VIMS_alpOri271I_analysis_v2

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0.1 VIMS_alpOri271I_analysis_v2.ipynb

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This Jupyter notebook contains an analysis of the Cassini VIMS observations of the alpOri271 occultation, using data provided by An Foster and Phil Nicholson.

The following steps are performed:

- 1) Perform model fits to the observations: isothermal fits isothermal + absorption model based on Goody random band model
- 2) Overplot Phil Nicholson's model predictions
- 3) Perform numerical inversion of observations to derive T(P) profile
- 4) Compare retrieved T(P) with CIRS observations
- 5) Construct synthetic lightcurve based on CIRS T(P) and compare with observations
- 6) Perform end-to-end test of inverting the synthetic lightcurve to show that it matches input T(P)

This is a non-interactive notebook - simply run all cells All required Python packages and data files are defined in the first code cell All individual functions and procedures are in separate cells, with a description of the purpose and method of each. The final cells perform the numbered steps above.

Revisions:

v2:

2023 Oct 10 - rfrench - Move all routines to top, retain tests for documentation, but start ne Liens:

1. g(r) not implemented - assumed to be constant over range of inversion (easy to fix)

```
[1]: # Load Python packages (ALL required dependencies are collected here)

from astropy import units as u
import astropy.constants as const
import csv
from lmfit import Parameters,minimize, fit_report
import matplotlib.pyplot as plt
import numpy as np
import os
from scipy.integrate import odeint
```

```
from scipy.interpolate import interp1d
from scipy.io import readsav # for IDL savefiles
from scipy.optimize import fsolve
from scipy.special import erfc # erfc for isothermal cap
import spiceypy as spice
\# specify paths to required data files, and define VIMS dictionary with eventu
\hookrightarrow information
path2inputfiles = './' # modify this to point to directory containing datafiles
path2kernels = './' # modify this to point to directory containing leapseconds<sub>□</sub>
 ⇔kernel file
path2outputfiles = './output/' # modify this to point to directory containing
 \rightarrow datafiles
if not os.path.exists(path2outputfiles): # create the output directory if it_
⇔doesn't exist
   os.makedirs(path2outputfiles)
VIMS_observed_lightcurve = 'lightcurvewithgoodbaseline.csv' # from An Foster
VIMS isothermal model = 'nicholso-isothermal-alpori271.out' # Phil Nicholson
 ⇒isothermal model
VIMS_isothermal_metadata = 'nicholso-isothermal-metadata-alpori271.out' # ...
 ⇔and metadata for documentation
tlsfile = 'naif0012.tls'
CIRS_TPprofiles = 'globaltemp.sav' # IDL savefile containing CIRS T(P) profiles_
⇔as function of JD and latitude
# (Obtained from PDS Atmos node? not sure of origin)
# define VIMS dictionary containing essential event geometry and information
VIMS = {
    'path2inputfiles':path2inputfiles,
    'path2outputfiles':path2outputfiles,
    'path2kernels':path2kernels,
    'VIMS observed lightcurve': VIMS observed lightcurve,
    'dtsec':1.68, # from An Foster
    'VIMS_isothermal_model':VIMS_isothermal_model,
    'VIMS_isothermal_metadata': VIMS_isothermal_metadata,
    'tlsfile':tlsfile,
    'CIRS_TPprofiles':CIRS_TPprofiles,
    'event':'VIMS alpOri271I',
    'UTC':"2017-116T21:20", #approximate only - used only to determine JD of \Box
 \rightarrow event for CIRS T(P)
    'g_ms2':11.90*u.m/u.s**2, # from PDN
    'H_km':44.*u.km, # from An
```

```
'half-light':930, # frame number, from metadata file...
't_cube':1.68*u.s,
't_pixel':0.021*u.s,
'rc':54914.3*u.km,
'vperp':-2.311*u.km/u.s,
'lat_c':-74.44*u.deg,
'lat_g':-77.23*u.deg,
'time':1566.6*u.s,
'range':6.8726e+05*u.km,
'alpha':-66.36*u.deg,
'r_curv':66043.5*u.km,
'mu':2.2* u.g/u.mol, # from PDN
'RSTP': 129.e-6 # refractivity at STP
}
```

