Example Programs for KINSOL v4.0.0-dev

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1 Introduction

This report is intended to serve as a companion document to the User Documentation of KINSOL [1]. It provides details, with listings, on the example programs supplied with the KINSOL distribution package.

The KINSOL distribution contains examples of types: serial C examples, parallel C examples, serial and parallel FORTRAN examples, and an OpenMP example. With the exception of "demo"-type example files, the names of all the examples distributed with SUNDIALS are of the form [slv][PbName]_[strat]_[ls]_[prec]_[p], where

[slv] identifies the solver (for KINSOL examples this is kin, while for FKINSOL examples, this is fkin);

[PbName] identifies the problem;

[strat] identifies the strategy (absent if "none" or "linesearch");

[ls] identifies the linear solver module used;

[prec] indicates the KINSOL preconditioner module used (only if applicable, for examples using a Krylov linear solver and the KINBBDPRE module, this will be bbd);

[p] indicates an example using the parallel vector module NVECTOR_PARALLEL.

The following lists summarize all examples distributed with KINSOL.

Supplied in the *srcdir*/examples/kinsol/serial directory are the following serial examples (using the NVECTOR_SERIAL module):

- kinRoberts_fp solves the backward Euler time step for a three-species chemical kinetics system, using the fixed point strategy.
- kinFerTron_dns solves the Ferraris-Tronconi problem.

 This program solves the problem with the KINDENSE linear solver and uses different combinations of globalization and Jacobian update strategies with different initial guesses.
- kinFerTron_klu solves the same problem as in kinFerTron_dns, but uses the sparse direct solver KLU.
- kinRoboKin_dns solves a nonlinear system from robot kinematics.

 This program solves the problem with the KINDENSE linear solver and a user-supplied Jacobian routine.
- kinRoboKin_slu is the same as kinRoboKin_dns but uses the SuperLUMT sparse direct linear solver.
- kinLaplace_bnd solves a simple 2-D elliptic PDE on a unit square. This program solves the problem with the KINBAND linear solver.
- kinLaplace_picard_bnd is the same as kinLaplace_bnd but uses the Picard strategy.

- kinFoodWeb_kry solves a food web model.

 This is a nonlinear system that arises from a system of partial differential equations describing a six-species food web population model, with predator-prey interaction and diffusion on the unit square in two dimensions. This program solves the problem with the KINSPGMR linear solver and a user-supplied preconditioner. The preconditioner is a block-diagonal matrix based on the partial derivatives of the interaction terms only.
- kinKrylovDemo_ls solves the same problem as kinFoodWeb_kry, but with three Krylov linear solvers: kinspgmr, kinspbcg, and kinsptfqmr.

Supplied in the *srcdir*/examples/kinsol/parallel directory are the following parallel examples (using the NVECTOR_PARALLEL module):

- kinFoodWeb_kry_p is a parallel implementation of kinFoodWeb_kry.
- kinFoodWeb_kry_bbd_p solves the same problem as kinFoodWeb_kry_p, with a block-diagonal matrix with banded blocks as a preconditioner, generated by difference quotients, using the KINBBDPRE module.

As part of the FKINSOL module, in the directories <code>srcdir/examples/kinsol/fcmix_serial</code> and <code>srcdir/examples/kinsol/fcmix_parallel</code>, respectively, are the following examples for the FORTRAN-C interface:

- fkinDiagon_kry is a serial example, which solves a nonlinear system of the form $u_i^2 = i^2$ using an approximate diagonal preconditioner.
- fkinDiagon_kry_p is a parallel implementation of fkinDiagon_kry.

Supplied in directory *srcdir*/examples/kinsol/C_openmp is an example using the OpenMP NVECTOR module. kin_FoodWeb_kry_omp solves the same problem as kin_FoodWeb_kry_but uses the OpenMP module.

