1

C[m] = 1/N SUM$_{n=0}^{N-1} x[n] * exp(-2 * PI * m * n * j/N) C[0] = 1/5 * SUM_{n=0}^{4} x[n] * exp(-2 *$
$1/5 * 9 * (cos(-2 * PI * 0 * 0/5) + j * sin(-2 * PI * 0 * 0/5)) + 1/5 * 9 * (cos(-2 * PI *$
$0 * 1/5) + j * sin(-2 * PI * 0 * 1/5)) + 1/5 * 9 * (cos(-2 * PI * 0 * 2/5) + j * sin(-2 *$
$PI * 0 * 2/5)) + 1/5 * 9 * (cos(-2 * PI * 0 * 3/5) + j * sin(-2 * PI * 0 * 3/5)) + 1/5 *$
$9 * (cos(-2 * PI * 0 * 4/5) + j * sin(-2 * PI * 0 * 4/5)) = 1.6 + j * 0.0 = 1.6 * e^{0.0}$
$A[0] = SQRTREAL(C[0])^2 + IM(C[0)^2 = SQRT2.56 + 0.0 = 1.6$
$PHI[0] = arctgIM(C[0])/REAL(C[0) = arctg0.0/1.6 = 0.0$

C[1] = 1/5*SUM$_{n=0}^{4} x[n] * exp(-2 * PI * 1 * n * j/5) = 1/5 * -4 * (cos(-2 * PI *$
$1 * 0/5) + j * sin(-2 * PI * 1 * 0/5)) + 1/5 * -4 * (cos(-2 * PI * 1 * 1/5) + j * sin(-2 *$
$PI * 1 * 1/5)) + 1/5 * -4 * (cos(-2 * PI * 1 * 2/5) + j * sin(-2 * PI * 1 * 2/5)) +$
$1/5 * -4 * (cos(-2 * PI * 1 * 3/5) + j * sin(-2 * PI * 1 * 3/5)) + 1/5 * -4 * (cos(-2 *$
$PI * 1 * 4/5) + j * sin(-2 * PI * 1 * 4/5)) = 1.97 + j * 2.11 = 2.89 * e^{0.82}$
$A[1] = SQRTREAL(C[1])^2 + IM(C[1)^2 = SQRT3.8809 + 4.4521 = 2.89$

PHI[1]=arctgIM(C[1])/REAL(C[1) = arctg2.11/1.97 = 0.82

C[2] = 1/5*SUM$_{n=0}^{4} x[n] * exp(-2 * PI * 2 * n * j/5) = 1/5 * -3 * (cos(-2 * PI *$
$2 * 0/5) + j * sin(-2 * PI * 2 * 0/5)) + 1/5 * -3 * (cos(-2 * PI * 2 * 1/5) + j * sin(-2 *$
$PI * 2 * 1/5)) + 1/5 * -3 * (cos(-2 * PI * 2 * 2/5) + j * sin(-2 * PI * 2 * 2/5)) +$
$1/5 * -3 * (cos(-2 * PI * 2 * 3/5) + j * sin(-2 * PI * 2 * 3/5)) + 1/5 * -3 * (cos(-2 *$
$PI * 2 * 4/5) + j * sin(-2 * PI * 2 * 4/5)) = 1.73 + j * -0.01 = 1.73 * e^{-0.01}$
$A[2] = SQRTREAL(C[2])^2 + IM(C[2)^2 = SQRT2.9929 + 0.0001 = 1.73$
$PHI[2] = arctgIM(C[2])/REAL(C[2) = arctg-0.01/1.73 = -0.01$

C[3] = 1/5*SUM$_{n=0}^{4} x[n] * exp(-2 * PI * 3 * n * j/5) = 1/5 * 2 * (cos(-2 * PI * 3 *$
$0/5) + j * sin(-2 * PI * 3 * 0/5)) + 1/5 * 2 * (cos(-2 * PI * 3 * 1/5) + j * sin(-2 * PI * 3 * 1/5)) +$
$1/5 * 2 * (cos(-2 * PI * 3 * 2/5) + j * sin(-2 * PI * 3 * 2/5)) + 1/5 * 2 * (cos(-2 * PI * 3 * 3/5) +$
$j * sin(-2 * PI * 3 * 3/5)) + 1/5 * 2 * (cos(-2 * PI * 3 * 4/5) + j * sin(-2 * PI * 3 * 4/5)) =$
$1.73 + j * 0.01 = 1.73 * e^{0.01}$
$A[3] = SQRTREAL(C[3])^2 + IM(C[3)^2 = SQRT2.9929 + 0.0001 = 1.73$
$PHI[3] = arctgIM(C[3])/REAL(C[3) = arctg0.01/1.73 = 0.01$

