

Detection of Copy-Rotate-Move Forgery Using Hu Moments

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Abstract

Copy-Rotate-Move forgery is a common image manipulation technique where specific regions of an image are duplicated and rotated to hide or alter content. This paper presents a novel approach for detecting such forgeries using Hu Moments as feature descriptors, enabling the identification of similar regions based on their shape and geometric invariants. The proposed method begins with edge detection using the Canny filter, followed by dilation to connect fragmented edges and segment regions of interest using connected component analysis. For each segmented region, Hu Moments are computed, which serve as a robust set of features invariant to rotation and scaling.

The similarity between regions is quantified using the Euclidean distance of their Hu Moment vectors, with regions exhibiting a distance below a defined threshold being flagged as potential forgeries. A binary mask is generated to visualize the detected regions, and the method's accuracy is evaluated against a ground truth mask using Precision, Recall, and F1-Measure metrics. The algorithm's performance is influenced by parameters such as block size, Gaussian smoothing (sigma), and kernel size for morphological operations, which allow for fine-tuning to reduce noise and enhance detection accuracy.

Keywords: Image Forgery Detection, Copy-Rotate-Move, Hu Moments, Canny Edge Detection, Region Similarity, Precision, Recall, F1-Measure.

1 Introduction

The rise of advanced image editing tools has made digital image forgery, particularly Copy-Rotate-Move (CRM), a significant challenge in digital forensics. This technique involves duplicating and rotating image regions to hide or alter content, making detection difficult using traditional methods. An example of such forgery is shown in Fig. 1a, where regions of an image have been copied, rotated, and moved to obscure original information.

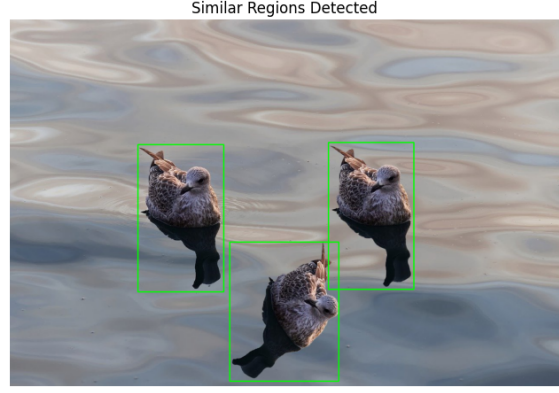
To address this, we propose a robust approach utilizing Hu Moments —geometric invariants that effectively describe region shapes regardless of rotation or scaling. As demonstrated in Fig. 1b, our method identifies similar regions based on their shape descriptors, making it well-suited for detecting geometrically transformed forgeries.

The proposed method integrates edge detection, morphological operations, and region segmentation, followed by the computation of Hu Moments for shape-based similarity comparison. Regions with matching descriptors are flagged as potential forgeries and visualized through a binary mask overlay.

The performance of the method is evaluated using Precision, Recall, and F1-Measure, demonstrating its effectiveness in detecting forgery while balancing sensitivity and robustness to noise. The results highlight the capability of our approach in detecting CRM forgeries under challenging conditions.



(a) Original image with potential Copy-Rotate-Move forgery regions.



(b) Detected similar regions highlighted using the proposed method.

Figure 1: Example of Copy-Rotate-Move (CRM) forgery and its detection.

2 Related Work

Image forgery detection methods can be categorized into block-based and keypoint-based approaches. Block-based methods, such as those using DCT and PCA, compare overlapping blocks but struggle with transformations like rotation or scaling. Keypoint-based methods, including SIFT and SURF, are more robust to such transformations but fail when duplicated regions lack texture or keypoints.

To address these limitations, shape-based descriptors like Hu Moments have been used for their invariance to rotation and scaling, making them effective for detecting shape similarity [2]. Combining edge detection, morphological operations, and moment-based features has shown promising results in identifying forged regions.