1 All defs loaded first

```
[2]: # non-dimensional bending angle (theta_hat = 1 at half-light)
def ftheta_hat(theta_hat,x):
    return np.log(theta_hat)+theta_hat - 1. - x
```

```
[3]: | # normalized flux as function of non-dimensional time \
     # (that vals in scale-heights in observer plane) for isothermal atmosphere
     def get_phi_vals(that_vals,xtol=1e-10):
         phi_vals = np.zeros(len(that_vals))
         for i,xval in enumerate(that_vals):
             if xval <= 0:</pre>
                 theta_hat0 = np.exp(xval)
             elif xval <= 4:</pre>
                 theta_hat0 = 2.
             elif xval <= 20:
                 theta hat 0 = 20.
             else:
                 theta hat0 = 1/xval
             theta_hat_val = fsolve(ftheta_hat,theta_hat0,xval,xtol=xtol)
             phi_iso = 1./(1.+theta_hat_val)
             phi_vals[i] = phi_iso
         return phi_vals
```

```
[4]: # normalized flux as function of non-dimensional time \
# (that_vals in scale-heights in observer plane)
# for isothermal atmosphere with optional random band model opacity
# tau_hl = half-light slant-path optical depth
# tau_gamma = random band model strength parameter
```

```
def
 -get_phi_tau_vals(that_vals,tau_hl,tau_gamma,IsothermalOnly=False,xtol=1e-10):
    phi_tau_vals = np.zeros(len(that_vals))
    for i,xval in enumerate(that vals):
        if xval <= 0:</pre>
            theta hat0 = np.exp(xval)
        elif xval <= 4:</pre>
            theta hat 0 = 2.
        elif xval <= 20:</pre>
            theta_hat0 = 20.
        else:
            theta_hat0 = 1/xval
        theta_hat_val = fsolve(ftheta_hat,theta_hat0,xval,xtol=xtol)
        phi_iso = 1./(1.+theta_hat_val)
        if IsothermalOnly:
            phi_tau_vals[i] = phi_iso
        else:
  # Random band model
            tau_gamma = max([0,tau_gamma]) # to avoid negative square root
            tau_exponent = -tau_hl * np.sqrt((1+tau_gamma)/
 →(1+tau_gamma*theta_hat_val)) * theta_hat_val
            # suppress exponent overflow message
            tau_exponent = np.clip(tau_exponent, -709.78, 709.78)
            tau_factor = np.exp(tau_exponent)
            refrac_factor = 1
            phi_tau_vals[i] = phi_iso * tau_factor * refrac_factor
    return phi tau vals
```

```
[5]: # compute isothermal model and residuals to observations, used by lmfit
     def fit_iso_lmfit(params,tsec,v_perp_kms,data,\
         returnModel=False, Verbose=False):
         Hkm = params['H_km']
         t_hl = params['t_hl']
         that_vals = -v_perp_kms * (tsec - t_hl)/Hkm
         neg_resid = []
         y_bkg = params['y_bkg']
         y_scale = params['y_scale']
         phi_vals = get_phi_vals(that_vals,xtol=1e-10)
         model = y_bkg + y_scale*phi_vals
         if returnModel:
             this_neg_resid = model
             this_neg_resid = model - data
         if len(neg_resid) == 0:
             neg_resid = this_neg_resid
         else:
```

```
neg_resid = np.concatenate((neg_resid,this_neg_resid))
return neg_resid
```

```
[6]: # compute isothermal + opacity model and residuals to observations, used by
      \hookrightarrow lmfit
     def fit_iso_tau_lmfit(params,tsec,v_perp_kms,tau_hl,tau_gamma,data,\
         returnModel=False, Verbose=False, IsothermalOnly=False):
         Hkm = params['H_km']
         t_hl = params['t_hl']
         that_vals = -v_perp_kms * (tsec - t_hl)/Hkm
         y bkg = params['y bkg']
         y_scale = params['y_scale']
         tau_hl = params['tau_hl']
         tau_gamma = params['tau_gamma']
         phi_vals = get_phi_tau_vals(that_vals,tau_hl,tau_gamma,\
                 IsothermalOnly=IsothermalOnly,xtol=1e-10)
         neg_resid = []
         model = y_bkg + y_scale*phi_vals
         if returnModel:
             this_neg_resid = model
         else:
             this_neg_resid = model - data
         if len(neg_resid) == 0:
             neg_resid = this_neg_resid
             neg_resid = np.concatenate((neg_resid,this_neg_resid))
         return neg_resid
```

```
[9]: # Read and plot the data...
     def plot_VIMS_data(infile,figfile_rawdata):
         with open(infile,newline='') as csvfile:
             reader = csv.reader(csvfile)
             counts = np.array(list(reader),dtype='float')[:,0]
     # three-panel plot of raw data, saved as figfile_rawdata
     # log scale
     # linear scale
     # zoomed linear cale
         fig1=plt.figure(1,figsize=(12,14)) # open up figure
         plt.rcParams.update({'font.size': 18})
         plt.subplot(3,1,1)
         plt.plot(counts)
         #plt.xlabel('Frame number')
         plt.ylabel('DN')
         plt.ylim(0.1,max(counts)*1.5)
         #plt.ylim(max(min([counts, .1]), max(counts)*1.5))
         plt.yscale('log')
         plt.title(VIMS['event']+' raw observations')
         plt.subplot(3,1,2)
         plt.plot(counts)
         #plt.xlabel('Frame number')
         plt.ylabel('DN')
         plt.ylim(0.0,200)
         plt.yscale('linear')
         plt.subplot(3,1,3)
```

```
plt.plot(counts)
plt.xlabel('Frame number')
plt.ylabel('DN')
plt.xlim(800,1450)
plt.ylim(-10,200)
plt.yscale('linear')

plt.show()
plt.savefig(figfile_rawdata)
print("Saved",figfile_rawdata,"\n")

return counts # for later use
```

```
[10]: #isothermal fits to the data
     def plot_isofits(VIMS,counts,figfile_isofits):
         params = Parameters()
     # specify initial guesses to fitted parameters
         Hkm = VIMS['H_km'].value
         t_hl = VIMS['half-light']*VIMS['t_cube'].value
         y_bkg = 0.
         y_scale = max(counts)
         vary_H_km = True
         vary_t_hl = True
         vary_y_bkg = False
         vary_y_scale = True
         params.add('H_km',value=Hkm ,vary=vary_H_km)
         params.add('t_hl',value=t_hl,vary=vary_t_hl)
         params.add('y_bkg',value=y_bkg,vary=vary_y_bkg)
         params.add('y_scale', value=y_scale, vary=vary_y_scale)
         frame = np.linspace(0,len(counts)-1,len(counts))
         L = np.where((frame >= 800) & (frame <= 1200)) # zoom specification
         tsec = frame[L]*VIMS['t_cube'] # time in sec from start of data
         data = counts[L] # the observed lightcurve
         yfit,fitted_params = fit_iso_to_observations(tsec.value,VIMS['vperp'].
       →value,data,params,\
             max_nfev=50,fit_xtol=1.e-6,Verbose=True)
         print('-----')
         print('Parameter
                                          Value
                                                        Stderr')
```

```
for name, param in fitted_params.params.items():
      try:
            print('{:18s} {:14.4f} {:14.4f}'.format(name, param.value, param.
⇒stderr))
      except:
            pass
  model_iso = fit_iso_lmfit(fitted_params.params,tsec.value,VIMS['vperp'].
⇔value,data,\
      returnModel=True, Verbose=False)
  # Plot isothermal fit, isothermal + absorption fits, and PDN model
  fig2=plt.figure(2,figsize=(12,8)); # open up figure
  plt.rcParams.update({'font.size': 18})
  plt.plot(tsec,data,label='Obs')
  # overplot PDN model
  result = np.genfromtxt( VIMS['path2inputfiles'] + L

¬VIMS['VIMS_isothermal_model'],
                         dtype='float',skip_header =_
dt = result['tsec']+fitted_params.params['t_hl']
  phi = result['phi']
  label='H = '+f'{VIMS["H_km"].value:.2f}'+' km (PDN)'
  Lfit = np.where((dt >0 ) & (dt < max(tsec.value)))</pre>
  plt.plot(dt[Lfit],fitted_params.params['y_scale']*phi[Lfit],label=label)
  # plot isothermal fit
  label='H = '+f'{fitted_params.params["H_km"].value:.2f}'+' km (fit)'
  plt.plot(tsec,model_iso,label=label)
  # now perform fit with band model
  params = Parameters()
  # specify initial quesses
  Hkm = VIMS['H_km'].value
  t_hl = VIMS['half-light']*VIMS['t_cube'].value
  y_bkg = 0.
  y_scale = max(counts)
  tau_hl = 0.2
```

```
# specify set of fixed values of tau_gamma
  tau_gammas = [0,100]
  vary_t_hl = True
  vary_y_bkg = False
  vary_H_km = True
  vary_y_scale = True
  vary_tau_hl = True
  vary_tau_gamma = False
  for imodel,tau_gamma in enumerate(tau_gammas):
      params.add('H_km',value=Hkm ,vary=vary_H_km)
      params.add('t_hl',value=t_hl,vary=vary_t_hl)
      params.add('y_bkg',value=y_bkg,vary=vary_y_bkg)
      params.add('y_scale', value=y_scale, vary=vary_y_scale)
      params.add('tau_hl',value=tau_hl,vary=vary_tau_hl)
      params.add('tau_gamma', value=tau_gamma, vary=vary_tau_gamma)
      yfit,fitted_params_tau = fit_iso_tau_to_observations(tsec.

¬value, VIMS['vperp'].value,
          tau hl, tau gamma, data, params, max nfev=50, fit xtol=1.
⇒e-6, Verbose=True)
      print('-----Band model
print('Parameter
                                        Value
                                                      Stderr')
      for name, param in fitted_params_tau.params.items():
          try:
                print('{:18s} {:14.4f} {:14.4f}'.format(name, param.value,
→param.stderr))
          except:
                pass
      model_tau = fit_iso_tau_lmfit(fitted_params_tau.params,tsec.
→value, VIMS['vperp'].value,
          tau_hl,tau_gamma,data,returnModel=True,Verbose=False)
      label='H = '+f'{fitted_params_tau.params["H km"].value:.2f}'+' km '+\
          r'$\tau {1/2}=$' +f'{fitted params_tau.params["tau_hl"].value:.2f}'__
→+\
          ', '+r"$\gamma=$" + str(tau_gamma)
      plt.plot(tsec.value,model_tau,label=label)
  plt.xlabel('t (sec)')
  plt.ylabel('DN')
```

```
plt.legend()
  plt.title(VIMS['event'])
  plt.show();
  plt.savefig(figfile_isofits);
  print("Saved",figfile_isofits,"\n")

  return tsec,data,model_iso, fitted_params, model_tau, fitted_params_tau

[11]: #isothermal fits with CIRS synthetic lightcurve overplotted
  def plot_isofits_CIRS(VIMS,tsec_CIRS,flux_CIRS,counts,figfile_isofits_CIRS):
     params = Parameters()
  # specify initial guesses to fitted parameters
     Hkm = VIMS['H_km'].value
```

```
t_hl = VIMS['half-light']*VIMS['t_cube'].value
  y_bkg = 0.
  y_scale = max(counts)
  vary_H_km = True
  vary_t_hl = True
  vary_y_bkg = False
  vary_y_scale = True
  params.add('H_km',value=Hkm ,vary=vary_H_km)
  params.add('t_hl',value=t_hl,vary=vary_t_hl)
  params.add('y_bkg',value=y_bkg,vary=vary_y_bkg)
  params.add('y_scale',value=y_scale,vary=vary_y_scale)
  frame = np.linspace(0,len(counts)-1,len(counts))
  L = np.where((frame >= 800) & (frame <= 1200)) # zoom specification
  tsec = frame[L]*VIMS['t_cube'] # time in sec from start of data
  data = counts[L] # the observed lightcurve
  yfit,fitted_params = fit_iso_to_observations(tsec.value,VIMS['vperp'].
⇔value,data,params,\
      max_nfev=50,fit_xtol=1.e-6,Verbose=True)
  print('-----'Isothermal model-----')
                                   Value
                                                 Stderr')
  print('Parameter
  for name, param in fitted_params.params.items():
            print('{:18s} {:14.4f} {:14.4f}'.format(name, param.value, param.
⇔stderr))
      except:
            pass
```

```
model_iso = fit_iso_lmfit(fitted_params.params,tsec.value,VIMS['vperp'].
⇔value,data,\
      returnModel=True, Verbose=False)
  # Plot isothermal fit, isothermal + absorption fits, and PDN model
  fig2=plt.figure(2,figsize=(12,8)); # open up figure
  plt.rcParams.update({'font.size': 18})
  plt.plot(tsec,data,label='Obs')
  # overplot PDN model
  result = np.genfromtxt( VIMS['path2inputfiles'] + L

¬VIMS['VIMS_isothermal_model'],
                         dtype='float',skip_header =_
dt = result['tsec']+fitted_params.params['t_hl']
  phi = result['phi']
  label='H = '+f'{VIMS["H_km"].value:.2f}'+' km (PDN)'
  Lfit = np.where((dt >0 ) & (dt < max(tsec.value)))</pre>
  plt.plot(dt[Lfit],fitted_params.params['y_scale']*phi[Lfit],label=label)
  # plot isothermal fit
  label='H = '+f'{fitted_params.params["H_km"].value:.2f}'+' km (fit)'
  plt.plot(tsec,model_iso,label=label)
  # now perform fit with band model
  params = Parameters()
  # specify initial quesses
  Hkm = VIMS['H_km'].value
  t_hl = VIMS['half-light']*VIMS['t_cube'].value
  y bkg = 0.
  y_scale = max(counts)
  tau_hl = 0.2
  # specify set of fixed values of tau_gamma
  tau_gammas = [0,100]
  vary_t_hl = True
  vary_y_bkg = False
  vary_H_km = True
```

```
vary_y_scale = True
  vary_tau_hl = True
  vary_tau_gamma = False
  for imodel,tau_gamma in enumerate(tau_gammas):
      params.add('H_km',value=Hkm ,vary=vary_H_km)
      params.add('t_hl',value=t_hl,vary=vary_t_hl)
      params.add('y_bkg',value=y_bkg,vary=vary_y_bkg)
      params.add('y_scale', value=y_scale, vary=vary_y_scale)
      params.add('tau_hl',value=tau_hl,vary=vary_tau_hl)
      params.add('tau_gamma', value=tau_gamma, vary=vary_tau_gamma)
      yfit,fitted_params_tau = fit_iso_tau_to_observations(tsec.

¬value, VIMS['vperp'].value,
          tau_hl,tau_gamma,data,params,max_nfev=50,fit_xtol=1.
→e-6, Verbose=True)
      print('-----Band model

    '+str(imodel+1)+'----')

      print('Parameter
                                        Value
                                                       Stderr')
      for name, param in fitted_params_tau.params.items():
          try:
                print('{:18s} {:14.4f} {:14.4f}'.format(name, param.value,
→param.stderr))
          except:
                pass
      model_tau = fit_iso_tau_lmfit(fitted_params_tau.params,tsec.
⇔value, VIMS['vperp'].value,
          tau_hl,tau_gamma,data,returnModel=True,Verbose=False)
      label='H = '+f'{fitted params tau.params["H km"].value:.2f}'+' km '+\
          r'$\tau_{1/2}=$' +f'{fitted_params_tau.params["tau_hl"].value:.2f}'u
→+\
          ', '+r"$\gamma=$" + str(tau_gamma)
      plt.plot(tsec.value,model tau,label=label)
  t_CIRS = tsec_CIRS.value+fitted_params.params['t_hl']
  model_CIRS = fitted_params.params['y_scale']*flux_CIRS + fitted_params.
→params['y_bkg']
  plt.plot(t_CIRS,model_CIRS,label='CIRS T(P)',linewidth=4)
  plt.xlabel('t (sec)')
  plt.ylabel('DN')
  plt.legend()
  plt.xlim(np.min(tsec.value),np.max(tsec.value))
  plt.title(VIMS['event'])
```

```
plt.show();
plt.savefig(figfile_isofits_CIRS);
print("Saved",figfile_isofits_CIRS,"\n")
return #tsec,model_iso, fitted_params, model_tau, fitted_params_tau
```

```
[12]: #rebin normalized input lightcurve into equal altitude bins
      def alt_bin(that_vals,flux,dh_bin,ha,D,tau_hl=0,Verbose=False):
          dt_av = that_vals[1] - that_vals[0]
          if Verbose == True:
              print('alt_bin debug:')
              print('that_vals[0:5]=',that_vals[0:5])
              print('in alt_bin: dt_av=',dt_av)
              print('that_vals=',that_vals)
              print('flux:',flux)
              print('dt_av:',dt_av)
          i = 0
          j = 0
          i_tot = 0
          t_start = dt_av/2.
          MAXSTORE = 500000 # modify this at some point to use append()
          profile_flux = np.zeros(MAXSTORE)
          profile_time = np.zeros(MAXSTORE)
          profile_dtheta = np.zeros(MAXSTORE)
          t_skip = that_vals[0] - dt_av # confirmed that this and t_start give proper
                                         # alignment of profile_flux and profile_time
          t_j = t_start + dt_av
          while(j < np.size(flux)):</pre>
              if Verbose == True:
                  print('\nj,np.size(flux)=',j,np.size(flux))
              dt_{phi} = (t_{j} - t_{start}) * flux[j]
              d_sum = dt_phi
              first = True
              if Verbose== True:
                  print('first,TP1: d_sum,dt_phi,dh_bin=',first,d_sum,dt_phi,dh_bin)
              if d_sum <= dh_bin: # altitude bin not yet full
                  first = False
                  while d_sum < dh_bin:</pre>
                      t_j += dt_av
                      j += 1
                      if Verbose == True:
                          print('TP2: t_j,j=',t_j,j)
                      if j >= np.size(flux):
                          break
```

```
dxs = d_sum - dh_bin
              ddt = dxs/flux[j]
              if first == True:
                  ddt = (t_j - t_start)*dxs/dt_phi
              t_{last} = t_{j} - ddt
              delta_t = np.abs(t_last - t_start)
              profile_flux[i] = dh_bin/delta_t
          # apply tau_hl if non-zero
              this_alt = ha-i*dh_bin - dh_bin/2 # to agree with below
              if tau_hl != 0:
                  this_tau = tau_hl * np.exp(-this_alt)
                  profile_flux[i] *= np.exp(-this_tau)
              profile_dtheta[i] = 1/profile_flux[i] - 1
              t_start = t_last
              t_now = t_skip + t_last
              profile_time[i] = t_now
              i_tot = i
              if Verbose == True:# and flux[j] < 0.99:</pre>
                  print('output⊔
       di,profile_flux[i],profile_time[i]',i,profile_flux[i],profile_time[i])
                  print('input j,flux[j] ',j,flux[j])
              i += 1
              if i > MAXSTORE-1:
                  break
          profile_flux = profile_flux[0:i_tot]
          profile_time = profile_time[0:i_tot]
          profile_dtheta = profile_dtheta[0:i_tot]/D # note the 1/D factor!
          profile_theta = np.cumsum(profile_dtheta)*dh_bin
          profile_alt = ha - np.linspace(0,i_tot-1,i_tot)*dh_bin - dh_bin/2
      # ??? not sure what this was supposed to be...
          profile_alt_theta = ha - np.linspace(0,i_tot-1,i_tot)*dh_bin - dh_bin
          return profile_flux,profile_time,profile_alt,profile_alt_theta,\
              profile_dtheta,profile_theta,i_tot
[13]: def__
       alt_bin_dimensional(that_vals_sec,flux,dh_bin_km,hakm,vperp_kms,Dkm,tau_hl=0,Verbose=False)
          dt_av = that_vals_sec[1] - that_vals_sec[0]
          i = 0
          j = 0
```