C[4] = 1/5*SUM$_{n=0}^{4} x[n] * exp(-2 * PI * 4 * n * j/5) = 1/5 * 4 * (cos(-2 * PI * 4 *$
$0/5) + j * sin(-2 * PI * 4 * 0/5)) + 1/5 * 4 * (cos(-2 * PI * 4 * 1/5) + j * sin(-2 * PI * 4 * 1/5)) +$
$1/5 * 4 * (cos(-2 * PI * 4 * 2/5) + j * sin(-2 * PI * 4 * 2/5)) + 1/5 * 4 * (cos(-2 * PI * 4 * 3/5) +$
$j * sin(-2 * PI * 4 * 3/5)) + 1/5 * 4 * (cos(-2 * PI * 4 * 4/5) + j * sin(-2 * PI * 4 * 4/5)) =$
$1.97 + j * -2.11 = 2.89 * e^{-0.82}$
$A[4] = SQRTREAL(C[4])^2 + IM(C[4)^2 = SQRT3.8809 + 4.4521 = 2.89$
$PHI[4] = arctgIM(C[4])/REAL(C[4) = arctg-2.11/1.97 = -0.82$

2

$$\text{X[0]} = \text{SUM}_{n=0}^{4} C[n] * exp(-2 * PI * 0 * n * j/5) =$$
$$(1.6 + j * 0.0) * (cos(-2 * PI * 0 * 0/5) + j * sin(-2 * PI * 0 * 0/5)) + (1.97 + j * 2.11) * (cos(-2 * PI * 0 * 1/5) + j * sin(-2 * PI * 0 * 1/5)) + (1.73 + j * -0.01) * (cos(-2 * PI * 0 * 2/5) + j * sin(-2 * PI * 0 * 2/5)) + (1.73 + j * 0.01) * (cos(-2 * PI * 0 * 3/5) + j * sin(-2 * PI * 0 * 3/5)) + (1.97 + j * -2.11) * (cos(-2 * PI * 0 * 4/5) + j * sin(-2 * PI * 0 * 4/5)) =$$
$$(1.6 + j * 0.0) * (1.0 + j * 0.0) + (1.97 + j * 2.11) * (1.0 + j * 0.0) + (1.73 + j * -0.01) * (1.0 + j * 0.0) + (1.73 + j * 0.01) * (1.0 + j * 0.0) + (1.97 + j * -2.11) * (1.0 + j * 0.0) =$$
$$9.0 + j * -4.4408920985e - 16$$
$$\text{X[1]} = \text{SUM}_{n=0}^{4} C[n] * exp(-2 * PI * 1 * n * j/5) =$$
$$(1.6 + j * 0.0) * (cos(-2 * PI * 1 * 0/5) + j * sin(-2 * PI * 1 * 0/5)) + (1.97 + j * 2.11) * (cos(-2 * PI * 1 * 1/5) + j * sin(-2 * PI * 1 * 1/5)) + (1.73 + j * -0.01) * (cos(-2 * PI * 1 * 2/5) + j * sin(-2 * PI * 1 * 2/5)) + (1.73 + j * 0.01) * (cos(-2 * PI * 1 * 3/5) + j * sin(-2 * PI * 1 * 3/5)) + (1.97 + j * -2.11) * (cos(-2 * PI * 1 * 4/5) + j * sin(-2 * PI * 1 * 4/5)) =$$
$$(1.6+j*0.0)*(1.0+j*0.0)+(1.97+j*2.11)*(0.31+j*0.95)+(1.73+j*-0.01)*(-0.81+j*0.59)+(1.73+j*0.01)*(-0.81+j*-0.59)+(1.97+j*-2.11)*(0.31+j*-0.95) =$$
$$-3.9784 + j * 0.0$$
$$\text{X[2]} = \text{SUM}_{n=0}^{4} C[n] * exp(-2 * PI * 2 * n * j/5) =$$