This work builds on these techniques by leveraging Hu Moments and edge-based segmentation to detect Copy-Rotate-Move forgeries, offering improved accuracy and robustness while ensuring computational efficiency.

3 Hu Moments

Hu Moments are a set of seven shape descriptors derived from central moments. They are invariant to translation, rotation, and scaling, making them robust for shape recognition and object matching. Hu Moments are widely used in tasks such as pattern recognition, image forgery detection, and shape-based analysis due to their ability to compactly represent global shape features while remaining unaffected by geometric transformations.

3.1 Definition

Hu Moments, introduced by *M.K. Hu* in 1962 [1], are a set of seven moment invariants derived from the central moments of an image. These moments are invariant to translation, rotation, and scaling, making them widely used in shape analysis and pattern recognition.

The central moments of an image $I(x, y)$ are calculated using the image's centroid (\bar{x}, \bar{y}) :

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q I(x, y), \quad (1)$$

where the centroid coordinates are defined as:

$$\bar{x} = \frac{\sum_x \sum_y x \cdot I(x, y)}{\sum_x \sum_y I(x, y)}, \quad \bar{y} = \frac{\sum_x \sum_y y \cdot I(x, y)}{\sum_x \sum_y I(x, y)}. \quad (2)$$

To ensure scale invariance, these central moments are normalized:

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma}, \quad \gamma = \frac{p+q}{2} + 1, \quad (3)$$

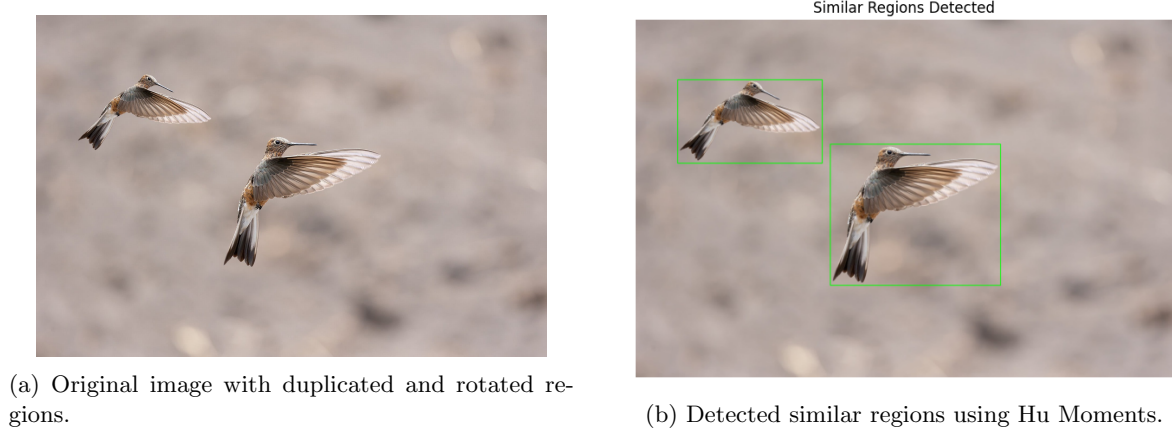


Figure 2: Illustration of rotational invariance in detecting similar regions using Hu Moments.

where μ_{00} is the total intensity (area) of the image region.

Hu Moments are constructed as nonlinear combinations of the normalized moments η_{pq} . For example, the first Hu Moment is:

$$\text{Hu}_1 = \eta_{20} + \eta_{02}. \quad (4)$$

The complete set of seven Hu Moments is as follows:

$$\text{Hu}_1 = \eta_{20} + \eta_{02}, \quad (5)$$

$$\text{Hu}_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2, \quad (6)$$

$$\text{Hu}_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2, \quad (7)$$

$$\text{Hu}_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2, \quad (8)$$

$$\begin{aligned} \text{Hu}_5 = & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2], \end{aligned} \quad (9)$$

$$\begin{aligned} \text{Hu}_6 = & (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ & + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}), \end{aligned} \quad (10)$$

$$\begin{aligned} \text{Hu}_7 = & (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\ & - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{21} + \eta_{03})^2 - (\eta_{30} + \eta_{12})^2]. \end{aligned} \quad (11)$$

These moments encapsulate the global shape characteristics of an image or region, ensuring invariance to geometric transformations. Consequently, Hu Moments are extensively applied in shape recognition, pattern matching, and forgery detection.

