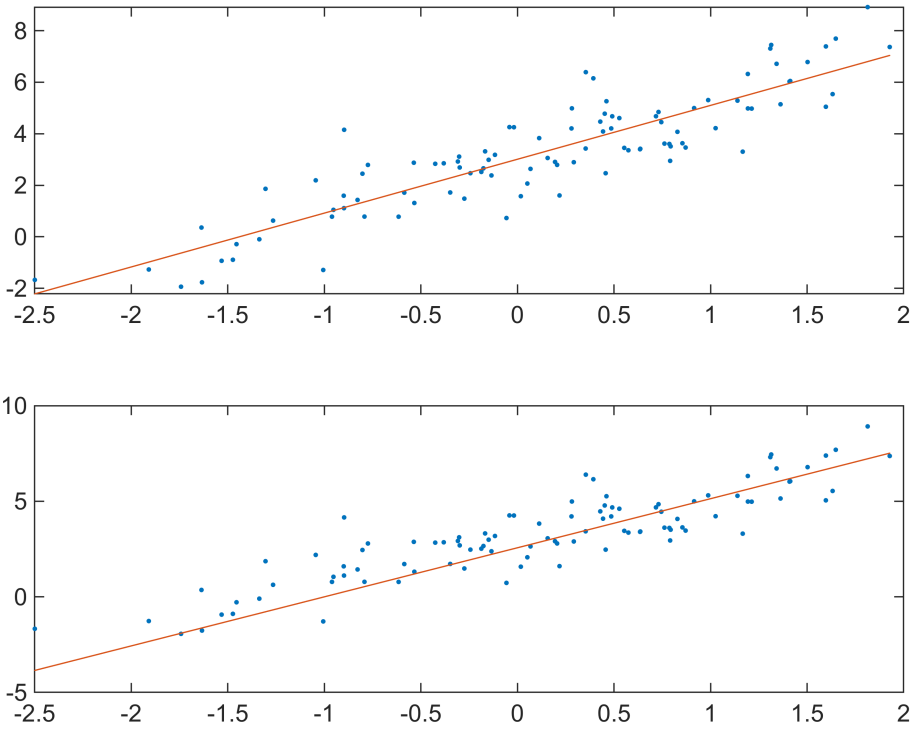
```
t0 = 2.5670
```

```
% MATLAB fit linear using fitlm(x,y)
mdl = fitlm(X, Y)   %%% ADD your code here
```

```
mdl =
선형 회귀 모델:
    y ~ 1 + x1

추정된 계수:
                   Estimate      SE       tStat       pValue
                   ________    _______    ______    __________

    (Intercept)     3.0111     0.10578    28.466    3.5158e-49
```

```
    x1              2.0901     0.11019     18.968     1.4282e-34

관측값 개수: 100, 오차 자유도: 98
RMS 오차: 1.05
결정계수: 0.786, 수정된 결정계수: 0.784
상수 모델에 대한 F-통계량: 360, p-값 = 1.43e-34
```

```
% Plot and compare predictions
figure
subplot(2,1,1)
plot(X,Y,'.',X,mdl.Fitted, '-')
subplot(2,1,2)
plot(X,Y,'.',X,ypred, '-')
```



Exercise 2 : Linear Regression with dim=2

Find the linear regression. Remove Outlier and predict a test value Xtest=[2;1];. You can use MATLAB fitlm()

