User Documentation for idas v3.0.0-dev (sundials v4.0.0-dev)

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Chapter 1

Introduction

IDAS is part of a software family called SUNDIALS: SUite of Nonlinear and DIfferential/ALgebraic equation Solvers [20]. This suite consists of CVODE, ARKODE, KINSOL, and IDA, and variants of these with sensitivity analysis capabilities, CVODES and IDAS.

IDAS is a general purpose solver for the initial value problem (IVP) for systems of differential-algebraic equations (DAEs). The name IDAS stands for Implicit Differential-Algebraic solver with Sensitivity capabilities. IDAS is an extension of the IDA solver within SUNDIALS, itself based on DASPK [5, 6]; however, like all SUNDIALS solvers, IDAS is written in ANSI-standard C rather than FORTRAN77. Its most notable features are that, (1) in the solution of the underlying nonlinear system at each time step, it offers a choice of Newton/direct methods and a choice of Inexact Newton/Krylov (iterative) methods; (2) it is written in a *data-independent* manner in that it acts on generic vectors without any assumptions on the underlying organization of the data; and (3) it provides a flexible, extensible framework for sensitivity analysis, using either *forward* or *adjoint* methods. Thus IDAS shares significant modules previously written within CASC at LLNL to support the ordinary differential equation (ODE) solvers CVODE [21, 12] and PVODE [8, 9], the DAE solver IDA [24] on which IDAS is based, the sensitivity-enabled ODE solver CVODES [22, 34], and also the nonlinear system solver KINSOL [13].

At present, IDAS may utilize a variety of Krylov methods provided in SUNDIALS that can be used in conjuction with Newton iteration: these include the GMRES (Generalized Minimal RESidual) [33], FGMRES (Flexible Generalized Minimum RESidual) [32], Bi-CGStab (Bi-Conjugate Gradient Stabilized) [36], TFQMR (Transpose-Free Quasi-Minimal Residual) [17], and PCG (Preconditioned Conjugate Gradient) [18] linear iterative methods. As Krylov methods, these require little matrix storage for solving the Newton equations as compared to direct methods. However, the algorithms allow for a user-supplied preconditioner matrix, and, for most problems, preconditioning is essential for an efficient solution.

For very large DAE systems, the Krylov methods are preferable over direct linear solver methods, and are often the only feasible choice. Among the Krylov methods in SUNDIALS, we recommend GMRES as the best overall choice. However, users are encouraged to compare all options, especially if encountering convergence failures with GMRES. Bi-CGFStab and TFQMR have an advantage in storage requirements, in that the number of workspace vectors they require is fixed, while that number for GMRES depends on the desired Krylov subspace size. FGMRES has an advantage in that it is designed to support preconditioners that vary between iterations (e.g. iterative methods). PCG exhibits rapid convergence and minimal workspace vectors, but only works for symmetric linear systems.

IDAS is written with a functionality that is a superset of that of IDA. Sensitivity analysis capabilities, both forward and adjoint, have been added to the main integrator. Enabling forward sensitivity computations in IDAS will result in the code integrating the so-called *sensitivity equations* simultaneously with the original IVP, yielding both the solution and its sensitivity with respect to parameters in the model. Adjoint sensitivity analysis, most useful when the gradients of relatively few functionals of the solution with respect to many parameters are sought, involves integration of the original IVP

forward in time followed by the integration of the so-called *adjoint equations* backward in time. IDAS provides the infrastructure needed to integrate any final-condition ODE dependent on the solution of the original IVP (in particular the adjoint system).

There are several motivations for choosing the C language for IDAS. First, a general movement away from FORTRAN and toward C in scientific computing was apparent. Second, the pointer, structure, and dynamic memory allocation features in C are extremely useful in software of this complexity, with the great variety of method options offered. Finally, we prefer C over C++ for IDAS because of the wider availability of C compilers, the potentially greater efficiency of C, and the greater ease of interfacing the solver to applications written in extended FORTRAN.

1.1   Changes from previous versions

Changes in v3.0.0-dev

Three fused vector operations and seven vector array operations have been added to the NVECTOR API. These *optional* operations are intended to increase data reuse in vector operations, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. The new operations are `N_VLinearCombination`, `N_VScaleAddMulti`, `N_VDotProdMulti`, `N_VLinearCombinationVectorArray`, `N_VScaleVectorArray`, `N_VConstVectorArray`, `N_VWrmsNormVectorArray`, `N_VWrmsNormMaskVectorArray`, `N_VScaleAddMultiVectorArray`, and `N_VLinearCombinationVectorArray`. If any of these operations are defined as `NULL` in an NVECTOR implementation the NVECTOR interface will automatically call standard NVECTOR operations as necessary. Details on the new operations can be found in Chapter 7.

Several changes were made to the build system. If MPI is enabled and MPI compiler wrappers are not set, the build system will check if `CMAKE_<language>_COMPILER` can compile MPI programs before trying to locate and use an MPI installation. The native CMake FindMPI module is now used to locate an MPI installation. The options for setting MPI compiler wrappers and the executable for running MPI programs have been updated to align with those in native CMake FindMPI module. This included changing `MPI_MPICC` to `MPI_C_COMPILER`, `MPI_MPICXX` to `MPI_CXX_COMPILER`, combining `MPI_MPIF77` and `MPI_MPIF90` to `MPI_Fortran_COMPILER`, and changing `MPI_RUN_COMMAND` to `MPIEXEC`. When a Fortran name-mangling scheme is needed (e.g., `LAPACK_ENABLE` is `ON`) the build system will infer the scheme from the Fortran compiler. If a Fortran compiler is not available or the inferred or default scheme needs to be overridden, the advanced options `SUNDIALS_F77_FUNC_CASE` and `SUNDIALS_F77_FUNC_UNDERSCORES` can be used to manually set the name-mangling scheme and bypass trying to infer the scheme. Additionally, parts of the main CMakeLists.txt file were moved to new files in the `src` and `example` directories to make the CMake configuration file structure more modular.

Changes in v2.1.1

The changes in this minor release include the following:

- Fixed a potential memory leak in the SPGMR and SPFGMR linear solvers: if "Initialize" was called multiple times then the solver memory was reallocated (without being freed).

- Updated KLU SUNLinearSolver module to use a `typedef` for the precision-specific solve function to be used (to avoid compiler warnings).

- Added missing typecasts for some (`void*`) pointers (again, to avoid compiler warnings).

- Bugfix in `sunmatrix_sparse.c` where we had used `int` instead of `sunindextype` in one location.

- Added missing `#include <stdio.h>` in NVECTOR and SUNMATRIX header files.

- Added missing prototype for `IDASpilsGetNumJTSetupEvals`.

- Fixed an indexing bug in the CUDA NVECTOR implementation of N_VWrmsNormMask and revised the RAJA NVECTOR implementation of N_VWrmsNormMask to work with mask arrays using values other than zero or one. Replaced `double` with `realtype` in the RAJA vector test functions.

In addition to the changes above, minor corrections were also made to the example programs, build system, and user documentation.

Changes in v2.1.0

Added NVECTOR print functions that write vector data to a specified file (e.g., N_VPrintFile_Serial).

Added `make test` and `make test_install` options to the build system for testing SUNDIALS after building with `make` and installing with `make install` respectively.

Changes in v2.0.0

All interfaces to matrix structures and linear solvers have been reworked, and all example programs have been updated. The goal of the redesign of these interfaces was to provide more encapsulation and to ease interfacing of custom linear solvers and interoperability with linear solver libraries. Specific changes include:

- Added generic SUNMATRIX module with three provided implementations: dense, banded and sparse. These replicate previous SUNDIALS Dls and Sls matrix structures in a single object-oriented API.

- Added example problems demonstrating use of generic SUNMATRIX modules.

- Added generic `SUNLinearSolver` module with eleven provided implementations: SUNDIALS native dense, SUNDIALS native banded, LAPACK dense, LAPACK band, KLU, SuperLU_MT, SPGMR, SPBCGS, SPTFQMR, SPFGMR, and PCG. These replicate previous SUNDIALS generic linear solvers in a single object-oriented API.

- Added example problems demonstrating use of generic `SUNLinearSolver` modules.

- Expanded package-provided direct linear solver (Dls) interfaces and scaled, preconditioned, iterative linear solver (Spils) interfaces to utilize generic SUNMATRIX and `SUNLinearSolver` objects.

- Removed package-specific, linear solver-specific, solver modules (e.g. `CVDENSE`, `KINBAND`, `IDAKLU`, `ARKSPGMR`) since their functionality is entirely replicated by the generic Dls/Spils interfaces and `SUNLinearSolver/SUNMATRIX` modules. The exception is `CVDIAG`, a diagonal approximate Jacobian solver available to CVODE and CVODES.

- Converted all SUNDIALS example problems and files to utilize the new generic SUNMATRIX and `SUNLinearSolver` objects, along with updated Dls and Spils linear solver interfaces.

- Added Spils interface routines to ARKODE, CVODE, CVODES, IDA, and IDAS to allow specification of a user-provided "JTSetup" routine. This change supports users who wish to set up data structures for the user-provided Jacobian-times-vector ("JTimes") routine, and where the cost of one JTSetup setup per Newton iteration can be amortized between multiple JTimes calls.

Two additional NVECTOR implementations were added – one for CUDA and one for RAJA vectors. These vectors are supplied to provide very basic support for running on GPU architectures. Users are advised that these vectors both move all data to the GPU device upon construction, and speedup will only be realized if the user also conducts the right-hand-side function evaluation on the device. In addition, these vectors assume the problem fits on one GPU. Further information about RAJA, users are referred to the web site, https://software.llnl.gov/RAJA/. These additions are accompanied by additions to various interface functions and to user documentation.

All indices for data structures were updated to a new `sunindextype` that can be configured to be a 32- or 64-bit integer data index type. `sunindextype` is defined to be `int32_t` or `int64_t` when

portable types are supported, otherwise it is defined as `int` or `long int`. The Fortran interfaces continue to use `long int` for indices, except for their sparse matrix interface that now uses the new `sunindextype`. This new flexible capability for index types includes interfaces to PETSc, hypre, SuperLU_MT, and KLU with either 32-bit or 64-bit capabilities depending how the user configures SUNDIALS.

To avoid potential namespace conflicts, the macros defining `booleantype` values `TRUE` and `FALSE` have been changed to `SUNTRUE` and `SUNFALSE` respectively.

Temporary vectors were removed from preconditioner setup and solve routines for all packages. It is assumed that all necessary data for user-provided preconditioner operations will be allocated and stored in user-provided data structures.

The file `include/sundials_fconfig.h` was added. This file contains SUNDIALS type information for use in Fortran programs.

The build system was expanded to support many of the xSDK-compliant keys. The xSDK is a movement in scientific software to provide a foundation for the rapid and efficient production of high-quality, sustainable extreme-scale scientific applications. More information can be found at, https://xsdk.info.

Added functions `SUNDIALSGetVersion` and `SUNDIALSGetVersionNumber` to get SUNDIALS release version information at runtime.

In addition, numerous changes were made to the build system. These include the addition of separate `BLAS_ENABLE` and `BLAS_LIBRARIES` CMake variables, additional error checking during CMake configuration, minor bug fixes, and renaming CMake options to enable/disable examples for greater clarity and an added option to enable/disable Fortran 77 examples. These changes included changing `EXAMPLES_ENABLE` to `EXAMPLES_ENABLE_C`, changing `CXX_ENABLE` to `EXAMPLES_ENABLE_CXX`, changing `F90_ENABLE` to `EXAMPLES_ENABLE_F90`, and adding an `EXAMPLES_ENABLE_F77` option.

A bug fix was done to add a missing prototype for `IDASetMaxBacksIC` in `ida.h`.

Corrections and additions were made to the examples, to installation-related files, and to the user documentation.

Changes in v1.3.0

Two additional NVECTOR implementations were added – one for Hypre (parallel) ParVector vectors, and one for PetSC vectors. These additions are accompanied by additions to various interface functions and to user documentation.

Each NVECTOR module now includes a function, `N_VGetVectorID`, that returns the NVECTOR module name.

An optional input function was added to set a maximum number of linesearch backtracks in the initial condition calculation, and four user-callable functions were added to support the use of LAPACK linear solvers in solving backward problems for adjoint sensitivity analysis.

For each linear solver, the various solver performance counters are now initialized to 0 in both the solver specification function and in solver `linit` function. This ensures that these solver counters are initialized upon linear solver instantiation as well as at the beginning of the problem solution.

A bug in for-loop indices was fixed in `IDAAckpntAllocVectors`. A bug was fixed in the interpolation functions used in solving backward problems.

A memory leak was fixed in the banded preconditioner interface. In addition, updates were done to return integers from linear solver and preconditioner 'free' functions.

In interpolation routines for backward problems, added logic to bypass sensitivity interpolation if input sensitivity argument is NULL.

The Krylov linear solver Bi-CGstab was enhanced by removing a redundant dot product. Various additions and corrections were made to the interfaces to the sparse solvers KLU and SuperLU_MT, including support for CSR format when using KLU.

New examples were added for use of the openMP vector and for use of sparse direct solvers within sensitivity integrations.

Minor corrections and additions were made to the IDAS solver, to the examples, to installation-related files, and to the user documentation.

Changes in v1.2.0

Two major additions were made to the linear system solvers that are available for use with the IDAS solver. First, in the serial case, an interface to the sparse direct solver KLU was added. Second, an interface to SuperLU_MT, the multi-threaded version of SuperLU, was added as a thread-parallel sparse direct solver option, to be used with the serial version of the NVECTOR module. As part of these additions, a sparse matrix (CSC format) structure was added to IDAS.

Otherwise, only relatively minor modifications were made to IDAS:

In `IDARootfind`, a minor bug was corrected, where the input array `rootdir` was ignored, and a line was added to break out of root-search loop if the initial interval size is below the tolerance `ttol`.

In `IDALapackBand`, the line `smu = MIN(N-1,mu+ml)` was changed to `smu = mu + ml` to correct an illegal input error for `DGBTRF/DGBTRS`.

An option was added in the case of Adjoint Sensitivity Analysis with dense or banded Jacobian: With a call to `IDADlsSetDenseJacFnBS` or `IDADlsSetBandJacFnBS`, the user can specify a user-supplied Jacobian function of type `IDADls***JacFnBS`, for the case where the backward problem depends on the forward sensitivities.

A minor bug was fixed regarding the testing of the input `tstop` on the first call to `IDASolve`.

For the Adjoint Sensitivity Analysis case in which the backward problem depends on the forward sensitivities, options have been added to allow for user-supplied `pset`, `psolve`, and `jtimes` functions.

In order to avoid possible name conflicts, the mathematical macro and function names `MIN`, `MAX`, `SQR`, `RAbs`, `RSqrt`, `RExp`, `RPowerI`, and `RPowerR` were changed to `SUNMIN`, `SUNMAX`, `SUNSQR`, `SUNRabs`, `SUNRsqrt`, `SUNRexp`, `SRpowerI`, and `SUNRpowerR`, respectively. These names occur in both the solver and in various example programs.

In the User Guide, a paragraph was added in Section 6.2.1 on `IDAAdjReInit`, and a paragraph was added in Section 6.2.9 on `IDAGetAdjY`.

Two new NVECTOR modules have been added for thread-parallel computing environments — one for openMP, denoted `NVECTOR_OPENMP`, and one for Pthreads, denoted `NVECTOR_PTHREADS`.

With this version of SUNDIALS, support and documentation of the Autotools mode of installation is being dropped, in favor of the CMake mode, which is considered more widely portable.

Changes in v1.1.0

One significant design change was made with this release: The problem size and its relatives, bandwidth parameters, related internal indices, pivot arrays, and the optional output `lsflag` have all been changed from type `int` to type `long int`, except for the problem size and bandwidths in user calls to routines specifying BLAS/LAPACK routines for the dense/band linear solvers. The function `NewIntArray` is replaced by a pair `NewIntArray/NewLintArray`, for `int` and `long int` arrays, respectively. In a minor change to the user interface, the type of the index `which` in IDAS was changed from `long int` to `int`.

Errors in the logic for the integration of backward problems were identified and fixed.

A large number of minor errors have been fixed. Among these are the following: A missing vector pointer setting was added in `IDASensLineSrch`. In `IDACompleteStep`, conditionals around lines loading a new column of three auxiliary divided difference arrays, for a possible order increase, were fixed. After the solver memory is created, it is set to zero before being filled. In each linear solver interface function, the linear solver memory is freed on an error return, and the `**Free` function now includes a line setting to NULL the main memory pointer to the linear solver memory. A memory leak was fixed in two of the `IDASp***Free` functions. In the rootfinding functions `IDARcheck1/IDARcheck2`, when an exact zero is found, the array `glo` of g values at the left endpoint is adjusted, instead of shifting the t location `tlo` slightly. In the installation files, we modified the treatment of the macro SUNDIALS_USE_GENERIC_MATH, so that the parameter GENERIC_MATH_LIB is either defined (with no value) or not defined.

1.2   Reading this User Guide

The structure of this document is as follows:

- In Chapter 2, we give short descriptions of the numerical methods implemented by IDAS for the solution of initial value problems for systems of DAEs, continue with short descriptions of preconditioning (§2.2) and rootfinding (§2.3), and then give an overview of the mathematical aspects of sensitivity analysis, both forward (§2.5) and adjoint (§2.6).

- The following chapter describes the structure of the SUNDIALS suite of solvers (§3.1) and the software organization of the IDAS solver (§3.2).

- Chapter 4 is the main usage document for IDAS for simulation applications. It includes a complete description of the user interface for the integration of DAE initial value problems. Readers that are not interested in using IDAS for sensitivity analysis can then skip the next two chapters.

- Chapter 5 describes the usage of IDAS for forward sensitivity analysis as an extension of its IVP integration capabilities. We begin with a skeleton of the user main program, with emphasis on the steps that are required in addition to those already described in Chapter 4. Following that we provide detailed descriptions of the user-callable interface routines specific to forward sensitivity analysis and of the additonal optional user-defined routines.

- Chapter 6 describes the usage of IDAS for adjoint sensitivity analysis. We begin by describing the IDAS checkpointing implementation for interpolation of the original IVP solution during integration of the adjoint system backward in time, and with an overview of a user's main program. Following that we provide complete descriptions of the user-callable interface routines for adjoint sensitivity analysis as well as descriptions of the required additional user-defined routines.

- Chapter 7 gives a brief overview of the generic NVECTOR module shared amongst the various components of SUNDIALS, as well as details on the NVECTOR implementations provided with SUNDIALS.

- Chapter 8 gives a brief overview of the generic SUNMATRIX module shared among the various components of SUNDIALS, and details on the SUNMATRIX implementations provided with SUNDIALS: a dense implementation (§8.1), a banded implementation (§8.2) and a sparse implementation (§8.3).

- Chapter 9 gives a brief overview of the generic SUNLINSOL module shared among the various components of SUNDIALS. This chapter contains details on the SUNLINSOL implementations provided with SUNDIALS. The chapter also contains details on the SUNLINSOL implementations provided with SUNDIALS that interface with external linear solver libraries.

- Finally, in the appendices, we provide detailed instructions for the installation of IDAS, within the structure of SUNDIALS (Appendix A), as well as a list of all the constants used for input to and output from IDAS functions (Appendix B).

Finally, the reader should be aware of the following notational conventions in this user guide: program listings and identifiers (such as `IDAInit`) within textual explanations appear in typewriter type style; fields in C structures (such as *content*) appear in italics; and packages or modules, such as IDADLS, are written in all capitals. Usage and installation instructions that constitute important warnings are marked with a triangular symbol in the margin.

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Chapter 2

Mathematical Considerations

IDAS solves the initial-value problem (IVP) for a DAE system of the general form

$$F(t, y, \dot{y}) = 0, \quad y(t_0) = y_0, \ \dot{y}(t_0) = \dot{y}_0, \tag{2.1}$$

where y, $\dot{y}$, and F are vectors in $\mathbf{R}^N$, t is the independent variable, $\dot{y} = dy/dt$, and initial values y_0, $\dot{y}_0$ are given. (Often t is time, but it certainly need not be.)

Additionally, if (2.1) depends on some parameters $p \in \mathbf{R}^{N_p}$, i.e.

$$\begin{aligned} F(t, y, \dot{y}, p) &= 0 \\ y(t_0) = y_0(p), \ \dot{y}(t_0) &= \dot{y}_0(p), \end{aligned} \tag{2.2}$$

IDAS can also compute first order derivative information, performing either *forward sensitivity analysis* or *adjoint sensitivity analysis*. In the first case, IDAS computes the sensitivities of the solution with respect to the parameters p, while in the second case, IDAS computes the gradient of a *derived function* with respect to the parameters p.

2.1 IVP solution

Prior to integrating a DAE initial-value problem, an important requirement is that the pair of vectors y_0 and $\dot{y}_0$ are both initialized to satisfy the DAE residual $F(t_0, y_0, \dot{y}_0) = 0$. For a class of problems that includes so-called semi-explicit index-one systems, IDAS provides a routine that computes consistent initial conditions from a user's initial guess [6]. For this, the user must identify sub-vectors of y (not necessarily contiguous), denoted y_d and y_a, which are its differential and algebraic parts, respectively, such that F depends on $\dot{y}_d$ but not on any components of $\dot{y}_a$. The assumption that the system is "index one" means that for a given t and y_d, the system $F(t, y, \dot{y}) = 0$ defines y_a uniquely. In this case, a solver within IDAS computes y_a and $\dot{y}_d$ at $t = t_0$, given y_d and an initial guess for y_a. A second available option with this solver also computes all of $y(t_0)$ given $\dot{y}(t_0)$; this is intended mainly for quasi-steady-state problems, where $\dot{y}(t_0) = 0$ is given. In both cases, IDA solves the system $F(t_0, y_0, \dot{y}_0) = 0$ for the unknown components of y_0 and $\dot{y}_0$, using Newton iteration augmented with a line search global strategy. In doing this, it makes use of the existing machinery that is to be used for solving the linear systems during the integration, in combination with certain tricks involving the step size (which is set artificially for this calculation). For problems that do not fall into either of these categories, the user is responsible for passing consistent values, or risks failure in the numerical integration.

The integration method used in IDAS is the variable-order, variable-coefficient BDF (Backward Differentiation Formula), in fixed-leading-coefficient form [3]. The method order ranges from 1 to 5, with the BDF of order q given by the multistep formula

$$\sum_{i=0}^{q} \alpha_{n,i} y_{n-i} = h_n \dot{y}_n, \tag{2.3}$$

where y_n and $\dot{y}_n$ are the computed approximations to $y(t_n)$ and $\dot{y}(t_n)$, respectively, and the step size is $h_n = t_n - t_{n-1}$. The coefficients $\alpha_{n,i}$ are uniquely determined by the order q, and the history of the step sizes. The application of the BDF (2.3) to the DAE system (2.1) results in a nonlinear algebraic system to be solved at each step:

$$G(y_n) \equiv F\left(t_n,\, y_n,\, h_n^{-1}\sum_{i=0}^{q}\alpha_{n,i}y_{n-i}\right) = 0\,. \tag{2.4}$$

Regardless of the method options, the solution of the nonlinear system (2.4) is accomplished with some form of Newton iteration. This leads to a linear system for each Newton correction, of the form

$$J[y_{n(m+1)} - y_{n(m)}] = -G(y_{n(m)})\,, \tag{2.5}$$

where $y_{n(m)}$ is the m-th approximation to y_n. Here J is some approximation to the system Jacobian

$$J = \frac{\partial G}{\partial y} = \frac{\partial F}{\partial y} + \alpha\frac{\partial F}{\partial \dot{y}}\,, \tag{2.6}$$

where $\alpha = \alpha_{n,0}/h_n$. The scalar α changes whenever the step size or method order changes.

For the solution of the linear systems within the Newton corrections, IDAS provides several choices, including the option of an user-supplied linear solver module. The linear solver modules distributed with SUNDIALS are organized in two families, a *direct* family comprising direct linear solvers for dense, banded, or sparse matrices and a *spils* family comprising scaled preconditioned iterative (Krylov) linear solvers. The methods offered through these modules are as follows:

- dense direct solvers, using either an internal implementation or a Blas/Lapack implementation (serial or threaded vector modules only),

- band direct solvers, using either an internal implementation or a Blas/Lapack implementation (serial or threaded vector modules only),

- sparse direct solver interfaces, using either the KLU sparse solver library [14, 1], or the thread-enabled SuperLU_MT sparse solver library [28, 15, 2] (serial or threaded vector modules only) [Note that users will need to download and install the KLU or SUPERLUMT packages independent of IDAS],

- SPGMR, a scaled preconditioned GMRES (Generalized Minimal Residual method) solver without restarts,

- SPFGMR, a scaled preconditioned FGMRES (Flexible Generalized Minimal Residual method) solver,

- SPBCGS, a scaled preconditioned Bi-CGStab (Bi-Conjugate Gradient Stable method) solver,

- SPTFQMR, a scaled preconditioned TFQMR (Transpose-Free Quasi-Minimal Residual method) solver, or

- PCG, a scaled preconditioned CG (Conjugate Gradient method) solver.

For large stiff systems, where direct methods are not feasible, the combination of a BDF integrator and any of the preconditioned Krylov methods (SPGMR, SPBCGS, or SPTFQMR) yields a powerful tool because it combines established methods for stiff integration, nonlinear iteration, and Krylov (linear) iteration with a problem-specific treatment of the dominant source of stiffness, in the form of the user-supplied preconditioner matrix [4]. For the *spils* linear solvers, preconditioning is allowed only on the left (see §2.2). Note that the dense, band, and sparse direct linear solvers can only be used with serial and threaded vector representations.

In the process of controlling errors at various levels, IDAS uses a weighted root-mean-square norm, denoted $\|\cdot\|_{\mathrm{WRMS}}$, for all error-like quantities. The multiplicative weights used are based on the current solution and on the relative and absolute tolerances input by the user, namely

$$W_i = 1/[\mathrm{RTOL} \cdot |y_i| + \mathrm{ATOL}_i]. \tag{2.7}$$

Because $1/W_i$ represents a tolerance in the component y_i, a vector whose norm is 1 is regarded as "small." For brevity, we will usually drop the subscript WRMS on norms in what follows.

In the case of a direct linear solver (dense, band, or sparse), the nonlinear iteration (2.5) is a Modified Newton iteration, in that the Jacobian J is fixed (and usually out of date), with a coefficient $\bar{\alpha}$ in place of α in J. When using one of the Krylov methods SPGMR, SPBCGS, or SPTFQMR as the linear solver, the iteration is an Inexact Newton iteration, using the current Jacobian (through matrix-free products Jv), in which the linear residual $J\Delta y + G$ is nonzero but controlled. The Jacobian matrix J (direct cases) or preconditioner matrix P (SPGMR/SPBCGS/SPTFQMR case) is updated when:

- starting the problem,

- the value $\bar{\alpha}$ at the last update is such that $\alpha/\bar{\alpha} < 3/5$ or $\alpha/\bar{\alpha} > 5/3$, or

- a non-fatal convergence failure occurred with an out-of-date J or P.

The above strategy balances the high cost of frequent matrix evaluations and preprocessing with the slow convergence due to infrequent updates. To reduce storage costs on an update, Jacobian information is always reevaluated from scratch.

We note that with the sparse direct solvers, the Jacobian *must* be supplied by a user routine in compressed-sparse-column format, as this is not approximated automatically within IDAS.

The stopping test for the Newton iteration in IDAS ensures that the iteration error $y_n - y_{n(m)}$ is small relative to y itself. For this, we estimate the linear convergence rate at all iterations $m > 1$ as

$$R = \left(\frac{\delta_m}{\delta_1}\right)^{\frac{1}{m-1}},$$

where the $\delta_m = y_{n(m)} - y_{n(m-1)}$ is the correction at iteration $m = 1, 2, \ldots$. The Newton iteration is halted if $R > 0.9$. The convergence test at the m-th iteration is then

$$S\|\delta_m\| < 0.33, \tag{2.8}$$

where $S = R/(R-1)$ whenever $m > 1$ and $R \le 0.9$. The user has the option of changing the constant in the convergence test from its default value of 0.33. The quantity S is set to $S = 20$ initially and whenever J or P is updated, and it is reset to $S = 100$ on a step with $\alpha \ne \bar{\alpha}$. Note that at $m = 1$, the convergence test (2.8) uses an old value for S. Therefore, at the first Newton iteration, we make an additional test and stop the iteration if $\|\delta_1\| < 0.33 \cdot 10^{-4}$ (since such a δ_1 is probably just noise and therefore not appropriate for use in evaluating R). We allow only a small number (default value 4) of Newton iterations. If convergence fails with J or P current, we are forced to reduce the step size h_n, and we replace h_n by $h_n/4$. The integration is halted after a preset number (default value 10) of convergence failures. Both the maximum allowable Newton iterations and the maximum nonlinear convergence failures can be changed by the user from their default values.

When SPGMR, SPBCGS, or SPTFQMR is used to solve the linear system, to minimize the effect of linear iteration errors on the nonlinear and local integration error controls, we require the preconditioned linear residual to be small relative to the allowed error in the Newton iteration, i.e., $\|P^{-1}(Jx + G)\| < 0.05 \cdot 0.33$. The safety factor 0.05 can be changed by the user.

In the direct linear solver cases, the Jacobian J defined in (2.6) can be either supplied by the user or have IDAS compute one internally by difference quotients. In the latter case, we use the approximation

$$J_{ij} = [F_i(t, y + \sigma_j e_j, \dot{y} + \alpha\sigma_j e_j) - F_i(t, y, \dot{y})]/\sigma_j, \text{ with}$$
$$\sigma_j = \sqrt{U} \max\{|y_j|, |h\dot{y}_j|, 1/W_j\} \operatorname{sign}(h\dot{y}_j),$$

where U is the unit roundoff, h is the current step size, and W_j is the error weight for the component y_j defined by (2.7). In the SPGMR/SPBCGS/SPTFQMR case, if a routine for Jv is not supplied, such products are approximated by

$$Jv = [F(t, y + \sigma v, \dot{y} + \alpha \sigma v) - F(t, y, \dot{y})]/\sigma \,,$$

where the increment σ is $1/\|v\|$. As an option, the user can specify a constant factor that is inserted into this expression for σ.

During the course of integrating the system, IDAS computes an estimate of the local truncation error, LTE, at the n-th time step, and requires this to satisfy the inequality

$$\|\text{LTE}\|_{\text{WRMS}} \leq 1 \,.$$

Asymptotically, LTE varies as h^{q+1} at step size h and order q, as does the predictor-corrector difference $\Delta_n \equiv y_n - y_{n(0)}$. Thus there is a constant C such that

$$\text{LTE} = C\Delta_n + O(h^{q+2}) \,,$$

and so the norm of LTE is estimated as $|C| \cdot \|\Delta_n\|$. In addition, IDAS requires that the error in the associated polynomial interpolant over the current step be bounded by 1 in norm. The leading term of the norm of this error is bounded by $\bar{C}\|\Delta_n\|$ for another constant $\bar{C}$. Thus the local error test in IDAS is

$$\max\{|C|, \bar{C}\}\|\Delta_n\| \leq 1 \,. \tag{2.9}$$

A user option is available by which the algebraic components of the error vector are omitted from the test (2.9), if these have been so identified.

In IDAS, the local error test is tightly coupled with the logic for selecting the step size and order. First, there is an initial phase that is treated specially; for the first few steps, the step size is doubled and the order raised (from its initial value of 1) on every step, until (a) the local error test (2.9) fails, (b) the order is reduced (by the rules given below), or (c) the order reaches 5 (the maximum). For step and order selection on the general step, IDAS uses a different set of local error estimates, based on the asymptotic behavior of the local error in the case of fixed step sizes. At each of the orders q' equal to q, $q-1$ (if $q > 1$), $q-2$ (if $q > 2$), or $q+1$ (if $q < 5$), there are constants $C(q')$ such that the norm of the local truncation error at order q' satisfies

$$\text{LTE}(q') = C(q')\|\phi(q' + 1)\| + O(h^{q'+2}) \,,$$

where $\phi(k)$ is a modified divided difference of order k that is retained by IDAS (and behaves asymptotically as h^k). Thus the local truncation errors are estimated as $\text{ELTE}(q') = C(q')\|\phi(q' + 1)\|$ to select step sizes. But the choice of order in IDAS is based on the requirement that the scaled derivative norms, $\|h^k y^{(k)}\|$, are monotonically decreasing with k, for k near q. These norms are again estimated using the $\phi(k)$, and in fact

$$\|h^{q'+1}y^{(q'+1)}\| \approx T(q') \equiv (q' + 1)\text{ELTE}(q') \,.$$

The step/order selection begins with a test for monotonicity that is made even *before* the local error test is performed. Namely, the order is reset to $q' = q-1$ if (a) $q = 2$ and $T(1) \leq T(2)/2$, or (b) $q > 2$ and $\max\{T(q-1), T(q-2)\} \leq T(q)$; otherwise $q' = q$. Next the local error test (2.9) is performed, and if it fails, the step is redone at order $q \leftarrow q'$ and a new step size h'. The latter is based on the h^{q+1} asymptotic behavior of $\text{ELTE}(q)$, and, with safety factors, is given by

$$\eta = h'/h = 0.9/[2\,\text{ELTE}(q)]^{1/(q+1)} \,.$$

The value of η is adjusted so that $0.25 \leq \eta \leq 0.9$ before setting $h \leftarrow h' = \eta h$. If the local error test fails a second time, IDAS uses $\eta = 0.25$, and on the third and subsequent failures it uses $q = 1$ and $\eta = 0.25$. After 10 failures, IDAS returns with a give-up message.

As soon as the local error test has passed, the step and order for the next step may be adjusted. No such change is made if $q' = q - 1$ from the prior test, if $q = 5$, or if q was increased on the previous step. Otherwise, if the last $q + 1$ steps were taken at a constant order $q < 5$ and a constant step size, IDAS considers raising the order to $q + 1$. The logic is as follows: (a) If $q = 1$, then reset $q = 2$ if $T(2) < T(1)/2$. (b) If $q > 1$ then

- reset $q \leftarrow q - 1$ if $T(q - 1) \leq \min\{T(q), T(q + 1)\}$;

- else reset $q \leftarrow q + 1$ if $T(q + 1) < T(q)$;

- leave q unchanged otherwise [then $T(q - 1) > T(q) \leq T(q + 1)$].

In any case, the new step size h' is set much as before:

$$\eta = h'/h = 1/[2\,\mathrm{ELTE}(q)]^{1/(q+1)}\,.$$

The value of η is adjusted such that (a) if $\eta > 2$, η is reset to 2; (b) if $\eta \leq 1$, η is restricted to $0.5 \leq \eta \leq 0.9$; and (c) if $1 < \eta < 2$ we use $\eta = 1$. Finally h is reset to $h' = \eta h$. Thus we do not increase the step size unless it can be doubled. See [3] for details.

IDAS permits the user to impose optional inequality constraints on individual components of the solution vector y. Any of the following four constraints can be imposed: $y_i > 0$, $y_i < 0$, $y_i \geq 0$, or $y_i \leq 0$. The constraint satisfaction is tested after a successful nonlinear system solution. If any constraint fails, we declare a convergence failure of the Newton iteration and reduce the step size. Rather than cutting the step size by some arbitrary factor, IDAS estimates a new step size h' using a linear approximation of the components in y that failed the constraint test (including a safety factor of 0.9 to cover the strict inequality case). These additional constraints are also imposed during the calculation of consistent initial conditions.

Normally, IDAS takes steps until a user-defined output value $t = t_{\mathrm{out}}$ is overtaken, and then computes $y(t_{\mathrm{out}})$ by interpolation. However, a "one step" mode option is available, where control returns to the calling program after each step. There are also options to force IDAS not to integrate past a given stopping point $t = t_{\mathrm{stop}}$.

2.2 Preconditioning

When using a Newton method to solve the nonlinear system (2.5), IDAS makes repeated use of a linear solver to solve linear systems of the form $J \Delta y = -G$. If this linear system solve is done with one of the scaled preconditioned iterative linear solvers, these solvers are rarely successful if used without preconditioning; it is generally necessary to precondition the system in order to obtain acceptable efficiency. A system $Ax = b$ can be preconditioned on the left, on the right, or on both sides. The Krylov method is then applied to a system with the matrix $P^{-1}A$, or AP^{-1}, or $P_L^{-1}AP_R^{-1}$, instead of A. However, within IDAS, preconditioning is allowed *only* on the left, so that the iterative method is applied to systems $(P^{-1}J)\Delta y = -P^{-1}G$. Left preconditioning is required to make the norm of the linear residual in the Newton iteration meaningful; in general, $\|J\Delta y + G\|$ is meaningless, since the weights used in the WRMS-norm correspond to y.

In order to improve the convergence of the Krylov iteration, the preconditioner matrix P should in some sense approximate the system matrix A. Yet at the same time, in order to be cost-effective, the matrix P should be reasonably efficient to evaluate and solve. Finding a good point in this tradeoff between rapid convergence and low cost can be very difficult. Good choices are often problem-dependent (for example, see [4] for an extensive study of preconditioners for reaction-transport systems).

Typical preconditioners used with IDAS are based on approximations to the Newton iteration matrix of the systems involved; in other words, $P \approx \frac{\partial F}{\partial y} + \alpha \frac{\partial F}{\partial \dot{y}}$, where α is a scalar inversely proportional to the integration step size h. Because the Krylov iteration occurs within a Newton iteration and further also within a time integration, and since each of these iterations has its own test for convergence, the preconditioner may use a very crude approximation, as long as it captures the dominant numerical feature(s) of the system. We have found that the combination of a preconditioner with the Newton-Krylov iteration, using even a fairly poor approximation to the Jacobian, can be surprisingly superior

to using the same matrix without Krylov acceleration (i.e., a modified Newton iteration), as well as to using the Newton-Krylov method with no preconditioning.

2.3   Rootfinding

The IDAS solver has been augmented to include a rootfinding feature. This means that, while integrating the Initial Value Problem (2.1), IDAS can also find the roots of a set of user-defined functions $g_i(t, y, \dot{y})$ that depend on t, the solution vector $y = y(t)$, and its t−derivative $\dot{y}(t)$. The number of these root functions is arbitrary, and if more than one g_i is found to have a root in any given interval, the various root locations are found and reported in the order that they occur on the t axis, in the direction of integration.

Generally, this rootfinding feature finds only roots of odd multiplicity, corresponding to changes in sign of $g_i(t, y(t), \dot{y}(t))$, denoted $g_i(t)$ for short. If a user root function has a root of even multiplicity (no sign change), it will probably be missed by IDAS. If such a root is desired, the user should reformulate the root function so that it changes sign at the desired root.

The basic scheme used is to check for sign changes of any $g_i(t)$ over each time step taken, and then (when a sign change is found) to home in on the root (or roots) with a modified secant method [19]. In addition, each time g is computed, IDAS checks to see if $g_i(t) = 0$ exactly, and if so it reports this as a root. However, if an exact zero of any g_i is found at a point t, IDAS computes g at $t + \delta$ for a small increment δ, slightly further in the direction of integration, and if any $g_i(t + \delta) = 0$ also, IDAS stops and reports an error. This way, each time IDAS takes a time step, it is guaranteed that the values of all g_i are nonzero at some past value of t, beyond which a search for roots is to be done.

At any given time in the course of the time-stepping, after suitable checking and adjusting has been done, IDAS has an interval $(t_{lo}, t_{hi}]$ in which roots of the $g_i(t)$ are to be sought, such that t_{hi} is further ahead in the direction of integration, and all $g_i(t_{lo}) \neq 0$. The endpoint t_{hi} is either t_n, the end of the time step last taken, or the next requested output time t_{out} if this comes sooner. The endpoint t_{lo} is either t_{n-1}, or the last output time t_{out} (if this occurred within the last step), or the last root location (if a root was just located within this step), possibly adjusted slightly toward t_n if an exact zero was found. The algorithm checks g at t_{hi} for zeros and for sign changes in (t_{lo}, t_{hi}). If no sign changes are found, then either a root is reported (if some $g_i(t_{hi}) = 0$) or we proceed to the next time interval (starting at t_{hi}). If one or more sign changes were found, then a loop is entered to locate the root to within a rather tight tolerance, given by

$$\tau = 100 * U * (|t_n| + |h|) \quad (U = \text{unit roundoff}) \ .$$

Whenever sign changes are seen in two or more root functions, the one deemed most likely to have its root occur first is the one with the largest value of $|g_i(t_{hi})|/|g_i(t_{hi}) - g_i(t_{lo})|$, corresponding to the closest to t_{lo} of the secant method values. At each pass through the loop, a new value t_{mid} is set, strictly within the search interval, and the values of $g_i(t_{mid})$ are checked. Then either t_{lo} or t_{hi} is reset to t_{mid} according to which subinterval is found to have the sign change. If there is none in (t_{lo}, t_{mid}) but some $g_i(t_{mid}) = 0$, then that root is reported. The loop continues until $|t_{hi} - t_{lo}| < \tau$, and then the reported root location is t_{hi}.

In the loop to locate the root of $g_i(t)$, the formula for t_{mid} is

$$t_{mid} = t_{hi} - (t_{hi} - t_{lo})g_i(t_{hi})/[g_i(t_{hi}) - \alpha g_i(t_{lo})] \ ,$$

where α a weight parameter. On the first two passes through the loop, α is set to 1, making t_{mid} the secant method value. Thereafter, α is reset according to the side of the subinterval (low vs high, i.e. toward t_{lo} vs toward t_{hi}) in which the sign change was found in the previous two passes. If the two sides were opposite, α is set to 1. If the two sides were the same, α is halved (if on the low side) or doubled (if on the high side). The value of t_{mid} is closer to t_{lo} when $\alpha < 1$ and closer to t_{hi} when $\alpha > 1$. If the above value of t_{mid} is within $\tau/2$ of t_{lo} or t_{hi}, it is adjusted inward, such that its fractional distance from the endpoint (relative to the interval size) is between .1 and .5 (.5 being the midpoint), and the actual distance from the endpoint is at least $\tau/2$.

2.4   Pure quadrature integration

In many applications, and most notably during the backward integration phase of an adjoint sensitivity analysis run (see §2.6) it is of interest to compute integral quantities of the form

$$z(t) = \int_{t_0}^t q(\tau, y(\tau), \dot{y}(\tau), p) \, d\tau \,. \tag{2.10}$$

The most effective approach to compute $z(t)$ is to extend the original problem with the additional ODEs (obtained by applying Leibnitz's differentiation rule):

$$\dot{z} = q(t, y, \dot{y}, p), \quad z(t_0) = 0 \,. \tag{2.11}$$

Note that this is equivalent to using a quadrature method based on the underlying linear multistep polynomial representation for $y(t)$.

This can be done at the "user level" by simply exposing to IDAS the extended DAE system (2.2)+(2.10). However, in the context of an implicit integration solver, this approach is not desirable since the nonlinear solver module will require the Jacobian (or Jacobian-vector product) of this extended DAE. Moreover, since the additional states, z, do not enter the right-hand side of the ODE (2.10) and therefore the residual of the extended DAE system does not depend on z, it is much more efficient to treat the ODE system (2.10) separately from the original DAE system (2.2) by "taking out" the additional states z from the nonlinear system (2.4) that must be solved in the correction step of the LMM. Instead, "corrected" values z_n are computed explicitly as

$$z_n = \frac{1}{\alpha_{n,0}} \left(h_n q(t_n, y_n, \dot{y}_n, p) - \sum_{i=1}^q \alpha_{n,i} z_{n-i} \right) \,,$$

once the new approximation y_n is available.

The quadrature variables z can be optionally included in the error test, in which case corresponding relative and absolute tolerances must be provided.

2.5   Forward sensitivity analysis

Typically, the governing equations of complex, large-scale models depend on various parameters, through the right-hand side vector and/or through the vector of initial conditions, as in (2.2). In addition to numerically solving the DAEs, it may be desirable to determine the sensitivity of the results with respect to the model parameters. Such sensitivity information can be used to estimate which parameters are most influential in affecting the behavior of the simulation or to evaluate optimization gradients (in the setting of dynamic optimization, parameter estimation, optimal control, etc.).

The *solution sensitivity* with respect to the model parameter p_i is defined as the vector $s_i(t) = \partial y(t)/\partial p_i$ and satisfies the following *forward sensitivity equations* (or *sensitivity equations* for short):

$$\begin{aligned} &\frac{\partial F}{\partial y} s_i + \frac{\partial F}{\partial \dot{y}} \dot{s}_i + \frac{\partial F}{\partial p_i} = 0 \\ &s_i(t_0) = \frac{\partial y_0(p)}{\partial p_i} \,, \ \dot{s}_i(t_0) = \frac{\partial \dot{y}_0(p)}{\partial p_i} \,, \end{aligned} \tag{2.12}$$

obtained by applying the chain rule of differentiation to the original DAEs (2.2).

When performing forward sensitivity analysis, IDAS carries out the time integration of the combined system, (2.2) and (2.12), by viewing it as a DAE system of size $N(N_s + 1)$, where N_s is the number of model parameters p_i, with respect to which sensitivities are desired ($N_s \le N_p$). However, major improvements in efficiency can be made by taking advantage of the special form of the sensitivity equations as linearizations of the original DAEs. In particular, the original DAE system and all sensitivity systems share the same Jacobian matrix J in (2.6).

The sensitivity equations are solved with the same linear multistep formula that was selected for the original DAEs and the same linear solver is used in the correction phase for both state and sensitivity variables. In addition, IDAS offers the option of including (*full error control*) or excluding (*partial error control*) the sensitivity variables from the local error test.

2.5.1 Forward sensitivity methods

In what follows we briefly describe three methods that have been proposed for the solution of the combined DAE and sensitivity system for the vector $\hat{y} = [y, s_1, \ldots, s_{N_s}]$.

- *Staggered Direct* In this approach [11], the nonlinear system (2.4) is first solved and, once an acceptable numerical solution is obtained, the sensitivity variables at the new step are found by directly solving (2.12) after the BDF discretization is used to eliminate $\dot{s}_i$. Although the system matrix of the above linear system is based on exactly the same information as the matrix J in (2.6), it must be updated and factored at every step of the integration, in contrast to an evaluation of J which is updated only occasionally. For problems with many parameters (relative to the problem size), the staggered direct method can outperform the methods described below [27]. However, the computational cost associated with matrix updates and factorizations makes this method unattractive for problems with many more states than parameters (such as those arising from semidiscretization of PDEs) and is therefore not implemented in IDAS.

- *Simultaneous Corrector* In this method [29], the discretization is applied simultaneously to both the original equations (2.2) and the sensitivity systems (2.12) resulting in an "extended" nonlinear system $\hat{G}(\hat{y}_n) = 0$ where $\hat{y}_n = [y_n, \ldots, s_i, \ldots]$. This combined nonlinear system can be solved using a modified Newton method as in (2.5) by solving the corrector equation

$$\hat{J}[\hat{y}_{n(m+1)} - \hat{y}_{n(m)}] = -\hat{G}(\hat{y}_{n(m)}) \tag{2.13}$$

at each iteration, where

$$\hat{J} = \begin{bmatrix} J & & & & \\ J_1 & J & & & \\ J_2 & 0 & J & & \\ \vdots & \vdots & \ddots & \ddots & \\ J_{N_s} & 0 & \ldots & 0 & J \end{bmatrix},$$

J is defined as in (2.6), and $J_i = (\partial/\partial y)\left[F_y s_i + F_{\dot{y}} \dot{s}_i + F_{p_i}\right]$. It can be shown that 2-step quadratic convergence can be retained by using only the block-diagonal portion of $\hat{J}$ in the corrector equation (2.13). This results in a decoupling that allows the reuse of J without additional matrix factorizations. However, the sum $F_y s_i + F_{\dot{y}} \dot{s}_i + F_{p_i}$ must still be reevaluated at each step of the iterative process (2.13) to update the sensitivity portions of the residual $\hat{G}$.

- *Staggered corrector* In this approach [16], as in the staggered direct method, the nonlinear system (2.4) is solved first using the Newton iteration (2.5). Then, for each sensitivity vector $\xi \equiv s_i$, a separate Newton iteration is used to solve the sensitivity system (2.12):

$$J[\xi_{n(m+1)} - \xi_{n(m)}] =$$
$$-\left[F_y(t_n, y_n, \dot{y}_n)\xi_{n(m)} + F_{\dot{y}}(t_n, y_n, \dot{y}_n) \cdot h_n^{-1}\left(\alpha_{n,0}\xi_{n(m)} + \sum_{i=1}^{q}\alpha_{n,i}\xi_{n-i}\right) + F_{p_i}(t_n, y_n, \dot{y}_n)\right].$$
$$\tag{2.14}$$

In other words, a modified Newton iteration is used to solve a linear system. In this approach, the matrices $\partial F/\partial y$, $\partial F/\partial \dot{y}$ and vectors $\partial F/\partial p_i$ need be updated only once per integration step, after the state correction phase (2.5) has converged.

IDAS implements both the simultaneous corrector method and the staggered corrector method.

An important observation is that the staggered corrector method, combined with a Krylov linear solver, effectively results in a staggered direct method. Indeed, the Krylov solver requires only the action of the matrix J on a vector, and this can be provided with the current Jacobian information. Therefore, the modified Newton procedure (2.14) will theoretically converge after one iteration.

2.5.2 Selection of the absolute tolerances for sensitivity variables

If the sensitivities are included in the error test, IDAS provides an automated estimation of absolute tolerances for the sensitivity variables based on the absolute tolerance for the corresponding state variable. The relative tolerance for sensitivity variables is set to be the same as for the state variables. The selection of absolute tolerances for the sensitivity variables is based on the observation that the sensitivity vector s_i will have units of $[y]/[p_i]$. With this, the absolute tolerance for the j-th component of the sensitivity vector s_i is set to $\text{ATOL}_j/|\bar{p}_i|$, where ATOL_j are the absolute tolerances for the state variables and $\bar{p}$ is a vector of scaling factors that are dimensionally consistent with the model parameters p and give an indication of their order of magnitude. This choice of relative and absolute tolerances is equivalent to requiring that the weighted root-mean-square norm of the sensitivity vector s_i with weights based on s_i be the same as the weighted root-mean-square norm of the vector of scaled sensitivities $\bar{s}_i = |\bar{p}_i|s_i$ with weights based on the state variables (the scaled sensitivities $\bar{s}_i$ being dimensionally consistent with the state variables). However, this choice of tolerances for the s_i may be a poor one, and the user of IDAS can provide different values as an option.

2.5.3 Evaluation of the sensitivity right-hand side

There are several methods for evaluating the residual functions in the sensitivity systems (2.12): analytic evaluation, automatic differentiation, complex-step approximation, and finite differences (or directional derivatives). IDAS provides all the software hooks for implementing interfaces to automatic differentiation (AD) or complex-step approximation; future versions will include a generic interface to AD-generated functions. At the present time, besides the option for analytical sensitivity right-hand sides (user-provided), IDAS can evaluate these quantities using various finite difference-based approximations to evaluate the terms $(\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i$ and $(\partial F/\partial p_i)$, or using directional derivatives to evaluate $[(\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i + (\partial F/\partial p_i)]$. As is typical for finite differences, the proper choice of perturbations is a delicate matter. IDAS takes into account several problem-related features: the relative DAE error tolerance RTOL, the machine unit roundoff U, the scale factor $\bar{p}_i$, and the weighted root-mean-square norm of the sensitivity vector s_i.

Using central finite differences as an example, the two terms $(\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i$ and $\partial F/\partial p_i$ in (2.12) can be evaluated either separately:

$$\frac{\partial F}{\partial y}s_i + \frac{\partial F}{\partial \dot{y}}\dot{s}_i \approx \frac{F(t, y + \sigma_y s_i, \dot{y} + \sigma_y \dot{s}_i, p) - F(t, y - \sigma_y s_i, \dot{y} - \sigma_y \dot{s}_i, p)}{2\,\sigma_y}\,, \tag{2.15}$$

$$\frac{\partial F}{\partial p_i} \approx \frac{F(t, y, \dot{y}, p + \sigma_i e_i) - F(t, y, \dot{y}, p - \sigma_i e_i)}{2\,\sigma_i}\,, \tag{2.15'}$$

$$\sigma_i = |\bar{p}_i|\sqrt{\max(\text{RTOL}, U)}\,, \quad \sigma_y = \frac{1}{\max(1/\sigma_i, \|s_i\|_{\text{WRMS}}/|\bar{p}_i|)}\,,$$

or simultaneously:

$$\frac{\partial F}{\partial y}s_i + \frac{\partial F}{\partial \dot{y}}\dot{s}_i + \frac{\partial F}{\partial p_i} \approx \frac{F(t, y + \sigma s_i, \dot{y} + \sigma \dot{s}_i, p + \sigma e_i) - F(t, y - \sigma s_i, \dot{y} - \sigma \dot{s}_i, p - \sigma e_i)}{2\,\sigma}\,, \tag{2.16}$$

$$\sigma = \min(\sigma_i, \sigma_y)\,,$$

or by adaptively switching between (2.15)+(2.15') and (2.16), depending on the relative size of the two finite difference increments σ_i and σ_y. In the adaptive scheme, if $\rho = \max(\sigma_i/\sigma_y, \sigma_y/\sigma_i)$, we use separate evaluations if $\rho > \rho_{\max}$ (an input value), and simultaneous evaluations otherwise.

These procedures for choosing the perturbations (σ_i, σ_y, σ) and switching between derivative formulas have also been implemented for one-sided difference formulas. Forward finite differences can be applied to $(\partial F/\partial y)s_i + (\partial F/\partial \dot{y})\dot{s}_i$ and $\frac{\partial F}{\partial p_i}$ separately, or the single directional derivative formula

$$\frac{\partial F}{\partial y}s_i + \frac{\partial F}{\partial \dot{y}}\dot{s}_i + \frac{\partial F}{\partial p_i} \approx \frac{F(t, y + \sigma s_i, \dot{y} + \sigma \dot{s}_i, p + \sigma e_i) - F(t, y, \dot{y}, p)}{\sigma}$$

can be used. In IDAS, the default value of $\rho_{\max} = 0$ indicates the use of the second-order centered directional derivative formula (2.16) exclusively. Otherwise, the magnitude of $\rho_{\max}$ and its sign (positive or negative) indicates whether this switching is done with regard to (centered or forward) finite differences, respectively.

2.5.4 Quadratures depending on forward sensitivities

If pure quadrature variables are also included in the problem definition (see §2.4), IDAS does *not* carry their sensitivities automatically. Instead, we provide a more general feature through which integrals depending on both the states y of (2.2) and the state sensitivities s_i of (2.12) can be evaluated. In other words, IDAS provides support for computing integrals of the form:

$$\bar{z}(t) = \int_{t_0}^{t} \bar{q}(\tau, y(\tau), \dot{y}(\tau), s_1(\tau), \ldots, s_{N_p}(\tau), p) \, d\tau \, .$$

If the sensitivities of the quadrature variables z of (2.10) are desired, these can then be computed by using:

$$\bar{q}_i = q_y s_i + q_{\dot{y}} \dot{s}_i + q_{p_i} \, , \quad i = 1, \ldots, N_p \, ,$$

as integrands for $\bar{z}$, where q_y, $q_{\dot{y}}$, and q_p are the partial derivatives of the integrand function q of (2.10).

As with the quadrature variables z, the new variables $\bar{z}$ are also excluded from any nonlinear solver phase and "corrected" values $\bar{z}_n$ are obtained through explicit formulas.

2.6 Adjoint sensitivity analysis

In the *forward sensitivity approach* described in the previous section, obtaining sensitivities with respect to N_s parameters is roughly equivalent to solving an DAE system of size $(1 + N_s)N$. This can become prohibitively expensive, especially for large-scale problems, if sensitivities with respect to many parameters are desired. In this situation, the *adjoint sensitivity method* is a very attractive alternative, provided that we do not need the solution sensitivities s_i, but rather the gradients with respect to model parameters of a relatively few derived functionals of the solution. In other words, if $y(t)$ is the solution of (2.2), we wish to evaluate the gradient dG/dp of

$$G(p) = \int_{t_0}^{T} g(t, y, p) dt \, , \tag{2.17}$$

or, alternatively, the gradient dg/dp of the function $g(t, y, p)$ at the final time $t = T$. The function g must be smooth enough that $\partial g/\partial y$ and $\partial g/\partial p$ exist and are bounded.

In what follows, we only sketch the analysis for the sensitivity problem for both G and g. For details on the derivation see [10].

2.6.1 Sensitivity of $G(p)$

We focus first on solving the sensitivity problem for $G(p)$ defined by (2.17). Introducing a Lagrange multiplier λ, we form the augmented objective function

$$I(p) = G(p) - \int_{t_0}^{T} \lambda^* F(t, y, \dot{y}, p) dt.$$

Since $F(t, y, \dot{y}, p) = 0$, the sensitivity of G with respect to p is

$$\frac{dG}{dp} = \frac{dI}{dp} = \int_{t_0}^{T} (g_p + g_y y_p) dt - \int_{t_0}^{T} \lambda^* (F_p + F_y y_p + F_{\dot{y}} \dot{y}_p) dt, \tag{2.18}$$

where subscripts on functions such as F or g are used to denote partial derivatives. By integration by parts, we have

$$\int_{t_0}^{T} \lambda^* F_{\dot{y}} \dot{y}_p dt = (\lambda^* F_{\dot{y}} y_p)|_{t_0}^{T} - \int_{t_0}^{T} (\lambda^* F_{\dot{y}})' y_p dt,$$

where $(\cdots)'$ denotes the $t-$derivative. Thus equation (2.18) becomes

$$\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p)\, dt - \int_{t_0}^{T} \left[-g_y + \lambda^* F_y - (\lambda^* F_{\dot{y}})' \right] y_p dt - (\lambda^* F_{\dot{y}} y_p)|_{t_0}^{T}. \qquad (2.19)$$

Now by requiring λ to satisfy

$$(\lambda^* F_{\dot{y}})' - \lambda^* F_y = -g_y, \qquad (2.20)$$

we obtain

$$\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p)\, dt - (\lambda^* F_{\dot{y}} y_p)|_{t_0}^{T}. \qquad (2.21)$$

Note that y_p at $t = t_0$ is the sensitivity of the initial conditions with respect to p, which is easily obtained. To find the initial conditions (at $t = T$) for the adjoint system, we must take into consideration the structure of the DAE system.

For index-0 and index-1 DAE systems, we can simply take

$$\lambda^* F_{\dot{y}}|_{t=T} = 0, \qquad (2.22)$$

yielding the sensitivity equation for dG/dp

$$\frac{dG}{dp} = \int_{t_0}^{T} (g_p - \lambda^* F_p)\, dt + (\lambda^* F_{\dot{y}} y_p)|_{t=t_0}. \qquad (2.23)$$

This choice will not suffice for a Hessenberg index-2 DAE system. For a derivation of proper final conditions in such cases, see [10].

The first thing to notice about the adjoint system (2.20) is that there is no explicit specification of the parameters p; this implies that, once the solution λ is found, the formula (2.21) can then be used to find the gradient of G with respect to any of the parameters p. The second important remark is that the adjoint system (2.20) is a terminal value problem which depends on the solution $y(t)$ of the original IVP (2.2). Therefore, a procedure is needed for providing the states y obtained during a forward integration phase of (2.2) to IDAS during the backward integration phase of (2.20). The approach adopted in IDAS, based on *checkpointing*, is described in §2.6.3 below.

2.6.2   Sensitivity of $g(T, p)$

Now let us consider the computation of $dg/dp(T)$. From $dg/dp(T) = (d/dT)(dG/dp)$ and equation (2.21), we have

$$\frac{dg}{dp} = (g_p - \lambda^* F_p)(T) - \int_{t_0}^{T} \lambda_T^* F_p dt + (\lambda_T^* F_{\dot{y}} y_p)|_{t=t_0} - \frac{d(\lambda^* F_{\dot{y}} y_p)}{dT} \qquad (2.24)$$

where λ_T denotes $\partial \lambda / \partial T$. For index-0 and index-1 DAEs, we obtain

$$\frac{d(\lambda^* F_{\dot{y}} y_p)|_{t=T}}{dT} = 0,$$

while for a Hessenberg index-2 DAE system we have

$$\frac{d(\lambda^* F_{\dot{y}} y_p)|_{t=T}}{dT} = - \left. \frac{d(g_{y^a}(CB)^{-1} f_p^2)}{dt} \right|_{t=T}.$$

The corresponding adjoint equations are

$$(\lambda_T^* F_{\dot{y}})' - \lambda_T^* F_y = 0. \tag{2.25}$$

For index-0 and index-1 DAEs (as shown above, the index-2 case is different), to find the boundary condition for this equation we write λ as $\lambda(t, T)$ because it depends on both t and T. Then

$$\lambda^*(T, T) F_{\dot{y}}|_{t=T} = 0.$$

Taking the total derivative, we obtain

$$(\lambda_t + \lambda_T)^*(T, T) F_{\dot{y}}|_{t=T} + \lambda^*(T, T) \frac{dF_{\dot{y}}}{dt}|_{t=T} = 0.$$

Since λ_t is just $\dot{\lambda}$, we have the boundary condition

$$(\lambda_T^* F_{\dot{y}})|_{t=T} = - \left[\lambda^*(T, T) \frac{dF_{\dot{y}}}{dt} + \dot{\lambda}^* F_{\dot{y}} \right]|_{t=T}.$$

For the index-one DAE case, the above relation and (2.20) yield

$$(\lambda_T^* F_{\dot{y}})|_{t=T} = [g_y - \lambda^* F_y]|_{t=T}. \tag{2.26}$$

For the regular implicit ODE case, $F_{\dot{y}}$ is invertible; thus we have $\lambda(T, T) = 0$, which leads to $\lambda_T(T) = -\dot{\lambda}(T)$. As with the final conditions for $\lambda(T)$ in (2.20), the above selection for $\lambda_T(T)$ is not sufficient for index-two Hessenberg DAEs (see [10] for details).

2.6.3   Checkpointing scheme

During the backward integration, the evaluation of the right-hand side of the adjoint system requires, at the current time, the states y which were computed during the forward integration phase. Since IDAS implements variable-step integration formulas, it is unlikely that the states will be available at the desired time and so some form of interpolation is needed. The IDAS implementation being also variable-order, it is possible that during the forward integration phase the order may be reduced as low as first order, which means that there may be points in time where only y and $\dot{y}$ are available. These requirements therefore limit the choices for possible interpolation schemes. IDAS implements two interpolation methods: a cubic Hermite interpolation algorithm and a variable-degree polynomial interpolation method which attempts to mimic the BDF interpolant for the forward integration.

However, especially for large-scale problems and long integration intervals, the number and size of the vectors y and $\dot{y}$ that would need to be stored make this approach computationally intractable. Thus, IDAS settles for a compromise between storage space and execution time by implementing a so-called *checkpointing scheme*. At the cost of at most one additional forward integration, this approach offers the best possible estimate of memory requirements for adjoint sensitivity analysis. To begin with, based on the problem size N and the available memory, the user decides on the number N_d of data pairs $(y, \dot{y})$ if cubic Hermite interpolation is selected, or on the number N_d of y vectors in the case of variable-degree polynomial interpolation, that can be kept in memory for the purpose of interpolation. Then, during the first forward integration stage, after every N_d integration steps a checkpoint is formed by saving enough information (either in memory or on disk) to allow for a hot restart, that is a restart which will exactly reproduce the forward integration. In order to avoid storing Jacobian-related data at each checkpoint, a reevaluation of the iteration matrix is forced before each checkpoint. At the end of this stage, we are left with N_c checkpoints, including one at t_0. During the backward integration stage, the adjoint variables are integrated backwards from T to t_0, going from one checkpoint to the previous one. The backward integration from checkpoint $i + 1$ to checkpoint i is preceded by a forward integration from i to $i + 1$ during which the N_d vectors y (and, if necessary $\dot{y}$) are generated and stored in memory for interpolation[1]

---

[1]The degree of the interpolation polynomial is always that of the current BDF order for the forward interpolation at
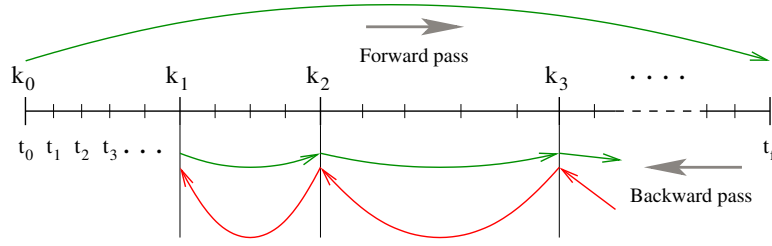
Figure 2.1: Illustration of the checkpointing algorithm for generation of the forward solution during the integration of the adjoint system.

This approach transfers the uncertainty in the number of integration steps in the forward integration phase to uncertainty in the final number of checkpoints. However, N_c is much smaller than the number of steps taken during the forward integration, and there is no major penalty for writing/reading the checkpoint data to/from a temporary file. Note that, at the end of the first forward integration stage, interpolation data are available from the last checkpoint to the end of the interval of integration. If no checkpoints are necessary (N_d is larger than the number of integration steps taken in the solution of (2.2)), the total cost of an adjoint sensitivity computation can be as low as one forward plus one backward integration. In addition, IDAS provides the capability of reusing a set of checkpoints for multiple backward integrations, thus allowing for efficient computation of gradients of several functionals (2.17).

Finally, we note that the adjoint sensitivity module in IDAS provides the necessary infrastructure to integrate backwards in time any DAE terminal value problem dependent on the solution of the IVP (2.2), including adjoint systems (2.20) or (2.25), as well as any other quadrature ODEs that may be needed in evaluating the integrals in (2.21). In particular, for DAE systems arising from semi-discretization of time-dependent PDEs, this feature allows for integration of either the discretized adjoint PDE system or the adjoint of the discretized PDE.

2.7 Second-order sensitivity analysis

In some applications (e.g., dynamically-constrained optimization) it may be desirable to compute second-order derivative information. Considering the DAE problem (2.2) and some model output functional[2] $g(y)$, the Hessian d^2g/dp^2 can be obtained in a forward sensitivity analysis setting as

$$\frac{d^2g}{dp^2} = \left(g_y \otimes I_{N_p}\right) y_{pp} + y_p^T g_{yy} y_p \,,$$

where $\otimes$ is the Kronecker product. The second-order sensitivities are solution of the matrix DAE system:

$$\left(F_{\dot{y}} \otimes I_{N_p}\right) \cdot \dot{y}_{pp} + \left(F_y \otimes I_{N_p}\right) \cdot y_{pp} + \left(I_N \otimes \dot{y}_p^T\right) \cdot (F_{\dot{y}\dot{y}}\dot{y}_p + F_{y\dot{y}}y_p) + \left(I_N \otimes y_p^T\right) \cdot (F_{y\dot{y}}\dot{y}_p + F_{yy}y_p) = 0$$

$$y_{pp}(t_0) = \frac{\partial^2 y_0}{\partial p^2} \,, \quad \dot{y}_{pp}(t_0) = \frac{\partial^2 \dot{y}_0}{\partial p^2} \,,$$

---

the first point to the right of the time at which the interpolated value is sought (unless too close to the i-th checkpoint, in which case it uses the BDF order at the right-most relevant point). However, because of the FLC BDF implementation (see §2.1), the resulting interpolation polynomial is only an approximation to the underlying BDF interpolant.

The Hermite cubic interpolation option is present because it was implemented chronologically first and it is also used by other adjoint solvers (e.g. DASPKADJOINT). The variable-degree polynomial is more memory-efficient (it requires only half of the memory storage of the cubic Hermite interpolation) and is more accurate.

[2]For the sake of simplicity in presentation, we do not include explicit dependencies of g on time t or parameters p. Moreover, we only consider the case in which the dependency of the original DAE (2.2) on the parameters p is through its initial conditions only. For details on the derivation in the general case, see [30].

where y_p denotes the first-order sensitivity matrix, the solution of N_p systems (2.12), and y_{pp} is a third-order tensor. It is easy to see that, except for situations in which the number of parameters N_p is very small, the computational cost of this so-called *forward-over-forward* approach is exorbitant as it requires the solution of $N_p + N_p^2$ additional DAE systems of the same dimension as (2.2).

A much more efficient alternative is to compute Hessian-vector products using a so-called *forward-over-adjoint* approach. This method is based on using the same "trick" as the one used in computing gradients of pointwise functionals with the adjoint method, namely applying a formal directional forward derivation to the gradient of (2.21) (or the equivalent one for a pointwise functional $g(T, y(T))$). With that, the cost of computing a full Hessian is roughly equivalent to the cost of computing the gradient with forward sensitivity analysis. However, Hessian-vector products can be cheaply computed with one additional adjoint solve.

As an illustration[3], consider the ODE problem

$$\dot{y} = f(t, y), \quad y(t_0) = y_0(p),$$

depending on some parameters p through the initial conditions only and consider the model functional output $G(p) = \int_{t_0}^{t_f} g(t, y)\, dt$. It can be shown that the product between the Hessian of G (with respect to the parameters p) and some vector u can be computed as

$$\frac{\partial^2 G}{\partial p^2} u = \left[\left(\lambda^T \otimes I_{N_p} \right) y_{pp} u + y_p^T \mu \right]_{t=t_0},$$

where λ and μ are solutions of

$$
\begin{aligned}
-\dot{\mu} &= f_y^T \mu + \left(\lambda^T \otimes I_n \right) f_{yy} s; \quad \mu(t_f) = 0 \\
-\dot{\lambda} &= f_y^T \lambda + g_y^T; \quad \lambda(t_f) = 0 \\
\dot{s} &= f_y s; \quad s(t_0) = y_{0p} u.
\end{aligned}
\tag{2.27}
$$

In the above equation, $s = y_p u$ is a linear combination of the columns of the sensitivity matrix y_p. The *forward-over-adjoint* approach hinges crucially on the fact that s can be computed at the cost of a forward sensitivity analysis with respect to a single parameter (the last ODE problem above) which is possible due to the linearity of the forward sensitivity equations (2.12).

Therefore (and this is also valid for the DAE case), the cost of computing the Hessian-vector product is roughly that of two forward and two backward integrations of a system of DAEs of size N. For more details, including the corresponding formulas for a pointwise model functional output, see the work by Ozyurt and Barton [30] who discuss this problem for ODE initial value problems. As far as we know, there is no published equivalent work on DAE problems. However, the derivations given in [30] for ODE problems can be extended to DAEs with some careful consideration given to the derivation of proper final conditions on the adjoint systems, following the ideas presented in [10].

To allow the *foward-over-adjoint* approach described above, IDAS provides support for:

- the integration of multiple backward problems depending on the same underlying forward problem (2.2), and

- the integration of backward problems and computation of backward quadratures depending on both the states y and forward sensitivities (for this particular application, s) of the original problem (2.2).

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[3]The derivation for the general DAE case is too involved for the purposes of this discussion.

Chapter 3

Code Organization

3.1 SUNDIALS organization

The family of solvers referred to as SUNDIALS consists of the solvers CVODE and ARKODE (for ODE systems), KINSOL (for nonlinear algebraic systems), and IDA (for differential-algebraic systems). In addition, SUNDIALS also includes variants of CVODE and IDA with sensitivity analysis capabilities (using either forward or adjoint methods), called CVODES and IDAS, respectively.

The various solvers of this family share many subordinate modules. For this reason, it is organized as a family, with a directory structure that exploits that sharing (see Figs. 3.1 and 3.2). The following is a list of the solver packages presently available, and the basic functionality of each:

- CVODE, a solver for stiff and nonstiff ODE systems $dy/dt = f(t, y)$ based on Adams and BDF methods;

- CVODES, a solver for stiff and nonstiff ODE systems with sensitivity analysis capabilities;

- ARKODE, a solver for ODE systems $M dy/dt = f_E(t, y) + f_I(t, y)$ based on additive Runge-Kutta methods;

- IDA, a solver for differential-algebraic systems $F(t, y, \dot{y}) = 0$ based on BDF methods;

- IDAS, a solver for differential-algebraic systems with sensitivity analysis capabilities;

- KINSOL, a solver for nonlinear algebraic systems $F(u) = 0$.

3.2 IDAS organization

The IDAS package is written in the ANSI C language. The following summarizes the basic structure of the package, although knowledge of this structure is not necessary for its use.

The overall organization of the IDAS package is shown in Figure 3.3. The central integration module, implemented in the files `idas.h`, `idas_impl.h`, and `idas.c`, deals with the evaluation of integration coefficients, the Newton iteration process, estimation of local error, selection of stepsize and order, and interpolation to user output points, among other issues. Although this module contains logic for the basic Newton iteration algorithm, it has no knowledge of the method being used to solve the linear systems that arise. For any given user problem, one of the linear system solver interfaces is specified, and is then invoked as needed during the integration.

In addition, if forward sensitivity analysis is turned on, the main module will integrate the forward sensitivity equations simultaneously with the original IVP. The sensitivity variables may be included in the local error control mechanism of the main integrator. IDAS provides two different strategies for dealing with the correction stage for the sensitivity variables: `IDA_SIMULTANEOUS IDA_STAGGERED` (see §2.5). The IDAS package includes an algorithm for the approximation of the sensitivity equations
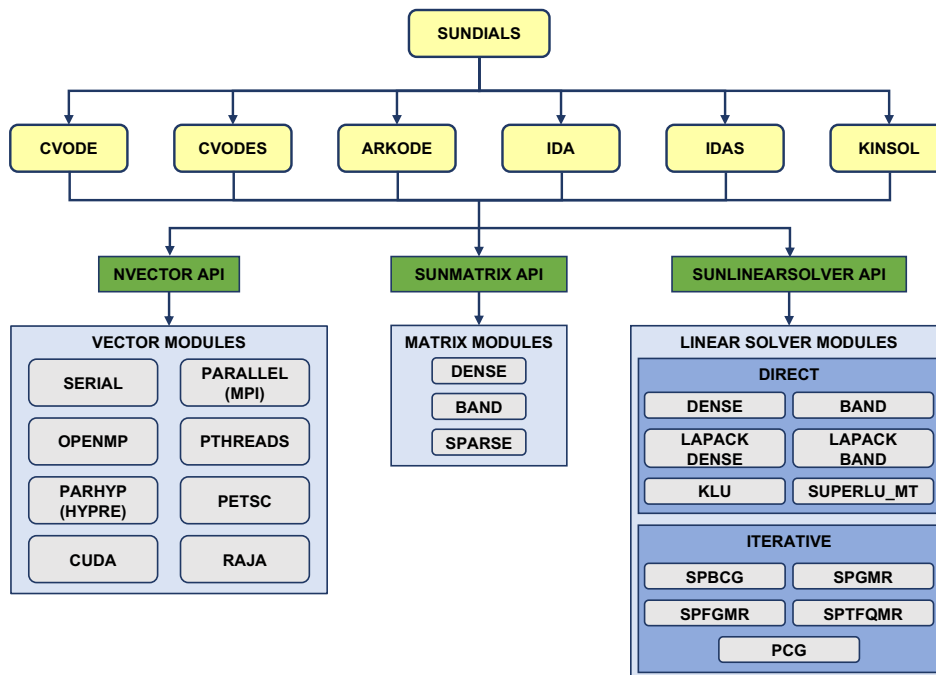
Figure 3.1: High-level diagram of the SUNDIALS suite

residuals by difference quotients, but the user has the option of supplying these residual functions directly.

The adjoint sensitivity module (file `idaa.c`) provides the infrastructure needed for the backward integration of any system of DAEs which depends on the solution of the original IVP, in particular the adjoint system and any quadratures required in evaluating the gradient of the objective functional. This module deals with the setup of the checkpoints, the interpolation of the forward solution during the backward integration, and the backward integration of the adjoint equations.

At present, the package includes two linear solver interfaces. The *direct* linear solver interface, IDADLS, supports SUNLINSOL implementations with type `SUNLINSOL_DIRECT` (see Chapter 9). These linear solvers utilize direct methods for the solution of linear systems stored using one of the SUN-DIALS generic SUNMATRIX implementations (dense, banded, or sparse; see Chapter 8). The *spils* linear solver interface, IDASPILS, supports SUNLINSOL implementations with type `SUNLINSOL_ITERATIVE` (see Chapter 9). These linear solvers utilize scaled preconditioned iterative methods. It is assumed that these methods are implemented in a "matrix-free" manner, wherein only the action of the matrix-vector product is required. Since IDAS can operate on any valid SUNLINSOL implementation of `SUNLINSOL_DIRECT` or `SUNLINSOL_ITERATIVE` types, the set of linear solver modules available to IDAS will expand as new SUNLINSOL modules are developed.

Within the IDADLS interface, the package includes algorithms for the approximation of dense or banded Jacobians through difference quotients, but the user also has the option of supplying the Jacobian (or an approximation to it) directly. This user-supplied routine is required when using sparse Jacobian matrices, since standard difference quotient approximations do not leverage the inherent sparsity of the problem.