dt_phi = flux[j] * dt_av

d_sum += dt_phi

if j >= np.size(flux):

break

 $i_tot = 0$

 $t_start = dt_av/2.$

```
MAXSTORE = 500000 # modify this at some point to use append()
  profile_flux = np.zeros(MAXSTORE)
  profile_time = np.zeros(MAXSTORE)*u.s
  profile_dtheta = np.zeros(MAXSTORE)
  t_skip = that_vals_sec[0] - dt_av # confirmed that this and t_start give_
\hookrightarrow proper
                                  # alignment of profile_flux and profile_tim
  t_j = t_start + dt_av
  while(j < np.size(flux)):</pre>
      if Verbose == True:
           print('j,np.size(flux)=',j,np.size(flux))
      dt_phi = (t_j - t_start) * flux[j]
      d_sum = dt_phi * abs(vperp_kms)
      first = True
      if Verbose== True:
                 print('TP1: d_sum,dt_phi,dh_bin=',d_sum,dt_phi,dh_bin)
      if d_sum <= dh_bin_km: # altitude bin not yet full</pre>
           first = False
           while d_sum < dh_bin_km:
               t_j += dt_av
               j+=1
               \#print('TP2: t_j, j=', t_j, j)
               if j >= np.size(flux):
                   break
               dt_phi = flux[j] * dt_av
               d_sum += dt_phi * abs(vperp_kms)
      if j >= np.size(flux):
          break
      dxs = (d_sum - dh_bin_km)/abs(vperp_kms)
      ddt = dxs/flux[j]
       print('ddt should have units of time:',ddt)
      if first == True:
           ddt = (t_j - t_start)*dxs/dt_phi
      t_{last} = t_{j} - ddt
      delta_t = np.abs(t_last - t_start)
      profile_flux[i] = dh_bin_km/(abs(vperp_kms)*delta_t)
  # apply tau_hl if non-zero
      this_alt = hakm-i*dh_bin_km - dh_bin_km/2 # to agree with below
         if tau_hl != O:
             this_tau = tau_hl * np.exp(-this_alt)
             profile_flux[i] *= np.exp(-this_tau)
      profile_dtheta[i] = 1/profile_flux[i] - 1
      t_start = t_last
      t_now = t_skip + t_last
```

```
profile_time[i] = t_now
              i tot = i
              i+=1
          profile_flux = profile_flux[0:i_tot]
          profile_time = profile_time[0:i_tot]
          profile_dtheta = profile_dtheta[0:i_tot]/Dkm # note the 1/D factor!
          profile_theta = np.cumsum(profile_dtheta)*dh_bin_km
      # print('check units of profile_theta',profile_theta)
          profile_alt = hakm - np.linspace(0,i_tot-1,i_tot)*dh_bin_km - dh_bin_km/2
      # ??? not sure what this was supposed to be...
          profile_alt_theta = hakm - np.linspace(0,i_tot-1,i_tot)*dh_bin_km -_u

→dh_bin_km
          return profile_flux,profile_time,profile_alt,profile_alt_theta,\
              profile_dtheta,profile_theta,i_tot
[14]: def lnbarometric(lnP, h, method_TofPmbar,mu,g):
          P = np.exp(lnP)
          T = method\_TofPmbar(P)*u.K
          H = (const.k_B * const.N_A *T/(mu*g)).to('km').value
          if T<0:
              print(P,T)
          deriv = -1/H \# dlnP/dh
          return deriv
[15]: def lnbarometric2(lnP, h, method_ToflogPmbar,mu,g):
          T = method_ToflogPmbar(lnP)*u.K
          H = (const.k_B * const.N_A *T/(mu*g)).to('km').value
          if T<0:
              P = np.exp(lnP)
              print(P,T)
          deriv = -1/H \# dlnP/dh
          return deriv
[16]: # altitude and bending angle to pressure, density, scale height, and temperature
      # using summations, but no isothermal cap to infinity
      def htheta2sclht(hvals,theta,D,denfac=1,prfac=1,Tfac=1*u.K/u.km):
          imax = np.size(theta)
          dh = np.abs(hvals[0] - hvals[1])
          hfac = 0.4 * dh
          dthet = D * np.gradient(-theta, hvals) # factor of D from Wasserman and
       → Ververka and C code quick_invert.c
          znum = np.zeros(imax)
          zden = np.zeros(imax)
          index = np.linspace(0,imax-1,imax)
          sqrt_index = np.sqrt(index)
```

```
for i in range(1,imax+1):
              j_index = np.array(index[1:i],dtype=int)
             fimj_vals = i - j_index
             fimj1_vals= fimj_vals - 1
             fac1_vals = sqrt_index[fimj_vals]*fimj_vals
             fac2_vals = sqrt_index[fimj1_vals]*fimj1_vals
             fac3_vals = fimj_vals * fac1_vals
             fac4_vals = fimj1_vals * fac2_vals
             znum[i-1] = sum(dthet[j index-1].value*(fac3 vals-fac4 vals))
             zden[i-1] = sum(dthet[j_index-1].value*(fac1_vals-fac2_vals))
         pr = znum * prfac
         den = zden * denfac
         zden[0] = zden[1] # to avoid divide by zero
         H = hfac*znum/zden
         T = H * Tfac
         return dthet,den,pr,H,T,znum,zden# invert the isothermal model curve
[17]: def HtoT(G,mu,H):
         Avogadro =const.N_A # 6.022141e23
         kBoltzCGS = const.k_B.to(u.erg/u.K) #1.380658e-16
                = G.to(u.cm/u.s**2)
         Tfac = (mu *Gcgs /(const.N_A * const.k_B)).to(u.K/u.km)
         T = (Tfac * H).to(u.K)
         return T # in K
[18]: def TtoH(G,mu,T):
         Avogadro =const.N_A # 6.022141e23
         kBoltzCGS = const.k_B.to(u.erg/u.K) #1.380658e-16
                = G.to(u.cm/u.s**2)
         Tfac = (mu *Gcgs /(const.N_A * const.k_B)).to(u.K/u.km)
         H = (T/Tfac).to(u.km)
         return H
[19]: # get physical units profile from refractivity
     def pro_refrac2profile(R,nu,Ttop,G,mu,refrac):
      #; input
      #;
                       radial refractivity profile (R in km, radius from center of
               nu(R)
      \hookrightarrowplanet)
      #;
               Ttop Temperature (K) at top of profile
                       gravitational acceleration (m/s^2)
      # ;
               Gref
      #;
                       scalar mean molecular wt
               mu
      #;
              refrac refractivity
     #; output
      #;
               T(R)
                      Kelvin
               P(R)
                      Pressure (Pascals)
      #;
```

```
#;
                  density (kg/m^3)
          den
#;
          H(R)
                  scale height (km)
#
      print('R=',R[0:4])
      print('nu=',nu[0:4])
#
#
     print('Ttop', Ttop)
      print('G',G)
#
#
     print('mu', mu)
      print('refrac', refrac)
   Avogadro =const.N_A # 6.022141e23
   kBoltzCGS = const.k_B.to(u.erg/u.K) #1.380658e-16
            = 1*u.g/const.N_A # 1.66053886e-24 # gm /mol
   Loschmidt = 2.6867811e+19 /u.cm**3 # const.amagat # 2.68684e19
   nR
            = len(nu)
            = nu*Loschmidt/refrac
   ncm3
            = np.zeros(nR,dtype=float)*u.mbar
   Pmbar
            = G.to(u.cm/u.s**2)
   Gcgs
   dencgs = ncm3 * mu / Avogadro # gm/cm^3
            = dencgs * 1000.0 # kg/m^3
   den
   Pmbar[0] = (ncm3[0] * kBoltzCGS * Ttop).to(u.mbar) # need pressure at top_
 ⇔of top shell
\neg print('ncm3[0], kBoltzCGS, Ttop, Pcgs[0], refrac', ncm3[0], kBoltzCGS, Ttop, Pmbar[0], refrac)
    # integrate hydrostatic equation to get dP
   ivals = np.linspace(1,nR-1,num=nR-1,dtype=int)
   for i in ivals:
          if i < 5:
#
#
              print(i)
              print('R[i]',R[i])
#
              print('nu[i]',nu[i])
#
              print('Pmbar[i-1]',Pmbar[i-1])
              print('ncm3[i-1]',ncm3[i-1])
              print('dencgs[i-1]', dencgs[i-1])
         Pmbar[i] = Pmbar[i-1] - (dencgs[i-1] * Gcgs * (R[i] - R[i-1])).to(u.
 \hookrightarrow mbar)
# this reduces the error considerably
        Pmbar[i] = Pmbar[i-1] - ((dencgs[i]+dencgs[i-1])/2. * Gcgs * (R[i] - 
 \hookrightarrow R[i-1])).to(u.mbar)
   ncm3[-1] = ncm3[-2] # fix off-scale endpints
   Pmbar[-1] = Pmbar[-2]
   den[-1] = den[-2]
```

```
T = (Pmbar/(kBoltzCGS * ncm3)).to(u.K)
Tfac = (mu *Gcgs /(const.N_A * const.k_B)).to(u.K/u.km)
H = T/Tfac
return ncm3,den,Pmbar,T,H
```

```
[20]: # convert altitude/bending angle arrays to lightcurve in time, accommodating
       ⇔ray crossing
      def hvalstheta2tsecflux(hvals,theta,vperp,tsec_start,tsec_stop,dt_sec,D_km):
          dh_km = abs(hvals[0] - hvals[1]) # input altitude spacing
          dlc_km = dt_sec * abs(vperp) # conversion between time and distance on_
       \hookrightarrow light curve
          dh_dlc = dh_km/dlc_km # non-dimensional
          ijmax = int((tsec_stop - tsec_start)/dt_sec+0.5)
          flux = np.zeros(ijmax)
          tsec_vals = tsec_start + np.linspace(0,ijmax-1,ijmax)*dt_sec
          thetaD = theta * D_km / dlc_km
          print('theta', theta)
          print('thetaD - should be dimensionless!', thetaD)
          imax = theta.size
          for i in range(1,imax):
              zji=(i-1)*dh_dlc + thetaD[i-1] - 1.5 # empirical factor
              zji1=i*dh dlc + thetaD[i] - 1.5
              zjmin=min([zji,zji1])
              zjmax=max([zji,zji1])
              jmin=int(1+zjmin)
              jmax=int(1+zjmax)