In the following sections, we give detailed descriptions of some (but not all) of these examples. We also give our output files for each of these examples, but users should be cautioned that their results may differ slightly from these. Differences in solution values may differ within the tolerances, and differences in cumulative counters, such as numbers of Newton iterations, may differ from one machine environment to another by as much as 10% to 20%.

In the descriptions below, we make frequent references to the KINSOL User Document [1]. All citations to specific sections (e.g. §4.2) are references to parts of that User Document, unless explicitly stated otherwise.

Note. The examples in the KINSOL distribution are written in such a way as to compile and run for any combination of configuration options used during the installation of SUNDIALS (see Appendix A in the User Guide). As a consequence, they contain portions of code that will not be typically present in a user program. For example, all C example programs make use of the variables SUNDIALS_EXTENDED_PRECISION and SUNDIALS_DOUBLE_PRECISION to test if the solver libraries were built in extended or double precision, and use the appropriate conversion specifiers in printf functions.

2 C example problems

2.1 A serial dense example: kinFerTron_dns

As an initial illustration of the use of the KINSOL package for the solution of nonlinear systems, we give a sample program called kinFerTron_dns.c. It uses the KINSOL dense linear solver module KINDENSE and the NVECTOR_SERIAL module (which provides a serial implementation of NVECTOR) for the solution of the Ferraris-Tronconi test problem [2].

This problem involves a blend of trigonometric and exponential terms:

$$0 = 0.5 \sin(x_1 x_2) - 0.25 x_2 / \pi - 0.5 x_1$$

$$0 = (1 - 0.25 / \pi) (e^{2x_1} - e) + ex_2 / \pi - 2ex_1$$
subject to
$$x_{1 \min} = 0.25 \le x_1 \le 1 = x_{1 \max}$$

$$x_{2 \min} = 1.5 \le x_2 \le 2\pi = x_{2 \max}.$$
(1)

The bounds constraints on x_1 and x_2 are treated by introducing four additional variables and using KINSOL's optional constraints feature to enforce non-positivity and non-negativity:

$$l_1 = x_1 - x_{1 \min} \ge 0$$

$$L_1 = x_1 - x_{1 \max} \le 0$$

$$l_2 = x_2 - x_{2 \min} \ge 0$$

$$L_2 = x_2 - x_{2 \max} \le 0$$

The Ferraris-Tronconi problem has two known solutions. We solve it with KINSOL using two sets of initial guesses for x_1 and x_2 (first their lower bounds and secondly the middle of their feasible regions), both with an exact and modified Newton method, with and without line search.

Following the initial comment block, this program has a number of #include lines, which allow access to useful items in CVODE header files. The kinsol.h file provides prototypes for the KINSOL functions to be called (excluding the linear solver selection function), and also a number of constants that are to be used in setting input arguments and testing the return value of KINSol. The nvector_serial.h file is the header file for the serial implementation of the NVECTOR module and includes definitions of the N_Vector type, a macro to access vector components, and prototypes for the serial implementation specific machine environment memory allocation and freeing functions. The kinsol_dense.h file provides the prototype for the KINDense function. The sundials_types.h file provides the definition of the type realtype (see §4.2 for details). For now, it suffices to read realtype as double. Finally, sundials_math.h is included for the definition of the exponential function RExp.

Next, the program defines some problem-specific constants, which are isolated to this early location to make it easy to change them as needed. This program includes a user-defined accessor macro, Ith, that is useful in writing the problem functions in a form closely matching the mathematical description of the system, i.e. with components numbered from 1 instead of from 0. The Ith macro is used to access components of a vector of type N_Vector with a serial implementation. It is defined using the NVECTOR_SERIAL accessor macro NV_Ith_S which numbers components starting with 0. The program prologue ends with prototypes of the user-supplied system function func and several private helper functions.