Within the IDASPILS interface, the package includes an algorithm for the approximation by difference quotients of the product between the Jacobian matrix and a vector. Again, the user has the option of providing routines for this operation, in two phases: setup (preprocessing of Jacobian data) and multiplication. For preconditioned iterative methods, the preconditioning must be supplied by the user, again in two phases: setup and solve. While there is no default choice of preconditioner
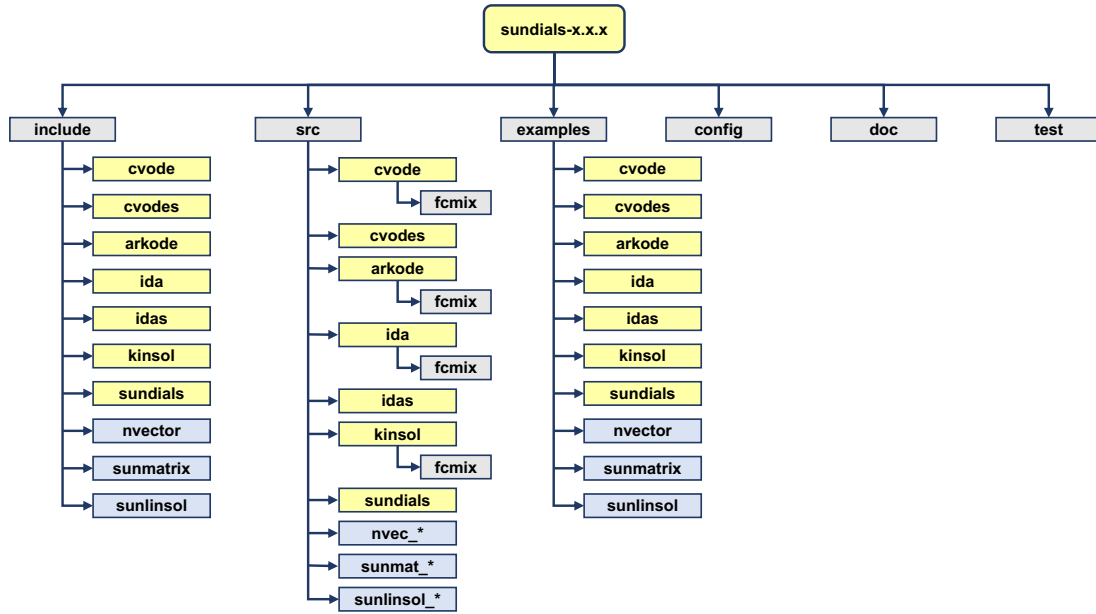
(a) Directory structure of the SUNDIALS source tree
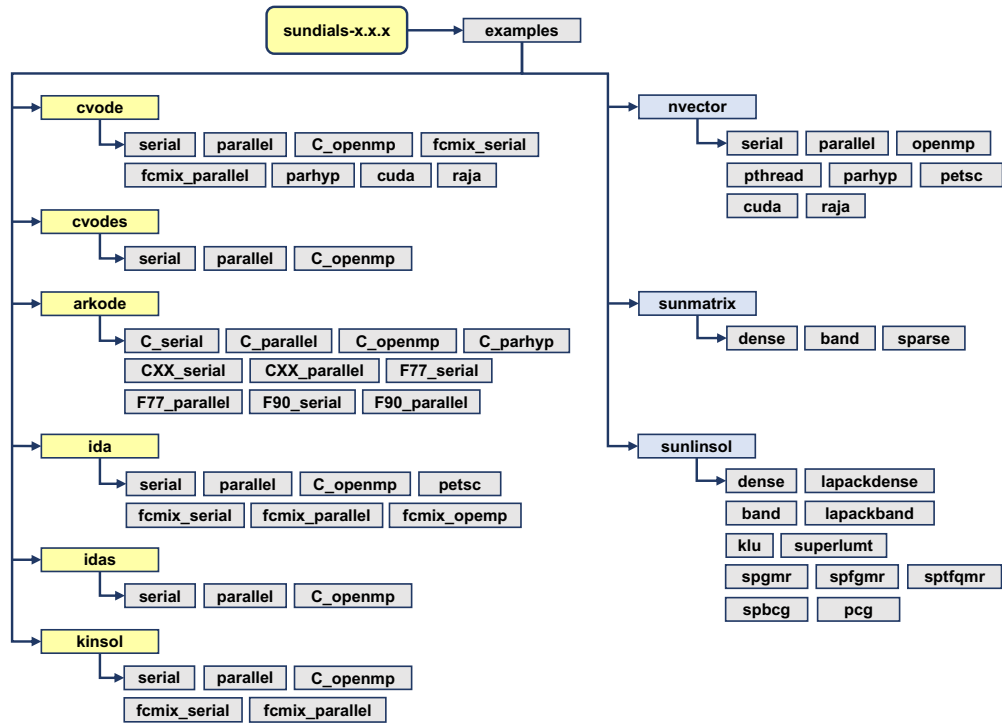
(b) Directory structure of the SUNDIALS examples

Figure 3.2: Organization of the SUNDIALS suite
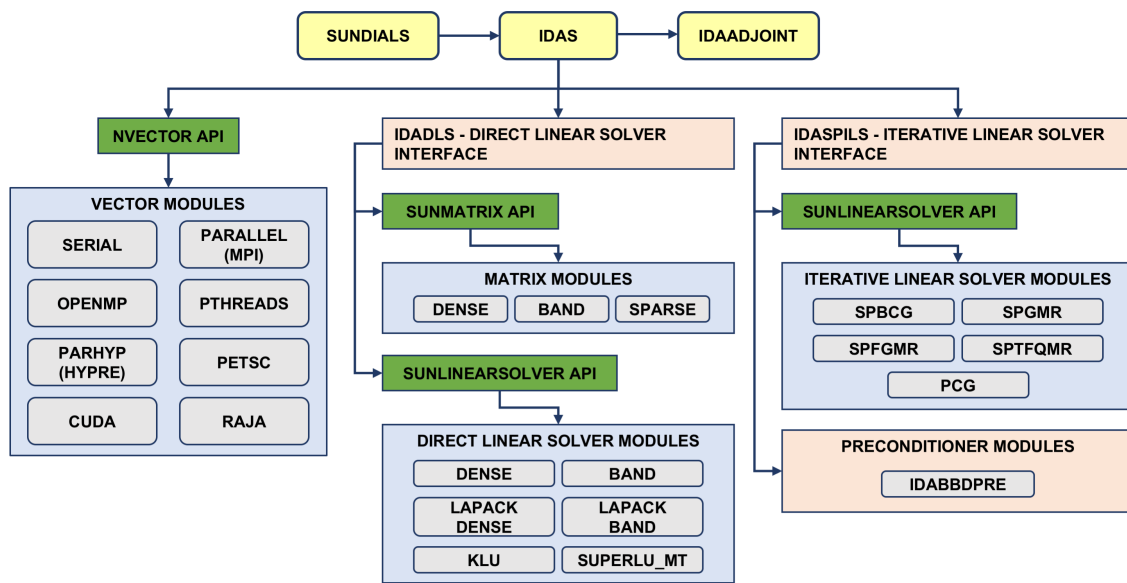
Figure 3.3: Overall structure diagram of the IDA package. Modules specific to IDA begin with "IDA" (IDADLS, IDASPILS, and IDABBDPRE), all other items correspond to generic solver and auxiliary modules. Note also that the LAPACK, KLU and SUPERLUMT support is through interfaces to external packages. Users will need to download and compile those packages independently.

analogous to the difference-quotient approximation in the direct case, the references [4, 7], together with the example and demonstration programs included with IDAS, offer considerable assistance in building preconditioners.

Each IDAS linear solver interface consists of four routines, devoted to (1) memory allocation and initialization, (2) setup of the matrix data involved, (3) solution of the system, and (4) freeing of memory. The setup and solution phases are separate because the evaluation of Jacobians and preconditioners is done only periodically during the integration, as required to achieve convergence. The call list within the central IDAS module to each of the four associated functions is fixed, thus allowing the central module to be completely independent of the linear system method.

These modules are also decomposed in another way. Each of the modules (IDADENSE etc.) consists of an interface built on top of a generic linear system solver (DENSE etc.). The interface deals with the use of the particular method in the IDAS context, whereas the generic solver is independent of the context. While some of the generic linear system solvers (DENSE, BAND, SPGMR, SPBCGS, and SPTFQMR) were written with SUNDIALS in mind, they are intended to be usable anywhere as general-purpose solvers. This separation also allows for any generic solver to be replaced by an improved version, with no necessity to revise the IDAS package elsewhere.

IDAS also provides a preconditioner module, IDABBDPRE, that works in conjunction with NVEC-TOR_PARALLEL and generates a preconditioner that is a block-diagonal matrix with each block being a banded matrix.

All state information used by IDAS to solve a given problem is saved in a structure, and a pointer to that structure is returned to the user. There is no global data in the IDAS package, and so, in this respect, it is reentrant. State information specific to the linear solver is saved in a separate structure, a pointer to which resides in the IDAS memory structure. The reentrancy of IDAS was motivated by the situation where two or more problems are solved by intermixed calls to the package from one user program.

Chapter 4

Using IDAS for IVP Solution

This chapter is concerned with the use of IDAS for the integration of DAEs. The following sections treat the header files, the layout of the user's main program, description of the IDAS user-callable functions, and description of user-supplied functions. This usage is essentially equivalent to using IDA [24].

The sample programs described in the companion document [35] may also be helpful. Those codes may be used as templates (with the removal of some lines involved in testing), and are included in the IDAS package.

The user should be aware that not all SUNLINSOL and SUNMATRIX modules are compatible with all NVECTOR implementations. Details on compatability are given in the documentation for each SUNMATRIX module (Chapter 8) and each SUNLINSOL module (Chapter 9). For example, NVECTOR_PARALLEL is not compatible with the dense, banded, or sparse SUNMATRIX types, or with the corresponding dense, banded, or sparse SUNLINSOL modules. Please check Chapters 8 and 9 to verify compatability between these modules. In addition to that documentation, we note that the preconditioner module IDABBDPRE can only be used with NVECTOR_PARALLEL. It is not recommended to use a threaded vector module with SuperLU_MT unless it is the NVECTOR_OPENMP module, and SuperLU_MT is also compiled with openMP.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

4.1 Access to library and header files

At this point, it is assumed that the installation of IDAS, following the procedure described in Appendix A, has been completed successfully.

Regardless of where the user's application program resides, its associated compilation and load commands must make reference to the appropriate locations for the library and header files required by IDAS. The relevant library files are

- *libdir*/**libsundials_idas.***lib*,

- *libdir*/**libsundials_nvec*.***lib* (one to four files),

where the file extension *.lib* is typically **.so** for shared libraries and **.a** for static libraries. The relevant header files are located in the subdirectories

- *incdir*/**include/idas**

- *incdir*/**include/sundials**

- *incdir*/**include/nvector**

- *incdir*/**include/sunmatrix**

- *incdir*/include/sunlinsol

The directories *libdir* and *incdir* are the install library and include directories, respectively. For a default installation, these are *instdir*/lib and *instdir*/include, respectively, where *instdir* is the directory where SUNDIALS was installed (see Appendix A).

Note that an application cannot link to both the IDA and IDAS libraries because both contain user-callable functions with the same names (to ensure that IDAS is backward compatible with IDA). Therefore, applications that contain both DAE problems and DAEs with sensitivity analysis, should use IDAS.

4.2   Data types

The sundials_types.h file contains the definition of the type realtype, which is used by the SUNDIALS solvers for all floating-point data, the definition of the integer type sunindextype, which is used for vector and matrix indices, and booleantype, which is used for certain logic operations within SUNDIALS.

4.2.1   Floating point types

The type realtype can be float, double, or long double, with the default being double. The user can change the precision of the SUNDIALS solvers arithmetic at the configuration stage (see §A.1.2).

Additionally, based on the current precision, sundials_types.h defines BIG_REAL to be the largest value representable as a realtype, SMALL_REAL to be the smallest value representable as a realtype, and UNIT_ROUNDOFF to be the difference between 1.0 and the minimum realtype greater than 1.0.

Within SUNDIALS, real constants are set by way of a macro called RCONST. It is this macro that needs the ability to branch on the definition realtype. In ANSI C, a floating-point constant with no suffix is stored as a double. Placing the suffix "F" at the end of a floating point constant makes it a float, whereas using the suffix "L" makes it a long double. For example,

```
#define A 1.0
#define B 1.0F
#define C 1.0L
```

defines A to be a double constant equal to 1.0, B to be a float constant equal to 1.0, and C to be a long double constant equal to 1.0. The macro call RCONST(1.0) automatically expands to 1.0 if realtype is double, to 1.0F if realtype is float, or to 1.0L if realtype is long double. SUNDIALS uses the RCONST macro internally to declare all of its floating-point constants.

A user program which uses the type realtype and the RCONST macro to handle floating-point constants is precision-independent except for any calls to precision-specific standard math library functions. (Our example programs use both realtype and RCONST.) Users can, however, use the type double, float, or long double in their code (assuming that this usage is consistent with the typedef for realtype). Thus, a previously existing piece of ANSI C code can use SUNDIALS without modifying the code to use realtype, so long as the SUNDIALS libraries use the correct precision (for details see §A.1.2).

4.2.2   Integer types used for vector and matrix indices

The type sunindextype can be either a 32- or 64-bit *signed* integer. The default is the portable int64_t type, and the user can change it to int32_t at the configuration stage. The configuration system will detect if the compiler does not support portable types, and will replace int32_t and int64_t with int and long int, respectively, to ensure use of the desired sizes on Linux, Mac OS X, and Windows platforms. SUNDIALS currently does not support *unsigned* integer types for vector and matrix indices, although these could be added in the future if there is sufficient demand.

A user program which uses sunindextype to handle vector and matrix indices will work with both index storage types except for any calls to index storage-specific external libraries. (Our C and C++

example programs use `sunindextype`.) Users can, however, use any one of `int`, `long int`, `int32_t`, `int64_t` or `long long int` in their code, assuming that this usage is consistent with the typedef for `sunindextype` on their architecture). Thus, a previously existing piece of ANSI C code can use SUNDIALS without modifying the code to use `sunindextype`, so long as the SUNDIALS libraries use the appropriate index storage type (for details see §A.1.2).

4.3 Header files

The calling program must include several header files so that various macros and data types can be used. The header file that is always required is:

- `idas.h`, the header file for IDAS, which defines the several types and various constants, and includes function prototypes.

Note that `idas.h` includes `sundials_types.h`, which defines the types `realtype`, `sunindextype`, and `booleantype` and the constants `SUNFALSE` and `SUNTRUE`.

The calling program must also include an NVECTOR implementation header file, of the form `nvector/nvector_***.h`. See Chapter 7 for the appropriate name. This file in turn includes the header file `sundials_nvector.h` which defines the abstract `N_Vector` data type.

Finally, a linear solver module header file is required. The header files corresponding to the various linear solver interfaces and linear solver modules available for use with IDAS are:

- `idas/idas_direct.h`, which is used with the IDADLS direct linear solver interface to access direct solvers with the following header files:

  - `sunlinsol/sunlinsol_dense.h`, which is used with the dense linear solver module, SUN-LINSOL_DENSE;

  - `sunlinsol/sunlinsol_band.h`, which is used with the banded linear solver module, SUN-LINSOL_BAND;

  - `sunlinsol/sunlinsol_lapackdense.h`, which is used with the LAPACK dense linear solver interface module, SUNLINSOL_LAPACKDENSE;

  - `sunlinsol/sunlinsol_lapackband.h`, which is used with the LAPACK banded linear solver interface module, SUNLINSOL_LAPACKBAND;

  - `sunlinsol/sunlinsol_klu.h`, which is used with the KLU sparse linear solver interface module, SUNLINSOL_KLU;

  - `sunlinsol/sunlinsol_superlumt.h`, which is used with the SUPERLUMT sparse linear solver interface module, SUNLINSOL_SUPERLUMT;

- `idas/idas_spils.h`, which is used with the IDASPILS iterative linear solver interface to access iterative solvers with the following header files:

  - `sunlinsol/sunlinsol_spgmr.h`, which is used with the scaled, preconditioned GMRES Krylov linear solver module, SUNLINSOL_SPGMR;

  - `sunlinsol/sunlinsol_spfgmr.h`, which is used with the scaled, preconditioned FGMRES Krylov linear solver module, SUNLINSOL_SPFGMR;

  - `sunlinsol/sunlinsol_spbcgs.h`, which is used with the scaled, preconditioned Bi-CGStab Krylov linear solver module, SUNLINSOL_SPBCGS;

  - `sunlinsol/sunlinsol_sptfqmr.h`, which is used with the scaled, preconditioned TFQMR Krylov linear solver module, SUNLINSOL_SPTFQMR;

  - `sunlinsol/sunlinsol_pcg.h`, which is used with the scaled, preconditioned CG Krylov linear solver module, SUNLINSOL_PCG;

The header files for the SUNLINSOL_DENSE and SUNLINSOL_LAPACKDENSE linear solver modules include the file `sunmatrix/sunmatrix_dense.h`, which defines the SUNMATRIX_DENSE matrix module, as as well as various functions and macros acting on such matrices.

The header files for the SUNLINSOL_BAND and SUNLINSOL_LAPACKBAND linear solver modules include the file `sunmatrix/sunmatrix_band.h`, which defines the SUNMATRIX_BAND matrix module, as as well as various functions and macros acting on such matrices.

The header files for the SUNLINSOL_KLU and SUNLINSOL_SUPERLUMT sparse linear solvers include the file `sunmatrix/sunmatrix_sparse.h`, which defines the SUNMATRIX_SPARSE matrix module, as well as various functions and macros acting on such matrices.

The header files for the Krylov iterative solvers include the file `sundials/sundials_iterative.h`, which enumerates the kind of preconditioning, and (for the SPGMR and SPFGMR solvers) the choices for the Gram-Schmidt process.

Other headers may be needed, according to the choice of preconditioner, etc. For example, in the `idasFoodWeb_kry_p` example (see [35]), preconditioning is done with a block-diagonal matrix. For this, even though the SUNLINSOL_SPGMR linear solver is used, the header `sundials/sundials_dense.h` is included for access to the underlying generic dense matrix arithmetic routines.

4.4   A skeleton of the user's main program

The following is a skeleton of the user's main program (or calling program) for the integration of a DAE IVP. Most of the steps are independent of the NVECTOR, SUNMATRIX, and SUNLINSOL implementations used. For the steps that are not, refer to Chapter 7, 8, and 9 for the specific name of the function to be called or macro to be referenced.

1. **Initialize parallel or multi-threaded environment, if appropriate**

   For example, call `MPI_Init` to initialize MPI if used, or set `num_threads`, the number of threads to use within the threaded vector functions, if used.

2. **Set problem dimensions etc.**

   This generally includes the problem size `N`, and may include the local vector length `Nlocal`.

   Note: The variables `N` and `Nlocal` should be of type `sunindextype`.

3. **Set vectors of initial values**

   To set the vectors `y0` and `yp0` to initial values for y and $\dot{y}$, use the appropriate functions defined by the particular NVECTOR implementation.

   For native SUNDIALS vector implementations (except the CUDA and RAJA-based ones), use a call of the form `y0 = N_VMake_***(..., ydata)` if the `realtype` array `ydata` containing the initial values of y already exists. Otherwise, create a new vector by making a call of the form `y0 = N_VNew_***(...)`, and then set its elements by accessing the underlying data with a call of the form `ydata = N_VGetArrayPointer(y0)`. See §7.1-7.4 for details.

   For the *hypre* and PETSc vector wrappers, first create and initialize the underlying vector and then create an NVECTOR wrapper with a call of the form `y0 = N_VMake_***(yvec)`, where `yvec` is a *hypre* or PETSc vector. Note that calls like `N_VNew_***(...)` and `N_VGetArrayPointer(...)` are not available for these vector wrappers. See §7.5 and §7.6 for details.

   If using either the CUDA- or RAJA-based vector implementations use a call of the form `y0 = N_VMake_***(..., c)` where `c` is a pointer to a `suncudavec` or `sunrajavec` vector class if this class already exists. Otherwise, create a new vector by making a call of the form `y0 = N_VNew_***(...)`, and then set its elements by accessing the underlying data where it is located with a call of the form `N_VGetDeviceArrayPointer_***` or `N_VGetHostArrayPointer_***`. Note that the vector class will allocate memory on both the host and device when instantiated. See §7.7-7.8 for details.

   Set the vector `yp0` of initial conditions for $\dot{y}$ similarly.

4. **Create IDAS object**

   Call ida_mem = IDACreate() to create the IDAS memory block. IDACreate returns a pointer to the IDAS memory structure. See §4.5.1 for details. This void * pointer must then be passed as the first argument to all subsequent IDAS function calls.

5. **Initialize IDAS solver**

   Call IDAInit(...) to provide required problem specifications (residual function, initial time, and initial conditions), allocate internal memory for IDAS, and initialize IDAS. IDAInit returns an error flag to indicate success or an illegal argument value. See §4.5.1 for details.

6. **Specify integration tolerances**

   Call IDASStolerances(...) or IDASVtolerances(...) to specify, respectively, a scalar relative tolerance and scalar absolute tolerance, or a scalar relative tolerance and a vector of absolute tolerances. Alternatively, call IDAWFtolerances to specify a function which sets directly the weights used in evaluating WRMS vector norms. See §4.5.2 for details.

7. **Set optional inputs**

   Optionally, call IDASet* functions to change from their default values any optional inputs that control the behavior of IDAS. See §4.5.7.1 for details.

8. **Create matrix object**

   If a direct linear solver is to be used within a Newton iteration then a template Jacobian matrix must be created by using the appropriate functions defined by the particular SUNMATRIX implementation.

   NOTE: The dense, banded, and sparse matrix objects are usable only in a serial or threaded environment.

9. **Create linear solver object**

   If a Newton iteration is chosen, then the desired linear solver object must be created by using the appropriate functions defined by the particular SUNLINSOL implementation.

10. **Set linear solver optional inputs**

    Optionally, call IDA*Set* functions from the selected linear solver module to change optional inputs specific to that linear solver. See §4.5.7.2 and §4.5.7.3 for details.

11. **Attach linear solver module**

    If a Newton iteration is chosen, initialize the IDADLS or IDASPILS linear solver interface by attaching the linear solver object (and matrix object, if applicable) with one of the following calls (for details see §4.5.3):

    ier = IDADlsSetLinearSolver(...);

    ier = IDASpilsSetLinearSolver(...);

12. **Set linear solver interface optional inputs**

    Call IDADlsSet* or IDASpilsSet* functions to change optional inputs specific to that linear solver interface. See §4.5.7 for details.

13. **Correct initial values**

    Optionally, call IDACalcIC to correct the initial values y0 and yp0 passed to IDAInit. See §4.5.4. Also see §4.5.7.4 for relevant optional input calls.

14. **Specify rootfinding problem**

Optionally, call `IDARootInit` to initialize a rootfinding problem to be solved during the integration of the DAE system. See §4.5.5 for details, and see §4.5.7.5 for relevant optional input calls.

15. **Advance solution in time**

    For each point at which output is desired, call `flag = IDASolve(ida_mem, tout, &tret, yret, ypret, itask)`. Here `itask` specifies the return mode. The vector `yret` (which can be the same as the vector `y0` above) will contain $y(t)$, while the vector `ypret` (which can be the same as the vector `yp0` above) will contain $\dot{y}(t)$. See §4.5.6 for details.

16. **Get optional outputs**

    Call `IDA*Get*` functions to obtain optional output. See §4.5.9 for details.

17. **Deallocate memory for solution vectors**

    Upon completion of the integration, deallocate memory for the vectors `yret` and `ypret` (or `y` and `yp`) by calling the appropriate destructor function defined by the NVECTOR implementation:

    `N_VDestroy(yret);`

    and similarly for `ypret`.

18. **Free solver memory**

    `IDAFree(&ida_mem)` to free the memory allocated for IDAS.

19. **Free linear solver and matrix memory**

    Call `SUNLinSolFree` and `SUNMatDestroy` to free any memory allocated for the linear solver and matrix objects created above.

20. **Finalize MPI, if used**

    Call `MPI_Finalize()` to terminate MPI.

SUNDIALS provides some linear solvers only as a means for users to get problems running and not as highly efficient solvers. For example, if solving a dense system, we suggest using the Lapack solvers if the size of the linear system is $> 50,000$. (Thanks to A. Nicolai for his testing and recommendation.) Table 4.1 shows the linear solver interfaces available as SUNLINSOL modules and the vector implementations required for use. As an example, one cannot use the dense direct solver interfaces with the MPI-based vector implementation. However, as discussed in Chapter 9 the SUNDIALS packages operate on generic SUNLINSOL objects, allowing a user to develop their own solvers should they so desire.

4.5   User-callable functions

This section describes the IDAS functions that are called by the user to set up and solve a DAE. Some of these are required. However, starting with §4.5.7, the functions listed involve optional inputs/outputs or restarting, and those paragraphs can be skipped for a casual use of IDAS. In any case, refer to §4.4 for the correct order of these calls.

On an error, each user-callable function returns a negative value and sends an error message to the error handler routine, which prints the message on `stderr` by default. However, the user can set a file as error output or can provide his own error handler function (see §4.5.7.1).

4.5.1   IDAS initialization and deallocation functions

The following three functions must be called in the order listed. The last one is to be called only after the DAE solution is complete, as it frees the IDAS memory block created and allocated by the first two calls.

Table 4.1: SUNDIALS linear solver interfaces and vector implementations that can be used for each.

Linear Solver	Serial	Parallel (MPI)	OpenMP	pThreads	hypre	PETSc	CUDA	RAJA	User Supp.
Dense	✓		✓	✓					✓
Band	✓		✓	✓					✓
LapackDense	✓		✓	✓					✓
LapackBand	✓		✓	✓					✓
KLU	✓		✓	✓					✓
SUPERLUMT	✓		✓	✓					✓
SPGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
SPFGMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
SPBCGS	✓	✓	✓	✓	✓	✓	✓	✓	✓
SPTFQMR	✓	✓	✓	✓	✓	✓	✓	✓	✓
PCG	✓	✓	✓	✓	✓	✓	✓	✓	✓
User Supp.	✓	✓	✓	✓	✓	✓	✓	✓	✓

`IDACreate`

Call   `ida_mem = IDACreate();`

Description   The function `IDACreate` instantiates an IDAS solver object.

Arguments   `IDACreate` has no arguments.

Return value If successful, `IDACreate` returns a pointer to the newly created IDAS memory block (of type `void *`). Otherwise it returns `NULL`.

`IDAInit`

Call   `flag = IDAInit(ida_mem, res, t0, y0, yp0);`

Description   The function `IDAInit` provides required problem and solution specifications, allocates internal memory, and initializes IDAS.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.

              `res`   (`IDAResFn`) is the C function which computes the residual function F in the DAE. This function has the form `res(t, yy, yp, resval, user_data)`. For full details see §4.6.1.

              `t0`   (`realtype`) is the initial value of t.

              `y0`   (`N_Vector`) is the initial value of y.

              `yp0`   (`N_Vector`) is the initial value of $\dot{y}$.

Return value The return value `flag` (of type `int`) will be one of the following:

              `IDA_SUCCESS`   The call to `IDAInit` was successful.

              `IDA_MEM_NULL`   The IDAS memory block was not initialized through a previous call to `IDACreate`.

              `IDA_MEM_FAIL`   A memory allocation request has failed.

              `IDA_ILL_INPUT` An input argument to `IDAInit` has an illegal value.

Notes   If an error occurred, `IDAInit` also sends an error message to the error handler function.

IDAFree

Call   `IDAFree(&ida_mem);`

Description   The function `IDAFree` frees the pointer allocated by a previous call to `IDACreate`.

Arguments   The argument is the pointer to the IDAS memory block (of type `void *`).

Return value The function `IDAFree` has no return value.

4.5.2   IDAS tolerance specification functions

One of the following three functions must be called to specify the integration tolerances (or directly specify the weights used in evaluating WRMS vector norms). Note that this call must be made after the call to `IDAInit`.

IDASStolerances

Call   `flag = IDASStolerances(ida_mem, reltol, abstol);`

Description   The function `IDASStolerances` specifies scalar relative and absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
              `reltol` (`realtype`) is the scalar relative error tolerance.
              `abstol` (`realtype`) is the scalar absolute error tolerance.

Return value The return value `flag` (of type `int`) will be one of the following:

              IDA_SUCCESS   The call to `IDASStolerances` was successful.
              IDA_MEM_NULL   The IDAS memory block was not initialized through a previous call to `IDACreate`.
              IDA_NO_MALLOC The allocation function `IDAInit` has not been called.
              IDA_ILL_INPUT One of the input tolerances was negative.

IDASVtolerances

Call   `flag = IDASVtolerances(ida_mem, reltol, abstol);`

Description   The function `IDASVtolerances` specifies scalar relative tolerance and vector absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
              `reltol` (`realtype`) is the scalar relative error tolerance.
              `abstol` (`N_Vector`) is the vector of absolute error tolerances.

Return value The return value `flag` (of type `int`) will be one of the following:

              IDA_SUCCESS   The call to `IDASVtolerances` was successful.
              IDA_MEM_NULL   The IDAS memory block was not initialized through a previous call to `IDACreate`.
              IDA_NO_MALLOC The allocation function `IDAInit` has not been called.
              IDA_ILL_INPUT The relative error tolerance was negative or the absolute tolerance had a negative component.

Notes   This choice of tolerances is important when the absolute error tolerance needs to be different for each component of the state vector y.

| IDAWFtolerances |

Call   `flag = IDAWFtolerances(ida_mem, efun);`

Description   The function `IDAWFtolerances` specifies a user-supplied function `efun` that sets the multiplicative error weights W_i for use in the weighted RMS norm, which are normally defined by Eq. (2.7).

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.

             `efun`   (`IDAEwtFn`) is the C function which defines the `ewt` vector (see §4.6.3).

Return value The return value `flag` (of type `int`) will be one of the following:

             IDA_SUCCESS   The call to `IDAWFtolerances` was successful.

             IDA_MEM_NULL   The IDAS memory block was not initialized through a previous call to `IDACreate`.

             IDA_NO_MALLOC   The allocation function `IDAInit` has not been called.

General advice on choice of tolerances. For many users, the appropriate choices for tolerance values in `reltol` and `abstol` are a concern. The following pieces of advice are relevant.

(1) The scalar relative tolerance `reltol` is to be set to control relative errors. So `reltol`$=10^{-4}$ means that errors are controlled to .01%. We do not recommend using `reltol` larger than 10^{-3}. On the other hand, `reltol` should not be so small that it is comparable to the unit roundoff of the machine arithmetic (generally around 10^{-15}).

(2) The absolute tolerances `abstol` (whether scalar or vector) need to be set to control absolute errors when any components of the solution vector `y` may be so small that pure relative error control is meaningless. For example, if `y[i]` starts at some nonzero value, but in time decays to zero, then pure relative error control on `y[i]` makes no sense (and is overly costly) after `y[i]` is below some noise level. Then `abstol` (if scalar) or `abstol[i]` (if a vector) needs to be set to that noise level. If the different components have different noise levels, then `abstol` should be a vector. See the example `idasRoberts_dns` in the IDAS package, and the discussion of it in the IDAS Examples document [35]. In that problem, the three components vary between 0 and 1, and have different noise levels; hence the `abstol` vector. It is impossible to give any general advice on `abstol` values, because the appropriate noise levels are completely problem-dependent. The user or modeler hopefully has some idea as to what those noise levels are.

(3) Finally, it is important to pick all the tolerance values conservatively, because they control the error committed on each individual time step. The final (global) errors are a sort of accumulation of those per-step errors. A good rule of thumb is to reduce the tolerances by a factor of .01 from the actual desired limits on errors. So if you want .01% accuracy (globally), a good choice is `reltol`$= 10^{-6}$. But in any case, it is a good idea to do a few experiments with the tolerances to see how the computed solution values vary as tolerances are reduced.

Advice on controlling unphysical negative values. In many applications, some components in the true solution are always positive or non-negative, though at times very small. In the numerical solution, however, small negative (hence unphysical) values can then occur. In most cases, these values are harmless, and simply need to be controlled, not eliminated. The following pieces of advice are relevant.

(1) The way to control the size of unwanted negative computed values is with tighter absolute tolerances. Again this requires some knowledge of the noise level of these components, which may or may not be different for different components. Some experimentation may be needed.

(2) If output plots or tables are being generated, and it is important to avoid having negative numbers appear there (for the sake of avoiding a long explanation of them, if nothing else), then eliminate them, but only in the context of the output medium. Then the internal values carried by the solver are unaffected. Remember that a small negative value in `yret` returned by IDAS, with magnitude comparable to `abstol` or less, is equivalent to zero as far as the computation is concerned.

(3) The user's residual routine `res` should never change a negative value in the solution vector `yy` to a non-negative value, as a "solution" to this problem. This can cause instability. If the `res` routine cannot tolerate a zero or negative value (e.g. because there is a square root or log of it), then the

offending value should be changed to zero or a tiny positive number in a temporary variable (not in the input yy vector) for the purposes of computing $F(t, y, \dot{y})$.

(4) IDAS provides the option of enforcing positivity or non-negativity on components. Also, such constraints can be enforced by use of the recoverable error return feature in the user-supplied residual function. However, because these options involve some extra overhead cost, they should only be exercised if the use of absolute tolerances to control the computed values is unsuccessful.

4.5.3 Linear solver interface functions

As previously explained, a Newton iteration requires the solution of linear systems of the form (2.5). There are two IDAS linear solvers currently available for this task: IDADLS and IDASPILS.

The first corresponds to the use of Direct Linear Solvers, and utilizes SUNMATRIX objects to store the Jacobian $J = \partial F/\partial y + \alpha \partial F/\partial \dot{y}$ and factorizations used throughout the solution process.

The second corresponds to the use of Scaled, Preconditioned, Iterative Linear Solvers, utilizing matrix-free Krylov methods to solve the Newton linear systems of equations. With most of these methods, preconditioning can be done on the left only, on the right only, on both the left and the right, or not at all. The exceptions to this rule are SPFGMR that supports right preconditioning only and PCG that performs symmetric preconditioning. For the specification of a preconditioner, see the iterative linear solver sections in §4.5.7 and §4.6. A preconditioner matrix P must approximate the Jacobian J, at least crudely.

To specify a generic linear solver to IDAS, after the call to IDACreate but before any calls to IDASolve, the user's program must create the appropriate SUNLINSOL object and call either of the functions IDADlsSetLinearSolver or IDASpilsSetLinearSolver, as documented below. The first argument passed to these functions is the IDAS memory pointer returned by IDACreate; the second argument passed to these functions is the desired SUNLINSOL object to use for solving Newton systems. A call to one of these functions initializes the appropriate IDAS linear solver interface, linking this to the main IDAS integrator, and allows the user to specify parameters which are specific to a particular solver interface. The use of each of the generic linear solvers involves certain constants and possibly some macros, that are likely to be needed in the user code. These are available in the corresponding header file associated with the specific SUNMATRIX or SUNLINSOL module in question, as described in Chapters 8 and 9.

IDADlsSetLinearSolver

Call	`flag = IDADlsSetLinearSolver(ida_mem, LS, J);`
Description	The function `IDADlsSetLinearSolver` attaches a direct SUNLINSOL object LS and corresponding template Jacobian SUNMATRIX object J to IDAS, initializing the IDADLS direct linear solver interface.
	The user's main program must include the `idas_direct.h` header file.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`LS` (`SUNLinearSolver`) SUNLINSOL object to use for solving Newton linear systems.
	`J` (`SUNMatrix`) SUNMATRIX object for used as a template for the Jacobian (must have a type compatible with the linear solver object).
Return value	The return value `flag` (of type `int`) is one of
	`IDADLS_SUCCESS` The IDADLS initialization was successful.
	`IDADLS_MEM_NULL` The `ida_mem` pointer is NULL.
	`IDADLS_ILL_INPUT` The IDADLS solver is not compatible with the current NVECTOR module.
	`IDADLS_MEM_FAIL` A memory allocation request failed.
Notes	The IDADLS linear solver is not compatible with all implementations of the SUNLINSOL and NVECTOR modules. Specifically, IDADLS requires use of a *direct* SUNLINSOL object

and a serial or theaded NVECTOR module. Additional compatibility limitations for each SUNLINSOL object (i.e. SUNMATRIX and NVECTOR object compatibility) are described in Chapter 9.

IDASpilsSetLinearSolver

Call   `flag = IDASpilsSetLinearSolver(ida_mem, LS);`

Description   The function `IDASpilsSetLinearSolver` attaches an iterative SUNLINSOL object `LS` to IDAS, initializing the IDASPILS scaled, preconditioned, iterative linear solver interface.

             The user's main program must include the `idas_spils.h` header file.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

             `LS`   (`SUNLinearSolver`) SUNLINSOL object to use for solving Newton linear systems.

Return value   The return value `flag` (of type `int`) is one of

             `IDASPILS_SUCCESS` The IDASPILS initialization was successful.

             `IDASPILS_MEM_NULL` The `ida_mem` pointer is `NULL`.

             `IDASPILS_ILL_INPUT` The IDASPILS solver is not compatible with the `LS` object or is incompatible with the current NVECTOR module.

             `IDASPILS_MEM_FAIL` A memory allocation request failed.

             `IDASPILS_SUNLS_FAIL` A call to the `LS` object failed.

Notes   The IDASPILS linear solver interface is not compatible with all implementations of the SUNLINSOL and NVECTOR modules. Specifically, IDASPILS requires use of an *iterative* SUNLINSOL object. Additional compatibility limitations for each SUNLINSOL object (i.e. required NVECTOR routines) are described in Chapter 9.

4.5.4   Initial condition calculation function

`IDACalcIC` calculates corrected initial conditions for the DAE system for certain index-one problems including a class of systems of semi-implicit form. (See §2.1 and Ref. [6].) It uses Newton iteration combined with a linesearch algorithm. Calling `IDACalcIC` is optional. It is only necessary when the initial conditions do not satisfy the given system. Thus if `y0` and `yp0` are known to satisfy $F(t_0, y_0, \dot{y}_0) = 0$, then a call to `IDACalcIC` is generally *not* necessary.

     A call to the function `IDACalcIC` must be preceded by successful calls to `IDACreate` and `IDAInit` (or `IDAReInit`), and by a successful call to the linear system solver specification function. The call to `IDACalcIC` should precede the call(s) to `IDASolve` for the given problem.

IDACalcIC

Call   `flag = IDACalcIC(ida_mem, icopt, tout1);`

Description   The function `IDACalcIC` corrects the initial values `y0` and `yp0` at time `t0`.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

             `icopt`   (`int`) is one of the following two options for the initial condition calculation.

                     `icopt=IDA_YA_YDP_INIT` directs `IDACalcIC` to compute the algebraic components of y and differential components of $\dot{y}$, given the differential components of y. This option requires that the `N_Vector id` was set through `IDASetId`, specifying the differential and algebraic components.

                     `icopt=IDA_Y_INIT` directs `IDACalcIC` to compute all components of y, given $\dot{y}$. In this case, `id` is not required.

             `tout1`   (`realtype`) is the first value of t at which a solution will be requested (from `IDASolve`). This value is needed here only to determine the direction of integration and rough scale in the independent variable t.

Return value   The return value `flag` (of type `int`) will be one of the following:

IDA_SUCCESS	IDASolve succeeded.
IDA_MEM_NULL	The argument `ida_mem` was NULL.
IDA_NO_MALLOC	The allocation function `IDAInit` has not been called.
IDA_ILL_INPUT	One of the input arguments was illegal.
IDA_LSETUP_FAIL	The linear solver's setup function failed in an unrecoverable manner.
IDA_LINIT_FAIL	The linear solver's initialization function failed.
IDA_LSOLVE_FAIL	The linear solver's solve function failed in an unrecoverable manner.
IDA_BAD_EWT	Some component of the error weight vector is zero (illegal), either for the input value of `y0` or a corrected value.
IDA_FIRST_RES_FAIL	The user's residual function returned a recoverable error flag on the first call, but `IDACalcIC` was unable to recover.
IDA_RES_FAIL	The user's residual function returned a nonrecoverable error flag.
IDA_NO_RECOVERY	The user's residual function, or the linear solver's setup or solve function had a recoverable error, but `IDACalcIC` was unable to recover.
IDA_CONSTR_FAIL	`IDACalcIC` was unable to find a solution satisfying the inequality constraints.
IDA_LINESEARCH_FAIL	The linesearch algorithm failed to find a solution with a step larger than `steptol` in weighted RMS norm, and within the allowed number of backtracks.
IDA_CONV_FAIL	`IDACalcIC` failed to get convergence of the Newton iterations.

Notes   All failure return values are negative and therefore a test `flag < 0` will trap all `IDACalcIC` failures.

Note that `IDACalcIC` will correct the values of $y(t_0)$ and $\dot{y}(t_0)$ which were specified in the previous call to `IDAInit` or `IDAReInit`. To obtain the corrected values, call `IDAGetconsistentIC` (see §4.5.9.3).

4.5.5   Rootfinding initialization function

While integrating the IVP, IDAS has the capability of finding the roots of a set of user-defined functions. To activate the rootfinding algorithm, call the following function. This is normally called only once, prior to the first call to `IDASolve`, but if the rootfinding problem is to be changed during the solution, `IDARootInit` can also be called prior to a continuation call to `IDASolve`.

IDARootInit

Call   `flag = IDARootInit(ida_mem, nrtfn, g);`

Description   The function `IDARootInit` specifies that the roots of a set of functions $g_i(t, y, \dot{y})$ are to be found while the IVP is being solved.

Arguments   `ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.

                `nrtfn`   (int) is the number of root functions g_i.

                `g`   (IDARootFn) is the C function which defines the `nrtfn` functions $g_i(t, y, \dot{y})$ whose roots are sought. See §4.6.4 for details.

Return value   The return value `flag` (of type `int`) is one of

IDA_SUCCESS	The call to `IDARootInit` was successful.
IDA_MEM_NULL	The `ida_mem` argument was NULL.
IDA_MEM_FAIL	A memory allocation failed.

IDA_ILL_INPUT The function g is NULL, but nrtfn> 0.

Notes   If a new IVP is to be solved with a call to IDAReInit, where the new IVP has no rootfinding problem but the prior one did, then call IDARootInit with nrtfn= 0.

4.5.6 IDAS solver function

This is the central step in the solution process, the call to perform the integration of the DAE. One of the input arguments (itask) specifies one of two modes as to where IDAS is to return a solution. But these modes are modified if the user has set a stop time (with IDASetStopTime) or requested rootfinding.

IDASolve

Call   `flag = IDASolve(ida_mem, tout, &tret, yret, ypret, itask);`

Description The function IDASolve integrates the DAE over an interval in t.

Arguments ida_mem (void *) pointer to the IDAS memory block.

      tout (realtype) the next time at which a computed solution is desired.

      tret (realtype) the time reached by the solver (output).

      yret (N_Vector) the computed solution vector y.

      ypret (N_Vector) the computed solution vector $\dot{y}$.

      itask (int) a flag indicating the job of the solver for the next user step. The IDA_NORMAL task is to have the solver take internal steps until it has reached or just passed the user specified tout parameter. The solver then interpolates in order to return approximate values of $y(\text{tout})$ and $\dot{y}(\text{tout})$. The IDA_ONE_STEP option tells the solver to just take one internal step and return the solution at the point reached by that step.

Return value IDASolve returns vectors yret and ypret and a corresponding independent variable value $t = $ tret, such that (yret, ypret) are the computed values of $(y(t), \dot{y}(t))$.

      In IDA_NORMAL mode with no errors, tret will be equal to tout and yret $= y(\text{tout})$, ypret $= \dot{y}(\text{tout})$.

      The return value flag (of type int) will be one of the following:

      IDA_SUCCESS   IDASolve succeeded.

      IDA_TSTOP_RETURN IDASolve succeeded by reaching the stop point specified through the optional input function IDASetStopTime.

      IDA_ROOT_RETURN IDASolve succeeded and found one or more roots. In this case, tret is the location of the root. If nrtfn > 1, call IDAGetRootInfo to see which g_i were found to have a root. See §4.5.9.4 for more information.

      IDA_MEM_NULL   The ida_mem argument was NULL.

      IDA_ILL_INPUT   One of the inputs to IDASolve was illegal, or some other input to the solver was either illegal or missing. The latter category includes the following situations: (a) The tolerances have not been set. (b) A component of the error weight vector became zero during internal time-stepping. (c) The linear solver initialization function (called by the user after calling IDACreate) failed to set the linear solver-specific lsolve field in ida_mem. (d) A root of one of the root functions was found both at a point t and also very near t. In any case, the user should see the printed error message for details.

      IDA_TOO_MUCH_WORK The solver took mxstep internal steps but could not reach tout. The default value for mxstep is MXSTEP_DEFAULT = 500.

IDA_TOO_MUCH_ACC	The solver could not satisfy the accuracy demanded by the user for some internal step.
IDA_ERR_FAIL	Error test failures occurred too many times (MXNEF = 10) during one internal time step or occurred with $|h| = h_{min}$.
IDA_CONV_FAIL	Convergence test failures occurred too many times (MXNCF = 10) during one internal time step or occurred with $|h| = h_{min}$.
IDA_LINIT_FAIL	The linear solver's initialization function failed.
IDA_LSETUP_FAIL	The linear solver's setup function failed in an unrecoverable manner.
IDA_LSOLVE_FAIL	The linear solver's solve function failed in an unrecoverable manner.
IDA_CONSTR_FAIL	The inequality constraints were violated and the solver was unable to recover.
IDA_REP_RES_ERR	The user's residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.
IDA_RES_FAIL	The user's residual function returned a nonrecoverable error flag.
IDA_RTFUNC_FAIL	The rootfinding function failed.

Notes   The vector yret can occupy the same space as the vector y0 of initial conditions that was passed to IDAInit, and the vector ypret can occupy the same space as yp0.

In the IDA_ONE_STEP mode, tout is used on the first call only, and only to get the direction and rough scale of the independent variable.

All failure return values are negative and therefore a test flag < 0 will trap all IDASolve failures.

On any error return in which one or more internal steps were taken by IDASolve, the returned values of tret, yret, and ypret correspond to the farthest point reached in the integration. On all other error returns, these values are left unchanged from the previous IDASolve return.

4.5.7   Optional input functions

There are numerous optional input parameters that control the behavior of the IDAS solver. IDAS provides functions that can be used to change these optional input parameters from their default values. Table 4.2 lists all optional input functions in IDAS which are then described in detail in the remainder of this section. For the most casual use of IDAS, the reader can skip to §4.6.

We note that, on an error return, all these functions also send an error message to the error handler function. We also note that all error return values are negative, so a test flag < 0 will catch any error.

4.5.7.1   Main solver optional input functions

The calls listed here can be executed in any order. However, if the user's program calls either IDASetErrFile or IDASetErrHandlerFn, then that call should appear first, in order to take effect for any later error message.

‖ IDASetErrFile ‖

Call   flag = IDASetErrFile(ida_mem, errfp);

Description   The function IDASetErrFile specifies the pointer to the file where all IDAS messages should be directed when the default IDAS error handler function is used.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
              errfp   (FILE *) pointer to output file.

Return value The return value flag (of type int) is one of

Table 4.2: Optional inputs for IDAS, IDADLS, and IDASPILS

Optional input	Function name	Default
IDAS main solver		
Pointer to an error file	`IDASetErrFile`	`stderr`
Error handler function	`IDASetErrHandlerFn`	internal fn.
User data	`IDASetUserData`	`NULL`
Maximum order for BDF method	`IDASetMaxOrd`	5
Maximum no. of internal steps before t_{out}	`IDASetMaxNumSteps`	500
Initial step size	`IDASetInitStep`	estimated
Maximum absolute step size	`IDASetMaxStep`	∞
Value of t_{stop}	`IDASetStopTime`	∞
Maximum no. of error test failures	`IDASetMaxErrTestFails`	10
Maximum no. of nonlinear iterations	`IDASetMaxNonlinIters`	4
Maximum no. of convergence failures	`IDASetMaxConvFails`	10
Maximum no. of error test failures	`IDASetMaxErrTestFails`	7
Coeff. in the nonlinear convergence test	`IDASetNonlinConvCoef`	0.33
Suppress alg. vars. from error test	`IDASetSuppressAlg`	`SUNFALSE`
Variable types (differential/algebraic)	`IDASetId`	`NULL`
Inequality constraints on solution	`IDASetConstraints`	`NULL`
Direction of zero-crossing	`IDASetRootDirection`	both
Disable rootfinding warnings	`IDASetNoInactiveRootWarn`	none
IDAS initial conditions calculation		
Coeff. in the nonlinear convergence test	`IDASetNonlinConvCoefIC`	0.0033
Maximum no. of steps	`IDASetMaxNumStepsIC`	5
Maximum no. of Jacobian/precond. evals.	`IDASetMaxNumJacsIC`	4
Maximum no. of Newton iterations	`IDASetMaxNumItersIC`	10
Max. linesearch backtracks per Newton iter.	`IDASetMaxBacksIC`	100
Turn off linesearch	`IDASetLineSearchOffIC`	`SUNFALSE`
Lower bound on Newton step	`IDASetStepToleranceIC`	uround$^{2/3}$
IDADLS linear solver interface		
Jacobian function	`IDADlsSetJacFn`	DQ
IDASPILS linear solver interface		
Preconditioner functions	`IDASpilsSetPreconditioner`	NULL, NULL
Jacobian-times-vector function	`IDASpilsSetJacTimes`	NULL, DQ
Ratio between linear and nonlinear tolerances	`IDASpilsSetEpsLin`	0.05

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The default value for errfp is stderr.

Passing a value NULL disables all future error message output (except for the case in which the IDAS memory pointer is NULL). This use of IDASetErrFile is strongly discouraged.

If IDASetErrFile is to be called, it should be called before any other optional input functions, in order to take effect for any later error message.

<u>IDASetErrHandlerFn</u>

Call   flag = IDASetErrHandlerFn(ida_mem, ehfun, eh_data);

Description   The function IDASetErrHandlerFn specifies the optional user-defined function to be used in handling error messages.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

ehfun   (IDAErrHandlerFn) is the user's C error handler function (see §4.6.2).

eh_data (void *) pointer to user data passed to ehfun every time it is called.

Return value   The return value flag (of type int) is one of

IDA_SUCCESS   The function ehfun and data pointer eh_data have been successfully set.

IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   Error messages indicating that the IDAS solver memory is NULL will always be directed to stderr.

<u>IDASetUserData</u>

Call   flag = IDASetUserData(ida_mem, user_data);

Description   The function IDASetUserData specifies the user data block user_data and attaches it to the main IDAS memory block.

Arguments   ida_mem   (void *) pointer to the IDAS memory block.

user_data (void *) pointer to the user data.

Return value   The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   If specified, the pointer to user_data is passed to all user-supplied functions that have it as an argument. Otherwise, a NULL pointer is passed.

If user_data is needed in user linear solver or preconditioner functions, the call to IDASetUserData must be made *before* the call to specify the linear solver.

<u>IDASetMaxOrd</u>

Call   flag = IDASetMaxOrd(ida_mem, maxord);

Description   The function IDASetMaxOrd specifies the maximum order of the linear multistep method.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

maxord (int) value of the maximum method order. This must be positive.

Return value   The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL   The ida_mem pointer is NULL.

IDA_ILL_INPUT The input value `maxord` is ≤ 0, or larger than its previous value.

Notes   The default value is 5. If the input value exceeds 5, the value 5 will be used. Since `maxord` affects the memory requirements for the internal IDAS memory block, its value cannot be increased past its previous value.

IDASetMaxNumSteps

Call   `flag = IDASetMaxNumSteps(ida_mem, mxsteps);`

Description   The function `IDASetMaxNumSteps` specifies the maximum number of steps to be taken by the solver in its attempt to reach the next output time.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

             `mxsteps` (`long int`) maximum allowed number of steps.

Return value The return value `flag` (of type `int`) is one of

             IDA_SUCCESS   The optional value has been successfully set.

             IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

Notes   Passing `mxsteps = 0` results in IDAS using the default value (500).

             Passing `mxsteps < 0` disables the test (*not recommended).*

IDASetInitStep

Call   `flag = IDASetInitStep(ida_mem, hin);`

Description   The function `IDASetInitStep` specifies the initial step size.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

             `hin`   (`realtype`) value of the initial step size to be attempted. Pass 0.0 to have IDAS use the default value.

Return value The return value `flag` (of type `int`) is one of

             IDA_SUCCESS   The optional value has been successfully set.

             IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

Notes   By default, IDAS estimates the initial step as the solution of $\|h\dot{y}\|_{\mathrm{WRMS}} = 1/2$, with an added restriction that $|h| \leq .001|\texttt{tout - t0}|$.

IDASetMaxStep

Call   `flag = IDASetMaxStep(ida_mem, hmax);`

Description   The function `IDASetMaxStep` specifies the maximum absolute value of the step size.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

             `hmax`   (`realtype`) maximum absolute value of the step size.

Return value The return value `flag` (of type `int`) is one of

             IDA_SUCCESS   The optional value has been successfully set.

             IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

             IDA_ILL_INPUT Either `hmax` is not positive or it is smaller than the minimum allowable step.

Notes   Pass `hmax= 0` to obtain the default value ∞.

IDASetStopTime

Call   `flag = IDASetStopTime(ida_mem, tstop);`

Description   The function `IDASetStopTime` specifies the value of the independent variable t past which the solution is not to proceed.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `tstop`   (`realtype`) value of the independent variable past which the solution should not proceed.

Return value The return value `flag` (of type `int`) is one of

              `IDA_SUCCESS`   The optional value has been successfully set.

              `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

              `IDA_ILL_INPUT` The value of `tstop` is not beyond the current t value, t_n.

Notes   The default, if this routine is not called, is that no stop time is imposed.

IDASetMaxErrTestFails

Call   `flag = IDASetMaxErrTestFails(ida_mem, maxnef);`

Description   The function `IDASetMaxErrTestFails` specifies the maximum number of error test failures in attempting one step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `maxnef` (`int`) maximum number of error test failures allowed on one step (> 0).

Return value The return value `flag` (of type `int`) is one of

              `IDA_SUCCESS`   The optional value has been successfully set.

              `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

Notes   The default value is 7.

IDASetMaxNonlinIters

Call   `flag = IDASetMaxNonlinIters(ida_mem, maxcor);`

Description   The function `IDASetMaxNonlinIters` specifies the maximum number of nonlinear solver iterations at one step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `maxcor` (`int`) maximum number of nonlinear solver iterations allowed on one step (> 0).

Return value The return value `flag` (of type `int`) is one of

              `IDA_SUCCESS`   The optional value has been successfully set.

              `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

Notes   The default value is 3.

IDASetMaxConvFails

Call   `flag = IDASetMaxConvFails(ida_mem, maxncf);`

Description   The function `IDASetMaxConvFails` specifies the maximum number of nonlinear solver convergence failures at one step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `maxncf` (`int`) maximum number of allowable nonlinear solver convergence failures on one step (> 0).

Return value The return value `flag` (of type `int`) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The default value is 10.

| IDASetNonlinConvCoef |

Call   flag = IDASetNonlinConvCoef(ida_mem, nlscoef);

Description   The function IDASetNonlinConvCoef specifies the safety factor in the nonlinear convergence test; see Chapter 2, Eq. (2.8).

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               nlscoef (realtype) coefficient in nonlinear convergence test (> 0.0).

Return value   The return value flag (of type int) is one of

               IDA_SUCCESS   The optional value has been successfully set.

               IDA_MEM_NULL The ida_mem pointer is NULL.

               IDA_ILL_INPUT The value of nlscoef is $<= 0.0$.

Notes   The default value is 0.33.

| IDASetSuppressAlg |

Call   flag = IDASetSuppressAlg(ida_mem, suppressalg);

Description   The function IDASetSuppressAlg indicates whether or not to suppress algebraic variables in the local error test.

Arguments   ida_mem   (void *) pointer to the IDAS memory block.

               suppressalg (booleantype) indicates whether to suppress (SUNTRUE) or not (SUNFALSE) the algebraic variables in the local error test.

Return value   The return value flag (of type int) is one of

               IDA_SUCCESS   The optional value has been successfully set.

               IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The default value is SUNFALSE.

               If suppressalg=SUNTRUE is selected, then the id vector must be set (through IDASetId) to specify the algebraic components.

               In general, the use of this option (with suppressalg = SUNTRUE) is *discouraged* when solving DAE systems of index 1, whereas it is generally *encouraged* for systems of index 2 or more. See pp. 146-147 of Ref. [3] for more on this issue.

| IDASetId |

Call   flag = IDASetId(ida_mem, id);

Description   The function IDASetId specifies algebraic/differential components in the y vector.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               id   (N_Vector) state vector. A value of 1.0 indicates a differential variable, while 0.0 indicates an algebraic variable.

Return value   The return value flag (of type int) is one of

               IDA_SUCCESS   The optional value has been successfully set.

               IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The vector id is required if the algebraic variables are to be suppressed from the local error test (see IDASetSuppressAlg) or if IDACalcIC is to be called with icopt = IDA_YA_YDP_INIT (see §4.5.4).

IDASetConstraints

Call   `flag = IDASetConstraints(ida_mem, constraints);`

Description   The function `IDASetConstraints` specifies a vector defining inequality constraints for each component of the solution vector y.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `constraints` (`N_Vector`) vector of constraint flags. If `constraints[i]` is

                        0.0 then no constraint is imposed on y_i.
                        1.0 then y_i will be constrained to be $y_i \geq 0.0$.
                        -1.0 then y_i will be constrained to be $y_i \leq 0.0$.
                        2.0 then y_i will be constrained to be $y_i > 0.0$.
                        -2.0 then y_i will be constrained to be $y_i < 0.0$.

Return value The return value `flag` (of type `int`) is one of

              `IDA_SUCCESS`   The optional value has been successfully set.

              `IDA_MEM_NULL`   The `ida_mem` pointer is `NULL`.

              `IDA_ILL_INPUT`   The constraints vector contains illegal values.

Notes   The presence of a non-`NULL` constraints vector that is not 0.0 in all components will cause constraint checking to be performed. However, a call with 0.0 in all components of `constraints` will result in an illegal input return.

4.5.7.2 Direct linear solver interface optional input functions

The IDADLS solver interface needs a function to compute an approximation to the Jacobian matrix $J(t, y, \dot{y})$. This function must be of type `IDADlsJacFn`. The user can supply a Jacobian function, or if using a dense or banded matrix J can use the default internal difference quotient approximation that comes with the IDADLS solver. To specify a user-supplied Jacobian function `jac`, IDADLS provides the function `IDADlsSetJacFn`. The IDADLS interface passes the pointer `user_data` to the Jacobian function. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied Jacobian function, without using global data in the program. The pointer `user_data` may be specified through `IDASetUserData`.

IDADlsSetJacFn

Call   `flag = IDADlsSetJacFn(ida_mem, jac);`

Description   The function `IDADlsSetJacFn` specifies the Jacobian approximation function to be used.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `jac`   (`IDADlsJacFn`) user-defined Jacobian approximation function.

Return value The return value `flag` (of type `int`) is one of

              `IDADLS_SUCCESS` The optional value has been successfully set.

              `IDADLS_MEM_NULL` The `ida_mem` pointer is `NULL`.

              `IDADLS_LMEM_NULL` The IDADLS linear solver interface has not been initialized.

Notes   By default, IDADLS uses an internal difference quotient function for dense and band matrices. If `NULL` is passed to `jac`, this default function is used. An error will occur if no `jac` is supplied when using a sparse matrix.

              The function type `IDADlsJacFn` is described in §4.6.5.

4.5.7.3 Iterative linear solver interface optional input functions

If the user will be doing preconditioning with the IDASPILS linear solver interface, then the user must supply a preconditioner solve function `psolve` and specify its name through a call to the routine `IDASpilsSetPreconditioner`. The evaluation and preprocessing of any Jacobian-related data needed by the user's preconditioner solve function is done in the optional user-supplied function `psetup`. Both of these functions are fully specified in §4.6. If used, the name of the `psetup` function should be specified in the call to `IDASpilsSetPreconditioner`.

The pointer `user_data` received through `IDASetUserData` (or a pointer to `NULL` if `user_data` was not specified) is passed to the preconditioner `psetup` and `psolve` functions. This allows the user to create an arbitrary structure with relevant problem data and access it during the execution of the user-supplied preconditioner functions without using global data in the program.

The IDASPILS solver interface requires a function to compute an approximation to the product between the Jacobian matrix $J(t, y)$ and a vector v. The user can supply his/her own Jacobian-times-vector approximation function, or use the default internal difference quotient function that comes with the IDASPILS solver interface. A user-defined Jacobian-vector function must be of type `IDASpilsJacTimesVecFn` and can be specified through a call to `IDASpilsSetJacTimes` (see §4.6.6 for specification details). As with the user-supplied preconditioner functions, the evaluation and processing of any Jacobian-related data needed by the user's Jacobian-times-vector function is done in the optional user-supplied function `jtsetup` (see §4.6.7 for specification details). As with the preconditioner functions, a pointer to the user-defined data structure, `user_data`, specified through `IDASetUserData` (or a `NULL` pointer otherwise) is passed to the Jacobian-times-vector setup and product functions, `jtsetup` and `jtimes`, each time they are called.

Finally, as described in Section 2.1, the IDASPILS interface requires that iterative linear solvers stop when the norm of the preconditioned residual is less than $0.05 \cdot (0.1\epsilon)$, where ϵ is the nonlinear solver tolerance. The user may adjust this linear solver tolerance by calling the function `IDASpilsSetEpsLin`.

`IDASpilsSetPreconditioner`

Call	`flag = IDASpilsSetPreconditioner(ida_mem, psetup, psolve);`
Description	The function `IDASpilsSetPreconditioner` specifies the preconditioner setup and solve functions.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`psetup` (`IDASpilsPrecSetupFn`) user-defined function to set up the preconditioner. Pass `NULL` if no setup is necessary.
	`psolve` (`IDASpilsPrecSolveFn`) user-defined preconditioner solve function.
Return value	The return value `flag` (of type `int`) is one of
	`IDASPILS_SUCCESS` The optional values have been successfully set.
	`IDASPILS_MEM_NULL` The `ida_mem` pointer is `NULL`.
	`IDASPILS_LMEM_NULL` The IDASPILS linear solver has not been initialized.
	`IDASPILS_SUNLS_FAIL` An error occurred when setting up preconditioning in the SUNLINSOL object used by the IDASPILS interface.
Notes	The function type `IDASpilsPrecSolveFn` is described in §4.6.8. The function type `IDASpilsPrecSetupFn` is described in §4.6.9.

`IDASpilsSetJacTimes`

Call	`flag = IDASpilsSetJacTimes(ida_mem, jsetup, jtimes);`
Description	The function `IDASpilsSetJacTimes` specifies the Jacobian-vector setup and product.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`jtsetup` (`IDASpilsJacTimesSetupFn`) user-defined function to set up the Jacobian-vector product. Pass `NULL` if no setup is necessary.

jtimes (IDASpilsJacTimesVecFn) user-defined Jacobian-vector product function.

Return value The return value flag (of type int) is one of

IDASPILS_SUCCESS   The optional value has been successfully set.

IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

IDASPILS_SUNLS_FAIL An error occurred when setting up the system matrix-times-vector routines in the SUNLINSOL object used by the IDASPILS interface.

Notes   By default, the IDASPILS solvers use the difference quotient function. If NULL is passed to jtimes, this default function is used.

The function type IDASpilsJacTimesSetupFn is described in §4.6.7.

The function type IDASpilsJacTimesVecFn is described in §4.6.6.

| IDASpilsSetEpsLin |

Call   flag = IDASpilsSetEpsLin(ida_mem, eplifac);

Description   The function IDASpilsSetEpsLin specifies the factor by which the Krylov linear solver's convergence test constant is reduced from the Newton iteration test constant.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

eplifac (realtype) linear convergence safety factor (≥ 0.0).

Return value The return value flag (of type int) is one of

IDASPILS_SUCCESS   The optional value has been successfully set.

IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

IDASPILS_ILL_INPUT   The factor eplifac is negative.

Notes   The default value is 0.05.

If eplifac$= 0.0$ is passed, the default value is used.

4.5.7.4 Initial condition calculation optional input functions

The following functions can be called just prior to calling IDACalcIC to set optional inputs controlling the initial condition calculation.

| IDASetNonlinConvCoefIC |

Call   flag = IDASetNonlinConvCoefIC(ida_mem, epiccon);

Description   The function IDASetNonlinConvCoefIC specifies the positive constant in the Newton iteration convergence test within the initial condition calculation.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

epiccon (realtype) coefficient in the Newton convergence test (> 0).

Return value The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL   The ida_mem pointer is NULL.

IDA_ILL_INPUT   The epiccon factor is $<= 0.0$.

Notes   The default value is $0.01 \cdot 0.33$.

This test uses a weighted RMS norm (with weights defined by the tolerances). For new initial value vectors y and $\dot{y}$ to be accepted, the norm of $J^{-1}F(t_0, y, \dot{y})$ must be $\leq$ epiccon, where J is the system Jacobian.

IDASetMaxNumStepsIC

Call   `flag = IDASetMaxNumStepsIC(ida_mem, maxnh);`

Description   The function `IDASetMaxNumStepsIC` specifies the maximum number of steps allowed when `icopt=IDA_YA_YDP_INIT` in `IDACalcIC`, where h appears in the system Jacobian, $J = \partial F/\partial y + (1/h)\partial F/\partial \dot{y}$.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

                `maxnh`   (`int`) maximum allowed number of values for h.

Return value   The return value `flag` (of type `int`) is one of

                IDA_SUCCESS   The optional value has been successfully set.

                IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

                IDA_ILL_INPUT `maxnh` is non-positive.

Notes   The default value is 5.

IDASetMaxNumJacsIC

Call   `flag = IDASetMaxNumJacsIC(ida_mem, maxnj);`

Description   The function `IDASetMaxNumJacsIC` specifies the maximum number of the approximate Jacobian or preconditioner evaluations allowed when the Newton iteration appears to be slowly converging.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

                `maxnj`   (`int`) maximum allowed number of Jacobian or preconditioner evaluations.

Return value   The return value `flag` (of type `int`) is one of

                IDA_SUCCESS   The optional value has been successfully set.

                IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

                IDA_ILL_INPUT `maxnj` is non-positive.

Notes   The default value is 4.

IDASetMaxNumItersIC

Call   `flag = IDASetMaxNumItersIC(ida_mem, maxnit);`

Description   The function `IDASetMaxNumItersIC` specifies the maximum number of Newton iterations allowed in any one attempt to solve the initial conditions calculation problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

                `maxnit` (`int`) maximum number of Newton iterations.

Return value   The return value `flag` (of type `int`) is one of

                IDA_SUCCESS   The optional value has been successfully set.

                IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

                IDA_ILL_INPUT `maxnit` is non-positive.

Notes   The default value is 10.

IDASetMaxBacksIC

Call   `flag = IDASetMaxBacksIC(ida_mem, maxbacks);`

Description   The function `IDASetMaxBacksIC` specifies the maximum number of linesearch backtracks allowed in any Newton iteration, when solving the initial conditions calculation problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

maxbacks (int) maximum number of linesearch backtracks per Newton step.

Return value The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL   The ida_mem pointer is NULL.

IDA_ILL_INPUT maxbacks is non-positive.

Notes   The default value is 100.

If IDASetMaxBacksIC is called in a Forward Sensitivity Analysis, the the limit maxbacks applies in the calculation of both the initial state values and the initial sensititivies.

IDASetLineSearchOffIC

Call   flag = IDASetLineSearchOffIC(ida_mem, lsoff);

Description The function IDASetLineSearchOffIC specifies whether to turn on or off the linesearch algorithm.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

lsoff   (booleantype) a flag to turn off (SUNTRUE) or keep (SUNFALSE) the linesearch algorithm.

Return value The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The default value is SUNFALSE.

IDASetStepToleranceIC

Call   flag = IDASetStepToleranceIC(ida_mem, steptol);

Description The function IDASetStepToleranceIC specifies a positive lower bound on the Newton step.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

steptol (int) Minimum allowed WRMS-norm of the Newton step (> 0.0).

Return value The return value flag (of type int) is one of

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL   The ida_mem pointer is NULL.

IDA_ILL_INPUT The steptol tolerance is $<= 0.0$.

Notes   The default value is (unit roundoff)$^{2/3}$.

4.5.7.5 Rootfinding optional input functions

The following functions can be called to set optional inputs to control the rootfinding algorithm.

IDASetRootDirection

Call   flag = IDASetRootDirection(ida_mem, rootdir);

Description The function IDASetRootDirection specifies the direction of zero-crossings to be located and returned to the user.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

rootdir (int *) state array of length nrtfn, the number of root functions g_i, as specified in the call to the function IDARootInit. A value of 0 for rootdir[i] indicates that crossing in either direction should be reported for g_i. A value of $+1$ or -1 indicates that the solver should report only zero-crossings where g_i is increasing or decreasing, respectively.

Return value The return value `flag` (of type `int`) is one of

> IDA_SUCCESS   The optional value has been successfully set.
>
> IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.
>
> IDA_ILL_INPUT rootfinding has not been activated through a call to `IDARootInit`.

Notes   The default behavior is to locate both zero-crossing directions.

| IDASetNoInactiveRootWarn |

Call   `flag = IDASetNoInactiveRootWarn(ida_mem);`

Description   The function `IDASetNoInactiveRootWarn` disables issuing a warning if some root function appears to be identically zero at the beginning of the integration.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

Return value The return value `flag` (of type `int`) is one of

> IDA_SUCCESS   The optional value has been successfully set.
>
> IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

Notes   IDAS will not report the initial conditions as a possible zero-crossing (assuming that one or more components g_i are zero at the initial time). However, if it appears that some g_i is identically zero at the initial time (i.e., g_i is zero at the initial time and after the first step), IDAS will issue a warning which can be disabled with this optional input function.

4.5.8   Interpolated output function

An optional function `IDAGetDky` is available to obtain additional output values. This function must be called after a successful return from `IDASolve` and provides interpolated values of y or its derivatives of order up to the last internal order used for any value of t in the last internal step taken by IDAS.

   The call to the `IDAGetDky` function has the following form:

| IDAGetDky |

Call   `flag = IDAGetDky(ida_mem, t, k, dky);`

Description   The function `IDAGetDky` computes the interpolated values of the kth derivative of y for any value of t in the last internal step taken by IDAS. The value of k must be non-negative and smaller than the last internal order used. A value of 0 for k means that the y is interpolated. The value of t must satisfy $t_n - h_u \le t \le t_n$, where t_n denotes the current internal time reached, and h_u is the last internal step size used successfully.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

> `t`   (`realtype`) time at which to interpolate.
>
> `k`   (`int`) integer specifying the order of the derivative of y wanted.
>
> `dky`   (`N_Vector`) vector containing the interpolated k^{th} derivative of $y(t)$.

Return value The return value `flag` (of type `int`) is one of

> IDA_SUCCESS   `IDAGetDky` succeeded.
>
> IDA_MEM_NULL The `ida_mem` argument was `NULL`.
>
> IDA_BAD_T   `t` is not in the interval $[t_n - h_u, t_n]$.
>
> IDA_BAD_K   `k` is not one of $\{0, 1, \dots, klast\}$.
>
> IDA_BAD_DKY   `dky` is `NULL`.

Notes   It is only legal to call the function `IDAGetDky` after a successful return from `IDASolve`. Functions `IDAGetCurrentTime`, `IDAGetLastStep` and `IDAGetLastOrder` (see §4.5.9.2) can be used to access t_n, h_u and $klast$.

4.5.9 Optional output functions

IDAS provides an extensive list of functions that can be used to obtain solver performance information. Table 4.3 lists all optional output functions in IDAS, which are then described in detail in the remainder of this section.

Some of the optional outputs, especially the various counters, can be very useful in determining how successful the IDAS solver is in doing its job. For example, the counters `nsteps` and `nrevals` provide a rough measure of the overall cost of a given run, and can be compared among runs with differing input options to suggest which set of options is most efficient. The ratio `nniters/nsteps` measures the performance of the Newton iteration in solving the nonlinear systems at each time step; typical values for this range from 1.1 to 1.8. The ratio `njevals/nniters` (in the case of a direct linear solver), and the ratio `npevals/nniters` (in the case of an iterative linear solver) measure the overall degree of nonlinearity in these systems, and also the quality of the approximate Jacobian or preconditioner being used. Thus, for example, `njevals/nniters` can indicate if a user-supplied Jacobian is inaccurate, if this ratio is larger than for the case of the corresponding internal Jacobian. The ratio `nliters/nniters` measures the performance of the Krylov iterative linear solver, and thus (indirectly) the quality of the preconditioner.

4.5.9.1 SUNDIALS version information

The following functions provide a way to get SUNDIALS version information at runtime.

SUNDIALSGetVersion

Call	`flag = SUNDIALSGetVersion(version, len);`
Description	The function `SUNDIALSGetVersion` fills a character array with SUNDIALS version information.
Arguments	`version` (`char *`) character array to hold the SUNDIALS version information.
	`len` (`int`) allocated length of the `version` character array.
Return value	If successful, `SUNDIALSGetVersion` returns 0 and `version` contains the SUNDIALS version information. Otherwise, it returns −1 and `version` is not set (the input character array is too short).
Notes	A string of 25 characters should be sufficient to hold the version information. Any trailing characters in the `version` array are removed.

SUNDIALSGetVersionNumber

Call	`flag = SUNDIALSGetVersionNumber(&major, &minor, &patch, label, len);`
Description	The function `SUNDIALSGetVersionNumber` set integers for the SUNDIALS major, minor, and patch release numbers and fills a character array with the release label if applicable.
Arguments	`major` (`int`) SUNDIALS release major version number.
	`minor` (`int`) SUNDIALS release minor version number.
	`patch` (`int`) SUNDIALS release patch version number.
	`label` (`char *`) character array to hold the SUNDIALS release label.
	`len` (`int`) allocated length of the `label` character array.
Return value	If successful, `SUNDIALSGetVersionNumber` returns 0 and the `major`, `minor`, `patch`, and `label` values are set. Otherwise, it returns −1 and the values are not set (the input character array is too short).
Notes	A string of 10 characters should be sufficient to hold the label information. If a label is not used in the release version, no information is copied to `label`. Any trailing characters in the `label` array are removed.

Table 4.3: Optional outputs from IDAS, IDADLS, and IDASPILS

Optional output	Function name
IDAS main solver	
Size of IDAS real and integer workspace	`IDAGetWorkSpace`
Cumulative number of internal steps	`IDAGetNumSteps`
No. of calls to residual function	`IDAGetNumResEvals`
No. of calls to linear solver setup function	`IDAGetNumLinSolvSetups`
No. of local error test failures that have occurred	`IDAGetNumErrTestFails`
Order used during the last step	`IDAGetLastOrder`
Order to be attempted on the next step	`IDAGetCurrentOrder`
Order reductions due to stability limit detection	`IDAGetNumStabLimOrderReds`
Actual initial step size used	`IDAGetActualInitStep`
Step size used for the last step	`IDAGetLastStep`
Step size to be attempted on the next step	`IDAGetCurrentStep`
Current internal time reached by the solver	`IDAGetCurrentTime`
Suggested factor for tolerance scaling	`IDAGetTolScaleFactor`
Error weight vector for state variables	`IDAGetErrWeights`
Estimated local errors	`IDAGetEstLocalErrors`
No. of nonlinear solver iterations	`IDAGetNumNonlinSolvIters`
No. of nonlinear convergence failures	`IDAGetNumNonlinSolvConvFails`
Array showing roots found	`IDAGetRootInfo`
No. of calls to user root function	`IDAGetNumGEvals`
Name of constant associated with a return flag	`IDAGetReturnFlagName`
IDAS initial conditions calculation	
Number of backtrack operations	`IDAGetNumBacktrackops`
Corrected initial conditions	`IDAGetConsistentIC`
IDADLS linear solver interface	
Size of real and integer workspace	`IDADlsGetWorkSpace`
No. of Jacobian evaluations	`IDADlsGetNumJacEvals`
No. of residual calls for finite diff. Jacobian evals.	`IDADlsGetNumResEvals`
Last return from a linear solver function	`IDADlsGetLastFlag`
Name of constant associated with a return flag	`IDADlsGetReturnFlagName`
IDASPILS linear solver interface	
Size of real and integer workspace	`IDASpilsGetWorkSpace`
No. of linear iterations	`IDASpilsGetNumLinIters`
No. of linear convergence failures	`IDASpilsGetNumConvFails`
No. of preconditioner evaluations	`IDASpilsGetNumPrecEvals`
No. of preconditioner solves	`IDASpilsGetNumPrecSolves`
No. of Jacobian-vector setup evaluations	`IDASpilsGetNumJTSetupEvals`
No. of Jacobian-vector product evaluations	`IDASpilsGetNumJtimesEvals`
No. of residual calls for finite diff. Jacobian-vector evals.	`IDASpilsGetNumResEvals`
Last return from a linear solver function	`IDASpilsGetLastFlag`
Name of constant associated with a return flag	`IDASpilsGetReturnFlagName`

4.5.9.2   Main solver optional output functions

IDAS provides several user-callable functions that can be used to obtain different quantities that may be of interest to the user, such as solver workspace requirements, solver performance statistics, as well as additional data from the IDAS memory block (a suggested tolerance scaling factor, the error weight vector, and the vector of estimated local errors). Also provided are functions to extract statistics related to the performance of the IDAS nonlinear solver being used. As a convenience, additional extraction functions provide the optional outputs in groups. These optional output functions are described next.

| IDAGetWorkSpace |

Call   `flag = IDAGetWorkSpace(ida_mem, &lenrw, &leniw);`

Description   The function `IDAGetWorkSpace` returns the IDAS real and integer workspace sizes.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `lenrw`   (`long int`) number of real values in the IDAS workspace.

              `leniw`   (`long int`) number of integer values in the IDAS workspace.

Return value The return value `flag` (of type `int`) is one of

        `IDA_SUCCESS`   The optional output value has been successfully set.

        `IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.

Notes   In terms of the problem size N, the maximum method order `maxord`, and the number `nrtfn` of root functions (see §4.5.5), the actual size of the real workspace, in `realtype` words, is given by the following:

        • base value: $\texttt{lenrw} = 55 + (m+6) * N_r + 3*\texttt{nrtfn}$;

        • with IDASVtolerances: $\texttt{lenrw} = \texttt{lenrw} + N_r$;

        • with constraint checking (see `IDASetConstraints`): $\texttt{lenrw} = \texttt{lenrw} + N_r$;

        • with `id` specified (see `IDASetId`): $\texttt{lenrw} = \texttt{lenrw} + N_r$;

    where $m = \max(\texttt{maxord}, 3)$, and N_r is the number of real words in one `N_Vector` ($\approx N$).

    The size of the integer workspace (without distinction between `int` and `long int` words) is given by:

        • base value: $\texttt{leniw} = 38 + (m+6) * N_i + \texttt{nrtfn}$;

        • with IDASVtolerances: $\texttt{leniw} = \texttt{leniw} + N_i$;

        • with constraint checking: $\texttt{lenrw} = \texttt{lenrw} + N_i$;

        • with `id` specified: $\texttt{lenrw} = \texttt{lenrw} + N_i$;

    where N_i is the number of integer words in one `N_Vector` ($= 1$ for NVECTOR_SERIAL and 2*`npes` for NVECTOR_PARALLEL on `npes` processors).

    For the default value of `maxord`, with no rootfinding, no `id`, no constraints, and with no call to `IDASVtolerances`, these lengths are given roughly by: $\texttt{lenrw} = 55 + 11N$, $\texttt{leniw} = 49$.

    Note that additional memory is allocated if quadratures and/or forward sensitivity integration is enabled. See §4.7.1 and §5.2.1 for more details.

IDAGetNumSteps

Call	`flag = IDAGetNumSteps(ida_mem, &nsteps);`
Description	The function `IDAGetNumSteps` returns the cumulative number of internal steps taken by the solver (total so far).
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nsteps` (`long int`) number of steps taken by IDAS.
Return value	The return value `flag` (of type `int`) is one of
	IDA_SUCCESS The optional output value has been successfully set.
	IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetNumResEvals

Call	`flag = IDAGetNumResEvals(ida_mem, &nrevals);`
Description	The function `IDAGetNumResEvals` returns the number of calls to the user's residual evaluation function.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nrevals` (`long int`) number of calls to the user's `res` function.
Return value	The return value `flag` (of type `int`) is one of
	IDA_SUCCESS The optional output value has been successfully set.
	IDA_MEM_NULL The `ida_mem` pointer is NULL.
Notes	The `nrevals` value returned by `IDAGetNumResEvals` does not account for calls made to `res` from a linear solver or preconditioner module.

IDAGetNumLinSolvSetups

Call	`flag = IDAGetNumLinSolvSetups(ida_mem, &nlinsetups);`
Description	The function `IDAGetNumLinSolvSetups` returns the cumulative number of calls made to the linear solver's setup function (total so far).
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nlinsetups` (`long int`) number of calls made to the linear solver setup function.
Return value	The return value `flag` (of type `int`) is one of
	IDA_SUCCESS The optional output value has been successfully set.
	IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetNumErrTestFails

Call	`flag = IDAGetNumErrTestFails(ida_mem, &netfails);`
Description	The function `IDAGetNumErrTestFails` returns the cumulative number of local error test failures that have occurred (total so far).
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`netfails` (`long int`) number of error test failures.
Return value	The return value `flag` (of type `int`) is one of
	IDA_SUCCESS The optional output value has been successfully set.
	IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetLastOrder

Call   `flag = IDAGetLastOrder(ida_mem, &klast);`

Description   The function `IDAGetLastOrder` returns the integration method order used during the last internal step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `klast`   (`int`) method order used on the last internal step.

Return value   The return value `flag` (of type `int`) is one of

        `IDA_SUCCESS`   The optional output value has been successfully set.

        `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

IDAGetCurrentOrder

Call   `flag = IDAGetCurrentOrder(ida_mem, &kcur);`

Description   The function `IDAGetCurrentOrder` returns the integration method order to be used on the next internal step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

        `kcur`   (`int`) method order to be used on the next internal step.

Return value   The return value `flag` (of type `int`) is one of

        `IDA_SUCCESS`   The optional output value has been successfully set.

        `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

IDAGetLastStep

Call   `flag = IDAGetLastStep(ida_mem, &hlast);`

Description   The function `IDAGetLastStep` returns the integration step size taken on the last internal step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

        `hlast`   (`realtype`) step size taken on the last internal step by IDAS, or last artificial step size used in `IDACalcIC`, whichever was called last.