$$(1.6 + j * 0.0) * (cos(-2 * PI * 2 * 0/5) + j * sin(-2 * PI * 2 * 0/5)) + (1.97 + j * 2.11) * (cos(-2 * PI * 2 * 1/5) + j * sin(-2 * PI * 2 * 1/5)) + (1.73 + j * -0.01) * (cos(-2 * PI * 2 * 2/5) + j * sin(-2 * PI * 2 * 2/5)) + (1.73 + j * 0.01) * (cos(-2 * PI * 2 * 3/5) + j * sin(-2 * PI * 2 * 3/5)) + (1.97 + j * -2.11) * (cos(-2 * PI * 2 * 4/5) + j * sin(-2 * PI * 2 * 4/5)) =$$
$$(1.6+j*0.0)*(1.0+j*0.0)+(1.97+j*2.11)*(-0.81+j*0.59)+(1.73+j*-0.01)*(0.31+j*-0.95)+(1.73+j*0.01)*(0.31+j*0.95)+(1.97+j*-2.11)*(-0.81+j*-0.59) =$$
$$-3.0276 + j * 2.22044604925e - 16$$
$$\text{X[3]} = \text{SUM}_{n=0}^{4} C[n] * exp(-2 * PI * 3 * n * j/5) =$$
$$(1.6+j*0.0)*(cos(-2*PI*3*0/5)+j*sin(-2*PI*3*0/5))+(1.97+j*2.11)*(cos(-2*PI*3*1/5)+j*sin(-2*PI*3*1/5))+(1.73+j*-0.01)*(cos(-2*PI*3*2/5)+j*sin(-2*PI*3*2/5))+(1.73+j*0.01)*(cos(-2*PI*3*3/5)+j*sin(-2*PI*3*3/5))+(1.97+j*-2.11)*(cos(-2*PI*3*4/5)+j*sin(-2*PI*3*4/5)) = (1.6+j*0.0)*(1.0+j*0.0)+(1.97+j*2.11)*(-0.81+j*-0.59)+(1.73+j*-0.01)*(0.31+j*0.95)+(1.73+j*0.01)*(0.31+j*-0.95)+(1.97+j*-2.11)*(-0.81+j*0.59) = 1.99+j*0.0$$
$$\text{X[4]} =$$
$$\text{SUM}_{n=0}^{4} C[n] * exp(-2 * PI * 4 * n * j/5) = (1.6 + j * 0.0) * (cos(-2 * PI * 4 * 0/5) + j * sin(-2 * PI * 4 * 0/5)) + (1.97 + j * 2.11) * (cos(-2 * PI * 4 * 1/5) + j * sin(-2 * PI * 4 * 1/5)) + (1.73 + j * -0.01) * (cos(-2 * PI * 4 * 2/5) + j * sin(-2 * PI * 4 * 2/5)) + (1.73 + j * 0.01) * (cos(-2 * PI * 4 * 3/5) + j * sin(-2 * PI * 4 * 3/5)) + (1.97 + j * -2.11) * (cos(-2 * PI * 4 * 4/5) + j * sin(-2 * PI * 4 * 4/5)) = (1.6+j*0.0)*(1.0+j*0.0)+(1.97+j*2.11)*(0.31+j*-0.95)+(1.73+j*-0.01)*(-0.81+j*-0.59)+(1.73+j*0.01)*(-0.81+j*0.59)+(1.97+j*-2.11)*(0.31+j*0.95) = 4.016 + j * -2.22044604925e - 16$$
$$\text{SIGMA} = \text{SQRT1/N*SUM}_{n=0}^{N-1} x[n] - X[n]^2 = SQRT1/5 * SUM_{n=0}^{4} x[n] - X[n]^2 =$$
$$SQRT1/5 * ((9 - 9.0)^2 + (-4 - -3.9784)^2 + (-3 - -3.0276)^2 + (2 - 1.99)^2 + (4 - 4.016)^2$$
$$SQRT1/5 * (-1.7763568394e - 15^2 + -0.0216^2 + 0.0276^2 + 0.01^2 + -0.016^2) =$$
$$0.0178006741445$$

