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1.1 Subproblem Variable Metric

For comparision: Subproblem proximal bundle

$$\min_{d\in\mathbb{R}^n,\xi\in\mathbb{R}} \xi + \frac{1}{2t_k}\|d\|^2 = \xi + \frac{1}{2}d^\top \left(\frac{1}{t_k}\mathbf{I}\right) d \tag{1}$$

$$\text{s.t.} \quad f(\hat{x}^k) + {g^j}^\top d - e_j^k - \xi \leq 0, \quad j \in J_k \tag{2}$$

Subproblem variable metric:

$$\min_{d\in\mathbb{R}^n,\xi in\mathbb{R}} \xi + \frac{1}{2}d^\top D_k d \tag{3}$$

$$\text{s.t.} \quad f(\hat{x}^k) + {g^j}^\top d - e_j^k - \xi \leq 0, \quad j \in J_k \tag{4}$$

These are $\mathbb{R}^{n+1}$ dimensional quadratic optimization problems.

Find out if D_k is diagonal matrix! Think not.

Approaches not so different. Instead of just scaling the identity $\rightarrow$ induce "curvature information" via past subgradients.

Dual proximal subproblem:

$$\min_{\alpha\in\mathbb{R}^{|J_k|}} \frac{1}{2}\left(\sum_{j\in J_k}\alpha_j g^j\right)^\top t_k\mathbf{I}\left(\sum_{j\in J_k}\alpha_j g^j\right) + \sum_{j\in J_k}\alpha_j e_j^k \tag{5}$$

$$\text{s.t.} \quad \sum_{j\in J_k}\alpha_j = 1 \text{ and } \alpha_j \geq 0 \; j \in J_k \tag{6}$$

Dual variable metric subproblem:

$$\min_{\alpha\in\mathbb{R}^{|J_k|}} \frac{1}{2}\left(\sum_{j\in J_k}\alpha_j g^j\right)^\top D_k^{-1}\left(\sum_{j\in J_k}\alpha_j g^j\right) + \sum_{j\in J_k}\alpha_j e_j^k \tag{7}$$

$$\text{s.t.} \quad \sum_{j\in J_k}\alpha_j = 1 \text{ and } \alpha_j \geq 0 \; j \in J_k \tag{8}$$

These are $\mathbb{R}^{|J_k|}$ dimensional quadratic optimization problems.

check linear independent g^j's.

References