              if jmin <= ijmax and jmax >= 1:
                  jmin1=max([jmin,1])
                  jmax1=min([jmax,ijmax])
                  fphi=dh_dlc/(zjmax-zjmin)
                  phifac=fphi
                  for j in range(jmin1,jmax1+1):
                      add=phifac*(min([zjmax,j])-max([zjmin,(j-1)]))
                      flux[j-1] += add
          flux[0:2] = flux[2] # since first bins not filled
          return tsec_vals,flux
```

```
[21]: def lightcurve_new(hvals,theta,tau_hl=0,pts_per_H=1000):
    dh = hvals[0] - hvals[1]
    print('in lightcurve_new: dh = ',dh)
    hhat = 1./dh
    that = pts_per_H
    dj = 1./that
    at = dj

# compute duration of lightcurve
```

```
ha = max(hvals)
    hb = -min(hvals)
    du = ha + np.exp(hb) + hb
   # print("in lightcurve: pts_per_H,hhat,ha,hb,du = ",pts_per_H,hhat,ha,hb,du)
    ijmax = int(du/at+0.5)
    du = at*ijmax
    flux = np.zeros(ijmax)
    #print("in lightcurve: pts_per_H, du, ijmax, at = ", \
           pts_per_H, du, i jmax, at)
\# This is the best I could come up with for alignment with variety of choices \sqcup
 \hookrightarrow of dh and dj
# not ideal but good enough.
# using golc() run 5 for a variety of choices
   \# that_vals = -max(hvals) + np.linspace(0,ijmax-1,ijmax)*dj - 1 + dh - 2*dj#u
⇔so zero at half-light
   \# that vals = -max(hvals) + np.linspace(0,ijmax-1,ijmax)*dj - 1 +2*dh - 1*dj
    that_vals = -max(hvals) + np.linspace(0,ijmax-1,ijmax)*dj - 1 +dh# -2*dh #-u
 →1*dj
    #print('in lightcurve: dj,that_vals[0:4]=',dj,that_vals[0:4])
# step through entire theta array and compute contribution to this part of \Box
 \hookrightarrow light curve
    imax = theta.size
    for i in range(1,imax):
        zji=((i-1)*dh+theta[i-1])/dj
        zji1=(i*dh+theta[i])/dj
        zjmin=min([zji,zji1])
        zjmax=max([zji,zji1])
        jmin=int(1+zjmin)
        jmax=int(1+zjmax)
        if jmin <= ijmax and jmax >= 1:
            jmin1=max([jmin,1])
            jmax1=min([jmax,ijmax])
            fphi=dh/(dj*(zjmax-zjmin))
            phifac=fphi
            for j in range(jmin1, jmax1+1):
                add=phifac*(min([zjmax,j])-max([zjmin,(j-1)]))
                flux[j-1] += add
    if tau hl>0:
        flux *= np.exp(-tau_hl * (1/flux-1))
    return that_vals,flux
```

```
[22]: # Use refractivity profile to determine bending angle
      def nu2theta(dh_km,nu_in):
      # integrate along line of sight through successively deeper rays...
          if nu_in[10] < nu_in[9]:</pre>
              print('Refractivity should increase with depth. Flipping...')
              nu=np.flip(nu_in)
          else:
              nu = nu_in
          imax = np.size(nu)
          theta = np.zeros(imax)
          index = np.linspace(0,imax-1,imax)
          fac2_all = np.sqrt(index)
          for i in range(imax):
              i_index= np.array(index[0:i+1],dtype=int)
              i_vals = i - i_index
              fac1_vals = -2*fac2_all[(i_vals-1).clip(min=0)]
              fac2_vals = fac2_all[i_vals]
              fac3_vals = fac2_all[(i_vals-2).clip(min=0)]
              facs = fac1_vals + fac2_vals + fac3_vals
              theta[i-1] = sum(nu[i index] * facs)
              print(i,end='\r')
          # scale theta by R_planet/dh
          theta *= (VIMS['r_curv']/abs(dh_km)).value
          return theta
```

```
[23]: def pro_lcgen_v2(R_in,nu_in,D_in):
          R = R_{in.value}
          nu = nu_in
          D = D_in.value
          nRp1 = len(R)
          nR = nRp1 - 1
          nRm1 = nR - 1
      # determine if arrays are increasing outward or not
          if R[0] < R[-1]:
              print('flipping input R array')
                      = np.flip(R)
              R
          if nu[0] > nu[-1]:
              print('flipping input nu array')
                      = np.flip(nu)
          print('First ten elements of R, nu: R should decrease, nu should increase')
          print(R[0:10])
          print(nu[0:10])
          print('len(R),len(nu)',len(R),len(nu))
          if R[-1] > R[100]:
```

```
print('R array is not in the correct order!')
    else:
        print('R array is in the correct order')
    if nu[-1] < nu[100]:
        print('nu array is not in the correct order!')
    else:
        print('nu array is in the correct order')
# compute coefficients without using 2-D arrays
            = np.abs(R[0:nRm1+1] - R[1:nR+1]) # note that dr is always positive
    dr
            = (R*R)
    Rsq
#; compute theta array
    theta = np.zeros(nRp1,dtype=object)
    rho = np.zeros(nRp1,dtype=object)
    dnudr = np.gradient(nu,R)
    dnudr = abs(np.diff(nu)/np.diff(R))
    ivals = np.linspace(1,nR,num=nR,dtype=int)
    for i in ivals:
        print(i,end='\r')
        Xji = np.array([np.sqrt(Rsq[0:i-1+1] - Rsq[i]), 0.0], dtype=object)
#
        Xji = np.sqrt(Rsq[0:i-1+1] - Rsq[i])
        Xji=np.append(Xji,0.0)
        dXji = -np.diff(Xji)
        print('i,Xji,dXji',i,Xji,dXji)
        vals = dXii[0:i] * dnudr[0:i]/R[0:i] * 2.0 * R[i]
         print('vals[0], np.sum(vals[0])', vals[0], np.
 ⇒sum(vals[0], keepdims=False))
#THIS IS VERY SLOW
         theta[i] = np.sum(vals[0], keepdims=False)
# try this instead
        theta[i] = sum(vals)
         print('i, theta[i], theta compare', i, theta[i], theta compare)
#
        rho[i] = R[i] - D*theta[i]
        print('theta[0:i+1]:',theta[0:i+1])
#
#
          if i \ll 4:
#
              print(' \nR[i], nu[i], dnudr[i]', R[i], nu[i], dnudr[i])
#
              print('Xji',Xji)
              print('dXji',dXji)
#
              print('vals',vals)
#
#
              print('theta[i]', theta[i])
#
             print('rho[i]',rho[i])
    phi_cyl = np.zeros(nRp1)
    drho = np.abs(np.diff(rho))
     print('drho', drho)
     phi_cyl = dr/drho
```

```
[24]: \parallel# determine refractivity and bending angle from numerical inversion of
       \hookrightarrow light curve
      def pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in):
           rho_in: distance in observer plane of ray from projected center of planet ⊔
       ⇒in km
          phi_cyl_in: normalized flux (cyl means no spherical focussing)
          phi_cyl = np.zeros(len(phi_cyl_in),dtype=float) + phi_cyl_in
           print(phi_cyl.flags.writeable)
          D = D_i
          nRp1 = len(rho_in)
          rho = np.zeros(nRp1,dtype=float) + rho_in
          rho[0] = rho[1] + rho[1] - rho[2]
          nR = nRp1-1
          nRm1 = nR -1
          if rho in [10] < rho in [-1]:
              rho = np.flip(rho)
              print('reversed order of rho')
          if phi_cyl[5]<phi_cyl[-1]:</pre>
              phi_cyl = np.flip(phi_cyl)
              print('reversed order of phi_cyl')
          if phi_cyl[1] < .1:</pre>
              phi_cyl[1] = 0.99999643
          R = np.zeros(nRp1, dtype = float)
          dr = np.zeros(nR, dtype = float)
          R[0] = rho[0] # align at top of
            print('rho[0:4]',rho[0:5])
            print('phi_cyl[0:4]',phi_cyl[0:5])
          ivals = np.linspace(1,nR-1,num=nR-1,dtype=int)
          for i in ivals:
              dr[i] = abs(rho[i-1] - rho[i]) * phi_cyl[i]
              if i == 1:
                  dr[0] = dr[1]
```

```
R[i] = R[i-1] - dr[i-1]
#
          if i <=5:
             print('i, dr[i-1]', i, dr[i-1])
#
              print('R[i-1],R[i]',R[i-1],R[i])
#
#
    if R[0] == R[1]:
        R[0] = R[1] - (R[2] - R[1])
           = R*R
   Rsq
   if dr[0] == 0:
        dr[0] = dr[1]
   theta = (R - rho)/D
            = np.zeros(nRp1,dtype=float)
   nu
           = np.zeros(nRp1,dtype=float)
   nu0
           = nuO in
   Aji_01 = 2.0*np.sqrt(Rsq[0] - Rsq[1])/dr[0]
   if Aji_01>0:
        dnu[0] = theta[1]/Aji_01
           = nu[0] + dnu[0]
   nu[1]
      print('rho[0:4]',rho[0:4])
#
     print('theta[0:4]', theta[0:4])
#
     print('phi_cyl[0:4]',phi_cyl[0:4])
     print('R[0:4]',R[0:4])
#
#
     print('dr[0:4]',dr[0:4])
#
     print('Aji_01=',Aji_01)
#
     print('dr[0]',dr[0])
     print('R[0:4]-rho[0:4]',R[0:4]-rho[0:4])
     print('D',D)
   for i in ivals:
       Xji1 = np.sqrt(Rsq[0:i+2] - Rsq[i+1])
              = 2. * R[i+1]/R[0:i+1] * (Xji1[0:i+1] - Xji1[1:i+2])/dr[0:i+1]
       Aji1
        if Aji1[i] >0:
            dnu[i] = (theta[i+1] - sum(Aji1[0:i-1+1]*dnu[0:i-1+1]))/Aji1[i]
       nu[i+1] = nu[i] + dnu[i]
          if i <= 4:
              print(i)
              print('Xji1',Xji1)
#
              print('Aji1',Aji1)
#
              print('dnu[i]',dnu[i])
#
             print('nu[i]',nu[i])
       print(i,end='\r')
   nu[0] = nu[1]
   nu[-1]=nu[-2]
   return nu, theta, R
```