The \mathtt{main} program begins with some dimensions and type declarations, including use of the type $\mathtt{N_Vector}$, initializations, and allocation and definitions for the user data structure

data which contains two arrays with lower and upper bounds for x_1 and x_2 . Next, we create five serial vectors of type N_Vector for the two different initial guesses, the solution vector \mathbf{u} , the scaling factors, and the constraint specifications.

The initial guess vectors u1 and u2 are set by the private functions SetInitialGuess1 and SetInitialGuess2 and the constraint vector c is initialized to [0,0,1,-1,1,-1] indicating that there are no additional constraints on the first two components of u (i.e. x_1 and x_2) and that the 3rd and 5th components should be non-negative, while the 4th and 6th should be non-positive.

The calls to N_VNew_Serial, and also later calls to various KIN*** functions, make use of a private function, check_flag, which examines the return value and prints a message if there was a failure. The check_flag function was written to be used for any serial SUNDIALS application.

The call to KINCreate creates the KINSOL solver memory block. Its return value is a pointer to that memory block for this problem. In the case of failure, the return value is NULL. This pointer must be passed in the remaining calls to KINSOL functions.

The next four calls to KINSOL optional input functions specify the pointer to the user data structure (to be passed to all subsequent calls to func), the vector of additional constraints, and the function and scaled step tolerances, fnormtol and scsteptol, respectively.

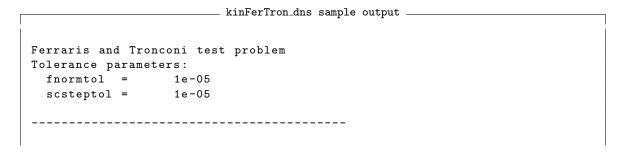
Solver memory is allocated through the call to KINInit which specifies the system function func and provides the vector **u** which will be used internally as a template for cloning additional necessary vectors of the same type as **u**. The use of the dense linear solver is specified by calling KINDense which also specifies the problem size NEQ.

The main program proceeds by solving the nonlinear system eight times, using each of the two initial guesses u1 and u2 (which are first copied into the vector u using N_VScale_Serial from the NVECTOR_SERIAL module), with and without globalization through line search (specified by setting glstr to KIN_LINESEARCH and KIN_NONE, respectively), and applying either an exact or a modified Newton method. The switch from exact to modified Newton is done by changing the number of nonlinear iterations after which a Jacobian evaluation is enforced, a value mset= 1 thus resulting in re-evaluating the Jacobian at every single iteration of the nonlinear solver (exact Newton method). Note that passing mset= 0 indicates using the default KINSOL value of 10.

The actual problem solution is carried out in the private function SolveIt which calls the main solver function KINSol after first setting the optional input mset. After a successful return from KINSol, the solution $[x_1, x_2]$ and some solver statistics are printed.

The function func is a straightforward expression of the extended nonlinear system. It uses the macro NV_DATA_S (defined by the NVECTOR_SERIAL module) to extract the pointers to the data arrays of the N_Vectors u and f and sets the components of fdata using the current values for the components of udata. See §4.6.1 for a detailed specification of f.

The output generated by kinFerTron_dns is shown below.



```
Initial guess on lower bounds
 [x1, x2] = 0.25 1.5
Exact Newton
Solution:
 [x1, x2] = 0.299449 2.83693
Final Statistics:
nni = 3 nfe =
         3 	 nfeD = 18
 nje =
Exact Newton with line search
Solution:
 [x1, x2] = 0.299449 2.83693
Final Statistics:
nni = 3 nfe = 4
nje = 3 nfeD = 18
Modified Newton
Solution:
 [x1, x2] = 0.299449 2.83693
Final Statistics:
nni = 11 nfe = 12
 nje = 2 nfeD = 12
Modified Newton with line search
Solution:
[x1, x2] = 0.299449 2.83693
Final Statistics:
nni = 11 nfe = 12
nje = 2 nfeD = 12
 nje =
-----
Initial guess in middle of feasible region
[x1, x2] = 0.625 \quad 3.89159
Exact Newton
Solution:
[x1,x2] = 0.5 \quad 3.14159
Final Statistics:
nni = 5 nfe = 6
nje = 5 nfeD = 30
 nje =
Exact Newton with line search
Solution:
 [x1, x2] = 0.5 \quad 3.14159
Final Statistics:
nni = 5 nfe =
         5 	 nfeD = 30
 nje =
Modified Newton
Solution:
 [x1, x2] = 0.500003 3.1416
Final Statistics:
nni = 12 nfe = 13
nje = 2 nfeD = 12
Modified Newton with line search
Solution:
```