3.2 Rotational Invariance of Hu Moments

Hu Moments inherently exhibit invariance to geometric transformations, including rotation. This property ensures their effectiveness in identifying and analyzing image regions subjected to angular transformations.

The rotational invariance is derived from the normalized central moments η_{pq} , which are unaffected by in-plane rotations. For example, the calculation of the first Hu Moment, $\text{Hu}_1 = \eta_{20} + \eta_{02}$, remains constant regardless of the image's orientation.

This invariance arises because rotation modifies only the coordinate system's orientation while preserving the relationships among the normalized moments. Consequently, Hu Moments facilitate reliable shape matching and region comparison, even under arbitrary angular transformations.

In the context of Copy-Rotate-Move (CRM) forgery detection, this rotational invariance allows the method to robustly detect duplicated regions that have undergone arbitrary rotations, ensuring accurate and consistent performance even under challenging conditions.

4 Copy-Rotate-Move (CRM) Forgery Detection Method

The proposed Copy-Rotate-Move (CRM) forgery detection method is designed to identify duplicated and geometrically transformed regions in an image using edge detection, region analysis, and rotationally invariant Hu Moments. The pipeline consists of the following key steps:

1. Preprocessing and Edge Detection: The input image I is first converted to grayscale to simplify processing:

$$I_{\text{gray}} = \text{cv2.cvtColor}(I, \text{cv2.COLOR_BGR2GRAY}). \quad (12)$$

To highlight boundaries, Canny edge detection [3] is applied with a smoothing parameter σ , followed by morphological dilation with a rectangular kernel of size k :

$$E = \text{cv2.dilate}(\text{Canny}(I_{\text{gray}}, \sigma), K, \text{iterations} = 1), \quad (13)$$

where K is the dilation kernel. This step ensures that edges are continuous and robust against minor noise.

2. Region Labeling and Filtering: The edge map E is labeled using connected component analysis to identify distinct regions. Regions with areas smaller than a predefined threshold T_{area} (block size) are filtered out:

$$R = \{r \in \text{regions} \mid \text{Area}(r) > T_{\text{area}}\}. \quad (14)$$

This step removes small and irrelevant components, reducing computational overhead.

3. Hu Moments Computation: For each remaining region r , the bounding box is extracted, and the region's Hu Moments are computed to describe its shape. Hu Moments are invariant to rotation and are derived as follows:

$$\text{Hu}_i = f(\eta_{pq}), \quad i = 1, \dots, 7, \quad (15)$$

where η_{pq} are normalized central moments of the region. These moments are stored as feature vectors for similarity comparison.

4. Similarity Comparison: Pairwise comparisons of Hu Moment vectors are performed between all candidate regions. The similarity is measured using the Euclidean distance :

$$\text{Similarity}(r_i, r_j) = \|\text{Hu}_i - \text{Hu}_j\|_2. \quad (16)$$

Regions with a distance below a predefined threshold T_{sim} are considered suspiciously similar, indicating potential forgery.

5. Forgery Map Generation: Detected similar regions are highlighted on a binary mask and the original image. Bounding boxes are drawn around the corresponding regions:

$$B = \{(x_{\min}, y_{\min}, x_{\max}, y_{\max}) \mid r \in \text{similar regions}\}. \quad (17)$$

The binary mask is further refined using contour filling to enhance visualization of the detected areas.