2 Load data, plot and perform isothermal and iso-tau band model fits

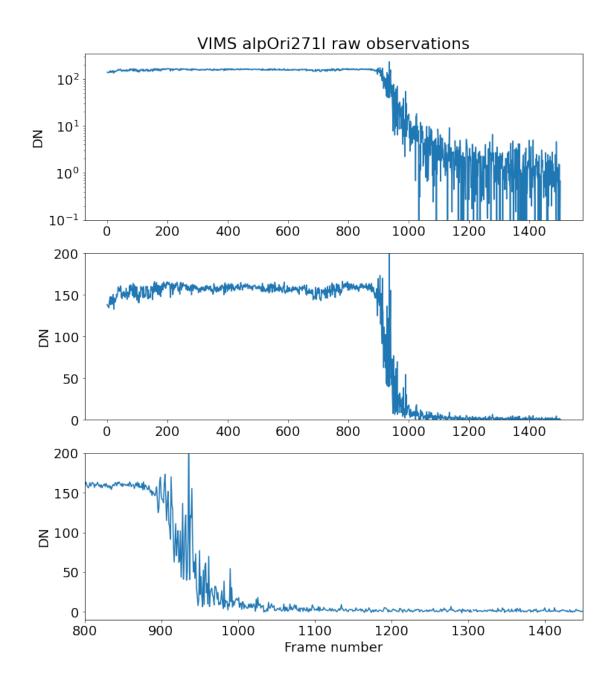
```
[25]: infile = VIMS['path2inputfiles'] + VIMS['VIMS_observed_lightcurve']
figfile_rawdata = VIMS['path2outputfiles'] + VIMS['event']+'_rawdata.jpg'

counts = plot_VIMS_data(infile,figfile_rawdata)

figfile_isofits = VIMS['path2outputfiles'] + VIMS['event']+'_isofits.jpg'
figfile_isofits_CIRS = VIMS['path2outputfiles'] + VIMS['event']+'_isofits_CIRS.

igfile_isofits_CIRS = VIMS['path2outputfiles'] + VIMS['event']+'_isofits_CIRS.

igfile_isofits_CIRS = VIMS['path2outputfiles'] + VIMS['event']+'_isofits_CIRS.
```

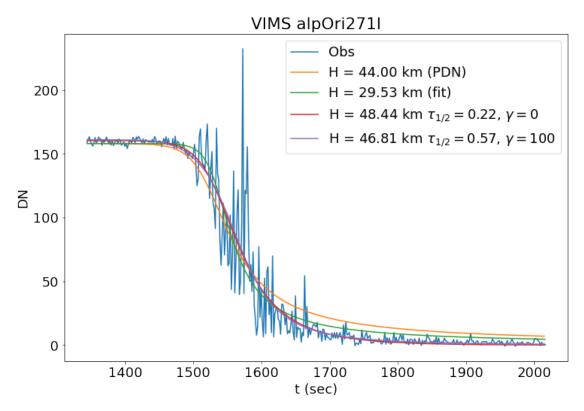


Saved ./output/VIMS alpOri271I_rawdata.jpg

Isothermal model				
Parameter	Value	Stderr		
H_km	29.5256	1.7366		
t_hl	1559.9046	1.8103		
y_bkg	0.0000	0.0000		
y_scale	158.0188	1.6116		
	-Band model 1			
Parameter	Value	Stderr		

H_km	48.4420	5.6023
t_hl	1577.7694	6.1565
y_bkg	0.0000	0.0000
y_scale	160.4330	1.7819
tau_hl	0.2164	0.0821
tau_gamma	0.0000	0.0000
	Band model 2	
Parameter	Value	Stderr
Parameter H_km	Value 46.8084	Stderr 3.9394
H_km	46.8084	3.9394
H_km t_hl	46.8084 1597.0443	3.9394 8.8826
H_km t_hl y_bkg	46.8084 1597.0443 0.0000	3.9394 8.8826 0.0000
H_km t_hl y_bkg y_scale	46.8084 1597.0443 0.0000 161.0295	3.9394 8.8826 0.0000 1.8444

<Figure size 432x288 with 0 Axes>



Saved ./output/VIMS alpOri271I_isofits.jpg

<Figure size 432x288 with 0 Axes>

3 Load CIRS data and construct T(P) profile

```
[26]: Isothermal = False
      Hhat = 50 # points per scale height
      #Hhat = 100 # points per scale height
      #Hhat = 200 # points per scale height
      #Hhat = 300 # points per scale height
      #Hhat = 500 # points per scale height
      Ha = 15.
      Hb = 7.05
      dh_bin_sec = 0.01*u.s
      # dh_bin_sec = 0.02*u.s
      # dh_bin_sec = 0.1*u.s
      dh_bin_km = dh_bin_sec * abs(VIMS['vperp'])
      bin_km = dh_bin_km
      # get CIRS profile
      IDL = readsav(VIMS['path2inputfiles']+VIMS['CIRS_TPprofiles'])
      CIRS Tarray = IDL['finaltemp']
      CIRS_pmbar = IDL['press']*1e3 * u.mbar
      CIRS_lat_list = IDL['newlat'] * u.deg
      CIRS_JD_list = IDL['newdays']
      VIMSlat_deg = VIMS['lat_c']
      spice.furnsh(VIMS['path2kernels']+VIMS['tlsfile'])
      VIMSUTC = VIMS['UTC']
      VIMSETsec = spice.str2et(VIMSUTC)
      VIMSJDocc = spice.j2000() + VIMSETsec/spice.spd()
      JD = VIMSJDocc
      dlat_deg = abs(VIMSlat_deg-CIRS_lat_list)
      index_lat = np.where(dlat_deg == min(dlat_deg))
      index_lat = int(index_lat[0])
      dJD = abs(JD - CIRS_JD_list)
      index_JD = np.where(dJD== min(dJD))
      index_JD = int(index_JD[0])
      CIRS_T = CIRS_Tarray[index_JD,index_lat,:]*u.K
      index = np.where(CIRS_T > 50*u.K)
      CIRS_Tindex = CIRS_T[index]
      CIRS_pmbarindex = CIRS_pmbar[index]
```