Return value   The return value `flag` (of type `int`) is one of

        `IDA_SUCCESS`   The optional output value has been successfully set.

        `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

IDAGetCurrentStep

Call   `flag = IDAGetCurrentStep(ida_mem, &hcur);`

Description   The function `IDAGetCurrentStep` returns the integration step size to be attempted on the next internal step.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

        `hcur`   (`realtype`) step size to be attempted on the next internal step.

Return value   The return value `flag` (of type `int`) is one of

        `IDA_SUCCESS`   The optional output value has been successfully set.

        `IDA_MEM_NULL` The `ida_mem` pointer is NULL.

IDAGetActualInitStep

Call	`flag = IDAGetActualInitStep(ida_mem, &hinused);`
Description	The function `IDAGetActualInitStep` returns the value of the integration step size used on the first step.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`hinused` (`realtype`) actual value of initial step size.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
Notes	Even if the value of the initial integration step size was specified by the user through a call to `IDASetInitStep`, this value might have been changed by IDAS to ensure that the step size is within the prescribed bounds ($h_{\min} \leq h_0 \leq h_{\max}$), or to meet the local error test.

IDAGetCurrentTime

Call	`flag = IDAGetCurrentTime(ida_mem, &tcur);`
Description	The function `IDAGetCurrentTime` returns the current internal time reached by the solver.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`tcur` (`realtype`) current internal time reached.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.

IDAGetTolScaleFactor

Call	`flag = IDAGetTolScaleFactor(ida_mem, &tolsfac);`
Description	The function `IDAGetTolScaleFactor` returns a suggested factor by which the user's tolerances should be scaled when too much accuracy has been requested for some internal step.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`tolsfac` (`realtype`) suggested scaling factor for user tolerances.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.

IDAGetErrWeights

Call	`flag = IDAGetErrWeights(ida_mem, eweight);`
Description	The function `IDAGetErrWeights` returns the solution error weights at the current time. These are the W_i given by Eq. (2.7) (or by the user's `IDAEwtFn`).
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`eweight` (`N_Vector`) solution error weights at the current time.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
Notes	The user must allocate space for `eweight`.

IDAGetEstLocalErrors

Call   flag = IDAGetEstLocalErrors(ida_mem, ele);

Description   The function IDAGetEstLocalErrors returns the estimated local errors.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
                ele   (N_Vector) estimated local errors at the current time.

Return value   The return value flag (of type int) is one of

                IDA_SUCCESS   The optional output value has been successfully set.
                IDA_MEM_NULL The ida_mem pointer is NULL.

Notes   The user must allocate space for ele.
                The values returned in ele are only valid if IDASolve returned a non-negative value.

                The ele vector, togther with the eweight vector from IDAGetErrWeights, can be used
                to determine how the various components of the system contributed to the estimated
                local error test. Specifically, that error test uses the RMS norm of a vector whose
                components are the products of the components of these two vectors. Thus, for example,
                if there were recent error test failures, the components causing the failures are those
                with largest values for the products, denoted loosely as eweight[i]*ele[i].

IDAGetIntegratorStats

Call   flag = IDAGetIntegratorStats(ida_mem, &nsteps, &nrevals, &nlinsetups,
                                              &netfails, &klast, &kcur, &hinused,
                                              &hlast, &hcur, &tcur);

Description   The function IDAGetIntegratorStats returns the IDAS integrator statistics as a group.

Arguments   ida_mem   (void *) pointer to the IDAS memory block.
                nsteps   (long int) cumulative number of steps taken by IDAS.
                nrevals   (long int) cumulative number of calls to the user's res function.
                nlinsetups   (long int) cumulative number of calls made to the linear solver setup
                             function.
                netfails   (long int) cumulative number of error test failures.
                klast   (int) method order used on the last internal step.
                kcur   (int) method order to be used on the next internal step.
                hinused   (realtype) actual value of initial step size.
                hlast   (realtype) step size taken on the last internal step.
                hcur   (realtype) step size to be attempted on the next internal step.
                tcur   (realtype) current internal time reached.

Return value   The return value flag (of type int) is one of

                IDA_SUCCESS   the optional output values have been successfully set.
                IDA_MEM_NULL the ida_mem pointer is NULL.

IDAGetNumNonlinSolvIters

Call   flag = IDAGetNumNonlinSolvIters(ida_mem, &nniters);

Description   The function IDAGetNumNonlinSolvIters returns the cumulative number of nonlinear
                (functional or Newton) iterations performed.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
                nniters (long int) number of nonlinear iterations performed.

Return value   The return value flag (of type int) is one of

                IDA_SUCCESS   The optional output value has been successfully set.
                IDA_MEM_NULL The ida_mem pointer is NULL.

IDAGetNumNonlinSolvConvFails

Call   `flag = IDAGetNumNonlinSolvConvFails(ida_mem, &nncfails);`

Description The function `IDAGetNumNonlinSolvConvFails` returns the cumulative number of non-linear convergence failures that have occurred.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `nncfails` (`long int`) number of nonlinear convergence failures.

Return value The return value `flag` (of type `int`) is one of

            IDA_SUCCESS   The optional output value has been successfully set.

            IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetNonlinSolvStats

Call   `flag = IDAGetNonlinSolvStats(ida_mem, &nniters, &nncfails);`

Description The function `IDAGetNonlinSolvStats` returns the IDAS nonlinear solver statistics as a group.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `nniters` (`long int`) cumulative number of nonlinear iterations performed.

            `nncfails` (`long int`) cumulative number of nonlinear convergence failures.

Return value The return value `flag` (of type `int`) is one of

            IDA_SUCCESS   The optional output value has been successfully set.

            IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetReturnFlagName

Call   `name = IDAGetReturnFlagName(flag);`

Description The function `IDAGetReturnFlagName` returns the name of the IDAS constant corresponding to `flag`.

Arguments   The only argument, of type `int`, is a return flag from an IDAS function.

Return value The return value is a string containing the name of the corresponding constant.

4.5.9.3 Initial condition calculation optional output functions

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IDAGetNumBcktrackOps

Call   `flag = IDAGetNumBacktrackOps(ida_mem, &nbacktr);`

Description The function `IDAGetNumBacktrackOps` returns the number of backtrack operations done in the linesearch algorithm in `IDACalcIC`.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `nbacktr` (`long int`) the cumulative number of backtrack operations.

Return value The return value `flag` (of type `int`) is one of

            IDA_SUCCESS   The optional output value has been successfully set.

            IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDAGetConsistentIC

Call	`flag = IDAGetConsistentIC(ida_mem, yy0_mod, yp0_mod);`
Description	The function `IDAGetConsistentIC` returns the corrected initial conditions calculated by `IDACalcIC`.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`yy0_mod` (`N_Vector`) consistent solution vector.
	`yp0_mod` (`N_Vector`) consistent derivative vector.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_ILL_INPUT` The function was not called before the first call to `IDASolve`.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
Notes	If the consistent solution vector or consistent derivative vector is not desired, pass `NULL` for the corresponding argument.
	The user must allocate space for `yy0_mod` and `yp0_mod` (if not `NULL`).

4.5.9.4 Rootfinding optional output functions

There are two optional output functions associated with rootfinding.

IDAGetRootInfo

Call	`flag = IDAGetRootInfo(ida_mem, rootsfound);`
Description	The function `IDAGetRootInfo` returns an array showing which functions were found to have a root.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`rootsfound` (`int *`) array of length `nrtfn` with the indices of the user functions g_i found to have a root. For $i = 0, \ldots,$ `nrtfn` -1, `rootsfound`$[i] \neq 0$ if g_i has a root, and $= 0$ if not.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output values have been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.
Notes	Note that, for the components g_i for which a root was found, the sign of `rootsfound`$[i]$ indicates the direction of zero-crossing. A value of $+1$ indicates that g_i is increasing, while a value of -1 indicates a decreasing g_i.
	The user must allocate memory for the vector `rootsfound`.

IDAGetNumGEvals

Call	`flag = IDAGetNumGEvals(ida_mem, &ngevals);`
Description	The function `IDAGetNumGEvals` returns the cumulative number of calls to the user root function g.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`ngevals` (`long int`) number of calls to the user's function `g` so far.
Return value	The return value `flag` (of type `int`) is one of
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is `NULL`.

4.5.9.5 Direct linear solver interface optional output functions

The following optional outputs are available from the IDADLS modules: workspace requirements, number of calls to the Jacobian routine, number of calls to the residual routine for finite-difference Jacobian approximation, and last return value from an IDADLS function. Note that, where the name of an output would otherwise conflict with the name of an optional output from the main solver, a suffix LS (for Linear Solver) has been added (e.g. `lenrwLS`).

| IDADlsGetWorkSpace |

Call	`flag = IDADlsGetWorkSpace(ida_mem, &lenrwLS, &leniwLS);`
Description	The function `IDADlsGetWorkSpace` returns the sizes of the real and integer workspaces used by the IDADLS linear solver interface.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`lenrwLS` (`long int`) the number of real values in the IDADLS workspace.
	`leniwLS` (`long int`) the number of integer values in the IDADLS workspace.
Return value	The return value `flag` (of type `int`) is one of
	`IDADLS_SUCCESS` The optional output value has been successfully set.
	`IDADLS_MEM_NULL` The `ida_mem` pointer is `NULL`.
	`IDADLS_LMEM_NULL` The IDADLS linear solver has not been initialized.
Notes	The workspace requirements reported by this routine correspond only to memory allocated within this interface and to memory allocated by the SUNLINSOL object attached to it. The template Jacobian matrix allocated by the user outside of IDADLS is not included in this report.

| IDADlsGetNumJacEvals |

Call	`flag = IDADlsGetNumJacEvals(ida_mem, &njevals);`
Description	The function `IDADlsGetNumJacEvals` returns the cumulative number of calls to the IDADLS Jacobian approximation function.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`njevals` (`long int`) the cumulative number of calls to the Jacobian function (total so far).
Return value	The return value `flag` (of type `int`) is one of
	`IDADLS_SUCCESS` The optional output value has been successfully set.
	`IDADLS_MEM_NULL` The `ida_mem` pointer is `NULL`.
	`IDADLS_LMEM_NULL` The IDADENSE linear solver has not been initialized.

| IDADlsGetNumResEvals |

Call	`flag = IDADlsGetNumResEvals(ida_mem, &nrevalsLS);`
Description	The function `IDADlsGetNumResEvals` returns the cumulative number of calls to the user residual function due to the finite difference Jacobian approximation.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nrevalsLS` (`long int`) the cumulative number of calls to the user residual function.
Return value	The return value `flag` (of type `int`) is one of
	`IDADLS_SUCCESS` The optional output value has been successfully set.
	`IDADLS_MEM_NULL` The `ida_mem` pointer is `NULL`.
	`IDADLS_LMEM_NULL` The IDADENSE linear solver has not been initialized.

Notes   The value nrevalsLS is incremented only if one of the default internal difference quotient
             functions (dense or banded) is used.

| IDADlsGetLastFlag |

Call   flag = IDADlsGetLastFlag(ida_mem, &lsflag);

Description The function IDADlsGetLastFlag returns the last return value from an IDADLS routine.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

             lsflag (long int) the value of the last return flag from an IDADLS function.

Return value The return value flag (of type int) is one of

             IDADLS_SUCCESS   The optional output value has been successfully set.

             IDADLS_MEM_NULL   The ida_mem pointer is NULL.

             IDADLS_LMEM_NULL The IDADENSE linear solver has not been initialized.

Notes   If the SUNLINSOL_DENSE or SUNLINSOL_BAND setup function failed (IDASolve returned
             IDA_LSETUP_FAIL), then the value of lsflag is equal to the column index (numbered
             from one) at which a zero diagonal element was encountered during the LU factorization
             of the (dense or banded) Jacobian matrix.

| IDADlsGetReturnFlagName |

Call   name = IDADlsGetReturnFlagName(lsflag);

Description The function IDADlsGetReturnFlagName returns the name of the IDADLS constant cor-
             responding to lsflag.

Arguments   The only argument, of type long int, is a return flag from an IDADLS function.

Return value The return value is a string containing the name of the corresponding constant. If $1 \leq$
             $\texttt{lsflag} \leq N$ (LU factorization failed), this function returns "NONE".

4.5.9.6 Iterative linear solver interface optional output functions

The following optional outputs are available from the IDASPILS modules: workspace requirements,
number of linear iterations, number of linear convergence failures, number of calls to the preconditioner
setup and solve routines, number of calls to the Jacobian-vector setup and product routines, number
of calls to the residual routine for finite-difference Jacobian-vector product approximation, and last
return value from a linear solver function. Note that, where the name of an output would otherwise
conflict with the name of an optional output from the main solver, a suffix LS (for Linear Solver) has
been added (e.g. lenrwLS).

| IDASpilsGetWorkSpace |

Call   flag = IDASpilsGetWorkSpace(ida_mem, &lenrwLS, &leniwLS);

Description The function IDASpilsGetWorkSpace returns the global sizes of the IDASPILS real and
             integer workspaces.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

             lenrwLS (long int) the number of realtype values in the IDASPILS workspace.

             leniwLS (long int) the number of integer values in the IDASPILS workspace.

Return value The return value flag (of type int) is one of

             IDASPILS_SUCCESS   The optional output value has been successfully set.

             IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

             IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

Notes   The workspace requirements reported by this routine correspond only to memory allo-
               cated within this interface and to memory allocated by the SUNLINSOL object attached
               to it.

               In a parallel setting, the above values are global (i.e., summed over all processors).

IDASpilsGetNumLinIters

Call   flag = IDASpilsGetNumLinIters(ida_mem, &nliters);

Description   The function IDASpilsGetNumLinIters returns the cumulative number of linear itera-
               tions.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               nliters (long int) the current number of linear iterations.

Return value   The return value flag (of type int) is one of

               IDASPILS_SUCCESS   The optional output value has been successfully set.

               IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

               IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

IDASpilsGetNumConvFails

Call   flag = IDASpilsGetNumConvFails(ida_mem, &nlcfails);

Description   The function IDASpilsGetNumConvFails returns the cumulative number of linear con-
               vergence failures.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               nlcfails (long int) the current number of linear convergence failures.

Return value   The return value flag (of type int) is one of

               IDASPILS_SUCCESS   The optional output value has been successfully set.

               IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

               IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

IDASpilsGetNumPrecEvals

Call   flag = IDASpilsGetNumPrecEvals(ida_mem, &npevals);

Description   The function IDASpilsGetNumPrecEvals returns the cumulative number of precondi-
               tioner evaluations, i.e., the number of calls made to psetup.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               npevals (long int) the cumulative number of calls to psetup.

Return value   The return value flag (of type int) is one of

               IDASPILS_SUCCESS   The optional output value has been successfully set.

               IDASPILS_MEM_NULL   The ida_mem pointer is NULL.

               IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

IDASpilsGetNumPrecSolves

Call   flag = IDASpilsGetNumPrecSolves(ida_mem, &npsolves);

Description   The function IDASpilsGetNumPrecSolves returns the cumulative number of calls made
               to the preconditioner solve function, psolve.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

               npsolves (long int) the cumulative number of calls to psolve.

Return value   The return value `flag` (of type `int`) is one of

    IDASPILS_SUCCESS   The optional output value has been successfully set.

    IDASPILS_MEM_NULL   The `ida_mem` pointer is NULL.

    IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASpilsGetNumJTSetupEvals

Call   `flag = IDASpilsGetNumJTSetupEvals(ida_mem, &njtsetup);`

Description   The function `IDASpilsGetNumJTSetupEvals` returns the cumulative number of calls made to the Jacobian-vector setup function `jtsetup`.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

    `njtsetup` (`long int`) the current number of calls to `jtsetup`.

Return value   The return value `flag` (of type `int`) is one of

    IDASPILS_SUCCESS The optional output value has been successfully set.

    IDASPILS_MEM_NULL The `ida_mem` pointer is NULL.

    IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASpilsGetNumJtimesEvals

Call   `flag = IDASpilsGetNumJtimesEvals(ida_mem, &njvevals);`

Description   The function `IDASpilsGetNumJtimesEvals` returns the cumulative number of calls made to the Jacobian-vector function, `jtimes`.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

    `njvevals` (`long int`) the cumulative number of calls to `jtimes`.

Return value   The return value `flag` (of type `int`) is one of

    IDASPILS_SUCCESS   The optional output value has been successfully set.

    IDASPILS_MEM_NULL   The `ida_mem` pointer is NULL.

    IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASpilsGetNumResEvals

Call   `flag = IDASpilsGetNumResEvals(ida_mem, &nrevalsLS);`

Description   The function `IDASpilsGetNumResEvals` returns the cumulative number of calls to the user residual function for finite difference Jacobian-vector product approximation.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

    `nrevalsLS` (`long int`) the cumulative number of calls to the user residual function.

Return value   The return value `flag` (of type `int`) is one of

    IDASPILS_SUCCESS   The optional output value has been successfully set.

    IDASPILS_MEM_NULL   The `ida_mem` pointer is NULL.

    IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

Notes   The value `nrevalsLS` is incremented only if the default `IDASpilsDQJtimes` difference quotient function is used.

IDASpilsGetLastFlag

Call	`flag = IDASpilsGetLastFlag(ida_mem, &lsflag);`
Description	The function `IDASpilsGetLastFlag` returns the last return value from an IDASPILS routine.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`lsflag` (`long int`) the value of the last return flag from an IDASPILS function.
Return value	The return value `flag` (of type `int`) is one of

IDASPILS_SUCCESS   The optional output value has been successfully set.

IDASPILS_MEM_NULL   The `ida_mem` pointer is `NULL`.

IDASPILS_LMEM_NULL   The IDASPILS linear solver has not been initialized.

Notes	If the IDASPILS setup function failed (`IDASolve` returned `IDA_LSETUP_FAIL`), `lsflag` will be `SUNLS_PSET_FAIL_UNREC`, `SUNLS_ASET_FAIL_UNREC`, or `SUNLS_PACKAGE_FAIL_UNREC`.

If the IDASPILS solve function failed (`IDA` returned `IDA_LSOLVE_FAIL`), `lsflag` contains the error return flag from the SUNLINSOL object, which will be one of: `SUNLS_MEM_NULL`, indicating that the SUNLINSOL memory is `NULL`; `SUNLS_ATIMES_FAIL_UNREC`, indicating an unrecoverable failure in the $J*v$ function; `SUNLS_PSOLVE_FAIL_UNREC`, indicating that the preconditioner solve function `psolve` failed unrecoverably; `SUNLS_GS_FAIL`, indicating a failure in the Gram-Schmidt procedure (generated only in SPGMR or SPFGMR); `SUNLS_QRSOL_FAIL`, indicating that the matrix R was found to be singular during the QR solve phase (SPGMR and SPFGMR only); or `SUNLS_PACKAGE_FAIL_UNREC`, indicating an unrecoverable failure in an external iterative linear solver package.

IDASpilsGetReturnFlagName

Call	`name = IDASpilsGetReturnFlagName(lsflag);`
Description	The function `IDASpilsGetReturnFlagName` returns the name of the IDASPILS constant corresponding to `lsflag`.
Arguments	The only argument, of type `long int`, is a return flag from an IDASPILS function.
Return value	The return value is a string containing the name of the corresponding constant.

4.5.10   IDAS reinitialization function

The function `IDAReInit` reinitializes the main IDAS solver for the solution of a new problem, where a prior call to `IDAInit` has been made. The new problem must have the same size as the previous one. `IDAReInit` performs the same input checking and initializations that `IDAInit` does, but does no memory allocation, as it assumes that the existing internal memory is sufficient for the new problem. A call to `IDAReInit` deletes the solution history that was stored internally during the previous integration. Following a successful call to `IDAReInit`, call `IDASolve` again for the solution of the new problem.

The use of `IDAReInit` requires that the maximum method order, `maxord`, is no larger for the new problem than for the problem specified in the last call to `IDAInit`. In addition, the same NVECTOR module set for the previous problem will be reused for the new problem.

If there are changes to the linear solver specifications, make the appropriate calls to either the linear solver objects themselves, or to the IDADLS or IDASPILS interface routines, as described in §4.5.3.

If there are changes to any optional inputs, make the appropriate `IDASet***` calls, as described in §4.5.7. Otherwise, all solver inputs set previously remain in effect.

One important use of the `IDAReInit` function is in the treating of jump discontinuities in the residual function. Except in cases of fairly small jumps, it is usually more efficient to stop at each point of discontinuity and restart the integrator with a readjusted DAE model, using a call to `IDAReInit`. To stop when the location of the discontinuity is known, simply make that location a value of tout. To

stop when the location of the discontinuity is determined by the solution, use the rootfinding feature. In either case, it is critical that the residual function *not* incorporate the discontinuity, but rather have a smooth extention over the discontinuity, so that the step across it (and subsequent rootfinding, if used) can be done efficiently. Then use a switch within the residual function (communicated through `user_data`) that can be flipped between the stopping of the integration and the restart, so that the restarted problem uses the new values (which have jumped). Similar comments apply if there is to be a jump in the dependent variable vector.

`IDAReInit`

Call	`flag = IDAReInit(ida_mem, t0, y0, yp0);`
Description	The function `IDAReInit` provides required problem specifications and reinitializes IDAS.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`t0` (`realtype`) is the initial value of t.
	`y0` (`N_Vector`) is the initial value of y.
	`yp0` (`N_Vector`) is the initial value of $\dot{y}$.
Return value	The return value `flag` (of type `int`) will be one of the following:
	`IDA_SUCCESS` The call to `IDAReInit` was successful.
	`IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
	`IDA_NO_MALLOC` Memory space for the IDAS memory block was not allocated through a previous call to `IDAInit`.
	`IDA_ILL_INPUT` An input argument to `IDAReInit` has an illegal value.
Notes	If an error occurred, `IDAReInit` also sends an error message to the error handler function.

4.6   User-supplied functions

The user-supplied functions consist of one function defining the DAE residual, (optionally) a function that handles error and warning messages, (optionally) a function that provides the error weight vector, (optionally) one or two functions that provide Jacobian-related information for the linear solver (if Newton iteration is chosen), and (optionally) one or two functions that define the preconditioner for use in any of the Krylov iteration algorithms.

4.6.1   Residual function

The user must provide a function of type `IDAResFn` defined as follows:

`IDAResFn`

Definition	`typedef int (*IDAResFn)(realtype tt, N_Vector yy, N_Vector yp,`
	`                        N_Vector rr, void *user_data);`
Purpose	This function computes the problem residual for given values of the independent variable t, state vector y, and derivative $\dot{y}$.
Arguments	`tt` is the current value of the independent variable.
	`yy` is the current value of the dependent variable vector, $y(t)$.
	`yp` is the current value of $\dot{y}(t)$.
	`rr` is the output residual vector $F(t, y, \dot{y})$.
	`user_data` is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.

Return value An `IDAResFn` function type should return a value of 0 if successful, a positive value if a recoverable error occurred (e.g. `yy` has an illegal value), or a negative value if a nonrecoverable error occurred. In the last case, the integrator halts. If a recoverable error occurred, the integrator will attempt to correct and retry.

Notes A recoverable failure error return from the `IDAResFn` is typically used to flag a value of the dependent variable `y` that is "illegal" in some way (e.g., negative where only a non-negative value is physically meaningful). If such a return is made, IDAS will attempt to recover (possibly repeating the Newton iteration, or reducing the step size) in order to avoid this recoverable error return.

For efficiency reasons, the DAE residual function is not evaluated at the converged solution of the nonlinear solver. Therefore, in general, a recoverable error in that converged value cannot be corrected. (It may be detected when the right-hand side function is called the first time during the following integration step, but a successful step cannot be undone.) However, if the user program also includes quadrature integration, the state variables can be checked for legality in the call to `IDAQuadRhsFn`, which is called at the converged solution of the nonlinear system, and therefore IDAS can be flagged to attempt to recover from such a situation. Also, if sensitivity analysis is performed with the staggered method, the DAE residual function is called at the converged solution of the nonlinear system, and a recoverable error at that point can be flagged, and IDAS will then try to correct it.

Allocation of memory for `yp` is handled within IDAS.

4.6.2 Error message handler function

As an alternative to the default behavior of directing error and warning messages to the file pointed to by `errfp` (see `IDASetErrFile`), the user may provide a function of type `IDAErrHandlerFn` to process any such messages. The function type `IDAErrHandlerFn` is defined as follows:

IDAErrHandlerFn

Definition
```
typedef void (*IDAErrHandlerFn)(int error_code, const char *module,
                                const char *function, char *msg,
                                void *eh_data);
```

Purpose This function processes error and warning messages from IDAS and its sub-modules.

Arguments `error_code` is the error code.

`module` is the name of the IDAS module reporting the error.

`function` is the name of the function in which the error occurred.

`msg` is the error message.

`eh_data` is a pointer to user data, the same as the `eh_data` parameter passed to `IDASetErrHandlerFn`.

Return value A `IDAErrHandlerFn` function has no return value.

Notes `error_code` is negative for errors and positive (`IDA_WARNING`) for warnings. If a function that returns a pointer to memory encounters an error, it sets `error_code` to 0.

4.6.3 Error weight function

As an alternative to providing the relative and absolute tolerances, the user may provide a function of type `IDAEwtFn` to compute a vector `ewt` containing the multiplicative weights W_i used in the WRMS norm $\| v \|_{\mathrm{WRMS}} = \sqrt{(1/N) \sum_1^N (W_i \cdot v_i)^2}$. These weights will used in place of those defined by Eq. (2.7). The function type `IDAEwtFn` is defined as follows:

> ⟦IDAEwtFn⟧

Definition	`typedef int (*IDAEwtFn)(N_Vector y, N_Vector ewt, void *user_data);`
Purpose	This function computes the WRMS error weights for the vector y.
Arguments	`y` is the value of the dependent variable vector at which the weight vector is to be computed.
	`ewt` is the output vector containing the error weights.
	`user_data` is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.
Return value	An `IDAEwtFn` function type must return 0 if it successfully set the error weights and -1 otherwise.
Notes	Allocation of memory for `ewt` is handled within IDAS.
	The error weight vector must have all components positive. It is the user's responsiblity to perform this test and return -1 if it is not satisfied.

4.6.4   Rootfinding function

If a rootfinding problem is to be solved during the integration of the DAE system, the user must supply a C function of type `IDARootFn`, defined as follows:

> ⟦IDARootFn⟧

Definition	`typedef int (*IDARootFn)(realtype t, N_Vector y, N_Vector yp,` `realtype *gout, void *user_data);`
Purpose	This function computes a vector-valued function $g(t, y, \dot{y})$ such that the roots of the `nrtfn` components $g_i(t, y, \dot{y})$ are to be found during the integration.
Arguments	`t` is the current value of the independent variable.
	`y` is the current value of the dependent variable vector, $y(t)$.
	`yp` is the current value of $\dot{y}(t)$, the $t-$derivative of y.
	`gout` is the output array, of length `nrtfn`, with components $g_i(t, y, \dot{y})$.
	`user_data` is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.
Return value	An `IDARootFn` should return 0 if successful or a non-zero value if an error occurred (in which case the integration is halted and `IDASolve` returns `IDA_RTFUNC_FAIL`).
Notes	Allocation of memory for `gout` is handled within IDAS.

4.6.5   Jacobian information (direct method Jacobian)

If the direct linear solver interface is used (i.e. `IDADlsSetLinearSolver` is called in the step described in §4.4), the user may provide a function of type `IDADlsJacFn` defined as follows:

> ⟦IDADlsJacFn⟧

Definition	`typedef int (*IDADlsJacFn)(realtype tt, realtype cj,`
	                             `N_Vector yy, N_Vector yp, N_Vector rr,`
	                             `SUNMatrix Jac, void *user_data,`
	                             `N_Vector tmp1, N_Vector tmp2, N_Vector tmp3);`

Purpose	This function computes the Jacobian matrix J of the DAE system (or an approximation to it), defined by Eq. (2.6).
Arguments	`tt` is the current value of the independent variable t.
	`cj` is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).

`yy`	is the current value of the dependent variable vector, $y(t)$.
`yp`	is the current value of $\dot{y}(t)$.
`rr`	is the current value of the residual vector $F(t, y, \dot{y})$.
`Jac`	is the output (approximate) Jacobian matrix (of type `SUNMatrix`), $J = \partial F/\partial y + cj\ \partial F/\partial \dot{y}$.
`user_data`	is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.
`tmp1`	
`tmp2`	
`tmp3`	are pointers to memory allocated for variables of type `N_Vector` which can be used by `IDADlsJacFn` function as temporary storage or work space.

Return value An `IDADlsJacFn` should return 0 if successful, a positive value if a recoverable error occurred, or a negative value if a nonrecoverable error occurred.

In the case of a recoverable eror return, the integrator will attempt to recover by reducing the stepsize, and hence changing α in (2.6).

Notes Information regarding the structure of the specific sunmatrix structure (e.g. number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific sunmatrix interface functions (see Chapter 8 for details).

Prior to calling the user-supplied Jacobian function, the Jacobian matrix $J(t, y)$ is zeroed out, so only nonzero elements need to be loaded into `Jac`.

If the user's `IDADlsJacFn` function uses difference quotient approximations, it may need to access quantities not in the call list. These quantities may include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_data` and then use the `IDAGet*` functions described in §4.5.9.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in `sundials_types.h`.

dense:
A user-supplied dense Jacobian function must load the `Neq` $\times$ `Neq` dense matrix `Jac` with an approximation to the Jacobian matrix $J(t, y, \dot{y})$ at the point (`tt`, `yy`, `yp`). The accessor macros `SM_ELEMENT_D` and `SM_COLUMN_D` allow the user to read and write dense matrix elements without making explicit references to the underlying representation of the sunmatrix_dense type. `SM_ELEMENT_D(J, i, j)` references the (`i`, `j`)-th element of the dense matrix `Jac` (with `i, j` $= 0 \ldots N - 1$). This macro is meant for small problems for which efficiency of access is not a major concern. Thus, in terms of the indices m and n ranging from 1 to N, the Jacobian element $J_{m,n}$ can be set using the statement `SM_ELEMENT_D(J, m-1, n-1)` $= J_{m,n}$. Alternatively, `SM_COLUMN_D(J, j)` returns a pointer to the first element of the `j`-th column of `Jac` (with `j` $= 0 \ldots N - 1$), and the elements of the `j`-th column can then be accessed using ordinary array indexing. Consequently, $J_{m,n}$ can be loaded using the statements `col_n = SM_COLUMN_D(J, n-1)`; `col_n[m-1]` $= J_{m,n}$. For large problems, it is more efficient to use `SM_COLUMN_D` than to use `SM_ELEMENT_D`. Note that both of these macros number rows and columns starting from 0. The sunmatrix_dense type and accessor macros are documented in §8.1.

banded:
A user-supplied banded Jacobian function must load the `Neq` $\times$ `Neq` banded matrix `Jac` with an approximation to the Jacobian matrix $J(t, y, \dot{y})$ at the point (`tt`, `yy`, `yp`). The accessor macros `SM_ELEMENT_B`, `SM_COLUMN_B`, and `SM_COLUMN_ELEMENT_B` allow the user to read and write banded matrix elements without making specific references to the underlying representation of the sunmatrix_band type. `SM_ELEMENT_B(J, i, j)` references the (`i`, `j`)-th element of the banded matrix `Jac`, counting from 0. This macro is meant for use in small problems for which efficiency of access is not a major concern. Thus, in terms of the indices m and n ranging from 1 to `N` with (m, n) within the band defined by `mupper` and `mlower`, the Jacobian element $J_{m,n}$ can be

loaded using the statement SM_ELEMENT_B(J, m-1, n-1) = $J_{m,n}$. The elements within the band are those with -mupper $\leq$ m-n $\leq$ mlower. Alternatively, SM_COLUMN_B(J, j) returns a pointer to the diagonal element of the j-th column of Jac, and if we assign this address to realtype *col_j, then the i-th element of the j-th column is given by SM_COLUMN_ELEMENT_B(col_j, i, j), counting from 0. Thus, for (m,n) within the band, $J_{m,n}$ can be loaded by setting col_n = SM_COLUMN_B(J, n-1); and SM_COLUMN_ELEMENT_B(col_n, m-1, n-1) = $J_{m,n}$. The elements of the j-th column can also be accessed via ordinary array indexing, but this approach requires knowledge of the underlying storage for a band matrix of type SUNMATRIX_BAND. The array col_n can be indexed from −mupper to mlower. For large problems, it is more efficient to use SM_COLUMN_B and SM_COLUMN_ELEMENT_B than to use the SM_ELEMENT_B macro. As in the dense case, these macros all number rows and columns starting from 0. The SUNMATRIX_BAND type and accessor macros are documented in §8.2.

sparse:
A user-supplied sparse Jacobian function must load the Neq $\times$ Neq compressed-sparse-column or compressed-sparse-row matrix Jac with an approximation to the Jacobian matrix $J(t, y, \dot{y})$ at the point (tt, yy, yp). Storage for Jac already exists on entry to this function, although the user should ensure that sufficient space is allocated in Jac to hold the nonzero values to be set; if the existing space is insufficient the user may reallocate the data and index arrays as needed. The amount of allocated space in a SUNMATRIX_SPARSE object may be accessed using the macro SM_NNZ_S or the routine SUNSparseMatrix_NNZ. The SUNMATRIX_SPARSE type and accessor macros are documented in §8.3.

4.6.6 Jacobian information (matrix-vector product)

If the IDASPILS solver interface is selected (i.e., IDASpilsSetLinearSolver is called in the steps described in §4.4), the user may provide a function of type IDASpilsJacTimesVecFn in the following form, to compute matrix-vector products Jv. If such a function is not supplied, the default is a difference quotient approximation to these products.

IDASpilsJacTimesVecFn

Definition
```
typedef int (*IDASpilsJacTimesVecFn)(realtype tt, N_Vector yy,
                                     N_Vector yp, N_Vector rr,
                                     N_Vector v, N_Vector Jv,
                                     realtype cj, void *user_data,
                                     N_Vector tmp1, N_Vector tmp2);
```

Purpose   This function computes the product Jv of the DAE system Jacobian J (or an approximation to it) and a given vector v, where J is defined by Eq. (2.6).

Arguments   tt   is the current value of the independent variable.

             yy   is the current value of the dependent variable vector, $y(t)$.

             yp   is the current value of $\dot{y}(t)$.

             rr   is the current value of the residual vector $F(t, y, \dot{y})$.

             v   is the vector by which the Jacobian must be multiplied to the right.

             Jv   is the computed output vector.

             cj   is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).

             user_data is a pointer to user data, the same as the user_data parameter passed to IDASetUserData.

             tmp1

<div style="margin-left:2em">

tmp2   are pointers to memory allocated for variables of type N_Vector which can
              be used by IDASpilsJacTimesVecFn as temporary storage or work space.

</div>

Return value The value returned by the Jacobian-times-vector function should be 0 if successful. A
              nonzero value indicates that a nonrecoverable error occurred.

Notes   This function must return a value of $J * v$ that uses the *current* value of J, i.e. as
              evaluated at the current $(t, y, \dot{y})$.

              If the user's IDASpilsJacTimesVecFn function uses difference quotient approximations,
              it may need to access quantities not in the call list. These include the current stepsize,
              the error weights, etc. To obtain these, the user will need to add a pointer to ida_mem to
              user_data and then use the IDAGet* functions described in §4.5.9.2. The unit roundoff
              can be accessed as UNIT_ROUNDOFF defined in sundials_types.h.

4.6.7   Jacobian information (matrix-vector setup)

If the user's Jacobian-times-vector requires that any Jacobian-related data be preprocessed or eval-
uated, then this needs to be done in a user-supplied function of type IDASpilsJacTimesSetupFn,
defined as follows:

| IDASpilsJacTimesSetupFn |

Definition   ```
              typedef int (*IDASpilsJacTimesSetupFn)(realtype tt, N_Vector yy,
                                                     N_Vector yp, N_Vector rr,
                                                     realtype cj, void *user_data);
              ```

Purpose       This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-
              times-vector routine.

Arguments     tt          is the current value of the independent variable.

              yy          is the current value of the dependent variable vector, $y(t)$.

              yp          is the current value of $\dot{y}(t)$.

              rr          is the current value of the residual vector $F(t, y, \dot{y})$.

              cj          is the scalar in the system Jacobian, proportional to the inverse of the step
                          size ($\alpha$ in Eq. (2.6) ).

              user_data   is a pointer to user data, the same as the user_data parameter passed to
                          IDASetUserData.

Return value  The value returned by the Jacobian-vector setup function should be 0 if successful,
              positive for a recoverable error (in which case the step will be retried), or negative for
              an unrecoverable error (in which case the integration is halted).

Notes         Each call to the Jacobian-vector setup function is preceded by a call to the IDARhsFn
              user function with the same (t,y, yp) arguments. Thus, the setup function can use any
              auxiliary data that is computed and saved during the evaluation of the DAE residual.

              If the user's IDASpilsJacTimesVecFn function uses difference quotient approximations,
              it may need to access quantities not in the call list. These include the current stepsize,
              the error weights, etc. To obtain these, the user will need to add a pointer to ida_mem to
              user_data and then use the IDAGet* functions described in §4.5.9.2. The unit roundoff
              can be accessed as UNIT_ROUNDOFF defined in sundials_types.h.

### 4.6.8   Preconditioning (linear system solution)

If preconditioning is used, then the user must provide a function to solve the linear system $Pz = r$
where $P$ is a left preconditioner matrix which approximates (at least crudely) the Jacobian matrix
$J = \partial F/\partial y + cj \, \partial F/\partial \dot{y}$. This function must be of type IDASpilsPrecSolveFn, defined as follows:

---

IDASpilsPrecSolveFn

| | |
|---|---|
| Definition | `typedef int (*IDASpilsPrecSolveFn)(realtype tt, N_Vector yy,`<br>`                                   N_Vector yp, N_Vector rr,`<br>`                                   N_Vector rvec, N_Vector zvec,`<br>`                                   realtype cj, realtype delta,`<br>`                                   void *user_data);` |

Purpose   This function solves the preconditioning system $Pz = r$.

Arguments
tt        is the current value of the independent variable.

yy        is the current value of the dependent variable vector, $y(t)$.

yp        is the current value of $\dot{y}(t)$.

rr        is the current value of the residual vector $F(t, y, \dot{y})$.

rvec      is the right-hand side vector $r$ of the linear system to be solved.

zvec      is the computed output vector.

cj        is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6) ).

delta     is an input tolerance to be used if an iterative method is employed in the solution. In that case, the residual vector $Res = r - Pz$ of the system should be made less than delta in weighted $l_2$ norm, i.e., $\sqrt{\sum_i (Res_i \cdot ewt_i)^2} <$ delta. To obtain the N_Vector ewt, call IDAGetErrWeights (see §4.5.9.2).

user_data is a pointer to user data, the same as the user_data parameter passed to the function IDASetUserData.

Return value  The value to be returned by the preconditioner solve function is a flag indicating whether it was successful. This value should be 0 if successful, positive for a recoverable error (in which case the step will be retried), negative for an unrecoverable error (in which case the integration is halted).

## 4.6.9   Preconditioning (Jacobian data)

If the user's preconditioner requires that any Jacobian-related data be evaluated or preprocessed, then this needs to be done in a user-supplied function of type IDASpilsPrecSetupFn, defined as follows:

---

IDASpilsPrecSetupFn

| | |
|---|---|
| Definition | `typedef int (*IDASpilsPrecSetupFn)(realtype tt, N_Vector yy,`<br>`                                   N_Vector yp, N_Vector rr,`<br>`                                   realtype cj, void *user_data);` |

Purpose   This function evaluates and/or preprocesses Jacobian-related data needed by the preconditioner.

Arguments
tt        is the current value of the independent variable.

yy        is the current value of the dependent variable vector, $y(t)$.

yp        is the current value of $\dot{y}(t)$.

rr        is the current value of the residual vector $F(t, y, \dot{y})$.

cj        is the scalar in the system Jacobian, proportional to the inverse of the step size ($\alpha$ in Eq. (2.6) ).

user_data is a pointer to user data, the same as the user_data parameter passed to the function IDASetUserData.

Return value  The value returned by the preconditioner setup function is a flag indicating whether it was successful. This value should be 0 if successful, positive for a recoverable error (in which case the step will be retried), negative for an unrecoverable error (in which case the integration is halted).

Notes  The operations performed by this function might include forming a crude approximate Jacobian, and performing an LU factorization on the resulting approximation.

Each call to the preconditioner setup function is preceded by a call to the `IDAResFn` user function with the same (`tt`, `yy`, `yp`) arguments. Thus the preconditioner setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

This function is not called in advance of every call to the preconditioner solve function, but rather is called only as often as needed to achieve convergence in the Newton iteration.

If the user's `IDASpilsPrecSetupFn` function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to `ida_mem` to `user_data` and then use the `IDAGet*` functions described in §4.5.9.2. The unit roundoff can be accessed as `UNIT_ROUNDOFF` defined in `sundials_types.h`.

## 4.7   Integration of pure quadrature equations

IDAS allows the DAE system to include *pure quadratures*. In this case, it is more efficient to treat the quadratures separately by excluding them from the nonlinear solution stage. To do this, begin by excluding the quadrature variables from the vectors `yy` and `yp` and the quadrature equations from within `res`. Thus a separate vector `yQ` of quadrature variables is to satisfy $(d/dt)\text{yQ} = f_Q(t, y, \dot{y})$. The following is an overview of the sequence of calls in a user's main program in this situation. Steps that are unchanged from the skeleton program presented in §4.4 are grayed out.

1. **Initialize parallel or multi-threaded environment, if appropriate**

2. **Set problem dimensions, etc.**

   This generally includes `N`, the problem size $N$ (excluding quadrature variables), `Nq`, the number of quadrature variables, and may include the local vector length `Nlocal` (excluding quadrature variables), and local number of quadrature variables `Nqlocal`.

3. **Set vectors of initial values**

4. **Create IDAS object**

5. **Allocate internal memory**

6. **Set optional inputs**

7. **Attach linear solver module**

8. **Set linear solver optional inputs**

9. **Set vector of initial values for quadrature variables**

   Typically, the quadrature variables should be initialized to 0.

10. **Initialize quadrature integration**

    Call `IDAQuadInit` to specify the quadrature equation right-hand side function and to allocate internal memory related to quadrature integration. See §4.7.1 for details.

11. **Set optional inputs for quadrature integration**

    Call `IDASetQuadErrCon` to indicate whether or not quadrature variables should be used in the step size control mechanism. If so, one of the `IDAQuad*tolerances` functions must be called to specify the integration tolerances for quadrature variables. See §4.7.4 for details.

12. **Advance solution in time**

13. **Extract quadrature variables**

    Call `IDAGetQuad` or `IDAGetQuadDky` to obtain the values of the quadrature variables or their derivatives at the current time. See §4.7.3 for details.

14. **Get optional outputs**

15. **Get quadrature optional outputs**

    Call `IDAGetQuad*` functions to obtain optional output related to the integration of quadratures. See §4.7.5 for details.

16. **Deallocate memory for solution vectors and for the vector of quadrature variables**

17. **Free solver memory**

18. **Finalize MPI, if used**

`IDAQuadInit` can be called and quadrature-related optional inputs (step 11 above) can be set, anywhere between steps 4 and 12.

## 4.7.1   Quadrature initialization and deallocation functions

The function `IDAQuadInit` activates integration of quadrature equations and allocates internal memory related to these calculations. The form of the call to this function is as follows:

---

| `IDAQuadInit` |
| --- |

| Call | `flag = IDAQuadInit(ida_mem, rhsQ, yQ0);` |
| --- | --- |
| Description | The function `IDAQuadInit` provides required problem specifications, allocates internal memory, and initializes quadrature integration. |
| Arguments | `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`. |
| | `rhsQ` (`IDAQuadRhsFn`) is the C function which computes $f_Q$, the right-hand side of the quadrature equations. This function has the form `fQ(t, yy, yp, rhsQ, user_data)` (for full details see §4.7.6). |
| | `yQ0` (`N_Vector`) is the initial value of $y_Q$. |
| Return value | The return value `flag` (of type `int`) will be one of the following: |
| | `IDA_SUCCESS`  The call to `IDAQuadInit` was successful. |
| | `IDA_MEM_NULL` The IDAS memory was not initialized by a prior call to `IDACreate`. |
| | `IDA_MEM_FAIL` A memory allocation request failed. |
| Notes | If an error occurred, `IDAQuadInit` also sends an error message to the error handler function. |

In terms of the number of quadrature variables $N_q$ and maximum method order `maxord`, the size of the real workspace is increased as follows:

- Base value: `lenrw = lenrw + (maxord+5)`$N_q$

- If `IDAQuadSVtolerances` is called: `lenrw = lenrw +`$N_q$

and the size of the integer workspace is increased as follows:

- Base value: `leniw = leniw + (maxord+5)`$N_q$

- If `IDAQuadSVtolerances` is called: `leniw = leniw +`$N_q$

The function `IDAQuadReInit`, useful during the solution of a sequence of problems of same size, reinitializes the quadrature-related internal memory and must follow a call to `IDAQuadInit` (and maybe a call to `IDAReInit`). The number `Nq` of quadratures is assumed to be unchanged from the prior call to `IDAQuadInit`. The call to the `IDAQuadReInit` function has the following form:

---

| `IDAQuadReInit` |
|---|

| | |
|---|---|
| Call | `flag = IDAQuadReInit(ida_mem, yQ0);` |
| Description | The function `IDAQuadReInit` provides required problem specifications and reinitializes the quadrature integration. |
| Arguments | `ida_mem` (`void *`) pointer to the IDAS memory block. |
| | `yQ0` (`N_Vector`) is the initial value of $y_Q$. |
| Return value | The return value `flag` (of type `int`) will be one of the following: |
| | `IDA_SUCCESS` The call to `IDAReInit` was successful. |
| | `IDA_MEM_NULL` The IDAS memory was not initialized by a prior call to `IDACreate`. |
| | `IDA_NO_QUAD` Memory space for the quadrature integration was not allocated by a prior call to `IDAQuadInit`. |
| Notes | If an error occurred, `IDAQuadReInit` also sends an error message to the error handler function. |

---

| `IDAQuadFree` |
|---|

| | |
|---|---|
| Call | `IDAQuadFree(ida_mem);` |
| Description | The function `IDAQuadFree` frees the memory allocated for quadrature integration. |
| Arguments | The argument is the pointer to the IDAS memory block (of type `void *`). |
| Return value | The function `IDAQuadFree` has no return value. |
| Notes | In general, `IDAQuadFree` need not be called by the user as it is invoked automatically by `IDAFree`. |

### 4.7.2   IDAS solver function

Even if quadrature integration was enabled, the call to the main solver function `IDASolve` is exactly the same as in §4.5.6. However, in this case the return value `flag` can also be one of the following:

| | |
|---|---|
| `IDA_QRHS_FAIL` | The quadrature right-hand side function failed in an unrecoverable manner. |
| `IDA_FIRST_QRHS_ERR` | The quadrature right-hand side function failed at the first call. |
| `IDA_REP_QRHS_ERR` | Convergence test failures occurred too many times due to repeated recoverable errors in the quadrature right-hand side function. This value will also be returned if the quadrature right-hand side function had repeated recoverable errors during the estimation of an initial step size (assuming the quadrature variables are included in the error tests). |

### 4.7.3   Quadrature extraction functions

If quadrature integration has been initialized by a call to `IDAQuadInit`, or reinitialized by a call to `IDAQuadReInit`, then IDAS computes both a solution and quadratures at time `t`. However, `IDASolve` will still return only the solution $y$ in `y`. Solution quadratures can be obtained using the following function:

---

IDAGetQuad

Call            `flag = IDAGetQuad(ida_mem, &tret, yQ);`

Description     The function `IDAGetQuad` returns the quadrature solution vector after a successful return
                from `IDASolve`.

Arguments       `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

                `tret`    (`realtype`) the time reached by the solver (output).

                `yQ`      (`N_Vector`) the computed quadrature vector.

Return value    The return value `flag` of `IDAGetQuad` is one of:

                `IDA_SUCCESS`   `IDAGetQuad` was successful.

                `IDA_MEM_NULL`  `ida_mem` was NULL.

                `IDA_NO_QUAD`   Quadrature integration was not initialized.

                `IDA_BAD_DKY`   `yQ` is NULL.

The function `IDAGetQuadDky` computes the k-th derivatives of the interpolating polynomials for the
quadrature variables at time `t`. This function is called by `IDAGetQuad` with `k = 0` and with the current
time at which `IDASolve` has returned, but may also be called directly by the user.

---

IDAGetQuadDky

Call            `flag = IDAGetQuadDky(ida_mem, t, k, dkyQ);`

Description     The function `IDAGetQuadDky` returns derivatives of the quadrature solution vector after
                a successful return from `IDASolve`.

Arguments       `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

                `t`       (`realtype`) the time at which quadrature information is requested. The time
                          `t` must fall within the interval defined by the last successful step taken by IDAS.

                `k`       (`int`) order of the requested derivative. This must be $\leq klast$.

                `dkyQ`    (`N_Vector`) the vector containing the derivative. This vector must be allocated
                          by the user.

Return value    The return value `flag` of `IDAGetQuadDky` is one of:

                `IDA_SUCCESS`   `IDAGetQuadDky` succeeded.

                `IDA_MEM_NULL`  The pointer to `ida_mem` was NULL.

                `IDA_NO_QUAD`   Quadrature integration was not initialized.

                `IDA_BAD_DKY`   The vector `dkyQ` is NULL.

                `IDA_BAD_K`     `k` is not in the range $0, 1, ..., klast$.

                `IDA_BAD_T`     The time `t` is not in the allowed range.

## 4.7.4   Optional inputs for quadrature integration

IDAS provides the following optional input functions to control the integration of quadrature equations.

---

IDASetQuadErrCon

Call            `flag = IDASetQuadErrCon(ida_mem, errconQ);`

Description     The function `IDASetQuadErrCon` specifies whether or not the quadrature variables are
                to be used in the step size control mechanism within IDAS. If they are, the user must
                call either `IDAQuadSStolerances` or `IDAQuadSVtolerances` to specify the integration
                tolerances for the quadrature variables.

Arguments       `ida_mem` (`void *`) pointer to the IDAS memory block.

errconQ (`booleantype`) specifies whether quadrature variables are included (`SUNTRUE`) or not (`SUNFALSE`) in the error control mechanism.

Return value The return value `flag` (of type `int`) is one of:

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL The `ida_mem` pointer is `NULL`

IDA_NO_QUAD   Quadrature integration has not been initialized.

Notes        By default, errconQ is set to `SUNFALSE`.

It is illegal to call `IDASetQuadErrCon` before a call to `IDAQuadInit`.                ⚠

If the quadrature variables are part of the step size control mechanism, one of the following functions must be called to specify the integration tolerances for quadrature variables.

---

| `IDAQuadSStolerances` |

Call        `flag = IDAQuadSVtolerances(ida_mem, reltolQ, abstolQ);`

Description  The function `IDAQuadSStolerances` specifies scalar relative and absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

reltolQ (`realtype`) is the scalar relative error tolerance.

abstolQ (`realtype`) is the scalar absolute error tolerance.

Return value The return value `flag` (of type `int`) is one of:

IDA_SUCCESS    The optional value has been successfully set.

IDA_NO_QUAD    Quadrature integration was not initialized.

IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

IDA_ILL_INPUT  One of the input tolerances was negative.

---

| `IDAQuadSVtolerances` |

Call        `flag = IDAQuadSVtolerances(ida_mem, reltolQ, abstolQ);`

Description  The function `IDAQuadSVtolerances` specifies scalar relative and vector absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

reltolQ (`realtype`) is the scalar relative error tolerance.

abstolQ (`N_Vector`) is the vector absolute error tolerance.

Return value The return value `flag` (of type `int`) is one of:

IDA_SUCCESS    The optional value has been successfully set.

IDA_NO_QUAD    Quadrature integration was not initialized.

IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

IDA_ILL_INPUT  One of the input tolerances was negative.

## 4.7.5   Optional outputs for quadrature integration

IDAS provides the following functions that can be used to obtain solver performance information related to quadrature integration.

---

| IDAGetQuadNumRhsEvals |
| --- |

Call            `flag = IDAGetQuadNumRhsEvals(ida_mem, &nrhsQevals);`

Description     The function `IDAGetQuadNumRhsEvals` returns the number of calls made to the user's quadrature right-hand side function.

Arguments       `ida_mem` (`void *`) pointer to the IDAS memory block.

                `nrhsQevals` (`long int`) number of calls made to the user's `rhsQ` function.

Return value    The return value `flag` (of type `int`) is one of:

      `IDA_SUCCESS`    The optional output value has been successfully set.

      `IDA_MEM_NULL`   The `ida_mem` pointer is NULL.

      `IDA_NO_QUAD`    Quadrature integration has not been initialized.

---

| IDAGetQuadNumErrTestFails |
| --- |

Call            `flag = IDAGetQuadNumErrTestFails(ida_mem, &nQetfails);`

Description     The function `IDAGetQuadNumErrTestFails` returns the number of local error test failures due to quadrature variables.

Arguments       `ida_mem` (`void *`) pointer to the IDAS memory block.

                `nQetfails` (`long int`) number of error test failures due to quadrature variables.

Return value    The return value `flag` (of type `int`) is one of:

      `IDA_SUCCESS`    The optional output value has been successfully set.

      `IDA_MEM_NULL`   The `ida_mem` pointer is NULL.

      `IDA_NO_QUAD`    Quadrature integration has not been initialized.

---

| IDAGetQuadErrWeights |
| --- |

Call            `flag = IDAGetQuadErrWeights(ida_mem, eQweight);`

Description     The function `IDAGetQuadErrWeights` returns the quadrature error weights at the current time.

Arguments       `ida_mem` (`void *`) pointer to the IDAS memory block.

                `eQweight` (`N_Vector`) quadrature error weights at the current time.

Return value    The return value `flag` (of type `int`) is one of:

      `IDA_SUCCESS`    The optional output value has been successfully set.

      `IDA_MEM_NULL`   The `ida_mem` pointer is NULL.

      `IDA_NO_QUAD`    Quadrature integration has not been initialized.

Notes           The user must allocate memory for `eQweight`.

                If quadratures were not included in the error control mechanism (through a call to `IDASetQuadErrCon` with `errconQ = SUNTRUE`), `IDAGetQuadErrWeights` does not set the `eQweight` vector.

---

| IDAGetQuadStats |
| --- |

Call            `flag = IDAGetQuadStats(ida_mem, &nrhsQevals, &nQetfails);`

Description     The function `IDAGetQuadStats` returns the IDAS integrator statistics as a group.

Arguments       `ida_mem` (`void *`) pointer to the IDAS memory block.

                `nrhsQevals` (`long int`) number of calls to the user's `rhsQ` function.

                `nQetfails` (`long int`) number of error test failures due to quadrature variables.

Return value    The return value `flag` (of type `int`) is one of

IDA_SUCCESS   the optional output values have been successfully set.

IDA_MEM_NULL the ida_mem pointer is NULL.

IDA_NO_QUAD   Quadrature integration has not been initialized.

### 4.7.6   User-supplied function for quadrature integration

For integration of quadrature equations, the user must provide a function that defines the right-hand side of the quadrature equations (in other words, the integrand function of the integral that must be evaluated). This function must be of type IDAQuadRhsFn defined as follows:

---

| IDAQuadRhsFn |
| --- |

| Definition | `typedef int (*IDAQuadRhsFn)(realtype t, N_Vector yy, N_Vector yp,`<br>`                            N_Vector rhsQ, void *user_data);` |
| Purpose | This function computes the quadrature equation right-hand side for a given value of the independent variable $t$ and state vectors $y$ and $\dot{y}$. |
| Arguments | t          is the current value of the independent variable. |
|  | yy         is the current value of the dependent variable vector, $y(t)$. |
|  | yp         is the current value of the dependent variable derivative vector, $\dot{y}(t)$. |
|  | rhsQ       is the output vector $f_Q(t, y, \dot{y})$. |
|  | user_data is the user_data pointer passed to IDASetUserData. |
| Return value | A IDAQuadRhsFn should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDA_QRHS_FAIL is returned). |
| Notes | Allocation of memory for rhsQ is automatically handled within IDAS. |
|  | Both y and rhsQ are of type N_Vector, but they typically have different internal representations. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their N_Vector arguments (see §7.1 and §7.2). |
|  | There is one situation in which recovery is not possible even if IDAQuadRhsFn function returns a recoverable error flag. This is when this occurs at the very first call to the IDAQuadRhsFn (in which case IDAS returns IDA_FIRST_QRHS_ERR). |

## 4.8   A parallel band-block-diagonal preconditioner module

A principal reason for using a parallel DAE solver such as IDAS lies in the solution of partial differential equations (PDEs). Moreover, the use of a Krylov iterative method for the solution of many such problems is motivated by the nature of the underlying linear system of equations (2.5) that must be solved at each time step. The linear algebraic system is large, sparse, and structured. However, if a Krylov iterative method is to be effective in this setting, then a nontrivial preconditioner needs to be used. Otherwise, the rate of convergence of the Krylov iterative method is usually unacceptably slow. Unfortunately, an effective preconditioner tends to be problem-specific.

However, we have developed one type of preconditioner that treats a rather broad class of PDE-based problems. It has been successfully used for several realistic, large-scale problems [26] and is included in a software module within the IDAS package. This module works with the parallel vector module NVECTOR_PARALLEL and generates a preconditioner that is a block-diagonal matrix with each block being a band matrix. The blocks need not have the same number of super- and sub-diagonals, and these numbers may vary from block to block. This Band-Block-Diagonal Preconditioner module is called IDABBDPRE.

One way to envision these preconditioners is to think of the domain of the computational PDE problem as being subdivided into $M$ non-overlapping sub-domains. Each of these sub-domains is then assigned to one of the $M$ processors to be used to solve the DAE system. The basic idea is to isolate the preconditioning so that it is local to each processor, and also to use a (possibly cheaper) approximate residual function. This requires the definition of a new function $G(t, y, \dot{y})$ which approximates the function $F(t, y, \dot{y})$ in the definition of the DAE system (2.1). However, the user may set $G = F$. Corresponding to the domain decomposition, there is a decomposition of the solution vectors $y$ and $\dot{y}$ into $M$ disjoint blocks $y_m$ and $\dot{y}_m$, and a decomposition of $G$ into blocks $G_m$. The block $G_m$ depends on $y_m$ and $\dot{y}_m$, and also on components of $y_{m'}$ and $\dot{y}_{m'}$ associated with neighboring sub-domains (so-called ghost-cell data). Let $\bar{y}_m$ and $\bar{\dot{y}}_m$ denote $y_m$ and $\dot{y}_m$ (respectively) augmented with those other components on which $G_m$ depends. Then we have

$$G(t, y, \dot{y}) = [G_1(t, \bar{y}_1, \bar{\dot{y}}_1), G_2(t, \bar{y}_2, \bar{\dot{y}}_2), \ldots, G_M(t, \bar{y}_M, \bar{\dot{y}}_M)]^T \ , \tag{4.1}$$

and each of the blocks $G_m(t, \bar{y}_m, \bar{\dot{y}}_m)$ is uncoupled from the others.

The preconditioner associated with this decomposition has the form

$$P = diag[P_1, P_2, \ldots, P_M] \tag{4.2}$$

where

$$P_m \approx \partial G_m / \partial y_m + \alpha \partial G_m / \partial \dot{y}_m \tag{4.3}$$

This matrix is taken to be banded, with upper and lower half-bandwidths `mudq` and `mldq` defined as the number of non-zero diagonals above and below the main diagonal, respectively. The difference quotient approximation is computed using `mudq` + `mldq` +2 evaluations of $G_m$, but only a matrix of bandwidth `mukeep` + `mlkeep` +1 is retained.

Neither pair of parameters need be the true half-bandwidths of the Jacobians of the local block of $G$, if smaller values provide a more efficient preconditioner. Such an efficiency gain may occur if the couplings in the DAE system outside a certain bandwidth are considerably weaker than those within the band. Reducing `mukeep` and `mlkeep` while keeping `mudq` and `mldq` at their true values, discards the elements outside the narrower band. Reducing both pairs has the additional effect of lumping the outer Jacobian elements into the computed elements within the band, and requires more caution and experimentation.

The solution of the complete linear system

$$Px = b \tag{4.4}$$

reduces to solving each of the equations

$$P_m x_m = b_m \tag{4.5}$$

and this is done by banded LU factorization of $P_m$ followed by a banded backsolve.

Similar block-diagonal preconditioners could be considered with different treatment of the blocks $P_m$. For example, incomplete LU factorization or an iterative method could be used instead of banded LU factorization.

The IDABBDPRE module calls two user-provided functions to construct $P$: a required function `Gres` (of type `IDABBDLocalFn`) which approximates the residual function $G(t, y, \dot{y}) \approx F(t, y, \dot{y})$ and which is computed locally, and an optional function `Gcomm` (of type `IDABBDCommFn`) which performs all inter-process communication necessary to evaluate the approximate residual $G$. These are in addition to the user-supplied residual function `res`. Both functions take as input the same pointer `user_data` as passed by the user to `IDASetUserData` and passed to the user's function `res`. The user is responsible for providing space (presumably within `user_data`) for components of `yy` and `yp` that are communicated by `Gcomm` from the other processors, and that are then used by `Gres`, which should not do any communication.

---

| IDABBDLocalFn |

Definition      `typedef int (*IDABBDLocalFn)(sunindextype Nlocal, realtype tt,`
                                          `N_Vector yy, N_Vector yp, N_Vector gval,`
                                          `void *user_data);`

Purpose         This `Gres` function computes $G(t, y, \dot{y})$. It loads the vector `gval` as a function of `tt`, `yy`, and `yp`.

Arguments       `Nlocal`     is the local vector length.

                `tt`           is the value of the independent variable.

                `yy`           is the dependent variable.

                `yp`           is the derivative of the dependent variable.

                `gval`        is the output vector.

                `user_data` is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.

Return value    An `IDABBDLocalFn` function type should return 0 to indicate success, 1 for a recoverable error, or -1 for a non-recoverable error.

Notes           This function must assume that all inter-processor communication of data needed to calculate `gval` has already been done, and this data is accessible within `user_data`.

                The case where $G$ is mathematically identical to $F$ is allowed.

---

| IDABBDCommFn |

Definition      `typedef int (*IDABBDCommFn)(sunindextype Nlocal, realtype tt,`
                                          `N_Vector yy, N_Vector yp, void *user_data);`

Purpose         This `Gcomm` function performs all inter-processor communications necessary for the execution of the `Gres` function above, using the input vectors `yy` and `yp`.

Arguments       `Nlocal`     is the local vector length.

                `tt`           is the value of the independent variable.

                `yy`           is the dependent variable.

                `yp`           is the derivative of the dependent variable.

                `user_data` is a pointer to user data, the same as the `user_data` parameter passed to `IDASetUserData`.

Return value    An `IDABBDCommFn` function type should return 0 to indicate success, 1 for a recoverable error, or -1 for a non-recoverable error.

Notes           The `Gcomm` function is expected to save communicated data in space defined within the structure `user_data`.

                Each call to the `Gcomm` function is preceded by a call to the residual function `res` with the same (`tt`, `yy`, `yp`) arguments. Thus `Gcomm` can omit any communications done by `res` if relevant to the evaluation of `Gres`. If all necessary communication was done in `res`, then `Gcomm = NULL` can be passed in the call to `IDABBDPrecInit` (see below).

Besides the header files required for the integration of the DAE problem (see §4.3), to use the IDABBDPRE module, the main program must include the header file `idas_bbdpre.h` which declares the needed function prototypes.

The following is a summary of the usage of this module and describes the sequence of calls in the user main program. Steps that are unchanged from the user main program presented in §4.4 are grayed-out.

1. **Initialize MPI**

2. **Set problem dimensions**

3. **Set vector of initial values**

4. **Create** IDAS **object**

5. **Allocate internal memory**

6. **Set optional inputs**

7. **Attach iterative linear solver, one of:**

    (a) `flag = IDASpgmr(ida_mem, maxl);`

    (b) `flag = IDASpbcg(ida_mem, maxl);`

    (c) `flag = IDASptfqmr(ida_mem, maxl);`

8. **Initialize the** IDABBDPRE **preconditioner module**

    Specify the upper and lower bandwidths `mudq`, `mldq` and `mukeep`, `mlkeep` and call

    ```
    flag = IDABBDPrecInit(ida_mem, Nlocal, mudq, mldq,
                          mukeep, mlkeep, dq_rel_yy, Gres, Gcomm);
    ```

    to allocate memory and initialize the internal preconditioner data. The last two arguments of `IDABBDPrecInit` are the two user-supplied functions described above.

9. **Set linear solver interface optional inputs**

    Note that the user should not overwrite the preconditioner setup function or solve function through calls to idIDASpilsSetPreconditioner optional input function.

10. **Correct initial values**

11. **Specify rootfinding problem**

12. **Advance solution in time**

13. **Get optional outputs**

    Additional optional outputs associated with IDABBDPRE are available by way of two routines described below, `IDABBDPrecGetWorkSpace` and `IDABBDPrecGetNumGfnEvals`.

14. **Deallocate memory for solution vector**

15. **Free solver memory**

16. **Free linear solver memory**

17. **Finalize MPI**

The user-callable functions that initialize (step 8 above) or re-initialize the IDABBDPRE preconditioner module are described next.

IDABBDPrecInit

Call	`flag = IDABBDPrecInit(ida_mem, Nlocal, mudq, mldq,` `                      mukeep, mlkeep, dq_rel_yy, Gres, Gcomm);`
Description	The function `IDABBDPrecInit` initializes and allocates (internal) memory for the ID-ABBDPRE preconditioner.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`Nlocal` (`sunindextype`) local vector dimension.
	`mudq` (`sunindextype`) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.

mldq	(`sunindextype`) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.	
mukeep	(`sunindextype`) upper half-bandwidth of the retained banded approximate Jacobian block.	
mlkeep	(`sunindextype`) lower half-bandwidth of the retained banded approximate Jacobian block.	
dq_rel_yy	(`realtype`) the relative increment in components of y used in the difference quotient approximations. The default is `dq_rel_yy=` $\sqrt{\text{unit roundoff}}$, which can be specified by passing `dq_rel_yy=` 0.0.	
Gres	(`IDABBDLocalFn`) the C function which computes the local residual approximation $G(t, y, \dot{y})$.	
Gcomm	(`IDABBDCommFn`) the optional C function which performs all inter-process communication required for the computation of $G(t, y, \dot{y})$.	

Return value   The return value `flag` (of type `int`) is one of

IDASPILS_SUCCESS	The call to `IDABBDPrecInit` was successful.
IDASPILS_MEM_NULL	The `ida_mem` pointer was NULL.
IDASPILS_MEM_FAIL	A memory allocation request has failed.
IDASPILS_LMEM_NULL	An IDASPILS linear solver memory was not attached.
IDASPILS_ILL_INPUT	The supplied vector implementation was not compatible with the block band preconditioner.

Notes   If one of the half-bandwidths `mudq` or `mldq` to be used in the difference-quotient calculation of the approximate Jacobian is negative or exceeds the value `Nlocal−1`, it is replaced by 0 or `Nlocal−1` accordingly.

The half-bandwidths `mudq` and `mldq` need not be the true half-bandwidths of the Jacobian of the local block of G, when smaller values may provide a greater efficiency.

Also, the half-bandwidths `mukeep` and `mlkeep` of the retained banded approximate Jacobian block may be even smaller, to reduce storage and computation costs further.

For all four half-bandwidths, the values need not be the same on every processor.

The IDABBDPRE module also provides a reinitialization function to allow for a sequence of problems of the same size, with the same linear solver choice, provided there is no change in `local_N`, `mukeep`, or `mlkeep`. After solving one problem, and after calling `IDAReInit` to re-initialize IDAS for a subsequent problem, a call to `IDABBDPrecReInit` can be made to change any of the following: the half-bandwidths `mudq` and `mldq` used in the difference-quotient Jacobian approximations, the relative increment `dq_rel_yy`, or one of the user-supplied functions `Gres` and `Gcomm`. If there is a change in any of the linear solver inputs, an additional call to the "Set" routines provided by the SUNLINSOL module, and/or one or more of the corresponding `IDASpilsSet***` functions, must also be made (in the proper order).

IDABBDPrecReInit

Call   `flag = IDABBDPrecReInit(ida_mem, mudq, mldq, dq_rel_yy);`

Description   The function `IDABBDPrecReInit` reinitializes the IDABBDPRE preconditioner.

Arguments

ida_mem	(`void *`) pointer to the IDAS memory block.
mudq	(`sunindextype`) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.
mldq	(`sunindextype`) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.
dq_rel_yy	(`realtype`) the relative increment in components of y used in the difference quotient approximations. The default is `dq_rel_yy` $= \sqrt{\text{unit roundoff}}$, which can be specified by passing `dq_rel_yy` $= 0.0$.

Return value   The return value `flag` (of type `int`) is one of

    `IDASPILS_SUCCESS`   The call to `IDABBDPrecReInit` was successful.

    `IDASPILS_MEM_NULL`   The `ida_mem` pointer was `NULL`.

    `IDASPILS_LMEM_NULL` An IDASPILS linear solver memory was not attached.

    `IDASPILS_PMEM_NULL` The function `IDABBDPrecInit` was not previously called.

Notes   If one of the half-bandwidths `mudq` or `mldq` is negative or exceeds the value `Nlocal`−1, it is replaced by 0 or `Nlocal`−1, accordingly.

The following two optional output functions are available for use with the IDABBDPRE module:

| `IDABBDPrecGetWorkSpace` |

Call   `flag = IDABBDPrecGetWorkSpace(ida_mem, &lenrwBBDP, &leniwBBDP);`

Description   The function `IDABBDPrecGetWorkSpace` returns the local sizes of the IDABBDPRE real and integer workspaces.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

    `lenrwBBDP` (`long int`) local number of real values in the IDABBDPRE workspace.

    `leniwBBDP` (`long int`) local number of integer values in the IDABBDPRE workspace.

Return value   The return value `flag` (of type `int`) is one of

    `IDASPILS_SUCCESS`   The optional output value has been successfully set.

    `IDASPILS_MEM_NULL`   The `ida_mem` pointer was `NULL`.

    `IDASPILS_PMEM_NULL` The IDABBDPRE preconditioner has not been initialized.

Notes   The workspace requirements reported by this routine correspond only to memory allocated within the IDABBDPRE module (the banded matrix approximation, banded SUN-LINSOL object, temporary vectors). These values are local to each process.

The workspaces referred to here exist in addition to those given by the corresponding function `IDASpilsGetWorkSpace`.

| `IDABBDPrecGetNumGfnEvals` |

Call   `flag = IDABBDPrecGetNumGfnEvals(ida_mem, &ngevalsBBDP);`

Description   The function `IDABBDPrecGetNumGfnEvals` returns the cumulative number of calls to the user `Gres` function due to the finite difference approximation of the Jacobian blocks used within IDABBDPRE's preconditioner setup function.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

    `ngevalsBBDP` (`long int`) the cumulative number of calls to the user `Gres` function.

Return value   The return value `flag` (of type `int`) is one of

    `IDASPILS_SUCCESS`   The optional output value has been successfully set.

    `IDASPILS_MEM_NULL`   The `ida_mem` pointer was `NULL`.

    `IDASPILS_PMEM_NULL` The IDABBDPRE preconditioner has not been initialized.

In addition to the `ngevalsBBDP` `Gres` evaluations, the costs associated with IDABBDPRE also include `nlinsetups` LU factorizations, `nlinsetups` calls to `Gcomm`, `npsolves` banded backsolve calls, and `nrevalsLS` residual function evaluations, where `nlinsetups` is an optional IDAS output (see §4.5.9.2), and `npsolves` and `nrevalsLS` are linear solver optional outputs (see §4.5.9.6).

Chapter 5

Using IDAS for Forward Sensitivity Analysis

This chapter describes the use of IDAS to compute solution sensitivities using forward sensitivity analysis. One of our main guiding principles was to design the IDAS user interface for forward sensitivity analysis as an extension of that for IVP integration. Assuming a user main program and user-defined support routines for IVP integration have already been defined, in order to perform forward sensitivity analysis the user only has to insert a few more calls into the main program and (optionally) define an additional routine which computes the residuals for sensitivity systems (2.12). The only departure from this philosophy is due to the `IDAResFn` type definition (§4.6.1). Without changing the definition of this type, the only way to pass values of the problem parameters to the DAE residual function is to require the user data structure `user_data` to contain a pointer to the array of real parameters p.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

We begin with a brief overview, in the form of a skeleton user program. Following that are detailed descriptions of the interface to the various user-callable routines and of the user-supplied routines that were not already described in Chapter 4.

5.1   A skeleton of the user's main program

The following is a skeleton of the user's main program (or calling program) as an application of IDAS. The user program is to have these steps in the order indicated, unless otherwise noted. For the sake of brevity, we defer many of the details to the later sections. As in §4.4, most steps are independent of the NVECTOR implementation used. For the steps that are not, refer to Chapter 7 for the specific names. Differences between the user main program in §4.4 and the one below start only at step (13). Steps that are unchanged from the skeleton program presented in §4.4 are grayed out.

First, note that no additional header files need be included for forward sensitivity analysis beyond those for IVP solution (§4.4).

1. **Initialize parallel or multi-threaded environment, if appropriate**

2. **Set problem dimensions etc.**

3. **Set vectors of initial values**

4. **Create IDAS object**

5. **Initialize IDAS**

6. **Specify integration tolerances**

7. **Set optional inputs**

8. **Create matrix object**

9. **Create linear solver object**

10. **Set linear solver optional inputs**

11. **Attach linear solver module**

12. **Initialize quadrature problem, if not sensitivity-dependent**

13. **Define the sensitivity problem**

   - •Number of sensitivities (required)

     Set `Ns` $= N_s$, the number of parameters with respect to which sensitivities are to be computed.

   - •Problem parameters (optional)

     If IDAS is to evaluate the residuals of the sensitivity systems, set `p`, an array of `Np` real parameters upon which the IVP depends. Only parameters with respect to which sensitivities are (potentially) desired need to be included. Attach `p` to the user data structure `user_data`. For example, `user_data->p = p;`

     If the user provides a function to evaluate the sensitivity residuals, `p` need not be specified.

   - •Parameter list (optional)

     If IDAS is to evaluate the sensitivity residuals, set `plist`, an array of `Ns` integers to specify the parameters `p` with respect to which solution sensitivities are to be computed. If sensitivities with respect to the j-th parameter `p[j]` ($0 \leq$ `j` $<$ `Np`) are desired, set $\text{plist}_i = j$, for some $i = 0, \ldots, N_s - 1$.

     If `plist` is not specified, IDAS will compute sensitivities with respect to the first `Ns` parameters; i.e., $\text{plist}_i = i$ $(i = 0, \ldots, N_s - 1)$.

     If the user provides a function to evaluate the sensitivity residuals, `plist` need not be specified.

   - •Parameter scaling factors (optional)

     If IDAS is to estimate tolerances for the sensitivity solution vectors (based on tolerances for the state solution vector) or if IDAS is to evaluate the residuals of the sensitivity systems using the internal difference-quotient function, the results will be more accurate if order of magnitude information is provided.

     Set `pbar`, an array of `Ns` positive scaling factors. Typically, if $p_i \neq 0$, the value $\bar{p}_i = |p_{\text{plist}_i}|$ can be used.

     If `pbar` is not specified, IDAS will use $\bar{p}_i = 1.0$.

     If the user provides a function to evaluate the sensitivity residual and specifies tolerances for the sensitivity variables, `pbar` need not be specified.

   Note that the names for `p`, `pbar`, `plist`, as well as the field p of `user_data` are arbitrary, but they must agree with the arguments passed to `IDASetSensParams` below.

14. **Set sensitivity initial conditions**

   Set the `Ns` vectors `yS0[i]` and `ypS0[i]` of initial values for sensitivities (for $i = 0, \ldots,$ `Ns` -1), using the appropriate functions defined by the particular NVECTOR implementation chosen.

   First, create an array of `Ns` vectors by making the appropriate call

   `yS0 = N_VCloneVectorArray_***(Ns, y0);`

   or

   `yS0 = N_VCloneVectorArrayEmpty_***(Ns, y0);`

Here the argument y0 serves only to provide the N_Vector type for cloning.

Then, for each $i = 0, \ldots,$Ns -1, load initial values for the i-th sensitivity vector yS0[i].

Set the initial conditions for the Ns sensitivity derivative vectors ypS0 of $\dot{y}$ similarly.

15. **Activate sensitivity calculations**

    Call flag = IDASensInit(...); to activate forward sensitivity computations and allocate internal memory for IDAS related to sensitivity calculations (see §5.2.1).

16. **Set sensitivity tolerances**

    Call IDASensSStolerances, IDASensSVtolerances, or IDASensEEtolerances. See §5.2.2.

17. **Set sensitivity analysis optional inputs**

    Call IDASetSens* routines to change from their default values any optional inputs that control the behavior of IDAS in computing forward sensitivities. See §5.2.6.

18. **Correct initial values**

19. **Specify rootfinding problem**

20. **Advance solution in time**

21. **Extract sensitivity solution**

    After each successful return from IDASolve, the solution of the original IVP is available in the y argument of IDASolve, while the sensitivity solution can be extracted into yS and ypS (which can be the same as yS0 and ypS0, respectively) by calling one of the following routines: IDAGetSens, IDAGetSens1, IDAGetSensDky or IDAGetSensDky1 (see §5.2.5).

22. **Get optional outputs**

23. **Deallocate memory for solution vector**

24. **Deallocate memory for sensitivity vectors**

    Upon completion of the integration, deallocate memory for the vectors contained in yS0 and ypS0:

    N_VDestroyVectorArray_***(yS0, Ns);

    If yS was created from realtype arrays yS_i, it is the user's responsibility to also free the space for the arrays yS_i, and likewise for ypS.

25. **Free user data structure**

26. **Free solver memory**

27. **Free vector specification memory**

28. **Free linear solver and matrix memory**

29. **Finalize MPI, if used**

5.2   User-callable routines for forward sensitivity analysis

This section describes the IDAS functions, in addition to those presented in §4.5, that are called by the user to set up and solve a forward sensitivity problem.

5.2.1   Forward sensitivity initialization and deallocation functions

Activation of forward sensitivity computation is done by calling IDASensInit. The form of the call to this routine is as follows:

IDASensInit

Call	`flag = IDASensInit(ida_mem, Ns, ism, resS, yS0, ypS0);`
Description	The routine `IDASensInit` activates forward sensitivity computations and allocates internal memory related to sensitivity calculations.
Arguments	`ida_mem` (void *) pointer to the IDAS memory block returned by `IDACreate`.
	`Ns` (int) the number of sensitivities to be computed.
	`ism` (int) a flag used to select the sensitivity solution method. Its value can be either `IDA_SIMULTANEOUS` or `IDA_STAGGERED`:
		• In the `IDA_SIMULTANEOUS` approach, the state and sensitivity variables are corrected at the same time. If `IDA_NEWTON` was selected as the nonlinear system solution method, this amounts to performing a modified Newton iteration on the combined nonlinear system;
		• In the `IDA_STAGGERED` approach, the correction step for the sensitivity variables takes place at the same time for all sensitivity equations, but only after the correction of the state variables has converged and the state variables have passed the local error test;
	`resS` (IDASensResFn) is the C function which computes the residual of the sensitivity DAE. For full details see §5.3.
	`yS0` (N_Vector *) a pointer to an array of `Ns` vectors containing the initial values of the sensitivities of y.
	`ypS0` (N_Vector *) a pointer to an array of `Ns` vectors containing the initial values of the sensitivities of $\dot{y}$.
Return value	The return value `flag` (of type int) will be one of the following:
	`IDA_SUCCESS` The call to `IDASensInit` was successful.
	`IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
	`IDA_MEM_FAIL` A memory allocation request has failed.
	`IDA_ILL_INPUT` An input argument to `IDASensInit` has an illegal value.
Notes	Passing `resS=NULL` indicates using the default internal difference quotient sensitivity residual routine.
	If an error occurred, `IDASensInit` also prints an error message to the file specified by the optional input `errfp`.