- Feature: 2-Dimensions, p=2
- True value: y = 2*X1+4*X2+3

Data Acquisition

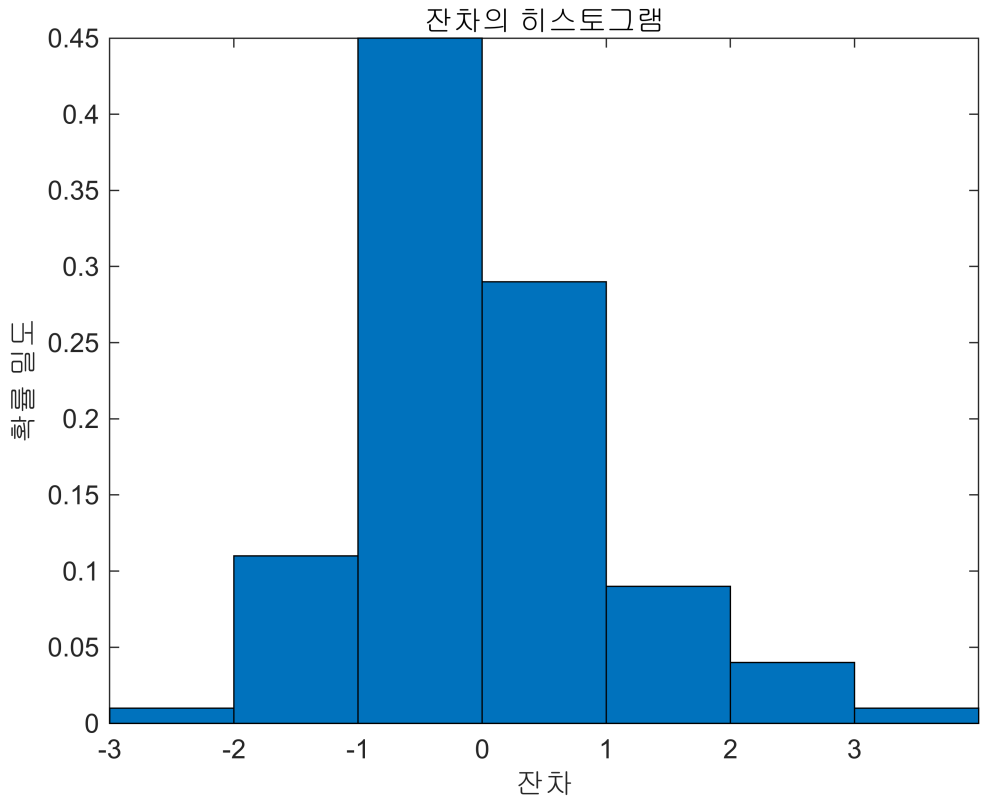
```
X = randn(100,2);
y = X*[2;4] + 3+ randn(100,1);
```

Fit Linear Regression

```
mdl=fitlm(X,y);
% Your code goes here
```

Analyze

```
% Remove outlier
%  -plot residual histogram
figure
plotResiduals(mdl)
```




```
% - remove outlier from histogram analysis
outl = find(mdl.Residuals.Raw > 2);
mdl.Residuals.Raw(outl)
```

```
ans = 5×1
    2.4975
    2.2773
    2.4835
    3.1499
    2.9978
```

```
% Fit linear regression (fitlm)
mdl2 = fitlm(X, y, 'Exclude', outl);

% Your code goes here
```

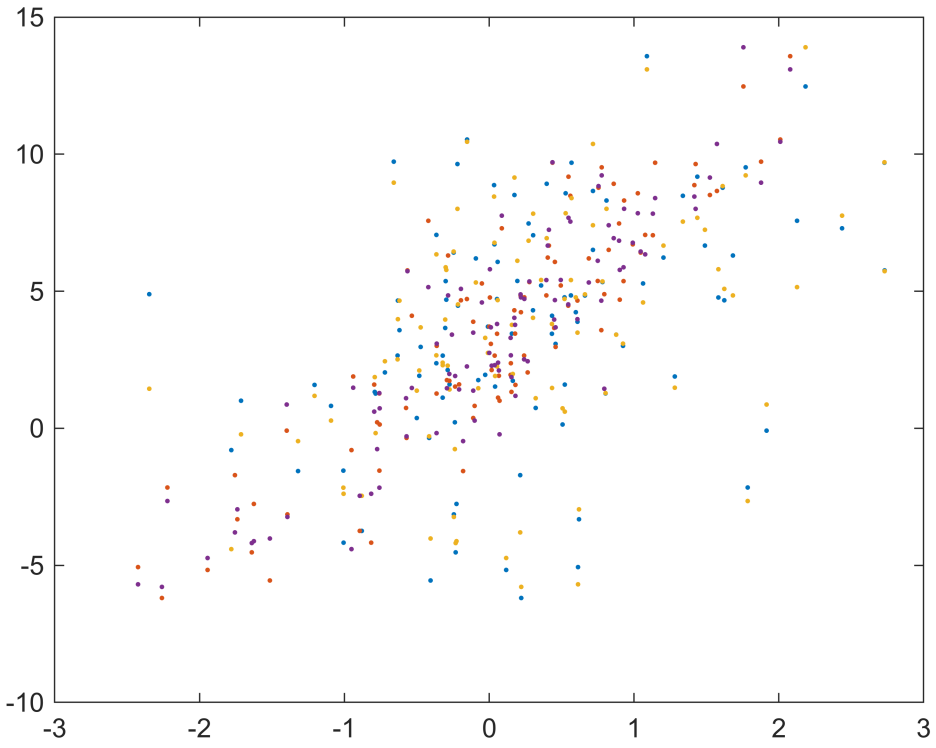
Predict

Predict for a Test value

```
Xnew=[2, 1];
Ynew=predict(mdl2,Xnew)
```

```
Ynew = 10.5491
```

```
plot(X,y,'.', X,mdl2.Fitted,'.')
```