2.2 A serial Krylov example: kinFoodWeb_kry

We give here an example that illustrates the use of KINSOL with the Krylov method SPGMR, in the KINSPGMR module, as the linear system solver.

This program solves a nonlinear system that arises from a discretized system of partial differential equations. The PDE system is a six-species food web population model, with predator-prey interaction and diffusion on the unit square in two dimensions. Given the dependent variable vector of species concentrations $c = [c_1, c_2, ..., c_{n_s}]^T$, where $n_s = 2n_p$ is the number of species and n_p is the number of predators and of prey, then the PDEs can be written as

$$d_i \cdot \left(\frac{\partial^2 c_i}{\partial x^2} + \frac{\partial^2 c_i}{\partial y^2}\right) + f_i(x, y, c) = 0 \quad (i = 1, ..., n_s),$$
(2)

where the subscripts i are used to distinguish the species, and where

$$f_i(x, y, c) = c_i \cdot \left(b_i + \sum_{j=1}^{n_s} a_{i,j} \cdot c_j \right). \tag{3}$$

The problem coefficients are given by

$$a_{ij} = \begin{cases} -1 & i = j \\ -0.5 \cdot 10^{-6} & i \le n_p, \ j > n_p \\ 10^4 & i > n_p, \ j \le n_p \\ 0 & \text{all other} \end{cases}$$

$$b_i = b_i(x, y) = \begin{cases} 1 + \alpha xy & i \le n_p \\ -1 - \alpha xy & i > n_p \end{cases},$$

and

$$d_i = \begin{cases} 1 & i \le n_p \\ 0.5 & i > n_p \end{cases}.$$

The spatial domain is the unit square $(x, y) \in [0, 1] \times [0, 1]$.

Homogeneous Neumann boundary conditions are imposed and the initial guess is constant in both x and y. For this example, the equations (2) are discretized spatially with standard central finite differences on a 8×8 mesh with $n_s = 6$, giving a system of size 384.

Among the initial #include lines in this case are lines to include kinsol_spgmr.h and sundials_math.h. The first contains constants and function prototypes associated with the SPGMR method. The inclusion of sundials_math.h is done to access the SUNMAX and SUNRabs macros, and the SUNRagrt function to compute the square root of a realtype number.

The main program calls KINCreate and then calls KINInit with the name of the user-supplied system function func and solution vector as arguments. The main program then calls a number of KINSet* routines to notify KINSOL of the user data pointer, the positivity constraints on the solution, and convergence tolerances on the system function and step size.

It calls KINSpgmr (see §4.5.2) to specify the KINSPGMR linear solver, and passes a value of 15 as the maximum Krylov subspace dimension, maxl. Next, a maximum value of maxlrst = 2 restarts is imposed and the user-supplied preconditioner setup and solve functions, PrecSetupBD and PrecSolveBD, are specified through a call to KINSpilsSetPreconditioner (see §4.5.4). The data pointer passed to KINSpilsSetUserData is passed to PrecSetupBD and PrecSolveBD whenever these are called.