In terms of the problem size N, number of sensitivity vectors N_s, and maximum method order maxord, the size of the real workspace is increased as follows:

- Base value: `lenrw = lenrw + (maxord+5)`$N_s N$

- With IDASensSVtolerances: `lenrw = lenrw +`$N_s N$

the size of the integer workspace is increased as follows:

- Base value: `leniw = leniw + (maxord+5)`$N_s N_i$

- With IDASensSVtolerances: `leniw = leniw +`$N_s N_i$,

where N_i is the number of integer words in one N_Vector.

The routine IDASensReInit, useful during the solution of a sequence of problems of same size, reinitializes the sensitivity-related internal memory and must follow a call to IDASensInit (and maybe a call to IDAReInit). The number Ns of sensitivities is assumed to be unchanged since the call to IDASensInit. The call to the IDASensReInit function has the form:

IDASensReInit

Call	`flag = IDASensReInit(ida_mem, ism, yS0, ypS0);`
Description	The routine `IDASensReInit` reinitializes forward sensitivity computations.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
	`ism` (`int`) a flag used to select the sensitivity solution method. Its value can be either `IDA_SIMULTANEOUS` or `IDA_STAGGERED`.
	`yS0` (`N_Vector *`) a pointer to an array of `Ns` variables of type `N_Vector` containing the initial values of the sensitivities of y.
	`ypS0` (`N_Vector *`) a pointer to an array of `Ns` variables of type `N_Vector` containing the initial values of the sensitivities of $\dot{y}$.
Return value	The return value `flag` (of type `int`) will be one of the following:
	`IDA_SUCCESS` The call to `IDAReInit` was successful.
	`IDA_MEM_NULL` The IDAS memory block was not initialized through a previous call to `IDACreate`.
	`IDA_NO_SENS` Memory space for sensitivity integration was not allocated through a previous call to `IDASensInit`.
	`IDA_ILL_INPUT` An input argument to `IDASensReInit` has an illegal value.
	`IDA_MEM_FAIL` A memory allocation request has failed.
Notes	All arguments of `IDASensReInit` are the same as those of `IDASensInit`.
	If an error occurred, `IDASensReInit` also prints an error message to the file specified by the optional input `errfp`.

To deallocate all forward sensitivity-related memory (allocated in a prior call to `IDASensInit`), the user must call

---

IDASensFree

Call	`IDASensFree(ida_mem);`
Description	The function `IDASensFree` frees the memory allocated for forward sensitivity computations by a previous call to `IDASensInit`.
Arguments	The argument is the pointer to the IDAS memory block (of type `void *`).
Return value	The function `IDASensFree` has no return value.
Notes	In general, `IDASensFree` need not be called by the user as it is invoked automatically by `IDAFree`.
	After a call to `IDASensFree`, forward sensitivity computations can be reactivated only by calling `IDASensInit` again.

To activate and deactivate forward sensitivity calculations for successive IDAS runs, without having to allocate and deallocate memory, the following function is provided:

IDASensToggleOff

Call	`IDASensToggleOff(ida_mem);`
Description	The function `IDASensToggleOff` deactivates forward sensitivity calculations. It does *not* deallocate sensitivity-related memory.
Arguments	`ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.
Return value	The return value `flag` of `IDASensToggle` is one of:
	`IDA_SUCCESS` `IDASensToggleOff` was successful.
	`IDA_MEM_NULL` `ida_mem` was NULL.
Notes	Since sensitivity-related memory is not deallocated, sensitivities can be reactivated at a later time (using `IDASensReInit`).

5.2.2   Forward sensitivity tolerance specification functions

One of the following three functions must be called to specify the integration tolerances for sensitivities.
Note that this call must be made after the call to `IDASensInit`.

IDASensSStolerances

Call	`flag = IDASensSStolerances(ida_mem, reltolS, abstolS);`
Description	The function `IDASensSStolerances` specifies scalar relative and absolute tolerances.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
	`reltolS` (`realtype`) is the scalar relative error tolerance.
	`abstolS` (`realtype*`) is a pointer to an array of length `Ns` containing the scalar absolute error tolerances.
Return value	The return flag `flag` (of type `int`) will be one of the following:

`IDA_SUCCESS`	The call to `IDASStolerances` was successful.
`IDA_MEM_NULL`	The IDAS memory block was not initialized through a previous call to `IDACreate`.
`IDA_NO_SENS`	The sensitivity allocation function `IDASensInit` has not been called.
`IDA_ILL_INPUT`	One of the input tolerances was negative.

IDASensSVtolerances

Call	`flag = IDASensSVtolerances(ida_mem, reltolS, abstolS);`
Description	The function `IDASensSVtolerances` specifies scalar relative tolerance and vector absolute tolerances.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
	`reltolS` (`realtype`) is the scalar relative error tolerance.
	`abstolS` (`N_Vector*`) is an array of `Ns` variables of type `N_Vector`. The `N_Vector` from `abstolS[is]` specifies the vector tolerances for `is`-th sensitivity.
Return value	The return flag `flag` (of type `int`) will be one of the following:

`IDA_SUCCESS`	The call to `IDASVtolerances` was successful.
`IDA_MEM_NULL`	The IDAS memory block was not initialized through a previous call to `IDACreate`.
`IDA_NO_SENS`	The sensitivity allocation function `IDASensInit` has not been called.
`IDA_ILL_INPUT`	The relative error tolerance was negative or one of the absolute tolerance vectors had a negative component.

Notes	This choice of tolerances is important when the absolute error tolerance needs to be different for each component of any vector `yS[i]`.

IDASensEEtolerances

Call	`flag = IDASensEEtolerances(ida_mem);`
Description	When `IDASensEEtolerances` is called, IDAS will estimate tolerances for sensitivity variables based on the tolerances supplied for states variables and the scaling factors $\bar{p}$.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
Return value	The return flag `flag` (of type `int`) will be one of the following:

`IDA_SUCCESS`	The call to `IDASensEEtolerances` was successful.
`IDA_MEM_NULL`	The IDAS memory block was not initialized through a previous call to `IDACreate`.
`IDA_NO_SENS`	The sensitivity allocation function `IDASensInit` has not been called.

5.2.3 Forward sensitivity initial condition calculation function

IDACalcIC also calculates corrected initial conditions for sensitivity variables of a DAE system. When used for initial conditions calculation of the forward sensitivities, IDACalcIC must be preceded by successful calls to IDASensInit (or IDASensReInit) and should precede the call(s) to IDASolve. For restrictions that apply for initial conditions calculation of the state variables, see §4.5.4.

Calling IDACalcIC is optional. It is only necessary when the initial conditions do not satisfy the sensitivity systems. Even if forward sensitivity analysis was enabled, the call to the initial conditions calculation function IDACalcIC is exactly the same as for state variables.

```
flag = IDACalcIC(ida_mem, icopt, tout1);
```

See §4.5.4 for a list of possible return values.

5.2.4 IDAS solver function

Even if forward sensitivity analysis was enabled, the call to the main solver function IDASolve is exactly the same as in §4.5.6. However, in this case the return value flag can also be one of the following:

IDA_SRES_FAIL   The sensitivity residual function failed in an unrecoverable manner.

IDA_REP_SRES_ERR The user's residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.

5.2.5 Forward sensitivity extraction functions

If forward sensitivity computations have been initialized by a call to IDASensInit, or reinitialized by a call to IDASensReInit, then IDAS computes both a solution and sensitivities at time t. However, IDASolve will still return only the solutions y and $\dot{y}$ in yret and ypret, respectively. Solution sensitivities can be obtained through one of the following functions:

IDAGetSens

Call	flag = IDAGetSens(ida_mem, &tret, yS);
Description	The function IDAGetSens returns the sensitivity solution vectors after a successful return from IDASolve.
Arguments	ida_mem (void *) pointer to the memory previously allocated by IDAInit.
	tret (realtype) the time reached by the solver (output).
	yS (N_Vector *) the array of Ns computed forward sensitivity vectors.
Return value	The return value flag of IDAGetSens is one of:
	IDA_SUCCESS IDAGetSens was successful.
	IDA_MEM_NULL ida_mem was NULL.
	IDA_NO_SENS Forward sensitivity analysis was not initialized.
	IDA_BAD_DKY yS is NULL.
Notes	Note that the argument tret is an output for this function. Its value will be the same as that returned at the last IDASolve call.

The function IDAGetSensDky computes the k-th derivatives of the interpolating polynomials for the sensitivity variables at time t. This function is called by IDAGetSens with k = 0, but may also be called directly by the user.

IDAGetSensDky

Call	`flag = IDAGetSensDky(ida_mem, t, k, dkyS);`
Description	The function `IDAGetSensDky` returns derivatives of the sensitivity solution vectors after a successful return from `IDASolve`.
Arguments	`ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

      `t`   (`realtype`) specifies the time at which sensitivity information is requested. The time `t` must fall within the interval defined by the last successful step taken by IDAS.

      `k`   (`int`) order of derivatives.

      `dkyS`   (`N_Vector *`) array of `Ns` vectors containing the derivatives on output. The space for `dkyS` must be allocated by the user.

Return value   The return value `flag` of `IDAGetSensDky` is one of:

      `IDA_SUCCESS`   `IDAGetSensDky` succeeded.

      `IDA_MEM_NULL` `ida_mem` was `NULL`.

      `IDA_NO_SENS`   Forward sensitivity analysis was not initialized.

      `IDA_BAD_DKY`   `dkyS` or one of the vectors `dkyS[i]` is `NULL`.

      `IDA_BAD_K`   `k` is not in the range $0, 1, ..., klast$.

      `IDA_BAD_T`   The time `t` is not in the allowed range.

Forward sensitivity solution vectors can also be extracted separately for each parameter in turn through the functions `IDAGetSens1` and `IDAGetSensDky1`, defined as follows:

IDAGetSens1

Call	`flag = IDAGetSens1(ida_mem, &tret, is, yS);`
Description	The function `IDAGetSens1` returns the `is`-th sensitivity solution vector after a successful return from `IDASolve`.
Arguments	`ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

      `tret`   (`realtype *`) the time reached by the solver (output).

      `is`   (`int`) specifies which sensitivity vector is to be returned ($0 \leq$ `is` $< N_s$).

      `yS`   (`N_Vector`) the computed forward sensitivity vector.

Return value   The return value `flag` of `IDAGetSens1` is one of:

      `IDA_SUCCESS`   `IDAGetSens1` was successful.

      `IDA_MEM_NULL` `ida_mem` was `NULL`.

      `IDA_NO_SENS`   Forward sensitivity analysis was not initialized.

      `IDA_BAD_IS`   The index `is` is not in the allowed range.

      `IDA_BAD_DKY`   `yS` is `NULL`.

      `IDA_BAD_T`   The time `t` is not in the allowed range.

Notes   Note that the argument `tret` is an output for this function. Its value will be the same as that returned at the last `IDASolve` call.

IDAGetSensDky1

Call	`flag = IDAGetSensDky1(ida_mem, t, k, is, dkyS);`
Description	The function `IDAGetSensDky1` returns the `k`-th derivative of the `is`-th sensitivity solution vector after a successful return from `IDASolve`.
Arguments	`ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

t	(`realtype`) specifies the time at which sensitivity information is requested. The time `t` must fall within the interval defined by the last successful step taken by IDAS.
k	(`int`) order of derivative.
is	(`int`) specifies the sensitivity derivative vector to be returned ($0 \leq$ `is` $< N_s$).
dkyS	(`N_Vector`) the vector containing the derivative on output. The space for `dkyS` must be allocated by the user.

Return value The return value `flag` of `IDAGetSensDky1` is one of:

IDA_SUCCESS	`IDAGetQuadDky1` succeeded.
IDA_MEM_NULL	`ida_mem` was NULL.
IDA_NO_SENS	Forward sensitivity analysis was not initialized.
IDA_BAD_DKY	`dkyS` is NULL.
IDA_BAD_IS	The index `is` is not in the allowed range.
IDA_BAD_K	`k` is not in the range $0, 1, ..., klast$.
IDA_BAD_T	The time `t` is not in the allowed range.

5.2.6 Optional inputs for forward sensitivity analysis

Optional input variables that control the computation of sensitivities can be changed from their default values through calls to `IDASetSens*` functions. Table 5.1 lists all forward sensitivity optional input functions in IDAS which are described in detail in the remainder of this section.

IDASetSensParams

Call	`flag = IDASetSensParams(ida_mem, p, pbar, plist);`
Description	The function `IDASetSensParams` specifies problem parameter information for sensitivity calculations.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	p (`realtype *`) a pointer to the array of real problem parameters used to evaluate $F(t, y, \dot{y}, p)$. If non-NULL, `p` must point to a field in the user's data structure `user_data` passed to the user's residual function. (See §5.1).
	pbar (`realtype *`) an array of `Ns` positive scaling factors. If non-NULL, `pbar` must have all its components > 0.0. (See §5.1).
	plist (`int *`) an array of `Ns` non-negative indices to specify which components of `p` to use in estimating the sensitivity equations. If non-NULL, `plist` must have all components ≥ 0. (See §5.1).
Return value	The return value `flag` (of type `int`) is one of:

IDA_SUCCESS	The optional value has been successfully set.
IDA_MEM_NULL	The `ida_mem` pointer is NULL.
IDA_NO_SENS	Forward sensitivity analysis was not initialized.
IDA_ILL_INPUT	An argument has an illegal value.

Notes	This function must be preceded by a call to `IDASensInit`.

⚠️

Table 5.1: Forward sensitivity optional inputs

Optional input	Routine name	Default
Sensitivity scaling factors	IDASetSensParams	NULL
DQ approximation method	IDASetSensDQMethod	centered,0.0
Error control strategy	IDASetSensErrCon	SUNFALSE
Maximum no. of nonlinear iterations	IDASetSensMaxNonlinIters	3

IDASetSensDQMethod

Call	`flag = IDASetSensDQMethod(ida_mem, DQtype, DQrhomax);`
Description	The function `IDASetSensDQMethod` specifies the difference quotient strategy in the case in which the residual of the sensitivity equations are to be computed by IDAS.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`DQtype` (`int`) specifies the difference quotient type and can be either `IDA_CENTERED` or `IDA_FORWARD`.
	`DQrhomax` (`realtype`) positive value of the selection parameter used in deciding switching between a simultaneous or separate approximation of the two terms in the sensitivity residual.
Return value	The return value `flag` (of type `int`) is one of:

   IDA_SUCCESS The optional value has been successfully set.

   IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

   IDA_ILL_INPUT An argument has an illegal value.

Notes	If `DQrhomax = 0.0`, then no switching is performed. The approximation is done simultaneously using either centered or forward finite differences, depending on the value of `DQtype`. For values of `DQrhomax ≥ 1.0`, the simultaneous approximation is used whenever the estimated finite difference perturbations for states and parameters are within a factor of `DQrhomax`, and the separate approximation is used otherwise. Note that a value `DQrhomax < 1.0` will effectively disable switching. See §2.5 for more details.

   The default value are `DQtype=IDA_CENTERED` and `DQrhomax= 0.0`.

IDASetSensErrCon

Call	`flag = IDASetSensErrCon(ida_mem, errconS);`
Description	The function `IDASetSensErrCon` specifies the error control strategy for sensitivity variables.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`errconS` (`booleantype`) specifies whether sensitivity variables are included (`SUNTRUE`) or not (`SUNFALSE`) in the error control mechanism.
Return value	The return value `flag` (of type `int`) is one of:

   IDA_SUCCESS The optional value has been successfully set.

   IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

Notes	By default, `errconS` is set to `SUNFALSE`. If `errconS=SUNTRUE` then both state variables and sensitivity variables are included in the error tests. If `errconS=SUNFALSE` then the sensitivity variables are excluded from the error tests. Note that, in any event, all variables are considered in the convergence tests.

IDASetSensMaxNonlinIters

Call	`flag = IDASetSensMaxNonlinIters(ida_mem, maxcorS);`
Description	The function `IDASetSensMaxNonlinIters` specifies the maximum number of nonlinear solver iterations for sensitivity variables per step.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`maxcorS` (`int`) maximum number of nonlinear solver iterations allowed per step (> 0).
Return value	The return value `flag` (of type `int`) is one of:

   IDA_SUCCESS The optional value has been successfully set.

   IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

Notes	The default value is 3.

5.2.7   Optional outputs for forward sensitivity analysis

5.2.7.1   Main solver optional output functions

Optional output functions that return statistics and solver performance information related to forward sensitivity computations are listed in Table 5.2 and described in detail in the remainder of this section.

$\boxed{\texttt{IDAGetSensNumResEvals}}$

Call	`flag = IDAGetSensNumResEvals(ida_mem, &nfSevals);`
Description	The function **IDAGetSensNumResEvals** returns the number of calls to the sensitivity residual function.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nfSevals` (`long int`) number of calls to the sensitivity residual function.
Return value	The return value `flag` (of type `int`) is one of:

> **IDA_SUCCESS**   The optional output value has been successfully set.
> **IDA_MEM_NULL** The `ida_mem` pointer is `NULL`.
> **IDA_NO_SENS**   Forward sensitivity analysis was not initialized.

$\boxed{\texttt{IDAGetNumResEvalsSens}}$

Call	`flag = IDAGetNumResEvalsSens(ida_mem, &nfevalsS);`
Description	The function **IDAGetNumResEvalsSEns** returns the number of calls to the user's residual function due to the internal finite difference approximation of the sensitivity residuals.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nfevalsS` (`long int`) number of calls to the user residual function for sensitivity residuals.
Return value	The return value `flag` (of type `int`) is one of:

> **IDA_SUCCESS**   The optional output value has been successfully set.
> **IDA_MEM_NULL** The `ida_mem` pointer is `NULL`.
> **IDA_NO_SENS**   Forward sensitivity analysis was not initialized.

Notes	This counter is incremented only if the internal finite difference approximation routines are used for the evaluation of the sensitivity residuals.

Table 5.2: Forward sensitivity optional outputs

Optional output	Routine name
No. of calls to sensitivity residual function	`IDAGetSensNumResEvals`
No. of calls to residual function for sensitivity	`IDAGetNumResEvalsSens`
No. of sensitivity local error test failures	`IDAGetSensNumErrTestFails`
No. of calls to lin. solv. setup routine for sens.	`IDAGetSensNumLinSolvSetups`
Sensitivity-related statistics as a group	`IDAGetSensStats`
Error weight vector for sensitivity variables	`IDAGetSensErrWeights`
No. of sens. nonlinear solver iterations	`IDAGetSensNumNonlinSolvIters`
No. of sens. convergence failures	`IDAGetSensNumNonlinSolvConvFails`
Sens. nonlinear solver statistics as a group	`IDAGetSensNonlinSolvStats`

IDAGetSensNumErrTestFails

Call   flag = IDAGetSensNumErrTestFails(ida_mem, &nSetfails);

Description   The function IDAGetSensNumErrTestFails returns the number of local error test fail-
                ures for the sensitivity variables that have occurred.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
                nSetfails (long int) number of error test failures.

Return value   The return value flag (of type int) is one of:

                IDA_SUCCESS   The optional output value has been successfully set.
                IDA_MEM_NULL The ida_mem pointer is NULL.
                IDA_NO_SENS   Forward sensitivity analysis was not initialized.

Notes   This counter is incremented only if the sensitivity variables have been included in the
                error test (see IDASetSensErrCon in §5.2.6). Even in that case, this counter is not
                incremented if the ism=IDA_SIMULTANEOUS sensitivity solution method has been used.

IDAGetSensNumLinSolvSetups

Call   flag = IDAGetSensNumLinSolvSetups(ida_mem, &nlinsetupsS);

Description   The function IDAGetSensNumLinSolvSetups returns the number of calls to the linear
                solver setup function due to forward sensitivity calculations.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
                nlinsetupsS (long int) number of calls to the linear solver setup function.

Return value   The return value flag (of type int) is one of:

                IDA_SUCCESS   The optional output value has been successfully set.
                IDA_MEM_NULL The ida_mem pointer is NULL.
                IDA_NO_SENS   Forward sensitivity analysis was not initialized.

Notes   This counter is incremented only if Newton iteration has been used and staggered sensi-
                tivity solution method (ism=IDA_STAGGERED) was specified in the call to IDASensInit
                (see §5.2.1).

IDAGetSensStats

Call   flag = IDAGetSensStats(ida_mem, &nfSevals, &nfevalsS, &nSetfails,
                                       &nlinsetupsS);

Description   The function IDAGetSensStats returns all of the above sensitivity-related solver statis-
                tics as a group.

Arguments   ida_mem (void *) pointer to the IDAS memory block.
                nfSevals (long int) number of calls to the sensitivity residual function.
                nfevalsS (long int) number of calls to the user-supplied residual function.
                nSetfails (long int) number of error test failures.
                nlinsetupsS (long int) number of calls to the linear solver setup function.

Return value   The return value flag (of type int) is one of:

                IDA_SUCCESS   The optional output values have been successfully set.
                IDA_MEM_NULL The ida_mem pointer is NULL.
                IDA_NO_SENS   Forward sensitivity analysis was not initialized.

IDAGetSensErrWeights

Call	`flag = IDAGetSensErrWeights(ida_mem, eSweight);`
Description	The function `IDAGetSensErrWeights` returns the sensitivity error weight vectors at the current time. These are the reciprocals of the W_i of (2.7) for the sensitivity variables.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`eSweight` (`N_Vector_S`) pointer to the array of error weight vectors.
Return value	The return value `flag` (of type `int`) is one of:
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is NULL.
	`IDA_NO_SENS` Forward sensitivity analysis was not initialized.
Notes	The user must allocate memory for `eweightS`.

IDAGetSensNumNonlinSolvIters

Call	`flag = IDAGetSensNumNonlinSolvIters(ida_mem, &nSniters);`
Description	The function `IDAGetSensNumNonlinSolvIters` returns the number of nonlinear iterations performed for sensitivity calculations.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nSniters` (`long int`) number of nonlinear iterations performed.
Return value	The return value `flag` (of type `int`) is one of:
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is NULL.
	`IDA_NO_SENS` Forward sensitivity analysis was not initialized.
Notes	This counter is incremented only if `ism` was `IDA_STAGGERED` in the call to `IDASensInit` (see §5.2.1).

IDAGetSensNumNonlinSolvConvFails

Call	`flag = IDAGetSensNumNonlinSolvConvFails(ida_mem, &nSncfails);`
Description	The function `IDAGetSensNumNonlinSolvConvFails` returns the number of nonlinear convergence failures that have occurred for sensitivity calculations.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nSncfails` (`long int`) number of nonlinear convergence failures.
Return value	The return value `flag` (of type `int`) is one of:
	`IDA_SUCCESS` The optional output value has been successfully set.
	`IDA_MEM_NULL` The `ida_mem` pointer is NULL.
	`IDA_NO_SENS` Forward sensitivity analysis was not initialized.
Notes	This counter is incremented only if `ism` was `IDA_STAGGERED` in the call to `IDASensInit` (see §5.2.1).

IDAGetSensNonlinSolvStats

Call	`flag = IDAGetSensNonlinSolvStats(ida_mem, &nSniters, &nSncfails);`
Description	The function `IDAGetSensNonlinSolvStats` returns the sensitivity-related nonlinear solver statistics as a group.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`nSniters` (`long int`) number of nonlinear iterations performed.

nSncfails (`long int`) number of nonlinear convergence failures.

Return value The return value flag (of type `int`) is one of:

IDA_SUCCESS   The optional output values have been successfully set.

IDA_MEM_NULL The `ida_mem` pointer is NULL.

IDA_NO_SENS   Forward sensitivity analysis was not initialized.

5.2.7.2 Initial condition calculation optional output functions

The sensitivity consistent initial conditions found by IDAS (after a successful call to `IDACalcIC`) can
be obtained by calling the following function:

IDAGetSensConsistentIC

Call   `flag = IDAGetSensConsistentIC(ida_mem, yyS0_mod, ypS0_mod);`

Description   The function `IDAGetSensConsistentIC` returns the corrected initial conditions calcu-
              lated by `IDACalcIC` for sensitivities variables.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `yyS0_mod` (`N_Vector *`) a pointer to an array of `Ns` vectors containing consistent sensi-
                          tivity vectors.

              `ypS0_mod` (`N_Vector *`) a pointer to an array of `Ns` vectors containing consistent sensi-
                          tivity derivative vectors.

Return value The return value flag (of type `int`) is one of

IDA_SUCCESS   `IDAGetSensConsistentIC` succeeded.

IDA_MEM_NULL   The `ida_mem` pointer is NULL.

IDA_NO_SENS   The function `IDASensInit` has not been previously called.

IDA_ILL_INPUT `IDASolve` has been already called.

Notes   If the consistent sensitivity vectors or consistent derivative vectors are not desired, pass
              NULL for the corresponding argument.

              The user must allocate space for `yyS0_mod` and `ypS0_mod` (if not NULL).

5.3   User-supplied routines for forward sensitivity analysis

In addition to the required and optional user-supplied routines described in §4.6, when using IDAS for
forward sensitivity analysis, the user has the option of providing a routine that calculates the residual
of the sensitivity equations (2.12).

   By default, IDAS uses difference quotient approximation routines for the residual of the sensitivity
equations. However, IDAS allows the option for user-defined sensitivity residual routines (which also
provides a mechanism for interfacing IDAS to routines generated by automatic differentiation).

   The user may provide the residuals of the sensitivity equations (2.12), for all sensitivity parameters
at once, through a function of type `IDASensResFn` defined by:

IDASensResFn

Definition   `typedef int (*IDASensResFn)(int Ns, realtype t,`
                                  `N_Vector yy, N_Vector yp, N_Vector resval,`
                                  `N_Vector *yS, N_Vector *ypS,`
                                  `N_Vector *resvalS, void *user_data,`
                                  `N_Vector tmp1, N_Vector tmp2, N_Vector tmp3);`

Purpose   This function computes the sensitivity residual for all sensitivity equations. It must com-
              pute the vectors $(\partial F/\partial y)s_i(t) + (\partial F/\partial \dot{y})\dot{s}_i(t) + (\partial F/\partial p_i)$ and store them in `resvalS[i]`.

Arguments Ns   is the number of sensitivities.

t   is the current value of the independent variable.

yy   is the current value of the state vector, $y(t)$.

yp   is the current value of $\dot{y}(t)$.

resval   contains the current value F of the original DAE residual.

yS   contains the current values of the sensitivities s_i.

ypS   contains the current values of the sensitivity derivatives $\dot{s}_i$.

resvalS   contains the output sensitivity residual vectors.

user_data is a pointer to user data.

tmp1

tmp2

tmp3   are N_Vectors of length N which can be used as temporary storage.

Return value An IDASensResFn should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDA_SRES_FAIL is returned).

Notes   There is one situation in which recovery is not possible even if IDASensResFn function returns a recoverable error flag. That is when this occurs at the very first call to the IDASensResFn, in which case IDAS returns IDA_FIRST_RES_FAIL.

5.4 Integration of quadrature equations depending on forward sensitivities

IDAS provides support for integration of quadrature equations that depends not only on the state variables but also on forward sensitivities.

The following is an overview of the sequence of calls in a user's main program in this situation. Steps that are unchanged from the skeleton program presented in §5.1 are grayed out. See also §4.7.

1. **Initialize parallel or multi-threaded environment**

2. **Set problem dimensions, etc.**

3. **Set vectors of initial values**

4. **Create IDAS object**

5. **Initialize IDAS solver**

6. **Specify integration tolerances**

7. **Set optional inputs**

8. **Create matrix object**

9. **Create linear solver object**

10. **Set linear solver optional inputs**

11. **Initialize sensitivity-independent quadrature problem**

12. **Define the sensitivity problem**

13. **Set sensitivity initial conditions**

14. **Activate sensitivity calculations**

15. Set sensitivity analysis optional inputs

16. **Set vector of initial values for quadrature variables**

    Typically, the quadrature variables should be initialized to 0.

17. **Initialize sensitivity-dependent quadrature integration**

    Call `IDAQuadSensInit` to specify the quadrature equation right-hand side function and to allocate internal memory related to quadrature integration. See §5.4.1 for details.

18. **Set optional inputs for sensitivity-dependent quadrature integration**

    Call `IDASetQuadSensErrCon` to indicate whether or not quadrature variables should be used in the step size control mechanism. If so, one of the `IDAQuadSens*tolerances` functions must be called to specify the integration tolerances for quadrature variables. See §5.4.4 for details.

19. Advance solution in time

20. **Extract sensitivity-dependent quadrature variables**

    Call `IDAGetQuadSens`, `IDAGetQuadSens1`, `IDAGetQuadSensDky` or `IDAGetQuadSensDky1` to obtain the values of the quadrature variables or their derivatives at the current time. See §5.4.3 for details.

21. Get optional outputs

22. Extract sensitivity solution

23. **Get sensitivity-dependent quadrature optional outputs**

    Call `IDAGetQuadSens*` functions to obtain optional output related to the integration of sensitivity-dependent quadratures. See §5.4.5 for details.

24. Deallocate memory for solutions vector

25. Deallocate memory for sensitivity vectors

26. **Deallocate memory for sensitivity-dependent quadrature variables**

27. Free solver memory

28. Free vector specification memory

29. Free linear solver and matrix memory

30. Finalize MPI, if used

Note: `IDAQuadSensInit` (step 17 above) can be called and quadrature-related optional inputs (step 18 above) can be set, anywhere between steps 12 and 19.

5.4.1 Sensitivity-dependent quadrature initialization and deallocation

The function `IDAQuadSensInit` activates integration of quadrature equations depending on sensitivities and allocates internal memory related to these calculations. If `rhsQS` is input as `NULL`, then IDAS uses an internal function that computes difference quotient approximations to the functions $\bar{q}_i = (\partial q/\partial y)s_i + (\partial q/\partial \dot{y})\dot{s}_i + \partial q/\partial p_i$, in the notation of (2.10). The form of the call to this function is as follows:

`IDAQuadSensInit`

Call	`flag = IDAQuadSensInit(ida_mem, rhsQS, yQS0);`
Description	The function `IDAQuadSensInit` provides required problem specifications, allocates internal memory, and initializes quadrature integration.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
	`rhsQS` (`IDAQuadSensRhsFn`) is the C function which computes f_{QS}, the right-hand side of the sensitivity-dependent quadrature equations (for full details see §5.4.6).
	`yQS0` (`N_Vector *`) contains the initial values of sensitivity-dependent quadratures.
Return value	The return value `flag` (of type `int`) will be one of the following:

`IDA_SUCCESS`	The call to `IDAQuadSensInit` was successful.
`IDA_MEM_NULL`	The IDAS memory was not initialized by a prior call to `IDACreate`.
`IDA_MEM_FAIL`	A memory allocation request failed.
`IDA_NO_SENS`	The sensitivities were not initialized by a prior call to `IDASensInit`.
`IDA_ILL_INPUT`	The parameter `yQS0` is `NULL`.

Notes	Before calling `IDAQuadSensInit`, the user must enable the sensitivites by calling `IDASensInit`.
	If an error occurred, `IDAQuadSensInit` also sends an error message to the error handler function.

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In terms of the number of quadrature variables N_q and maximum method order `maxord`, the size of the real workspace is increased as follows:

- Base value: `lenrw = lenrw + (maxord+5)`N_q

- If `IDAQuadSensSVtolerances` is called: `lenrw = lenrw` $+ N_q N_s$

and the size of the integer workspace is increased as follows:

- Base value: `leniw = leniw + (maxord+5)`N_q

- If `IDAQuadSensSVtolerances` is called: `leniw = leniw` $+ N_q N_s$

The function `IDAQuadSensReInit`, useful during the solution of a sequence of problems of same size, reinitializes the quadrature related internal memory and must follow a call to `IDAQuadSensInit`. The number `Nq` of quadratures as well as the number `Ns` of sensitivities are assumed to be unchanged from the prior call to `IDAQuadSensInit`. The call to the `IDAQuadSensReInit` function has the form:

`IDAQuadSensReInit`

Call	`flag = IDAQuadSensReInit(ida_mem, yQS0);`
Description	The function `IDAQuadSensReInit` provides required problem specifications and reinitializes the sensitivity-dependent quadrature integration.
Arguments	`ida_mem` (`void *`) pointer to the IDAS memory block.
	`yQS0` (`N_Vector *`) contains the initial values of sensitivity-dependent quadratures.
Return value	The return value `flag` (of type `int`) will be one of the following:

`IDA_SUCCESS`	The call to `IDAQuadSensReInit` was successful.
`IDA_MEM_NULL`	The IDAS memory was not initialized by a prior call to `IDACreate`.
`IDA_NO_SENS`	Memory space for the sensitivity calculation was not allocated by a prior call to `IDASensInit`.
`IDA_NO_QUADSENS`	Memory space for the sensitivity quadratures integration was not allocated by a prior call to `IDAQuadSensInit`.
`IDA_ILL_INPUT`	The parameter `yQS0` is `NULL`.

Notes	If an error occurred, `IDAQuadSensReInit` also sends an error message to the error handler function.

IDAQuadSensFree

Call   `IDAQuadSensFree(ida_mem);`

Description   The function `IDAQuadSensFree` frees the memory allocated for sensitivity quadrature integration.

Arguments   The argument is the pointer to the IDAS memory block (of type `void *`).

Return value The function `IDAQuadSensFree` has no return value.

Notes   In general, `IDAQuadSensFree` need not be called by the user as it is called automatically by `IDAFree`.

5.4.2   IDAS solver function

Even if quadrature integration was enabled, the call to the main solver function `IDASolve` is exactly the same as in §4.5.6. However, in this case the return value `flag` can also be one of the following:

IDA_QSRHS_FAIL   The sensitivity quadrature right-hand side function failed in an unrecoverable manner.

IDA_FIRST_QSRHS_ERR The sensitivity quadrature right-hand side function failed at the first call.

IDA_REP_QSRHS_ERR   Convergence test failures occurred too many times due to repeated recoverable errors in the quadrature right-hand side function. The IDA_REP_RES_ERR will also be returned if the quadrature right-hand side function had repeated recoverable errors during the estimation of an initial step size (assuming the sensitivity quadrature variables are included in the error tests).

5.4.3   Sensitivity-dependent quadrature extraction functions

If sensitivity-dependent quadratures have been initialized by a call to `IDAQuadSensInit`, or reinitialized by a call to `IDAQuadSensReInit`, then IDAS computes a solution, sensitivities, and quadratures depending on sensitivities at time `t`. However, `IDASolve` will still return only the solutions y and $\dot{y}$. Sensitivity-dependent quadratures can be obtained using one of the following functions:

IDAGetQuadSens

Call   `flag = IDAGetQuadSens(ida_mem, &tret, yQS);`

Description   The function `IDAGetQuadSens` returns the quadrature sensitivity solution vectors after a successful return from `IDASolve`.

Arguments   `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

              `tret`   (`realtype`) the time reached by the solver (output).

              `yQS`   (`N_Vector *`) array of `Ns` computed sensitivity-dependent quadrature vectors.

Return value The return value `flag` of `IDAGetQuadSens` is one of:

IDA_SUCCESS	IDAGetQuadSens was successful.
IDA_MEM_NULL	ida_mem was NULL.
IDA_NO_SENS	Sensitivities were not activated.
IDA_NO_QUADSENS	Quadratures depending on the sensitivities were not activated.
IDA_BAD_DKY	yQS or one of the yQS[i] is NULL.

The function `IDAGetQuadSensDky` computes the `k`-th derivatives of the interpolating polynomials for the sensitivity-dependent quadrature variables at time `t`. This function is called by `IDAGetQuadSens` with `k = 0`, but may also be called directly by the user.

IDAGetQuadSensDky

Call   `flag = IDAGetQuadSensDky(ida_mem, t, k, dkyQS);`

Description   The function `IDAGetQuadSensDky` returns derivatives of the quadrature sensitivities solution vectors after a successful return from `IDASolve`.

Arguments   `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

        `t`   (`realtype`) the time at which information is requested. The time `t` must fall within the interval defined by the last successful step taken by IDAS.

        `k`   (`int`) order of the requested derivative.

        `dkyQS`   (`N_Vector *`) array of `Ns` vectors containing the derivatives. This vector array must be allocated by the user.

Return value The return value `flag` of `IDAGetQuadSensDky` is one of:

        `IDA_SUCCESS`   `IDAGetQuadSensDky` succeeded.

        `IDA_MEM_NULL`   `ida_mem` was NULL.

        `IDA_NO_SENS`   Sensitivities were not activated.

        `IDA_NO_QUADSENS` Quadratures depending on the sensitivities were not activated.

        `IDA_BAD_DKY`   `dkyQS` or one of the vectors `dkyQS[i]` is NULL.

        `IDA_BAD_K`   `k` is not in the range $0, 1, ..., klast$.

        `IDA_BAD_T`   The time `t` is not in the allowed range.

Quadrature sensitivity solution vectors can also be extracted separately for each parameter in turn through the functions `IDAGetQuadSens1` and `IDAGetQuadSensDky1`, defined as follows:

IDAGetQuadSens1

Call   `flag = IDAGetQuadSens1(ida_mem, &tret, is, yQS);`

Description   The function `IDAGetQuadSens1` returns the `is`-th sensitivity of quadratures after a successful return from `IDASolve`.

Arguments   `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

        `tret`   (`realtype`) the time reached by the solver (output).

        `is`   (`int`) specifies which sensitivity vector is to be returned ($0 \leq$ `is` $< N_s$).

        `yQS`   (`N_Vector`) the computed sensitivity-dependent quadrature vector.

Return value The return value `flag` of `IDAGetQuadSens1` is one of:

        `IDA_SUCCESS`   `IDAGetQuadSens1` was successful.

        `IDA_MEM_NULL`   `ida_mem` was NULL.

        `IDA_NO_SENS`   Forward sensitivity analysis was not initialized.

        `IDA_NO_QUADSENS` Quadratures depending on the sensitivities were not activated.

        `IDA_BAD_IS`   The index `is` is not in the allowed range.

        `IDA_BAD_DKY`   `yQS` is NULL.

IDAGetQuadSensDky1

Call   `flag = IDAGetQuadSensDky1(ida_mem, t, k, is, dkyQS);`

Description   The function `IDAGetQuadSensDky1` returns the `k`-th derivative of the `is`-th sensitivity solution vector after a successful return from `IDASolve`.

Arguments   `ida_mem` (`void *`) pointer to the memory previously allocated by `IDAInit`.

        `t`   (`realtype`) specifies the time at which sensitivity information is requested. The time `t` must fall within the interval defined by the last successful step taken by IDAS.

        `k`   (`int`) order of derivative.

is   (int) specifies the sensitivity derivative vector to be returned ($0 \leq$ is $< N_s$).

dkyQS   (N_Vector) the vector containing the derivative. The space for dkyQS must be
         allocated by the user.

Return value   The return value flag of IDAGetQuadSensDky1 is one of:

IDA_SUCCESS   IDAGetQuadDky1 succeeded.

IDA_MEM_NULL   ida_mem was NULL.

IDA_NO_SENS   Forward sensitivity analysis was not initialized.

IDA_NO_QUADSENS Quadratures depending on the sensitivities were not activated.

IDA_BAD_DKY   dkyQS is NULL.

IDA_BAD_IS   The index is is not in the allowed range.

IDA_BAD_K   k is not in the range $0, 1, ..., klast$.

IDA_BAD_T   The time t is not in the allowed range.

5.4.4   Optional inputs for sensitivity-dependent quadrature integration

IDAS provides the following optional input functions to control the integration of sensitivity-dependent
quadrature equations.

| IDASetQuadSensErrCon |

Call   flag = IDASetQuadSensErrCon(ida_mem, errconQS)

Description   The function IDASetQuadSensErrCon specifies whether or not the quadrature variables
               are to be used in the local error control mechanism. If they are, the user must specify
               the error tolerances for the quadrature variables by calling IDAQuadSensSStolerances,
               IDAQuadSensSVtolerances, or IDAQuadSensEEtolerances.

Arguments   ida_mem   (void *) pointer to the IDAS memory block.

              errconQS (booleantype) specifies whether sensitivity quadrature variables are included
                        (SUNTRUE) or not (SUNFALSE) in the error control mechanism.

Return value   The return value flag (of type int) is one of:

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL   The ida_mem pointer is NULL.

IDA_NO_SENS   Sensitivities were not activated.

IDA_NO_QUADSENS Quadratures depending on the sensitivities were not activated.

Notes   By default, errconQS is set to SUNFALSE.

              It is illegal to call IDASetQuadSensErrCon before a call to IDAQuadSensInit.

   If the quadrature variables are part of the step size control mechanism, one of the following
functions must be called to specify the integration tolerances for quadrature variables.

| IDAQuadSensSStolerances |

Call   flag = IDAQuadSensSVtolerances(ida_mem, reltolQS, abstolQS);

Description   The function IDAQuadSensSStolerances specifies scalar relative and absolute toler-
               ances.

Arguments   ida_mem   (void *) pointer to the IDAS memory block.

              reltolQS (realtype) is the scalar relative error tolerance.

              abstolQS (realtype*) is a pointer to an array containing the Ns scalar absolute error
                        tolerances.

Return value   The return value flag (of type int) is one of:

IDA_SUCCESS   The optional value has been successfully set.

IDA_MEM_NULL	The `ida_mem` pointer is NULL.
IDA_NO_SENS	Sensitivities were not activated.
IDA_NO_QUADSENS	Quadratures depending on the sensitivities were not activated.
IDA_ILL_INPUT	One of the input tolerances was negative.

IDAQuadSensSVtolerances

Call   `flag = IDAQuadSensSVtolerances(ida_mem, reltolQS, abstolQS);`

Description   The function `IDAQuadSensSVtolerances` specifies scalar relative and vector absolute tolerances.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

                `reltolQS` (`realtype`) is the scalar relative error tolerance.

                `abstolQS` (`N_Vector*`) is an array of `Ns` variables of type `N_Vector`. The `N_Vector` from `abstolS[is]` specifies the vector tolerances for `is`-th quadrature sensitivity.

Return value   The return value `flag` (of type `int`) is one of:

IDA_SUCCESS	The optional value has been successfully set.
IDA_NO_QUAD	Quadrature integration was not initialized.
IDA_MEM_NULL	The `ida_mem` pointer is NULL.
IDA_NO_SENS	Sensitivities were not activated.
IDA_NO_QUADSENS	Quadratures depending on the sensitivities were not activated.
IDA_ILL_INPUT	One of the input tolerances was negative.

IDAQuadSensEEtolerances

Call   `flag = IDAQuadSensEEtolerances(ida_mem);`

Description   The function `IDAQuadSensEEtolerances` specifies that the tolerances for the sensitivity-dependent quadratures should be estimated from those provided for the pure quadrature variables.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

Return value   The return value `flag` (of type `int`) is one of:

IDA_SUCCESS	The optional value has been successfully set.
IDA_MEM_NULL	The `ida_mem` pointer is NULL.
IDA_NO_SENS	Sensitivities were not activated.
IDA_NO_QUADSENS	Quadratures depending on the sensitivities were not activated.

Notes   When `IDAQuadSensEEtolerances` is used, before calling `IDASolve`, integration of pure quadratures must be initialized (see 4.7.1) and tolerances for pure quadratures must be also specified (see 4.7.4).

5.4.5   Optional outputs for sensitivity-dependent quadrature integration

IDAS provides the following functions that can be used to obtain solver performance information related to quadrature integration.

IDAGetQuadSensNumRhsEvals

Call   `flag = IDAGetQuadSensNumRhsEvals(ida_mem, &nrhsQSevals);`

Description   The function `IDAGetQuadSensNumRhsEvals` returns the number of calls made to the user's quadrature right-hand side function.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

nrhsQSevals (`long int`) number of calls made to the user's `rhsQS` function.

Return value   The return value `flag` (of type `int`) is one of:

IDA_SUCCESS   The optional output value has been successfully set.

IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

IDA_NO_QUADSENS   Sensitivity-dependent quadrature integration has not been initialized.

IDAGetQuadSensNumErrTestFails

Call   `flag = IDAGetQuadSensNumErrTestFails(ida_mem, &nQSetfails);`

Description   The function `IDAGetQuadSensNumErrTestFails` returns the number of local error test failures due to quadrature variables.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

nQSetfails (`long int`) number of error test failures due to quadrature variables.

Return value   The return value `flag` (of type `int`) is one of:

IDA_SUCCESS   The optional output value has been successfully set.

IDA_MEM_NULL   The `ida_mem` pointer is `NULL`.

IDA_NO_QUADSENS   Sensitivity-dependent quadrature integration has not been initialized.

IDAGetQuadSensErrWeights

Call   `flag = IDAGetQuadSensErrWeights(ida_mem, eQSweight);`

Description   The function `IDAGetQuadSensErrWeights` returns the quadrature error weights at the current time.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

eQSweight (`N_Vector *`) array of quadrature error weight vectors at the current time.

Return value   The return value `flag` (of type `int`) is one of:

IDA_SUCCESS   The optional output value has been successfully set.

IDA_MEM_NULL The `ida_mem` pointer is `NULL`.

IDA_NO_QUADSENS Sensitivity-dependent quadrature integration has not been initialized.

Notes   The user must allocate memory for `eQSweight`.

If quadratures were not included in the error control mechanism (through a call to `IDASetQuadSensErrCon` with `errconQS=SUNTRUE`), `IDAGetQuadSensErrWeights` does not set the `eQSweight` vector.

IDAGetQuadSensStats

Call   `flag = IDAGetQuadSensStats(ida_mem, &nrhsQSevals, &nQSetfails);`

Description   The function `IDAGetQuadSensStats` returns the IDAS integrator statistics as a group.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

nrhsQSevals (`long int`) number of calls to the user's `rhsQS` function.

nQSetfails   (`long int`) number of error test failures due to quadrature variables.

Return value   The return value `flag` (of type `int`) is one of

IDA_SUCCESS   the optional output values have been successfully set.

IDA_MEM_NULL   the `ida_mem` pointer is `NULL`.

IDA_NO_QUADSENS   Sensitivity-dependent quadrature integration has not been initialized.

5.4.6 User-supplied function for sensitivity-dependent quadrature integration

For the integration of sensitivity-dependent quadrature equations, the user must provide a function that defines the right-hand side of the sensitivity quadrature equations. For sensitivities of quadratures (2.10) with integrands q, the appropriate right-hand side functions are given by $\bar{q}_i = (\partial q/\partial y)s_i + (\partial q/\partial \dot{y})\dot{s}_i + \partial q/\partial p_i$. This user function must be of type `IDAQuadSensRhsFn`, defined as follows:

`IDAQuadSensRhsFn`

Definition	`typedef int (*IDAQuadSensRhsFn)(int Ns, realtype t, N_Vector yy,`
	`                                 N_Vector yp, N_Vector *yyS, N_Vector *ypS,`
	`                                 N_Vector rrQ, N_Vector *rhsvalQS,`
	`                                 void *user_data, N_Vector tmp1,`
	`                                 N_Vector tmp2, N_Vector tmp3)`

Purpose	This function computes the sensitivity quadrature equation right-hand side for a given value of the independent variable t and state vector y.
Arguments	`Ns` is the number of sensitivity vectors.
	`t` is the current value of the independent variable.
	`yy` is the current value of the dependent variable vector, $y(t)$.
	`yp` is the current value of the dependent variable vector, $\dot{y}(t)$.
	`yyS` is an array of `Ns` variables of type `N_Vector` containing the dependent sensitivity vectors s_i.
	`ypS` is an array of `Ns` variables of type `N_Vector` containing the dependent sensitivity derivatives $\dot{s}_i$.
	`rrQ` is the current value of the quadrature right-hand side q.
	`rhsvalQS` contains the `Ns` output vectors.
	`user_data` is the `user_data` pointer passed to `IDASetUserData`.
	`tmp1`
	`tmp2`
	`tmp3` are `N_Vector`s which can be used as temporary storage.
Return value	An `IDAQuadSensRhsFn` should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and `IDA_QRHS_FAIL` is returned).
Notes	Allocation of memory for `rhsvalQS` is automatically handled within IDAS.
	Both `yy` and `yp` are of type `N_Vector` and both `yyS` and `ypS` are pointers to an array containing `Ns` vectors of type `N_Vector`. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their `N_Vector` arguments (see §7.1 and §7.2).
	There is one situation in which recovery is not possible even if `IDAQuadSensRhsFn` function returns a recoverable error flag. That is when this occurs at the very first call to the `IDAQuadSensRhsFn`, in which case IDAS returns `IDA_FIRST_QSRHS_ERR`).

5.5 Note on using partial error control

For some problems, when sensitivities are excluded from the error control test, the behavior of IDAS may appear at first glance to be erroneous. One would expect that, in such cases, the sensitivity variables would not influence in any way the step size selection.

The short explanation of this behavior is that the step size selection implemented by the error control mechanism in IDAS is based on the magnitude of the correction calculated by the nonlinear solver. As mentioned in §5.2.1, even with partial error control selected in the call to `IDASensInit`, the sensitivity variables are included in the convergence tests of the nonlinear solver.

When using the simultaneous corrector method (§2.5), the nonlinear system that is solved at each step involves both the state and sensitivity equations. In this case, it is easy to see how the sensitivity variables may affect the convergence rate of the nonlinear solver and therefore the step size selection. The case of the staggered corrector approach is more subtle. The sensitivity variables at a given step are computed only once the solver for the nonlinear state equations has converged. However, if the nonlinear system corresponding to the sensitivity equations has convergence problems, IDAS will attempt to improve the initial guess by reducing the step size in order to provide a better prediction of the sensitivity variables. Moreover, even if there are no convergence failures in the solution of the sensitivity system, IDAS may trigger a call to the linear solver's setup routine which typically involves reevaluation of Jacobian information (Jacobian approximation in the case of IDADENSE and IDABAND, or preconditioner data in the case of the Krylov solvers). The new Jacobian information will be used by subsequent calls to the nonlinear solver for the state equations and, in this way, potentially affect the step size selection.

When using the simultaneous corrector method it is not possible to decide whether nonlinear solver convergence failures or calls to the linear solver setup routine have been triggered by convergence problems due to the state or the sensitivity equations. When using one of the staggered corrector methods, however, these situations can be identified by carefully monitoring the diagnostic information provided through optional outputs. If there are no convergence failures in the sensitivity nonlinear solver, and none of the calls to the linear solver setup routine were made by the sensitivity nonlinear solver, then the step size selection is not affected by the sensitivity variables.

Finally, the user must be warned that the effect of appending sensitivity equations to a given system of DAEs on the step size selection (through the mechanisms described above) is problem-dependent and can therefore lead to either an increase or decrease of the total number of steps that IDAS takes to complete the simulation. At first glance, one would expect that the impact of the sensitivity variables, if any, would be in the direction of increasing the step size and therefore reducing the total number of steps. The argument for this is that the presence of the sensitivity variables in the convergence test of the nonlinear solver can only lead to additional iterations (and therefore a smaller iteration error), or to additional calls to the linear solver setup routine (and therefore more up-to-date Jacobian information), both of which will lead to larger steps being taken by IDAS. However, this is true only locally. Overall, a larger integration step taken at a given time may lead to step size reductions at later times, due to either nonlinear solver convergence failures or error test failures.

Chapter 6

Using IDAS for Adjoint Sensitivity Analysis

This chapter describes the use of IDAS to compute sensitivities of derived functions using adjoint sensitivity analysis. As mentioned before, the adjoint sensitivity module of IDAS provides the infrastructure for integrating backward in time any system of DAEs that depends on the solution of the original IVP, by providing various interfaces to the main IDAS integrator, as well as several supporting user-callable functions. For this reason, in the following sections we refer to the *backward problem* and not to the *adjoint problem* when discussing details relevant to the DAEs that are integrated backward in time. The backward problem can be the adjoint problem (2.20) or (2.25), and can be augmented with some quadrature differential equations.

IDAS uses various constants for both input and output. These are defined as needed in this chapter, but for convenience are also listed separately in Appendix B.

We begin with a brief overview, in the form of a skeleton user program. Following that are detailed descriptions of the interface to the various user-callable functions and of the user-supplied functions that were not already described in Chapter 4.

6.1 A skeleton of the user's main program

The following is a skeleton of the user's main program as an application of IDAS. The user program is to have these steps in the order indicated, unless otherwise noted. For the sake of brevity, we defer many of the details to the later sections. As in §4.4, most steps are independent of the NVECTOR implementation used. Where this is not the case, refer to Chapter 7 for specific names. Steps that are unchanged from the skeleton programs presented in §4.4, §5.1, and §5.4, are grayed out.

1. **Include necessary header files**

   The `idas.h` header file also defines additional types, constants, and function prototypes for the adjoint sensitivity module user-callable functions. In addition, the main program should include an NVECTOR implementation header file (for the particular implementation used) and, if Newton iteration was selected, the main header file of the desired linear solver module.

2. **Initialize parallel or multi-threaded environment**

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Forward problem

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3. **Set problem dimensions etc. for the forward problem**

4. **Set initial conditions for the forward problem**

5. **Create IDAS object for the forward problem**

6. Initialize IDAS solver for the forward problem

7. Specify integration tolerances for forward problem

8. Set optional inputs for the forward problem

9. Create matrix object for the forward problem

10. Create linear solver object for the forward problem

11. Set linear solver optional inputs for the forward problem

12. Attach linear solver module for the forward problem

13. Initialize quadrature problem or problems for forward problems, using `IDAQuadInit` and/or `IDAQuadSensInit`.

14. Initialize forward sensitivity problem

15. Specify rootfinding

16. **Allocate space for the adjoint computation**

    Call `IDAAdjInit()` to allocate memory for the combined forward-backward problem (see §6.2.1 for details). This call requires `Nd`, the number of steps between two consecutive checkpoints. `IDAAdjInit` also specifies the type of interpolation used (see §2.6.3).

17. **Integrate forward problem**

    Call `IDASolveF`, a wrapper for the IDAS main integration function `IDASolve`, either in `IDA_NORMAL` mode to the time `tout` or in `IDA_ONE_STEP` mode inside a loop (if intermediate solutions of the forward problem are desired (see §6.2.3)). The final value of `tret` is then the maximum allowable value for the endpoint T of the backward problem.

Backward problem(s)

18. **Set problem dimensions etc. for the backward problem**

    This generally includes `NB`, the number of variables in the backward problem and possibly the local vector length `NBlocal`.

19. **Set initial values for the backward problem**

    Set the endpoint time $\mathtt{tB0} = T$, and set the corresponding vectors `yB0` and `ypB0` at which the backward problem starts.

20. **Create the backward problem**

    Call `IDACreateB`, a wrapper for `IDACreate`, to create the IDAS memory block for the new backward problem. Unlike `IDACreate`, the function `IDACreateB` does not return a pointer to the newly created memory block (see §6.2.4). Instead, this pointer is attached to the internal adjoint memory block (created by `IDAAdjInit`) and returns an identifier called `which` that the user must later specify in any actions on the newly created backward problem.

21. **Allocate memory for the backward problem**

    Call `IDAInitB` (or `IDAInitBS`, when the backward problem depends on the forward sensitivities). The two functions are actually wrappers for `IDAInit` and allocate internal memory, specify problem data, and initialize IDAS at `tB0` for the backward problem (see §6.2.4).

22. **Specify integration tolerances for backward problem**

Call `IDASStolerancesB(...)` or `IDASVtolerancesB(...)` to specify a scalar relative tolerance and scalar absolute tolerance, or a scalar relative tolerance and a vector of absolute tolerances, respectively. The functions are wrappers for `IDASStolerances(...)` and `IDASVtolerances(...)` but they require an extra argument `which`, the identifier of the backward problem returned by `IDACreateB`. See §6.2.5 for more information.

23. **Set optional inputs for the backward problem**

    Call `IDASet*B` functions to change from their default values any optional inputs that control the behavior of IDAS. Unlike their counterparts for the forward problem, these functions take an extra argument `which`, the identifier of the backward problem returned by `IDACreateB` (see §6.2.9).

24. **Create matrix object for the backward problem**

    If a direct linear solver is to be used within a Newton iteration then a template Jacobian matrix must be created by using the appropriate functions defined by the particular SUNMATRIX implementation.

    NOTE: The dense, banded, and sparse matrix objects are usable only in a serial or threaded environment.

    Note also that it is not required to use the same matrix type for both the forward and the backward problems.

25. **Create linear solver object for the backward problem**

    Create the linear solver object for the backward problem by using the appropriate functions defined by the particular SUNLINSOL implementation desired.

    Note that it is not required to use the same linear solver module for both the forward and the backward problems; for example, the forward problem could be solved with the IDADLS linear solver module and the backward problem with IDASPILS linear solver module.

26. **Set linear solver interface optional inputs for the backward problem**

    Call `IDADlsSet*B` or `IDASpilsSet*B` functions to change optional inputs specific to that linear solver interface. See §6.2.9 for details.

27. **Initialize quadrature calculation**

    If additional quadrature equations must be evaluated, call `IDAQuadInitB` or `IDAQuadInitBS` (if quadrature depends also on the forward sensitivities) as shown in §6.2.11.1. These functions are wrappers around `IDAQuadInit` and can be used to initialize and allocate memory for quadrature integration. Optionally, call `IDASetQuad*B` functions to change from their default values optional inputs that control the integration of quadratures during the backward phase.

28. **Integrate backward problem**

    Call `IDASolveB`, a second wrapper around the IDAS main integration function `IDASolve`, to integrate the backward problem from `tB0` (see §6.2.8). This function can be called either in IDA_NORMAL or IDA_ONE_STEP mode. Typically, `IDASolveB` will be called in IDA_NORMAL mode with an end time equal to the initial time t_0 of the forward problem.

29. **Extract quadrature variables**

    If applicable, call `IDAGetQuadB`, a wrapper around `IDAGetQuad`, to extract the values of the quadrature variables at the time returned by the last call to `IDASolveB`. See §6.2.11.2.

30. **Deallocate memory**

    Upon completion of the backward integration, call all necessary deallocation functions. These include appropriate destructors for the vectors `y` and `yB`, a call to `IDAFree` to free the IDAS memory block for the forward problem. If one or more additional adjoint sensitivity analyses are

to be done for this problem, a call to `IDAAdjFree` (see §6.2.1) may be made to free and deallocate the memory allocated for the backward problems, followed by a call to `IDAAdjInit`.

31. **Free linear solver and matrix memory for the backward problem**

32. Finalize MPI, if used

The above user interface to the adjoint sensitivity module in IDAS was motivated by the desire to keep it as close as possible in look and feel to the one for DAE IVP integration. Note that if steps (18)-(29) are not present, a program with the above structure will have the same functionality as one described in §4.4 for integration of DAEs, albeit with some overhead due to the checkpointing scheme.

If there are multiple backward problems associated with the same forward problem, repeat steps (18)-(29) above for each successive backward problem. In the process, each call to `IDACreateB` creates a new value of the identifier `which`.

6.2 User-callable functions for adjoint sensitivity analysis

6.2.1 Adjoint sensitivity allocation and deallocation functions

After the setup phase for the forward problem, but before the call to `IDASolveF`, memory for the combined forward-backward problem must be allocated by a call to the function `IDAAdjInit`. The form of the call to this function is

---

`IDAAdjInit`	
Call	`flag = IDAAdjInit(ida_mem, Nd, interpType);`
Description	The function `IDAAdjInit` updates IDAS memory block by allocating the internal memory needed for backward integration. Space is allocated for the `Nd` $= N_d$ interpolation data points, and a linked list of checkpoints is initialized.
Arguments	`ida_mem` (`void *`) is the pointer to the IDAS memory block returned by a previous call to `IDACreate`.
	`Nd` (`long int`) is the number of integration steps between two consecutive checkpoints.
	`interpType` (`int`) specifies the type of interpolation used and can be `IDA_POLYNOMIAL` or `IDA_HERMITE`, indicating variable-degree polynomial and cubic Hermite interpolation, respectively (see §2.6.3).
Return value	The return value `flag` (of type `int`) is one of:
	`IDA_SUCCESS` `IDAAdjInit` was successful.
	`IDA_MEM_FAIL` A memory allocation request has failed.
	`IDA_MEM_NULL` `ida_mem` was NULL.
	`IDA_ILL_INPUT` One of the parameters was invalid: `Nd` was not positive or `interpType` is not one of the `IDA_POLYNOMIAL` or `IDA_HERMITE`.
Notes	The user must set `Nd` so that all data needed for interpolation of the forward problem solution between two checkpoints fits in memory. `IDAAdjInit` attempts to allocate space for (2`Nd`+3) variables of type `N_Vector`.
	If an error occurred, `IDAAdjInit` also sends a message to the error handler function.

`IDAAdjReInit`	
Call	`flag = IDAAdjReInit(ida_mem);`
Description	The function `IDAAdjReInit` reinitializes the IDAS memory block for ASA, assuming that the number of steps between check points and the type of interpolation remain unchanged.

Arguments   ida mem (void *) is the pointer to the IDAS memory block returned by a previous call to IDACreate.

Return value The return value flag (of type int) is one of:

IDA SUCCESS IDAAdjReInit was successful.

IDA MEM NULL ida mem was NULL.

IDA NO ADJ   The function IDAAdjInit was not previously called.

Notes   The list of check points (and associated memory) is deleted.

The list of backward problems is kept. However, new backward problems can be added to this list by calling IDACreateB. If a new list of backward problems is also needed, then free the adjoint memory (by calling IDAAdjFree) and reinitialize ASA with IDAAdjInit.

The IDAS memory for the forward and backward problems can be reinitialized separately by calling IDAReInit and IDAReInitB, respectively.

IDAAdjFree

Call   IDAAdjFree(ida mem);

Description   The function IDAAdjFree frees the memory related to backward integration allocated by a previous call to IDAAdjInit.

Arguments   The only argument is the IDAS memory block pointer returned by a previous call to IDACreate.

Return value The function IDAAdjFree has no return value.

Notes   This function frees all memory allocated by IDAAdjInit. This includes workspace memory, the linked list of checkpoints, memory for the interpolation data, as well as the IDAS memory for the backward integration phase.

Unless one or more further calls to IDAAdjInit are to be made, IDAAdjFree should not be called by the user, as it is invoked automatically by IDAFree.

6.2.2   Adjoint sensitivity optional input

At any time during the integration of the forward problem, the user can disable the checkpointing of the forward sensitivities by calling the following function:

IDAAdjSetNoSensi

Call   flag = IDAAdjSetNoSensi(ida mem);

Description   The function IDAAdjSetNoSensi instructs IDASolveF not to save checkpointing data for forward sensitivities any more.

Arguments   ida mem (void *) pointer to the IDAS memory block.

Return value The return flag (of type int) is one of:

IDA SUCCESS   The call to IDACreateB was successful.

IDA MEM NULL The ida mem was NULL.

IDA NO ADJ   The function IDAAdjInit has not been previously called.

6.2.3   Forward integration function

The function IDASolveF is very similar to the IDAS function IDASolve (see §4.5.6) in that it integrates the solution of the forward problem and returns the solution $(y, \dot{y})$. At the same time, however, IDASolveF stores checkpoint data every Nd integration steps. IDASolveF can be called repeatedly by the user. Note that IDASolveF is used only for the forward integration pass within an Adjoint Sensitivity Analysis. It is not for use in Forward Sensitivity Analysis; for that, see Chapter 5. The call to this function has the form

---

IDASolveF

Call   `flag = IDASolveF(ida_mem, tout, &tret, yret, ypret, itask, &ncheck);`

Description   The function `IDASolveF` integrates the forward problem over an interval in t and saves checkpointing data.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

`tout`   (`realtype`) the next time at which a computed solution is desired.

`tret`   (`realtype`) the time reached by the solver (output).

`yret`   (`N_Vector`) the computed solution vector y.

`ypret`   (`N_Vector`) the computed solution vector $\dot{y}$.

`itask`   (`int`) a flag indicating the job of the solver for the next step. The `IDA_NORMAL` task is to have the solver take internal steps until it has reached or just passed the user-specified `tout` parameter. The solver then interpolates in order to return an approximate value of $y(\texttt{tout})$ and $\dot{y}(\texttt{tout})$. The `IDA_ONE_STEP` option tells the solver to take just one internal step and return the solution at the point reached by that step.

`ncheck` (`int`) the number of (internal) checkpoints stored so far.

Return value On return, `IDASolveF` returns vectors `yret`, `ypret` and a corresponding independent variable value $t = \texttt{tret}$, such that `yret` is the computed value of $y(t)$ and `ypret` the value of $\dot{y}(t)$. Additionally, it returns in `ncheck` the number of internal checkpoints saved; the total number of checkpoint intervals is `ncheck+1`. The return value `flag` (of type `int`) will be one of the following. For more details see §4.5.6.

IDA_SUCCESS   `IDASolveF` succeeded.

IDA_TSTOP_RETURN   `IDASolveF` succeeded by reaching the optional stopping point.

IDA_ROOT_RETURN   `IDASolveF` succeeded and found one or more roots. In this case, `tret` is the location of the root. If $\texttt{nrtfn} > 1$, call `IDAGetRootInfo` to see which g_i were found to have a root.

IDA_NO_MALLOC   The function `IDAInit` has not been previously called.

IDA_ILL_INPUT   One of the inputs to `IDASolveF` is illegal.

IDA_TOO_MUCH_WORK The solver took `mxstep` internal steps but could not reach `tout`.

IDA_TOO_MUCH_ACC   The solver could not satisfy the accuracy demanded by the user for some internal step.

IDA_ERR_FAILURE   Error test failures occurred too many times during one internal time step or occurred with $|h| = h_{min}$.

IDA_CONV_FAILURE   Convergence test failures occurred too many times during one internal time step or occurred with $|h| = h_{min}$.

IDA_LSETUP_FAIL   The linear solver's setup function failed in an unrecoverable manner.

IDA_LSOLVE_FAIL   The linear solver's solve function failed in an unrecoverable manner.

IDA_NO_ADJ   The function `IDAAdjInit` has not been previously called.

IDA_MEM_FAIL   A memory allocation request has failed (in an attempt to allocate space for a new checkpoint).

Notes   All failure return values are negative and therefore a test `flag`< 0 will trap all `IDASolveF` failures.

At this time, `IDASolveF` stores checkpoint information in memory only. Future versions will provide for a safeguard option of dumping checkpoint data into a temporary file as needed. The data stored at each checkpoint is basically a snapshot of the IDAS internal memory block and contains enough information to restart the integration from that time and to proceed with the same step size and method order sequence as during the forward integration.

In addition, IDASolveF also stores interpolation data between consecutive checkpoints so that, at the end of this first forward integration phase, interpolation information is already available from the last checkpoint forward. In particular, if no checkpoints were necessary, there is no need for the second forward integration phase.

It is illegal to change the integration tolerances between consecutive calls to IDASolveF, as this information is not captured in the checkpoint data.

6.2.4 Backward problem initialization functions

The functions IDACreateB and IDAInitB (or IDAInitBS) must be called in the order listed. They instantiate an IDAS solver object, provide problem and solution specifications, and allocate internal memory for the backward problem.

| IDACreateB |

Call	flag = IDACreateB(ida_mem, &which);
Description	The function IDACreateB instantiates an IDAS solver object for the backward problem.
Arguments	ida_mem (void *) pointer to the IDAS memory block returned by IDACreate.
	which (int) contains the identifier assigned by IDAS for the newly created backward problem. Any call to IDA*B functions requires such an identifier.
Return value	The return flag (of type int) is one of:

        IDA_SUCCESS The call to IDACreateB was successful.

        IDA_MEM_NULL The ida_mem was NULL.

        IDA_NO_ADJ   The function IDAAdjInit has not been previously called.

        IDA_MEM_FAIL A memory allocation request has failed.

There are two initialization functions for the backward problem – one for the case when the backward problem does not depend on the forward sensitivities, and one for the case when it does. These two functions are described next.

The function IDAInitB initializes the backward problem when it does not depend on the forward sensitivities. It is essentially wrapper for IDAInit with some particularization for backward integration, as described below.

| IDAInitB |

Call	flag = IDAInitB(ida_mem, which, resB, tB0, yB0, ypB0);
Description	The function IDAInitB provides problem specification, allocates internal memory, and initializes the backward problem.
Arguments	ida_mem (void *) pointer to the IDAS memory block returned by IDACreate.
	which (int) represents the identifier of the backward problem.
	resB (IDAResFnB) is the C function which computes fB, the residual of the backward DAE problem. This function has the form resB(t, y, yp, yB, ypB, resvalB, user_dataB) (for full details see §6.3.1).
	tB0 (realtype) specifies the endpoint T where final conditions are provided for the backward problem, normally equal to the endpoint of the forward integration.
	yB0 (N_Vector) is the initial value (at $t = $ tB0) of the backward solution.
	ypB0 (N_Vector) is the initial derivative value (at $t = $ tB0) of the backward solution.
Return value	The return flag (of type int) will be one of the following:

        IDA_SUCCESS   The call to IDAInitB was successful.

        IDA_NO_MALLOC The function IDAInit has not been previously called.

        IDA_MEM_NULL   The ida_mem was NULL.

	IDA_NO_ADJ	The function IDAAdjInit has not been previously called.
	IDA_BAD_TB0	The final time tB0 was outside the interval over which the forward problem was solved.
	IDA_ILL_INPUT	The parameter which represented an invalid identifier, or one of yB0, ypB0, resB was NULL.
Notes	The memory allocated by IDAInitB is deallocated by the function IDAAdjFree.	

For the case when backward problem also depends on the forward sensitivities, user must call IDAInitBS instead of IDAInitB. Only the third argument of each function differs between these functions.

IDAInitBS

Call	flag = IDAInitBS(ida_mem, which, resBS, tB0, yB0, ypB0);
Description	The function IDAInitBS provides problem specification, allocates internal memory, and initializes the backward problem.
Arguments	ida_mem (void *) pointer to the IDAS memory block returned by IDACreate.
	which (int) represents the identifier of the backward problem.
	resBS (IDAResFnBS) is the C function which computes fB, the residual or the backward DAE problem. This function has the form resBS(t, y, yp, yS, ypS, yB, ypB, resvalB, user_dataB) (for full details see §6.3.2).
	tB0 (realtype) specifies the endpoint T where final conditions are provided for the backward problem.
	yB0 (N_Vector) is the initial value (at $t = $ tB0) of the backward solution.
	ypB0 (N_Vector) is the initial derivative value (at $t = $ tB0) of the backward solution.
Return value	The return flag (of type int) will be one of the following:

	IDA_SUCCESS	The call to IDAInitB was successful.
	IDA_NO_MALLOC	The function IDAInit has not been previously called.
	IDA_MEM_NULL	The ida_mem was NULL.
	IDA_NO_ADJ	The function IDAAdjInit has not been previously called.
	IDA_BAD_TB0	The final time tB0 was outside the interval over which the forward problem was solved.
	IDA_ILL_INPUT	The parameter which represented an invalid identifier, or one of yB0, ypB0, resB was NULL, or sensitivities were not active during the forward integration.
Notes	The memory allocated by IDAInitBS is deallocated by the function IDAAdjFree.	

The function IDAReInitB reinitializes IDAS for the solution of a series of backward problems, each identified by a value of the parameter which. IDAReInitB is essentially a wrapper for IDAReInit, and so all details given for IDAReInit in §4.5.10 apply here. Also, IDAReInitB can be called to reinitialize a backward problem even if it has been initialized with the sensitivity-dependent version IDAInitBS. Before calling IDAReInitB for a new backward problem, call any desired solution extraction functions IDAGet** associated with the previous backward problem. The call to the IDAReInitB function has the form

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IDAReInitB

Call	flag = IDAReInitB(ida_mem, which, tB0, yB0, ypB0)
Description	The function IDAReInitB reinitializes an IDAS backward problem.
Arguments	ida_mem (void *) pointer to IDAS memory block returned by IDACreate.
	which (int) represents the identifier of the backward problem.

tB0   (`realtype`) specifies the endpoint T where final conditions are provided for the backward problem.

yB0   (`N_Vector`) is the initial value (at $t = $ `tB0`) of the backward solution.

ypB0   (`N_Vector`) is the initial derivative value (at $t = $ `tB0`) of the backward solution.

Return value   The return value `flag` (of type `int`) will be one of the following:

IDA_SUCCESS   The call to `IDAReInitB` was successful.

IDA_NO_MALLOC   The function `IDAInit` has not been previously called.

IDA_MEM_NULL   The `ida_mem` memory block pointer was `NULL`.

IDA_NO_ADJ   The function `IDAAdjInit` has not been previously called.

IDA_BAD_TB0   The final time `tB0` is outside the interval over which the forward problem was solved.

IDA_ILL_INPUT   The parameter `which` represented an invalid identifier, or one of `yB0`, `ypB0` was `NULL`.

6.2.5   Tolerance specification functions for backward problem

One of the following two functions must be called to specify the integration tolerances for the backward problem. Note that this call must be made after the call to `IDAInitB` or `IDAInitBS`.

`IDASStolerancesB`

Call   `flag = IDASStolerances(ida_mem, which, reltolB, abstolB);`

Description   The function `IDASStolerancesB` specifies scalar relative and absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
`which`   (`int`) represents the identifier of the backward problem.
`reltolB` (`realtype`) is the scalar relative error tolerance.
`abstolB` (`realtype`) is the scalar absolute error tolerance.

Return value   The return `flag` (of type `int`) will be one of the following:

IDA_SUCCESS   The call to `IDASStolerancesB` was successful.

IDA_MEM_NULL   The IDAS memory block was not initialized through a previous call to `IDACreate`.

IDA_NO_MALLOC   The allocation function `IDAInit` has not been called.

IDA_NO_ADJ   The function `IDAAdjInit` has not been previously called.

IDA_ILL_INPUT   One of the input tolerances was negative.

`IDASVtolerancesB`

Call   `flag = IDASVtolerancesB(ida_mem, which, reltolB, abstolB);`

Description   The function `IDASVtolerancesB` specifies scalar relative tolerance and vector absolute tolerances.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.
`which`   (`int`) represents the identifier of the backward problem.
`reltol` (`realtype`) is the scalar relative error tolerance.
`abstol` (`N_Vector`) is the vector of absolute error tolerances.

Return value   The return `flag` (of type `int`) will be one of the following:

IDA_SUCCESS   The call to `IDASVtolerancesB` was successful.

IDA_MEM_NULL   The IDAS memory block was not initialized through a previous call to `IDACreate`.

IDA_NO_MALLOC   The allocation function `IDAInit` has not been called.

IDA_NO_ADJ   The function IDAAdjInit has not been previously called.

IDA_ILL_INPUT The relative error tolerance was negative or the absolute tolerance had
                   a negative component.

Notes   This choice of tolerances is important when the absolute error tolerance needs to be
                  different for each component of the DAE state vector y.

6.2.6 Linear solver initialization functions for backward problem

All IDAS linear solver modules available for forward problems are available for the backward problem.
They should be created as for the forward problem then attached to the memory structure for the
backward problem using one of the following functions.

IDADlsSetLinearSolverB

Call   `flag = IDADlsSetLinearSolverB(ida_mem, which, LS, A);`

Description   The function `IDADlsSetLinearSolverB` attaches a direct SUNLINSOL object `LS` and
              corresponding template Jacobian SUNMATRIX object `A` to IDAS, initializing the IDADLS
              direct linear solver interface for solution of the backward problem.

              The user's main program must include the `idas_direct.h` header file.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `which`   (`int`) represents the identifier of the backward problem returned by `IDACreateB`.

              `LS`   (`SUNLinearSolver`) SUNLINSOL object to use for solving Newton linear systems
                        for the backward problem.

              `A`   (`SUNMatrix`) SUNMATRIX object for used as a template for the Jacobian for
                        the backward problem (must have a type compatible with the linear solver
                        object).

Return value The return value `flag` (of type `int`) is one of

              `IDADLS_SUCCESS` The IDADLS initialization was successful.

              `IDADLS_MEM_NULL` The `ida_mem` pointer is `NULL`.

              `IDADLS_ILL_INPUT` The IDADLS solver is not compatible with the current NVECTOR
                  module.

              `IDADLS_MEM_FAIL` A memory allocation request failed.

              `IDADLS_NO_ADJ` The function `IDAAdjInit` has not been previously called.

              `IDADLS_ILL_INPUT` The parameter `which` represented an invalid identifier.

Notes   The IDADLS linear solver is not compatible with all implementations of the SUNLINSOL
              and NVECTOR modules. Specifically, IDADLS requires use of a *direct* SUNLINSOL object
              and a serial or theaded NVECTOR module. Additional compatibility limitations for each
              SUNLINSOL object (i.e. SUNMATRIX and NVECTOR object compatibility) are described
              in Chapter 9.

IDASpilsSetLinearSolverB

Call   `flag = IDASpilsSetLinearSolverB(ida_mem, which, LS);`

Description   The function `IDASpilsSetLinearSolver` attaches an iterative SUNLINSOL object `LS` to
              IDAS, initializing the IDASPILS scaled, preconditioned, iterative linear solver interface to
              use for the backward problem.

              The user's main program must include the `idas_spils.h` header file.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `which`   (`int`) represents the identifier of the backward problem returned by `IDACreateB`.

LS   (`SUNLinearSolver`) SUNLINSOL object to use for solving Newton linear systems for the backward problem.

Return value   The return value `flag` (of type `int`) is one of

         IDASPILS_SUCCESS The IDASPILS initialization was successful.

         IDASPILS_MEM_NULL The `ida_mem` pointer is `NULL`.

         IDASPILS_ILL_INPUT The IDASPILS solver is not compatible with the current NVECTOR module.

         IDASPILS_MEM_FAIL A memory allocation request failed.

         IDASPILS_NO_ADJ The function `IDAAdjInit` has not been previously called.

         IDASPILS_ILL_INPUT The parameter `which` represented an invalid identifier.

Notes   The IDASPILS linear solver interface is not compatible with all implementations of the SUNLINSOL and NVECTOR modules. Specifically, IDASPILS requires use of an *iterative* SUNLINSOL object. Additional compatibility limitations for each SUNLINSOL object (i.e. required NVECTOR routines) are described in Chapter 9.

6.2.7   Initial condition calculation functions for backward problem

IDAS provides support for calculation of consistent initial conditions for certain backward index-one problems of semi-implicit form through the functions `IDACalcICB` and `IDACalcICBS`. Calling them is optional. It is only necessary when the initial conditions do not satisfy the adjoint system.

The above functions provide the same functionality for backward problems as `IDACalcIC` with parameter `icopt = IDA_YA_YDP_INIT` provides for forward problems (see §4.5.4): compute the algebraic components of yB and differential components of $\dot{y}B$, given the differential components of yB. They require that the `IDASetIdB` was previously called to specify the differential and algebraic components.

Both functions require forward solutions at the final time `tB0`. `IDACalcICBS` also needs forward sensitivities at the final time `tB0`.

IDACalcICB

Call   `flag = IDACalcICB(ida_mem, which, tBout1, N_Vector yfin, N_Vector ypfin);`

Description   The function `IDACalcICB` corrects the initial values `yB0` and `ypB0` at time `tB0` for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `which`   (`int`) is the identifier of the backward problem.

            `tBout1` (`realtype`) is the first value of t at which a solution will be requested (from `IDASolveB`). This value is needed here only to determine the direction of integration and rough scale in the independent variable t.

            `yfin`   (`N_Vector`) the forward solution at the final time `tB0`.

            `ypfin`   (`N_Vector`) the forward solution derivative at the final time `tB0`.

