Noise Fundamentals: where noise *is* the signal

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Abstract

The measurement and theoretical examination of noise is an important and underrepresented topic in the advanced laboratory. Noise is inherent in all measurements and sets the lower limit of any physical measurement. Noise can also be used to determine fundamental constants. The Johnson voltage noise from a resistor ($v_n^2 = 4\ R\ k_B T\ \Delta f$) gives a measure of the Boltzmann constant (k_B) if the resistance (R) and temperature (T) are known. The charge of the electron (e) can be determined from a DC current (I) with full shot noise ($i_n^2 = 2\ e\ I\ \Delta f$).

The apparatus that will be demonstrated can be used to measure the Johnson noise from resistors in the range from 10 Ω to 10 MΩ, over a calibrated frequency range of 3 Hz to 100 kHz and temperatures from 77 to 373 K. Shot noise can be measured in currents ranging from 10 nA to 1 mA. The currents can be derived from a variety of sources: a photodiode illuminated by an incandescent light bulb or an LED; a forward biased PN junction; a diode-connected bipolar junction transistor; or from a zener diode. Some of the experimental subtleties of noise measurement will be presented, especially the effect of capacitance.

Introduction

What is noise? Where does it come from? Can you get rid of it? (and can you learn something from the bits you can't get rid of?)

We are only talking about electronic noise: the noise caused by the motion of electrons. Noise is generally an unwanted signal that exists 'on top' of the signal you want to measure.

Several sources of noise are observed in the lab:

Technical noise:
- Electro'static' (fluorescent lights)
- Magneto'static' (ground loops and AC transformers)
- E&M radiation (room lights into PN diodes, sparks)
- Acoustic, thermoelectric, triboelectric……

Fundamental noise*
- Thermal motion of electrons (Johnson noise)
- Discrete size of moving electrons (Shot noise)

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*Rolf Landauer would like us to understand these separate types of fundamental noise as being related. Both are due to the motion of electrons driven either thermally or by an emf. In certain devices (diodes) it is possible to go continuously from thermal noise to shot noise.

Theory

Any noise signal has time average $\langle V(t)\rangle = 0$, but the mean-square value $\langle V^2(t)\rangle$ is non-zero, and can serve as a measure of the noise. Because noise is distributed in frequency space, the noise measured will depend on the bandwidth Δf. White noise is characterized by having uniform noise power per unit bandwidth, with natural units Volts² / Hertz (V²/Hz). Two fundamental mechanisms do generate (Gaussian) white noise. All resistors (R) at non-zero temperature (T) show Johnson noise,

$$\langle V_n^2\rangle = 4\ k_B\ T\ R\ \Delta f.$$

Some currents show shot noise,

$$\langle I_n^2\rangle = 2\ e\ I_{DC}\ \Delta f,$$

where I_{DC} is the average current.

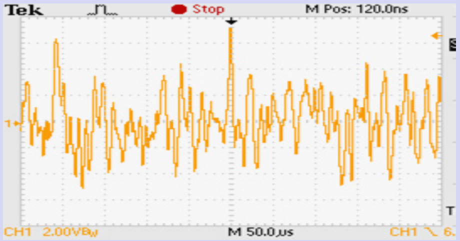

Fig. 1 An oscilloscope trace of noise

Apparatus

The signal chain starts in the preamp stage. The first amplifier is configured appropriately for the type of measurement, and is followed by a x100 gain stage. The bandwidth Δf is defined by adjustable low-side and high-side filters. There follows a further gain stage, adjustable from x10 to x10⁴. An analog multiplier generates the real-time square of the filtered and amplified voltage. That square is time-averaged to yield an output voltage. A model of the whole chain is

$$V_{in} \rightarrow GV_{in} \rightarrow (GV_{in})^2 \rightarrow G^2\langle V_{in}^2\rangle = V_{out}$$

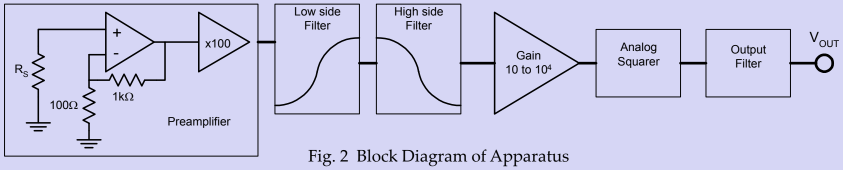

Fig. 2 Block Diagram of Apparatus

Data

Figure 3 shows a test of the claim that Johnson noise gives an output signal proportional to the bandwidth Δf used in the signal chain.