Next, KINSo1 is called, the return value is tested for error conditions, and the approximate solution vector is printed via a call to PrintOutput. After that, PrintFinalStats is called to get and print final statistics, and memory is freed by calls to N_VDestroy_Serial, FreeUserData and KINFree. The statistics printed are the total numbers of nonlinear iterations (nni), of func evaluations (excluding those for Jv product evaluations) (nfe), of func evaluations for Jv evaluations (nfeSG), of linear (Krylov) iterations (nli), of preconditioner evaluations (npe), and of preconditioner solves (nps). All of these optional outputs and others are described in §4.5.5.

Mathematically, the dependent variable has three dimensions: species number, x mesh point, and y mesh point. But in NVECTOR_SERIAL, a vector of type N_Vector works with a one-dimensional contiguous array of data components. The macro IJ_Vptr isolates the translation from three dimensions to one. Its use results in clearer code and makes it easy to change the underlying layout of the three-dimensional data. Here the problem size is 384, so we use the NV_DATA_S macro for efficient N_Vector access. The NV_DATA_S macro gives a pointer to the first component of a serial N_Vector which is then passed to the IJ_Vptr macro.

The preconditioner used here is the block-diagonal part of the true Newton matrix and is based only on the partial derivatives of the interaction terms f in (3) and hence its diagonal blocks are $n_s \times n_s$ matrices ($n_s = 6$). It is generated and factored in the PrecSetupBD routine and backsolved in the PrecSolveBD routine. See §4.6.7 for detailed descriptions of these preconditioner functions.

The program kinFoodWeb_kry.c uses the "small" dense functions for all operations on the 6×6 preconditioner blocks. Thus it includes sundials_smalldense.h, and calls the small dense matrix functions denalloc, denallocpiv, denfree, denfreepiv, denGETRF, and denGETRS. The small dense functions are generally available for KINSOL user programs (for more information, see §?? or the comments in the header file sundials_smalldense.h).

In addition to the functions called by KINSOL, kinFoodWeb_kry.c includes definitions of several private functions. These are: AllocUserData to allocate space for the preconditioner and the pivot arrays; InitUserData to load problem constants in the data block; FreeUserData to free that block; SetInitialProfiles to load the initial values in cc; PrintHeader to print the heading for the output; PrintOutput to retreive and print selected solution values; PrintFinalStats to print statistics; and check_flag to check return values for error conditions.

The output generated by kinFoodWeb_kry is shown below. Note that the solution involved 10 Newton iterations, with an average of about 38 Krylov iterations per Newton iteration.

```
kinFoodWeb_kry sample output

Predator-prey test problem -- KINSol (serial version)

Mesh dimensions = 8 X 8

Number of species = 6

Total system size = 384
```

```
Flag globalstrategy = 0 (0 = None, 1 = Linesearch)
Linear solver is SPGMR with maxl = 15, maxlrst = 2
Preconditioning uses interaction-only block-diagonal matrix
Positivity constraints imposed on all components
Tolerance parameters: fnormtol = 1e-07
                                         scsteptol = 1e-13
Initial profile of concentration
At all mesh points: 1 1 1
                             30000 30000 30000
Computed equilibrium species concentrations:
At bottom left:
1.16428 1.16428 1.16428 34927.5 34927.5 34927.5
At top right:
1.25797 1.25797 1.25797 37736.7 37736.7 37736.7
Final Statistics..
            9
                            329
nni
                  nli
nfe
            10
                  nfeSG =
                            338
nps
           338
                  npe
                                     ncfl
                                                 6
```

2.3 A parallel example: kinFoodWeb_kry_bbd_p

In this example, kinFoodWeb_kry_bbd_p, we solve the same problem as with kinFoodWeb_kry above, but in parallel, and instead of supplying the preconditioner we use the KINBBDPRE module.

In this case, we think of the parallel MPI processes as being laid out in a rectangle, and each process being assigned a subgrid of size MXSUB \times MYSUB of the x-y grid. If there are NPEX processes in the x direction and NPEY processes in the y direction, then the overall grid size is MX \times MY with MX=NPEX \times MXSUB and MY=NPEY \times MYSUB, and the size of the nonlinear system is NUM_SPECIES·MX·MY.