Return value   The return value `flag` (of type `int`) can be any that is returned by `IDACalcIC` (see §4.5.4). However `IDACalcICB` can also return one of the following:

         IDA_NO_ADJ   `IDAAdjInit` has not been previously called.

         IDA_ILL_INPUT Parameter `which` represented an invalid identifier.

Notes   All failure return values are negative and therefore a test `flag < 0` will trap all `IDACalcICB` failures.

           Note that `IDACalcICB` will correct the values of $yB(tB_0)$ and $\dot{y}B(tB_0)$ which were specified in the previous call to `IDAInitB` or `IDAReInitB`. To obtain the corrected values, call `IDAGetconsistentICB` (see §6.2.10.2).

In the case where the backward problem also depends on the forward sensitivities, user must call the following function to correct the initial conditions:

IDACalcICBS

Call	`flag = IDACalcICBS(ida_mem, which, tBout1, N_Vector yfin, N_Vector ypfin,` `                     N_Vector ySfin, N_Vector ypSfin);`
Description	The function `IDACalcICBS` corrects the initial values `yB0` and `ypB0` at time `tB0` for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `which`   (`int`) is the identifier of the backward problem.

            `tBout1` (`realtype`) is the first value of t at which a solution will be requested (from `IDASolveB`).This value is needed here only to determine the direction of integration and rough scale in the independent variable t.

            `yfin`   (`N_Vector`) the forward solution at the final time `tB0`.

            `ypfin`   (`N_Vector`) the forward solution derivative at the final time `tB0`.

            `ySfin`   (`N_Vector *`) a pointer to an array of `Ns` vectors containing the sensitivities of the forward solution at the final time `tB0`.

            `ypSfin` (`N_Vector *`) a pointer to an array of `Ns` vectors containing the derivatives of the forward solution sensitivities at the final time `tB0`.

Return value   The return value `flag` (of type `int`) can be any that is returned by `IDACalcIC` (see §4.5.4). However `IDACalcICBS` can also return one of the following:

            `IDA_NO_ADJ`   `IDAAdjInit` has not been previously called.

            `IDA_ILL_INPUT` Parameter `which` represented an invalid identifier, sensitivities were not active during forward integration, or `IDAInitBS` (or `IDAReInitBS`) has not been previously called.

Notes   All failure return values are negative and therefore a test `flag < 0` will trap all `IDACalcICBS` failures.

            Note that `IDACalcICBS` will correct the values of $yB(tB_0)$ and $\dot{y}B(tB_0)$ which were specified in the previous call to `IDAInitBS` or `IDAReInitBS`. To obtain the corrected values, call `IDAGetConsistentICB` (see §6.2.10.2).

6.2.8   Backward integration function

The function `IDASolveB` performs the integration of the backward problem. It is essentially a wrapper for the IDAS main integration function `IDASolve` and, in the case in which checkpoints were needed, it evolves the solution of the backward problem through a sequence of forward-backward integration pairs between consecutive checkpoints. In each pair, the first run integrates the original IVP forward in time and stores interpolation data; the second run integrates the backward problem backward in time and performs the required interpolation to provide the solution of the IVP to the backward problem.

     The function `IDASolveB` does not return the solution `yB` itself. To obtain that, call the function `IDAGetB`, which is also described below.

     The `IDASolveB` function does not support rootfinding, unlike `IDASoveF`, which supports the finding of roots of functions of $(t, y, \dot{y})$. If rootfinding was performed by `IDASolveF`, then for the sake of efficiency, it should be disabled for `IDASolveB` by first calling `IDARootInit` with `nrtfn = 0`.

     The call to `IDASolveB` has the form

IDASolveB

Call	`flag = IDASolveB(ida_mem, tBout, itaskB);`
Description	The function `IDASolveB` integrates the backward DAE problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory returned by `IDACreate`.

            `tBout`   (`realtype`) the next time at which a computed solution is desired.

itaskB   (int) a flag indicating the job of the solver for the next step. The `IDA_NORMAL` task is to have the solver take internal steps until it has reached or just passed the user-specified value `tBout`. The solver then interpolates in order to return an approximate value of $yB(\texttt{tBout})$. The `IDA_ONE_STEP` option tells the solver to take just one internal step in the direction of `tBout` and return.

Return value   The return value `flag` (of type `int`) will be one of the following. For more details see §4.5.6.

`IDA_SUCCESS`	`IDASolveB` succeeded.
`IDA_MEM_NULL`	The `ida_mem` was NULL.
`IDA_NO_ADJ`	The function `IDAAdjInit` has not been previously called.
`IDA_NO_BCK`	No backward problem has been added to the list of backward problems by a call to `IDACreateB`
`IDA_NO_FWD`	The function `IDASolveF` has not been previously called.
`IDA_ILL_INPUT`	One of the inputs to `IDASolveB` is illegal.
`IDA_BAD_ITASK`	The `itaskB` argument has an illegal value.
`IDA_TOO_MUCH_WORK`	The solver took `mxstep` internal steps but could not reach `tBout`.
`IDA_TOO_MUCH_ACC`	The solver could not satisfy the accuracy demanded by the user for some internal step.
`IDA_ERR_FAILURE`	Error test failures occurred too many times during one internal time step.
`IDA_CONV_FAILURE`	Convergence test failures occurred too many times during one internal time step.
`IDA_LSETUP_FAIL`	The linear solver's setup function failed in an unrecoverable manner.
`IDA_SOLVE_FAIL`	The linear solver's solve function failed in an unrecoverable manner.
`IDA_BCKMEM_NULL`	The IDAS memory for the backward problem was not created with a call to `IDACreateB`.
`IDA_BAD_TBOUT`	The desired output time `tBout` is outside the interval over which the forward problem was solved.
`IDA_REIFWD_FAIL`	Reinitialization of the forward problem failed at the first checkpoint (corresponding to the initial time of the forward problem).
`IDA_FWD_FAIL`	An error occurred during the integration of the forward problem.

Notes   All failure return values are negative and therefore a test `flag`< 0 will trap all `IDASolveB` failures.

In the case of multiple checkpoints and multiple backward problems, a given call to `IDASolveB` in `IDA_ONE_STEP` mode may not advance every problem one step, depending on the relative locations of the current times reached. But repeated calls will eventually advance all problems to `tBout`.

To obtain the solution `yB` to the backward problem, call the function `IDAGetB` as follows:

`IDAGetB`

Call   `flag = IDAGetB(ida_mem, which, &tret, yB, ypB);`

Description The function `IDAGetB` provides the solution `yB` of the backward DAE problem.

Arguments   `ida_mem` (void *) pointer to the IDAS memory returned by `IDACreate`.
        `which`   (int) the identifier of the backward problem.
        `tret`   (realtype) the time reached by the solver (output).
        `yB`   (N_Vector) the backward solution at time `tret`.
        `ypB`   (N_Vector) the backward solution derivative at time `tret`.

Return value The return value `flag` (of type `int`) will be one of the following.

> IDA_SUCCESS   IDAGetB was successful.
>
> IDA_MEM_NULL   ida_mem is NULL.
>
> IDA_NO_ADJ   The function IDAAdjInit has not been previously called.
>
> IDA_ILL_INPUT The parameter which is an invalid identifier.

Notes   The user must allocate space for `yB` and `ypB`.

6.2.9 Optional input functions for the backward problem

6.2.9.1 Main solver optional input functions

The adjoint module in IDAS provides wrappers for most of the optional input functions defined in
§4.5.7.1. The only difference is that the user must specify the identifier `which` of the backward
problem within the list managed by IDAS.

   The optional input functions defined for the backward problem are:

```
flag = IDASetUserDataB(ida_mem, which, user_dataB);
flag = IDASetMaxOrdB(ida_mem, which, maxordB);
flag = IDASetMaxNumStepsB(ida_mem, which, mxstepsB);
flag = IDASetInitStepB(ida_mem, which, hinB)
flag = IDASetMaxStepB(ida_mem, which, hmaxB);
flag = IDASetSuppressAlgB(ida_mem, which, suppressalgB);
flag = IDASetIdB(ida_mem, which, idB);
flag = IDASetConstraintsB(ida_mem, which, constraintsB);
```

Their return value `flag` (of type `int`) can have any of the return values of their counterparts, but it
can also be `IDA_NO_ADJ` if `IDAAdjInit` has not been called, or `IDA_ILL_INPUT` if `which` was an invalid
identifier.

6.2.9.2 Direct linear solver interface optional input functions

If using a direct linear solver interface for the Jacobian of the backward problem, the linear solver
will need to be attached to the memory structure through a call to `IDADlsSetLinearSolverB`.
The Jacobian evaluation function can be attached through a call to either `IDADlsSetJacFnB` or
IDAIDADlsSetJacFnBS, with the second used when the backward problem depends on the forwrad
sensitivities.

`IDADlsSetJacFnB`

Call   `flag = IDADlsSetJacFnB(ida_mem, which, jacB);`

Description   The function `IDADlsSetJacFnB` specifies the Jacobian approximation function to be
              used for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

              `which`   (`int`) represents the identifier of the backward problem.

              `jacB`   (`IDADlsJacFnB`) user-defined Jacobian approximation function.

Return value The return value `flag` (of type `int`) is one of

> IDADLS_SUCCESS IDADlsSetDlsJacFnB succeeded.
>
> IDADLS_MEM_NULL The ida_mem was NULL.
>
> IDADLS_NO_ADJ The function IDAAdjInit has not been previously called.
>
> IDADLS_LMEM_NULL The linear solver has not been initialized with a call to IDADlsSetLinearSolverB.
>
> IDADLS_ILL_INPUT The parameter which represented an invalid identifier.

Notes   The function type `IDADlsJacFnB` is described in §6.3.5.

IDADlsSetJacFnBS

Call   `flag = IDADlsSetJacFnBS(ida_mem, which, jacBS);`

Description The function `IDADlsSetJacFnB` specifies the Jacobian approximation function to be used for the backward problem in the case where the backward problem depends on the forward sensitivities.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

`which`   (`int`) represents the identifier of the backward problem.

`jacBS`   (`IDADlsJacFnBS`) user-defined Jacobian approximation function.

Return value The return value `flag` (of type `int`) is one of

`IDADLS_SUCCESS` `IDADlsSetDlsJacFnB` succeeded.

`IDADLS_MEM_NULL` The `ida_mem` was NULL.

`IDADLS_NO_ADJ` The function `IDAAdjInit` has not been previously called.

`IDADLS_LMEM_NULL` The linear solver has not been initialized with a call to `IDADlsSetLinearSolverB`.

`IDADLS_ILL_INPUT` The parameter `which` represented an invalid identifier.

Notes   The function type `IDADlsJacFnBS` is described in §6.3.5.

6.2.9.3   SPILS linear solvers

Optional inputs for the IDASPILS linear solver module can be set for the backward problem through the following functions:

IDASpilsSetPreconditionerB

Call   `flag = IDASpilsSetPreconditionerB(ida_mem, which, psetupB, psolveB);`

Description The function `IDASpilsSetPrecSolveFnB` specifies the preconditioner setup and solve functions for the backward integration.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

`which`   (`int`) the identifier of the backward problem.

`psetupB` (`IDASpilsPrecSetupFnB`) user-defined preconditioner setup function.

`psolveB` (`IDASpilsPrecSolveFnB`) user-defined preconditioner solve function.

Return value The return value `flag` (of type `int`) is one of:

`IDASPILS_SUCCESS`   The optional value has been successfully set.

`IDASPILS_MEM_NULL`   The `ida_mem` memory block pointer was NULL.

`IDASPILS_LMEM_NULL` The IDASPILS linear solver has not been initialized.

`IDASPILS_NO_ADJ`   The function `IDAAdjInit` has not been previously called.

`IDASPILS_ILL_INPUT` The parameter `which` represented an invalid identifier.

Notes   The function types `IDASpilsPrecSolveFnB` and `IDASpilsPrecSetupFnB` are described in §6.3.8 and §6.3.9, respectively. The `psetupB` argument may be NULL if no setup operation is involved in the preconditioner.

IDASpilsSetPreconditionerBS

Call   `flag = IDASpilsSetPreconditionerBS(ida_mem, which, psetupBS, psolveBS);`

Description The function `IDASpilsSetPrecSolveFnBS` specifies the preconditioner setup and solve functions for the backward integration, in the case where the backward problem depends on the forward sensitivities.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

`which`   (`int`) the identifier of the backward problem.

psetupBS (IDASpilsPrecSetupFnBS) user-defined preconditioner setup function.

psolveBS (IDASpilsPrecSolveFnBS) user-defined preconditioner solve function.

Return value The return value flag (of type int) is one of:

IDASPILS_SUCCESS   The optional value has been successfully set.

IDASPILS_MEM_NULL   The ida_mem memory block pointer was NULL.

IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASPILS_NO_ADJ   The function IDAAdjInit has not been previously called.

IDASPILS_ILL_INPUT The parameter which represented an invalid identifier.

Notes   The function types IDASpilsPrecSolveFnBS and IDASpilsPrecSetupFnBS are described in §6.3.8 and §6.3.9, respectively. The psetupBS argument may be NULL if no setup operation is involved in the preconditioner.

IDASpilsSetJacTimesB

Call   flag = IDASpilsSetJacTimesB(ida_mem, which, jsetupB, jtimesB);

Description The function IDASpilsSetJacTimesB specifies the Jacobian-vector setup and product functions to be used.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

which   (int) the identifier of the backward problem.

jtsetupB (IDASpilsJacTimesSetupFnB) user-defined function to set up the Jacobian-vector product. Pass NULL if no setup is necessary.

jtimesB (IDASpilsJacTimesVecFnB) user-defined Jacobian-vector product function.

Return value The return value flag (of type int) is one of:

IDASPILS_SUCCESS   The optional value has been successfully set.

IDASPILS_MEM_NULL   The ida_mem memory block pointer was NULL.

IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASPILS_NO_ADJ   The function IDAAdjInit has not been previously called.

IDASPILS_ILL_INPUT The parameter which represented an invalid identifier.

Notes   The function types IDASpilsJacTimesVecFnB and IDASpilsJacTimesSetupFnB are described in §6.3.6.

IDASpilsSetJacTimesBS

Call   flag = IDASpilsSetJacTimesBS(ida_mem, which, jsetupBS, jtimesBS);

Description The function IDASpilsSetJacTimesBS specifies the Jacobian-vector product setup and evaluation functions to be used, in the case where the backward problem depends on the forward sensitivities.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

which   (int) the identifier of the backward problem.

jtsetupBS (IDASpilsJacTimesSetupFnBS) user-defined function to set up the Jacobian-vector product. Pass NULL if no setup is necessary.

jtimesBS (IDASpilsJacTimesVecFnBS) user-defined Jacobian-vector product function.

Return value The return value flag (of type int) is one of:

IDASPILS_SUCCESS   The optional value has been successfully set.

IDASPILS_MEM_NULL   The ida_mem memory block pointer was NULL.

IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

IDASPILS_NO_ADJ   The function IDAAdjInit has not been previously called.

IDASPILS_ILL_INPUT The parameter `which` represented an invalid identifier.

Notes   The function types `IDASpilsJacTimesVecFnBS` and `IDASpilsJacTimesSetupFnBS` are described in §6.3.6.

| IDASpilsSetEpsLinB |

Call   `flag = IDASpilsSetEpsLinB(ida_mem, which, eplifacB);`

Description   The function `IDASpilsSetEpsLinB` specifies the factor by which the Krylov linear solver's convergence test constant is reduced from the Newton iteration test constant. (See §2.1). This routine can be used in both the cases wherethe backward problem does and does not depend on the forward sensitvities.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

   `which`   (`int`) the identifier of the backward problem.

   `eplifacB` (`realtype`) linear convergence safety factor ($>= 0.0$).

Return value   The return value `flag` (of type `int`) is one of

   IDASPILS_SUCCESS   The optional value has been successfully set.

   IDASPILS_MEM_NULL   The `ida_mem` pointer is NULL.

   IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

   IDASPILS_NO_ADJ   The function `IDAAdjInit` has not been previously called.

   IDASPILS_ILL_INPUT   The value of `eplifacB` is negative.

   IDASPILS_ILL_INPUT   The parameter `which` represented an invalid identifier.

Notes   The default value is 0.05.

   Passing a value `eplifacB`= 0.0 also indicates using the default value.

| IDASpilsSetIncrementFactorB |

Call   `flag = IDASpilsSetIncrementFactorB(ida_mem, which, dqincfacB);`

Description   The function `IDASpilsSetIncrementFactorB` specifies the factor in the increments used in the difference quotient approximations to matrix-vector products for the backward problem.

   This routine can be used in both the cases wherethe backward problem does and does not depend on the forward sensitvities.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

   `which`   (`int`) the identifier of the backward problem.

   `dqincfacB` (`realtype`) difference quotient approximation factor.

Return value   The return value `flag` (of type `int`) is one of

   IDASPILS_SUCCESS   The optional value has been successfully set.

   IDASPILS_MEM_NULL   The `ida_mem` pointer is NULL.

   IDASPILS_LMEM_NULL The IDASPILS linear solver has not been initialized.

   IDASPILS_NO_ADJ   The function `IDAAdjInit` has not been previously called.

   IDASPILS_ILL_INPUT   The value of `eplifacB` is negative.

   IDASPILS_ILL_INPUT   The parameter `which` represented an invalid identifier.

Notes   The default value is 1.0.

   Passing a value `dqincfacB`= 0.0 also indicates using the default value.

6.2.10   Optional output functions for the backward problem

6.2.10.1   Main solver optional output functions

The user of the adjoint module in IDAS has access to any of the optional output functions described in §4.5.9, both for the main solver and for the linear solver modules. The first argument of these `IDAGet*` and `IDA*Get*` functions is the pointer to the IDAS memory block for the backward problem. In order to call any of these functions, the user must first call the following function to obtain this pointer:

`IDAGetAdjIDABmem`

Call   `ida_memB = IDAGetAdjIDABmem(ida_mem, which);`

Description   The function `IDAGetAdjIDABmem` returns a pointer to the IDAS memory block for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block created by `IDACreate`.
                `which`   (`int`) the identifier of the backward problem.

Return value   The return value, `ida_memB` (of type `void *`), is a pointer to the IDAS memory for the backward problem.

Notes   The user should not modify `ida_memB` in any way.

                Optional output calls should pass `ida_memB` as the first argument; thus, for example, to get the number of integration steps: `flag = IDAGetNumSteps(idas_memB,&nsteps)`.

To get values of the *forward* solution during a backward integration, use the following function. The input value of `t` would typically be equal to that at which the backward solution has just been obtained with `IDAGetB`. In any case, it must be within the last checkpoint interval used by `IDASolveB`.

`IDAGetAdjY`

Call   `flag = IDAGetAdjY(ida_mem, t, y, yp);`

Description   The function `IDAGetAdjY` returns the interpolated value of the forward solution y and its derivative during a backward integration.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block created by `IDACreate`.
                `t`   (`realtype`) value of the independent variable at which y is desired (input).
                `y`   (`N_Vector`) forward solution $y(t)$.
                `yp`   (`N_Vector`) forward solution derivative $\dot{y}(t)$.

Return value   The return value `flag` (of type `int`) is one of:

                `IDA_SUCCESS`   `IDAGetAdjY` was successful.

                `IDA_MEM_NULL`   `ida_mem` was `NULL`.

                `IDA_GETY_BADT` The value of `t` was outside the current checkpoint interval.

Notes   The user must allocate space for `y` and `yp`.

`IDAGetAdjCheckPointsInfo`

Call   `flag = IDAGetAdjCheckPointsInfo(ida_mem, IDAadjCheckPointRec *ckpnt);`

Description   The function `IDAGetAdjCheckPointsInfo` loads an array of `ncheck+1` records of type `IDAadjCheckPointRec`. The user must allocate space for the array `ckpnt`.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block created by `IDACreate`.
                `ckpnt`   (`IDAadjCheckPointRec *`) array of `ncheck+1` checkpoint records, each of type `IDAadjCheckPointRec`.

Return value   The return value is `IDA_SUCCESS` if successful, or `IDA_MEM_NULL` if `ida_mem` is `NULL`, or `IDA_NO_ADJ` if ASA was not initialized.

Notes   The members of each record `ckpnt[i]` are:

- : `ckpnt[i].my_addr` (`void *`) address of current checkpoint in `ida_mem->ida_adj_mem`
- : `ckpnt[i].next_addr` (`void *`) address of next checkpoint
- : `ckpnt[i].t0` (`realtype`) start of checkpoint interval
- : `ckpnt[i].t1` (`realtype`) end of checkpoint interval
- : `ckpnt[i].nstep` (`long int`) step counter at ckeckpoint `t0`
- : `ckpnt[i].order` (`int`) method order at checkpoint `t0`
- : `ckpnt[i].step` (`realtype`) step size at checkpoint `t0`

6.2.10.2   Initial condition calculation optional output function

`IDAGetConsistentICB`

Call   `flag = IDAGetConsistentICB(ida_mem, which, yB0_mod, ypB0_mod);`

Description   The function `IDAGetConsistentICB` returns the corrected initial conditions for backward problem calculated by `IDACalcICB`.

Arguments   `ida_mem`   (`void *`) pointer to the IDAS memory block.

              `which`   is the identifier of the backward problem.

              `yB0_mod`   (`N_Vector`) consistent initial vector.

              `ypB0_mod` (`N_Vector`) consistent initial derivative vector.

Return value The return value `flag` (of type `int`) is one of

              `IDA_SUCCESS`   The optional output value has been successfully set.

              `IDA_MEM_NULL`   The `ida_mem` pointer is `NULL`.

              `IDA_NO_ADJ`   `IDAAdjInit` has not been previously called.

              `IDA_ILL_INPUT` Parameter `which` did not refer a valid backward problem identifier.

Notes   If the consistent solution vector or consistent derivative vector is not desired, pass `NULL` for the corresponding argument.

              The user must allocate space for `yB0_mod` and `ypB0_mod` (if not `NULL`).

6.2.11   Backward integration of quadrature equations

Not only the backward problem but also the backward quadrature equations may or may not depend on the forward sensitivities. Accordingly, one of the `IDAQuadInitB` or `IDAQuadInitBS` should be used to allocate internal memory and to initialize backward quadratures. For any other operation (extraction, optional input/output, reinitialization, deallocation), the same function is called regardless of whether or not the quadratures are sensitivity-dependent.

6.2.11.1   Backward quadrature initialization functions

The function `IDAQuadInitB` initializes and allocates memory for the backward integration of quadrature equations that do not depende on forward sensititvities. It has the following form:

`IDAQuadInitB`

Call   `flag = IDAQuadInitB(ida_mem, which, rhsQB, yQB0);`

Description   The function `IDAQuadInitB` provides required problem specifications, allocates internal memory, and initializes backward quadrature integration.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

which   (int) the identifier of the backward problem.

rhsQB   (IDAQuadRhsFnB) is the C function which computes fQB, the residual of the backward quadrature equations. This function has the form rhsQB(t, y, yp, yB, ypB, rhsvalBQ, user_dataB) (see §6.3.3).

yQB0   (N_Vector) is the value of the quadrature variables at tB0.

Return value The return value flag (of type int) will be one of the following:

IDA_SUCCESS   The call to IDAQuadInitB was successful.

IDA_MEM_NULL   ida_mem was NULL.

IDA_NO_ADJ   The function IDAAdjInit has not been previously called.

IDA_MEM_FAIL   A memory allocation request has failed.

IDA_ILL_INPUT The parameter which is an invalid identifier.

The function IDAQuadInitBS initializes and allocates memory for the backward integration of quadrature equations that depend on the forward sensitivities.

IDAQuadInitBS

Call   flag = IDAQuadInitBS(ida_mem, which, rhsQBS, yQBS0);

Description   The function IDAQuadInitBS provides required problem specifications, allocates internal memory, and initializes backward quadrature integration.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

which   (int) the identifier of the backward problem.

rhsQBS (IDAQuadRhsFnBS) is the C function which computes $fQBS$, the residual of the backward quadrature equations. This function has the form rhsQBS(t, y, yp, yS, ypS, yB, ypB, rhsvalBQS, user_dataB) (see §6.3.4).

yQBS0   (N_Vector) is the value of the sensitivity-dependent quadrature variables at tB0.

Return value The return value flag (of type int) will be one of the following:

IDA_SUCCESS   The call to IDAQuadInitBS was successful.

IDA_MEM_NULL   ida_mem was NULL.

IDA_NO_ADJ   The function IDAAdjInit has not been previously called.

IDA_MEM_FAIL   A memory allocation request has failed.

IDA_ILL_INPUT The parameter which is an invalid identifier.

The integration of quadrature equations during the backward phase can be re-initialized by calling the following function. Before calling IDAQuadReInitB for a new backward problem, call any desired solution extraction functions IDAGet** associated with the previous backward problem.

IDAQuadReInitB

Call   flag = IDAQuadReInitB(ida_mem, which, yQB0);

Description   The function IDAQuadReInitB re-initializes the backward quadrature integration.

Arguments   ida_mem (void *) pointer to the IDAS memory block.

which   (int) the identifier of the backward problem.

yQB0   (N_Vector) is the value of the quadrature variables at tB0.

Return value The return value flag (of type int) will be one of the following:

IDA_SUCCESS   The call to IDAQuadReInitB was successful.

IDA_MEM_NULL   ida_mem was NULL.

IDA_NO_ADJ   The function IDAAdjInit has not been previously called.

IDA_MEM_FAIL   A memory allocation request has failed.

IDA_NO_QUAD   Quadrature integration was not activated through a previous call to
               IDAQuadInitB.

IDA_ILL_INPUT The parameter `which` is an invalid identifier.

Notes   `IDAQuadReInitB` can be used after a call to either `IDAQuadInitB` or `IDAQuadInitBS`.

6.2.11.2 Backward quadrature extraction function

To extract the values of the quadrature variables at the last return time of `IDASolveB`, IDAS provides
a wrapper for the function `IDAGetQuad` (see §4.7.3). The call to this function has the form

---

| IDAGetQuad B |

Call   `flag = IDAGetQuadB(ida_mem, which, &tret, yQB);`

Description   The function `IDAGetQuadB` returns the quadrature solution vector after a successful
              return from `IDASolveB`.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory.

              `tret`   (`realtype`) the time reached by the solver (output).

              `yQB`   (`N_Vector`) the computed quadrature vector.

Return value

Notes   T

he user must allocate space for `yQB`. The return value `flag` of `IDAGetQuadB` is one of:

IDA_SUCCESS   `IDAGetQuadB` was successful.

IDA_MEM_NULL   `ida_mem` is NULL.

IDA_NO_ADJ   The function `IDAAdjInit` has not been previously called.

IDA_NO_QUAD   Quadrature integration was not initialized.

IDA_BAD_DKY   `yQB` was NULL.

IDA_ILL_INPUT The parameter `which` is an invalid identifier.

6.2.11.3 Optional input/output functions for backward quadrature integration

Optional values controlling the backward integration of quadrature equations can be changed from
their default values through calls to one of the following functions which are wrappers for the corre-
sponding optional input functions defined in §4.7.4. The user must specify the identifier `which` of the
backward problem for which the optional values are specified.

```
flag = IDASetQuadErrConB(ida_mem, which, errconQ);
flag = IDAQuadSStolerancesB(ida_mem, which, reltolQ, abstolQ);
flag = IDAQuadSVtolerancesB(ida_mem, which, reltolQ, abstolQ);
```

Their return value `flag` (of type `int`) can have any of the return values of its counterparts, but it
can also be `IDA_NO_ADJ` if the function `IDAAdjInit` has not been previously called or `IDA_ILL_INPUT`
if the parameter `which` was an invalid identifier.

Access to optional outputs related to backward quadrature integration can be obtained by calling
the corresponding `IDAGetQuad*` functions (see §4.7.5). A pointer `ida_memB` to the IDAS memory block
for the backward problem, required as the first argument of these functions, can be obtained through
a call to the functions `IDAGetAdjIDABmem` (see §6.2.10).

6.3   User-supplied functions for adjoint sensitivity analysis

In addition to the required DAE residual function and any optional functions for the forward problem, when using the adjoint sensitivity module in IDAS, the user must supply one function defining the backward problem DAE and, optionally, functions to supply Jacobian-related information and one or two functions that define the preconditioner (if one of the IDASPILS solvers is selected) for the backward problem. Type definitions for all these user-supplied functions are given below.

6.3.1   DAE residual for the backward problem

The user must provide a `resB` function of type `IDAResFnB` defined as follows:

| `IDAResFnB` |

Definition	`typedef int (*IDAResFnB)(realtype t, N_Vector y, N_Vector yp,`
	`                          N_Vector yB, N_Vector ypB,`
	`                          N_Vector resvalB, void *user_dataB);`

Purpose	This function evaluates the residual of the backward problem DAE system. This could be (2.20) or (2.25).
Arguments	`t` is the current value of the independent variable.
	`y` is the current value of the forward solution vector.
	`yp` is the current value of the forward solution derivative vector.
	`yB` is the current value of the backward dependent variable vector.
	`ypB` is the current value of the backward dependent derivative vector.
	`resvalB` is the output vector containing the residual for the backward DAE problem.
	`user_dataB` is a pointer to user data, same as passed to `IDASetUserDataB`.
Return value	An `IDAResFnB` should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if an unrecoverabl failure occurred (in which case the integration stops and `IDASolveB` returns `IDA_RESFUNC_FAIL`).
Notes	Allocation of memory for `resvalB` is handled within IDAS.
	The `y`, `yp`, `yB`, `ypB`, and `resvalB` arguments are all of type `N_Vector`, but `yB`, `ypB`, and `resvalB` typically have different internal representations from `y` and `yp`. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their `N_Vector` arguments (see §7.1 and §7.2).
	The `user_dataB` pointer is passed to the user's `resB` function every time it is called and can be the same as the `user_data` pointer used for the forward problem.
	Before calling the user's `resB` function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the residual function which will halt the integration and `IDASolveB` will return `IDA_RESFUNC_FAIL`.

6.3.2   DAE residual for the backward problem depending on the forward sensitivities

The user must provide a `resBS` function of type `IDAResFnBS` defined as follows:

IDAResFnBS

Definition	`typedef int (*IDAResFnBS)(realtype t, N_Vector y, N_Vector yp,`
	`                          N_Vector *yS, N_Vector *ypS,`
	`                          N_Vector yB, N_Vector ypB,`
	`                          N_Vector resvalB, void *user_dataB);`

Purpose   This function evaluates the residual of the backward problem DAE system. This could be (2.20) or (2.25).

Arguments
`t`	is the current value of the independent variable.
`y`	is the current value of the forward solution vector.
`yp`	is the current value of the forward solution derivative vector.
`yS`	a pointer to an array of `Ns` vectors containing the sensitivities of the forward solution.
`ypS`	a pointer to an array of `Ns` vectors containing the derivatives of the forward sensitivities.
`yB`	is the current value of the backward dependent variable vector.
`ypB`	is the current value of the backward dependent derivative vector.
`resvalB`	is the output vector containing the residual for the backward DAE problem.
`user_dataB`	is a pointer to user data, same as passed to `IDASetUserDataB`.

Return value   An `IDAResFnBS` should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if an unrecoverable error occurred (in which case the integration stops and `IDASolveB` returns `IDA_RESFUNC_FAIL`).

Notes   Allocation of memory for `resvalB` is handled within IDAS.

The `y`, `yp`, `yB`, `ypB`, and `resvalB` arguments are all of type `N_Vector`, but `yB`, `ypB`, and `resvalB` typically have different internal representations from `y` and `yp`. Likewise for each `yS[i]` and `ypS[i]`. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with respect to their `N_Vector` arguments (see §7.1 and §7.2).

The `user_dataB` pointer is passed to the user's `resBS` function every time it is called and can be the same as the `user_data` pointer used for the forward problem.

Before calling the user's `resBS` function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the residual function which will halt the integration and `IDASolveB` will return `IDA_RESFUNC_FAIL`.

6.3.3 Quadrature right-hand side for the backward problem

The user must provide an `fQB` function of type `IDAQuadRhsFnB` defined by

---

IDAQuadRhsFnB

Definition	`typedef int (*IDAQuadRhsFnB)(realtype t, N_Vector y, N_Vector yp,`
	`                             N_Vector yB, N_Vector ypB,`
	`                             N_Vector rhsvalBQ, void *user_dataB);`

Purpose   This function computes the quadrature equation right-hand side for the backward problem.

Arguments
`t`	is the current value of the independent variable.
`y`	is the current value of the forward solution vector.

yp   is the current value of the forward solution derivative vector.

yB   is the current value of the backward dependent variable vector.

ypB   is the current value of the backward dependent derivative vector.

rhsvalBQ   is the output vector containing the residual for the backward quadrature
            equations.

user_dataB is a pointer to user data, same as passed to IDASetUserDataB.

Return value An IDAQuadRhsFnB should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns IDA_QRHSFUNC_FAIL).

Notes   Allocation of memory for rhsvalBQ is handled within IDAS.

The y, yp, yB, ypB, and rhsvalBQ arguments are all of type N_Vector, but they typically all have different internal representations. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with repsect to their N_Vector arguments (see §7.1 and §7.2).

The user_dataB pointer is passed to the user's fQB function every time it is called and can be the same as the user_data pointer used for the forward problem.

Before calling the user's fQB function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the quadrature right-hand side function which will halt the integration and IDASolveB will return IDA_QRHSFUNC_FAIL.

6.3.4 Sensitivity-dependent quadrature right-hand side for the backward problem

The user must provide an fQBS function of type IDAQuadRhsFnBS defined by

IDAQuadRhsFnBS

Definition   typedef int (*IDAQuadRhsFnBS)(realtype t, N_Vector y, N_Vector yp,
                                          N_Vector *yS, N_Vector *ypS,
                                          N_Vector yB, N_Vector ypB,
                                          N_Vector rhsvalBQS, void *user_dataB);

Purpose   This function computes the quadrature equation residual for the backward problem.

Arguments   t   is the current value of the independent variable.

             y   is the current value of the forward solution vector.

             yp   is the current value of the forward solution derivative vector.

             yS   a pointer to an array of Ns vectors containing the sensitivities of the forward
                        solution.

             ypS   a pointer to an array of Ns vectors containing the derivatives of the forward
                        sensitivities.

             yB   is the current value of the backward dependent variable vector.

             ypB   is the current value of the backward dependent derivative vector.

             rhsvalBQS is the output vector containing the residual for the backward quadrature
                        equations.

             user_dataB is a pointer to user data, same as passed to IDASetUserDataB.

Return value An `IDAQuadRhsFnBS` should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and `IDASolveB` returns `IDA_QRHSFUNC_FAIL`).

Notes Allocation of memory for `rhsvalBQS` is handled within IDAS.

The `y`, `yp`, `yB`, `ypB`, and `rhsvalBQS` arguments are all of type `N_Vector`, but they typically do not all have the same internal representations. Likewise for each `yS[i]` and `ypS[i]`. It is the user's responsibility to access the vector data consistently (including the use of the correct accessor macros from each NVECTOR implementation). For the sake of computational efficiency, the vector functions in the two NVECTOR implementations provided with IDAS do not perform any consistency checks with repsect to their `N_Vector` arguments (see §7.1 and §7.2).

The `user_dataB` pointer is passed to the user's `fQBS` function every time it is called and can be the same as the `user_data` pointer used for the forward problem.

Before calling the user's `fQBS` function, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the quadrature right-hand side function which will halt the integration and `IDASolveB` will return `IDA_QRHSFUNC_FAIL`.

6.3.5 Jacobian information for the backward problem (direct method Jacobian)

If the direct linear solver interface is used for the backward problem (i.e. `IDADlsSetLinearSolverB` is called in the step described in §6.1), the user may provide a function of type `IDADlsJacFnB` or `IDADlsJacFnBS` (see §6.2.9), defined as follows:

$\boxed{\text{IDADlsJacFnB}}$

Definition
```
typedef int (*IDADlsJacFnB)(realtype tt, realtype cjB,
                            N_Vector yy, N_Vector yp,
                            N_Vector yB, N_Vector ypB,
                            N_Vector resvalB,
                            SUNMatrix JacB, void *user_dataB,
                            N_Vector tmp1B, N_Vector tmp2B,
                            N_Vector tmp3B);
```

Purpose This function computes the Jacobian of the backward problem (or an approximation to it).

Arguments
`tt`	is the current value of the independent variable.
`cjB`	is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).
`yy`	is the current value of the forward solution vector.
`yp`	is the current value of the forward solution derivative vector.
`yB`	is the current value of the backward dependent variable vector.
`ypB`	is the current value of the backward dependent derivative vector.
`resvalB`	is the current value of the residual for the backward problem.
`JacB`	is the output approximate Jacobian matrix.
`user_dataB`	is a pointer to user data — the parameter passed to `IDASetUserDataB`.
`tmp1B`	
`tmp2B`	
`tmp3B`	are pointers to memory allocated for variables of type `N_Vector` which can be used by the `IDADlsJacFnB` function as temporary storage or work space.

Return value   An IDADlsJacFnB should return 0 if successful, a positive value if a recoverable error
               occurred (in which case IDAS will attempt to correct, while IDADLS sets last_flag to
               IDADLS_JACFUNC_RECVR), or a negative value if it failed unrecoverably (in which case the
               integration is halted, IDASolveB returns IDA_LSETUP_FAIL and IDADLS sets last_flag
               to IDADLS_JACFUNC_UNRECVR).

Notes   A user-supplied Jacobian function must load the matrix JacB with an approximation
               to the Jacobian matrix at the point (tt,yy,yB), where yy is the solution of the original
               IVP at time tt, and yB is the solution of the backward problem at the same time.
               Information regarding the structure of the specific SUNMATRIX structure (e.g. number
               of rows, upper/lower bandwidth, sparsity type) may be obtained through using the
               implementation-specific SUNMATRIX interface functions (see Chapter 8 for details). Only
               nonzero elements need to be loaded into JacB as this matrix is set to zero before the
               call to the Jacobian function.

               Before calling the user's IDADlsJacFnB, IDAS needs to evaluate (through interpolation)
               the values of the states from the forward integration. If an error occurs in the interpo-
               lation, IDAS triggers an unrecoverable failure in the Jacobian function which will halt
               the integration (IDASolveB returns IDA_LSETUP_FAIL and IDADLS sets last_flag to
               IDADLS_JACFUNC_UNRECVR).

| IDADlsJacFnBS |

Definition   ```
               typedef int (*IDADlsDenseJacFnBS)(realtype tt, realtype cjB,
                                                 N_Vector yy, N_Vector yp,
                                                 N_Vector *yS, N_Vector *ypS,
                                                 N_Vector yB, N_Vector ypB,
                                                 N_Vector resvalB,
                                                 SUNMatrix JacB, void *user_dataB,
                                                 N_Vector tmp1B, N_Vector tmp2B,
                                                 N_Vector tmp3B);
               ```

Purpose   This function computes the Jacobian of the backward problem (or an approximation to
               it), in the case where the backward problem depends on the forward sensitivities.

Arguments   tt   is the current value of the independent variable.
               cjB   is the scalar in the system Jacobian, proportional to the inverse of the step
                           size (α in Eq. (2.6)).
               yy   is the current value of the forward solution vector.
               yp   is the current value of the forward solution derivative vector.
               yS   a pointer to an array of Ns vectors containing the sensitivities of the forward
                           solution.
               ypS   a pointer to an array of Ns vectors containing the derivatives of the forward
                           solution sensitivities.
               yB   is the current value of the backward dependent variable vector.
               ypB   is the current value of the backward dependent derivative vector.
               resvalB   is the current value of the residual for the backward problem.
               JacB   is the output approximate Jacobian matrix.
               user_dataB is a pointer to user data — the parameter passed to IDASetUserDataB.
               tmp1B
               tmp2B
               tmp3B   are pointers to memory allocated for variables of type N_Vector which can
                           be used by IDADlsJacFnBS as temporary storage or work space.

Return value   An IDADlsJacFnBS should return 0 if successful, a positive value if a recoverable error
               occurred (in which case IDAS will attempt to correct, while IDADLS sets last_flag to

IDADLS_JACFUNC_RECVR), or a negative value if it failed unrecoverably (in which case the integration is halted, IDASolveB returns IDA_LSETUP_FAIL and IDADLS sets last_flag to IDADLS_JACFUNC_UNRECVR).

Notes   A user-supplied dense Jacobian function must load the matrix JacB with an approximation to the Jacobian matrix at the point (tt,yy,yS,yB), where yy is the solution of the original IVP at time tt, yS is the array of forward sensitivities at time tt, and yB is the solution of the backward problem at the same time. Information regarding the structure of the specific SUNMATRIX structure (e.g. number of rows, upper/lower bandwidth, sparsity type) may be obtained through using the implementation-specific SUNMATRIX interface functions (see Chapter 8 for details). Only nonzero elements need to be loaded into JacB as this matrix is set to zero before the call to the Jacobian function.

Before calling the user's IDADlsJacFnBS, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the Jacobian function which will halt the integration (IDASolveB returns IDA_LSETUP_FAIL and IDADENSE sets last_flag to IDADLS_JACFUNC_UNRECVR).

6.3.6   Jacobian information for the backward problem (matrix-vector product)

If the IDASPILS solver interface is selected for the backward problem (i.e., IDASpilsSetLinearSolverB is called in the steps described in §6.1), the user may provide a function of type IDASpilsJacTimesVecFnB or IDASpilsJacTimesVecFnBS in the following form, to compute matrix-vector products Jv. If such a function is not supplied, the default is a difference quotient approximation to these products.

IDASpilsJacTimesVecFnB

Definition
```
typedef int (*IDASpilsJacTimesVecFnB)(realtype t,
                                      N_Vector yy, N_Vector yp,
                                      N_Vector yB, N_Vector ypB,
                                      N_Vector resvalB,
                                      N_Vector vB, N_Vector JvB,
                                      realtype cjB, void *user_dataB,
                                      N_Vector tmp1B, N_Vector tmp2B);
```

Purpose   This function computes the action of the backward problem Jacobian JB on a given vector vB.

Arguments   t   is the current value of the independent variable.

             yy   is the current value of the forward solution vector.

             yp   is the current value of the forward solution derivative vector.

             yB   is the current value of the backward dependent variable vector.

             ypB   is the current value of the backward dependent derivative vector.

             resvalB   is the current value of the residual for the backward problem.

             vB   is the vector by which the Jacobian must be multiplied.

             JvB   is the computed output vector, JB*vB.

             cjB   is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).

             user_dataB   is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

             tmp1B

             tmp2B   are pointers to memory allocated for variables of type N_Vector which can be used by IDASpilsJacTimesVecFnB as temporary storage or work space.

Return value The return value of a function of type `IDASpilsJtimesVecFnB` should be 0 if successful
or nonzero if an error was encountered, in which case the integration is halted.

Notes   A user-supplied Jacobian-vector product function must load the vector `JvB` with the
product of the Jacobian of the backward problem at the point (`t`, `y`, `yB`) and the vector
`vB`. Here, `y` is the solution of the original IVP at time `t` and `yB` is the solution of the
backward problem at the same time. The rest of the arguments are equivalent to those
passed to a function of type `IDASpilsJacTimesVecFn` (see §4.6.6). If the backward
problem is the adjoint of $\dot{y} = f(t, y)$, then this function is to compute $-(\partial f/\partial y)^T v_B$.

`IDASpilsJacTimesVecFnBS`

Definition `typedef int (*IDASpilsJacTimesVecFnBS)(realtype t,`
`                                        N_Vector yy, N_Vector yp,`
`                                        N_Vector *yyS, N_Vector *ypS,`
`                                        N_Vector yB, N_Vector ypB,`
`                                        N_Vector resvalB,`
`                                        N_Vector vB, N_Vector JvB,`
`                                        realtype cjB, void *user_dataB,`
`                                        N_Vector tmp1B, N_Vector tmp2B);`

Purpose   This function computes the action of the backward problem Jacobian `JB` on a given
vector `vB`, in the case where the backward problem depends on the forward sensitivities.

Arguments   `t`   is the current value of the independent variable.

            `yy`   is the current value of the forward solution vector.

            `yp`   is the current value of the forward solution derivative vector.

            `yyS`   a pointer to an array of `Ns` vectors containing the sensitivities of the forward
                         solution.

            `ypS`   a pointer to an array of `Ns` vectors containing the derivatives of the forward
                         sensitivities.

            `yB`   is the current value of the backward dependent variable vector.

            `ypB`   is the current value of the backward dependent derivative vector.

            `resvalB`   is the current value of the residual for the backward problem.

            `vB`   is the vector by which the Jacobian must be multiplied.

            `JvB`   is the computed output vector, `JB*vB`.

            `cjB`   is the scalar in the system Jacobian, proportional to the inverse of the step
                         size (α in Eq. (2.6)).

            `user_dataB` is a pointer to user data — the same as the `user_dataB` parameter passed
                         to `IDASetUserDataB`.

            `tmp1B`

            `tmp2B`   are pointers to memory allocated for variables of type `N_Vector` which
                         can be used by `IDASpilsJacTimesVecFnBS` as temporary storage or work
                         space.

Return value The return value of a function of type `IDASpilsJtimesVecFnBS` should be 0 if successful
or nonzero if an error was encountered, in which case the integration is halted.

Notes   A user-supplied Jacobian-vector product function must load the vector `JvB` with the
product of the Jacobian of the backward problem at the point (`t`, `y`, `yB`) and the vector
`vB`. Here, `y` is the solution of the original IVP at time `t` and `yB` is the solution of the
backward problem at the same time. The rest of the arguments are equivalent to those
passed to a function of type `IDASpilsJacTimesVecFn` (see §4.6.6).

6.3.7 Jacobian information for the backward problem (matrix-vector setup)

If the user's Jacobian-times-vector requires that any Jacobian-related data be preprocessed or evaluated, then this needs to be done in a user-supplied function of type IDASpilsJacTimesSetupFnB, defined as follows:

IDASpilsJacTimesSetupFnB

Definition	`typedef int (*IDASpilsJacTimesSetupFnB)(realtype tt,`
	`                                         N_Vector yy, N_Vector yp,`
	`                                         N_Vector yB, N_Vector ypB,`
	`                                         N_Vector resvalB,`
	`                                         realtype cjB, void *user_dataB);`

Purpose	This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-times-vector routine for the backward problem.

Arguments
tt   is the current value of the independent variable.
yy   is the current value of the dependent variable vector, $y(t)$.
yp   is the current value of $\dot{y}(t)$.
yB   is the current value of the backward dependent variable vector.
ypB   is the current value of the backward dependent derivative vector.
resvalB   is the current value of the residual for the backward problem.
cjB   is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).
user_dataB is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

Return value The value returned by the Jacobian-vector setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes Each call to the Jacobian-vector setup function is preceded by a call to the backward problem residual user function with the same (t,y, yp, yB, ypB) arguments. Thus, the setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

If the user's IDASpilsJacTimesVecFnB function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to ida_mem to user_dataB and then use the IDAGet* functions described in §4.5.9.2. The unit roundoff can be accessed as UNIT_ROUNDOFF defined in sundials_types.h.

IDASpilsJacTimesSetupFnBS

Definition	`typedef int (*IDASpilsJacTimesSetupFnBS)(realtype tt,`
	`                                          N_Vector yy, N_Vector yp,`
	`                                          N_Vector *yyS, N_Vector *ypS,`
	`                                          N_Vector yB, N_Vector ypB,`
	`                                          N_Vector resvalB,`
	`                                          realtype cjB, void *user_dataB);`

Purpose	This function preprocesses and/or evaluates Jacobian data needed by the Jacobian-times-vector routine for the backward problem, in the case that the backward problem depends on the forward sensitivities.

Arguments
tt   is the current value of the independent variable.
yy   is the current value of the dependent variable vector, $y(t)$.

yp	is the current value of $\dot{y}(t)$.
yyS	a pointer to an array of Ns vectors containing the sensitivities of the forward solution.
ypS	a pointer to an array of Ns vectors containing the derivatives of the forward sensitivities.
yB	is the current value of the backward dependent variable vector.
ypB	is the current value of the backward dependent derivative vector.
resvalB	is the current value of the residual for the backward problem.
cjB	is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).
user_dataB	is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

Return value The value returned by the Jacobian-vector setup function should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

Notes   Each call to the Jacobian-vector setup function is preceded by a call to the backward problem residual user function with the same (t,y, yp, yyS, ypS, yB, ypB) arguments. Thus, the setup function can use any auxiliary data that is computed and saved during the evaluation of the DAE residual.

If the user's IDASpilsJacTimesVecFnB function uses difference quotient approximations, it may need to access quantities not in the call list. These include the current stepsize, the error weights, etc. To obtain these, the user will need to add a pointer to ida_mem to user_dataB and then use the IDAGet* functions described in §4.5.9.2. The unit roundoff can be accessed as UNIT_ROUNDOFF defined in sundials_types.h.

6.3.8   Preconditioning for the backward problem (linear system solution)

If preconditioning is used during integration of the backward problem, then the user must provide a C function to solve the linear system $Pz = r$, where P is a left preconditioner matrix. This function must have one of the following two forms:

IDASpilsPrecSolveFnB

Definition   `typedef int (*IDASpilsPrecSolveFnB)(realtype t,`
                                                  `N_Vector yy, N_Vector yp,`
                                                  `N_Vector yB, N_Vector ypB,`
                                                  `N_Vector resvalB,`
                                                  `N_Vector rvecB, N_Vector zvecB,`
                                                  `realtype cjB, realtype deltaB,`
                                                  `void *user_dataB);`

Purpose   This function solves the preconditioning system $Pz = r$ for the backward problem.

Arguments   t   is the current value of the independent variable.

yy	is the current value of the forward solution vector.
yp	is the current value of the forward solution derivative vector.
yB	is the current value of the backward dependent variable vector.
ypB	is the current value of the backward dependent derivative vector.
resvalB	is the current value of the residual for the backward problem.
rvecB	is the right-hand side vector r of the linear system to be solved.
zvecB	is the computed output vector.
cjB	is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).

deltaB	is an input tolerance to be used if an iterative method is employed in the solution.
user_dataB	is a pointer to user data — the same as the user_dataB parameter passed to the function IDASetUserDataB.

Return value   The return value of a preconditioner solve function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

IDASpilsPrecSolveFnBS

Definition
```
typedef int (*IDASpilsPrecSolveFnBS)(realtype t,
                                     N_Vector yy, N_Vector yp,
                                     N_Vector *yyS, N_Vector *ypS,
                                     N_Vector yB, N_Vector ypB,
                                     N_Vector resvalB,
                                     N_Vector rvecB, N_Vector zvecB,
                                     realtype cjB, realtype deltaB,
                                     void *user_dataB);
```

Purpose   This function solves the preconditioning system $Pz = r$ for the backward problem, for the case in which the backward problem depends on the forward sensitivities.

Arguments
t	is the current value of the independent variable.
yy	is the current value of the forward solution vector.
yp	is the current value of the forward solution derivative vector.
yyS	a pointer to an array of Ns vectors containing the sensitivities of the forward solution.
ypS	a pointer to an array of Ns vectors containing the derivatives of the forward sensitivities.
yB	is the current value of the backward dependent variable vector.
ypB	is the current value of the backward dependent derivative vector.
resvalB	is the current value of the residual for the backward problem.
rvecB	is the right-hand side vector r of the linear system to be solved.
zvecB	is the computed output vector.
cjB	is the scalar in the system Jacobian, proportional to the inverse of the step size (α in Eq. (2.6)).
deltaB	is an input tolerance to be used if an iterative method is employed in the solution.
user_dataB	is a pointer to user data — the same as the user_dataB parameter passed to the function IDASetUserDataB.

Return value   The return value of a preconditioner solve function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

6.3.9   Preconditioning for the backward problem (Jacobian data)

If the user's preconditioner requires that any Jacobian-related data be preprocessed or evaluated, then this needs to be done in a user-supplied C function of one of the following two types:

IDASpilsPrecSetupFnB

Definition   `typedef int (*IDASpilsPrecSetupFnB)(realtype t,`
                                        `N_Vector yy, N_Vector yp,`
                                        `N_Vector yB, N_Vector ypB,`
                                        `N_Vector resvalB,`
                                        `realtype cjB, void *user_dataB);`

Purpose   This function preprocesses and/or evaluates Jacobian-related data needed by the pre-
                conditioner for the backward problem.

Arguments   The arguments of an `IDASpilsPrecSetupFnB` are as follows:

                t   is the current value of the independent variable.

                yy   is the current value of the forward solution vector.

                yp   is the current value of the forward solution vector.

                yB   is the current value of the backward dependent variable vector.

                ypB   is the current value of the backward dependent derivative vector.

                resvalB   is the current value of the residual for the backward problem.

                cjB   is the scalar in the system Jacobian, proportional to the inverse of the step
                            size (α in Eq. (2.6)).

                user_dataB is a pointer to user data — the same as the `user_dataB` parameter passed
                            to the function `IDASetUserDataB`.

Return value   The return value of a preconditioner setup function for the backward problem should
                be 0 if successful, positive for a recoverable error (in which case the step will be retried),
                or negative for an unrecoverable error (in which case the integration is halted).

IDASpilsPrecSetupFnBS

Definition   `typedef int (*IDASpilsPrecSetupFnBS)(realtype t,`
                                        `N_Vector yy, N_Vector yp,`
                                        `N_Vector *yyS, N_Vector *ypS,`
                                        `N_Vector yB, N_Vector ypB,`
                                        `N_Vector resvalB,`
                                        `realtype cjB, void *user_dataB);`

Purpose   This function preprocesses and/or evaluates Jacobian-related data needed by the pre-
                conditioner for the backward problem, in the case where the backward problem depends
                on the forward sensitivities.

Arguments   The arguments of an `IDASpilsPrecSetupFnBS` are as follows:

                t   is the current value of the independent variable.

                yy   is the current value of the forward solution vector.

                yp   is the current value of the forward solution vector.

                yyS   a pointer to an array of `Ns` vectors containing the sensitivities of the forward
                            solution.

                ypS   a pointer to an array of `Ns` vectors containing the derivatives of the forward
                            sensitivities.

                yB   is the current value of the backward dependent variable vector.

                ypB   is the current value of the backward dependent derivative vector.

                resvalB   is the current value of the residual for the backward problem.

                cjB   is the scalar in the system Jacobian, proportional to the inverse of the step
                            size (α in Eq. (2.6)).

                user_dataB is a pointer to user data — the same as the `user_dataB` parameter passed
                            to the function `IDASetUserDataB`.

Return value The return value of a preconditioner setup function for the backward problem should be 0 if successful, positive for a recoverable error (in which case the step will be retried), or negative for an unrecoverable error (in which case the integration is halted).

6.4 Using the band-block-diagonal preconditioner for backward problems

As on the forward integration phase, the efficiency of Krylov iterative methods for the solution of linear systems can be greatly enhanced through preconditioning. The band-block-diagonal preconditioner module IDABBDPRE, provides interface functions through which it can be used on the backward integration phase.

The adjoint module in IDAS offers an interface to the band-block-diagonal preconditioner module IDABBDPRE described in section §4.8. This generates a preconditioner that is a block-diagonal matrix with each block being a band matrix and can be used with one of the Krylov linear solvers and with the MPI-parallel vector module NVECTOR_PARALLEL.

In order to use the IDABBDPRE module in the solution of the backward problem, the user must define one or two additional functions, described at the end of this section.

6.4.1 Usage of IDABBDPRE for the backward problem

The IDABBDPRE module is initialized by calling the following function, *after* one of the IDASPILS linear solvers has been specified, by calling the appropriate function (see §6.2.6).

IDABBDPrecInitB

Call   `flag = IDABBDPrecInitB(ida_mem, which, NlocalB, mudqB, mldqB,`
            `                        mukeepB, mlkeepB, dqrelyB, GresB, GcommB);`

Description The function `IDABBDPrecInitB` initializes and allocates memory for the IDABBDPRE preconditioner for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block.

            `which`   (`int`) the identifier of the backward problem.

            `NlocalB` (`sunindextype`) local vector dimension for the backward problem.

            `mudqB`   (`sunindextype`) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.

            `mldqB`   (`sunindextype`) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.

            `mukeepB` (`sunindextype`) upper half-bandwidth of the retained banded approximate Jacobian block.

            `mlkeepB` (`sunindextype`) lower half-bandwidth of the retained banded approximate Jacobian block.

            `dqrelyB` (`realtype`) the relative increment in components of `yB` used in the difference quotient approximations. The default is `dqrelyB=` $\sqrt{\text{unit roundoff}}$, which can be specified by passing `dqrely=` 0.0.

            `GresB`   (`IDABBDLocalFnB`) the C function which computes $G_B(t, y, \dot{y}, y_B, \dot{y}_B)$, the function approximating the residual of the backward problem.

            `GcommB` (`IDABBDCommFnB`) the optional C function which performs all interprocess communication required for the computation of G_B.

Return value If successful, `IDABBDPrecInitB` creates, allocates, and stores (internally in the IDAS solver block) a pointer to the newly created IDABBDPRE memory block. The return value `flag` (of type `int`) is one of:

            `IDASPILS_SUCCESS`   The call to `IDABBDPrecInitB` was successful.

IDASPILS_MEM_FAIL	A memory allocation request has failed.
IDASPILS_MEM_NULL	The ida_mem argument was NULL.
IDASPILS_LMEM_NULL	No linear solver has been attached.
IDASPILS_ILL_INPUT	An invalid parameter has been passed.

To reinitialize the IDABBDPRE preconditioner module for the backward problem, possibly with a change in mudqB, mldqB, or dqrelyB, call the following function:

IDABBDPrecReInitB

Call   `flag = IDABBDPrecReInitB(ida_mem, which, mudqB, mldqB, dqrelyB);`

Description   The function `IDABBDPrecReInitB` reinitializes the IDABBDPRE preconditioner for the backward problem.

Arguments   `ida_mem` (`void *`) pointer to the IDAS memory block returned by `IDACreate`.

              `which`   (`int`) the identifier of the backward problem.

              `mudqB`   (`sunindextype`) upper half-bandwidth to be used in the difference-quotient Jacobian approximation.

              `mldqB`   (`sunindextype`) lower half-bandwidth to be used in the difference-quotient Jacobian approximation.

              `dqrelyB` (`realtype`) the relative increment in components of yB used in the difference quotient approximations.

Return value The return value `flag` (of type `int`) is one of:

IDASPILS_SUCCESS	The call to IDABBDPrecReInitB was successful.
IDASPILS_MEM_FAIL	A memory allocation request has failed.
IDASPILS_MEM_NULL	The ida_mem argument was NULL.
IDASPILS_PMEM_NULL	The IDABBDPrecInitB has not been previously called.
IDASPILS_LMEM_NULL	No linear solver has been attached.
IDASPILS_ILL_INPUT	An invalid parameter has been passed.

For more details on IDABBDPRE see §4.8.

6.4.2   User-supplied functions for IDABBDPRE

To use the IDABBDPRE module, the user must supply one or two functions which the module calls to construct the preconditioner: a required function `GresB` (of type `IDABBDLocalFnB`) which approximates the residual of the backward problem and which is computed locally, and an optional function `GcommB` (of type `IDABBDCommFnB`) which performs all interprocess communication necessary to evaluate this approximate residual (see §4.8). The prototypes for these two functions are described below.

IDABBDLocalFnB

Definition   `typedef int (*IDABBDLocalFnB)(sunindextype NlocalB, realtype t,`
              `                              N_Vector y, N_Vector yp,`
              `                              N_Vector yB, N_Vector ypB,`
              `                              N_Vector gB, void *user_dataB);`

Purpose   This `GresB` function loads the vector `gB`, an approximation to the residual of the backward problem, as a function of t, y, yp, and yB and ypB.

Arguments   `NlocalB`   is the local vector length for the backward problem.

              `t`   is the value of the independent variable.

              `y`   is the current value of the forward solution vector.

              `yp`   is the current value of the forward solution derivative vector.

              `yB`   is the current value of the backward dependent variable vector.

ypB	is the current value of the backward dependent derivative vector.
gB	is the output vector, $G_B(t, y, \dot{y}, y_B, \dot{y}_B)$.
user_dataB	is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

Return value An IDABBDLocalFnB should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns IDA_LSETUP_FAIL).

Notes This routine must assume that all interprocess communication of data needed to calculate gB has already been done, and this data is accessible within user_dataB.

Before calling the user's IDABBDLocalFnB, IDAS needs to evaluate (through interpolation) the values of the states from the forward integration. If an error occurs in the interpolation, IDAS triggers an unrecoverable failure in the preconditioner setup function which will halt the integration (IDASolveB returns IDA_LSETUP_FAIL).

⚠️

IDABBDCommFnB

Definition
```
typedef int (*IDABBDCommFnB)(sunindextype NlocalB, realtype t,
                             N_Vector y, N_Vector yp,
                             N_Vector yB, N_Vector ypB,
                             void *user_dataB);
```

Purpose This GcommB function performs all interprocess communications necessary for the execution of the GresB function above, using the input vectors y, yp, yB and ypB.

Arguments
NlocalB	is the local vector length.
t	is the value of the independent variable.
y	is the current value of the forward solution vector.
yp	is the current value of the forward solution derivative vector.
yB	is the current value of the backward dependent variable vector.
ypB	is the current value of the backward dependent derivative vector.
user_dataB	is a pointer to user data — the same as the user_dataB parameter passed to IDASetUserDataB.

Return value An IDABBDCommFnB should return 0 if successful, a positive value if a recoverable error occurred (in which case IDAS will attempt to correct), or a negative value if it failed unrecoverably (in which case the integration is halted and IDASolveB returns IDA_LSETUP_FAIL).

Notes The GcommB function is expected to save communicated data in space defined within the structure user_dataB.

Each call to the GcommB function is preceded by a call to the function that evaluates the residual of the backward problem with the same t, y, yp, yB and ypB arguments. If there is no additional communication needed, then pass GcommB = NULL to IDABBDPrecInitB.

Chapter 7

Description of the NVECTOR module

The SUNDIALS solvers are written in a data-independent manner. They all operate on generic vectors (of type N_Vector) through a set of operations defined by the particular NVECTOR implementation. Users can provide their own specific implementation of the NVECTOR module, or use one of the implementations provided with SUNDIALS. The generic operations are described below and the implementations provided with SUNDIALS are described in the following sections.

The generic N_Vector type is a pointer to a structure that has an implementation-dependent *content* field containing the description and actual data of the vector, and an *ops* field pointing to a structure with generic vector operations. The type N_Vector is defined as

```
typedef struct _generic_N_Vector *N_Vector;

struct _generic_N_Vector {
    void *content;
    struct _generic_N_Vector_Ops *ops;
};
```

The _generic_N_Vector_Ops structure is essentially a list of pointers to the various actual vector operations, and is defined as

```
struct _generic_N_Vector_Ops {
  N_Vector_ID (*nvgetvectorid)(N_Vector);
  N_Vector    (*nvclone)(N_Vector);
  N_Vector    (*nvcloneempty)(N_Vector);
  void        (*nvdestroy)(N_Vector);
  void        (*nvspace)(N_Vector, sunindextype *, sunindextype *);
  realtype*   (*nvgetarraypointer)(N_Vector);
  void        (*nvsetarraypointer)(realtype *, N_Vector);
  void        (*nvlinearsum)(realtype, N_Vector, realtype, N_Vector, N_Vector);
  void        (*nvconst)(realtype, N_Vector);
  void        (*nvprod)(N_Vector, N_Vector, N_Vector);
  void        (*nvdiv)(N_Vector, N_Vector, N_Vector);
  void        (*nvscale)(realtype, N_Vector, N_Vector);
  void        (*nvabs)(N_Vector, N_Vector);
  void        (*nvinv)(N_Vector, N_Vector);
  void        (*nvaddconst)(N_Vector, realtype, N_Vector);
  realtype    (*nvdotprod)(N_Vector, N_Vector);
  realtype    (*nvmaxnorm)(N_Vector);
  realtype    (*nvwrmsnorm)(N_Vector, N_Vector);
```

```
realtype     (*nvwrmsnormmask)(N_Vector, N_Vector, N_Vector);
realtype     (*nvmin)(N_Vector);
realtype     (*nvwl2norm)(N_Vector, N_Vector);
realtype     (*nvl1norm)(N_Vector);
void         (*nvcompare)(realtype, N_Vector, N_Vector);
booleantype  (*nvinvtest)(N_Vector, N_Vector);
booleantype  (*nvconstrmask)(N_Vector, N_Vector, N_Vector);
realtype     (*nvminquotient)(N_Vector, N_Vector);
int          (*nvlinearcombination)(int, realtype*, N_Vector*, N_Vector);
int          (*nvscaleaddmulti)(int, realtype*, N_Vector, N_Vector*, N_Vector*);
int          (*nvdotprodmulti)(int, N_Vector, N_Vector*, realtype*);
int          (*nvlinearsumvectorarray)(int, realtype, N_Vector*, realtype,
                                       N_Vector*, N_Vector*);
int          (*nvscalevectorarray)(int, realtype*, N_Vector*, N_Vector*);
int          (*nvconstvectorarray)(int, realtype, N_Vector*);
int          (*nvwrmsnomrvectorarray)(int, N_Vector*, N_Vector*, realtype*);
int          (*nvwrmsnomrmaskvectorarray)(int, N_Vector*, N_Vector*, N_Vector,
                                          realtype*);
int          (*nvscaleaddmultivectorarray)(int, int, realtype*, N_Vector*,
                                           N_Vector**, N_Vector**);
int          (*nvlinearcombinationvectorarray)(int, int, realtype*, N_Vector**,
                                              N_Vector*);
};
```

The generic NVECTOR module defines and implements the vector operations acting on an `N_Vector`. These routines are nothing but wrappers for the vector operations defined by a particular NVECTOR implementation, which are accessed through the *ops* field of the `N_Vector` structure. To illustrate this point we show below the implementation of a typical vector operation from the generic NVECTOR module, namely `N_VScale`, which performs the scaling of a vector `x` by a scalar `c`:

```
void N_VScale(realtype c, N_Vector x, N_Vector z)
{
   z->ops->nvscale(c, x, z);
}
```

Table 7.2 contains a complete list of all standard vector operations defined by the generic NVEC-TOR module. Tables 7.3 and 7.4 list *optional* fused and vector array operations respectively. These operations are intended to increase data reuse, reduce parallel communication on distributed memory systems, and lower the number of kernel launches on systems with accelerators. If a particular NVECTOR implementation defines one of the fused or vector array operations as `NULL`, the NVECTOR interface will call one of the standard vector operations as necessary.

Finally, note that the generic NVECTOR module defines the functions `N_VCloneVectorArray` and `N_VCloneVectorArrayEmpty`. Both functions create (by cloning) an array of `count` variables of type `N_Vector`, each of the same type as an existing `N_Vector`. Their prototypes are

```
N_Vector *N_VCloneVectorArray(int count, N_Vector w);
N_Vector *N_VCloneVectorArrayEmpty(int count, N_Vector w);
```

and their definitions are based on the implementation-specific `N_VClone` and `N_VCloneEmpty` operations, respectively.

An array of variables of type `N_Vector` can be destroyed by calling `N_VDestroyVectorArray`, whose prototype is

```
void N_VDestroyVectorArray(N_Vector *vs, int count);
```

and whose definition is based on the implementation-specific `N_VDestroy` operation.

A particular implementation of the NVECTOR module must:

Table 7.1: Vector Identifications associated with vector kernels supplied with SUNDIALS.

Vector ID	Vector type	ID Value
SUNDIALS_NVEC_SERIAL	Serial	0
SUNDIALS_NVEC_PARALLEL	Distributed memory parallel (MPI)	1
SUNDIALS_NVEC_OPENMP	OpenMP shared memory parallel	2
SUNDIALS_NVEC_PTHREADS	PThreads shared memory parallel	3
SUNDIALS_NVEC_PARHYP	*hypre* ParHyp parallel vector	4
SUNDIALS_NVEC_PETSC	PETSc parallel vector	5
SUNDIALS_NVEC_CUSTOM	User-provided custom vector	6

- Specify the *content* field of N_Vector.

- Define and implement the vector operations. Note that the names of these routines should be unique to that implementation in order to permit using more than one NVECTOR module (each with different N_Vector internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free an N_Vector with the new *content* field and with *ops* pointing to the new vector operations.

- Optionally, define and implement additional user-callable routines acting on the newly defined N_Vector (e.g., a routine to print the content for debugging purposes).

- Optionally, provide accessor macros as needed for that particular implementation to be used to access different parts in the *content* field of the newly defined N_Vector.

Each NVECTOR implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 7.1. It is recommended that a user-supplied NVECTOR implementation use the SUNDIALS_NVEC_CUSTOM identifier.

Table 7.2: Description of the NVECTOR operations

Name	Usage and Description
N_VGetVectorID	id = N_VGetVectorID(w); Returns the vector type identifier for the vector w. It is used to determine the vector implementation type (e.g. serial, parallel,...) from the abstract N_Vector interface. Returned values are given in Table 7.1.
N_VClone	v = N_VClone(w); Creates a new N_Vector of the same type as an existing vector w and sets the *ops* field. It does not copy the vector, but rather allocates storage for the new vector.

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Name	**Usage and Description**
N_VCloneEmpty	v = N_VCloneEmpty(w); Creates a new N_Vector of the same type as an existing vector w and sets the *ops* field. It does not allocate storage for data.
N_VDestroy	N_VDestroy(v); Destroys the N_Vector v and frees memory allocated for its internal data.
N_VSpace	N_VSpace(nvSpec, &lrw, &liw); Returns storage requirements for one N_Vector. lrw contains the number of realtype words and liw contains the number of integer words. This function is advisory only, for use in determining a user's total space requirements; it could be a dummy function in a user-supplied NVECTOR module if that information is not of interest.
N_VGetArrayPointer	vdata = N_VGetArrayPointer(v); Returns a pointer to a realtype array from the N_Vector v. Note that this assumes that the internal data in N_Vector is a contiguous array of realtype. This routine is only used in the solver-specific interfaces to the dense and banded (serial) linear solvers, the sparse linear solvers (serial and threaded), and in the interfaces to the banded (serial) and band-block-diagonal (parallel) preconditioner modules provided with SUNDIALS.
N_VSetArrayPointer	N_VSetArrayPointer(vdata, v); Overwrites the data in an N_Vector with a given array of realtype. Note that this assumes that the internal data in N_Vector is a contiguous array of realtype. This routine is only used in the interfaces to the dense (serial) linear solver, hence need not exist in a user-supplied NVECTOR module for a parallel environment.
N_VLinearSum	N_VLinearSum(a, x, b, y, z); Performs the operation $z = ax + by$, where a and b are realtype scalars and x and y are of type N_Vector: $z_i = ax_i + by_i$, $i = 0, \ldots, n-1$.
N_VConst	N_VConst(c, z); Sets all components of the N_Vector z to realtype c: $z_i = c, i = 0, \ldots, n-1$.
N_VProd	N_VProd(x, y, z); Sets the N_Vector z to be the component-wise product of the N_Vector inputs x and y: $z_i = x_i y_i$, $i = 0, \ldots, n-1$.
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Name	**Usage and Description**
N_VDiv	N_VDiv(x, y, z); Sets the N_Vector z to be the component-wise ratio of the N_Vector inputs x and y: $z_i = x_i/y_i$, $i = 0, \ldots, n-1$. The y_i may not be tested for 0 values. It should only be called with a y that is guaranteed to have all nonzero components.
N_VScale	N_VScale(c, x, z); Scales the N_Vector x by the realtype scalar c and returns the result in z: $z_i = cx_i$, $i = 0, \ldots, n-1$.
N_VAbs	N_VAbs(x, z); Sets the components of the N_Vector z to be the absolute values of the components of the N_Vector x: $y_i = |x_i|$, $i = 0, \ldots, n-1$.
N_VInv	N_VInv(x, z); Sets the components of the N_Vector z to be the inverses of the components of the N_Vector x: $z_i = 1.0/x_i$, $i = 0, \ldots, n-1$. This routine may not check for division by 0. It should be called only with an x which is guaranteed to have all nonzero components.
N_VAddConst	N_VAddConst(x, b, z); Adds the realtype scalar b to all components of x and returns the result in the N_Vector z: $z_i = x_i + b$, $i = 0, \ldots, n-1$.
N_VDotProd	d = N_VDotProd(x, y); Returns the value of the ordinary dot product of x and y: $d = \sum_{i=0}^{n-1} x_i y_i$.
N_VMaxNorm	m = N_VMaxNorm(x); Returns the maximum norm of the N_Vector x: $m = \max_i |x_i|$.
N_VWrmsNorm	m = N_VWrmsNorm(x, w) Returns the weighted root-mean-square norm of the N_Vector x with realtype weight vector w: $m = \sqrt{\left(\sum_{i=0}^{n-1}(x_i w_i)^2\right)/n}$.
N_VWrmsNormMask	m = N_VWrmsNormMask(x, w, id); Returns the weighted root mean square norm of the N_Vector x with realtype weight vector w built using only the elements of x corresponding to positive elements of the N_Vector id: $m = \sqrt{\left(\sum_{i=0}^{n-1}(x_i w_i H(id_i))^2\right)/n}$, where $H(\alpha) = \begin{cases} 1 & \alpha > 0 \\ 0 & \alpha \leq 0 \end{cases}$
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Name	**Usage and Description**
`N_VMin`	`m = N_VMin(x);` Returns the smallest element of the `N_Vector` x: $m = \min_i x_i$.
`N_VWL2Norm`	`m = N_VWL2Norm(x, w);` Returns the weighted Euclidean ℓ_2 norm of the `N_Vector` x with `realtype` weight vector w: $m = \sqrt{\sum_{i=0}^{n-1}(x_i w_i)^2}$.
`N_VL1Norm`	`m = N_VL1Norm(x);` Returns the ℓ_1 norm of the `N_Vector` x: $m = \sum_{i=0}^{n-1}|x_i|$.
`N_VCompare`	`N_VCompare(c, x, z);` Compares the components of the `N_Vector` x to the `realtype` scalar c and returns an `N_Vector` z such that: $z_i = 1.0$ if $|x_i| \geq c$ and $z_i = 0.0$ otherwise.
`N_VInvTest`	`t = N_VInvTest(x, z);` Sets the components of the `N_Vector` z to be the inverses of the components of the `N_Vector` x, with prior testing for zero values: $z_i = 1.0/x_i$, $i = 0, \ldots, n-1$. This routine returns a boolean assigned to `SUNTRUE` if all components of x are nonzero (successful inversion) and returns `SUNFALSE` otherwise.
`N_VConstrMask`	`t = N_VConstrMask(c, x, m);` Performs the following constraint tests: $x_i > 0$ if $c_i = 2$, $x_i \geq 0$ if $c_i = 1$, $x_i \leq 0$ if $c_i = -1$, $x_i < 0$ if $c_i = -2$. There is no constraint on x_i if $c_i = 0$. This routine returns a boolean assigned to `SUNFALSE` if any element failed the constraint test and assigned to `SUNTRUE` if all passed. It also sets a mask vector m, with elements equal to 1.0 where the constraint test failed, and 0.0 where the test passed. This routine is used only for constraint checking.
`N_VMinQuotient`	`minq = N_VMinQuotient(num, denom);` This routine returns the minimum of the quotients obtained by term-wise dividing num_i by $denom_i$. A zero element in `denom` will be skipped. If no such quotients are found, then the large value `BIG_REAL` (defined in the header file `sundials_types.h`) is returned.

Table 7.3: Description of the NVECTOR fused operations

Name	Usage and Description
N_VLinearCombination	`ier = N_VLinearCombination(nv, c, X, z);` This routine computes the linear combination of nv vectors with n elements: $$z_i = \sum_{j=1}^{nv} c_j x_{j,i}, \quad i = 1, \dots, n,$$ where c is an array of nv scalars (type `realtype*`), X is an array of n vectors (type `N_Vector*`), and z is the output vector (type `N_Vector`). If the output vector z is one of the vectors in X, then it *must* be the first vector in the vector array. The operation returns `0` for success and a non-zero value otherwise.
N_VScaleAddMulti	`ier = N_VScaleAddMulti(nv, c, x, Y, Z);` This routine scales and adds one vector to nv vectors with n elements: $$z_{j,i} = c_j x_i + y_{j,i}, \quad j = 1, \dots, nv \quad i = 1, \dots, n,$$ where c is an array of nv scalars (type `realtype*`), x is the vector (type `N_Vector`) to be scaled and added to each vector in the vector array of nv vectors Y (type `N_Vector*`), and Z (type `N_Vector*`) is a vector array of nv output vectors. The operation returns `0` for success and a non-zero value otherwise.
N_VDotProdMulti	`ier = N_VDotProdMulti(nv, x, Y, d);` This routine computes the dot product of a vector with nv other vectors: $$d_j = \sum_{i}^{n} x_i y_{j,i}, \quad j = 1, \dots, nv,$$ where d (type `realtype*`) is an array of nv scalars containing the dot products of the vector x (type `N_Vector`) with each of the nv vectors in the vector array Y (type `N_Vector*`). The operation returns `0` for success and a non-zero value otherwise.
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Name	**Usage and Description**

Table 7.4: Description of the NVECTOR vector array operations

Name	Usage and Description
N_VLinearSumVectorArray	`ier = N_VLinearSumVectorArray(nv, a, X, b, Y, Z);` This routine comuptes the linear sum of two vector arrays containing nv vectors of n elements: $$z_{j,i} = ax_{j,i} + by_{j,i}, \quad i = 1, \ldots, n \quad j = 1, \ldots, nv,$$ where a and b are `realtype` scalars and X, Y, and Z are arrays of nv vectors (type `N_Vector*`). The operation returns `0` for success and a non-zero value otherwise.
N_VScaleVectorArray	`ier = N_VScaleVectorArray(nv, c, X, Z);` This routine scales each vector of n elements in a vector array of nv vectors by a potentially different constant: $$z_{j,i} = c_j x_{j,i}, \quad i = 1, \ldots, n \quad j = 1, \ldots, nv,$$ where c is an array of nv scalars (type `realtype*`) and X and Z are arrays of nv vectors (type `N_Vector*`). The operation returns `0` for success and a non-zero value otherwise.
N_VConstVectorArray	`ier = N_VConstVectorArray(nv, c, X);` This routine sets each element in a vector of n elements in a vector array of nv vectors to the same value: $$z_{j,i} = c, \quad i = 1, \ldots, n \quad j = 1, \ldots, nv,$$ where c is a `realtype` scalar and X is an array of nv vectors (type `N_Vector*`). The operation returns `0` for success and a non-zero value otherwise.
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Name	**Usage and Description**
`N_VWrmsNormVectorArray`	`ier = N_VWrmsNormVectorArray(nv, X, W, m);` This routine computes the weighted root mean square norm of nv vectors with n elements: $$m_j = \left(\frac{1}{n} \sum_{i=1}^{n} (x_{j,i} w_{j,i})^2 \right)^{1/2}, \quad j = 1, \ldots, nvec,$$ where m (type `realtype*`) contains the nv norms of the vectors in the vector array X (type `N_Vector*`) with corresponding weight vectors W (type `N_Vector*`). The operation returns `0` for success and a non-zero value otherwise.
`N_VWrmsNormMaskVectorArray`	`ier = N_VWrmsNormMaskVectorArray(nv, X, W, id, m);` This routine computes the masked weighted root mean square norm of nv vectors with n elements: $$m_j = \left(\frac{1}{n} \sum_{i=1}^{n} (x_{j,i} w_{j,i} H(id_i))^2 \right)^{1/2}, \quad j = 1, \ldots, nvec,$$ $H(id_i) = 1$ for $id_i > 0$ and is zero otherwise, m (type `realtype*`) contains the nv norms of the vectors in the vector array X (type `N_Vector*`) with corresponding weight vectors W (type `N_Vector*`) and mask vector id (type `N_Vector`). The operation returns `0` for success and a non-zero value otherwise.
`N_VScaleAddMultiVectorArray`	`ier = N_VScaleAddMultiVectorArray(nv, ns, c, X, YY, ZZ);` This routine scales and adds a vector in a vector array of nv vectors to the corresponding vector in ns vector arrays: $$z_{j,i} = \sum_{k=1}^{ns} c_k x_{k,j,i}, \quad i = 1, \ldots, n \quad j = 1, \ldots, nv,$$ where c is an array of ns scalars (type `realtype*`), X is a vector array of nv vectors (type idN_Vector*) to be scaled and added to the corresponding vector in each of the ns vector arrays in the array of vector arrays YY (type `N_Vector**`) and stored in the output array of vector arrays ZZ (type `N_Vector**`). The operation returns `0` for success and a non-zero value otherwise.
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Name	**Usage and Description**
N_VLinearCombinationVectorArray	`ier = N_VLinearCombinationVectorArray(nv, ns, c, XX, Z);` This routine computes the linear combination of ns vector arrays containing nv vectors with n elements: $$z_{j,i} = \sum_{k=1}^{ns} c_k x_{k,j,i}, \quad i = 1,\ldots,n \quad j = 1,\ldots,nv,$$ where c is an array of ns scalars (type `realtype*`), XX (type `N_Vector**`) is an array of ns vector arrays each containing nv vectors to be summed into the output vector array of nv vectors ZZ (type `N_Vector*`). If the output vector array ZZ is one of the vector arrays in XX, then it *must* be the first vector array in XX. The operation returns `0` for success and a non-zero value otherwise.