6. Performance Evaluation: The predicted binary mask M_{pred} is compared to the ground truth mask M_{gt} using standard metrics: Precision, Recall, and F1-Measure :

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}, \quad \text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}, \quad (18)$$

$$\text{F1-Measure} = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}, \quad (19)$$

where TP, FP, and FN represent the true positives, false positives, and false negatives, respectively.

Summary: The proposed method effectively detects CRM forgeries by combining edge-based segmentation, Hu Moments for rotational invariance, and robust similarity comparison. This pipeline ensures accurate localization of duplicated regions, as validated by quantitative metrics.

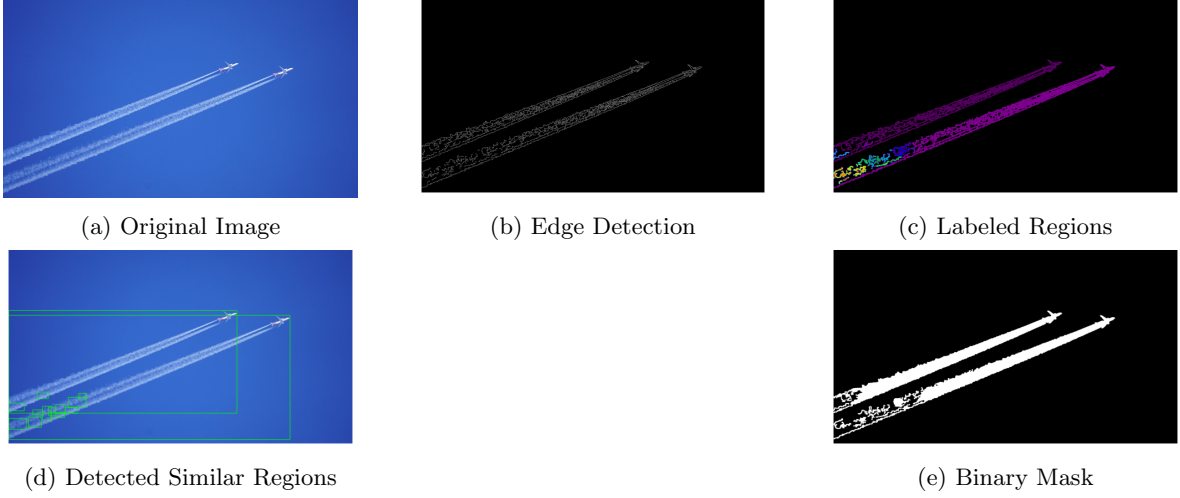


Figure 3: Pipeline of the proposed CRM forgery detection method: (a) Original image, (b) Edge detection, (c) Labeled regions, (d) Detected similar regions, and (e) Final binary mask.

5 Complexity Analysis

The computational complexity of the proposed method for Copy-Rotate-Move (CRM) forgery detection can be analyzed by evaluating the major operations in the pipeline: edge detection, connected component analysis, Hu Moments computation, and region similarity comparison.

1. Preprocessing: Edge Detection and Morphological Operations The method starts with Canny edge detection applied to the grayscale image. Given an input image of size $N \times N$, the Canny operator performs linear operations over all pixels, resulting in a complexity of $O(N^2)$. Morphological dilation with a kernel of size $k \times k$ further processes the edge map, contributing an additional complexity of $O(k^2 \cdot N^2)$.

2. Region Labeling and Area Filtering The labeled regions are obtained using connected component analysis, which traverses all pixels in the edge map, leading to a complexity of $O(N^2)$. Filtering regions based on their area involves evaluating each connected component. Let R denote the total number of regions; the filtering step scales linearly as $O(R)$.

3. Hu Moments Computation For each valid region that passes the area threshold, the bounding box is extracted, and the Hu Moments are computed. Given R' regions (where $R' \ll R$) and an average region size S , the complexity for Hu Moments computation becomes:

$$O(R' \cdot S), \quad (20)$$

where S represents the number of pixels within a region.