The evaluation of the nonlinear system function is performed in func. In this parallel setting, the processes first communicate the subgrid boundary data and then compute the local components of the nonlinear system function. The MPI communication is isolated in the private function ccomm (which in turn calls BRecvPost, BSend, and BRecvWait) and the subgrid boundary data received from neighboring processes is loaded into the work array cext. The computation of the nonlinear system function is done in func_local which starts by copying the local segment of the cc vector into cext, and then by imposing the boundary conditions by copying the first interior mesh line from cc into cext. After this, the nonlinear system function is evaluated by using central finite-difference approximations using the data in cext exclusively.

KINBBDPRE uses a band-block-diagonal preconditioner, generated by difference quotients. The upper and lower half-bandwidths of the Jacobian block generated on each process are both equal to $2 \cdot n_s - 1$, and that is the value passed as mudq and mldq in the call to KINBBDPrecInit. These values are much less than the true half-bandwidths of the Jacobian blocks, which are n_s · MXSUB. However, an even narrower band matrix is retained as the preconditioner, with half-bandwidths equal to n_s , and this is the value passed to KINBBDPrecInit for mukeep and mlkeep.

The function func_local is also passed as the gloc argument to KINBBDPrecInit. Since all communication needed for the evaluation of the local approximation of f used in building the band-block-diagonal preconditioner is already done for the evaluation of f in func, a NULL pointer is passed as the gcomm argument to KINBBDPrecInit.

The main program resembles closely that of the kinFoodWeb_kry example, with particularization arising from the use of the parallel MPI NVECTOR_PARALLEL module. It begins by initializing MPI and obtaining the total number of processes and the rank of the local process. The local length of the solution vector is then computed as NUM_SPECIES·MXSUB·MYSUB. Distributed vectors are created by calling the constructor defined in NVECTOR_PARALLEL with the MPI communicator and the local and global problem sizes as arguments. All output is performed only from the process with id equal to 0. Finally, after KINSol is called and the results are printed, all memory is deallocated, and the MPI environment is terminated by calling MPI_Finalize.

The output generated by kinFoodWeb_kry_bbd_p is shown below. Note that 9 Newton iterations were required, with an average of about 51.6 Krylov iterations per Newton iteration.

```
_ kinFoodWeb_kry_bbd_p sample output .
Predator-prey test problem -- KINSol (parallel-BBD version)
Mesh dimensions = 20 X 20
Number of species = 6
Total system size = 2400
Subgrid dimensions = 10 \times 10
Processor array is 2 X 2
Flag globalstrategy = 0 (0 = None, 1 = Linesearch)
Linear solver is SPGMR with max1 = 20, max1rst = 2
Preconditioning uses band-block-diagonal matrix from KINBBDPRE
  Difference quotient half-bandwidths: mudq = 11, mldq = 11
  Retained band block half-bandwidths: mukeep = 6, mlkeep = 6
Tolerance parameters: fnormtol = 1e-07
                                          scsteptol = 1e-13
Initial profile of concentration
At all mesh points: 1 1 1
                             30000 30000 30000
Computed equilibrium species concentrations:
At bottom left:
 1.165 1.165 1.165 34949 34949 34949
At top right:
1.25552 1.25552 1.25552 37663.2 37663.2 37663.2
Final Statistics..
           9
                            464
nni
                 nli
           10
                 nfeSG =
                            473
nfe
                  npe =
           473
                                                 6
nps
                              1
                                    ncfl =
```

3 Fortran example problems

The Fortran example problem programs supplied with the KINSOL package are all written in standard F77 Fortran and use double precision arithmetic. Before running any of these examples, the user should make sure that the Fortran data types for real and integer variables appropriately match the C types. See §5.2 in the KINSOL User Document for details.