7.1 The NVECTOR_SERIAL implementation

The serial implementation of the NVECTOR module provided with SUNDIALS, NVECTOR_SERIAL, defines the *content* field of N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, and a boolean flag *own_data* which specifies the ownership of *data*.

```
struct _N_VectorContent_Serial {
  sunindextype length;
  booleantype own_data;
  realtype *data;
};
```

The header file to include when using this module is nvector_serial.h. The installed module library to link to is libsundials_nvecserial.*lib* where *.lib* is typically .so for shared libraries and .a for static libraries.

The following macros are provided to access the content of an NVECTOR_SERIAL vector. The suffix _S in the names denotes the serial version.

- NV_CONTENT_S

  This routine gives access to the contents of the serial vector N_Vector.

  The assignment v_cont = NV_CONTENT_S(v) sets v_cont to be a pointer to the serial N_Vector content structure.

  Implementation:

  `#define NV_CONTENT_S(v) ( (N_VectorContent_Serial)(v->content) )`

- NV_OWN_DATA_S, NV_DATA_S, NV_LENGTH_S

  These macros give individual access to the parts of the content of a serial N_Vector.

  The assignment v_data = NV_DATA_S(v) sets v_data to be a pointer to the first component of the data for the N_Vector v. The assignment NV_DATA_S(v) = v_data sets the component array of v to be v_data by storing the pointer v_data.

  The assignment v_len = NV_LENGTH_S(v) sets v_len to be the length of v. On the other hand, the call NV_LENGTH_S(v) = len_v sets the length of v to be len_v.

Implementation:

```
#define NV_OWN_DATA_S(v) ( NV_CONTENT_S(v)->own_data )
#define NV_DATA_S(v) ( NV_CONTENT_S(v)->data )
#define NV_LENGTH_S(v) ( NV_CONTENT_S(v)->length )
```

- NV_Ith_S

  This macro gives access to the individual components of the data array of an N_Vector.

  The assignment r = NV_Ith_S(v,i) sets r to be the value of the i-th component of v. The assignment NV_Ith_S(v,i) = r sets the value of the i-th component of v to be r.

  Here i ranges from 0 to $n-1$ for a vector of length n.

  Implementation:

  ```
  #define NV_Ith_S(v,i) ( NV_DATA_S(v)[i] )
  ```

The NVECTOR_SERIAL module defines serial implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4. Their names are obtained from those in Tables 7.2, 7.3, and 7.4. by appending the suffix _Serial (e.g. N_VDestroy_Serial). The module NVECTOR_SERIAL provides the following additional user-callable routines:

- N_VNew_Serial

  This function creates and allocates memory for a serial N_Vector. Its only argument is the vector length.

  ```
  N_Vector N_VNew_Serial(sunindextype vec_length);
  ```

- N_VNewEmpty_Serial

  This function creates a new serial N_Vector with an empty (NULL) data array.

  ```
  N_Vector N_VNewEmpty_Serial(sunindextype vec_length);
  ```

- N_VMake_Serial

  This function creates and allocates memory for a serial vector with user-provided data array.

  (This function does *not* allocate memory for v_data itself.)

  ```
  N_Vector N_VMake_Serial(sunindextype vec_length, realtype *v_data);
  ```

- N_VCloneVectorArray_Serial

  This function creates (by cloning) an array of count serial vectors.

  ```
  N_Vector *N_VCloneVectorArray_Serial(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_Serial

  This function creates (by cloning) an array of count serial vectors, each with an empty (NULL) data array.

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Serial(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_Serial

  This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_Serial or with N_VCloneVectorArrayEmpty_Serial.

  ```
  void N_VDestroyVectorArray_Serial(N_Vector *vs, int count);
  ```

- N_VGetLength_Serial

  This function returns the number of vector elements.

  ```
  sunindextype N_VGetLength_Serial(N_Vector v);
  ```

- N_VPrint_Serial

  This function prints the content of a serial vector to `stdout`.

  ```
  void N_VPrint_Serial(N_Vector v);
  ```

- N_VPrintFile_Serial

  This function prints the content of a serial vector to `outfile`.

  ```
  void N_VPrintFile_Serial(N_Vector v, FILE *outfile);
  ```

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via v_data = NV_DATA_S(v) and then access v_data[i] within the loop than it is to use NV_Ith_S(v,i) within the loop.

- N_VNewEmpty_Serial, N_VMake_Serial, and N_VCloneVectorArrayEmpty_Serial set the field *own_data* = SUNFALSE. N_VDestroy_Serial and N_VDestroyVectorArray_Serial will not attempt to free the pointer *data* for any N_Vector with *own_data* set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the *data* pointer.

- To maximize efficiency, vector operations in the NVECTOR_SERIAL implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

For solvers that include a Fortran interface module, the NVECTOR_SERIAL module also includes a Fortran-callable function FNVINITS(code, NEQ, IER), to initialize this NVECTOR_SERIAL module. Here `code` is an input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type `long int`); and IER is an error return flag equal 0 for success and -1 for failure.

7.2   The NVECTOR_PARALLEL implementation

The NVECTOR_PARALLEL implementation of the NVECTOR module provided with SUNDIALS is based on MPI. It defines the *content* field of N_Vector to be a structure containing the global and local lengths of the vector, a pointer to the beginning of a contiguous local data array, an MPI communicator, and a boolean flag *own_data* indicating ownership of the data array *data*.

```
struct _N_VectorContent_Parallel {
  sunindextype local_length;
  sunindextype global_length;
  booleantype own_data;
  realtype *data;
  MPI_Comm comm;
};
```

The header file to include when using this module is `nvector_parallel.h`. The installed module library to link to is `libsundials_nvecparallel.`*`lib`* where *`.lib`* is typically `.so` for shared libraries and `.a` for static libraries.

The following macros are provided to access the content of a NVECTOR_PARALLEL vector. The suffix _P in the names denotes the distributed memory parallel version.

- NV_CONTENT_P

  This macro gives access to the contents of the parallel vector N_Vector.

  The assignment v_cont = NV_CONTENT_P(v) sets v_cont to be a pointer to the N_Vector content structure of type struct _N_VectorContent_Parallel.

Implementation:

```
#define NV_CONTENT_P(v) ( (N_VectorContent_Parallel)(v->content) )
```

- NV_OWN_DATA_P, NV_DATA_P, NV_LOCLENGTH_P, NV_GLOBLENGTH_P

These macros give individual access to the parts of the content of a parallel N_Vector.

The assignment v_data = NV_DATA_P(v) sets v_data to be a pointer to the first component of the local data for the N_Vector v. The assignment NV_DATA_P(v) = v_data sets the component array of v to be v_data by storing the pointer v_data.

The assignment v_llen = NV_LOCLENGTH_P(v) sets v_llen to be the length of the local part of v. The call NV_LENGTH_P(v) = llen_v sets the local length of v to be llen_v.

The assignment v_glen = NV_GLOBLENGTH_P(v) sets v_glen to be the global length of the vector v. The call NV_GLOBLENGTH_P(v) = glen_v sets the global length of v to be glen_v.

Implementation:

```
#define NV_OWN_DATA_P(v)   ( NV_CONTENT_P(v)->own_data )
#define NV_DATA_P(v)       ( NV_CONTENT_P(v)->data )
#define NV_LOCLENGTH_P(v)  ( NV_CONTENT_P(v)->local_length )
#define NV_GLOBLENGTH_P(v) ( NV_CONTENT_P(v)->global_length )
```

- NV_COMM_P

This macro provides access to the MPI communicator used by the NVECTOR_PARALLEL vectors.

Implementation:

```
#define NV_COMM_P(v) ( NV_CONTENT_P(v)->comm )
```

- NV_Ith_P

This macro gives access to the individual components of the local data array of an N_Vector.

The assignment r = NV_Ith_P(v,i) sets r to be the value of the i-th component of the local part of v. The assignment NV_Ith_P(v,i) = r sets the value of the i-th component of the local part of v to be r.

Here i ranges from 0 to $n-1$, where n is the local length.

Implementation:

```
#define NV_Ith_P(v,i) ( NV_DATA_P(v)[i] )
```

The NVECTOR_PARALLEL module defines parallel implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4. Their names are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _Parallel (e.g. N_VDestroy_Parallel). The module NVECTOR_PARALLEL provides the following additional user-callable routines:

- N_VNew_Parallel

This function creates and allocates memory for a parallel vector.

```
N_Vector N_VNew_Parallel(MPI_Comm comm,
                         sunindextype local_length,
                         sunindextype global_length);
```

- N_VNewEmpty_Parallel

This function creates a new parallel N_Vector with an empty (NULL) data array.

```
N_Vector N_VNewEmpty_Parallel(MPI_Comm comm,
                              sunindextype local_length,
                              sunindextype global_length);
```

- N_VMake_Parallel

  This function creates and allocates memory for a parallel vector with user-provided data array.
  (This function does *not* allocate memory for v_data itself.)

  ```
  N_Vector N_VMake_Parallel(MPI_Comm comm,
                            sunindextype local_length,
                            sunindextype global_length,
                            realtype *v_data);
  ```

- N_VCloneVectorArray_Parallel

  This function creates (by cloning) an array of count parallel vectors.

  ```
  N_Vector *N_VCloneVectorArray_Parallel(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_Parallel

  This function creates (by cloning) an array of count parallel vectors, each with an empty (NULL)
  data array.

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Parallel(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_Parallel

  This function frees memory allocated for the array of count variables of type N_Vector created
  with N_VCloneVectorArray_Parallel or with N_VCloneVectorArrayEmpty_Parallel.

  ```
  void N_VDestroyVectorArray_Parallel(N_Vector *vs, int count);
  ```

- N_VGetLength_Parallel

  This function returns the number of vector elements (global vector length).

  ```
  sunindextype N_VGetLength_Parallel(N_Vector v);
  ```

- N_VGetLocalLength_Parallel

  This function returns the local vector length.

  ```
  sunindextype N_VGetLocalLength_Parallel(N_Vector v);
  ```

- N_VPrint_Parallel

  This function prints the local content of a parallel vector to stdout.

  ```
  void N_VPrint_Parallel(N_Vector v);
  ```

- N_VPrintFile_Parallel

  This function prints the local content of a parallel vector to outfile.

  ```
  void N_VPrintFile_Parallel(N_Vector v, FILE *outfile);
  ```

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the local
  component array via v_data = NV_DATA_P(v) and then access v_data[i] within the loop than
  it is to use NV_Ith_P(v,i) within the loop.

- N_VNewEmpty_Parallel, N_VMake_Parallel, and N_VCloneVectorArrayEmpty_Parallel set the
  field *own_data* = SUNFALSE. N_VDestroy_Parallel and N_VDestroyVectorArray_Parallel will
  not attempt to free the pointer *data* for any N_Vector with *own_data* set to SUNFALSE. In such
  a case, it is the user's responsibility to deallocate the *data* pointer.

⚠️

- To maximize efficiency, vector operations in the NVECTOR_PARALLEL implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

For solvers that include a Fortran interface module, the NVECTOR_PARALLEL module also includes a Fortran-callable function FNVINITP(COMM, code, NLOCAL, NGLOBAL, IER), to initialize this NVECTOR_PARALLEL module. Here COMM is the MPI communicator, code is an input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NLOCAL and NGLOBAL are the local and global vector sizes, respectively (declared so as to match C type long int); and IER is an error return flag equal 0 for success and -1 for failure. NOTE: If the header file sundials_config.h defines SUNDIALS_MPI_COMM_F2C to be 1 (meaning the MPI implementation used to build SUNDIALS includes the MPI_Comm_f2c function), then COMM can be any valid MPI communicator. Otherwise, MPI_COMM_WORLD will be used, so just pass an integer value as a placeholder.

⚠️

7.3 The NVECTOR_OPENMP implementation

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of NVECTOR using OpenMP, called NVECTOR_OPENMP, and an implementation using Pthreads, called NVECTOR_PTHREADS. Testing has shown that vectors should be of length at least $100,000$ before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.

The OpenMP NVECTOR implementation provided with SUNDIALS, NVECTOR_OPENMP, defines the *content* field of N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag *own_data* which specifies the ownership of *data*, and the number of threads. Operations on the vector are threaded using OpenMP.

```
struct _N_VectorContent_OpenMP {
  sunindextype length;
  booleantype own_data;
  realtype *data;
  int num_threads;
};
```

The header file to include when using this module is nvector_openmp.h. The installed module library to link to is libsundials_nvecopenmp.*lib* where *.lib* is typically .so for shared libraries and .a for static libraries.

The following macros are provided to access the content of an NVECTOR_OPENMP vector. The suffix _OMP in the names denotes the OpenMP version.

- NV_CONTENT_OMP

  This routine gives access to the contents of the OpenMP vector N_Vector.

  The assignment v_cont = NV_CONTENT_OMP(v) sets v_cont to be a pointer to the OpenMP N_Vector content structure.

  Implementation:

  #define NV_CONTENT_OMP(v) ((N_VectorContent_OpenMP)(v->content))

- NV_OWN_DATA_OMP, NV_DATA_OMP, NV_LENGTH_OMP, NV_NUM_THREADS_OMP

  These macros give individual access to the parts of the content of a OpenMP N_Vector.

  The assignment v_data = NV_DATA_OMP(v) sets v_data to be a pointer to the first component of the data for the N_Vector v. The assignment NV_DATA_OMP(v) = v_data sets the component array of v to be v_data by storing the pointer v_data.

  The assignment v_len = NV_LENGTH_OMP(v) sets v_len to be the length of v. On the other hand, the call NV_LENGTH_OMP(v) = len_v sets the length of v to be len_v.

The assignment v_num_threads = NV_NUM_THREADS_OMP(v) sets v_num_threads to be the number of threads from v. On the other hand, the call NV_NUM_THREADS_OMP(v) = num_threads_v sets the number of threads for v to be num_threads_v.

Implementation:

```
#define NV_OWN_DATA_OMP(v) ( NV_CONTENT_OMP(v)->own_data )
```

```
#define NV_DATA_OMP(v) ( NV_CONTENT_OMP(v)->data )
```

```
#define NV_LENGTH_OMP(v) ( NV_CONTENT_OMP(v)->length )
```

```
#define NV_NUM_THREADS_OMP(v) ( NV_CONTENT_OMP(v)->num_threads )
```

- NV_Ith_OMP

  This macro gives access to the individual components of the data array of an N_Vector.

  The assignment r = NV_Ith_OMP(v,i) sets r to be the value of the i-th component of v. The assignment NV_Ith_OMP(v,i) = r sets the value of the i-th component of v to be r.

  Here i ranges from 0 to $n-1$ for a vector of length n.

  Implementation:

  ```
  #define NV_Ith_OMP(v,i) ( NV_DATA_OMP(v)[i] )
  ```

The NVECTOR_OPENMP module defines OpenMP implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4. Their names are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _OpenMP (e.g. N_VDestroy_OpenMP). The module NVECTOR_OPENMP provides the following additional user-callable routines:

- N_VNew_OpenMP

  This function creates and allocates memory for a OpenMP N_Vector. Arguments are the vector length and number of threads.

  ```
  N_Vector N_VNew_OpenMP(sunindextype vec_length, int num_threads);
  ```

- N_VNewEmpty_OpenMP

  This function creates a new OpenMP N_Vector with an empty (NULL) data array.

  ```
  N_Vector N_VNewEmpty_OpenMP(sunindextype vec_length, int num_threads);
  ```

- N_VMake_OpenMP

  This function creates and allocates memory for a OpenMP vector with user-provided data array.

  (This function does *not* allocate memory for v_data itself.)

  ```
  N_Vector N_VMake_OpenMP(sunindextype vec_length, realtype *v_data, int num_threads);
  ```

- N_VCloneVectorArray_OpenMP

  This function creates (by cloning) an array of count OpenMP vectors.

  ```
  N_Vector *N_VCloneVectorArray_OpenMP(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_OpenMP

  This function creates (by cloning) an array of count OpenMP vectors, each with an empty (NULL) data array.

  ```
  N_Vector *N_VCloneVectorArrayEmpty_OpenMP(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_OpenMP

  This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_OpenMP or with N_VCloneVectorArrayEmpty_OpenMP.

  ```
  void N_VDestroyVectorArray_OpenMP(N_Vector *vs, int count);
  ```

- N_VGetLength_OpenMP

  This function returns number of vector elements.

  ```
  sunindextype N_VGetLength_OpenMP(N_Vector v);
  ```

- N_VPrint_OpenMP

  This function prints the content of an OpenMP vector to `stdout`.

  ```
  void N_VPrint_OpenMP(N_Vector v);
  ```

- N_VPrintFile_OpenMP

  This function prints the content of an OpenMP vector to `outfile`.

  ```
  void N_VPrintFile_OpenMP(N_Vector v, FILE *outfile);
  ```

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via v_data = NV_DATA_OMP(v) and then access v_data[i] within the loop than it is to use NV_Ith_OMP(v,i) within the loop.

- N_VNewEmpty_OpenMP, N_VMake_OpenMP, and N_VCloneVectorArrayEmpty_OpenMP set the field *own_data* = SUNFALSE. N_VDestroy_OpenMP and N_VDestroyVectorArray_OpenMP will not attempt to free the pointer *data* for any N_Vector with *own_data* set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the *data* pointer.

- To maximize efficiency, vector operations in the NVECTOR_OPENMP implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

For solvers that include a Fortran interface module, the NVECTOR_OPENMP module also includes a Fortran-callable function FNVINITOMP(code, NEQ, NUMTHREADS, IER), to initialize this module. Here code is an input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type `long int`); NUMTHREADS is the number of threads; and IER is an error return flag equal 0 for success and -1 for failure.

7.4   The NVECTOR_PTHREADS implementation

In situations where a user has a multi-core processing unit capable of running multiple parallel threads with shared memory, SUNDIALS provides an implementation of NVECTOR using OpenMP, called NVECTOR_OPENMP, and an implementation using Pthreads, called NVECTOR_PTHREADS. Testing has shown that vectors should be of length at least $100,000$ before the overhead associated with creating and using the threads is made up by the parallelism in the vector calculations.

The Pthreads NVECTOR implementation provided with SUNDIALS, denoted NVECTOR_PTHREADS, defines the *content* field of N_Vector to be a structure containing the length of the vector, a pointer to the beginning of a contiguous data array, a boolean flag *own_data* which specifies the ownership of *data*, and the number of threads. Operations on the vector are threaded using POSIX threads (Pthreads).

```
struct _N_VectorContent_Pthreads {
  sunindextype length;
  booleantype own_data;
  realtype *data;
  int num_threads;
};
```

The header file to include when using this module is `nvector_pthreads.h`. The installed module library to link to is `libsundials_nvecpthreads.`*`lib`* where *`.lib`* is typically `.so` for shared libraries and `.a` for static libraries.

The following macros are provided to access the content of an NVECTOR_PTHREADS vector. The suffix _PT in the names denotes the Pthreads version.

- NV_CONTENT_PT

  This routine gives access to the contents of the Pthreads vector N_Vector.

  The assignment `v_cont = NV_CONTENT_PT(v)` sets `v_cont` to be a pointer to the Pthreads N_Vector content structure.

  Implementation:

  ```
  #define NV_CONTENT_PT(v) ( (N_VectorContent_Pthreads)(v->content) )
  ```

- NV_OWN_DATA_PT, NV_DATA_PT, NV_LENGTH_PT, NV_NUM_THREADS_PT

  These macros give individual access to the parts of the content of a Pthreads N_Vector.

  The assignment `v_data = NV_DATA_PT(v)` sets `v_data` to be a pointer to the first component of the data for the N_Vector v. The assignment `NV_DATA_PT(v) = v_data` sets the component array of v to be `v_data` by storing the pointer `v_data`.

  The assignment `v_len = NV_LENGTH_PT(v)` sets `v_len` to be the length of v. On the other hand, the call `NV_LENGTH_PT(v) = len_v` sets the length of v to be `len_v`.

  The assignment `v_num_threads = NV_NUM_THREADS_PT(v)` sets `v_num_threads` to be the number of threads from v. On the other hand, the call `NV_NUM_THREADS_PT(v) = num_threads_v` sets the number of threads for v to be `num_threads_v`.

  Implementation:

  ```
  #define NV_OWN_DATA_PT(v) ( NV_CONTENT_PT(v)->own_data )

  #define NV_DATA_PT(v) ( NV_CONTENT_PT(v)->data )

  #define NV_LENGTH_PT(v) ( NV_CONTENT_PT(v)->length )

  #define NV_NUM_THREADS_PT(v) ( NV_CONTENT_PT(v)->num_threads )
  ```

- NV_Ith_PT

  This macro gives access to the individual components of the data array of an N_Vector.

  The assignment `r = NV_Ith_PT(v,i)` sets `r` to be the value of the `i`-th component of v. The assignment `NV_Ith_PT(v,i) = r` sets the value of the `i`-th component of v to be `r`.

  Here i ranges from 0 to $n - 1$ for a vector of length n.

  Implementation:

  ```
  #define NV_Ith_PT(v,i) ( NV_DATA_PT(v)[i] )
  ```

The NVECTOR_PTHREADS module defines Pthreads implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4. Their names are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _Pthreads (e.g. N_VDestroy_Pthreads). The module NVECTOR_PTHREADS provides the following additional user-callable routines:

- N_VNew_Pthreads

  This function creates and allocates memory for a Pthreads N_Vector. Arguments are the vector length and number of threads.

  ```
  N_Vector N_VNew_Pthreads(sunindextype vec_length, int num_threads);
  ```

- N_VNewEmpty_Pthreads

  This function creates a new Pthreads N_Vector with an empty (NULL) data array.

  ```
  N_Vector N_VNewEmpty_Pthreads(sunindextype vec_length, int num_threads);
  ```

- N_VMake_Pthreads

  This function creates and allocates memory for a Pthreads vector with user-provided data array. (This function does *not* allocate memory for v_data itself.)

  ```
  N_Vector N_VMake_Pthreads(sunindextype vec_length, realtype *v_data, int num_threads);
  ```

- N_VCloneVectorArray_Pthreads

  This function creates (by cloning) an array of count Pthreads vectors.

  ```
  N_Vector *N_VCloneVectorArray_Pthreads(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_Pthreads

  This function creates (by cloning) an array of count Pthreads vectors, each with an empty (NULL) data array.

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Pthreads(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_Pthreads

  This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_Pthreads or with N_VCloneVectorArrayEmpty_Pthreads.

  ```
  void N_VDestroyVectorArray_Pthreads(N_Vector *vs, int count);
  ```

- N_VGetLength_Pthreads

  This function returns the number of vector elements.

  ```
  sunindextype N_VGetLength_Pthreads(N_Vector v);
  ```

- N_VPrint_Pthreads

  This function prints the content of a Pthreads vector to stdout.

  ```
  void N_VPrint_Pthreads(N_Vector v);
  ```

- N_VPrintFile_Pthreads

  This function prints the content of a Pthreads vector to outfile.

  ```
  void N_VPrintFile_Pthreads(N_Vector v, FILE *outfile);
  ```

Notes

- When looping over the components of an N_Vector v, it is more efficient to first obtain the component array via v_data = NV_DATA_PT(v) and then access v_data[i] within the loop than it is to use NV_Ith_PT(v,i) within the loop.

- N_VNewEmpty_Pthreads, N_VMake_Pthreads, and N_VCloneVectorArrayEmpty_Pthreads set the field *own_data* = SUNFALSE. N_VDestroy_Pthreads and N_VDestroyVectorArray_Pthreads will not attempt to free the pointer *data* for any N_Vector with *own_data* set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the *data* pointer.

- To maximize efficiency, vector operations in the NVECTOR_PTHREADS implementation that have more than one N_Vector argument do not check for consistent internal representation of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

For solvers that include a Fortran interface module, the NVECTOR_PTHREADS module also includes a Fortran-callable function FNVINITPTS(code, NEQ, NUMTHREADS, IER), to initialize this module. Here code is an input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); NEQ is the problem size (declared so as to match C type long int); NUMTHREADS is the number of threads; and IER is an error return flag equal 0 for success and -1 for failure.

7.5   The NVECTOR_PARHYP implementation

The NVECTOR_PARHYP implementation of the NVECTOR module provided with SUNDIALS is a wrapper around *hypre*'s ParVector class. Most of the vector kernels simply call *hypre* vector operations. The implementation defines the *content* field of N_Vector to be a structure containing the global and local lengths of the vector, a pointer to an object of type hypre_ParVector, an MPI communicator, and a boolean flag *own_parvector* indicating ownership of the *hypre* parallel vector object *x*.

```
struct _N_VectorContent_ParHyp {
  sunindextype local_length;
  sunindextype global_length;
  booleantype own_parvector;
  MPI_Comm comm;
  hypre_ParVector *x;
};
```

The header file to include when using this module is nvector_parhyp.h. The installed module library to link to is libsundials_nvecparhyp.*lib* where .*lib* is typically .so for shared libraries and .a for static libraries.

Unlike native SUNDIALS vector types, NVECTOR_PARHYP does not provide macros to access its member variables. Note that NVECTOR_PARHYP requires SUNDIALS to be built with MPI support.

The NVECTOR_PARHYP module defines implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4, except for N_VSetArrayPointer and N_VGetArrayPointer, because accessing raw vector data is handled by low-level *hypre* functions. As such, this vector is not available for use with SUNDIALS Fortran interfaces. When access to raw vector data is needed, one should extract the *hypre* vector first, and then use *hypre* methods to access the data. Usage examples of NVECTOR_PARHYP are provided in the cvAdvDiff_non_ph.c example program for CVODE [25] and the ark_diurnal_kry_ph.c example program for ARKODE [31].

The names of parhyp methods are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _ParHyp (e.g. N_VDestroy_ParHyp). The module NVECTOR_PARHYP provides the following additional user-callable routines:

- N_VNewEmpty_ParHyp

  This function creates a new parhyp N_Vector with the pointer to the *hypre* vector set to NULL.

  ```
  N_Vector N_VNewEmpty_ParHyp(MPI_Comm comm,
                              sunindextype local_length,
                              sunindextype global_length);
  ```

- N_VMake_ParHyp

  This function creates an N_Vector wrapper around an existing *hypre* parallel vector. It does *not* allocate memory for x itself.

  ```
  N_Vector N_VMake_ParHyp(hypre_ParVector *x);
  ```

- N_VGetVector_ParHyp

  This function returns a pointer to the underlying *hypre* vector.

  ```
  hypre_ParVector *N_VGetVector_ParHyp(N_Vector v);
  ```

- N_VCloneVectorArray_ParHyp

  This function creates (by cloning) an array of count parallel vectors.

  ```
  N_Vector *N_VCloneVectorArray_ParHyp(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_ParHyp

  This function creates (by cloning) an array of `count` parallel vectors, each with an empty (`NULL`) data array.

  ```
  N_Vector *N_VCloneVectorArrayEmpty_ParHyp(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_ParHyp

  This function frees memory allocated for the array of `count` variables of type `N_Vector` created with N_VCloneVectorArray_ParHyp or with N_VCloneVectorArrayEmpty_ParHyp.

  ```
  void N_VDestroyVectorArray_ParHyp(N_Vector *vs, int count);
  ```

- N_VPrint_ParHyp

  This function prints the local content of a parhyp vector to `stdout`.

  ```
  void N_VPrint_ParHyp(N_Vector v);
  ```

- N_VPrintFile_ParHyp

  This function prints the local content of a parhyp vector to `outfile`.

  ```
  void N_VPrintFile_ParHyp(N_Vector v, FILE *outfile);
  ```

Notes

- When there is a need to access components of an `N_Vector_ParHyp`, v, it is recommended to extract the *hypre* vector via `x_vec = N_VGetVector_ParHyp(v)` and then access components using appropriate *hypre* functions.

- N_VNewEmpty_ParHyp, N_VMake_ParHyp, and N_VCloneVectorArrayEmpty_ParHyp set the field *own_parvector* to `SUNFALSE`. N_VDestroy_ParHyp and N_VDestroyVectorArray_ParHyp will not attempt to delete an underlying *hypre* vector for any `N_Vector` with *own_parvector* set to `SUNFALSE`. In such a case, it is the user's responsibility to delete the underlying vector.

- To maximize efficiency, vector operations in the NVECTOR_PARHYP implementation that have more than one `N_Vector` argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with `N_Vector` arguments that were all created with the same internal representations.

7.6 The NVECTOR_PETSC implementation

The NVECTOR_PETSC module is an NVECTOR wrapper around the PETSc vector. It defines the *content* field of a `N_Vector` to be a structure containing the global and local lengths of the vector, a pointer to the PETSc vector, an MPI communicator, and a boolean flag *own_data* indicating ownership of the wrapped PETSc vector.

```
struct _N_VectorContent_Petsc {
  sunindextype local_length;
  sunindextype global_length;
  booleantype own_data;
  Vec *pvec;
  MPI_Comm comm;
};
```

The header file to include when using this module is `nvector_petsc.h`. The installed module library to link to is `libsundials_nvecpetsc.`*`lib`* where *`.lib`* is typically `.so` for shared libraries and `.a` for static libraries.

Unlike native SUNDIALS vector types, NVECTOR_PETSC does not provide macros to access its member variables. Note that NVECTOR_PETSC requires SUNDIALS to be built with MPI support.

The NVECTOR_PETSC module defines implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4, except for N_VGetArrayPointer and N_VSetArrayPointer. As such, this vector cannot be used with SUNDIALS Fortran interfaces. When access to raw vector data is needed, it is recommended to extract the PETSc vector first, and then use PETSc methods to access the data. Usage examples of NVECTOR_PETSC are provided in example programs for IDA [23].

The names of vector operations are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _Petsc (e.g. N_VDestroy_Petsc). The module NVECTOR_PETSC provides the following additional user-callable routines:

- N_VNewEmpty_Petsc

  This function creates a new NVECTOR wrapper with the pointer to the wrapped PETSc vector set to (NULL). It is used by the N_VMake_Petsc and N_VClone_Petsc implementations.

  ```
  N_Vector N_VNewEmpty_Petsc(MPI_Comm comm,
                             sunindextype local_length,
                             sunindextype global_length);
  ```

- N_VMake_Petsc

  This function creates and allocates memory for an NVECTOR_PETSC wrapper around a user-provided PETSc vector. It does *not* allocate memory for the vector pvec itself.

  ```
  N_Vector N_VMake_Petsc(Vec *pvec);
  ```

- N_VGetVector_Petsc

  This function returns a pointer to the underlying PETSc vector.

  ```
  Vec *N_VGetVector_Petsc(N_Vector v);
  ```

- N_VCloneVectorArray_Petsc

  This function creates (by cloning) an array of count NVECTOR_PETSC vectors.

  ```
  N_Vector *N_VCloneVectorArray_Petsc(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_Petsc

  This function creates (by cloning) an array of count NVECTOR_PETSC vectors, each with pointers to PETSc vectors set to (NULL).

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Petsc(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_Petsc

  This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_Petsc or with N_VCloneVectorArrayEmpty_Petsc.

  ```
  void N_VDestroyVectorArray_Petsc(N_Vector *vs, int count);
  ```

- N_VPrint_Petsc

  This function prints the global content of a wrapped PETSc vector to stdout.

  ```
  void N_VPrint_Petsc(N_Vector v);
  ```

- N_VPrintFile_Petsc

  This function prints the global content of a wrapped PETSc vector to fname.

  ```
  void N_VPrintFile_Petsc(N_Vector v, const char fname[]);
  ```

Notes

- When there is a need to access components of an N_Vector_Petsc, v, it is recommeded to extract the PETSc vector via x_vec = N_VGetVector_Petsc(v) and then access components using appropriate PETSc functions.

- The functions N_VNewEmpty_Petsc, N_VMake_Petsc, and N_VCloneVectorArrayEmpty_Petsc set the field *own_data* to SUNFALSE. N_VDestroy_Petsc and N_VDestroyVectorArray_Petsc will not attempt to free the pointer *pvec* for any N_Vector with *own_data* set to SUNFALSE. In such a case, it is the user's responsibility to deallocate the *pvec* pointer.

- To maximize efficiency, vector operations in the NVECTOR_PETSC implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

7.7 The NVECTOR_CUDA implementation

The NVECTOR_CUDA module is an experimental NVECTOR implementation in the CUDA language. The module allows for SUNDIALS vector kernels to run on GPU devices. It is intended for users who are already familiar with CUDA and GPU programming. Building this vector module requires a CUDA compiler and, by extension, a C++ compiler. The class Vector in namespace suncudavec manages vector data layout:

```
template <class T, class I>
class Vector {
  I size_;
  I mem_size_;
  T* h_vec_;
  T* d_vec_;
  StreamPartitioning<T, I>* partStream_;
  ReducePartitioning<T, I>* partReduce_;
  bool ownPartitioning_;

  ...
};
```

The class members are vector size (length), size of the vector data memory block, pointers to vector data on the host and the device, pointers to classes StreamPartitioning and ReducePartitioning, which handle thread partitioning for streaming and reduction vector kernels, respectively, and a boolean flag that signals if the vector owns the thread partitioning. The class Vector inherits from the empty structure

```
struct _N_VectorContent_Cuda {
};
```

to interface the C++ class with the NVECTOR C code. When instantiated, the class Vector will allocate memory on both the host and the device. Due to the rapid progress of CUDA development, we expect that the suncudavec::Vector class will change frequently in future SUNDIALS releases. The code is structured so that it can tolerate significant changes in the suncudavec::Vector class without requiring changes to the user API.

The header file to include when using this module is nvector_cuda.h. The installed module library to link to is libsundials_nveccuda.*lib* where .*lib* is typically .so for shared libraries and .a for static libraries.

Unlike other native SUNDIALS vector types, NVECTOR_CUDA does not provide macros to access its member variables.

The NVECTOR_CUDA module defines implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4, except for `N_VGetArrayPointer` and `N_VSetArrayPointer`. As such, this vector cannot be used with SUNDIALS Fortran interfaces, nor with SUNDIALS direct solvers and preconditioners. This support will be added in subsequent SUNDIALS releases. The NVECTOR_CUDA module provides separate functions to access data on the host and on the device. It also provides methods for copying from the host to the device and vice versa. Usage examples of NVECTOR_CUDA are provided in some example programs for CVODE [25].

The names of vector operations are obtained from those in Tables 7.2, 7.3, and 7.4 by appending the suffix _Cuda (e.g. `N_VDestroy_Cuda`). The module NVECTOR_CUDA provides the following additional user-callable routines:

- `N_VNew_Cuda`

  This function creates and allocates memory for a CUDA `N_Vector`. The memory is allocated on both host and device. Its only argument is the vector length.

  ```
  N_Vector N_VNew_Cuda(sunindextype vec_length);
  ```

- `N_VNewEmpty_Cuda`

  This function creates a new NVECTOR wrapper with the pointer to the wrapped CUDA vector set to (NULL). It is used by the `N_VNew_Cuda`, `N_VMake_Cuda`, and `N_VClone_Cuda` implementations.

  ```
  N_Vector N_VNewEmpty_Cuda(sunindextype vec_length);
  ```

- `N_VMake_Cuda`

  This function creates and allocates memory for an NVECTOR_CUDA wrapper around a user-provided `suncudavec::Vector` class. Its only argument is of type `N_VectorContent_Cuda`, which is the pointer to the class.

  ```
  N_Vector N_VMake_Cuda(N_VectorContent_Cuda c);
  ```

- `N_VCloneVectorArray_Cuda`

  This function creates (by cloning) an array of `count` NVECTOR_CUDA vectors.

  ```
  N_Vector *N_VCloneVectorArray_Cuda(int count, N_Vector w);
  ```

- `N_VCloneVectorArrayEmpty_Cuda`

  This function creates (by cloning) an array of `count` NVECTOR_CUDA vectors, each with pointers to CUDA vectors set to (NULL).

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Cuda(int count, N_Vector w);
  ```

- `N_VDestroyVectorArray_Cuda`

  This function frees memory allocated for the array of `count` variables of type `N_Vector` created with `N_VCloneVectorArray_Cuda` or with `N_VCloneVectorArrayEmpty_Cuda`.

  ```
  void N_VDestroyVectorArray_Cuda(N_Vector *vs, int count);
  ```

- `N_VGetLength_Cuda`

  This function returns the length of the vector.

  ```
  sunindextype N_VGetLength_Cuda(N_Vector v);
  ```

- N_VGetHostArrayPointer_Cuda

  This function returns a pointer to the vector data on the host.

  `realtype *N_VGetHostArrayPointer_Cuda(N_Vector v);`

- N_VGetDeviceArrayPointer_Cuda

  This function returns a pointer to the vector data on the device.

  `realtype *N_VGetDeviceArrayPointer_Cuda(N_Vector v);`

- N_VCopyToDevice_Cuda

  This function copies host vector data to the device.

  `realtype *N_VCopyToDevice_Cuda(N_Vector v);`

- N_VCopyFromDevice_Cuda

  This function copies vector data from the device to the host.

  `realtype *N_VCopyFromDevice_Cuda(N_Vector v);`

- N_VPrint_Cuda

  This function prints the content of a CUDA vector to `stdout`.

  `void N_VPrint_Cuda(N_Vector v);`

- N_VPrintFile_Cuda

  This function prints the content of a CUDA vector to `outfile`.

  `void N_VPrintFile_Cuda(N_Vector v, FILE *outfile);`

Notes

- When there is a need to access components of an N_Vector_Cuda, v, it is recommeded to use functions N_VGetDeviceArrayPointer_Cuda or N_VGetHostArrayPointer_Cuda.

- To maximize efficiency, vector operations in the NVECTOR_CUDA implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

7.8 The NVECTOR_RAJA implementation

The NVECTOR_RAJA module is an experimental NVECTOR implementation using the RAJA hardware abstraction layer, https://software.llnl.gov/RAJA/. In this implementation, RAJA allows for SUNDIALS vector kernels to run on GPU devices. The module is intended for users who are already familiar with RAJA and GPU programming. Building this vector module requires a C++11 compliant compiler and a CUDA software development toolkit. Besides the CUDA backend, RAJA has other backends such as serial, OpenMP, and OpenAC. These backends are not used in this SUNDIALS release. Class `Vector` in namespace `sunrajavec` manages the vector data layout:

```
template <class T, class I>
class Vector {
  I size_;
  I mem_size_;
  T* h_vec_;
  T* d_vec_;

  ...
};
```

The class members are: vector size (length), size of the vector data memory block, and pointers to vector data on the host and on the device. The class `Vector` inherits from an empty structure

```
struct _N_VectorContent_Raja {
};
```

to interface the C++ class with the NVECTOR C code. When instantiated, the class `Vector` will allocate memory on both the host and the device. Due to the rapid progress of RAJA development, we expect that the `sunrajavec::Vector` class will change frequently in future SUNDIALS releases. The code is structured so that it can tolerate significant changes in the `sunrajavec::Vector` class without requiring changes to the user API.

The header file to include when using this module is `nvector_raja.h`. The installed module library to link to is `libsundials_nvecraja.lib` where `.lib` is typically `.so` for shared libraries and `.a` for static libraries.

Unlike other native SUNDIALS vector types, NVECTOR_RAJA does not provide macros to access its member variables.

The NVECTOR_RAJA module defines the implementations of all vector operations listed in Tables 7.2, 7.3, and 7.4, except for `N_VDotProdMulti`, `N_VWrmsNormVectorArray`, and `N_VWrmsNormMaskVectorArray` as support for arrays of reduction vectors is not yet supported in RAJA. These function will be added to the NVECTOR_RAJA implementation in the futrue. Additionally vector `N_VGetArrayPointer` and `N_VSetArrayPointer` are not implemented by the RAJA vector. As such, this vector cannot be used with SUNDIALS Fortran interfaces, nor with SUNDIALS direct solvers and preconditioners. The NVECTOR_RAJA module provides separate functions to access data on the host and on the device. It also provides methods for copying data from the host to the device and vice versa. Usage examples of NVECTOR_RAJA are provided in some example programs for CVODE [25].

The names of vector operations are obtained from those in Tables 7.2, 7.3, and 7.4, by appending the suffix _Raja (e.g. `N_VDestroy_Raja`). The module NVECTOR_RAJA provides the following additional user-callable routines:

- `N_VNew_Raja`

  This function creates and allocates memory for a RAJA `N_Vector`. The memory is allocated on both the host and the device. Its only argument is the vector length.

  ```
  N_Vector N_VNew_Raja(sunindextype vec_length);
  ```

- `N_VNewEmpty_Raja`

  This function creates a new NVECTOR wrapper with the pointer to the wrapped RAJA vector set to (NULL). It is used by the `N_VNew_Raja`, `N_VMake_Raja`, and `N_VClone_Raja` implementations.

  ```
  N_Vector N_VNewEmpty_Raja(sunindextype vec_length);
  ```

- `N_VMake_Raja`

  This function creates and allocates memory for an NVECTOR_RAJA wrapper around a user-provided `sunrajavec::Vector` class. Its only argument is of type `N_VectorContent_Raja`, which is the pointer to the class.

  ```
  N_Vector N_VMake_Raja(N_VectorContent_Raja c);
  ```

- `N_VCloneVectorArray_Raja`

  This function creates (by cloning) an array of `count` NVECTOR_RAJA vectors.

  ```
  N_Vector *N_VCloneVectorArray_Raja(int count, N_Vector w);
  ```

- N_VCloneVectorArrayEmpty_Raja

  This function creates (by cloning) an array of count NVECTOR_RAJA vectors, each with pointers to RAJA vectors set to (NULL).

  ```
  N_Vector *N_VCloneVectorArrayEmpty_Raja(int count, N_Vector w);
  ```

- N_VDestroyVectorArray_Raja

  This function frees memory allocated for the array of count variables of type N_Vector created with N_VCloneVectorArray_Raja or with N_VCloneVectorArrayEmpty_Raja.

  ```
  void N_VDestroyVectorArray_Raja(N_Vector *vs, int count);
  ```

- N_VGetLength_Raja

  This function returns the length of the vector.

  ```
  sunindextype N_VGetLength_Raja(N_Vector v);
  ```

- N_VGetHostArrayPointer_Raja

  This function returns a pointer to the vector data on the host.

  ```
  realtype *N_VGetHostArrayPointer_Raja(N_Vector v);
  ```

- N_VGetDeviceArrayPointer_Raja

  This function returns a pointer to the vector data on the device.

  ```
  realtype *N_VGetDeviceArrayPointer_Raja(N_Vector v);
  ```

- N_VCopyToDevice_Raja

  This function copies host vector data to the device.

  ```
  realtype *N_VCopyToDevice_Raja(N_Vector v);
  ```

- N_VCopyFromDevice_Raja

  This function copies vector data from the device to the host.

  ```
  realtype *N_VCopyFromDevice_Raja(N_Vector v);
  ```

- N_VPrint_Raja

  This function prints the content of a RAJA vector to stdout.

  ```
  void N_VPrint_Raja(N_Vector v);
  ```

- N_VPrintFile_Raja

  This function prints the content of a RAJA vector to outfile.

  ```
  void N_VPrintFile_Raja(N_Vector v, FILE *outfile);
  ```

Notes

- When there is a need to access components of an N_Vector_Raja, v, it is recommeded to use functions N_VGetDeviceArrayPointer_Raja or N_VGetHostArrayPointer_Raja.

- To maximize efficiency, vector operations in the NVECTOR_RAJA implementation that have more than one N_Vector argument do not check for consistent internal representations of these vectors. It is the user's responsibility to ensure that such routines are called with N_Vector arguments that were all created with the same internal representations.

7.9   NVECTOR Examples

There are `NVector` examples that may be installed for the implementations provided with SUNDIALS. Each implementation makes use of the functions in `test_nvector.c`. These example functions show simple usage of the `NVector` family of functions. The input to the examples are the vector length, number of threads (if threaded implementation), and a print timing flag.
The following is a list of the example functions in `test_nvector.c`:

- `Test_N_VClone`: Creates clone of vector and checks validity of clone.

- `Test_N_VCloneEmpty`: Creates clone of empty vector and checks validity of clone.

- `Test_N_VCloneVectorArray`: Creates clone of vector array and checks validity of cloned array.

- `Test_N_VCloneVectorArray`: Creates clone of empty vector array and checks validity of cloned array.

- `Test_N_VGetArrayPointer`: Get array pointer.

- `Test_N_VSetArrayPointer`: Allocate new vector, set pointer to new vector array, and check values.

- `Test_N_VLinearSum` Case 1a: Test $y = x + y$

- `Test_N_VLinearSum` Case 1b: Test $y = -x + y$

- `Test_N_VLinearSum` Case 1c: Test $y = ax + y$

- `Test_N_VLinearSum` Case 2a: Test $x = x + y$

- `Test_N_VLinearSum` Case 2b: Test $x = x - y$

- `Test_N_VLinearSum` Case 2c: Test $x = x + by$

- `Test_N_VLinearSum` Case 3: Test $z = x + y$

- `Test_N_VLinearSum` Case 4a: Test $z = x - y$

- `Test_N_VLinearSum` Case 4b: Test $z = -x + y$

- `Test_N_VLinearSum` Case 5a: Test $z = x + by$

- `Test_N_VLinearSum` Case 5b: Test $z = ax + y$

- `Test_N_VLinearSum` Case 6a: Test $z = -x + by$

- `Test_N_VLinearSum` Case 6b: Test $z = ax - y$

- `Test_N_VLinearSum` Case 7: Test $z = a(x + y)$

- `Test_N_VLinearSum` Case 8: Test $z = a(x - y)$

- `Test_N_VLinearSum` Case 9: Test $z = ax + by$

- `Test_N_VConst`: Fill vector with constant and check result.

- `Test_N_VProd`: Test vector multiply: $z = x * y$

- `Test_N_VDiv`: Test vector division: $z = x / y$

- `Test_N_VScale`: Case 1: scale: $x = cx$

- `Test_N_VScale`: Case 2: copy: $z = x$

- `Test_N_VScale`: Case 3: negate: z = -x

- `Test_N_VScale`: Case 4: combination: z = cx

- `Test_N_VAbs`: Create absolute value of vector.

- `Test_N_VAddConst`: add constant vector: z = c + x

- `Test_N_VDotProd`: Calculate dot product of two vectors.

- `Test_N_VMaxNorm`: Create vector with known values, find and validate the max norm.

- `Test_N_VWrmsNorm`: Create vector of known values, find and validate the weighted root mean square.

- `Test_N_VWrmsNormMask`: Create vector of known values, find and validate the weighted root mean square using all elements except one.

- `Test_N_VMin`: Create vector, find and validate the min.

- `Test_N_VWL2Norm`: Create vector, find and validate the weighted Euclidean L2 norm.

- `Test_N_VL1Norm`: Create vector, find and validate the L1 norm.

- `Test_N_VCompare`: Compare vector with constant returning and validating comparison vector.

- `Test_N_VInvTest`: Test z[i] = 1 / x[i]

- `Test_N_VConstrMask`: Test mask of vector x with vector c.

- `Test_N_VMinQuotient`: Fill two vectors with known values. Calculate and validate minimum quotient.

- `Test_N_VLinearCombination` Case 1a: Test x = a x

- `Test_N_VLinearCombination` Case 1b: Test z = a x

- `Test_N_VLinearCombination` Case 2a: Test x = a x + b y

- `Test_N_VLinearCombination` Case 2b: Test z = a x + b y

- `Test_N_VLinearCombination` Case 3a: Test x = x + a y + b z

- `Test_N_VLinearCombination` Case 3b: Test x = a x + b y + c z

- `Test_N_VLinearCombination` Case 3c: Test w = a x + b y + c z

- `Test_N_VScaleAddMulti` Case 1a: y = a x + y

- `Test_N_VScaleAddMulti` Case 1b: z = a x + y

- `Test_N_VScaleAddMulti` Case 2a: Y[i] = c[i] x + Y[i], i = 1,2,3

- `Test_N_VScaleAddMulti` Case 2b: Z[i] = c[i] x + Y[i], i = 1,2,3

- `Test_N_VDotProdMulti` Case 1: Calculate the dot product of two vectors

- `Test_N_VDotProdMulti` Case 2: Calculate the dot product of one vector with three other vectors in a vector array.

- `Test_N_VLinearSumVectorArray` Case 1: z = a x + b y

- `Test_N_VLinearSumVectorArray` Case 2a: Z[i] = a X[i] + b Y[i]

- `Test_N_VLinearSumVectorArray` Case 2b: X[i] = a X[i] + b Y[i]

- `Test_N_VLinearSumVectorArray` Case 2c: $Y[i] = a\,X[i] + b\,Y[i]$

- `Test_N_VScaleVectorArray` Case 1a: $y = c\,y$

- `Test_N_VScaleVectorArray` Case 1b: $z = c\,y$

- `Test_N_VScaleVectorArray` Case 2a: $Y[i] = c[i]\,Y[i]$

- `Test_N_VScaleVectorArray` Case 2b: $Z[i] = c[i]\,Y[i]$

- `Test_N_VScaleVectorArray` Case 1a: $z = c$

- `Test_N_VScaleVectorArray` Case 1b: $Z[i] = c$

- `Test_N_VWrmsNormVectorArray` Case 1a: Create a vector of know values, find and validate the weighted root mean square norm.

- `Test_N_VWrmsNormVectorArray` Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each.

- `Test_N_VWrmsNormMaskVectorArray` Case 1a: Create a vector of know values, find and validate the weighted root mean square norm using all elements except one.

- `Test_N_VWrmsNormMaskVectorArray` Case 1b: Create a vector array of three vectors of know values, find and validate the weighted root mean square norm of each using all elements except one.

- `Test_N_VScaleAddMultiVectorArray` Case 1a: $y = a\,x + y$

- `Test_N_VScaleAddMultiVectorArray` Case 1b: $z = a\,x + y$

- `Test_N_VScaleAddMultiVectorArray` Case 2a: $Y[j][0] = a[j]\,X[0] + Y[j][0]$

- `Test_N_VScaleAddMultiVectorArray` Case 2b: $Z[j][0] = a[j]\,X[0] + Y[j][0]$

- `Test_N_VScaleAddMultiVectorArray` Case 3a: $Y[0][i] = a[0]\,X[i] + Y[0][i]$

- `Test_N_VScaleAddMultiVectorArray` Case 3b: $Z[0][i] = a[0]\,X[i] + Y[0][i]$

- `Test_N_VScaleAddMultiVectorArray` Case 4a: $Y[j][i] = a[j]\,X[i] + Y[j][i]$

- `Test_N_VScaleAddMultiVectorArray` Case 4b: $Z[j][i] = a[j]\,X[i] + Y[j][i]$

- `Test_N_VLinearCombinationVectorArray` Case 1a: $x = a\,x$

- `Test_N_VLinearCombinationVectorArray` Case 1b: $z = a\,x$

- `Test_N_VLinearCombinationVectorArray` Case 2a: $x = a\,x + b\,y$

- `Test_N_VLinearCombinationVectorArray` Case 2b: $z = a\,x + b\,y$

- `Test_N_VLinearCombinationVectorArray` Case 3a: $x = a\,x + b\,y + c\,z$

- `Test_N_VLinearCombinationVectorArray` Case 3b: $w = a\,x + b\,y + c\,z$

- `Test_N_VLinearCombinationVectorArray` Case 4a: $X[0][i] = c[0]\,X[0][i]$

- `Test_N_VLinearCombinationVectorArray` Case 4b: $Z[i] = c[0]\,X[0][i]$

- `Test_N_VLinearCombinationVectorArray` Case 5a: $X[0][i] = c[0]\,X[0][i] + c[1]\,X[1][i]$

- `Test_N_VLinearCombinationVectorArray` Case 5b: $Z[i] = c[0]\,X[0][i] + c[1]\,X[1][i]$

- `Test_N_VLinearCombinationVectorArray` Case 6a: $X[0][i] = X[0][i] + c[1]\,X[1][i] + c[2]\,X[2][i]$

- `Test_N_VLinearCombinationVectorArray` Case 6b: $X[0][i] = c[0]\,X[0][i] + c[1]\,X[1][i] + c[2]\,X[2][i]$

- `Test_N_VLinearCombinationVectorArray` Case 6c: $Z[i] = c[0]\,X[0][i] + c[1]\,X[1][i] + c[2]\,X[2][i]$

7.10   NVECTOR functions used by IDAS

In Table 7.5 below, we list the vector functions used in the NVECTOR module used by the IDAS package. The table also shows, for each function, which of the code modules uses the function. The IDAS column shows function usage within the main integrator module, while the remaining columns show function usage within each of the IDAS linear solvers interfaces, the IDABBDPRE preconditioner module, and the IDAA module. Here, IDADLS stands for the direct linear solver interface in IDAS, and IDASPILS stands for the scaled, preconditioned, iterative linear solver interface in IDAS.

Table 7.5: List of vector functions usage by IDAS code modules

	IDAS	IDADLS	IDASPILS	IDABBDPRE	IDAA
N_VGetVectorID					
N_VClone	✓		✓	✓	✓
N_VDestroy	✓		✓	✓	✓
N_VCloneVectorArray	✓				✓
N_VDestroyVectorArray	✓				✓
N_VSpace	✓				
N_VGetArrayPointer		✓		✓	
N_VSetArrayPointer		✓			
N_VLinearSum	✓	✓	✓		✓
N_VConst	✓		✓		✓
N_VProd	✓		✓		
N_VDiv	✓		✓		
N_VScale	✓	✓	✓	✓	✓
N_VAbs	✓				
N_VInv	✓				
N_VAddConst	✓				
N_VDotProd			✓		
N_VMaxNorm	✓				
N_VWrmsNorm	✓		✓		
N_VMin	✓				
N_VMinQuotient	✓				
N_VConstrMask	✓				
N_VWrmsNormMask	✓				
N_VCompare	✓				
N_VLinearCombination	✓		✓		
N_VScaleAddMulti	✓				
N_VDotProdMulti			✓		
N_VLinearSumVectorArray	✓				
N_VScaleVectorArray	✓				
N_VConstVectorArray	✓				
N_VWrmsNormVectorArray	✓				
N_VWrmsNormMaskVectorArray	✓				
N_VScaleAddMultiVectorArray	✓				
N_VLinearCombinationVectorArray	✓				

Of the functions listed in Table 7.2, N_VWL2Norm, N_VL1Norm, N_VCloneEmpty, and N_VInvTest are *not* used by IDAS. Therefore a user-supplied NVECTOR module for IDAS could omit these four

functions.

The optional function N␣VDotProdMulti is only used when Classical Gram-Schmidt is enabled with SPGMR or SPFGMR. The remaining operations from Tables 7.3 and 7.4 not listed above are unused and a user-supplied NVECTOR module for IDAS could omit these operations.

Chapter 8

Description of the SUNMatrix module

For problems that involve direct methods for solving linear systems, the SUNDIALS solvers not only operate on generic vectors, but also on generic matrices (of type SUNMatrix), through a set of operations defined by the particular SUNMATRIX implementation. Users can provide their own specific implementation of the SUNMATRIX module, particularly in cases where they provide their own NVECTOR and/or linear solver modules, and require matrices that are compatible with those implementations. Alternately, we provide three SUNMATRIX implementations: dense, banded, and sparse. The generic operations are described below, and descriptions of the implementations provided with SUNDIALS follow.

The generic SUNMatrix type has been modeled after the object-oriented style of the generic N_Vector type. Specifically, a generic SUNMatrix is a pointer to a structure that has an implementation-dependent *content* field containing the description and actual data of the matrix, and an *ops* field pointing to a structure with generic matrix operations. The type SUNMatrix is defined as

```
typedef struct _generic_SUNMatrix *SUNMatrix;

struct _generic_SUNMatrix {
    void *content;
    struct _generic_SUNMatrix_Ops *ops;
};
```

The _generic_SUNMatrix_Ops structure is essentially a list of pointers to the various actual matrix operations, and is defined as

```
struct _generic_SUNMatrix_Ops {
  SUNMatrix_ID (*getid)(SUNMatrix);
  SUNMatrix    (*clone)(SUNMatrix);
  void         (*destroy)(SUNMatrix);
  int          (*zero)(SUNMatrix);
  int          (*copy)(SUNMatrix, SUNMatrix);
  int          (*scaleadd)(realtype, SUNMatrix, SUNMatrix);
  int          (*scaleaddi)(realtype, SUNMatrix);
  int          (*matvec)(SUNMatrix, N_Vector, N_Vector);
  int          (*space)(SUNMatrix, long int*, long int*);
};
```

The generic SUNMATRIX module defines and implements the matrix operations acting on SUNMatrix objects. These routines are nothing but wrappers for the matrix operations defined by a particular SUNMATRIX implementation, which are accessed through the *ops* field of the SUNMatrix structure. To

Table 8.1: Identifiers associated with matrix kernels supplied with SUNDIALS.

Matrix ID	Matrix type	ID Value
SUNMATRIX_DENSE	Dense $M \times N$ matrix	0
SUNMATRIX_BAND	Band $M \times M$ matrix	1
SUNMATRIX_SPARSE	Sparse (CSR or CSC) $M \times N$ matrix	2
SUNMATRIX_CUSTOM	User-provided custom matrix	3

illustrate this point we show below the implementation of a typical matrix operation from the generic SUNMATRIX module, namely SUNMatZero, which sets all values of a matrix A to zero, returning a flag denoting a successful/failed operation:

```
int SUNMatZero(SUNMatrix A)
{
  return((int) A->ops->zero(A));
}
```

Table 8.2 contains a complete list of all matrix operations defined by the generic SUNMATRIX module. A particular implementation of the SUNMATRIX module must:

- Specify the *content* field of the SUNMatrix object.

- Define and implement a minimal subset of the matrix operations. See the documentation for each SUNDIALS solver to determine which SUNMATRIX operations they require.

  Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNMATRIX module (each with different SUNMatrix internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free a SUNMatrix with the new *content* field and with *ops* pointing to the new matrix operations.

- Optionally, define and implement additional user-callable routines acting on the newly defined SUNMatrix (e.g., a routine to print the content for debugging purposes).

- Optionally, provide accessor macros or functions as needed for that particular implementation to access different parts of the *content* field of the newly defined SUNMatrix.

Each SUNMATRIX implementation included in SUNDIALS has a unique identifier specified in enumeration and shown in Table 8.1. It is recommended that a user-supplied SUNMATRIX implementation use the SUNMATRIX_CUSTOM identifier.

Table 8.2: Description of the SUNMatrix operations

Name	Usage and Description
SUNMatGetID	id = SUNMatGetID(A); Returns the type identifier for the matrix A. It is used to determine the matrix implementation type (e.g. dense, banded, sparse,...) from the abstract SUNMatrix interface. This is used to assess compatibility with SUNDIALS-provided linear solver implementations. Returned values are given in the Table 8.1.
	continued on next page

Name	Usage and Description
SUNMatClone	`B = SUNMatClone(A);` Creates a new **SUNMatrix** of the same type as an existing matrix **A** and sets the *ops* field. It does not copy the matrix, but rather allocates storage for the new matrix.
SUNMatDestroy	`SUNMatDestroy(A);` Destroys the **SUNMatrix A** and frees memory allocated for its internal data.
SUNMatSpace	`ier = SUNMatSpace(A, &lrw, &liw);` Returns the storage requirements for the matrix **A**. **lrw** is a **long int** containing the number of realtype words and **liw** is a **long int** containing the number of integer words. The return value is an integer flag denoting success/failure of the operation. This function is advisory only, for use in determining a user's total space requirements; it could be a dummy function in a user-supplied SUNMATRIX module if that information is not of interest.
SUNMatZero	`ier = SUNMatZero(A);` Performs the operation $A_{ij} = 0$ for all entries of the matrix A. The return value is an integer flag denoting success/failure of the operation.
SUNMatCopy	`ier = SUNMatCopy(A,B);` Performs the operation $B_{ij} = A_{i,j}$ for all entries of the matrices A and B. The return value is an integer flag denoting success/failure of the operation.
SUNMatScaleAdd	`ier = SUNMatScaleAdd(c, A, B);` Performs the operation $A = cA + B$. The return value is an integer flag denoting success/failure of the operation.
SUNMatScaleAddI	`ier = SUNMatScaleAddI(c, A);` Performs the operation $A = cA + I$. The return value is an integer flag denoting success/failure of the operation.
SUNMatMatvec	`ier = SUNMatMatvec(A, x, y);` Performs the matrix-vector product operation, $y = Ax$. It should only be called with vectors **x** and **y** that are compatible with the matrix **A** – both in storage type and dimensions. The return value is an integer flag denoting success/failure of the operation.

We note that not all SUNMATRIX types are compatible with all NVECTOR types provided with SUNDIALS. This is primarily due to the need for compatibility within the **SUNMatMatvec** routine; however, compatibility between SUNMATRIX and NVECTOR implementations is more crucial when considering their interaction within SUNLINSOL objects, as will be described in more detail in Chapter 9. More specifically, in Table 8.3 we show the matrix interfaces available as SUNMATRIX modules, and the compatible vector implementations.

Table 8.3: SUNDIALS matrix interfaces and vector implementations that can be used for each.

Matrix Interface	Serial	Parallel (MPI)	OpenMP	pThreads	*hypre* Vec.	PETSc Vec.	CUDA	RAJA	User Suppl.
Dense	✓		✓	✓					✓
continued on next page									

Matrix Interface	Serial	Parallel (MPI)	OpenMP	pThreads	*hypre* Vec.	PETSc Vec.	CUDA	RAJA	User Suppl.
Band	✓		✓	✓					✓
Sparse	✓		✓	✓					✓
User supplied	✓	✓	✓	✓	✓	✓	✓	✓	✓

8.1   The SUNMatrix_Dense implementation

The dense implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX_DENSE, defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Dense {
  sunindextype M;
  sunindextype N;
  realtype *data;
  sunindextype ldata;
  realtype **cols;
};
```

These entries of the *content* field contain the following information:

M - number of rows

N - number of columns

data - pointer to a contiguous block of `realtype` variables. The elements of the dense matrix are stored columnwise, i.e. the (i,j)-th element of a dense SUNMATRIX A (with $0 \leq$ i $<$ M and $0 \leq$ j $<$ N) may be accessed via `data[j*M+i]`.

ldata - length of the data array (= M·N).

cols - array of pointers. `cols[j]` points to the first element of the j-th column of the matrix in the array `data`. The (i,j)-th element of a dense SUNMATRIX A (with $0 \leq$ i $<$ M and $0 \leq$ j $<$ N) may be accessed via `cols[j][i]`.

The header file to include when using this module is **sunmatrix/sunmatrix_dense.h**. The SUNMA-TRIX_DENSE module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunmatrixdense` module library.

The following macros are provided to access the content of a SUNMATRIX_DENSE matrix. The prefix SM_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix _D denotes that these are specific to the *dense* version.

- **SM_CONTENT_D**

  This macro gives access to the contents of the dense `SUNMatrix`.

  The assignment A_cont = SM_CONTENT_D(A) sets A_cont to be a pointer to the dense `SUNMatrix` content structure.

  Implementation:

  ```
  #define SM_CONTENT_D(A)    ( (SUNMatrixContent_Dense)(A->content) )
  ```

- **SM_ROWS_D**, **SM_COLUMNS_D**, and **SM_LDATA_D**

  These macros give individual access to various lengths relevant to the content of a dense `SUNMatrix`.

These may be used either to retrieve or to set these values. For example, the assignment A_rows = SM_ROWS_D(A) sets A_rows to be the number of rows in the matrix A. Similarly, the assignment SM_COLUMNS_D(A) = A_cols sets the number of columns in A to equal A_cols.

Implementation:

```
#define SM_ROWS_D(A)        ( SM_CONTENT_D(A)->M )

#define SM_COLUMNS_D(A)     ( SM_CONTENT_D(A)->N )

#define SM_LDATA_D(A)       ( SM_CONTENT_D(A)->ldata )
```

- SM_DATA_D and SM_COLS_D

  These macros give access to the data and cols pointers for the matrix entries.

  The assignment A_data = SM_DATA_D(A) sets A_data to be a pointer to the first component of the data array for the dense SUNMatrix A. The assignment SM_DATA_D(A) = A_data sets the data array of A to be A_data by storing the pointer A_data.

  Similarly, the assignment A_cols = SM_COLS_D(A) sets A_cols to be a pointer to the array of column pointers for the dense SUNMatrix A. The assignment SM_COLS_D(A) = A_cols sets the column pointer array of A to be A_cols by storing the pointer A_cols.

  Implementation:

```
#define SM_DATA_D(A)        ( SM_CONTENT_D(A)->data )

#define SM_COLS_D(A)        ( SM_CONTENT_D(A)->cols )
```

- SM_COLUMN_D and SM_ELEMENT_D

  These macros give access to the individual columns and entries of the data array of a dense SUNMatrix.

  The assignment col_j = SM_COLUMN_D(A,j) sets col_j to be a pointer to the first entry of the j-th column of the M × N dense matrix A (with $0 \leq j < N$). The type of the expression SM_COLUMN_D(A,j) is realtype *. The pointer returned by the call SM_COLUMN_D(A,j) can be treated as an array which is indexed from 0 to $M - 1$.

  The assignments SM_ELEMENT_D(A,i,j) = a_ij and a_ij = SM_ELEMENT_D(A,i,j) reference the (i,j)-th element of the M × N dense matrix A (with $0 \leq i < M$ and $0 \leq j < N$).

  Implementation:

```
#define SM_COLUMN_D(A,j)    ( (SM_CONTENT_D(A)->cols)[j] )

#define SM_ELEMENT_D(A,i,j) ( (SM_CONTENT_D(A)->cols)[j][i] )
```

The SUNMATRIX_DENSE module defines dense implementations of all matrix operations listed in Table 8.2. Their names are obtained from those in Table 8.2 by appending the suffix _Dense (e.g. SUNMatCopy_Dense). The module SUNMATRIX_DENSE provides the following additional user-callable routines:

- SUNDenseMatrix

  This constructor function creates and allocates memory for a dense SUNMatrix. Its arguments are the number of rows, M, and columns, N, for the dense matrix.

```
SUNMatrix SUNDenseMatrix(sunindextype M, sunindextype N);
```

- SUNDenseMatrix_Print

  This function prints the content of a dense SUNMatrix to the output stream specified by outfile. Note: stdout or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.

```
void SUNDenseMatrix_Print(SUNMatrix A, FILE* outfile);
```

- SUNDenseMatrix_Rows

  This function returns the number of rows in the dense SUNMatrix.

  `sunindextype SUNDenseMatrix_Rows(SUNMatrix A);`

- SUNDenseMatrix_Columns

  This function returns the number of columns in the dense SUNMatrix.

  `sunindextype SUNDenseMatrix_Columns(SUNMatrix A);`

- SUNDenseMatrix_LData

  This function returns the length of the data array for the dense SUNMatrix.

  `sunindextype SUNDenseMatrix_LData(SUNMatrix A);`

- SUNDenseMatrix_Data

  This function returns a pointer to the data array for the dense SUNMatrix.

  `realtype* SUNDenseMatrix_Data(SUNMatrix A);`

- SUNDenseMatrix_Cols

  This function returns a pointer to the cols array for the dense SUNMatrix.

  `realtype** SUNDenseMatrix_Cols(SUNMatrix A);`

- SUNDenseMatrix_Column

  This function returns a pointer to the first entry of the jth column of the dense SUNMatrix. The resulting pointer should be indexed over the range 0 to $M-1$.

  `realtype* SUNDenseMatrix_Column(SUNMatrix A, sunindextype j);`

Notes

- When looping over the components of a dense SUNMatrix A, the most efficient approaches are to:

  - First obtain the component array via A_data = SM_DATA_D(A) or
    A_data = SUNDenseMatrix_Data(A) and then access A_data[i] within the loop.
  - First obtain the array of column pointers via A_cols = SM_COLS_D(A) or
    A_cols = SUNDenseMatrix_Cols(A), and then access A_cols[j][i] within the loop.
  - Within a loop over the columns, access the column pointer via
    A_colj = SUNDenseMatrix_Column(A,j) and then to access the entries within that column using A_colj[i] within the loop.

  All three of these are more efficient than using SM_ELEMENT_D(A,i,j) within a double loop.

⚠️

- Within the SUNMatMatvec_Dense routine, internal consistency checks are performed to ensure that the matrix is called with consistent NVECTOR implementations. These are currently limited to: NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

For solvers that include a Fortran interface module, the SUNMATRIX_DENSE module also includes the Fortran-callable function FSUNDenseMatInit(code, M, N, ier) to initialize this SUNMATRIX_DENSE module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); M and N are the corresponding dense matrix construction arguments (declared to match C type long int); and ier is an error return flag equal to 0 for success and -1 for failure. Both code and ier are declared to match C type int. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNDenseMassMatInit(M, N, ier) initializes this SUNMATRIX_DENSE module for storing the mass matrix.

8.2   The SUNMatrix_Band implementation

The banded implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX_BAND, defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Band {
  sunindextype M;
  sunindextype N;
  sunindextype mu;
  sunindextype ml;
  sunindextype s_mu;
  sunindextype ldim;
  realtype *data;
  sunindextype ldata;
  realtype **cols;
};
```

A diagram of the underlying data representation in a banded matrix is shown in Figure 8.1. A more complete description of the parts of this *content* field is given below:

M - number of rows

N - number of columns (N = M)

mu - upper half-bandwidth, $0 \leq$ mu $<$ N

ml - lower half-bandwidth, $0 \leq$ ml $<$ N

s_mu - storage upper bandwidth, mu $\leq$ s_mu $<$ N. The LU decomposition routines in the associated SUNLINSOL_BAND and SUNLINSOL_LAPACKBAND modules write the LU factors into the storage for A. The upper triangular factor U, however, may have an upper bandwidth as big as min(N-1,mu+ml) because of partial pivoting. The s_mu field holds the upper half-bandwidth allocated for A.

ldim - leading dimension (ldim $\geq$ s_mu+ml+1)

data - pointer to a contiguous block of realtype variables. The elements of the banded matrix are stored columnwise (i.e. columns are stored one on top of the other in memory). Only elements within the specified half-bandwidths are stored. data is a pointer to ldata contiguous locations which hold the elements within the band of A.

ldata - length of the data array ($=$ ldim·N)

cols - array of pointers. cols[j] is a pointer to the uppermost element within the band in the j-th column. This pointer may be treated as an array indexed from s_mu−mu (to access the uppermost element within the band in the j-th column) to s_mu+ml (to access the lowest element within the band in the j-th column). Indices from 0 to s_mu−mu−1 give access to extra storage elements required by the LU decomposition function. Finally, cols[j][i-j+s_mu] is the (i, j)-th element with $j-$mu $\leq i \leq j+$ml.

The header file to include when using this module is sunmatrix/sunmatrix_band.h. The SUNMATRIX_BAND module is accessible from all SUNDIALS solvers *without* linking to the libsundials_sunmatrixband module library.

The following macros are provided to access the content of a SUNMATRIX_BAND matrix. The prefix SM_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix _B denotes that these are specific to the *banded* version.
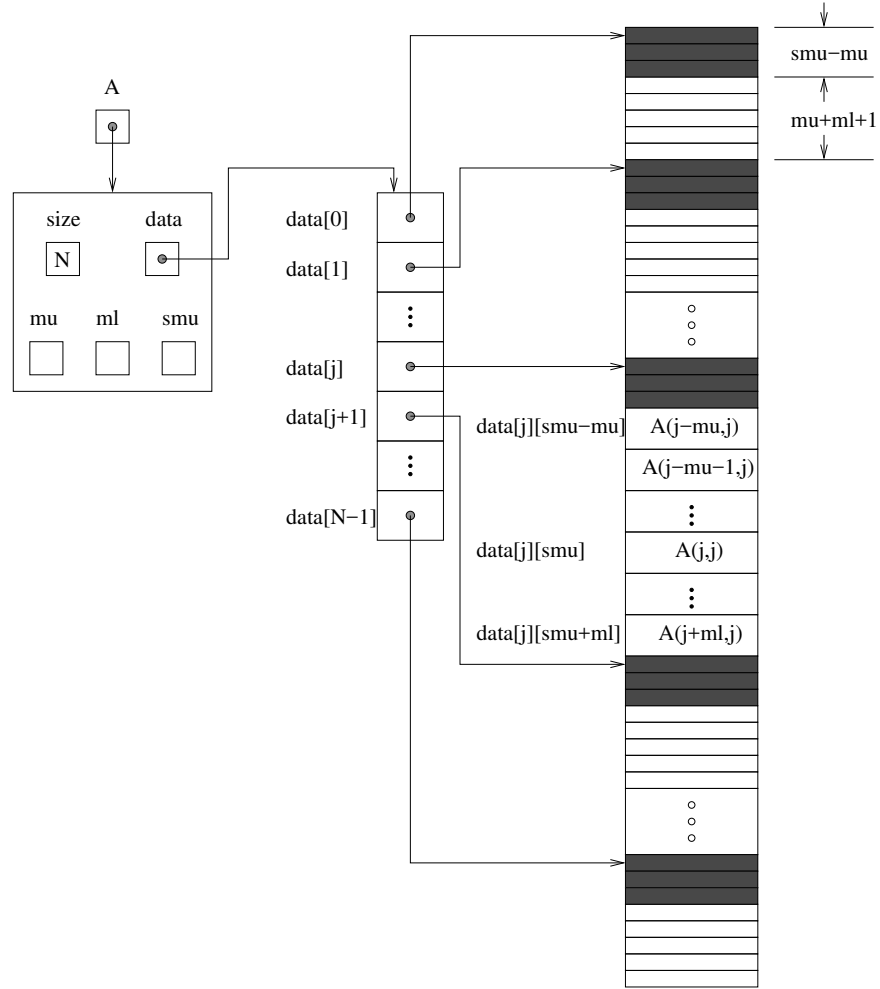
Figure 8.1: Diagram of the storage for the SUNMATRIX_BAND module. Here A is an N × N band matrix with upper and lower half-bandwidths mu and ml, respectively. The rows and columns of A are numbered from 0 to N − 1 and the (i, j)-th element of A is denoted A(i,j). The greyed out areas of the underlying component storage are used by the associated SUNLINSOL_BAND linear solver.

- SM_CONTENT_B

  This routine gives access to the contents of the banded SUNMatrix.

  The assignment A_cont = SM_CONTENT_B(A) sets A_cont to be a pointer to the banded SUNMatrix content structure.

  Implementation:

  ```
  #define SM_CONTENT_B(A)     ( (SUNMatrixContent_Band)(A->content) )
  ```

- SM_ROWS_B, SM_COLUMNS_B, SM_UBAND_B, SM_LBAND_B, SM_SUBAND_B, SM_LDIM_B, and SM_LDATA_B

  These macros give individual access to various lengths relevant to the content of a banded SUNMatrix.

  These may be used either to retrieve or to set these values. For example, the assignment A_rows = SM_ROWS_B(A) sets A_rows to be the number of rows in the matrix A. Similarly, the assignment SM_COLUMNS_B(A) = A_cols sets the number of columns in A to equal A_cols.

  Implementation:

  ```
  #define SM_ROWS_B(A)        ( SM_CONTENT_B(A)->M )
  #define SM_COLUMNS_B(A)     ( SM_CONTENT_B(A)->N )
  #define SM_UBAND_B(A)       ( SM_CONTENT_B(A)->mu )
  #define SM_LBAND_B(A)       ( SM_CONTENT_B(A)->ml )
  #define SM_SUBAND_B(A)      ( SM_CONTENT_B(A)->s_mu )
  #define SM_LDIM_B(A)        ( SM_CONTENT_B(A)->ldim )
  #define SM_LDATA_B(A)       ( SM_CONTENT_B(A)->ldata )
  ```

- SM_DATA_B and SM_COLS_B

  These macros give access to the `data` and `cols` pointers for the matrix entries.

  The assignment A_data = SM_DATA_B(A) sets A_data to be a pointer to the first component of the data array for the banded SUNMatrix A. The assignment SM_DATA_B(A) = A_data sets the data array of A to be A_data by storing the pointer A_data.

  Similarly, the assignment A_cols = SM_COLS_B(A) sets A_cols to be a pointer to the array of column pointers for the banded SUNMatrix A. The assignment SM_COLS_B(A) = A_cols sets the column pointer array of A to be A_cols by storing the pointer A_cols.

  Implementation:

  ```
  #define SM_DATA_B(A)        ( SM_CONTENT_B(A)->data )
  #define SM_COLS_B(A)        ( SM_CONTENT_B(A)->cols )
  ```

- SM_COLUMN_B, SM_COLUMN_ELEMENT_B, and SM_ELEMENT_B

  These macros give access to the individual columns and entries of the data array of a banded SUNMatrix.

  The assignments SM_ELEMENT_B(A,i,j) = a_ij and a_ij = SM_ELEMENT_B(A,i,j) reference the (i,j)-th element of the $N \times N$ band matrix A, where $0 \leq i, j \leq N - 1$. The location (i,j) should further satisfy j$-$mu $\leq$ i $\leq$ j+ml.

  The assignment col_j = SM_COLUMN_B(A,j) sets col_j to be a pointer to the diagonal element of the j-th column of the $N \times N$ band matrix A, $0 \leq j \leq N - 1$. The type of the expression SM_COLUMN_B(A,j) is `realtype *`. The pointer returned by the call SM_COLUMN_B(A,j) can be treated as an array which is indexed from $-$mu to ml.

  The assignments SM_COLUMN_ELEMENT_B(col_j,i,j) = a_ij and a_ij = SM_COLUMN_ELEMENT_B(col_j,i,j) reference the (i,j)-th entry of the band matrix A when used in conjunction with SM_COLUMN_B to reference the j-th column through col_j. The index (i,j) should satisfy j$-$mu $\leq$ i $\leq$ j+ml.

Implementation:

```
#define SM_COLUMN_B(A,j)     ( ((SM_CONTENT_B(A)->cols)[j])+SM_SUBAND_B(A) )

#define SM_COLUMN_ELEMENT_B(col_j,i,j) (col_j[(i)-(j)])

#define SM_ELEMENT_B(A,i,j)
                ( (SM_CONTENT_B(A)->cols)[j][(i)-(j)+SM_SUBAND_B(A)] )
```

The SUNMATRIX_BAND module defines banded implementations of all matrix operations listed in Table 8.2. Their names are obtained from those in Table 8.2 by appending the suffix _Band (e.g. SUNMatCopy_Band). The module SUNMATRIX_BAND provides the following additional user-callable routines:

- SUNBandMatrix

  This constructor function creates and allocates memory for a banded SUNMatrix. Its arguments are the matrix size, N, the upper and lower half-bandwidths of the matrix, mu and ml, and the stored upper bandwidth, smu. When creating a band SUNMatrix, this value should be

  - at least min(N-1,mu+ml) if the matrix will be used by the SUNLINSOL_BAND module;
  - exactly equal to mu+ml if the matrix will be used by the SUNLINSOL_LAPACKBAND module;
  - at least mu if used in some other manner.