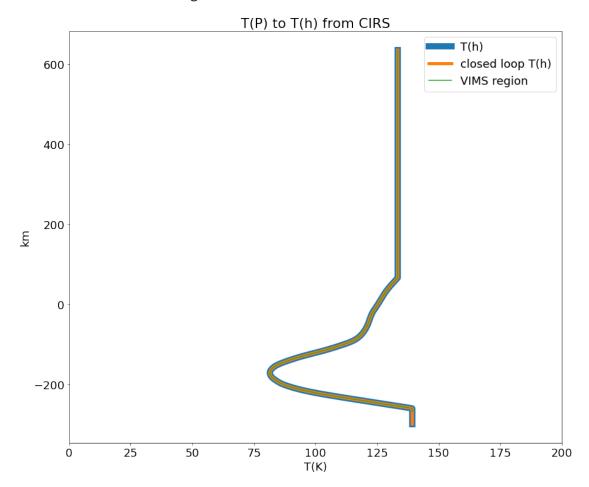
```
fill Thigh P= CIRS Tindex[0]
fill_Tlow_P = CIRS_Tindex[-1]
fill_Tbelow = CIRS_Tindex[0]
fill_Tabove = CIRS_Tindex[-1]
if Isothermal == True:
    H_{km} = VIMS['H_{km}']
    T_VIMS = HtoT(VIMS['g_ms2'],VIMS['mu'],H_km)
    method TofPmbar=
 interp1d(CIRS_pmbarindex,CIRS_Tindex*0+T_VIMS,'cubic',bounds_error=False,
        fill_value = (T_VIMS,T_VIMS))
    method_ToflogPmbar= interp1d(np.log(CIRS_pmbarindex.value),CIRS_Tindex*0_
 →+T_VIMS, 'cubic', bounds_error=False,\
        fill_value=(T_VIMS,T_VIMS))
else:
    H_km = TtoH(VIMS['g_ms2'], VIMS['mu'], fill_Tlow_P) # in km already
    method_TofPmbar=_
 sinterp1d(CIRS_pmbarindex,CIRS_Tindex,'cubic',bounds_error=False,
       fill_value=(fill_Tlow_P,fill_Thigh_P))
    method_ToflogPmbar= interp1d(np.log(CIRS_pmbarindex.
 ⇔value),CIRS_Tindex,'cubic',bounds_error=False,\
       fill value=(fill Tlow P, fill Thigh P))
Haphb = Ha + Hb
NH = int(Hhat*Haphb)
imax = NH
dH = 1.0/Hhat
print('dH=',dH,' in scale heights')
Hvals = -Hb + np.linspace(0,NH-1,NH)*dH # this is positive upward
Hkm = H_km.value
hvalskm = Hvals * H_km # array of altitudes in km
hvalskm_ = hvalskm.value
g = VIMS['g_ms2'].to('km/s**2')
Losch = (2.6867811e25/u.m**3).to('1/km**3')
RSTP = VIMS['RSTP']
mu = VIMS['mu']
R_curv = VIMS['r_curv']
N_A = const.N_A
D_km = VIMS['range']
denfac0 = 2*Losch/(np.pi*np.sqrt(2*R_curv)*RSTP)
denfac = denfac0 * bin_km**1.5 * 2/(3*D_km) # number density /km^3
denfac_cgs = denfac.to(1/u.cm**3)
den_halflight = (denfac0*H_km**1.5/D_km).to(1/u.cm**3)*np.sqrt(np.pi)/2
```

```
prfac0 = 4*g*mu*Losch/(3*np.pi*np.sqrt(2*R_curv)*RSTP*N_A)
prfac = prfac0 * bin_km**2.5 * 2/(5*D_km)
prfac_mbar = prfac.to(u.mbar)
phalf_mbar = (prfac0 * H_km**2.5/D_km * 3 * np.sqrt(np.pi)/4).to(u.mbar)
dhkm = hvalskm[1]-hvalskm[0]
#specify initial condition (boundary condition )
Pmbar_initial = phalf_mbar.value * np.exp(Hb)
PO_mb = Pmbar_initial
print('Deepest pressure level P0_mb=',P0_mb)
g_kms2 = VIMS['g_ms2'].to('km/s**2')
lnPmbarinitial = np.log(Pmbar_initial)
lnPsolution = odeint(lnbarometric, lnPmbarinitial,__
 →hvalskm,args=(method_TofPmbar,mu,g_kms2),rtol=1e-12)
lnPodeint = lnPsolution[:,0] # Pressure as a function of height
Podeint = np.exp(lnPodeint)
lnPsolution2 = odeint(lnbarometric2, lnPmbarinitial,_
 →hvalskm_,args=(method_ToflogPmbar,mu,g_kms2),rtol=1e-12)
lnPodeint2 = lnPsolution2[:,0] # Pressure as a function of height
Podeint2 = np.exp(lnPodeint2)
Tofh = method TofPmbar(Podeint)
Pofh = Podeint
Tofh2 = method_ToflogPmbar(lnPodeint2)
Pofh2 = Podeint2
if Isothermal == True:
   fill_value=(T_VIMS,T_VIMS)
else:
   fill_value=(fill_Tlow_P,fill_Thigh_P)
method_Tofhkm = interp1d(hvalskm_,Tofh,bounds_error=False,
   fill_value=fill_value)
method_Pofhkm= interp1d(hvalskm_,Podeint,bounds_error=False)
method_lnPofhkm= interp1d(hvalskm_,lnPodeint,bounds_error=False)
method_hkmofPmbar= interp1d(Podeint,hvalskm_,bounds_error=False)
```

```
method_hkmoflnPmbar= interp1d(lnPodeint,hvalskm_,bounds_error=False)
testTofh = method_TofPmbar(method_Pofhkm(hvalskm_))
testTofh2 = method_ToflogPmbar(method_lnPofhkm(hvalskm_))
# confirm that starting Podeint is our intial value
# print(Pofh[0],Pmbar_initial)
# compute number density and then refractivy
nofh = (Pofh*u.mbar/(const.k B * Tofh*u.K)).to("1/cm**3")
# print("min, max nofh=", min(nofh), max(nofh))
# print("min, max Pofh=", min(Pofh), max(Pofh))
# print("min, max Tofh=", min(Tofh), max(Tofh))
Nofh = nofh *RSTP/Losch.to('1/cm**3')
T_new = method_TofPmbar(Pmbar_initial)*u.K
P_new = np.exp(method_lnPofhkm(0))*u.mbar
print("\nPmbar at 0 altitude from method_lnPofhkm:",P_new)
print("Compared to phalf_mbar.value",phalf_mbar.value,'\n')
n_new = (P_new/(const.k_B * T_new)).to("1/cm**3")
print('Compute half-light nden: ',n new,den halflight)
nofh new = (Pofh*u.mbar/(const.k B * Tofh*u.K)).to("1/cm**3")
dnofh = abs(nofh_new - den_halflight)
index = np.where(dnofh == min(dnofh))[0]
h0 = hvalskm[index]
print('index,half-light altitude from number density =',index,h0)
#determine altitude where Nofh is closest to half light pred value
print(min(Nofh), max(Nofh))
Nofh_new = nofh_new * RSTP / Losch.to('1/cm**3')
H_km_VIMS = H_km
fig1=plt.figure(figsize=(14,12)) # open up figure
nhalf_pred = (Losch * H_km_VIMS**1.5/(np.sqrt(2*np.pi*R_curv)*D_km*RSTP)).to(1/
 \rightarrowu.cm**3)
Nhalf_pred = (nhalf_pred * VIMS['RSTP']/Losch.to('1/cm**3')).value
phalf_pred = (nhalf_pred * H_km_VIMS * mu * g_kms2 * 1/N_A).to(u.mbar)
print(min(Nofh_new),max(Nofh_new))
dNofh = abs(Nofh_new.value-Nhalf_pred)
index = np.where(dNofh == min(dNofh))[0]
```

```
h0 = hvalskm[index]
print('index,half-light altitude from refractivity =',index,h0)
Phalf_odeint = Pofh[index]
print("from odeint, pred: Phalf = ",Phalf_odeint,phalf_pred)
# refractivity at hvalskm of 0
l=np.where(np.abs(hvalskm) == min(np.abs(hvalskm)))
Nofh_new_Okm = Nofh_new[1]
print('Refractivity at zero altitude', Nofh new 0km)
VIMS Pmbar max = Pmbar initial*.5*u.mbar # placeholdeer
index = np.where(hvalskm_ > method_hkmofPmbar(VIMS_Pmbar_max))
hvals = hvalskm[index]
xvals = method_Tofhkm(hvals_.value)
#print('xvals=',xvals)
#print('hvals=',hvals)
fig1=plt.figure(1,figsize=(10,6)) # open up figure
plt.rcParams.update({'font.size': 18})
plt.plot(Tofh,hvalskm_,linewidth=10,label='T(h)')
plt.plot(testTofh,hvalskm_,linewidth=5,label='closed loop T(h)')
plt.plot(xvals,hvals_,label='VIMS region')
plt.xlim(0,200)
plt.xlabel('T(K)')
plt.ylabel('km')
#plt.ylim(200,600)
plt.title('T(P) to T(h) from CIRS')
plt.legend()
# now try predicting flux level at top of CIRS data
phi_upper = 1/(1+min(CIRS_pmbarindex)/phalf_pred)
phi_lower = 1/(1+VIMS_Pmbar_max/phalf_pred)
print('predicted flux at upper range of CIRS data=',phi_upper)
print('predicted flux at lower range of VIMS data=',phi_lower)
dH= 0.02 in scale heights
Deepest pressure level P0_mb= 2752.9373374100487
Pmbar at 0 altitude from method lnPofhkm: 0.4332912618002866 mbar
Compared to phalf_mbar.value 2.3879225053743665
Compute half-light nden: 2.2533064737770332e+16 1 / cm3 1.2968433102469112e+17
1 / cm3
index, half-light altitude from number density = [279] [-62.26318281] km
3.374104069928733e-14 0.0006873755583793674
3.374104069928733e-14 0.0006873755583793674
index, half-light altitude from refractivity = [279] [-62.26318281] km
```

from odeint, pred: Phalf = [2.14633839] 2.387922505374367 mbar Refractivity at zero altitude [1.21589141e-07] predicted flux at upper range of CIRS data= 0.965154682781798 predicted flux at lower range of VIMS data= 0.0017318135334713684



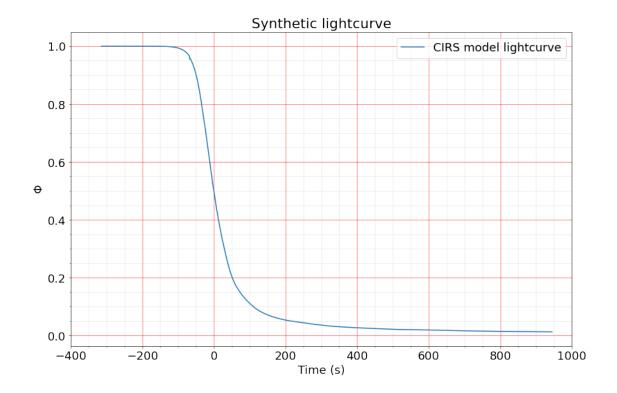
4 Construct synthetic lightcurve based on CIRS T(P) and refractivity

```
[27]: print('Computing lightcurve...')
Rt = hvalskm + VIMS['r_curv']
R_in = Rt
nu_in = Nofh_new
rho2,theta2,phi_cyl2 = pro_lcgen_v2(R_in,nu_in,VIMS['range'])
print('Finished computing lightcurve')
```