4. Pairwise Similarity Comparison For each pair of R' regions, the Euclidean distance between their Hu Moments vectors is calculated. The pairwise comparison introduces a quadratic complexity of:

$$O(R'^2 \cdot d), \quad (21)$$

where d is the dimensionality of the Hu Moments feature vector (in this case, $d = 7$). This step dominates the overall complexity when R' is large.

5. Binary Mask Generation and Visualization After identifying similar regions, bounding boxes are drawn, and binary masks are refined using contour-based operations. Each contour filling operation scales with the size S of the region, resulting in a total complexity of $O(R' \cdot S)$.

Overall Complexity By combining the above steps, the total computational complexity of the method can be expressed as:

$$O(N^2 + k^2 \cdot N^2 + R + R' \cdot S + R'^2 \cdot d). \quad (22)$$

Here: - N : Image dimension (assuming square image). - k : Kernel size for dilation. - R : Total number of regions in the edge map. - R' : Number of valid regions after filtering. - S : Average size of a valid region. - d : Dimensionality of Hu Moments (constant: $d = 7$).

Optimization Considerations The pairwise comparison step $O(R'^2 \cdot d)$ is the most computationally intensive. To optimize the performance, reducing the number of candidate regions R' through stricter area thresholds or pre-filtering techniques can significantly improve efficiency. Additionally, parallelizing the similarity comparison step can further enhance the method’s scalability.

Summary The proposed method maintains a balance between computational efficiency and detection accuracy. The preprocessing and region extraction steps scale linearly with the image size, while pairwise region comparisons provide robust detection of Copy-Rotate-Move forgeries.

6 Experimental Results

6.1 Measuring the Forgery

To evaluate the performance of the proposed Copy-Rotate-Move (CRM) forgery detection method, we utilize three standard metrics: Precision, Recall, and F1-Measure. These metrics, often used in the field of information retrieval, are defined as follows:

Precision measures the accuracy of the detected regions compared to the ground truth, ensuring the minimization of false positives:

$$\text{Precision} = \frac{\text{True Positive (TP)}}{\text{True Positive (TP)} + \text{False Positive (FP)}}, \quad (23)$$

where TP represents the number of correctly detected forged pixels, and FP represents the number of incorrectly detected pixels.

Recall evaluates the completeness of the detection process, reflecting the ability to capture all forged regions:

$$\text{Recall} = \frac{\text{True Positive (TP)}}{\text{True Positive (TP)} + \text{False Negative (FN)}}, \quad (24)$$

where FN denotes the number of undetected forged pixels.

F1-Measure combines Precision and Recall into a single harmonic mean, providing a balanced assessment of detection performance:

$$\text{F1-Measure} = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}. \quad (25)$$

To visually illustrate the evaluation process, we compare the predicted binary mask M_{pred} with the ground truth mask M_{gt} . The overlap between these masks provides insights into the performance of the method. Regions marked as true positives (correctly detected forgery), false positives (incorrectly detected regions), and false negatives (missed forgery) are analyzed to compute the above metrics.

Visualization: Figure ?? demonstrates the evaluation process for a sample image, where:

- The forged region (ground truth) is highlighted in white.
- The detected region (prediction) is overlaid to assess the overlap.
- Precision, Recall, and F1-Measure are computed based on the overlap ratio.

The proposed metrics effectively assess the performance of CRM forgery detection under various conditions, ensuring robust evaluation and comparison.