  ```
  SUNMatrix SUNBandMatrix(sunindextype N, sunindextype mu,
                          sunindextype ml, sunindextype smu);
  ```

- SUNBandMatrix_Print

  This function prints the content of a banded SUNMatrix to the output stream specified by outfile. Note: stdout or stderr may be used as arguments for outfile to print directly to standard output or standard error, respectively.

  ```
  void SUNBandMatrix_Print(SUNMatrix A, FILE* outfile);
  ```

- SUNBandMatrix_Rows

  This function returns the number of rows in the banded SUNMatrix.

  ```
  sunindextype SUNBandMatrix_Rows(SUNMatrix A);
  ```

- SUNBandMatrix_Columns

  This function returns the number of columns in the banded SUNMatrix.

  ```
  sunindextype SUNBandMatrix_Columns(SUNMatrix A);
  ```

- SUNBandMatrix_LowerBandwidth

  This function returns the lower half-bandwidth of the banded SUNMatrix.

  ```
  sunindextype SUNBandMatrix_LowerBandwidth(SUNMatrix A);
  ```

- SUNBandMatrix_UpperBandwidth

  This function returns the upper half-bandwidth of the banded SUNMatrix.

  ```
  sunindextype SUNBandMatrix_UpperBandwidth(SUNMatrix A);
  ```

- SUNBandMatrix_StoredUpperBandwidth

  This function returns the stored upper half-bandwidth of the banded SUNMatrix.

  ```
  sunindextype SUNBandMatrix_StoredUpperBandwidth(SUNMatrix A);
  ```

- SUNBandMatrix_LDim

  This function returns the length of the leading dimension of the banded SUNMatrix.

  `sunindextype SUNBandMatrix_LDim(SUNMatrix A);`

- SUNBandMatrix_Data

  This function returns a pointer to the data array for the banded SUNMatrix.

  `realtype* SUNBandMatrix_Data(SUNMatrix A);`

- SUNBandMatrix_Cols

  This function returns a pointer to the cols array for the banded SUNMatrix.

  `realtype** SUNBandMatrix_Cols(SUNMatrix A);`

- SUNBandMatrix_Column

  This function returns a pointer to the diagonal entry of the j-th column of the banded SUNMatrix. The resulting pointer should be indexed over the range $-$mu to ml.

  `realtype* SUNBandMatrix_Column(SUNMatrix A, sunindextype j);`

Notes

- When looping over the components of a banded SUNMatrix A, the most efficient approaches are to:

  - First obtain the component array via A_data = SM_DATA_B(A) or
    A_data = SUNBandMatrix_Data(A) and then access A_data[i] within the loop.

  - First obtain the array of column pointers via A_cols = SM_COLS_B(A) or
    A_cols = SUNBandMatrix_Cols(A), and then access A_cols[j][i] within the loop.

  - Within a loop over the columns, access the column pointer via
    A_colj = SUNBandMatrix_Column(A,j) and then to access the entries within that column using SM_COLUMN_ELEMENT_B(A_colj,i,j).

  All three of these are more efficient than using SM_ELEMENT_B(A,i,j) within a double loop.

- Within the SUNMatMatvec_Band routine, internal consistency checks are performed to ensure that the matrix is called with consistent NVECTOR implementations. These are currently limited to: NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

For solvers that include a Fortran interface module, the SUNMATRIX_BAND module also includes the Fortran-callable function FSUNBandMatInit(code, N, mu, ml, smu, ier) to initialize this SUNMATRIX_BAND module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); N, mu, ml and smu are the corresponding band matrix construction arguments (declared to match C type long int); and ier is an error return flag equal to 0 for success and -1 for failure. Both code and ier are declared to match C type int. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNBandMassMatInit(N, mu, ml, smu, ier) initializes this SUNMATRIX_BAND module for storing the mass matrix.

8.3   The SUNMatrix_Sparse implementation

The sparse implementation of the SUNMATRIX module provided with SUNDIALS, SUNMATRIX_SPARSE, is designed to work with either *compressed-sparse-column* (CSC) or *compressed-sparse-row* (CSR) sparse matrix formats. To this end, it defines the *content* field of SUNMatrix to be the following structure:

```
struct _SUNMatrixContent_Sparse {
  sunindextype M;
  sunindextype N;
  sunindextype NNZ;
  sunindextype NP;
  realtype *data;
  int sparsetype;
  sunindextype *indexvals;
  sunindextype *indexptrs;
  /* CSC indices */
  sunindextype **rowvals;
  sunindextype **colptrs;
  /* CSR indices */
  sunindextype **colvals;
  sunindextype **rowptrs;
};
```

A diagram of the underlying data representation for a CSC matrix is shown in Figure 8.2 (the CSR format is similar). A more complete description of the parts of this *content* field is given below:

M - number of rows

N - number of columns

NNZ - maximum number of nonzero entries in the matrix (allocated length of `data` and `indexvals` arrays)

NP - number of index pointers (e.g. number of column pointers for CSC matrix). For CSC matrices `NP = N`, and for CSR matrices `NP = M`. This value is set automatically based the input for `sparsetype`.

data - pointer to a contiguous block of `realtype` variables (of length `NNZ`), containing the values of the nonzero entries in the matrix

sparsetype - type of the sparse matrix (`CSC_MAT` or `CSR_MAT`)

indexvals - pointer to a contiguous block of `int` variables (of length `NNZ`), containing the row indices (if CSC) or column indices (if CSR) of each nonzero matrix entry held in `data`

indexptrs - pointer to a contiguous block of `int` variables (of length `NP+1`). For CSC matrices each entry provides the index of the first column entry into the `data` and `indexvals` arrays, e.g. if `indexptr[3]=7`, then the first nonzero entry in the fourth column of the matrix is located in `data[7]`, and is located in row `indexvals[7]` of the matrix. The last entry contains the total number of nonzero values in the matrix and hence points one past the end of the active data in the `data` and `indexvals` arrays. For CSR matrices, each entry provides the index of the first row entry into the `data` and `indexvals` arrays.

The following pointers are added to the `SlsMat` type for user convenience, to provide a more intuitive interface to the CSC and CSR sparse matrix data structures. They are set automatically when creating a sparse SUNMATRIX, based on the sparse matrix storage type.

rowvals - pointer to `indexvals` when `sparsetype` is `CSC_MAT`, otherwise set to `NULL`.

colptrs - pointer to `indexptrs` when `sparsetype` is `CSC_MAT`, otherwise set to `NULL`.

colvals - pointer to `indexvals` when `sparsetype` is `CSR_MAT`, otherwise set to `NULL`.

rowptrs - pointer to `indexptrs` when `sparsetype` is `CSR_MAT`, otherwise set to `NULL`.

For example, the 5×4 CSC matrix

$$
\begin{bmatrix}
0 & 3 & 1 & 0 \\
3 & 0 & 0 & 2 \\
0 & 7 & 0 & 0 \\
1 & 0 & 0 & 9 \\
0 & 0 & 0 & 5
\end{bmatrix}
$$

could be stored in this structure as either

```
M = 5;
N = 4;
NNZ = 8;
NP = N;
data = {3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0};
sparsetype = CSC_MAT;
indexvals = {1, 3, 0, 2, 0, 1, 3, 4};
indexptrs = {0, 2, 4, 5, 8};
```

or

```
M = 5;
N = 4;
NNZ = 10;
NP = N;
data = {3.0, 1.0, 3.0, 7.0, 1.0, 2.0, 9.0, 5.0, *, *};
sparsetype = CSC_MAT;
indexvals = {1, 3, 0, 2, 0, 1, 3, 4, *, *};
indexptrs = {0, 2, 4, 5, 8};
```

where the first has no unused space, and the second has additional storage (the entries marked with * may contain any values). Note in both cases that the final value in `indexptrs` is 8, indicating the total number of nonzero entries in the matrix.

Similarly, in CSR format, the same matrix could be stored as

```
M = 5;
N = 4;
NNZ = 8;
NP = N;
data = {3.0, 1.0, 3.0, 2.0, 7.0, 1.0, 9.0, 5.0};
sparsetype = CSR_MAT;
indexvals = {1, 2, 0, 3, 1, 0, 3, 3};
indexptrs = {0, 2, 4, 5, 7, 8};
```

The header file to include when using this module is `sunmatrix/sunmatrix_sparse.h`. The SUNMA-TRIX_SPARSE module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunmatrixsparse` module library.

The following macros are provided to access the content of a SUNMATRIX_SPARSE matrix. The prefix SM_ in the names denotes that these macros are for *SUNMatrix* implementations, and the suffix _S denotes that these are specific to the *sparse* version.

- SM_CONTENT_S

  This routine gives access to the contents of the sparse `SUNMatrix`.

  The assignment `A_cont = SM_CONTENT_S(A)` sets `A_cont` to be a pointer to the sparse `SUNMatrix` content structure.

  Implementation:

  ```
  #define SM_CONTENT_S(A)     ( (SUNMatrixContent_Sparse)(A->content) )
  ```
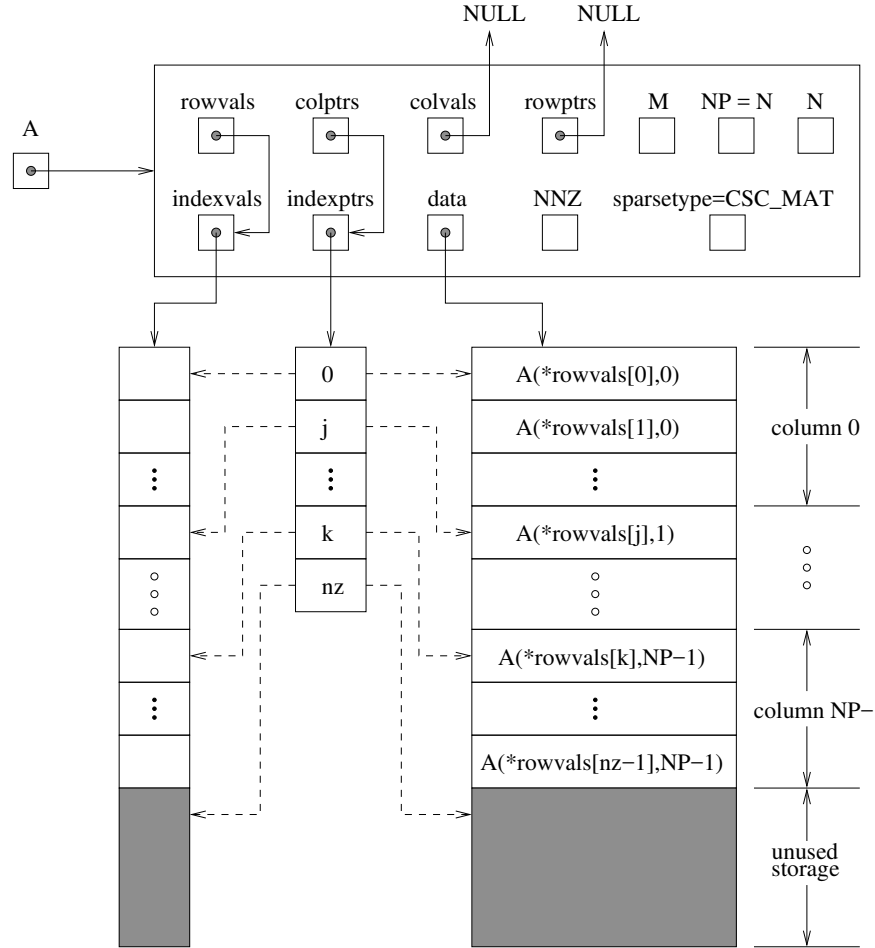
Figure 8.2: Diagram of the storage for a compressed-sparse-column matrix. Here A is an M × N sparse matrix with storage for up to NNZ nonzero entries (the allocated length of both data and indexvals). The entries in indexvals may assume values from 0 to M − 1, corresponding to the row index (zero-based) of each nonzero value. The entries in data contain the values of the nonzero entries, with the row i, column j entry of A (again, zero-based) denoted as A(i,j). The indexptrs array contains N + 1 entries; the first N denote the starting index of each column within the indexvals and data arrays, while the final entry points one past the final nonzero entry. Here, although NNZ values are allocated, only nz are actually filled in; the greyed-out portions of data and indexvals indicate extra allocated space.

- SM_ROWS_S, SM_COLUMNS_S, SM_NNZ_S, SM_NP_S, and SM_SPARSETYPE_S

  These macros give individual access to various lengths relevant to the content of a sparse
  SUNMatrix.

  These may be used either to retrieve or to set these values. For example, the assignment A_rows
  = SM_ROWS_S(A) sets A_rows to be the number of rows in the matrix A. Similarly, the assignment
  SM_COLUMNS_S(A) = A_cols sets the number of columns in A to equal A_cols.

  Implementation:

  ```
  #define SM_ROWS_S(A)        ( SM_CONTENT_S(A)->M )
  #define SM_COLUMNS_S(A)     ( SM_CONTENT_S(A)->N )
  #define SM_NNZ_S(A)         ( SM_CONTENT_S(A)->NNZ )
  #define SM_NP_S(A)          ( SM_CONTENT_S(A)->NP )
  #define SM_SPARSETYPE_S(A)  ( SM_CONTENT_S(A)->sparsetype )
  ```

- SM_DATA_S, SM_INDEXVALS_S, and SM_INDEXPTRS_S

  These macros give access to the data and index arrays for the matrix entries.

  The assignment A_data = SM_DATA_S(A) sets A_data to be a pointer to the first component of
  the data array for the sparse SUNMatrix A. The assignment SM_DATA_S(A) = A_data sets the
  data array of A to be A_data by storing the pointer A_data.

  Similarly, the assignment A_indexvals = SM_INDEXVALS_S(A) sets A_indexvals to be a pointer
  to the array of index values (i.e. row indices for a CSC matrix, or column indices for a CSR
  matrix) for the sparse SUNMatrix A. The assignment A_indexptrs = SM_INDEXPTRS_S(A) sets
  A_indexptrs to be a pointer to the array of index pointers (i.e. the starting indices in the
  data/indexvals arrays for each row or column in CSR or CSC formats, respectively).

  Implementation:

  ```
  #define SM_DATA_S(A)        ( SM_CONTENT_S(A)->data )
  #define SM_INDEXVALS_S(A)   ( SM_CONTENT_S(A)->indexvals )
  #define SM_INDEXPTRS_S(A)   ( SM_CONTENT_S(A)->indexptrs )
  ```

The SUNMATRIX_SPARSE module defines sparse implementations of all matrix operations listed in
Table 8.2. Their names are obtained from those in Table 8.2 by appending the suffix _Sparse (e.g.
SUNMatCopy_Sparse). The module SUNMATRIX_SPARSE provides the following additional user-callable
routines:

- SUNSparseMatrix

  This function creates and allocates memory for a sparse SUNMatrix. Its arguments are the
  number of rows and columns of the matrix, M and N, the maximum number of nonzeros to be
  stored in the matrix, NNZ, and a flag sparsetype indicating whether to use CSR or CSC format
  (valid arguments are CSR_MAT or CSC_MAT).

  ```
  SUNMatrix SUNSparseMatrix(sunindextype M, sunindextype N,
                            sunindextype NNZ, int sparsetype);
  ```

- SUNSparseFromDenseMatrix

  This function creates a new sparse matrix from an existing dense matrix by copying all values
  with magnitude larger than droptol into the sparse matrix structure.

  Requirements:

    – A must have type SUNMATRIX_DENSE;

– `droptol` must be non-negative;

– `sparsetype` must be either CSC_MAT or CSR_MAT.

The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

```
SUNMatrix SUNSparseFromDenseMatrix(SUNMatrix A, realtype droptol,
                                   int sparsetype);
```

- `SUNSparseFromBandMatrix`

  This function creates a new sparse matrix from an existing band matrix by copying all values with magnitude larger than `droptol` into the sparse matrix structure.

  Requirements:

  – `A` must have type SUNMATRIX_BAND;

  – `droptol` must be non-negative;

  – `sparsetype` must be either CSC_MAT or CSR_MAT.

  The function returns NULL if any requirements are violated, or if the matrix storage request cannot be satisfied.

```
SUNMatrix SUNSparseFromBandMatrix(SUNMatrix A, realtype droptol,
                                  int sparsetype);
```

- `SUNSparseMatrix_Realloc`

  This function reallocates internal storage arrays in a sparse matrix so that the resulting sparse matrix has no wasted space (i.e. the space allocated for nonzero entries equals the actual number of nonzeros, `indexptrs[NP]`). Returns 0 on success and 1 on failure (e.g. if the input matrix is not sparse).

  ```
  int SUNSparseMatrix_Realloc(SUNMatrix A);
  ```

- `SUNSparseMatrix_Print`

  This function prints the content of a sparse SUNMatrix to the output stream specified by `outfile`. Note: `stdout` or `stderr` may be used as arguments for `outfile` to print directly to standard output or standard error, respectively.

  ```
  void SUNSparseMatrix_Print(SUNMatrix A, FILE* outfile);
  ```

- `SUNSparseMatrix_Rows`

  This function returns the number of rows in the sparse SUNMatrix.

  ```
  sunindextype SUNSparseMatrix_Rows(SUNMatrix A);
  ```

- `SUNSparseMatrix_Columns`

  This function returns the number of columns in the sparse SUNMatrix.

  ```
  sunindextype SUNSparseMatrix_Columns(SUNMatrix A);
  ```

- `SUNSparseMatrix_NNZ`

  This function returns the number of entries allocated for nonzero storage for the sparse matrix SUNMatrix.

  ```
  sunindextype SUNSparseMatrix_NNZ(SUNMatrix A);
  ```

- SUNSparseMatrix_NP

  This function returns the number of columns/rows for the sparse SUNMatrix, depending on whether the matrix uses CSC/CSR format, respectively. The indexptrs array has NP+1 entries.

  ```
  sunindextype SUNSparseMatrix_NP(SUNMatrix A);
  ```

- SUNSparseMatrix_SparseType

  This function returns the storage type (CSR_MAT or CSC_MAT) for the sparse SUNMatrix.

  ```
  int SUNSparseMatrix_SparseType(SUNMatrix A);
  ```

- SUNSparseMatrix_Data

  This function returns a pointer to the data array for the sparse SUNMatrix.

  ```
  realtype* SUNSparseMatrix_Data(SUNMatrix A);
  ```

- SUNSparseMatrix_IndexValues

  This function returns a pointer to index value array for the sparse SUNMatrix: for CSR format this is the column index for each nonzero entry, for CSC format this is the row index for each nonzero entry.

  ```
  sunindextype* SUNSparseMatrix_IndexValues(SUNMatrix A);
  ```

- SUNSparseMatrix_IndexPointers

  This function returns a pointer to the index pointer array for the sparse SUNMatrix: for CSR format this is the location of the first entry of each row in the data and indexvalues arrays, for CSC format this is the location of the first entry of each column.

  ```
  sunindextype* SUNSparseMatrix_IndexPointers(SUNMatrix A);
  ```

Within the SUNMatMatvec_Sparse routine, internal consistency checks are performed to ensure that the matrix is called with consistent NVECTOR implementations. These are currently limited to: NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS. As additional compatible vector implementations are added to SUNDIALS, these will be included within this compatibility check.

For solvers that include a Fortran interface module, the SUNMATRIX_SPARSE module also includes the Fortran-callable function FSUNSparseMatInit(code, M, N, NNZ, sparsetype, ier) to initialize this SUNMATRIX_SPARSE module for a given SUNDIALS solver. Here code is an integer input for the solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); M, N and NNZ are the corresponding sparse matrix construction arguments (declared to match C type long int); sparsetype is an integer flag indicating the sparse storage type (0 for CSC, 1 for CSR); and ier is an error return flag equal to 0 for success and -1 for failure. Each of code, sparsetype and ier are declared so as to match C type int. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNSparseMassMatInit(M, N, NNZ, sparsetype, ier) initializes this SUNMATRIX_SPARSE module for storing the mass matrix.

8.4 SUNMatrix Examples

There are SUNMatrix examples that may be installed for each implementation: dense, banded, and sparse. Each implementation makes use of the functions in test_sunmatrix.c. These example functions show simple usage of the SUNMatrix family of functions. The inputs to the examples depend on the matrix type, and are output to stdout if the example is run without the appropriate number of command-line arguments.

The following is a list of the example functions in test_sunmatrix.c:

- Test_SUNMatGetID: Verifies the returned matrix ID against the value that should be returned.

- Test_SUNMatClone: Creates clone of an existing matrix, copies the data, and checks that their values match.

- `Test_SUNMatZero`: Zeros out an existing matrix and checks that each entry equals 0.0.

- `Test_SUNMatCopy`: Clones an input matrix, copies its data to a clone, and verifies that all values match.

- `Test_SUNMatScaleAdd`: Given an input matrix A and an input identity matrix I, this test clones and copies A to a new matrix B, computes $B = -B + B$, and verifies that the resulting matrix entries equal 0.0. Additionally, if the matrix is square, this test clones and copies A to a new matrix D, clones and copies I to a new matrix C, computes $D = D + I$ and $C = C + A$ using `SUNMatScaleAdd`, and then verifies that $C == D$.

- `Test_SUNMatScaleAddI`: Given an input matrix A and an input identity matrix I, this clones and copies I to a new matrix B, computes $B = -B + I$ using `SUNMatScaleAddI`, and verifies that the resulting matrix entries equal 0.0.

- `Test_SUNMatMatvec` Given an input matrix A and input vectors x and y such that $y = Ax$, this test has different behavior depending on whether A is square. If it is square, it clones and copies A to a new matrix B, computes $B = 3B + I$ using `SUNMatScaleAddI`, clones y to new vectors w and z, computes $z = Bx$ using `SUNMatMatvec`, computes $w = 3y + x$ using `N_VLinearSum`, and verifies that $w == z$. If A is not square, it just clones y to a new vector z, computes $z = Ax$ using `SUNMatMatvec`, and verifies that $y == z$.

- `Test_SUNMatSpace` verifies that `SUNMatSpace` can be called, and outputs the results to `stdout`.

8.5   SUNMatrix functions used by IDAS

In Table 8.4 below, we list the matrix functions in the SUNMATRIX module used within the IDAS package. The table also shows, for each function, which of the code modules uses the function. Neither the main IDAS integrator or the IDASPILS interface call SUNMATRIX functions directly, so the table columns are specific to the IDADLS direct solver interface and the IDABBDPRE preconditioner module.

    At this point, we should emphasize that the IDAS user does not need to know anything about the usage of matrix functions by the IDAS code modules in order to use IDAS. The information is presented as an implementation detail for the interested reader.

Table 8.4: List of matrix functions usage by IDAS code modules

	IDADLS	IDABBDPRE
`SUNMatGetID`	✓	
`SUNMatDestroy`		✓
`SUNMatZero`	✓	✓
`SUNMatSpace`		†

    The matrix functions listed in Table 8.2 with a † symbol are optionally used, in that these are only called if they are implemented in the SUNMATRIX module that is being used (i.e. their function pointers are non-`NULL`). The matrix functions listed in Table 8.2 that are *not* used by IDAS are: `SUNMatCopy`, `SUNMatClone`, `SUNMatScaleAdd`, `SUNMatScaleAddI` and `SUNMatMatvec`. Therefore a user-supplied SUNMATRIX module for IDAS could omit these functions.

Chapter 9

Description of the SUNLinearSolver module

For problems that involve the solution of linear systems of equations, the SUNDIALS solvers operate using generic linear solver modules (of type `SUNLinearSolver`), through a set of operations defined by the particular SUNLINSOL implementation. These work in coordination with the SUNDIALS generic NVECTOR and SUNMATRIX modules to provide a set of compatible data structures and solvers for the solution of linear systems using direct or iterative methods. Moreover, users can provide their own specific SUNLINSOL implementation to each SUNDIALS solver, particularly in cases where they provide their own NVECTOR and/or SUNMATRIX modules, and the customized linear solver leverages these additional data structures to create highly efficient and/or scalable solvers for their particular problem. Additionally, SUNDIALS provides native implementations SUNLINSOL modules, as well as SUNLINSOL modules that interface between SUNDIALS and external linear solver libraries.

The various SUNDIALS solvers have been designed to specifically leverage the use of either *direct linear solvers* or *scaled, preconditioned, iterative linear solvers*, through their "Dls" and "Spils" interfaces, respectively. Additionally, SUNDIALS solvers can make use of user-supplied custom linear solvers, whether these are problem-specific or come from external solver libraries.

For iterative (and possibly custom) linear solvers, the SUNDIALS solvers leverage scaling and preconditioning, as applicable, to balance error between solution components and to accelerate convergence of the linear solver. To this end, instead of solving the linear system $Ax = b$ directly, we apply the underlying iterative algorithm to the transformed system

$$\tilde{A}\tilde{x} = \tilde{b} \tag{9.1}$$

where

$$
\begin{aligned}
\tilde{A} &= S_1 P_1^{-1} A P_2^{-1} S_2^{-1}, \\
\tilde{b} &= S_1 P_1^{-1} b, \\
\tilde{x} &= S_2 P_2 x,
\end{aligned}
\tag{9.2}
$$

and where

- P_1 is the left preconditioner,

- P_2 is the right preconditioner,

- S_1 is a diagonal matrix of scale factors for $P_1^{-1}b$,

- S_2 is a diagonal matrix of scale factors for $P_2 x$.

The SUNDIALS solvers request that iterative linear solvers stop based on the 2-norm of the scaled preconditioned residual meeting a prescribed tolerance

$$\left\| \tilde{b} - \tilde{A}\tilde{x} \right\|_2 < \text{tol}.$$

We note that not all of the iterative linear solvers implemented in SUNDIALS support the full range of the above options. Similarly, some of the SUNDIALS integrators only utilize a subset of these options. Exceptions to the operators shown above are described in the documentation for each SUNLINSOL implementation, or for each SUNDIALS solver "Spils" interface.

   The generic SUNLinearSolver type has been modeled after the object-oriented style of the generic N_Vector type. Specifically, a generic SUNLinearSolver is a pointer to a structure that has an implementation-dependent *content* field containing the description and actual data of the linear solver, and an *ops* field pointing to a structure with generic linear solver operations. The type SUNLinearSolver is defined as

```
typedef struct _generic_SUNLinearSolver *SUNLinearSolver;
```

```
struct _generic_SUNLinearSolver {
  void *content;
  struct _generic_SUNLinearSolver_Ops *ops;
};
```

The _generic_SUNLinearSolver_Ops structure is essentially a list of pointers to the various actual linear solver operations, and is defined as

```
struct _generic_SUNLinearSolver_Ops {
  SUNLinearSolver_Type (*gettype)(SUNLinearSolver);
  int                  (*setatimes)(SUNLinearSolver, void*, ATimesFn);
  int                  (*setpreconditioner)(SUNLinearSolver, void*,
                                            PSetupFn, PSolveFn);
  int                  (*setscalingvectors)(SUNLinearSolver,
                                            N_Vector, N_Vector);
  int                  (*initialize)(SUNLinearSolver);
  int                  (*setup)(SUNLinearSolver, SUNMatrix);
  int                  (*solve)(SUNLinearSolver, SUNMatrix, N_Vector,
                                N_Vector, realtype);
  int                  (*numiters)(SUNLinearSolver);
  realtype             (*resnorm)(SUNLinearSolver);
  long int             (*lastflag)(SUNLinearSolver);
  int                  (*space)(SUNLinearSolver, long int*, long int*);
  N_Vector             (*resid)(SUNLinearSolver);
  int                  (*free)(SUNLinearSolver);
};
```

   The generic SUNLINSOL module defines and implements the linear solver operations acting on SUNLinearSolver objects. These routines are in fact only wrappers for the linear solver operations defined by a particular SUNLINSOL implementation, which are accessed through the *ops* field of the SUNLinearSolver structure. To illustrate this point we show below the implementation of a typical linear solver operation from the generic SUNLINSOL module, namely SUNLinSolInitialize, which initializes a SUNLINSOL object for use after it has been created and configured, and returns a flag denoting a successful/failed operation:

```
int SUNLinSolInitialize(SUNLinearSolver S)
{
  return ((int) S->ops->initialize(S));
}
```

Table 9.2 contains a complete list of all linear solver operations defined by the generic SUNLINSOL module. In order to support both direct and iterative linear solver types, the generic SUNLINSOL module defines linear solver routines (or arguments) that may be specific to individual use cases. As such, for each routine we specify its intended use. If a custom SUNLINSOL module is provided, the function pointers for non-required routines may be set to NULL to indicate that they are not provided.

   A particular implementation of the SUNLINSOL module must:

Table 9.1: Identifiers associated with linear solver kernels supplied with SUNDIALS.

Linear Solver ID	Solver type	ID Value
SUNLINEARSOLVER_DIRECT	Direct solvers	0
SUNLINEARSOLVER_ITERATIVE	Iterative solvers	1
SUNLINEARSOLVER_CUSTOM	Custom solvers	2

- Specify the *content* field of the `SUNLinearSolver` object.

- Define and implement a minimal subset of the linear solver operations. See the documentation for each SUNDIALS linear solver interface to determine which SUNLINSOL operations they require.

  Note that the names of these routines should be unique to that implementation in order to permit using more than one SUNLINSOL module (each with different `SUNLinearSolver` internal data representations) in the same code.

- Define and implement user-callable constructor and destructor routines to create and free a `SUNLinearSolver` with the new *content* field and with *ops* pointing to the new linear solver operations.

- Optionally, define and implement additional user-callable routines acting on the newly defined `SUNLinearSolver` (e.g., routines to set various configuration options for tuning the linear solver to a particular problem).

- Optionally, provide functions as needed for that particular implementation to access different parts in the *content* field of the newly defined `SUNLinearSolver` object (e.g., routines to return various statistics from the solver).

Each SUNLINSOL implementation included in SUNDIALS has a "type" identifier specified in enumeration and shown in Table 9.1. It is recommended that a user-supplied SUNLINSOL implementation set this identifier based on the SUNDIALS solver interface they intend to use: "Dls" interfaces require the `SUNLINEARSOLVER_DIRECT` SUNLINSOL objects and "Spils" interfaces require the `SUNLINEARSOLVER_ITERATIVE` objects.

Table 9.2: Description of the `SUNLinearSolver` operations

Name	Usage and Description
SUNLinSolGetType	`type = SUNLinSolGetType(LS);` Returns the type identifier for the linear solver `LS`. It is used to determine the solver type (direct, iterative, or custom) from the abstract `SUNLinearSolver` interface. This is used to assess compatibility with SUNDIALS-provided linear solver interfaces. Returned values are given in the Table 9.1.
	continued on next page

Name	Usage and Description
SUNLinSolInitialize	`ier = SUNLinSolInitialize(LS);` Performs linear solver initialization (assumes that all solver-specific options have been set). This should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 9.4.
SUNLinSolSetup	`ier = SUNLinSolSetup(LS, A);` Performs any linear solver setup needed, based on an updated system SUNMATRIX `A`. This may be called frequently (e.g. with a full Newton method) or infrequently (for a modified Newton method), based on the type of integrator and/or nonlinear solver requesting the solves. This should return zero for a successful call, a positive value for a recoverable failure and a negative value for an unrecoverable failure, ideally returning one of the generic error codes listed in Table 9.4.
SUNLinSolSolve	`ier = SUNLinSolSolve(LS, A, x, b, tol);` Solves a linear system $Ax = b$. This should return zero for a successful call, a positive value for a recoverable failure and a negative value for an unrecoverable failure, ideally returning one of the generic error codes listed in Table 9.4. **Direct solvers:** can ignore the `realtype` argument `tol`. **Iterative solvers:** can ignore the SUNMATRIX input `A` since a `NULL` argument will be passed (these should instead rely on the matrix-vector product function supplied through the routine `SUNLinSolSetATimes`). These should attempt to solve to the specified `realtype` tolerance `tol` in a weighted 2-norm. If the solver does not support scaling then it should just use a 2-norm. **Custom solvers:** all arguments will be supplied, and if the solver is approximate then it should attempt to solve to the specified `realtype` tolerance `tol` in a weighted 2-norm. If the solver does not support scaling then it should just use a 2-norm.
SUNLinSolFree	`ier = SUNLinSolFree(LS);` Frees memory allocated by the linear solver. This should return zero for a successful call, and a negative value for a failure.
SUNLinSolSetATimes	`ier = SUNLinSolSetATimes(LS, A_data, ATimes);` (Iterative/Custom linear solvers only) Provides `ATimesFn` function pointer, as well as a `void *` pointer to a data structure used by this routine, to a linear solver object. SUNDIALS solvers will call this function to set the matrix-vector product function to either a solver-provided difference-quotient via vector operations or a user-supplied solver-specific routine. This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 9.4.

Name	Usage and Description
SUNLinSolSetPreconditioner	`ier = SUNLinSolSetPreconditioner(LS, Pdata, Pset, Psol);` (Optional; Iterative/Custom linear solvers only) Provides `PSetupFn` and `PSolveFn` function pointers that implement the preconditioner solves P_1^{-1} and P_2^{-1} from equations (9.1)-(9.2). This routine will be called by a SUNDIALS solver, which will provide translation between the generic `Pset` and `Psol` calls and the integrator-specific and integrator- or user-supplied routines. This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 9.4.
SUNLinSolSetScalingVectors	`ier = SUNLinSolSetScalingVectors(LS, s1, s2);` (Optional; Iterative/Custom linear solvers only) Sets pointers to left/right scaling vectors for the linear system solve. Here, `s1` is an NVECTOR of positive scale factors containing the diagonal of the matrix S_1 from equations (9.1)-(9.2). Similarly, `s2` is an NVECTOR containing the diagonal of S_2 from equations (9.1)-(9.2). Neither of these vectors are tested for positivity, and a NULL argument for either indicates that the corresponding scaling matrix is the identity. This routine should return zero for a successful call, and a negative value for a failure, ideally returning one of the generic error codes listed in Table 9.4.
SUNLinSolNumIters	`its = SUNLinSolNumIters(LS);` (Optional; Iterative/Custom linear solvers only) Should return the `int` number of linear iterations performed in the last 'solve' call.
SUNLinSolResNorm	`rnorm = SUNLinSolResNorm(LS);` (Optional; Iterative/Custom linear solvers only) Should return the `realtype` final residual norm from the last 'solve' call.
SUNLinSolResid	`rvec = SUNLinSolResid(LS);` (Optional; Iterative/Custom linear solvers only) If an iterative method computes the preconditioned initial residual and returns with a successful solve without performing any iterations (i.e. either the initial guess or the preconditioner is sufficiently accurate), then this function may be called by the SUNDIALS solver. This routine should return the NVECTOR containing the preconditioned initial residual vector.
	continued on next page

Name	Usage and Description
SUNLinLastFlag	lflag = SUNLinLastFlag(LS); (Optional) Should return the last error flag encountered within the linear solver. This is not called by the SUNDIALS solvers directly; it allows the user to investigate linear solver issues after a failed solve.
SUNLinSolSpace	ier = SUNLinSolSpace(LS, &lrw, &liw); (Optional) Returns the storage requirements for the linear solver LS. lrw is a long int containing the number of re-altype words and liw is a long int containing the number of integer words. The return value is an integer flag denoting success/failure of the operation. This function is advisory only, for use in determining a user's total space requirements.

9.1   Description of the client-supplied SUNLinearSolver routines

The SUNDIALS packages provide the ATimes, Pset and Psol routines utilized by the SUNLINSOL modules. These function types are defined in the header file sundials/sundials_iterative.h, and are described here in case a user wishes to interact directly with an iterative SUNLINSOL object.

ATimesFn

Definition   typedef int (*ATimesFn)(void *A_data, N_Vector v, N_Vector z);

Purpose   These functions compute the action of a matrix on a vector, performing the operation $z = Av$. Memory for z should already be alloced prior to calling this function. The vector v should be left unchanged.

Arguments   A_data is a pointer to client data, the same as that supplied to SUNLinSolSetATimes.

                 v   is the input vector to multiply.

                 z   is the output vector computed.

Return value This routine should return 0 if successful and a non-zero value if unsuccessful.

Notes

PSetupFn

Definition   typedef int (*PSetupFn)(void *P_data)

Purpose   These functions set up any requisite problem data in preparation for calls to the corresponding PSolveFn.

Arguments   P_data is a pointer to client data, the same pointer as that supplied to the routine SUNLinSolSetPreconditioner.

Return value This routine should return 0 if successful and a non-zero value if unsuccessful.

Notes

PSolveFn

Definition   `typedef int (*PSolveFn)(void *P_data, N_Vector r, N_Vector z,`
                              `realtype tol, int lr)`

Purpose   These functions solve the preconditioner equation $Pz = r$ for the vector z. Memory for
              `z` should already be alloced prior to calling this function. The parameter `P_data` is a
              pointer to any information about P which the function needs in order to do its job (set
              up by the corresponding `PSetupFn`. The parameter `lr` is input, and indicates whether
              P is to be taken as the left preconditioner or the right preconditioner: `lr` $= 1$ for left
              and `lr` $= 2$ for right. If preconditioning is on one side only, `lr` can be ignored. If the
              preconditioner is iterative, then it should strive to solve the preconditioner equation so
              that
              $$\|Pz - r\|_{\mathrm{wrms}} < tol$$
              where the weight vector for the WRMS norm may be accessed from the main package
              memory structure. The vector `r` should not be modified by the `PSolveFn`.

Arguments   `P_data` is a pointer to client data, the same pointer as that supplied to the routine
                  `SUNLinSolSetPreconditioner`.
              `r`   is the right-hand side vector for the preconditioner system
              `z`   is the solution vector for the preconditioner system
              `tol` is the desired tolerance for an iterative preconditioner
              `lr` is flag indicating whether the routine should perform left (1) or right (2) precondi-
                  tioning.

Return value This routine should return 0 if successful and a non-zero value if unsuccessful. On a
              failure, a negative return value indicates an unrecoverable condition, while a positive
              value indicates a recoverable one, in which the calling routine may reattempt the solution
              after updating preconditioner data.

Notes

9.2 Compatibility of SUNLinearSolver modules

We note that not all SUNLINSOL types are compatible with all SUNMATRIX and NVECTOR types provided
with SUNDIALS. In Table 9.3 we show the direct linear solvers available as SUNLINSOL modules, and
the compatible matrix implementations. Recall that Table 4.1 shows the compatibility between all
SUNLINSOL modules and vector implementations.

Table 9.3: SUNDIALS direct linear solvers and matrix implementations that can be used for each.

Linear Solver Interface	Dense Matrix	Banded Matrix	Sparse Matrix	User Supplied
Dense	✓			✓
Band		✓		✓
LapackDense	✓			✓
LapackBand		✓		✓
KLU			✓	✓
SUPERLUMT			✓	✓
continued on next page				

Linear Solver Interface	Dense Matrix	Banded Matrix	Sparse Matrix	User Supplied
User supplied	✓	✓	✓	✓

The functions within the SUNDIALS-provided `SUNLinearSolver` implementations return a common set of error codes, shown below in the Table 9.4.

Table 9.4: Description of the `SUNLinearSolver` error codes

Name	Value	Description
SUNLS_SUCCESS	0	successful call or converged solve
SUNLS_MEM_NULL	-1	the memory argument to the function is NULL
SUNLS_ILL_INPUT	-2	an illegal input has been provided to the function
SUNLS_MEM_FAIL	-3	failed memory access or allocation
SUNLS_ATIMES_FAIL_UNREC	-4	an unrecoverable failure occurred in the `ATimes` routine
SUNLS_PSET_FAIL_UNREC	-5	an unrecoverable failure occurred in the `Pset` routine
SUNLS_PSOLVE_FAIL_UNREC	-6	an unrecoverable failure occurred in the `Psolve` routine
SUNLS_PACKAGE_FAIL_UNREC	-7	an unrecoverable failure occurred in an external linear solver package
SUNLS_GS_FAIL	-8	a failure occurred during Gram-Schmidt orthogonalization (SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)
SUNLS_QRSOL_FAIL	-9	a singular R matrix was encountered in a QR factorization (SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)
SUNLS_RES_REDUCED	1	an iterative solver reduced the residual, but did not converge to the desired tolerance
SUNLS_CONV_FAIL	2	an iterative solver did not converge (and the residual was not reduced)
SUNLS_ATIMES_FAIL_REC	3	a recoverable failure occurred in the `ATimes` routine
SUNLS_PSET_FAIL_REC	4	a recoverable failure occurred in the `Pset` routine
SUNLS_PSOLVE_FAIL_REC	5	a recoverable failure occurred in the `Psolve` routine
SUNLS_PACKAGE_FAIL_REC	6	a recoverable failure occurred in an external linear solver package
SUNLS_QRFACT_FAIL	7	a singular matrix was encountered during a QR factorization (SUNLINSOL_SPGMR/SUNLINSOL_SPFGMR)
SUNLS_LUFACT_FAIL	8	a singular matrix was encountered during a LU factorization (SUNLINSOL_DENSE/SUNLINSOL_BAND)

9.3   The SUNLinearSolver_Dense implementation

The dense implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_DENSE, is designed to be used with the corresponding SUNMATRIX_DENSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP or NVECTOR_PTHREADS). The SUNLINSOL_DENSE module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_Dense {
  sunindextype N;
  sunindextype *pivots;
```

```
  long int last_flag;
};
```

These entries of the *content* field contain the following information:

N - size of the linear system,

pivots - index array for partial pivoting in LU factorization,

last_flag - last error return flag from internal function evaluations.

This solver is constructed to perform the following operations:

- The "setup" call performs a LU factorization with partial (row) pivoting ($\mathcal{O}(N^3)$ cost), $PA = LU$, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_DENSE object A, with pivoting information encoding P stored in the `pivots` array.

- The "solve" call performs pivoting and forward and backward substitution using the stored `pivots` array and the LU factors held in the SUNMATRIX_DENSE object ($\mathcal{O}(N^2)$ cost).

The header file to include when using this module is `sunlinsol/sunlinsol_dense.h`. The SUNLINSOL_DENSE module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunlinsoldense` module library.

The SUNLINSOL_DENSE module defines dense implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_Dense`

- `SUNLinSolInitialize_Dense` – this does nothing, since all consistency checks are performed at solver creation.

- `SUNLinSolSetup_Dense` – this performs the LU factorization.

- `SUNLinSolSolve_Dense` – this uses the LU factors and `pivots` array to perform the solve.

- `SUNLinSolLastFlag_Dense`

- `SUNLinSolSpace_Dense` – this only returns information for the storage *within* the solver object, i.e. storage for N, `last_flag`, and `pivots`.

- `SUNLinSolFree_Dense`

The module SUNLINSOL_DENSE provides the following additional user-callable constructor routine:

- `SUNDenseLinearSolver`

  This function creates and allocates memory for a dense `SUNLinearSolver`. Its arguments are an NVECTOR and SUNMATRIX, that it uses to determine the linear system size and to assess compatibility with the linear solver implementation.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_DENSE matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  If either `A` or `y` are incompatible then this routine will return `NULL`.

  `SUNLinearSolver SUNDenseLinearSolver(N_Vector y, SUNMatrix A);`

For solvers that include a Fortran interface module, the SUNLINSOL_DENSE module also includes the Fortran-callable function `FSUNDenseLinSolInit(code, ier)` to initialize this SUNLINSOL_DENSE module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); `ier` is an error return flag equal to 0 for success and -1 for failure. Both `code` and `ier` are declared to match C type `int`. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function `FSUNMassDenseLinSolInit(ier)` initializes this SUNLINSOL_DENSE module for solving mass matrix linear systems.

9.4   The SUNLinearSolver_Band implementation

The band implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_BAND, is designed to be used with the corresponding SUNMATRIX_BAND matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP or NVECTOR_PTHREADS). The SUNLINSOL_BAND module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_Band {
  sunindextype N;
  sunindextype *pivots;
  long int last_flag;
};
```

These entries of the *content* field contain the following information:

N - size of the linear system,

pivots - index array for partial pivoting in LU factorization,

last_flag - last error return flag from internal function evaluations.

   This solver is constructed to perform the following operations:

- The "setup" call performs a *LU* factorization with partial (row) pivoting, $PA = LU$, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_BAND object A, with pivoting information encoding P stored in the `pivots` array.

- The "solve" call performs pivoting and forward and backward substitution using the stored `pivots` array and the *LU* factors held in the SUNMATRIX_BAND object.

- A must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More precisely, if A is a band matrix with upper bandwidth `mu` and lower bandwidth `ml`, then the upper triangular factor U can have upper bandwidth as big as `smu = MIN(N-1,mu+ml)`. The lower triangular factor L has lower bandwidth `ml`.

The header file to include when using this module is `sunlinsol/sunlinsol_band.h`. The SUNLINSOL_BAND module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunlinsolband` module library.

The SUNLINSOL_BAND module defines band implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_Band`

- `SUNLinSolInitialize_Band` – this does nothing, since all consistency checks are performed at solver creation.

- `SUNLinSolSetup_Band` – this performs the *LU* factorization.

- SUNLinSolSolve_Band – this uses the *LU* factors and `pivots` array to perform the solve.

- SUNLinSolLastFlag_Band

- SUNLinSolSpace_Band – this only returns information for the storage *within* the solver object, i.e. storage for N, `last_flag`, and `pivots`.

- SUNLinSolFree_Band

The module SUNLINSOL_BAND provides the following additional user-callable constructor routine:

- SUNBandLinearSolver

  This function creates and allocates memory for a band `SUNLinearSolver`. Its arguments are an NVECTOR and SUNMATRIX, that it uses to determine the linear system size and to assess compatibility with the linear solver implementation.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_BAND matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  Additionally, this routine will verify that the input matrix A is allocated with appropriate upper bandwidth storage for the *LU* factorization.

  If either A or y are incompatible then this routine will return NULL.

  `SUNLinearSolver SUNBandLinearSolver(N_Vector y, SUNMatrix A);`

For solvers that include a Fortran interface module, the SUNLINSOL_BAND module also includes the Fortran-callable function `FSUNBandLinSolInit(code, ier)` to initialize this SUNLINSOL_BAND module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); `ier` is an error return flag equal to 0 for success and -1 for failure. Both `code` and `ier` are declared to match C type `int`. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function `FSUNMassBandLinSolInit(ier)` initializes this SUNLINSOL_BAND module for solving mass matrix linear systems.

9.5 The SUNLinearSolver_LapackDense implementation

The LAPACK dense implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_LAPACKDENSE, is designed to be used with the corresponding SUNMATRIX_DENSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS). The SUNLINSOL_LAPACKDENSE module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_Dense {
  sunindextype N;
  sunindextype *pivots;
  long int last_flag;
};
```

These entries of the *content* field contain the following information:

N - size of the linear system,

pivots - index array for partial pivoting in LU factorization,

last_flag - last error return flag from internal function evaluations.

⚠️

The SUNLINSOL_LAPACKDENSE module is a SUNLINSOL wrapper for the LAPACK dense matrix factorization and solve routines, `*GETRF` and `*GETRS`, where `*` is either `D` or `S`, depending on whether SUNDIALS was configured to have `realtype` set to `double` or `single`, respectively (see Section 4.2). In order to use the SUNLINSOL_LAPACKDENSE module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see Appendix A for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using `extended` precision for `realtype`. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLINSOL_LAPACKDENSE module also cannot be compiled when using `int64_t` for the `sunindextype`.

This solver is constructed to perform the following operations:

- The "setup" call performs a LU factorization with partial (row) pivoting ($\mathcal{O}(N^3)$ cost), $PA = LU$, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_DENSE object A, with pivoting information encoding P stored in the `pivots` array.

- The "solve" call performs pivoting and forward and backward substitution using the stored `pivots` array and the LU factors held in the SUNMATRIX_DENSE object ($\mathcal{O}(N^2)$ cost).

The header file to include when using this module is `sunlinsol/sunlinsol_lapackdense.h`. The installed module library to link to is `libsundials_sunlinsollapackdense.`*`lib`* where *`.lib`* is typically `.so` for shared libraries and `.a` for static libraries.

The SUNLINSOL_LAPACKDENSE module defines dense implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_LapackDense`

- `SUNLinSolInitialize_LapackDense` – this does nothing, since all consistency checks are performed at solver creation.

- `SUNLinSolSetup_LapackDense` – this calls either `DGETRF` or `SGETRF` to perform the LU factorization.

- `SUNLinSolSolve_LapackDense` – this calls either `DGETRS` or `SGETRS` to use the LU factors and `pivots` array to perform the solve.

- `SUNLinSolLastFlag_LapackDense`

- `SUNLinSolSpace_LapackDense` – this only returns information for the storage *within* the solver object, i.e. storage for `N`, `last_flag`, and `pivots`.

- `SUNLinSolFree_LapackDense`

The module SUNLINSOL_LAPACKDENSE provides the following additional user-callable constructor routine:

- `SUNLapackDense`

  This function creates and allocates memory for a LAPACK dense `SUNLinearSolver`. Its arguments are an NVECTOR and SUNMATRIX, that it uses to determine the linear system size and to assess compatibility with the linear solver implementation.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_DENSE matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  If either `A` or `y` are incompatible then this routine will return `NULL`.

  `SUNLinearSolver SUNLapackDense(N_Vector y, SUNMatrix A);`

For solvers that include a Fortran interface module, the SUNLINSOL_LAPACKDENSE module also includes the Fortran-callable function FSUNLapackDenseInit(code, ier) to initialize this SUNLINSOL_LAPACKDENSE module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); ier is an error return flag equal to 0 for success and -1 for failure. Both code and ier are declared to match C type int. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNMassLapackDenseInit(ier) initializes this SUNLINSOL_LAPACKDENSE module for solving mass matrix linear systems.

9.6 The SUNLinearSolver_LapackBand implementation

The LAPACK band implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_LAPACKBAND, is designed to be used with the corresponding SUNMATRIX_BAND matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS). The SUNLINSOL_LAPACKBAND module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_Band {
  sunindextype N;
  sunindextype *pivots;
  long int last_flag;
};
```

These entries of the *content* field contain the following information:

N - size of the linear system,

pivots - index array for partial pivoting in LU factorization,

last_flag - last error return flag from internal function evaluations.

The SUNLINSOL_LAPACKBAND module is a SUNLINSOL wrapper for the LAPACK band matrix factorization and solve routines, *GBTRF and *GBTRS, where * is either D or S, depending on whether SUNDIALS was configured to have realtype set to double or single, respectively (see Section 4.2). In order to use the SUNLINSOL_LAPACKBAND module it is assumed that LAPACK has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with LAPACK (see Appendix A for details). We note that since there do not exist 128-bit floating-point factorization and solve routines in LAPACK, this interface cannot be compiled when using extended precision for realtype. Similarly, since there do not exist 64-bit integer LAPACK routines, the SUNLINSOL_LAPACKBAND module also cannot be compiled when using int64_t for the sunindextype.
This solver is constructed to perform the following operations:

- The "setup" call performs a *LU* factorization with partial (row) pivoting, $PA = LU$, where P is a permutation matrix, L is a lower triangular matrix with 1's on the diagonal, and U is an upper triangular matrix. This factorization is stored in-place on the input SUNMATRIX_BAND object A, with pivoting information encoding P stored in the pivots array.

- The "solve" call performs pivoting and forward and backward substitution using the stored pivots array and the *LU* factors held in the SUNMATRIX_BAND object.

- A must be allocated to accommodate the increase in upper bandwidth that occurs during factorization. More precisely, if A is a band matrix with upper bandwidth mu and lower bandwidth ml, then the upper triangular factor U can have upper bandwidth as big as smu = MIN(N-1,mu+ml). The lower triangular factor L has lower bandwidth ml.

The header file to include when using this module is `sunlinsol/sunlinsol_lapackband.h`. The installed module library to link to is `libsundials_sunlinsollapackband.`*`lib`* where *`.lib`* is typically `.so` for shared libraries and `.a` for static libraries.

The SUNLINSOL_LAPACKBAND module defines band implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_LapackBand`

- `SUNLinSolInitialize_LapackBand` – this does nothing, since all consistency checks are performed at solver creation.

- `SUNLinSolSetup_LapackBand` – this calls either `DGBTRF` or `SGBTRF` to perform the *LU* factorization.

- `SUNLinSolSolve_LapackBand` – this calls either `DGBTRS` or `SGBTRS` to use the *LU* factors and `pivots` array to perform the solve.

- `SUNLinSolLastFlag_LapackBand`

- `SUNLinSolSpace_LapackBand` – this only returns information for the storage *within* the solver object, i.e. storage for `N`, `last_flag`, and `pivots`.

- `SUNLinSolFree_LapackBand`

The module SUNLINSOL_LAPACKBAND provides the following additional user-callable routine:

- `SUNLapackBand`

  This function creates and allocates memory for a LAPACK band `SUNLinearSolver`. Its arguments are an NVECTOR and SUNMATRIX, that it uses to determine the linear system size and to assess compatibility with the linear solver implementation.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_BAND matrix type and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  Additionally, this routine will verify that the input matrix `A` is allocated with appropriate upper bandwidth storage for the *LU* factorization.

  If either `A` or `y` are incompatible then this routine will return `NULL`.

  `SUNLinearSolver SUNLapackBand(N_Vector y, SUNMatrix A);`

For solvers that include a Fortran interface module, the SUNLINSOL_LAPACKBAND module also includes the Fortran-callable function `FSUNLapackBandInit(code, ier)` to initialize this SUNLINSOL_LAPACKBAND module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); `ier` is an error return flag equal to 0 for success and -1 for failure. Both `code` and `ier` are declared to match C type `int`. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function `FSUNMassLapackBandInit(ier)` initializes this SUNLINSOL_LAPACKBAND module for solving mass matrix linear systems.

9.7   The SUNLinearSolver_KLU implementation

The KLU implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_KLU, is designed to be used with the corresponding SUNMATRIX_SPARSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS). The SUNLINSOL_KLU module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_KLU {
  long int          last_flag;
  int               first_factorize;
  sun_klu_symbolic *symbolic;
  sun_klu_numeric  *numeric;
  sun_klu_common    common;
  sunindextype      (*klu_solver)(sun_klu_symbolic*, sun_klu_numeric*,
                                  sunindextype, sunindextype,
                                  double*, sun_klu_common*);
};
```

These entries of the *content* field contain the following information:

last_flag - last error return flag from internal function evaluations,

first_factorize - flag indicating whether the factorization has ever been performed,

Symbolic - KLU storage structure for symbolic factorization components,

Numeric - KLU storage structure for numeric factorization components,

Common - storage structure for common KLU solver components,

klu_solver – pointer to the appropriate KLU solver function (depending on whether it is using a CSR or CSC sparse matrix).

The SUNLINSOL_KLU module is a SUNLINSOL wrapper for the KLU sparse matrix factorization and solver library written by Tim Davis [1, 14]. In order to use the SUNLINSOL_KLU interface to KLU, it is assumed that KLU has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with KLU (see Appendix A for details). Additionally, this wrapper only supports double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have `realtype` set to either `extended` or `single` (see Section 4.2). Since the KLU library supports both 32-bit and 64-bit integers, this interface will be compiled for either of the available `sunindextype` options.

The KLU library has a symbolic factorization routine that computes the permutation of the linear system matrix to block triangular form and the permutations that will pre-order the diagonal blocks (the only ones that need to be factored) to reduce fill-in (using AMD, COLAMD, CHOLAMD, natural, or an ordering given by the user). Of these ordering choices, the default value in the SUNLINSOL_KLU module is the COLAMD ordering.

KLU breaks the factorization into two separate parts. The first is a symbolic factorization and the second is a numeric factorization that returns the factored matrix along with final pivot information. KLU also has a refactor routine that can be called instead of the numeric factorization. This routine will reuse the pivot information. This routine also returns diagnostic information that a user can examine to determine if numerical stability is being lost and a full numerical factorization should be done instead of the refactor.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLINSOL_KLU module is constructed to perform the following operations:

- The first time that the "setup" routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.

- On subsequent calls to the "setup" routine, it calls the appropriate KLU "refactor" routine, followed by estimates of the numerical conditioning using the relevant "rcond", and if necessary "condest", routine(s). If these estimates of the condition number are larger than $\varepsilon^{-2/3}$ (where ε is the double-precision unit roundoff), then a new factorization is performed.

- The module includes the routine `SUNKLUReInit`, that can be called by the user to force a full refactorization at the next "setup" call.

- The "solve" call performs pivoting and forward and backward substitution using the stored KLU data structures. We note that in this solve KLU operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

The header file to include when using this module is `sunlinsol/sunlinsol_klu.h`. The installed module library to link to is `libsundials_sunlinsolklu.`*lib* where *.lib* is typically `.so` for shared libraries and `.a` for static libraries.

The SUNLINSOL_KLU module defines implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_KLU`

- `SUNLinSolInitialize_KLU` – this sets the `first_factorize` flag to 1, forcing both symbolic and numerical factorizations on the subsequent "setup" call.

- `SUNLinSolSetup_KLU` – this performs either a *LU* factorization or refactorization of the input matrix.

- `SUNLinSolSolve_KLU` – this calls the appropriate KLU solve routine to utilize the *LU* factors to solve the linear system.

- `SUNLinSolLastFlag_KLU`

- `SUNLinSolSpace_KLU` – this only returns information for the storage within the solver *interface*, i.e. storage for the integers `last_flag` and `first_factorize`. For additional space requirements, see the KLU documentation.

- `SUNLinSolFree_KLU`

The module SUNLINSOL_KLU provides the following additional user-callable routines:

- `SUNKLU`

  This constructor function creates and allocates memory for a SUNLINSOL_KLU object. Its arguments are an NVECTOR and SUNMATRIX, that it uses to determine the linear system size and to assess compatibility with the linear solver implementation.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_SPARSE matrix type (using either CSR or CSC storage formats) and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  If either `A` or `y` are incompatible then this routine will return `NULL`.

  `SUNLinearSolver SUNKLU(N_Vector y, SUNMatrix A);`

- `SUNKLUReInit`

  This function reinitializes memory and flags for a new factorization (symbolic and numeric) to be conducted at the next solver setup call. This routine is useful in the cases where the number of nonzeroes has changed or if the structure of the linear system has changed which would require a new symbolic (and numeric factorization).

  The `reinit_type` argument governs the level of reinitialization. The allowed values are:

  1 The Jacobian matrix will be destroyed and a new one will be allocated based on the `nnz` value passed to this call. New symbolic and numeric factorizations will be completed at the next solver setup.

  2 Only symbolic and numeric factorizations will be completed. It is assumed that the Jacobian size has not exceeded the size of `nnz` given in the sparse matrix provided to the original constructor routine (or the previous `SUNKLUReInit` call).

This routine assumes no other changes to solver use are necessary.

The return values from this function are SUNLS_MEM_NULL (either S or A are NULL), SUNLS_ILL_INPUT (A does not have type SUNMATRIX_SPARSE or reinit_type is invalid), SUNLS_MEM_FAIL (reallocation of the sparse matrix failed) or SUNLS_SUCCESS.

```
int SUNKLUReInit(SUNLinearSolver S, SUNMatrix A,
                 sunindextype nnz, int reinit_type);
```

- SUNKLUSetOrdering

  This function sets the ordering used by KLU for reducing fill in the linear solve. Options for ordering_choice are:

  0 AMD,

  1 COLAMD, and

  2 the natural ordering.

  The default is 1 for COLAMD.

  The return values from this function are SUNLS_MEM_NULL (S is NULL), SUNLS_ILL_INPUT (invalid ordering_choice), or SUNLS_SUCCESS.

  ```
  int SUNKLUSetOrdering(SUNLinearSolver S, int ordering_choice);
  ```

For solvers that include a Fortran interface module, the SUNLINSOL_KLU module also includes the Fortran-callable function FSUNKLUInit(code, ier) to initialize this SUNLINSOL_KLU module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); ier is an error return flag equal to 0 for success and -1 for failure. Both code and ier are declared to match C type int. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNMassKLUInit(ier) initializes this SUNLINSOL_KLU module for solving mass matrix linear systems.

The SUNKLUReInit and SUNKLUSetOrdering routines also support Fortran interfaces for the system and mass matrix solvers:

- FSUNKLUReInit(code, NNZ, reinit_type, ier) – NNZ should be commensurate with a C long int and reinit_type should be commensurate with a C int

- FSUNMassKLUReInit(NNZ, reinit_type, ier)

- FSUNKLUSetOrdering(code, ordering, ier) – ordering should be commensurate with a C int

- FSUNMassKLUSetOrdering(ordering, ier)

9.8   The SUNLinearSolver_SuperLUMT implementation

The SUPERLUMT implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SUPERLUMT, is designed to be used with the corresponding SUNMATRIX_SPARSE matrix type, and one of the serial or shared-memory NVECTOR implementations (NVECTOR_SERIAL, NVECTOR_OPENMP, or NVECTOR_PTHREADS). While these are compatible, it is not recommended to use a threaded vector module with SUNLINSOL_SUPERLUMT unless it is the NVECTOR_OPENMP module and the SUPERLUMT library has also been compiled with OpenMP. The SUNLINSOL_SUPERLUMT module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SuperLUMT {
  long int      last_flag;
  int           first_factorize;
  SuperMatrix  *A, *AC, *L, *U, *B;
  Gstat_t      *Gstat;
  sunindextype *perm_r, *perm_c;
  sunindextype N;
  int           num_threads;
  realtype      diag_pivot_thresh;
  int           ordering;
  superlumt_options_t *options;
};
```

These entries of the *content* field contain the following information:

last_flag - last error return flag from internal function evaluations,

first_factorize - flag indicating whether the factorization has ever been performed,

A, AC, L, U, B - `SuperMatrix` pointers used in solve,

Gstat - `GStat_t` object used in solve,

perm_r, perm_c - permutation arrays used in solve,

N - size of the linear system,

num_threads - number of OpenMP/Pthreads threads to use,

diag_pivot_thresh - threshold on diagonal pivoting,

ordering - flag for which reordering algorithm to use,

options - pointer to SUPERLUMT options structure.

⚠️ The SUNLINSOL_SUPERLUMT module is a SUNLINSOL wrapper for the SUPERLUMT sparse matrix factorization and solver library written by X. Sherry Li [2, 28, 15]. The package performs matrix factorization using threads to enhance efficiency in shared memory parallel environments. It should be noted that threads are only used in the factorization step. In order to use the SUNLINSOL_SUPERLUMT interface to SUPERLUMT, it is assumed that SUPERLUMT has been installed on the system prior to installation of SUNDIALS, and that SUNDIALS has been configured appropriately to link with SUPERLUMT (see Appendix A for details). Additionally, this wrapper only supports single- and double-precision calculations, and therefore cannot be compiled if SUNDIALS is configured to have `realtype` set to `extended` (see Section 4.2). Moreover, since the SUPERLUMT library may be installed to support either 32-bit or 64-bit integers, it is assumed that the SUPERLUMT library is installed using the same integer precision as the SUNDIALS `sunindextype` option.

The SUPERLUMT library has a symbolic factorization routine that computes the permutation of the linear system matrix to reduce fill-in on subsequent LU factorizations (using COLAMD, minimal degree ordering on $A^T * A$, minimal degree ordering on $A^T + A$, or natural ordering). Of these ordering choices, the default value in the SUNLINSOL_SUPERLUMT module is the COLAMD ordering.

Since the linear systems that arise within the context of SUNDIALS calculations will typically have identical sparsity patterns, the SUNLINSOL_SUPERLUMT module is constructed to perform the following operations:

- The first time that the "setup" routine is called, it performs the symbolic factorization, followed by an initial numerical factorization.

- On subsequent calls to the "setup" routine, it skips the symbolic factorization, and only refactors the input matrix.

- The "solve" call performs pivoting and forward and backward substitution using the stored SUPERLUMT data structures. We note that in this solve SUPERLUMT operates on the native data arrays for the right-hand side and solution vectors, without requiring costly data copies.

The header file to include when using this module is `sunlinsol/sunlinsol_superlumt.h`. The installed module library to link to is `libsundials_sunlinsolsuperlumt.lib` where `.lib` is typically `.so` for shared libraries and `.a` for static libraries.

The SUNLINSOL_SUPERLUMT module defines implementations of all "direct" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_SuperLUMT`

- `SUNLinSolInitialize_SuperLUMT` – this sets the `first_factorize` flag to 1 and resets the internal SUPERLUMT statistics variables.

- `SUNLinSolSetup_SuperLUMT` – this performs either a LU factorization or refactorization of the input matrix.

- `SUNLinSolSolve_SuperLUMT` – this calls the appropriate SUPERLUMT solve routine to utilize the LU factors to solve the linear system.

- `SUNLinSolLastFlag_SuperLUMT`

- `SUNLinSolSpace_SuperLUMT` – this only returns information for the storage within the solver *interface*, i.e. storage for the integers `last_flag` and `first_factorize`. For additional space requirements, see the SUPERLUMT documentation.

- `SUNLinSolFree_SuperLUMT`

The module SUNLINSOL_SUPERLUMT provides the following additional user-callable routines:

- `SUNSuperLUMT`

  This constructor function creates and allocates memory for a SUNLINSOL_SUPERLUMT object. Its arguments are an NVECTOR, a SUNMATRIX, and a desired number of threads (OpenMP or Pthreads, depending on how SUPERLUMT was installed) to use during the factorization steps. This routine analyzes the input matrix and vector to determine the linear system size and to assess compatibility with the SUPERLUMT library.

  This routine will perform consistency checks to ensure that it is called with consistent NVECTOR and SUNMATRIX implementations. These are currently limited to the SUNMATRIX_SPARSE matrix type (using either CSR or CSC storage formats) and the NVECTOR_SERIAL, NVECTOR_OPENMP, and NVECTOR_PTHREADS vector types. As additional compatible matrix and vector implementations are added to SUNDIALS, these will be included within this compatibility check.

  If either `A` or `y` are incompatible then this routine will return NULL. The `num_threads` argument is not checked and is passed directly to SUPERLUMT routines.

  `SUNLinearSolver SUNSuperLUMT(N_Vector y, SUNMatrix A, int num_threads);`

- `SUNSuperLUMTSetOrdering`

  This function sets the ordering used by SUPERLUMT for reducing fill in the linear solve. Options for `ordering_choice` are:

    0 natural ordering

    1 minimal degree ordering on $A^T A$

    2 minimal degree ordering on $A^T + A$

    3 COLAMD ordering for unsymmetric matrices

The default is 3 for COLAMD.

The return values from this function are SUNLS_MEM_NULL (S is NULL), SUNLS_ILL_INPUT (invalid ordering_choice), or SUNLS_SUCCESS.

```
int SUNSuperLUMTSetOrdering(SUNLinearSolver S, int ordering_choice);
```

For solvers that include a Fortran interface module, the SUNLINSOL_SUPERLUMT module also includes the Fortran-callable function FSUNSuperLUMTInit(code, num_threads, ier) to initialize this SUN-LINSOL_SUPERLUMT module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); num_threads is the desired number of Open-MP/Pthreads threads to use in the factorization; ier is an error return flag equal to 0 for success and -1 for failure. All of these arguments should be declared so as to match C type int. This routine must be called *after* both the NVECTOR and SUNMATRIX objects have been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNMassSuperLUMTInit(num_threads, ier) initializes this SUNLINSOL_SUPERLUMT module for solving mass matrix linear systems.

The SUNSuperLUMTSetOrdering routine also supports Fortran interfaces for the system and mass matrix solvers:

- FSUNSuperLUMTSetOrdering(code, ordering, ier) – ordering should be commensurate with a C int

- FSUNMassSuperLUMTSetOrdering(ordering, ier)

9.9   The SUNLinearSolver_SPGMR implementation

The SPGMR (Scaled, Preconditioned, Generalized Minimum Residual [33]) implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SPGMR, is an iterative linear solver that is designed to be compatible with any NVECTOR implementation (serial, threaded, parallel, and user-supplied) that supports a minimal subset of operations (N_VClone, N_VDotProd, N_VScale, N_VLinearSum, N_VProd, N_VConst, N_VDiv, and N_VDestroy).

The SUNLINSOL_SPGMR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPGMR {
  int maxl;
  int pretype;
  int gstype;
  int max_restarts;
  int numiters;
  realtype resnorm;
  long int last_flag;
  ATimesFn ATimes;
  void* ATData;
  PSetupFn Psetup;
  PSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector *V;
  realtype **Hes;
  realtype *givens;
  N_Vector xcor;
  realtype *yg;
  N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

maxl - number of GMRES basis vectors to use (default is 5),

pretype - flag for type of preconditioning to employ (default is none),

gstype - flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),

max_restarts - number of GMRES restarts to allow (default is 0),

numiters - number of iterations from the most-recent solve,

resnorm - final linear residual norm from the most-recent solve,

last_flag - last error return flag from an internal function,

ATimes - function pointer to perform Av product,

ATData - pointer to structure for `ATimes`,

Psetup - function pointer to preconditioner setup routine,

Psolve - function pointer to preconditioner solve routine,

PData - pointer to structure for `Psetup` and `Psolve`,

s1, s2 - vector pointers for supplied scaling matrices (default is `NULL`),

V - the array of Krylov basis vectors $v_1, \ldots, v_{\texttt{maxl}+1}$, stored in `V[0]`, ..., `V[maxl]`. Each v_i is a vector of type NVECTOR.,

Hes - the $(\texttt{maxl} + 1) \times \texttt{maxl}$ Hessenberg matrix. It is stored row-wise so that the (i,j)th element is given by `Hes[i][j]`.,

givens - a length `2*maxl` array which represents the Givens rotation matrices that arise in the GMRES algorithm. These matrices are $F_0, F_1, \ldots, F_j$, where $F_i = \begin{bmatrix} 1 & & & & & & & \\ & \ddots & & & & & & \\ & & 1 & & & & & \\ & & & c_i & -s_i & & & \\ & & & s_i & c_i & & & \\ & & & & & 1 & & \\ & & & & & & \ddots & \\ & & & & & & & 1 \end{bmatrix}$,

are represented in the `givens` vector as `givens[0]` $= c_0$, `givens[1]` $= s_0$, `givens[2]` $= c_1$, `givens[3]` $= s_1$, ... `givens[2j]` $= c_j$, `givens[2j+1]` $= s_j$.,

xcor - a vector which holds the scaled, preconditioned correction to the initial guess,

yg - a length `(maxl+1)` array of `realtype` values used to hold "short" vectors (e.g. y and g),

vtemp - temporary vector storage.

This solver is constructed to perform the following operations:

- During construction, the `xcor` and `vtemp` arrays are cloned from a template NVECTOR that is input, and default solver parameters are set.

- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPGMR to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s1` and `s2` scaling vectors.

- In the "initialize" call, the remaining solver data is allocated (V, Hes, givens, and yg)

- In the "setup" call, any non-NULL PSetup function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic PSetup function and the solver-specific routine (solver-supplied or user-supplied).

- In the "solve" call, the GMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

The header file to include when using this module is sunlinsol/sunlinsol_spgmr.h. The SUNLIN-SOL_SPGMR module is accessible from all SUNDIALS solvers *without* linking to the
    libsundials_sunlinsolspgmr module library.
The SUNLINSOL_SPGMR module defines implementations of all "iterative" linear solver operations listed in Table 9.2:

- SUNLinSolGetType_SPGMR

- SUNLinSolInitialize_SPGMR

- SUNLinSolSetATimes_SPGMR

- SUNLinSolSetPreconditioner_SPGMR

- SUNLinSolSetScalingVectors_SPGMR

- SUNLinSolSetup_SPGMR

- SUNLinSolSolve_SPGMR

- SUNLinSolNumIters_SPGMR

- SUNLinSolResNorm_SPGMR

- SUNLinSolResid_SPGMR

- SUNLinSolLastFlag_SPGMR

- SUNLinSolSpace_SPGMR

- SUNLinSolFree_SPGMR

The module SUNLINSOL_SPGMR provides the following additional user-callable routines:

- SUNSPGMR

  This constructor function creates and allocates memory for a SPGMR SUNLinearSolver. Its arguments are an NVECTOR, the desired type of preconditioning, and the number of Krylov basis vectors to use.

  This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.

  A maxl argument that is ≤ 0 will result in the default value (5).

  Allowable inputs for pretype are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2) and PREC_BOTH (3); any other integer input will result in the default (no preconditioning). We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLINSOL_SPGMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

  SUNLinearSolver SUNSPGMR(N_Vector y, int pretype, int maxl);

- SUNSPGMRSetPrecType

  This function updates the type of preconditioning to use. Supported values are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2) and PREC_BOTH (3).

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal `pretype`), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPGMRSetPrecType(SUNLinearSolver S, int pretype);
  ```

- SUNSPGMRSetGSType

  This function sets the type of Gram-Schmidt orthogonalization to use. Supported values are MODIFIED_GS (1) and CLASSICAL_GS (2). Any other integer input will result in a failure, returning error code SUNLS_ILL_INPUT.

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal `gstype`), SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPGMRSetGSType(SUNLinearSolver S, int gstype);
  ```

- SUNSPGMRSetMaxRestarts

  This function sets the number of GMRES restarts to allow. A negative input will result in the default of 0.

  This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPGMRSetMaxRestarts(SUNLinearSolver S, int maxrs);
  ```

For solvers that include a Fortran interface module, the SUNLINSOL_SPGMR module also includes the Fortran-callable function FSUNSPGMRInit(code, pretype, maxl, ier) to initialize this SUNLIN-SOL_SPGMR module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); pretype and maxl are the same as for the C function SUNSPGMR; ier is an error return flag equal to 0 for success and -1 for failure. All of these input arguments should be declared so as to match C type int. This routine must be called *after* the NVECTOR object has been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNMassSPGMRInit(pretype, maxl, ier) initializes this SUNLINSOL_SPGMR module for solving mass matrix linear systems.

The SUNSPGMRSetPrecType, SUNSPGMRSetGSType and SUNSPGMRSetMaxRestarts routines also support Fortran interfaces for the system and mass matrix solvers (all arguments should be commensurate with a C int):

- FSUNSPGMRSetGSType(code, gstype, ier)

- FSUNMassSPGMRSetGSType(gstype, ier)

- FSUNSPGMRSetPrecType(code, pretype, ier)

- FSUNMassSPGMRSetPrecType(pretype, ier)

- FSUNSPGMRSetMaxRS(code, maxrs, ier)

- FSUNMassSPGMRSetMaxRS(maxrs, ier)

9.10   The SUNLinearSolver_SPFGMR implementation

The SPFGMR (Scaled, Preconditioned, Flexible, Generalized Minimum Residual [32]) implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SPFGMR, is an iterative linear solver that is designed to be compatible with any NVECTOR implementation (serial, threaded, parallel, and user-supplied) that supports a minimal subset of operations (N_VClone, N_VDotProd, N_VScale, N_VLinearSum, N_VProd, N_VConst, N_VDiv, and N_VDestroy). Unlike the other Krylov iterative linear

solvers supplied with SUNDIALS, FGMRES is specifically designed to work with a changing preconditioner (e.g. from an iterative method).

The SUNLINSOL_SPFGMR module defines the *content* field of a SUNLinearSolver to be the following structure:

```
struct _SUNLinearSolverContent_SPFGMR {
  int maxl;
  int pretype;
  int gstype;
  int max_restarts;
  int numiters;
  realtype resnorm;
  long int last_flag;
  ATimesFn ATimes;
  void* ATData;
  PSetupFn Psetup;
  PSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector *V;
  N_Vector *Z;
  realtype **Hes;
  realtype *givens;
  N_Vector xcor;
  realtype *yg;
  N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

maxl - number of FGMRES basis vectors to use (default is 5),

pretype - flag for use of preconditioning (default is none),

gstype - flag for type of Gram-Schmidt orthogonalization (default is modified Gram-Schmidt),

max_restarts - number of FGMRES restarts to allow (default is 0),

numiters - number of iterations from the most-recent solve,

resnorm - final linear residual norm from the most-recent solve,

last_flag - last error return flag from an internal function,

ATimes - function pointer to perform Av product,

ATData - pointer to structure for ATimes,

Psetup - function pointer to preconditioner setup routine,

Psolve - function pointer to preconditioner solve routine,

PData - pointer to structure for Psetup and Psolve,

s1, s2 - vector pointers for supplied scaling matrices (default is NULL),

V - the array of Krylov basis vectors $v_1, \ldots, v_{\texttt{maxl}+1}$, stored in V[0], ..., V[maxl]. Each v_i is a vector of type NVECTOR.,

Z - the array of preconditioned Krylov basis vectors $z_1, \ldots, z_{\texttt{maxl}+1}$, stored in `Z[0]`, ..., `Z[maxl]`. Each z_i is a vector of type NVECTOR.,

Hes - the $(\texttt{maxl}+1) \times \texttt{maxl}$ Hessenberg matrix. It is stored row-wise so that the (i,j)th element is given by `Hes[i][j]`.,

givens - a length `2*maxl` array which represents the Givens rotation matrices that arise in the FGM-RES algorithm. These matrices are $F_0, F_1, \ldots, F_j$, where $F_i = \begin{bmatrix} 1 & & & & & & & \\ & \ddots & & & & & & \\ & & 1 & & & & & \\ & & & c_i & -s_i & & & \\ & & & s_i & c_i & & & \\ & & & & & 1 & & \\ & & & & & & \ddots & \\ & & & & & & & 1 \end{bmatrix}$,

are represented in the `givens` vector as `givens[0]` $= c_0$, `givens[1]` $= s_0$, `givens[2]` $= c_1$, `givens[3]` $= s_1$, ... `givens[2j]` $= c_j$, `givens[2j+1]` $= s_j$.,

xcor - a vector which holds the scaled, preconditioned correction to the initial guess,

yg - a length `(maxl+1)` array of `realtype` values used to hold "short" vectors (e.g. y and g),

vtemp - temporary vector storage.

This solver is constructed to perform the following operations:

- During construction, the `xcor` and `vtemp` arrays are cloned from a template NVECTOR that is input, and default solver parameters are set.

- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPFGMR to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s1` and `s2` scaling vectors.

- In the "initialize" call, the remaining solver data is allocated (`V`, `Hes`, `givens`, and `yg`)

- In the "setup" call, any non-`NULL` `PSetup` function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic `PSetup` function and the solver-specific routine (solver-supplied or user-supplied).

- In the "solve" call, the FGMRES iteration is performed. This will include scaling, preconditioning, and restarts if those options have been supplied.

The header file to include when using this module is `sunlinsol/sunlinsol_spfgmr.h`. The SUNLINSOL_SPFGMR module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunlinsolspfgmr` module library.

The SUNLINSOL_SPFGMR module defines implementations of all "iterative" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_SPFGMR`

- `SUNLinSolInitialize_SPFGMR`

- `SUNLinSolSetATimes_SPFGMR`

- `SUNLinSolSetPreconditioner_SPFGMR`

- `SUNLinSolSetScalingVectors_SPFGMR`

- `SUNLinSolSetup_SPFGMR`

- SUNLinSolSolve_SPFGMR

- SUNLinSolNumIters_SPFGMR

- SUNLinSolResNorm_SPFGMR

- SUNLinSolResid_SPFGMR

- SUNLinSolLastFlag_SPFGMR

- SUNLinSolSpace_SPFGMR

- SUNLinSolFree_SPFGMR

The module SUNLINSOL_SPFGMR provides the following additional user-callable routines:

- SUNSPFGMR

  This constructor function creates and allocates memory for a SPFGMR SUNLinearSolver. Its arguments are an NVECTOR, a flag indicating to use preconditioning, and the number of Krylov basis vectors to use.

  This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.

  A maxl argument that is ≤ 0 will result in the default value (5).

  Since the FGMRES algorithm is designed to only support right preconditioning, then any of the pretype inputs PREC_LEFT (1), PREC_RIGHT (2), or PREC_BOTH (3) will result in use of PREC_RIGHT; any other integer input will result in the default (no preconditioning). We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS). While it is possible to use a right-preconditioned SUNLINSOL_SPFGMR object for these packages, this use mode is not supported and may result in inferior performance.