Computing lightcurve... flipping input R array flipping input nu array

```
First ten elements of R, nu: R should decrease, nu should increase [66677.56792294 66676.7208048 66675.87368667 66675.02656854 66674.1794504 66673.33233227 66672.48521414 66671.638096 66670.79097787 66669.94385974]
[3.37410407e-14 3.44226549e-14 3.51180387e-14 3.58274701e-14 3.65512330e-14 3.72896169e-14 3.80429172e-14 3.88114351e-14 3.95954781e-14 4.03953598e-14]
len(R),len(nu) 1102 1102
R array is in the correct order nu array is in the correct order
Finished computing lightcurve
```

```
[28]: offset2 = 0*u.s
      xvals2 = -(-np.array(rho2,dtype=float)*u.km+VIMS['r_curv'])/VIMS['vperp'] +u
       ⊶offset2
      xvals2[0] = xvals2[1] - (xvals2[2]-xvals2[1])
      tscale = xvals2
      L = np.where(tscale < 1000*u.s)
      tscale = tscale[L]
      flux = phi_cyl2[L]
      dt_interp = 0.1*u.s
      start = np.ceil(tscale[0])
      stop = np.floor(tscale[-1] )
      Npts = int((stop-start)/dt_interp +1)
      t_interpol = np.linspace(start,stop,Npts,endpoint=True)
      f interpol = interp1d(tscale,flux,kind='linear')
      flux_interpol = f_interpol(t_interpol)
      L = np.min(np.where(flux_interpol <= 0.5))
      t_half = t_interpol[L]
      fig,ax=plt.subplots(figsize=(14,9)) # open up figure
      plt.plot(t interpol-t half,flux interpol,label='CIRS model lightcurve')
      plt.xlabel('Time (s)')
      plt.ylabel(r'$\Phi$')
      plt.title('Synthetic lightcurve')
      plt.legend()
      ax.grid(linestyle='-', linewidth='0.5', color='red')
      ax.minorticks_on()
      ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
      plt.xlim(-400,1000)
      plt.show()
```



5 Invert synthetic lightcurve to confirm retrieval agrees with input T(P)

1100

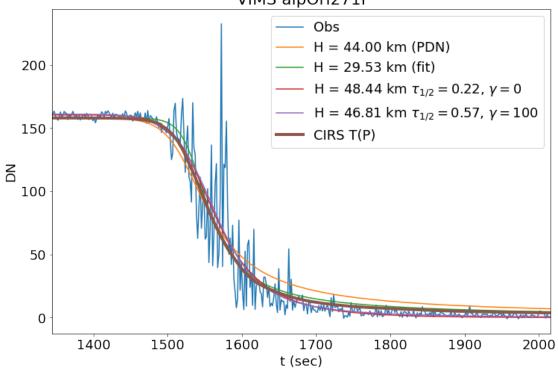
6 Overplot CIRS model lightcurve on isofit/isotau VIMS observations

```
[30]: tsec_CIRS = t_interpol-t_half
flux_CIRS = flux_interpol
plot_isofits_CIRS(VIMS,tsec_CIRS,flux_CIRS,counts,figfile_isofits_CIRS)
```

-----Isothermal model-----

Parameter	Value	Stderr
H_km	29.5256	1.7366
t_hl	1559.9046	1.8103
y_bkg	0.0000	0.0000
y_scale	158.0188	1.6116
Ban	d model 1	
Parameter	Value	Stderr
H_km	48.4420	5.6023
t_hl	1577.7694	6.1565
y_bkg	0.0000	0.0000
y_scale	160.4330	1.7819
tau_hl	0.2164	0.0821
tau_gamma	0.0000	0.0000
Ban	d model 2	
Parameter	Value	Stderr
H_km	46.8084	3.9394
t_hl	1597.0443	8.8826
y_bkg	0.0000	0.0000
y_scale	161.0295	1.8444
tau_hl	0.5699	0.1534
tau_gamma	100.0000	0.0000

VIMS alpOri271I

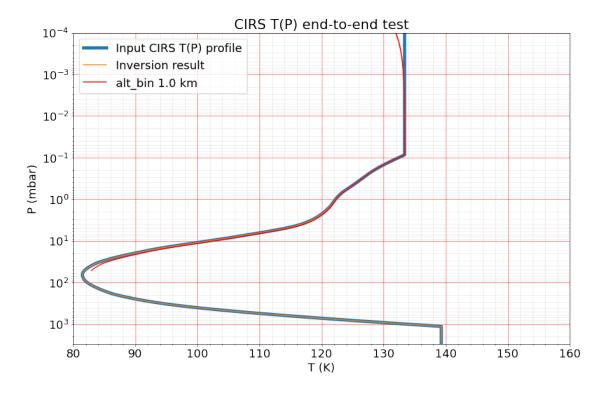


Saved ./output/VIMS alpOri271I_isofits_CIRS.jpg

7 Invert CIRS interpolated lightcurve to confirm it matches input T(P)

```
[31]: vperp_kms = abs(VIMS['vperp'])
     Dkm = VIMS['range']
     tau_hl = 0.
     dh_bin_km = 1*u.km # desired altitude resolution
     hakm = 400*u.km # not sure what this should be
     #lightcurve to altitude-binned values
     profile_flux,profile_time,profile_alt,profile_alt_theta,\
         profile_dtheta,profile_theta,i_tot = \
       →alt_bin_dimensional(tsec_CIRS,flux_CIRS,dh_bin_km,hakm,vperp_kms,Dkm,tau_hl=tau_hl,Verbose=
     # inversion to retrieve refractivity and bending angle
     rho_in = (profile_time*VIMS['vperp']+VIMS['r_curv']).value
     phi_cyl_in = profile_flux
     nu0_in = 0.
     D in = Dkm.value
     nu,theta,R = pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in)
     # conversion to physical units for vertical profile
     ncm3_,den_,Pmbar_,T_,H_ = \
         pro_refrac2profile(R*u.km,nu,Tofh[-1]*u.
       # plot results
     fig,ax=plt.subplots(figsize=(14,9)) # open up figure
     plt.plot(Tofh,Pofh,label='Input CIRS T(P) profile',linewidth=5)
     #plt.plot(T2,Pmbar_2,label='Inversion result')
     plt.plot(T2a,Pmbar_2a,label='Inversion result')
     plt.plot(T_,Pmbar_,color='r',label='alt_bin '+str(dh_bin_km))
     plt.yscale('log')
     plt.ylim(3000,.0001)
```

```
plt.xlim(80,160)
plt.ylabel('P (mbar)')
plt.xlabel('T (K)')
plt.legend()
plt.title('CIRS T(P) end-to-end test')
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks_on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.show();
```

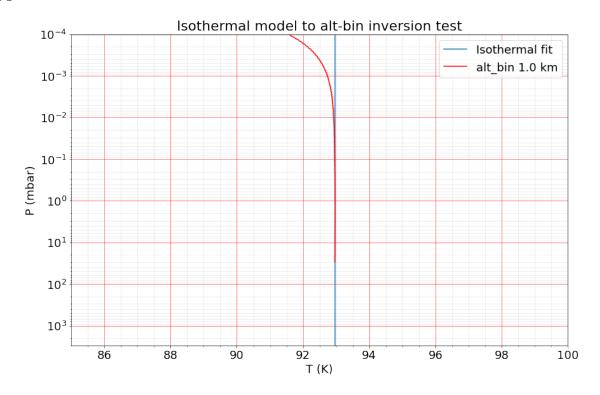


8 Invert isothermal model lightcurve as end-to-end test of alt-bin

```
dh_bin_km = 1*u.km # desired altitude resolution
hakm = 400*u.km # not sure what this should be
#lightcurve to altitude-binned values
profile_flux,profile_time,profile_alt,profile_alt_theta,\
   profile_dtheta,profile_theta,i_tot = \
 alt_bin_dimensional(t_iso,flux_iso,dh_bin_km,hakm,vperp_kms,Dkm,tau_hl=tau_hl,Verbose=False
# plt.plot(t_iso, flux_iso)
# plt.plot(profile_time,profile_flux)
# inversion to retrieve refractivity and bending angle
rho_in = (profile_time*VIMS['vperp']+VIMS['r_curv']).value
phi_cyl_in = profile_flux
nu0_in = 0.
D_in = Dkm.value
nu,theta,R = pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in)
# conversion to physical units for vertical profile
ncm3_,den_,Pmbar_,T_,H_ = \
   pro_refrac2profile(R*u.km,nu,Tofh[-1]*u.

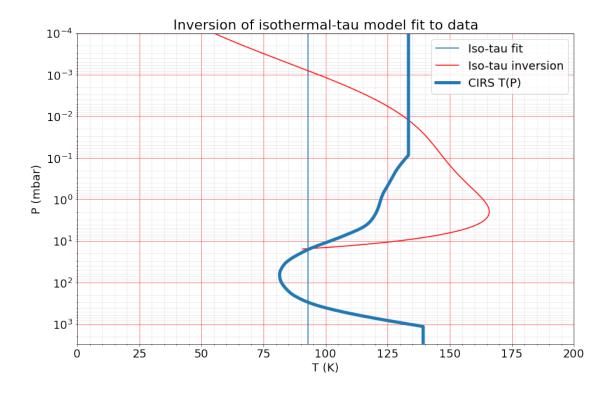
→K,VIMS['g_ms2'],VIMS['mu'],VIMS['RSTP'])
# plot results
fig,ax=plt.subplots(figsize=(14,9)) # open up figure
T_iso = HtoT(g,mu,fitted_params.params['H_km']*u.km)
plt.axvline(T_iso.value,label='Isothermal fit')
# #plt.plot(T2,Pmbar_2,label='Inversion result')
# plt.plot(T2a,Pmbar_2a,label='Inversion result')
plt.plot(T_,Pmbar_,color='r',label='alt_bin '+str(dh_bin_km))
plt.yscale('log')
plt.ylim(3000,.0001)
plt.xlim(85,100)
plt.ylabel('P (mbar)')
plt.xlabel('T (K)')
plt.legend()
plt.title('Isothermal model to alt-bin inversion test')
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks_on()
```

```
ax.grid(visible=True, which='minor', color='#9999999', linestyle='-', alpha=0.2)
plt.show();
```