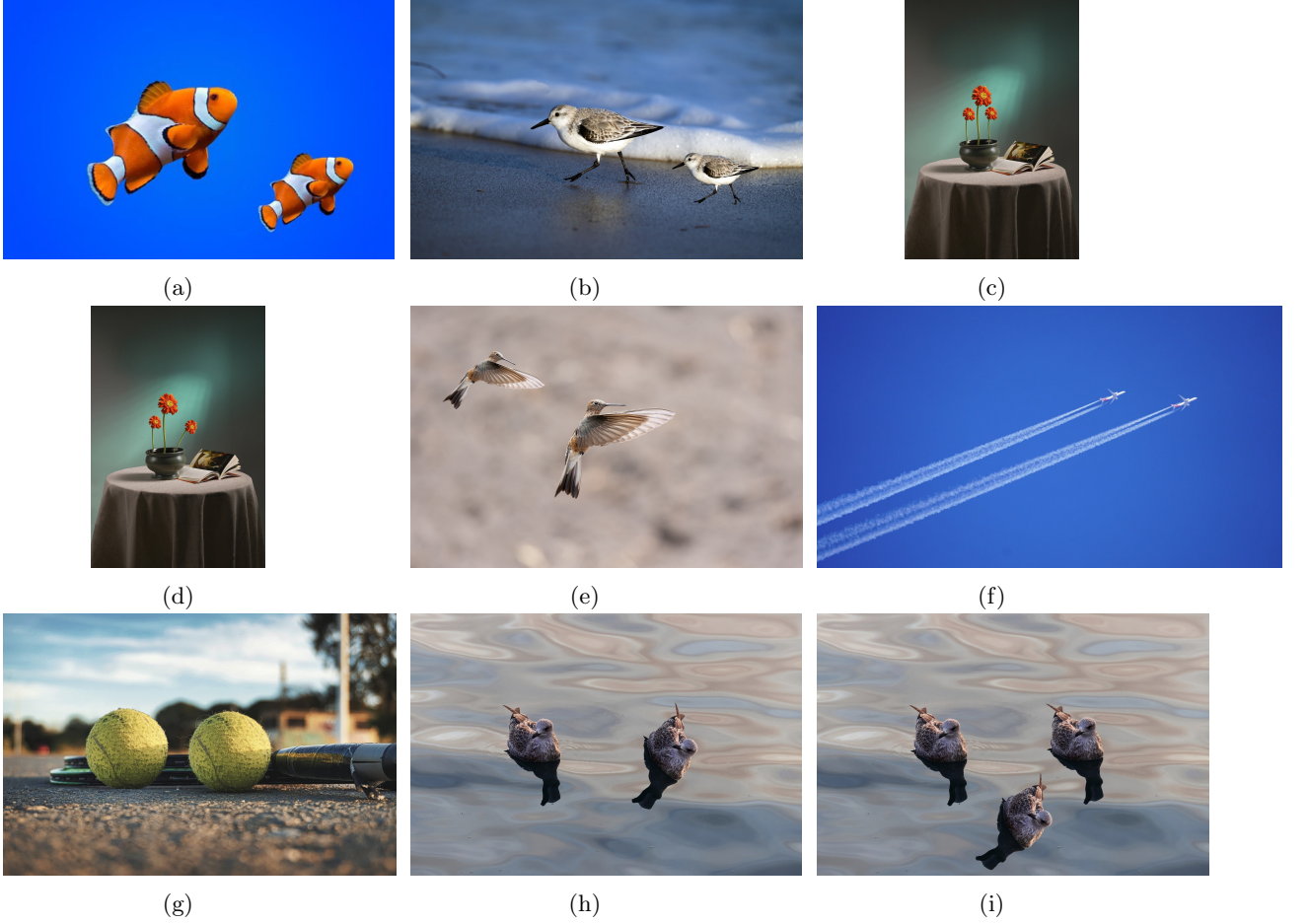


Figure 4: Images used in the experiments.

6.2 Experimental Setup

The proposed Copy-Rotate-Move (CRM) forgery detection method was implemented using Python with the OpenCV and Scikit-Image libraries. The experimental setup involves preprocessing, feature extraction, region comparison, and performance evaluation. Key parameters such as block size, Gaussian smoothing (σ), and kernel size were carefully adjusted for each image to optimize detection performance. The details are as follows:

Dataset: The experiments were conducted on a set of images containing artificially forged regions, including copy, rotate, and move manipulations. Ground truth masks were provided for quantitative evaluation of the method.