  ```
  SUNLinearSolver SUNSPFGMR(N_Vector y, int pretype, int maxl);
  ```

- SUNSPFGMRSetPrecType

  This function updates the flag indicating use of preconditioning. Since the FGMRES algorithm is designed to only support right preconditioning, then any of the pretype inputs PREC_LEFT (1), PREC_RIGHT (2), or PREC_BOTH (3) will result in use of PREC_RIGHT; any other integer input will result in the default (no preconditioning).

  This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPFGMRSetPrecType(SUNLinearSolver S, int pretype);
  ```

- SUNSPFGMRSetGSType

  This function sets the type of Gram-Schmidt orthogonalization to use. Supported values are MODIFIED_GS (1) and CLASSICAL_GS (2). Any other integer input will result in a failure, returning error code SUNLS_ILL_INPUT.

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal gstype), SUNLS_MEM_NULL (S is NULL), or SUNLS_SUCCESS.

  ```
  int SUNSPFGMRSetGSType(SUNLinearSolver S, int gstype);
  ```

- SUNSPFGMRSetMaxRestarts

  This function sets the number of FGMRES restarts to allow. A negative input will result in the default of 0.

  This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPFGMRSetMaxRestarts(SUNLinearSolver S, int maxrs);
  ```

For solvers that include a Fortran interface module, the SUNLINSOL_SPFGMR module also includes the Fortran-callable function `FSUNSPFGMRInit(code, pretype, maxl, ier)` to initialize this SUN-LINSOL_SPFGMR module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); `pretype` and `maxl` are the same as for the C function `SUNSPFGMR`; `ier` is an error return flag equal to 0 for success and -1 for failure. All of these input arguments should be declared so as to match C type `int`. This routine must be called *after* the NVECTOR object has been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function `FSUNMassSPFGMRInit(pretype, maxl, ier)` initializes this SUNLINSOL_SPFGMR module for solving mass matrix linear systems.

The `SUNSPFGMRSetPrecType`, `SUNSPFGMRSetGSType`, and `SUNSPFGMRSetMaxRestarts` routines also support Fortran interfaces for the system and mass matrix solvers (all arguments should be commensurate with a C `int`):

- `FSUNSPFGMRSetGSType(code, gstype, ier)`

- `FSUNMassSPFGMRSetGSType(gstype, ier)`

- `FSUNSPFGMRSetPrecType(code, pretype, ier)`

- `FSUNMassSPFGMRSetPrecType(pretype, ier)`

- `FSUNSPFGMRSetMaxRS(code, maxrs, ier)`

- `FSUNMassSPFGMRSetMaxRS(maxrs, ier)`

9.11 The SUNLinearSolver_SPBCGS implementation

The SPBCGS (Scaled, Preconditioned, Bi-Conjugate Gradient, Stabilized [36]) implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SPBCGS, is an iterative linear solver that is designed to be compatible with any NVECTOR implementation (serial, threaded, parallel, and user-supplied) that supports a minimal subset of operations (`N_VClone`, `N_VDotProd`, `N_VScale`, `N_VLinearSum`, `N_VProd`, `N_VDiv`, and `N_VDestroy`). Unlike the SPGMR and SPFGMR algorithms, SP-BCGS requires a fixed amount of memory that does not increase with the number of allowed iterations.

The SUNLINSOL_SPBCGS module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_SPBCGS {
  int maxl;
  int pretype;
  int numiters;
  realtype resnorm;
  long int last_flag;
  ATimesFn ATimes;
  void* ATData;
  PSetupFn Psetup;
  PSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector r;
  N_Vector r_star;
  N_Vector p;
  N_Vector q;
  N_Vector u;
  N_Vector Ap;
  N_Vector vtemp;
};
```

These entries of the *content* field contain the following information:

maxl - number of SPBCGS iterations to allow (default is 5),

pretype - flag for type of preconditioning to employ (default is none),

numiters - number of iterations from the most-recent solve,

resnorm - final linear residual norm from the most-recent solve,

last_flag - last error return flag from an internal function,

ATimes - function pointer to perform Av product,

ATData - pointer to structure for `ATimes`,

Psetup - function pointer to preconditioner setup routine,

Psolve - function pointer to preconditioner solve routine,

PData - pointer to structure for `Psetup` and `Psolve`,

s1, s2 - vector pointers for supplied scaling matrices (default is `NULL`),

r - a NVECTOR which holds the current scaled, preconditioned linear system residual,

r_star - a NVECTOR which holds the initial scaled, preconditioned linear system residual,

p, q, u, Ap, vtemp - NVECTORs used for workspace by the SPBCGS algorithm.

This solver is constructed to perform the following operations:

- During construction all NVECTOR solver data is allocated, with vectors cloned from a template NVECTOR that is input, and default solver parameters are set.

- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPBCGS to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s1` and `s2` scaling vectors.

- In the "initialize" call, the solver parameters are checked for validity.

- In the "setup" call, any non-`NULL` `PSetup` function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic `PSetup` function and the solver-specific routine (solver-supplied or user-supplied).

- In the "solve" call the SPBCGS iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The header file to include when using this module is `sunlinsol/sunlinsol_spbcgs.h`. The SUNLIN-SOL_SPBCGS module is accessible from all SUNDIALS solvers *without* linking to the
`libsundials_sunlinsolspbcgs` module library.
The SUNLINSOL_SPBCGS module defines implementations of all "iterative" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_SPBCGS`

- `SUNLinSolInitialize_SPBCGS`

- `SUNLinSolSetATimes_SPBCGS`

- `SUNLinSolSetPreconditioner_SPBCGS`

- `SUNLinSolSetScalingVectors_SPBCGS`

- SUNLinSolSetup_SPBCGS

- SUNLinSolSolve_SPBCGS

- SUNLinSolNumIters_SPBCGS

- SUNLinSolResNorm_SPBCGS

- SUNLinSolResid_SPBCGS

- SUNLinSolLastFlag_SPBCGS

- SUNLinSolSpace_SPBCGS

- SUNLinSolFree_SPBCGS

The module SUNLINSOL_SPBCGS provides the following additional user-callable routines:

- SUNSPBCGS

  This constructor function creates and allocates memory for a SPBCGS SUNLinearSolver. Its arguments are an NVECTOR, the desired type of preconditioning, and the number of linear iterations to allow.

  This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.

  A maxl argument that is ≤ 0 will result in the default value (5).

  Allowable inputs for pretype are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2) and PREC_BOTH (3); any other integer input will result in the default (no preconditioning). We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUNLINSOL_SPBCGS object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

  ```
  SUNLinearSolver SUNSPBCGS(N_Vector y, int pretype, int maxl);
  ```

- SUNSPBCGSSetPrecType

  This function updates the type of preconditioning to use. Supported values are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2), and PREC_BOTH (3).

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL), or SUNLS_SUCCESS.

  ```
  int SUNSPBCGSSetPrecType(SUNLinearSolver S, int pretype);
  ```

- SUNSPBCGSSetMaxl

  This function updates the number of linear solver iterations to allow.

  A maxl argument that is ≤ 0 will result in the default value (5).

  This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  ```
  int SUNSPBCGSSetMaxl(SUNLinearSolver S, int maxl);
  ```

For solvers that include a Fortran interface module, the SUNLINSOL_SPBCGS module also includes the Fortran-callable function FSUNSPBCGSInit(code, pretype, maxl, ier) to initialize this SUN-LINSOL_SPBCGS module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); pretype and maxl are the same as for the C function SUNSPBCGS; ier is an error return flag equal to 0 for success and -1 for failure. All of these input arguments should be declared so as to match C type int. This routine must be called *after* the NVECTOR object has been initialized. Additionally, when using ARKODE with a non-identity

mass matrix, the Fortran-callable function `FSUNMassSPBCGSInit(pretype, maxl, ier)` initializes this SUNLINSOL_SPBCGS module for solving mass matrix linear systems.

The `SUNSPBCGSSetPrecType` and `SUNSPBCGSSetMaxl` routines also support Fortran interfaces for the system and mass matrix solvers (all arguments should be commensurate with a C `int`):

- `FSUNSPBCGSSetPrecType(code, pretype, ier)`

- `FSUNMassSPBCGSSetPrecType(pretype, ier)`

- `FSUNSPBCGSSetMaxl(code, maxl, ier)`

- `FSUNMassSPBCGSSetMaxl(maxl, ier)`

9.12   The SUNLinearSolver_SPTFQMR implementation

The SPTFQMR (Scaled, Preconditioned, Transpose-Free Quasi-Minimum Residual [17]) implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_SPTFQMR, is an iterative linear solver that is designed to be compatible with any NVECTOR implementation (serial, threaded, parallel, and user-supplied) that supports a minimal subset of operations (N_VClone, N_VDotProd, N_VScale, N_VLinearSum, N_VProd, N_VConst, N_VDiv, and N_VDestroy). Unlike the SPGMR and SPFGMR algorithms, SPTFQMR requires a fixed amount of memory that does not increase with the number of allowed iterations.

The SUNLINSOL_SPTFQMR module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_SPTFQMR {
  int maxl;
  int pretype;
  int numiters;
  realtype resnorm;
  long int last_flag;
  ATimesFn ATimes;
  void* ATData;
  PSetupFn Psetup;
  PSolveFn Psolve;
  void* PData;
  N_Vector s1;
  N_Vector s2;
  N_Vector r_star;
  N_Vector q;
  N_Vector d;
  N_Vector v;
  N_Vector p;
  N_Vector *r;
  N_Vector u;
  N_Vector vtemp1;
  N_Vector vtemp2;
  N_Vector vtemp3;
};
```

These entries of the *content* field contain the following information:

maxl - number of TFQMR iterations to allow (default is 5),

pretype - flag for type of preconditioning to employ (default is none),

numiters - number of iterations from the most-recent solve,

resnorm - final linear residual norm from the most-recent solve,

last_flag - last error return flag from an internal function,

ATimes - function pointer to perform Av product,

ATData - pointer to structure for `ATimes`,

Psetup - function pointer to preconditioner setup routine,

Psolve - function pointer to preconditioner solve routine,

PData - pointer to structure for `Psetup` and `Psolve`,

s1, s2 - vector pointers for supplied scaling matrices (default is `NULL`),

r_star - a NVECTOR which holds the initial scaled, preconditioned linear system residual,

q, d, v, p, u - NVECTORs used for workspace by the SPTFQMR algorithm,

r - array of two NVECTORs used for workspace within the SPTFQMR algorithm,

vtemp1, vtemp2, vtemp3 - temporary vector storage.

This solver is constructed to perform the following operations:

- During construction all NVECTOR solver data is allocated, with vectors cloned from a template NVECTOR that is input, and default solver parameters are set.

- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_SPTFQMR to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s1` and `s2` scaling vectors.

- In the "initialize" call, the solver parameters are checked for validity.

- In the "setup" call, any non-`NULL` `PSetup` function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic `PSetup` function and the solver-specific routine (solver-supplied or user-supplied).

- In the "solve" call the TFQMR iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The header file to include when using this module is `sunlinsol/sunlinsol_sptfqmr.h`. The SUNLINSOL_SPTFQMR module is accessible from all SUNDIALS solvers *without* linking to the `libsundials_sunlinsolsptfqmr` module library.

The SUNLINSOL_SPTFQMR module defines implementations of all "iterative" linear solver operations listed in Table 9.2:

- `SUNLinSolGetType_SPTFQMR`

- `SUNLinSolInitialize_SPTFQMR`

- `SUNLinSolSetATimes_SPTFQMR`

- `SUNLinSolSetPreconditioner_SPTFQMR`

- `SUNLinSolSetScalingVectors_SPTFQMR`

- `SUNLinSolSetup_SPTFQMR`

- `SUNLinSolSolve_SPTFQMR`

- `SUNLinSolNumIters_SPTFQMR`

- SUNLinSolResNorm_SPTFQMR

- SUNLinSolResid_SPTFQMR

- SUNLinSolLastFlag_SPTFQMR

- SUNLinSolSpace_SPTFQMR

- SUNLinSolFree_SPTFQMR

The module SUNLINSOL_SPTFQMR provides the following additional user-callable routines:

- SUNSPTFQMR

  This constructor function creates and allocates memory for a SPTFQMR SUNLinearSolver. Its arguments are an NVECTOR, the desired type of preconditioning, and the number of linear iterations to allow.

  This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR implementation (i.e. that it supplies the requisite vector operations). If y is incompatible, then this routine will return NULL.

  A maxl argument that is ≤ 0 will result in the default value (5).

  Allowable inputs for pretype are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2) and PREC_BOTH (3); any other integer input will result in the default (no preconditioning). We note that some SUNDIALS solvers are designed to only work with left preconditioning (IDA and IDAS) and others with only right preconditioning (KINSOL). While it is possible to configure a SUN-LINSOL_SPTFQMR object to use any of the preconditioning options with these solvers, this use mode is not supported and may result in inferior performance.

  SUNLinearSolver SUNSPTFQMR(N_Vector y, int pretype, int maxl);

- SUNSPTFQMRSetPrecType

  This function updates the type of preconditioning to use. Supported values are PREC_NONE (0), PREC_LEFT (1), PREC_RIGHT (2), and PREC_BOTH (3).

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL (S is NULL), or SUNLS_SUCCESS.

  int SUNSPTFQMRSetPrecType(SUNLinearSolver S, int pretype);

- SUNSPTFQMRSetMaxl

  This function updates the number of linear solver iterations to allow.

  A maxl argument that is ≤ 0 will result in the default value (5).

  This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

  int SUNSPTFQMRSetMaxl(SUNLinearSolver S, int maxl);

For solvers that include a Fortran interface module, the SUNLINSOL_SPTFQMR module also includes the Fortran-callable function FSUNSPTFQMRInit(code, pretype, maxl, ier) to initialize this SUN-LINSOL_SPTFQMR module for a given SUNDIALS solver. Here code is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); pretype and maxl are the same as for the C function SUNSPTFQMR; ier is an error return flag equal to 0 for success and -1 for failure. All of these input arguments should be declared so as to match C type int. This routine must be called *after* the NVECTOR object has been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function FSUNMassSPTFQMRInit(pretype, maxl, ier) initializes this SUNLINSOL_SPTFQMR module for solving mass matrix linear systems.

The SUNSPTFQMRSetPrecType and SUNSPTFQMRSetMaxl routines also support Fortran interfaces for the system and mass matrix solvers (all arguments should be commensurate with a C int):

- FSUNSPTFQMRSetPrecType(code, pretype, ier)

- `FSUNMassSPTFQMRSetPrecType(pretype, ier)`

- `FSUNSPTFQMRSetMaxl(code, maxl, ier)`

- `FSUNMassSPTFQMRSetMaxl(maxl, ier)`

9.13   The SUNLinearSolver_PCG implementation

The PCG (Preconditioned Conjugate Gradient [18]) implementation of the SUNLINSOL module provided with SUNDIALS, SUNLINSOL_PCG, is an iterative linear solver that is designed to be compatible with any NVECTOR implementation (serial, threaded, parallel, and user-supplied) that supports a minimal subset of operations (`N_VClone`, `N_VDotProd`, `N_VScale`, `N_VLinearSum`, `N_VProd`, and `N_VDestroy`). Unlike the SPGMR and SPFGMR algorithms, PCG requires a fixed amount of memory that does not increase with the number of allowed iterations.

Unlike all of the other iterative linear solvers supplied with SUNDIALS, PCG should only be used on *symmetric* linear systems (e.g. mass matrix linear systems encountered in ARKODE). As a result, the explanation of the role of scaling and preconditioning matrices given in general must be modified in this scenario. The PCG algorithm solves a linear system $Ax = b$ where A is a symmetric ($A^T = A$), real-valued matrix. Preconditioning is allowed, and is applied in a symmetric fashion on both the right and left. Scaling is also allowed and is applied symmetrically. We denote the preconditioner and scaling matrices as follows:

- P is the preconditioner (assumed symmetric),

- S is a diagonal matrix of scale factors.

The matrices A and P are not required explicitly; only routines that provide A and P^{-1} as operators are required. The diagonal of the matrix S is held in a single NVECTOR, supplied by the user.

In this notation, PCG applies the underlying CG algorithm to the equivalent transformed system

$$\tilde{A}\tilde{x} = \tilde{b} \tag{9.3}$$

where

$$\begin{aligned} \tilde{A} &= SP^{-1}AP^{-1}S, \\ \tilde{b} &= SP^{-1}b, \\ \tilde{x} &= S^{-1}Px. \end{aligned} \tag{9.4}$$

The scaling matrix must be chosen so that the vectors $SP^{-1}b$ and $S^{-1}Px$ have dimensionless components.

The stopping test for the PCG iterations is on the L2 norm of the scaled preconditioned residual:

$$\|\tilde{b} - \tilde{A}\tilde{x}\|_2 < \delta$$
$$\Leftrightarrow$$
$$\|SP^{-1}b - SP^{-1}Ax\|_2 < \delta$$
$$\Leftrightarrow$$
$$\|P^{-1}b - P^{-1}Ax\|_S < \delta$$

where $\|v\|_S = \sqrt{v^T S^T S v}$, with an input tolerance δ.

The SUNLINSOL_PCG module defines the *content* field of a `SUNLinearSolver` to be the following structure:

```
struct _SUNLinearSolverContent_PCG {
  int maxl;
  int pretype;
```

```
    int numiters;
    realtype resnorm;
    long int last_flag;
    ATimesFn ATimes;
    void* ATData;
    PSetupFn Psetup;
    PSolveFn Psolve;
    void* PData;
    N_Vector s;
    N_Vector r;
    N_Vector p;
    N_Vector z;
    N_Vector Ap;
};
```

These entries of the *content* field contain the following information:

maxl - number of PCG iterations to allow (default is 5),

pretype - flag for use of preconditioning (default is none),

numiters - number of iterations from the most-recent solve,

resnorm - final linear residual norm from the most-recent solve,

last_flag - last error return flag from an internal function,

ATimes - function pointer to perform Av product,

ATData - pointer to structure for `ATimes`,

Psetup - function pointer to preconditioner setup routine,

Psolve - function pointer to preconditioner solve routine,

PData - pointer to structure for `Psetup` and `Psolve`,

s - vector pointer for supplied scaling matrix (default is `NULL`),

r - a NVECTOR which holds the preconditioned linear system residual,

p, z, Ap - NVECTORs used for workspace by the PCG algorithm.

This solver is constructed to perform the following operations:

- During construction all NVECTOR solver data is allocated, with vectors cloned from a template NVECTOR that is input, and default solver parameters are set.

- User-facing "set" routines may be called to modify default solver parameters.

- Additional "set" routines are called by the SUNDIALS solver that interfaces with SUNLINSOL_PCG to supply the `ATimes`, `PSetup`, and `Psolve` function pointers and `s` scaling vector.

- In the "initialize" call, the solver parameters are checked for validity.

- In the "setup" call, any non-`NULL` `PSetup` function is called. Typically, this is provided by the SUNDIALS solver itself, that translates between the generic `PSetup` function and the solver-specific routine (solver-supplied or user-supplied).

- In the "solve" call the PCG iteration is performed. This will include scaling and preconditioning if those options have been supplied.

The header file to include when using this module is sunlinsol/sunlinsol_pcg.h. The SUNLIN-
SOL_PCG module is accessible from all SUNDIALS solvers *without* linking to the
    libsundials_sunlinsolpcg module library.
The SUNLINSOL_PCG module defines implementations of all "iterative" linear solver operations listed
in Table 9.2:

- SUNLinSolGetType_PCG

- SUNLinSolInitialize_PCG

- SUNLinSolSetATimes_PCG

- SUNLinSolSetPreconditioner_PCG

- SUNLinSolSetScalingVectors_PCG – since PCG only supports symmetric scaling, the second
  NVECTOR argument to this function is ignored

- SUNLinSolSetup_PCG

- SUNLinSolSolve_PCG

- SUNLinSolNumIters_PCG

- SUNLinSolResNorm_PCG

- SUNLinSolResid_PCG

- SUNLinSolLastFlag_PCG

- SUNLinSolSpace_PCG

- SUNLinSolFree_PCG

The module SUNLINSOL_PCG provides the following additional user-callable routines:

- SUNPCG

  This constructor function creates and allocates memory for a PCG SUNLinearSolver. Its ar-
  guments are an NVECTOR, a flag indicating to use preconditioning, and the number of linear
  iterations to allow.

  This routine will perform consistency checks to ensure that it is called with a consistent NVECTOR
  implementation (i.e. that it supplies the requisite vector operations). If y is incompatible then
  this routine will return NULL.

  A maxl argument that is ≤ 0 will result in the default value (5).

  Since the PCG algorithm is designed to only support symmetric preconditioning, then any of
  the pretype inputs PREC_LEFT (1), PREC_RIGHT (2), or PREC_BOTH (3) will result in use of the
  symmetric preconditioner; any other integer input will result in the default (no preconditioning).
  Although some SUNDIALS solvers are designed to only work with left preconditioning (IDA and
  IDAS) and others with only right preconditioning (KINSOL), PCG should *only* be used with these
  packages when the linear systems are known to be *symmetric*. Since the scaling of matrix rows
  and columns must be identical in a symmetric matrix, symmetric preconditioning should work
  appropriately even for packages designed with one-sided preconditioning in mind.

  SUNLinearSolver SUNPCG(N_Vector y, int pretype, int maxl);

- SUNPCGSetPrecType

  This function updates the flag indicating use of preconditioning. As above, any one of the input
  values, PREC_LEFT (1), PREC_RIGHT (2), or PREC_BOTH (3) will enable preconditioning; PREC_NONE
  (0) disables preconditioning.

  This routine will return with one of the error codes SUNLS_ILL_INPUT (illegal pretype), SUNLS_MEM_NULL
  (S is NULL), or SUNLS_SUCCESS.

  int SUNPCGSetPrecType(SUNLinearSolver S, int pretype);

- SUNPCGSetMaxl

   This function updates the number of linear solver iterations to allow.

   A `maxl` argument that is ≤ 0 will result in the default value (5).

   This routine will return with one of the error codes SUNLS_MEM_NULL (S is NULL) or SUNLS_SUCCESS.

   `int SUNPCGSetMaxl(SUNLinearSolver S, int maxl);`

For solvers that include a Fortran interface module, the SUNLINSOL_PCG module also includes the Fortran-callable function `FSUNPCGInit(code, pretype, maxl, ier)` to initialize this SUNLINSOL_PCG module for a given SUNDIALS solver. Here `code` is an integer input solver id (1 for CVODE, 2 for IDA, 3 for KINSOL, 4 for ARKODE); `pretype` and `maxl` are the same as for the C function SUNPCG; `ier` is an error return flag equal to 0 for success and -1 for failure. All of these input arguments should be declared so as to match C type `int`. This routine must be called *after* the NVECTOR object has been initialized. Additionally, when using ARKODE with a non-identity mass matrix, the Fortran-callable function `FSUNMassPCGInit(pretype, maxl, ier)` initializes this SUNLINSOL_PCG module for solving mass matrix linear systems.

   The `SUNPCGSetPrecType` and `SUNPCGSetMaxl` routines also support Fortran interfaces for the system and mass matrix solvers (all arguments should be commensurate with a C `int`):

- FSUNPCGSetPrecType(code, pretype, ier)

- FSUNMassPCGSetPrecType(pretype, ier)

- FSUNPCGSetMaxl(code, maxl, ier)

- FSUNMassPCGSetMaxl(maxl, ier)

9.14   SUNLinearSolver Examples

There are SUNLinearSolver examples that may be installed for each implementation; these make use of the functions in `test_sunlinsol.c`. These example functions show simple usage of the SUNLinearSolver family of functions. The inputs to the examples depend on the linear solver type, and are output to `stdout` if the example is run without the appropriate number of command-line arguments.

The following is a list of the example functions in `test_sunlinsol.c`:

- Test_SUNLinSolGetType: Verifies the returned solver type against the value that should be returned.

- Test_SUNLinSolInitialize: Verifies that `SUNLinSolInitialize` can be called and returns successfully.

- Test_SUNLinSolSetup: Verifies that `SUNLinSolSetup` can be called and returns successfully.

- Test_SUNLinSolSolve: Given a SUNMATRIX object A, NVECTOR objects x and b (where $Ax = b$) and a desired solution tolerance `tol`, this routine clones x into a new vector y, calls `SUNLinSolSolve` to fill y as the solution to $Ay = b$ (to the input tolerance), verifies that each entry in x and y match to within `10*tol`, and overwrites x with y prior to returning (in case the calling routine would like to investigate further).

- Test_SUNLinSolSetATimes (iterative solvers only): Verifies that `SUNLinSolSetATimes` can be called and returns successfully.

- Test_SUNLinSolSetPreconditioner (iterative solvers only): Verifies that `SUNLinSolSetPreconditioner` can be called and returns successfully.

- Test_SUNLinSolSetScalingVectors (iterative solvers only): Verifies that `SUNLinSolSetScalingVectors` can be called and returns successfully.

- `Test_SUNLinSolLastFlag`: Verifies that `SUNLinSolLastFlag` can be called, and outputs the result to `stdout`.

- `Test_SUNLinSolNumIters` (iterative solvers only): Verifies that `SUNLinSolNumIters` can be called, and outputs the result to `stdout`.

- `Test_SUNLinSolResNorm` (iterative solvers only): Verifies that `SUNLinSolResNorm` can be called, and that the result is non-negative.

- `Test_SUNLinSolResid` (iterative solvers only): Verifies that `SUNLinSolResid` can be called.

- `Test_SUNLinSolSpace` verifies that `SUNLinSolSpace` can be called, and outputs the results to `stdout`.

We'll note that these tests should be performed in a particular order. For either direct or iterative linear solvers, `Test_SUNLinSolInitialize` must be called before `Test_SUNLinSolSetup`, which must be called before `Test_SUNLinSolSolve`. Additionally, for iterative linear solvers `Test_SUNLinSolSetATimes`, `Test_SUNLinSolSetPreconditioner` and `Test_SUNLinSolSetScalingVectors` should be called before `Test_SUNLinSolInitialize`; similarly `Test_SUNLinSolNumIters`, `Test_SUNLinSolResNorm` and `Test_SUNLinSolResid` should be called after `Test_SUNLinSolSolve`. These are called in the appropriate order in all of the example problems.

9.15   SUNLinearSolver functions used by IDAS

In Table 9.5 below, we list the linear solver functions in the SUNLINSOL module used within the IDAS package. The table also shows, for each function, which of the code modules uses the function. In general, the main IDAS integrator considers three categories of linear solvers, *direct*, *iterative* and *custom*, with interfaces accessible in the IDAS header files `idas_direct.h` (IDADLS), `idas_spils.h` (IDASPILS) and `idas_customls.h` (IDACLS), respectively. Hence, the table columns reference the use of SUNLINSOL functions by each of these solver interfaces.

As with the SUNMATRIX module, we emphasize that the IDAS user does not need to know detailed usage of linear solver functions by the IDAS code modules in order to use IDAS. This information is presented as an implementation detail for the interested reader.

Table 9.5: List of linear solver functions usage by IDAS code modules

	IDADLS	IDASPILS	IDACLS
SUNLinSolGetType	✓	✓	†
SUNLinSolSetATimes		✓	†
SUNLinSolSetPreconditioner		✓	†
SUNLinSolSetScalingVectors		✓	†
SUNLinSolInitialize	✓	✓	✓
SUNLinSolSetup	✓	✓	✓
SUNLinSolSolve	✓	✓	✓
SUNLinSolNumIters		✓	†
SUNLinSolResNorm		✓	†
SUNLinSolResid		✓	†
SUNLinSolLastFlag			
SUNLinSolFree			
SUNLinSolSpace	†	†	†

The linear solver functions listed in Table 9.2 with a † symbol are optionally used, in that these are only called if they are implemented in the SUNLINSOL module that is being used (i.e. their function

pointers are non-NULL). Also, although IDAS does not call the `SUNLinSolLastFlag` or `SUNLinSolFree` routines directly, these are available for users to query linear solver issues and free linear solver memory directly.

Appendix A

SUNDIALS Package Installation Procedure

The installation of any SUNDIALS package is accomplished by installing the SUNDIALS suite as a whole, according to the instructions that follow. The same procedure applies whether or not the downloaded file contains one or all solvers in SUNDIALS.

The SUNDIALS suite (or individual solvers) are distributed as compressed archives (`.tar.gz`). The name of the distribution archive is of the form *solver*-`x.y.z.tar.gz`, where *solver* is one of: `sundials`, `cvode`, `cvodes`, `arkode`, `ida`, `idas`, or `kinsol`, and `x.y.z` represents the version number (of the SUNDIALS suite or of the individual solver). To begin the installation, first uncompress and expand the sources, by issuing

```
% tar xzf solver-x.y.z.tar.gz
```

This will extract source files under a directory *solver*-`x.y.z`.

Starting with version `2.6.0` of SUNDIALS, CMake is the only supported method of installation. The explanations of the installation procedure begins with a few common observations:

- The remainder of this chapter will follow these conventions:

  solverdir is the directory *solver*-`x.y.z` created above; i.e., the directory containing the SUNDIALS sources.

  builddir is the (temporary) directory under which SUNDIALS is built.

  instdir is the directory under which the SUNDIALS exported header files and libraries will be installed. Typically, header files are exported under a directory *instdir*/`include` while libraries are installed under *instdir*/`lib`, with *instdir* specified at configuration time.

- For SUNDIALS CMake-based installation, in-source builds are prohibited; in other words, the build directory *builddir* can **not** be the same as *solverdir* and such an attempt will lead to an error. This prevents "polluting" the source tree and allows efficient builds for different configurations and/or options.

- The installation directory *instdir* can **not** be the same as the source directory *solverdir*.

- By default, only the libraries and header files are exported to the installation directory *instdir*. If enabled by the user (with the appropriate toggle for CMake), the examples distributed with SUNDIALS will be built together with the solver libraries but the installation step will result in exporting (by default in a subdirectory of the installation directory) the example sources and sample outputs together with automatically generated configuration files that reference the *installed* SUNDIALS headers and libraries. As such, these configuration files for the SUNDIALS examples can be used as "templates" for your own problems. CMake installs `CMakeLists.txt` files and also (as an option available only under Unix/Linux) `Makefile` files. Note this installation

approach also allows the option of building the SUNDIALS examples without having to install them. (This can be used as a sanity check for the freshly built libraries.)

- Even if generation of shared libraries is enabled, only static libraries are created for the FCMIX modules. (Because of the use of fixed names for the Fortran user-provided subroutines, FCMIX shared libraries would result in "undefined symbol" errors at link time.)

A.1   CMake-based installation

CMake-based installation provides a platform-independent build system. CMake can generate Unix and Linux Makefiles, as well as KDevelop, Visual Studio, and (Apple) XCode project files from the same configuration file. In addition, CMake also provides a GUI front end and which allows an interactive build and installation process.

The SUNDIALS build process requires CMake version `3.0.2` or higher and a working C compiler. On Unix-like operating systems, it also requires Make (and `curses`, including its development libraries, for the GUI front end to CMake, `ccmake`), while on Windows it requires Visual Studio. While many Linux distributions offer CMake, the version included may be out of date. Many new CMake features have been added recently, and you should download the latest version from `http://www.cmake.org`. Build instructions for CMake (only necessary for Unix-like systems) can be found on the CMake website. Once CMake is installed, Linux/Unix users will be able to use `ccmake`, while Windows users will be able to use `CMakeSetup`.

As previously noted, when using CMake to configure, build and install SUNDIALS, it is always required to use a separate build directory. While in-source builds are possible, they are explicitly prohibited by the SUNDIALS CMake scripts (one of the reasons being that, unlike autotools, CMake does not provide a `make distclean` procedure and it is therefore difficult to clean-up the source tree after an in-source build). By ensuring a separate build directory, it is an easy task for the user to clean-up all traces of the build by simply removing the build directory. CMake does generate a `make clean` which will remove files generated by the compiler and linker.

A.1.1   Configuring, building, and installing on Unix-like systems

The default CMake configuration will build all included solvers and associated examples and will build static and shared libraries. The *instdir* defaults to `/usr/local` and can be changed by setting the `CMAKE_INSTALL_PREFIX` variable. Support for FORTRAN and all other options are disabled.

CMake can be used from the command line with the `cmake` command, or from a `curses`-based GUI by using the `ccmake` command. Examples for using both methods will be presented. For the examples shown it is assumed that there is a top level SUNDIALS directory with appropriate source, build and install directories:

```
% mkdir (...)sundials/instdir
% mkdir (...)sundials/builddir
% cd (...)sundials/builddir
```

Building with the GUI

Using CMake with the GUI follows this general process:

- Select and modify values, run configure (`c` key)

- New values are denoted with an asterisk

- To set a variable, move the cursor to the variable and press enter

  - If it is a boolean (ON/OFF) it will toggle the value
  - If it is string or file, it will allow editing of the string

- For file and directories, the `<tab>` key can be used to complete

- Repeat until all values are set as desired and the generate option is available (`g` key)

- Some variables (advanced variables) are not visible right away

- To see advanced variables, toggle to advanced mode (`t` key)

- To search for a variable press `/` key, and to repeat the search, press the `n` key

To build the default configuration using the GUI, from the *builddir* enter the ccmake command and point to the *solverdir*:

```
% ccmake ../solverdir
```

The default configuration screen is shown in Figure A.1.
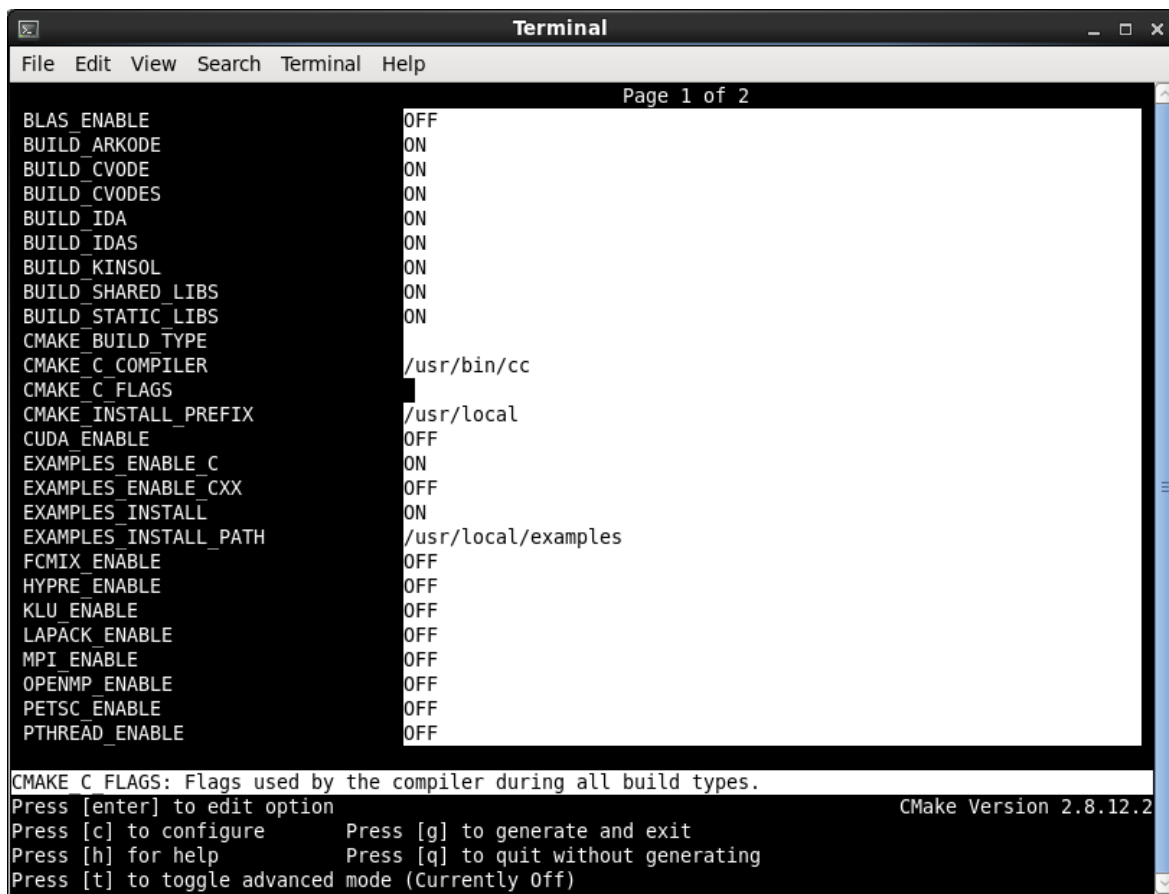
Figure A.1: Default configuration screen. Note: Initial screen is empty. To get this default configuration, press 'c' repeatedly (accepting default values denoted with asterisk) until the 'g' option is available.

The default *instdir* for both SUNDIALS and corresponding examples can be changed by setting the CMAKE_INSTALL_PREFIX and the EXAMPLES_INSTALL_PATH as shown in figure A.2.

Pressing the (`g` key) will generate makefiles including all dependencies and all rules to build SUNDIALS on this system. Back at the command prompt, you can now run:

```
% make
```

To install SUNDIALS in the installation directory specified in the configuration, simply run:

```
% make install
```

```
                                              Terminal                                    _ □ ×
 File  Edit  View  Search  Terminal  Help
                                       Page 1 of 2
   BLAS_ENABLE                       OFF
   BUILD_ARKODE                      ON
   BUILD_CVODE                       ON
   BUILD_CVODES                      ON
   BUILD_IDA                         ON
   BUILD_IDAS                        ON
   BUILD_KINSOL                      ON
   BUILD_SHARED_LIBS                 ON
   BUILD_STATIC_LIBS                 ON
   CMAKE_BUILD_TYPE
   CMAKE_C_COMPILER                  /usr/bin/cc
   CMAKE_C_FLAGS                     █
   CMAKE_INSTALL_PREFIX              /usr/casc/sundials/instdir
   CUDA_ENABLE                       OFF
   EXAMPLES_ENABLE_C                 ON
   EXAMPLES_ENABLE_CXX               OFF
   EXAMPLES_INSTALL                  ON
   EXAMPLES_INSTALL_PATH             /usr/casc/sundials/instdir/examples
   FCMIX_ENABLE                      OFF
   HYPRE_ENABLE                      OFF
   KLU_ENABLE                        OFF
   LAPACK_ENABLE                     OFF
   MPI_ENABLE                        OFF
   OPENMP_ENABLE                     OFF
   PETSC_ENABLE                      OFF
   PTHREAD_ENABLE                    OFF

 CMAKE_C_FLAGS: Flags used by the compiler during all build types.
 Press [enter] to edit option                                   CMake Version 2.8.12.2
 Press [c] to configure       Press [g] to generate and exit
 Press [h] for help           Press [q] to quit without generating
 Press [t] to toggle advanced mode (Currently Off)
```

Figure A.2: Changing the *instdir* for SUNDIALS and corresponding examples

Building from the command line

Using CMake from the command line is simply a matter of specifying CMake variable settings with the cmake command. The following will build the default configuration:

```
% cmake -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> ../solverdir
% make
% make install
```

A.1.2   Configuration options (Unix/Linux)

A complete list of all available options for a CMake-based SUNDIALS configuration is provide below. Note that the default values shown are for a typical configuration on a Linux system and are provided as illustration only.

BLAS_ENABLE - Enable BLAS support
    Default: OFF
    Note: Setting this option to ON will trigger additional CMake options. See additional information on building with BLAS enabled in A.1.4.

BLAS_LIBRARIES - BLAS library
    Default: /usr/lib/libblas.so

Note: CMake will search for libraries in your `LD LIBRARY PATH` prior to searching default system paths.

`BUILD ARKODE` - Build the ARKODE library
Default: ON

`BUILD CVODE` - Build the CVODE library
Default: ON

`BUILD CVODES` - Build the CVODES library
Default: ON

`BUILD IDA` - Build the IDA library
Default: ON

`BUILD IDAS` - Build the IDAS library
Default: ON

`BUILD KINSOL` - Build the KINSOL library
Default: ON

`BUILD SHARED LIBS` - Build shared libraries
Default: ON

`BUILD STATIC LIBS` - Build static libraries
Default: ON

`CMAKE BUILD TYPE` - Choose the type of build, options are: `None` (CMAKE_C_FLAGS used), `Debug`, `Release`, `RelWithDebInfo`, and `MinSizeRel`
Default:
Note: Specifying a build type will trigger the corresponding build type specific compiler flag options below which will be appended to the flags set by CMAKE_<language>_FLAGS.

`CMAKE C COMPILER` - C compiler
Default: /usr/bin/cc

`CMAKE C FLAGS` - Flags for C compiler
Default:

`CMAKE C FLAGS DEBUG` - Flags used by the C compiler during debug builds
Default: -g

`CMAKE C FLAGS MINSIZEREL` - Flags used by the C compiler during release minsize builds
Default: -Os -DNDEBUG

`CMAKE C FLAGS RELEASE` - Flags used by the C compiler during release builds
Default: -O3 -DNDEBUG

`CMAKE CXX COMPILER` - C++ compiler
Default: /usr/bin/c++
Note: A C++ compiler (and all related options) are only triggered if C++ examples are enabled (`EXAMPLES ENABLE CXX` is ON). All sundials solvers can be used from C++ applications by default without setting any additional configuration options.

`CMAKE CXX FLAGS` - Flags for C++ compiler
Default:

`CMAKE CXX FLAGS DEBUG` - Flags used by the C++ compiler during debug builds
Default: -g

CMAKE_CXX_FLAGS_MINSIZEREL - Flags used by the C++ compiler during release minsize builds
      Default: -Os -DNDEBUG

CMAKE_CXX_FLAGS_RELEASE - Flags used by the C++ compiler during release builds
      Default: -O3 -DNDEBUG

CMAKE_Fortran_COMPILER - Fortran compiler
      Default: /usr/bin/gfortran
      Note: Fortran support (and all related options) are triggered only if either Fortran-C support is enabled (FCMIX_ENABLE is ON) or BLAS/LAPACK support is enabled (BLAS_ENABLE or LAPACK_ENABLE is ON).

CMAKE_Fortran_FLAGS - Flags for Fortran compiler
      Default:

CMAKE_Fortran_FLAGS_DEBUG - Flags used by the Fortran compiler during debug builds
      Default: -g

CMAKE_Fortran_FLAGS_MINSIZEREL - Flags used by the Fortran compiler during release minsize builds
      Default: -Os

CMAKE_Fortran_FLAGS_RELEASE - Flags used by the Fortran compiler during release builds
      Default: -O3

CMAKE_INSTALL_PREFIX - Install path prefix, prepended onto install directories
      Default: /usr/local
      Note: The user must have write access to the location specified through this option. Exported SUNDIALS header files and libraries will be installed under subdirectories `include` and `lib` of CMAKE_INSTALL_PREFIX, respectively.

CUDA_ENABLE - Build the SUNDIALS CUDA vector module.
      Default: OFF

EXAMPLES_ENABLE_C - Build the SUNDIALS C examples
      Default: ON

EXAMPLES_ENABLE_CUDA - Build the SUNDIALS CUDA examples
      Default: OFF
      Note: You need to enable CUDA support to build these examples.

EXAMPLES_ENABLE_CXX - Build the SUNDIALS C++ examples
      Default: OFF

EXAMPLES_ENABLE_RAJA - Build the SUNDIALS RAJA examples
      Default: OFF
      Note: You need to enable CUDA and RAJA support to build these examples.

EXAMPLES_ENABLE_F77 - Build the SUNDIALS Fortran77 examples
      Default: ON (if FCMIX_ENABLE is ON)

EXAMPLES_ENABLE_F90 - Build the SUNDIALS Fortran90 examples
      Default: OFF

EXAMPLES_INSTALL - Install example files
      Default: ON
      Note: This option is triggered when any of the SUNDIALS example programs are enabled (EXAMPLES_ENABLE_<language> is ON). If the user requires installation of example programs then the sources and sample output files for all SUNDIALS modules that are currently enabled will be exported to the directory specified by EXAMPLES_INSTALL_PATH. A CMake configuration

script will also be automatically generated and exported to the same directory. Additionally, if the configuration is done under a Unix-like system, makefiles for the compilation of the example programs (using the installed SUNDIALS libraries) will be automatically generated and exported to the directory specified by EXAMPLES_INSTALL_PATH.

EXAMPLES_INSTALL_PATH - Output directory for installing example files
> Default: /usr/local/examples
> Note: The actual default value for this option will be an examples subdirectory created under CMAKE_INSTALL_PREFIX.

FCMIX_ENABLE - Enable Fortran-C support
> Default: OFF

HYPRE_ENABLE - Enable *hypre* support
> Default: OFF
> Note: See additional information on building with *hypre* enabled in A.1.4.

HYPRE_INCLUDE_DIR - Path to *hypre* header files

HYPRE_LIBRARY_DIR - Path to *hypre* installed library files

KLU_ENABLE - Enable KLU support
> Default: OFF
> Note: See additional information on building with KLU enabled in A.1.4.

KLU_INCLUDE_DIR - Path to SuiteSparse header files

KLU_LIBRARY_DIR - Path to SuiteSparse installed library files

LAPACK_ENABLE - Enable LAPACK support
> Default: OFF
> Note: Setting this option to ON will trigger additional CMake options. See additional information on building with LAPACK enabled in A.1.4.

LAPACK_LIBRARIES - LAPACK (and BLAS) libraries
> Default: /usr/lib/liblapack.so;/usr/lib/libblas.so
> Note: CMake will search for libraries in your LD_LIBRARY_PATH prior to searching default system paths.

MPI_ENABLE - Enable MPI support (build the parallel nvector).
> Default: OFF
> Note: Setting this option to ON will trigger several additional options related to MPI.

MPI_C_COMPILER - mpicc program
> Default:

MPI_CXX_COMPILER - mpicxx program
> Default:
> Note: This option is triggered only if MPI is enabled (MPI_ENABLE is ON) and C++ examples are enabled (EXAMPLES_ENABLE_CXX is ON). All SUNDIALS solvers can be used from C++ MPI applications by default without setting any additional configuration options other than MPI_ENABLE.

MPI_Fortran_COMPILER - mpif77 or mpif90 program
> Default:
> Note: This option is triggered only if MPI is enabled (MPI_ENABLE is ON), Fortran-C support is enabled (FCMIX_ENABLE is ON), and Fortran77 or Fortran90 examples are enabled (EXAMPLES_ENABLE_F77 or EXAMPLES_ENABLE_F90 are ON).

MPIEXEC - Specify the executable for running MPI programs
> Default: `mpirun`
> Note: This option is triggered only if MPI is enabled (`MPI_ENABLE` is ON).

OPENMP_ENABLE - Enable OpenMP support (build the OpenMP nvector).
> Default: OFF

PETSC_ENABLE - Enable PETSc support
> Default: OFF
> Note: See additional information on building with PETSc enabled in A.1.4.

PETSC_INCLUDE_DIR - Path to PETSc header files

PETSC_LIBRARY_DIR - Path to PETSc installed library files

PTHREAD_ENABLE - Enable Pthreads support (build the Pthreads nvector).
> Default: OFF

RAJA_ENABLE - Enable RAJA support (build the RAJA nvector).
> Default: OFF
> Note: You need to enable CUDA in order to build the RAJA vector module.

SUNDIALS_F77_FUNC_CASE - **advanced option** - Specify the case to use in the Fortran name-mangling scheme, options are: `lower` or `upper`
> Default:
> Note: The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available or to override the inferred or default (`lower`) scheme if one can not be determined. If used, `SUNDIALS_F77_FUNC_UNDERSCORES` must also be set.

SUNDIALS_F77_FUNC_UNDERSCORES - **advanced option** - Specify the number of underscores to append in the Fortran name-mangling scheme, options are: `none`, `one`, or `two`
> Default:
> Note: The build system will attempt to infer the Fortran name-mangling scheme using the Fortran compiler. This option should only be used if a Fortran compiler is not available or to override the inferred or default (`one`) scheme if one can not be determined. If used, `SUNDIALS_F77_FUNC_CASE` must also be set.

SUNDIALS_INDEX_TYPE - Integer type used for SUNDIALS indices, options are: `int32_t` or `int64_t`
> Default: `int64_t`

SUNDIALS_PRECISION - Precision used in SUNDIALS, options are: `double`, `single`, or `extended`
> Default: `double`

SUPERLUMT_ENABLE - Enable SuperLU_MT support
> Default: OFF
> Note: See additional information on building with SuperLU_MT enabled in A.1.4.

SUPERLUMT_INCLUDE_DIR - Path to SuperLU_MT header files (typically SRC directory)

SUPERLUMT_LIBRARY_DIR - Path to SuperLU_MT installed library files

SUPERLUMT_THREAD_TYPE - Must be set to Pthread or OpenMP
> Default: Pthread

USE_GENERIC_MATH - Use generic (stdc) math libraries
> Default: ON

xSDK Configuration Options

SUNDIALS supports CMake configuration options defined by the Extreme-scale Scientific Software Development Kit (xSDK) community policies (see `https://xsdk.info` for more information). xSDK CMake options are unused by default but may be activated by setting `USE_XSDK_DEFAULTS` to ON.

⚠️

When xSDK options are active, they will overwrite the corresponding SUNDIALS option and may have different default values (see details below). As such the equivalent SUNDIALS options should not be used when configuring with xSDK options. In the GUI front end to CMake (`ccmake`), setting `USE_XSDK_DEFAULTS` to ON will hide the corresponding SUNDIALS options as advanced CMake variables. During configuration, messages are output detailing which xSDK flags are active and the equivalent SUNDIALS options that are replaced. Below is a complete list xSDK options and the corresponding SUNDIALS options if applicable.

`TPL_BLAS_LIBRARIES` - BLAS library
> Default: /usr/lib/libblas.so
> SUNDIALS equivalent: `BLAS_LIBRARIES`
> Note: CMake will search for libraries in your `LD_LIBRARY_PATH` prior to searching default system paths.

`TPL_ENABLE_BLAS` - Enable BLAS support
> Default: OFF
> SUNDIALS equivalent: `BLAS_ENABLE`

`TPL_ENABLE_HYPRE` - Enable *hypre* support
> Default: OFF
> SUNDIALS equivalent: `HYPRE_ENABLE`

`TPL_ENABLE_KLU` - Enable KLU support
> Default: OFF
> SUNDIALS equivalent: `KLU_ENABLE`

`TPL_ENABLE_PETSC` - Enable PETSc support
> Default: OFF
> SUNDIALS equivalent: `PETSC_ENABLE`

`TPL_ENABLE_LAPACK` - Enable LAPACK support
> Default: OFF
> SUNDIALS equivalent: `LAPACK_ENABLE`

`TPL_ENABLE_SUPERLUMT` - Enable SuperLU_MT support
> Default: OFF
> SUNDIALS equivalent: `SUPERLUMT_ENABLE`

`TPL_HYPRE_INCLUDE_DIRS` - Path to *hypre* header files
> SUNDIALS equivalent: `HYPRE_INCLUDE_DIR`

`TPL_HYPRE_LIBRARIES` - *hypre* library
> SUNDIALS equivalent: N/A

`TPL_KLU_INCLUDE_DIRS` - Path to KLU header files
> SUNDIALS equivalent: `KLU_INCLUDE_DIR`

`TPL_KLU_LIBRARIES` - KLU library
> SUNDIALS equivalent: N/A

`TPL_LAPACK_LIBRARIES` - LAPACK (and BLAS) libraries
> Default: /usr/lib/liblapack.so;/usr/lib/libblas.so
> SUNDIALS equivalent: `LAPACK_LIBRARIES`
> Note: CMake will search for libraries in your `LD_LIBRARY_PATH` prior to searching default system paths.

TPL_PETSC_INCLUDE_DIRS - Path to PETSc header files
     SUNDIALS equivalent: PETSC_INCLUDE_DIR

TPL_PETSC_LIBRARIES - PETSc library
     SUNDIALS equivalent: N/A

TPL_SUPERLUMT_INCLUDE_DIRS - Path to SuperLU_MT header files
     SUNDIALS equivalent: SUPERLUMT_INCLUDE_DIR

TPL_SUPERLUMT_LIBRARIES - SuperLU_MT library
     SUNDIALS equivalent: N/A

TPL_SUPERLUMT_THREAD_TYPE - SuperLU_MT library thread type
     SUNDIALS equivalent: SUPERLUMT_THREAD_TYPE

USE_XSDK_DEFAULTS - Enable xSDK default configuration settings
     Default: OFF
     SUNDIALS equivalent: N/A
     Note: Enabling xSDK defaults also sets CMAKE_BUILD_TYPE to Debug

XSDK_ENABLE_FORTRAN - Enable SUNDIALS Fortran interface
     Default: OFF
     SUNDIALS equivalent: FCMIX_ENABLE

XSDK_INDEX_SIZE - Integer size (bits) used for indices in SUNDIALS, options are: 32 or 64
     Default: 32
     SUNDIALS equivalent: SUNDIALS_INDEX_TYPE

XSDK_PRECISION - Precision used in SUNDIALS, options are: double, single, or quad
     Default: double
     SUNDIALS equivalent: SUNDIALS_PRECISION

A.1.3 Configuration examples

The following examples will help demonstrate usage of the CMake configure options.
To configure SUNDIALS using the default C and Fortran compilers, and default mpicc and mpif77
parallel compilers, enable compilation of examples, and install libraries, headers, and example sources
under subdirectories of /home/myname/sundials/, use:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DMPI_ENABLE=ON \
> -DFCMIX_ENABLE=ON \
> /home/myname/sundials/solverdir
%
% make install
%
```

To disable installation of the examples, use:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DMPI_ENABLE=ON \
> -DFCMIX_ENABLE=ON \
> -DEXAMPLES_INSTALL=OFF \
> /home/myname/sundials/solverdir
```

```
%
% make install
%
```

A.1.4   Working with external Libraries

The SUNDIALS suite contains many options to enable implementation flexibility when developing solutions. The following are some notes addressing specific configurations when using the supported third party libraries. When building SUNDIALS as a shared library external libraries any used with SUNDIALS must also be build as a shared library or as a static library compiled with the `-fPIC` flag.

Building with BLAS

SUNDIALS does not utilize BLAS directly but it may be needed by other external libraries that SUNDIALS can be built with (e.g. LAPACK, PETSc, SuperLU_MT, etc.). To enable BLAS, set the `BLAS_ENABLE` option to `ON`. If the directory containing the BLAS library is in the `LD_LIBRARY_PATH` environment variable, CMake will set the `BLAS_LIBRARIES` variable accordingly, otherwise CMake will attempt to find the BLAS library in standard system locations. To explicitly tell CMake what libraries to use, the `BLAS_LIBRARIES` variable can be set to the desired library. Example:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DBLAS_ENABLE=ON \
> -DBLAS_LIBRARIES=/myblaspath/lib/libblas.so \
> -DSUPERLUMT_ENABLE=ON \
> -DSUPERLUMT_INCLUDE_DIR=/mysuperlumtpath/SRC
> -DSUPERLUMT_LIBRARY_DIR=/mysuperlumtpath/lib
> /home/myname/sundials/solverdir
%
% make install
%
```

When allowing CMake to automatically locate the LAPACK library, CMake *may* also locate the corresponding BLAS library.

If a working Fortran compiler is not available to infer the Fortran name-mangling scheme, the options `SUNDIALS_F77_FUNC_CASE` and `SUNDIALS_F77_FUNC_UNDERSCORES` *must* be set in order to bypass the check for a Fortran compiler and define the name-mangling scheme. The defaults for these options in earlier versions of SUNDIALS were `lower` and `one` respectively.

Building with LAPACK

To enable LAPACK, set the `LAPACK_ENABLE` option to `ON`. If the directory containing the LAPACK library is in the `LD_LIBRARY_PATH` environment variable, CMake will set the `LAPACK_LIBRARIES` variable accordingly, otherwise CMake will attempt to find the LAPACK library in standard system locations. To explicitly tell CMake what library to use, the `LAPACK_LIBRARIES` variable can be set to the desired libraries. When setting the LAPACK location explicitly the location of the corresponding BLAS library will also need to be set. Example:

```
% cmake \
> -DCMAKE_INSTALL_PREFIX=/home/myname/sundials/instdir \
> -DEXAMPLES_INSTALL_PATH=/home/myname/sundials/instdir/examples \
> -DBLAS_ENABLE=ON \
> -DBLAS_LIBRARIES=/mylapackpath/lib/libblas.so \
> -DLAPACK_ENABLE=ON \
> -DLAPACK_LIBRARIES=/mylapackpath/lib/liblapack.so \
```

```
> /home/myname/sundials/solverdir
%
% make install
%
```

⚠️ When allowing CMake to automatically locate the LAPACK library, CMake *may* also locate the corresponding BLAS library.

If a working Fortran compiler is not available to infer the Fortran name-mangling scheme, the options SUNDIALS_F77_FUNC_CASE and SUNDIALS_F77_FUNC_UNDERSCORES *must* be set in order to bypass the check for a Fortran compiler and define the name-mangling scheme. The defaults for these options in earlier versions of SUNDIALS were `lower` and `one` respectively.

Building with KLU

The KLU libraries are part of SuiteSparse, a suite of sparse matrix software, available from the Texas A&M University website: `http://faculty.cse.tamu.edu/davis/suitesparse.html`. SUNDIALS has been tested with SuiteSparse version 4.5.3. To enable KLU, set KLU_ENABLE to ON, set KLU_INCLUDE_DIR to the `include` path of the KLU installation and set KLU_LIBRARY_DIR to the `lib` path of the KLU installation. The CMake configure will result in populating the following variables: AMD_LIBRARY, AMD_LIBRARY_DIR, BTF_LIBRARY, BTF_LIBRARY_DIR, COLAMD_LIBRARY, COLAMD_LIBRARY_DIR, and KLU_LIBRARY.

Building with SuperLU_MT

The SuperLU_MT libraries are available for download from the Lawrence Berkeley National Laboratory website: `http://crd-legacy.lbl.gov/~xiaoye/SuperLU/#superlu_mt`. SUNDIALS has been tested with SuperLU_MT version 3.1. To enable SuperLU_MT, set SUPERLUMT_ENABLE to ON, set SUPERLUMT_INCLUDE_DIR to the SRC path of the SuperLU_MT installation, and set the variable SUPERLUMT_LIBRARY_DIR to the `lib` path of the SuperLU_MT installation. At the same time, the variable SUPERLUMT_THREAD_TYPE must be set to either `Pthread` or `OpenMP`.

⚠️ Do not mix thread types when building SUNDIALS solvers. If threading is enabled for SUNDIALS by having either OPENMP_ENABLE or PTHREAD_ENABLE set to ON then SuperLU_MT should be set to use the same threading type.

Building with PETSc

The PETSc libraries are available for download from the Argonne National Laboratory website: `http://www.mcs.anl.gov/petsc`. SUNDIALS has been tested with PETSc version 3.7.2. To enable PETSc, set PETSC_ENABLE to ON, set PETSC_INCLUDE_DIR to the `include` path of the PETSc installation, and set the variable PETSC_LIBRARY_DIR to the `lib` path of the PETSc installation.

Building with *hypre*

The *hypre* libraries are available for download from the Lawrence Livermore National Laboratory website: `http://computation.llnl.gov/projects/hypre`. SUNDIALS has been tested with *hypre* version 2.11.1. To enable *hypre*, set HYPRE_ENABLE to ON, set HYPRE_INCLUDE_DIR to the `include` path of the *hypre* installation, and set the variable HYPRE_LIBRARY_DIR to the `lib` path of the *hypre* installation.

Building with CUDA

SUNDIALS CUDA modules and examples have been tested with version 8.0 of the CUDA toolkit. To build them, you need to install the Toolkit and compatible NVIDIA drivers. Both are available for download from the NVIDIA website: `https://developer.nvidia.com/cuda-downloads`. To enable CUDA, set CUDA_ENABLE to ON. If CUDA is installed in a nonstandard location, you may be prompted to

set the variable `CUDA_TOOLKIT_ROOT_DIR` with your CUDA Toolkit installation path. To enable CUDA examples, set `EXAMPLES_ENABLE_CUDA` to `ON`.

Building with RAJA

RAJA is a performance portability layer developed by Lawrence Livermore National Laboratory and can be obtained from `https://github.com/LLNL/RAJA`. SUNDIALS RAJA modules and examples have been tested with RAJA version 0.3. Building SUNDIALS RAJA modules requires a CUDA-enabled RAJA installation. To enable RAJA, set `CUDA_ENABLE` and `RAJA_ENABLE` to `ON`. If RAJA is installed in a nonstandard location you will be prompted to set the variable `RAJA_DIR` with the path to the RAJA CMake configuration file. To enable building the RAJA examples set `EXAMPLES_ENABLE_RAJA` to `ON`.

A.1.5 Testing the build and installation

If SUNDIALS was configured with `EXAMPLES_ENABLE_<language>` options to `ON`, then a set of regression tests can be run after building with the `make` command by running:

```
% make test
```

Additionally, if `EXAMPLES_INSTALL` was also set to `ON`, then a set of smoke tests can be run after installing with the `make install` command by running:

```
% make test_install
```

A.2 Building and Running Examples

Each of the SUNDIALS solvers is distributed with a set of examples demonstrating basic usage. To build and install the examples, set at least of the `EXAMPLES_ENABLE_<language>` options to `ON`, and set `EXAMPLES_INSTALL` to `ON`. Specify the installation path for the examples with the variable `EXAMPLES_INSTALL_PATH`. CMake will generate `CMakeLists.txt` configuration files (and `Makefile` files if on Linux/Unix) that reference the *installed* SUNDIALS headers and libraries.

Either the `CMakeLists.txt` file or the traditional `Makefile` may be used to build the examples as well as serve as a template for creating user developed solutions. To use the supplied `Makefile` simply run `make` to compile and generate the executables. To use CMake from within the installed example directory, run `cmake` (or `ccmake` to use the GUI) followed by `make` to compile the example code. Note that if CMake is used, it will overwrite the traditional `Makefile` with a new CMake-generated `Makefile`. The resulting output from running the examples can be compared with example output bundled in the SUNDIALS distribution.

NOTE: There will potentially be differences in the output due to machine architecture, compiler versions, use of third party libraries etc.

⚠

A.3 Configuring, building, and installing on Windows

CMake can also be used to build SUNDIALS on Windows. To build SUNDIALS for use with Visual Studio the following steps should be performed:

1. Unzip the downloaded tar file(s) into a directory. This will be the *solverdir*

2. Create a separate *builddir*

3. Open a Visual Studio Command Prompt and cd to *builddir*

4. Run cmake-gui ../*solverdir*

    (a) Hit Configure

    (b) Check/Uncheck solvers to be built

(c) Change CMAKE_INSTALL_PREFIX to *instdir*

(d) Set other options as desired

(e) Hit Generate

5. Back in the VS Command Window:

(a) Run msbuild ALL_BUILD.vcxproj

(b) Run msbuild INSTALL.vcxproj

The resulting libraries will be in the *instdir*. The SUNDIALS project can also now be opened in Visual Studio. Double click on the ALL_BUILD.vcxproj file to open the project. Build the whole *solution* to create the SUNDIALS libraries. To use the SUNDIALS libraries in your own projects, you must set the include directories for your project, add the SUNDIALS libraries to your project solution, and set the SUNDIALS libraries as dependencies for your project.

A.4   Installed libraries and exported header files

Using the CMake SUNDIALS build system, the command

```
% make install
```

will install the libraries under *libdir* and the public header files under *includedir*. The values for these directories are *instdir*/`lib` and *instdir*/`include`, respectively. The location can be changed by setting the CMake variable `CMAKE_INSTALL_PREFIX`. Although all installed libraries reside under *libdir*/`lib`, the public header files are further organized into subdirectories under *includedir*/`include`.

   The installed libraries and exported header files are listed for reference in Table A.1. The file extension *.lib* is typically `.so` for shared libraries and `.a` for static libraries. Note that, in the Tables, names are relative to *libdir* for libraries and to *includedir* for header files.

   A typical user program need not explicitly include any of the shared SUNDIALS header files from under the *includedir*/`include/sundials` directory since they are explicitly included by the appropriate solver header files (*e.g.*, `cvode_dense.h` includes `sundials_dense.h`). However, it is both legal and safe to do so, and would be useful, for example, if the functions declared in `sundials_dense.h` are to be used in building a preconditioner.

Table A.1: SUNDIALS libraries and header files

SHARED	Libraries	n/a	
	Header files	sundials/sundials_config.h	sundials/sundials_fconfig.h
		sundials/sundials_types.h	sundials/sundials_math.h
		sundials/sundials_nvector.h	sundials/sundials_fnvector.h
		sundials/sundials_iterative.h	sundials/sundials_direct.h
		sundials/sundials_dense.h	sundials/sundials_band.h
		sundials/sundials_matrix.h	sundials/sundials_version.h
		sundials/sundials_linearsolver.h	
NVECTOR_SERIAL	Libraries	libsundials_nvecserial.*lib*	libsundials_fnvecserial.a
	Header files	nvector/nvector_serial.h	
NVECTOR_PARALLEL	Libraries	libsundials_nvecparallel.*lib*	libsundials_fnvecparallel.a
	Header files	nvector/nvector_parallel.h	
			continued on next page

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NVECTOR_OPENMP	Libraries	libsundials_nvecopenmp.*lib* libsundials_fnvecopenmp.a	
	Header files	nvector/nvector_openmp.h	
NVECTOR_PTHREADS	Libraries	libsundials_nvecpthreads.*lib* libsundials_fnvecpthreads.a	
	Header files	nvector/nvector_pthreads.h	
NVECTOR_PARHYP	Libraries	libsundials_nvecparhyp.*lib*	
	Header files	nvector/nvector_parhyp.h	
NVECTOR_PETSC	Libraries	libsundials_nvecpetsc.*lib*	
	Header files	nvector/nvector_petsc.h	
NVECTOR_CUDA	Libraries	libsundials_nveccuda.*lib*	
	Header files	nvector/nvector_cuda.h	
		nvector/cuda/ThreadPartitioning.hpp	
		nvector/cuda/Vector.hpp	
		nvector/cuda/VectorKernels.cuh	
NVECTOR_RAJA	Libraries	libsundials_nvecraja.*lib*	
	Header files	nvector/nvector_raja.h	
		nvector/raja/Vector.hpp	
SUNMATRIX_BAND	Libraries	libsundials_sunmatrixband.*lib*	
		libsundials_fsunmatrixband.a	
	Header files	sunmatrix/sunmatrix_band.h	
SUNMATRIX_DENSE	Libraries	libsundials_sunmatrixdense.*lib*	
		libsundials_fsunmatrixdense.a	
	Header files	sunmatrix/sunmatrix_dense.h	
SUNMATRIX_SPARSE	Libraries	libsundials_sunmatrixsparse.*lib*	
		libsundials_fsunmatrixsparse.a	
	Header files	sunmatrix/sunmatrix_sparse.h	
SUNLINSOL_BAND	Libraries	libsundials_sunlinsolband.*lib*	
		libsundials_fsunlinsolband.a	
	Header files	sunlinsol/sunlinsol_band.h	
SUNLINSOL_DENSE	Libraries	libsundials_sunlinsoldense.*lib*	
		libsundials_fsunlinsoldense.a	
	Header files	sunlinsol/sunlinsol_dense.h	
SUNLINSOL_KLU	Libraries	libsundials_sunlinsolklu.*lib*	
		libsundials_fsunlinsolklu.a	
	Header files	sunlinsol/sunlinsol_klu.h	
SUNLINSOL_LAPACKBAND	Libraries	libsundials_sunlinsollapackband.*lib*	
		libsundials_fsunlinsollapackband.a	
	Header files	sunlinsol/sunlinsol_lapackband.h	
SUNLINSOL_LAPACKDENSE	Libraries	libsundials_sunlinsollapackdense.*lib*	
		libsundials_fsunlinsollapackdense.a	
	Header files	sunlinsol/sunlinsol_lapackdense.h	
SUNLINSOL_PCG	Libraries	libsundials_sunlinsolpcg.*lib*	
			continued on next page

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		libsundials_fsunlinsolpcg.a	
	Header files	sunlinsol/sunlinsol_pcg.h	
SUNLINSOL_SPBCGS	Libraries	libsundials_sunlinsolspbcgs.*lib*	
		libsundials_fsunlinsolspbcgs.a	
	Header files	sunlinsol/sunlinsol_spbcgs.h	
SUNLINSOL_SPFGMR	Libraries	libsundials_sunlinsolspfgmr.*lib*	
		libsundials_fsunlinsolspfgmr.a	
	Header files	sunlinsol/sunlinsol_spfgmr.h	
SUNLINSOL_SPGMR	Libraries	libsundials_sunlinsolspgmr.*lib*	
		libsundials_fsunlinsolspgmr.a	
	Header files	sunlinsol/sunlinsol_spgmr.h	
SUNLINSOL_SPTFQMR	Libraries	libsundials_sunlinsolsptfqmr.*lib*	
		libsundials_fsunlinsolsptfqmr.a	
	Header files	sunlinsol/sunlinsol_sptfqmr.h	
SUNLINSOL_SUPERLUMT	Libraries	libsundials_sunlinsolsuperlumt.*lib*	
		libsundials_fsunlinsolsuperlumt.a	
	Header files	sunlinsol/sunlinsol_superlumt.h	
CVODE	Libraries	libsundials_cvode.*lib*	libsundials_fcvode.a
	Header files	cvode/cvode.h	cvode/cvode_impl.h
		cvode/cvode_direct.h	cvode/cvode_spils.h
		cvode/cvode_bandpre.h	cvode/cvode_bbdpre.h
CVODES	Libraries	libsundials_cvodes.*lib*	
	Header files	cvodes/cvodes.h	cvodes/cvodes_impl.h
		cvodes/cvodes_direct.h	cvodes/cvodes_spils.h
		cvodes/cvodes_bandpre.h	cvodes/cvodes_bbdpre.h
ARKODE	Libraries	libsundials_arkode.*lib*	libsundials_farkode.a
	Header files	arkode/arkode.h	arkode/arkode_impl.h
		arkode/arkode_direct.h	arkode/arkode_spils.h
		arkode/arkode_bandpre.h	arkode/arkode_bbdpre.h
IDA	Libraries	libsundials_ida.*lib*	libsundials_fida.a
	Header files	ida/ida.h	ida/ida_impl.h
		ida/ida_direct.h	ida/ida_spils.h
		ida/ida_bbdpre.h	
IDAS	Libraries	libsundials_idas.*lib*	
	Header files	idas/idas.h	idas/idas_impl.h
		idas/idas_direct.h	idas/idas_spils.h
		idas/idas_bbdpre.h	
KINSOL	Libraries	libsundials_kinsol.*lib*	libsundials_fkinsol.a
	Header files	kinsol/kinsol.h	kinsol/kinsol_impl.h
		kinsol/kinsol_direct.h	kinsol/kinsol_spils.h
		kinsol/kinsol_bbdpre.h	

Appendix B

IDAS Constants

Below we list all input and output constants used by the main solver and linear solver modules, together with their numerical values and a short description of their meaning.

B.1 IDAS input constants

IDAS **main solver module**		
IDA_NORMAL	1	Solver returns at specified output time.
IDA_ONE_STEP	2	Solver returns after each successful step.
IDA_SIMULTANEOUS	1	Simultaneous corrector forward sensitivity method.
IDA_STAGGERED	2	Staggered corrector forward sensitivity method.
IDA_CENTERED	1	Central difference quotient approximation (2nd order) of the sensitivity RHS.
IDA_FORWARD	2	Forward difference quotient approximation (1st order) of the sensitivity RHS.
IDA_YA_YDP_INIT	1	Compute y_a and $\dot{y}_d$, given y_d.
IDA_Y_INIT	2	Compute y, given $\dot{y}$.

IDAS **adjoint solver module**		
IDA_HERMITE	1	Use Hermite interpolation.
IDA_POLYNOMIAL	2	Use variable-degree polynomial interpolation.

Iterative linear solver module		
PREC_NONE	0	No preconditioning
PREC_LEFT	1	Preconditioning on the left.
MODIFIED_GS	1	Use modified Gram-Schmidt procedure.
CLASSICAL_GS	2	Use classical Gram-Schmidt procedure.

B.2 IDAS output constants

IDAS **main solver module**

IDA_SUCCESS	0	Successful function return.
IDA_TSTOP_RETURN	1	`IDASolve` succeeded by reaching the specified stopping point.
IDA_ROOT_RETURN	2	`IDASolve` succeeded and found one or more roots.
IDA_WARNING	99	`IDASolve` succeeded but an unusual situation occurred.
IDA_TOO_MUCH_WORK	-1	The solver took `mxstep` internal steps but could not reach tout.
IDA_TOO_MUCH_ACC	-2	The solver could not satisfy the accuracy demanded by the user for some internal step.
IDA_ERR_FAIL	-3	Error test failures occurred too many times during one internal time step or minimum step size was reached.
IDA_CONV_FAIL	-4	Convergence test failures occurred too many times during one internal time step or minimum step size was reached.
IDA_LINIT_FAIL	-5	The linear solver's initialization function failed.
IDA_LSETUP_FAIL	-6	The linear solver's setup function failed in an unrecoverable manner.
IDA_LSOLVE_FAIL	-7	The linear solver's solve function failed in an unrecoverable manner.
IDA_RES_FAIL	-8	The user-provided residual function failed in an unrecoverable manner.
IDA_REP_RES_FAIL	-9	The user-provided residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.
IDA_RTFUNC_FAIL	-10	The rootfinding function failed in an unrecoverable manner.
IDA_CONSTR_FAIL	-11	The inequality constraints were violated and the solver was unable to recover.
IDA_FIRST_RES_FAIL	-12	The user-provided residual function failed recoverably on the first call.
IDA_LINESEARCH_FAIL	-13	The line search failed.
IDA_NO_RECOVERY	-14	The residual function, linear solver setup function, or linear solver solve function had a recoverable failure, but `IDACalcIC` could not recover.
IDA_MEM_NULL	-20	The `ida_mem` argument was NULL.
IDA_MEM_FAIL	-21	A memory allocation failed.
IDA_ILL_INPUT	-22	One of the function inputs is illegal.
IDA_NO_MALLOC	-23	The IDAS memory was not allocated by a call to `IDAInit`.
IDA_BAD_EWT	-24	Zero value of some error weight component.
IDA_BAD_K	-25	The k-th derivative is not available.
IDA_BAD_T	-26	The time t is outside the last step taken.
IDA_BAD_DKY	-27	The vector argument where derivative should be stored is NULL.
IDA_NO_QUAD	-30	Quadratures were not initialized.
IDA_QRHS_FAIL	-31	The user-provided right-hand side function for quadratures failed in an unrecoverable manner.
IDA_FIRST_QRHS_ERR	-32	The user-provided right-hand side function for quadratures failed in an unrecoverable manner on the first call.
IDA_REP_QRHS_ERR	-33	The user-provided right-hand side repeatedly returned a recoverable error flag, but the solver was unable to recover.

IDA_NO_SENS	-40	Sensitivities were not initialized.
IDA_SRES_FAIL	-41	The user-provided sensitivity residual function failed in an unrecoverable manner.
IDA_REP_SRES_ERR	-42	The user-provided sensitivity residual function repeatedly returned a recoverable error flag, but the solver was unable to recover.
IDA_BAD_IS	-43	The sensitivity identifier is not valid.
IDA_NO_QUADSENS	-50	Sensitivity-dependent quadratures were not initialized.
IDA_QSRHS_FAIL	-51	The user-provided sensitivity-dependent quadrature right-hand side function failed in an unrecoverable manner.
IDA_FIRST_QSRHS_ERR	-52	The user-provided sensitivity-dependent quadrature right-hand side function failed in an unrecoverable manner on the first call.
IDA_REP_QSRHS_ERR	-53	The user-provided sensitivity-dependent quadrature right-hand side repeatedly returned a recoverable error flag, but the solver was unable to recover.

<table>
<tr><td colspan="3" align="center">IDAS adjoint solver module</td></tr>
<tr><td>IDA_NO_ADJ</td><td>-101</td><td>The combined forward-backward problem has not been initialized.</td></tr>
<tr><td>IDA_NO_FWD</td><td>-102</td><td>IDASolveF has not been previously called.</td></tr>
<tr><td>IDA_NO_BCK</td><td>-103</td><td>No backward problem was specified.</td></tr>
<tr><td>IDA_BAD_TB0</td><td>-104</td><td>The desired output for backward problem is outside the interval over which the forward problem was solved.</td></tr>
<tr><td>IDA_REIFWD_FAIL</td><td>-105</td><td>No checkpoint is available for this hot start.</td></tr>
<tr><td>IDA_FWD_FAIL</td><td>-106</td><td>IDASolveB failed because IDASolve was unable to store data between two consecutive checkpoints.</td></tr>
<tr><td>IDA_GETY_BADT</td><td>-107</td><td>Wrong time in interpolation function.</td></tr>
</table>

<table>
<tr><td colspan="3" align="center">IDADLS linear solver modules</td></tr>
<tr><td>IDADLS_SUCCESS</td><td>0</td><td>Successful function return.</td></tr>
<tr><td>IDADLS_MEM_NULL</td><td>-1</td><td>The ida_mem argument was NULL.</td></tr>
<tr><td>IDADLS_LMEM_NULL</td><td>-2</td><td>The IDADLS linear solver has not been initialized.</td></tr>
<tr><td>IDADLS_ILL_INPUT</td><td>-3</td><td>The IDADLS solver is not compatible with the current NVECTOR module.</td></tr>
<tr><td>IDADLS_MEM_FAIL</td><td>-4</td><td>A memory allocation request failed.</td></tr>
<tr><td>IDADLS_JACFUNC_UNRECVR</td><td>-5</td><td>The Jacobian function failed in an unrecoverable manner.</td></tr>
<tr><td>IDADLS_JACFUNC_RECVR</td><td>-6</td><td>The Jacobian function had a recoverable error.</td></tr>
<tr><td>IDADLS_SUNMAT_FAIL</td><td>-7</td><td>An error occurred with the current SUNMATRIX module.</td></tr>
<tr><td>IDADLS_NO_ADJ</td><td>-101</td><td>The combined forward-backward problem has not been initialized.</td></tr>
<tr><td>IDADLS_LMEMB_NULL</td><td>-102</td><td>The linear solver was not initialized for the backward phase.</td></tr>
</table>

	IDASPILS **linear solver modules**	

IDASPILS_SUCCESS	0	Successful function return.
IDASPILS_MEM_NULL	-1	The `ida_mem` argument was NULL.
IDASPILS_LMEM_NULL	-2	The IDASPILS linear solver has not been initialized.
IDASPILS_ILL_INPUT	-3	The IDASPILS solver is not compatible with the current NVEC-TOR module, or an input value was illegal.
IDASPILS_MEM_FAIL	-4	A memory allocation request failed.
IDASPILS_PMEM_NULL	-5	The preconditioner module has not been initialized.
IDASPILS_SUNLS_FAIL	-6	An error occurred with the current SUNLINSOL module.
IDASPILS_NO_ADJ	-101	The combined forward-backward problem has not been initialized.
IDASPILS_LMEMB_NULL	-102	The linear solver was not initialized for the backward phase.

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