However, when the FORTRAN examples are built, the source code is automatically modified according to the configure options supplied by the user and the system type. Integer variables are declared as INTEGER*n, where n denotes the number of bytes in the corresponding C type (long int or int). Floating-point variable declarations remain unchanged if double precision is used, but are changed to REAL*n, where n denotes the number of bytes in the SUNDIALS type realtype, if using single precision. Also, if using single precision, declarations of floating-point constants are appropriately modified, e.g. 0.5D-4 is changed to 0.5E-4.

The two examples supplied with the FKINSOL module are very simple tests of the FORTRAN-C interface module. They solve the nonlinear system

$$F(u) = 0$$
, where $f_i(u) = u_i^2 - i^2$, $1 \le i \le N$.

3.1 A serial example: fkinDiagon_kry

The fkinDiagon_kry program solves the above problem using the NVECTOR_SERIAL module. The main program begins by calling fnvinits to initialize computations with the NVECTOR_SERIAL module. Next, the array uu is set to contain the initial guess $u_i = 2i$, the array scale is set with all components equal to 1.0 (meaning that no scaling is done), and the array constr is set with all components equal to 0.0 to indicate that no inequality constraints should be imposed on the solution vector.

The KINSOL solver is initialized and memory for it is allocated by calling fkinmalloc, which also specifies the iout and rout arrays which are used to store integer and real outputs, respectively (see Table 5.2). Also, various integer, real, and vector parameters are specified by calling the fkinsetiin, fkinsetrin, and fkinsetvin subroutines, respectively. In particular, the maximum number of iterations between calls to the preconditioner setup routine (msbpre = 5), the tolerance for stopping based on the function norm (fnormtol = 10^{-5}), and the tolerance for stopping based on the step length (scsteptol = 10^{-4}) are specified.

Next, the KINSPGMR linear solver module is attached to KINSOL by calling fkinspgmr, which also specifies the maximum Krylov subspace dimension (maxl = 10) and the maximum number of restarts allowed for SPGMR (maxlrst = 2). The KINSPGMR module is directed to use the supplied preconditioner by calling the fkinspilssetprec routine with a first argument equal to 1. The solution of the nonlinear system is obtained after a successful return from fkinsol, which is then printed to unit 6 (stdout). Finally, memory allocated for the KINSOL solver is released by calling fkinfree.

The user-supplied routine fkfun contains a straightforward transcription of the nonlinear system function f, while the routine fkpset sets the array pp (in the common block pcom) to contain an approximation to the reciprocals of the Jacobian diagonal elements. The components of pp are then used in fkpsol to solve the preconditioner linear system Px = v through simple multiplications.