Exercise 3: Linear Regression with dim=4

Find linear relationship of

- MPG ~ 1 + Acceleration + Displacement + Horsepower + Weight

Choose a test data and predict

Data Acquisition

```
clear
load carsmall
```

```
tbl = table(MPG,Acceleration,Displacement,Horsepower,Weight);
```

Fit Linear Regression

```
mdl = fitlm(tbl, 'MPG~Acceleration+Displacement+Horsepower+Weight')
```

```
mdl =
선형 회귀 모델:
    MPG ~ 1 + Acceleration + Displacement + Horsepower + Weight

추정된 계수:
                    Estimate        SE         tStat        pValue
                   _________    _________    ________    _________

    (Intercept)        48.117       3.9008      12.335    6.9194e-21
    Acceleration     -0.060312      0.21167    -0.28493       0.77636
    Displacement    -0.0066826     0.011594    -0.57638       0.56583
    Horsepower       -0.037547     0.026139     -1.4364       0.15442
    Weight           -0.006084    0.0013823     -4.4014      3.01e-05


관측값 개수: 93, 오차 자유도: 88
RMS 오차: 4.11
결정계수: 0.753, 수정된 결정계수: 0.742
상수 모델에 대한 F-통계량: 67.1, p-값 = 6.49e-26
```

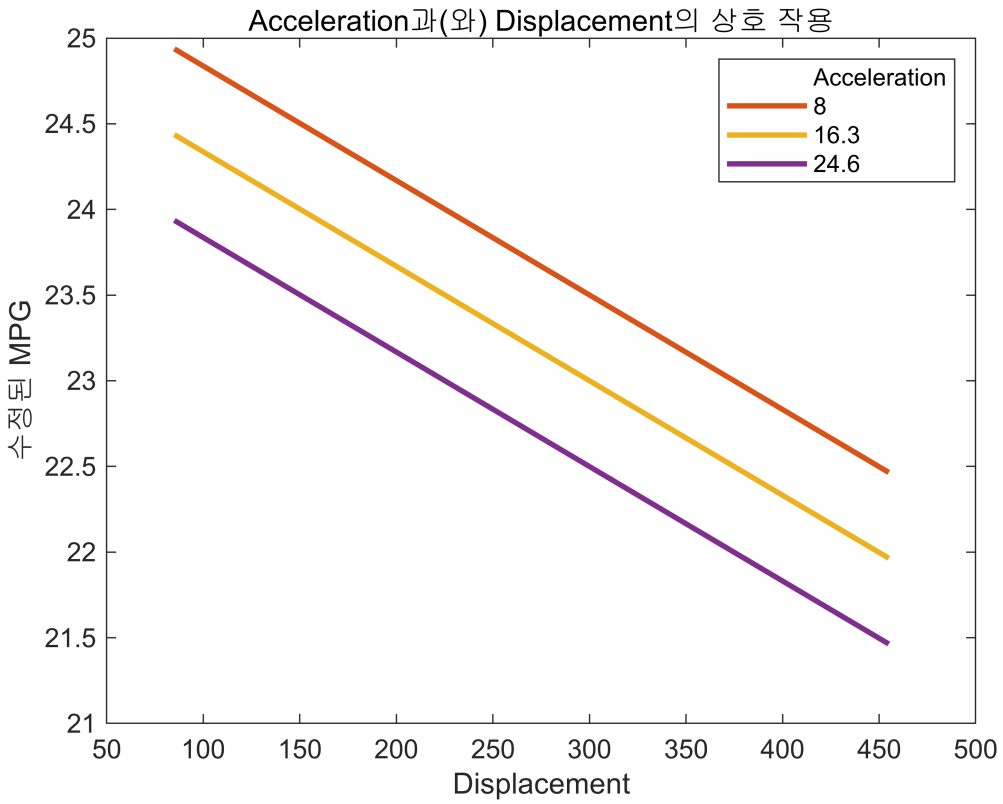
Analyze

```
% Remove outlier
%  -plot residual histogram
% - remove outlier from histogram analysis
outl = find(abs(mdl.Residuals.Raw) > 9);


% Fit linear regression (fitlm)
mdl2 = fitlm(tbl,'MPG~Acceleration+Displacement+Horsepower+Weight','Exclude',outl);
```

Predict

Predict for a Test value

```
Xnew = [10 300 150 3500];
Ynew= predict(mdl2, Xnew);
plotInteraction(mdl,'Acceleration','Displacement','predictions')
```


Acceleration과(와) Displacement의 상호 작용
Acceleration
8
16.3
24.6
수정된 MPG
Displacement
25
24.5
24
23.5
23
22.5
22
21.5
21
50
100
150
200
250
300
350
400
450
500