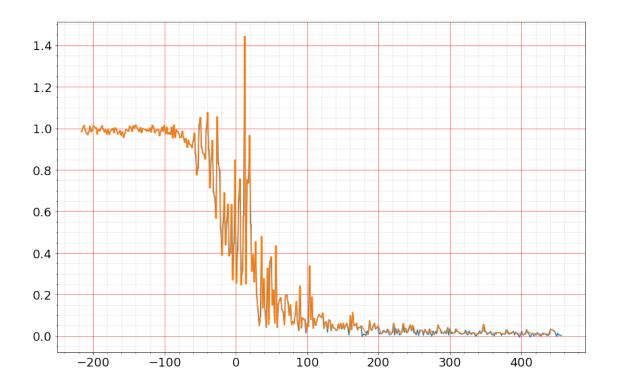
9 Invert isothermal tau model and compare to CIRS T(P)

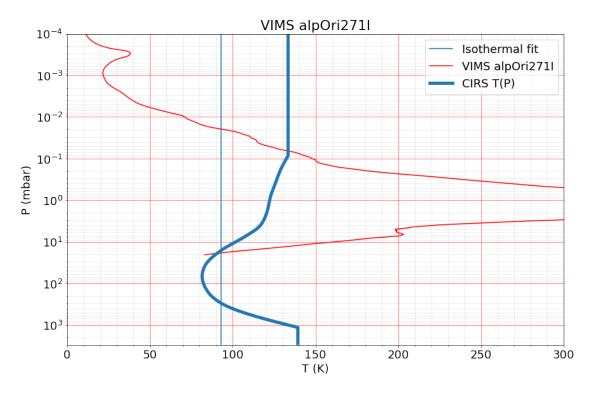
```
walt bin dimensional(t_isotau,flux_isotau,dh_bin km,hakm,vperp_kms,Dkm,tau_hl=tau_hl,Verbose
# plt.plot(t_iso,flux_iso)
# plt.plot(profile_time,profile_flux)
# inversion to retrieve refractivity and bending angle
rho_in = (profile_time*VIMS['vperp']+VIMS['r_curv']).value
phi_cyl_in = profile_flux
nu0_in = 0.
D_in = Dkm.value
nu,theta,R = pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in)
# conversion to physical units for vertical profile
ncm3_,den_,Pmbar_,T_,H_ = \
    pro_refrac2profile(R*u.km,nu,Tofh[-1]*u.
 →K, VIMS['g_ms2'], VIMS['mu'], VIMS['RSTP'])
# plot results
fig,ax=plt.subplots(figsize=(14,9)) # open up figure
T_isotau = HtoT(g,mu,fitted_params_tau.params['H_km']*u.km)
plt.axvline(T_iso.value,label='Iso-tau fit')
# #plt.plot(T2,Pmbar_2,label='Inversion result')
# plt.plot(T2a,Pmbar_2a,label='Inversion result')
plt.plot(T_,Pmbar_,color='r',label='Iso-tau inversion')
plt.plot(Tofh,Pofh,label='CIRS T(P)',linewidth=5)
plt.yscale('log')
plt.ylim(3000,.0001)
plt.xlim(0,200)
plt.ylabel('P (mbar)')
plt.xlabel('T (K)')
plt.legend()
plt.title('Inversion of isothermal-tau model fit to data')
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks_on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.show();
```



10 Inversion of VIMS lightcurve - no isothermal cap yet

```
plt.plot(profile_time,profile_flux)
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks_on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.show()
# inversion to retrieve refractivity and bending angle
rho_in = (profile_time*VIMS['vperp']+VIMS['r_curv']).value
phi_cyl_in = profile_flux
nu0_in = 0.
D in = Dkm.value
nu,theta,R = pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in)
# conversion to physical units for vertical profile
ncm3_obs,den_obs,Pmbar_obs,T_obs,H_obs = \
   pro_refrac2profile(R*u.km,nu,Tofh[-1]*u.
 →K,VIMS['g_ms2'],VIMS['mu'],VIMS['RSTP'])
# plot results
fig,ax=plt.subplots(figsize=(14,9)) # open up figure
T_iso = HtoT(g,mu,fitted_params.params['H_km']*u.km)
plt.axvline(T_iso.value,label='Isothermal fit')
plt.plot(T_obs,Pmbar_obs,color='r',label=VIMS['event'])
plt.plot(Tofh,Pofh,label='CIRS T(P)',linewidth=5)
plt.yscale('log')
plt.ylim(3000,.0001)
plt.xlim(0,300)
plt.ylabel('P (mbar)')
plt.xlabel('T (K)')
plt.legend()
plt.title(VIMS['event'])
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.show();
```

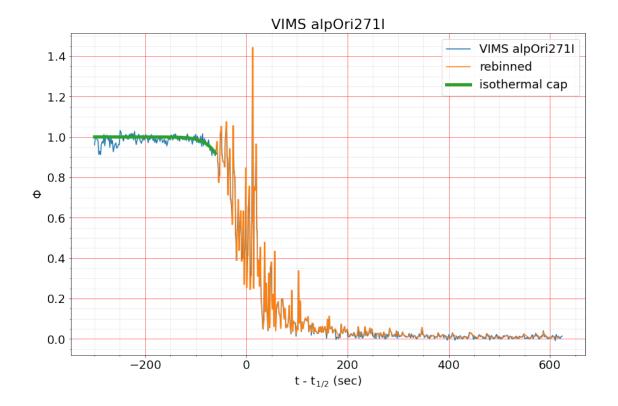


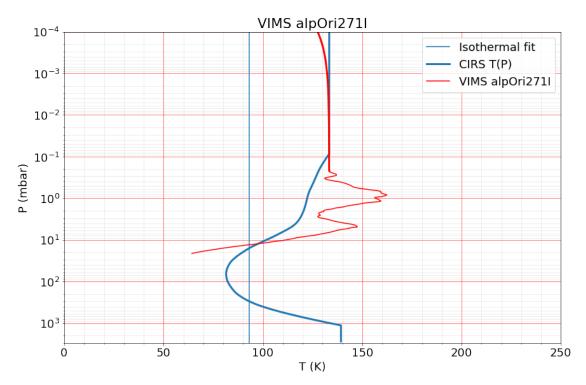


11 isothermal cap

```
[35]: frame = np.linspace(0,len(counts)-1,len(counts))
     L = np.where((frame >= 700) & (frame <= 1400)) # zoom specification
     L = np.where((frame >= 750) & (frame <= 1300)) # zoom specification
     tsec_all = frame[L]*VIMS['t_cube'] # time in sec from start of data
     data_all = counts[L] # the observed lightcurve
      # Normalized flux and halflight-relative time:
     t_obs = tsec_all-fitted_params.params['t_hl']*u.s
     flux_obs = 0.98*(data_all-fitted_params_tau.params['y_bkg'])/fitted_params.
      →params['y_scale']
     temp = 0.98*(data_all-fitted_params_tau.params['y_bkg'])/fitted_params.
       →params['y_scale']
     L = np.where(t_obs > (-40)*u.s)
     L = np.where(t_obs > (-60)*u.s)
     tL = t_obs[L[0]]
     H_{extrap} = TtoH(g,mu,Tofh[-1]*u.K)
     thatextrap = t_obs[0:L[0][0]]*abs(VIMS['vperp'])/H_extrap
     flux_cap = flux_obs
     flux_cap[0:L[0][0]] = get_phi_vals(thatextrap)
     vperp_kms = abs(VIMS['vperp'])
     Dkm = VIMS['range']
     tau hl = 0.
     dh_bin_km = 0.1*u.km # desired altitude resolution
     hakm = 600*u.km
      #lightcurve to altitude-binned values
     profile_flux,profile_time,profile_alt,profile_alt_theta,\
         profile_dtheta,profile_theta,i_tot = \
       -alt_bin_dimensional(t_obs,flux_cap,dh_bin_km,hakm,vperp_kms,Dkm,tau_hl=tau_hl,Verbose=False
     fig,ax=plt.subplots(figsize=(14,9)) # open up figure
     plt.plot(t_obs,temp,label=VIMS['event'])
     plt.plot(profile_time,profile_flux,label='rebinned')
     plt.plot(t_obs[0:L[0][0]],flux_cap[0:L[0][0]],linewidth=5,label='isothermalu
     ax.grid(linestyle='-', linewidth='0.5', color='red')
```

```
ax.minorticks_on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.legend()
plt.xlabel(r't - t_{1/2}\ (sec)')
plt.ylabel(r'$\Phi$')
plt.title(VIMS['event'])
plt.show()
# inversion to retrieve refractivity and bending angle
rho_in = (profile_time*VIMS['vperp']+VIMS['r_curv']).value
phi_cyl_in = profile_flux
nu0_in = 0.
D_in = Dkm.value
nu,theta,R = pro_lcinvert(rho_in,phi_cyl_in,nu0_in,D_in)
# conversion to physical units for vertical profile
ncm3_obs,den_obs,Pmbar_obs,T_obs,H_obs = \
    pro_refrac2profile(R*u.km,nu,Tofh[-1]*u.
 →K, VIMS['g_ms2'], VIMS['mu'], VIMS['RSTP'])
# plot results
fig,ax=plt.subplots(figsize=(14,9)) # open up figure
T_iso = HtoT(g,mu,fitted_params.params['H_km']*u.km)
plt.axvline(T_iso.value,label='Isothermal fit')
plt.plot(Tofh,Pofh,label='CIRS T(P)',linewidth=3)
plt.plot(T_obs,Pmbar_obs,color='r',label=VIMS['event'])
L = np.where(profile_time < (-60)*u.s)
plt.plot(T_obs[L],Pmbar_obs[L],color='r',linewidth=3) # thick line for_
 \hookrightarrow isothermal cap
plt.yscale('log')
plt.ylim(3000,.0001)
plt.xlim(0,250)
plt.ylabel('P (mbar)')
plt.xlabel('T (K)')
plt.legend()
plt.title(VIMS['event'])
ax.grid(linestyle='-', linewidth='0.5', color='red')
ax.minorticks on()
ax.grid(visible=True, which='minor', color='#999999', linestyle='-', alpha=0.2)
plt.show();
```





```
[36]: print('All done!')
```

All done!