The following is sample output from fkinDiagon_kry, using N = 128.

```
_{-} fkinDiagon_kry sample output _{-}
Example program fkinDiagon_kry:
This FKINSOL example solves a 128 eqn diagonal algebraic system.
Its purpose is to demonstrate the use of the Fortran interface
in a serial environment.
 globalstrategy = KIN_NONE
FKINSOL return code is
The resultant values of uu are:
                            3.000000
      1.000000
                 2.000000
                                       4.000000
      5.000000
                6.000000
                            7.000000
                                       8.000000
      9.000000 10.000000 11.000000
                                     12.000000
     13.000000 14.000000 15.000000
                                      16.000000
  13
  17
     17.000000 18.000000 19.000000
                                      20.000000
  21
     21.000000 22.000000
                           23.000000
                                      24.000000
     25.000000 26.000000
                           27.000000
                                      28.000000
 25
     29.000000 30.000000
  29
                           31.000000
                                      32.000000
     33.000000 34.000000
 33
                           35.000000
                                      36.000000
     37.000000 38.000000
  37
                           39.000000
                                      40.000000
     41.000000 42.000000
                           43.000000
                                      44.000000
  45
     45.000000 46.000000
                           47.000000
                                      48.000000
  49
     49.000000 50.000000 51.000000
                                     52.000000
  53 53.000000 54.000000 55.000000
                                     56.000000
     57.000000 58.000000 59.000000
  57
                                     60.000000
    61.000000 62.000000 63.000000
  61
                                     64.000000
     65.000000 66.000000 67.000000
                                      68.000000
  69 69.000000 70.000000 71.000000
                                      72.000000
 73
     73.000000 74.000000
                           75.000000
                                      76.000000
 77
    77.000000 78.000000 79.000000
                                      80.000000
     81.000000 82.000000 83.000000
 81
                                      84.000000
     85.000000 86.000000
                           87.000000
 85
                                      88.000000
 89
     89.000000 90.000000
                           91.000000
                                      92.000000
     93.000000
                94.000000
                           95.000000
                                      96.000000
     97.000000 98.000000
 97
                           99.000000 100.000000
101 101.000000 102.000000 103.000000 104.000000
105 105.000000 106.000000 107.000000 108.000000
109 109.000000 110.000000 111.000000 112.000000
113 113.000000 114.000000 115.000000 116.000000
117 117.000000 118.000000 119.000000 120.000000
121 121.000000 122.000000 123.000000 124.000000
125 125.000000 126.000000 127.000000 128.000000
Final statistics:
         7, nli
 nni =
 nfe =
         8, npe
 nps = 28, ncfl =
```

3.2 A parallel example: fkinDiagon_kry_p

The program fkinDiagon_kry_p is a straightforward modification of fkinDiagon_kry to use the MPI-enabled NVECTOR_PARALLEL module.

After initialization of MPI, the NVECTOR_PARALLEL module is initialized by calling fnvinitp with the default MPI communicator mpi_comm_world and the local and global vector sizes as its first three arguments. The rank of the local process, mype, is used in both the initial guess and the system function, inasmuch as the global and local indices to the vector u are related by the equation iglobal = ilocal + mype*nlocal. In other respects, the problem setup (KINSOL initialization, KINSPGMR specification) and solution steps are the same as in fkinDiagon_kry. Upon successful return from fkinsol, the solution segment local to the process with id equal to 0 is printed to unit 6. Finally, the KINSOL memory is released and the MPI environent is terminated.

For this simple example, no inter-process communication is required to evaluate the non-linear system function f or the preconditioner. As a consequence, the user-supplied routines fkfun, fkpset, and fkpsol are basically identical to those in fkinDiagon_kry.

Sample output from fkinDiagon_kry_p, for N = 128, follows.

```
_ fkinDiagon_kry_p sample output .
Example program fkinDiagon_kry_p:
This FKINSOL example solves a 128 eqn diagonal algebraic system.
Its purpose is to demonstrate the use of the Fortran interface
in a parallel environment.
FKINSOL return code is
The resultant values of uu (process 0) are:
      1.000000
                2.000000
                          3.000000
                                     4.000000
  5
      5.000000
               6.000000
                          7.000000
                                    8.000000
  9
      9.000000 10.000000 11.000000
                                   12.000000
 13
    13.000000 14.000000 15.000000 16.000000
    17.000000 18.000000 19.000000 20.000000
 17
 21 21.000000 22.000000 23.000000 24.000000
 29 29.000000 30.000000 31.000000 32.000000
Final statistics:
                    21
 nni =
         7, nli
       8, npe
 nfe =
                     2
 nps = 28,
            ncfl =
```

References

- [1] A. M. Collier, A. C. Hindmarsh, R. Serban, and C.S. Woodward. User Documentation for KINSOL v4.0.0-dev. Technical Report UCRL-SM-208116, LLNL, 2017.
- [2] C. Floudas, P. Pardalos, C. Adjiman, W. Esposito, Z. Gumus, S. Harding, J. Klepeis, C. Meyer, and C. Schweiger. *Handbook of Test Problems in Local and Global Optimization*. Kluwer Academic Publishers, Dordrecht, 1999.