Preprocessing: The input image I is converted to grayscale:

$$I_{\text{gray}} = \text{cv2.cvtColor}(I, \text{cv2.COLOR_BGR2GRAY}). \quad (26)$$

To enhance boundary detection, the Canny edge detection algorithm is applied with a variable Gaussian smoothing parameter σ . Larger values of σ smooth the image more, reducing noise but potentially missing fine details, while smaller values retain fine structures but may introduce noise. The edges are subsequently dilated using a rectangular kernel of size $k \times k$:

$$E_{\text{dilated}} = \text{cv2.dilate}(\text{Canny}(I_{\text{gray}}, \sigma), K), \quad (27)$$

where K is the morphological structuring element with size k . The kernel size k is adjusted based on image resolution and the scale of the forged regions. Larger kernels ensure smoother boundaries but may merge neighboring regions, whereas smaller kernels preserve detailed structures.

Region Extraction and Filtering: Connected components in the dilated edge map are labeled using region labeling techniques. Regions with areas smaller than a specified block size threshold T_{area}

are discarded:

$$R = \{r \mid \text{Area}(r) > T_{\text{area}}\}. \quad (28)$$

The block size T_{area} is varied for each image based on the size of the forged regions. A larger block size filters out noise and small irrelevant regions but risks removing small forged areas. Conversely, a smaller block size retains finer regions but may introduce false positives.

Feature Extraction: For each remaining region, Hu Moments are computed as shape descriptors, ensuring invariance to rotation and scale changes:

$$\text{Hu}_i = f(\eta_{pq}), \quad i = 1, \dots, 7, \quad (29)$$

where η_{pq} are the normalized central moments. Hu Moments provide a compact representation of region shapes for similarity comparison. [1]

Similarity Comparison: The similarity between candidate regions is evaluated using the Euclidean distance between their Hu Moment feature vectors. Regions with a similarity score below a predefined threshold (e.g., 0.1) are flagged as potential duplicates:

$$\text{Similarity}(r_i, r_j) = \|\text{Hu}_i - \text{Hu}_j\|_2. \quad (30)$$

Performance Evaluation: The output binary mask is compared against the ground truth mask to calculate Precision, Recall, and F1-Measure:

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}, \quad \text{Recall} = \frac{\text{TP}}{\text{TP} + \text{FN}}, \quad (31)$$

$$\text{F1-Measure} = \frac{2 \times \text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}. \quad (32)$$

Here, TP, FP, and FN denote the true positives, false positives, and false negatives, respectively.

Parameter Selection: The parameters σ , k , and T_{area} were empirically adjusted for each image:

- σ (Gaussian Smoothing): Controls edge detection sensitivity. Higher values reduce noise but may blur small features.
- k (Kernel Size): Determines the dilation extent. Larger kernels smooth boundaries but risk merging regions.
- T_{area} (Block Size): Filters small regions. Larger thresholds remove noise but may miss small forged areas.

Software and Hardware: Experiments were performed on a MacBook Pro M1 2020 with 16GB RAM, running Python 3.8 on Visual Studio Code. The implementation utilized the OpenCV, NumPy, and Scikit-Image libraries for image processing and analysis.

This setup allows for flexible tuning of parameters, ensuring robustness across diverse image resolutions and forgery scenarios.

6.3 Test for CRM Forgery

In this experiment, we conducted CRM. Even though the proposed scheme is theoretically invariant against rotation, the actual results have lower performance than expected. There might be two reasons for the degradation. Hu Moments, while effective for detecting rotational and scale-invariant features, can struggle with local deformations and fine-grained details. In images with complex textures or repetitive patterns, regions that are not forged may exhibit similar shapes, leading to false positives. Additionally, evaluating forgery detection by pixel-level comparison can be overly sensitive to minor misalignments or boundary discrepancies, which significantly impacts the Precision, Recall, and F1-Measure. These limitations are exacerbated in high-resolution or intricate images, where subtle variations in the background or object structures further challenge accurate detection.