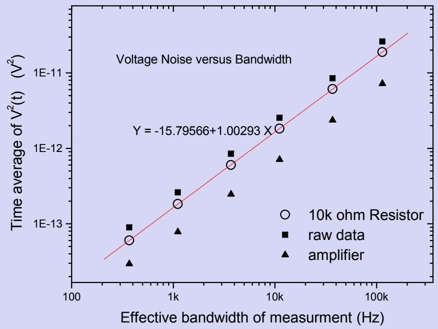

Fig. 3 Voltage noise versus Bandwidth

Note this log-log plot shows a linear fit with slope 1.003, indicating $\langle V_n^2\rangle \sim \Delta f$ consistent with expectations. Such a fit motivates forming the quotient $\langle V_n^2\rangle / \Delta f$, with units Volts²/ Hertz, in the graphs below.

Figure 4 shows a test of the claim that Johnson noise is 'white', ie. that the noise density is independent of the center frequency of the band pass filter.

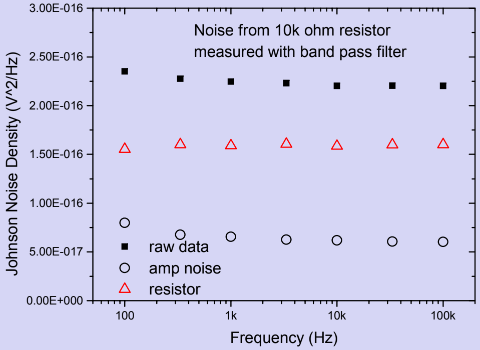

Fig. 4 Voltage noise versus Center Frequency

Note the vertical axis in Fig 4. has a linear scale, and the corrected data display a scatter of only +/- 1% over three decades of frequency.

Figure 5 tests the claim that Johnson noise gives a voltage noise power proportional to source resistance.

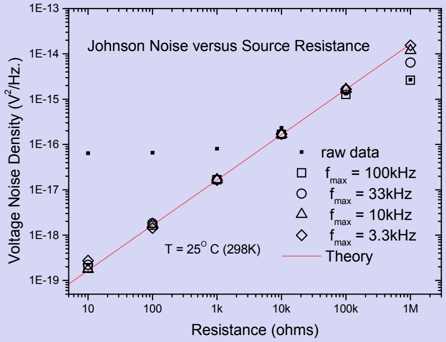

Fig. 5 Voltage noise versus Source Resistance

Note that the amplifier noise can be successfully subtracted to reveal the Johnson noise generated by a 10 Ω resistor. For a large source resistance, capacitive effects reduce the noise content for measurements with higher maximum frequencies. The points do indeed fall on a line computed from the textbook value of Boltzmann's constant and the ambient temperature.

Figure 6 shows a measurement of the shot noise current, generated in a silicon photodiode illuminated by an incandescent light bulb. The photocurrent's average value varies over a range of 10⁴, and again the input-stage amplifier noise can be successfully subtracted to show that $\langle I_n^2\rangle$ is proportional to I_{DC}. The corrected points fall on a line computed from the textbook value of e, the charge of the electron.

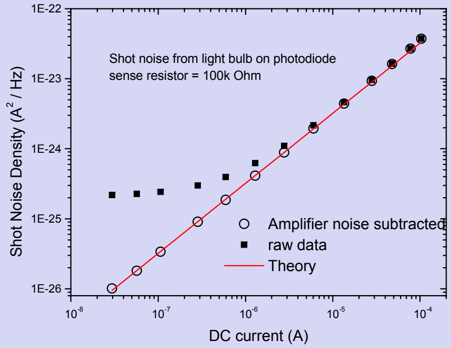

Fig. 6 Current noise versus DC Current