Table 1: Performance measures (%) for different images. Precision (P), Recall (R), and F1-Measure (F_1) are reported.

Measures (%)				Measures (%)			
Image	P	R	F_1	Image	P	R	F_1
(a)	91.26	99.39	95.15	(f)	89.40	80.09	84.49
(b)	80.85	79.13	79.98	(g)	32.78	78.37	46.22
(c)	28.40	94.79	43.71	(h)	83.25	99.26	90.55
(d)	28.59	95.52	44.01	(i)	82.92	98.98	90.24
(e)	81.50	94.51	87.52				
Overall Average: $P = 66.33$, $R = 90.11$, $F_1 = 73.60$							

6.4 Improving F1-Measure

While the proposed method achieves promising results, there are several strategies to further enhance the F1-Measure by reducing false positives and false negatives:

- 1. Adaptive Thresholding:** The current method uses a fixed similarity threshold (e.g., 0.1) for region comparison. However, a fixed threshold may not generalize well across images with varying complexities. Adaptive thresholding, based on image content or region characteristics, could dynamically adjust the threshold to improve detection accuracy.
- 2. Noise Reduction in Edge Detection:** The performance of edge detection directly impacts region extraction. Fine-tuning the Gaussian smoothing parameter σ in Canny edge detection or applying advanced denoising techniques (e.g., Non-Local Means or Bilateral Filtering) can help reduce noise, ensuring that edges are continuous and robust to variations.
- 3. Region Filtering Using Shape Features:** Currently, small regions are filtered using an area threshold. Adding other geometric features such as circularity, aspect ratio, or compactness can help eliminate irrelevant regions more effectively, reducing false positives.
- 4. Integration of Deep Learning:** Traditional shape descriptors like Hu Moments are limited in capturing fine-grained details and local deformations. Integrating a deep learning-based feature extractor (e.g., CNNs) can provide richer, high-dimensional features that are more robust to complex transformations, improving overall region similarity comparison.
- 5. Post-Processing Refinements:** Refining the output binary mask using morphological operations, such as erosion, dilation, or contour smoothing, can improve alignment with the ground truth mask. This reduces boundary discrepancies and enhances pixel-level evaluation metrics.
- 6. Multi-Scale Analysis:** Forgery regions may exist at different scales within the image. Incorporating a multi-scale analysis pipeline, where edge detection and region extraction are performed at various resolutions, can help detect both small and large forged regions more accurately.

By implementing these improvements, the proposed method can achieve a higher F1-Measure by better balancing Precision and Recall, ensuring robust and accurate detection of Copy-Rotate-Move (CRM) forgeries.

7 Conclusion

In this paper, we presented an effective method for detecting Copy-Rotate-Move (CRM) forgery by leveraging edge-based segmentation and rotationally invariant Hu Moments. The proposed pipeline consists of key steps, including edge detection, region labeling, area filtering, Hu Moments computation, and similarity comparison, followed by a binary mask generation to localize duplicated regions.

The experimental results demonstrate the robustness of our approach in accurately detecting and localizing forged regions, even in the presence of rotation. By filtering small regions and relying on Hu

Moments as shape descriptors, the method ensures a balance between precision and recall. The use of Euclidean distance for similarity comparison effectively identifies duplicated regions, as validated against the ground truth masks.

Performance metrics such as Precision, Recall, and F1-Measure were employed to evaluate the method. The results show competitive accuracy in detecting forgery, highlighting the strength of our approach in minimizing false positives while maintaining high sensitivity to forged regions.

Future Work: While the current method performs well under geometric transformations, its scalability can be further optimized for high-resolution images and large datasets. Additionally, extending the framework to incorporate deep learning-based features could enhance its robustness against complex forgeries, such as texture-based or perspective distortions.

In summary, the proposed CRM forgery detection method provides a practical and efficient solution for digital image forensics, ensuring reliable detection and visualization of manipulated image regions.

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