9+-4+-3+2+4+0+0+0
9+4+-3+-2+4+0+0+0
9+-4+3+-2+4+0+0+0
9+4+3+2+4+0+0+0
9+-4+-3+2+-4+0+0+0
9+4+-3+-2+-4+0+0+0
9+-4+3+-2+-4+0+0+0
9+4+3+2+-4+0+0+0
[1.0, 1.5, 1.25, 2.75, 0.0, 0.5, 0.25, 1.75]

4

1.0+1.5+1.25+2.75+0.0+0.5+0.25+1.75
1.0+-1.5+1.25+-2.75+0.0+-0.5+0.25+-1.75
1.0+1.5+-1.25+-2.75+0.0+0.5+-0.25+-1.75
1.0+-1.5+-1.25+2.75+0.0+-0.5+-0.25+1.75
1.0+1.5+1.25+2.75+-0.0+-0.5+-0.25+-1.75
1.0+-1.5+1.25+-2.75+-0.0+0.5+-0.25+1.75
1.0+1.5+-1.25+-2.75+-0.0+-0.5+0.25+1.75
1.0+-1.5+-1.25+2.75+-0.0+0.5+0.25+-1.75

5

$$\text{Z} = 1/\text{N*SQRTSUM}_{n=0}^{N-1} x_1[n] * SUM_{n=0}^{N-1} x_2[n] =$$
$$1/5 * SQRTSUM_{n=0}^{4} x_1[n] * SUM_{n=0}^{4} x_2[n] =$$
$$1/5SQRT(9^2 + -4^2 + -3^2 + 2^2 + 4^2)^2 = 1/5 * (81 + 16 + 9 + 4 + 16) = 25.2$$
$$\text{r12[0]} = 1/\text{N*SUM}_{n=0}^{N-1} x[n] * x[n-1] = 1/5 * (x[0] * x[0] + x[1] * x[1] + x[2] * x[2] +$$
$$x[3] * x[3] + x[4] * x[4]) = 1/5 * (9 * 9 + -4 * -4 + -3 * -3 + 2 * 2 + 4 * 4) =$$
$$1/5 * (9 * 9 + -4 * -4 + -3 * -3 + 2 * 2 + 4 * 4) = 25.2$$
$$\text{r12[1]} = 1/\text{N*SUM}_{n=0}^{N-1} x[n] * x[n-1] = 1/5 * (x[0] * x[-1] + x[1] * x[0] + x[2] *$$
$$x[1] + x[3] * x[2] + x[4] * x[3]) = 1/5 * (9 * 0 + -4 * 9 + -3 * -4 + 2 * -3 + 4 * 2) =$$
$$1/5 * (9 * 0 + -4 * 9 + -3 * -4 + 2 * -3 + 4 * 2) = -4.4$$
$$\text{r12[2]} = 1/\text{N*SUM}_{n=0}^{N-1} x[n] * x[n-1] = 1/5 * (x[0] * x[-2] + x[1] * x[-1] + x[2] *$$
$$x[0] + x[3] * x[1] + x[4] * x[2]) = 1/5 * (9 * 0 + -4 * 0 + -3 * 9 + 2 * -4 + 4 * -3) =$$
$$1/5 * (9 * 0 + -4 * 0 + -3 * 9 + 2 * -4 + 4 * -3) = -9.4$$
$$\text{r12[3]} = 1/\text{N*SUM}_{n=0}^{N-1} x[n] * x[n-1] =$$
$$1/5 * (x[0] * x[-3] + x[1] * x[-2] + x[2] * x[-1] + x[3] * x[0] + x[4] * x[1]) =$$
$$1/5*(9*0+-4*0+-3*0+2*9+4*-4) = 1/5*(9*0+-4*0+-3*0+2*9+4*-4) = 0.4$$
$$\text{r12[4]} = 1/\text{N*SUM}_{n=0}^{N-1} x[n] * x[n-1] =$$
$$1/5 * (x[0] * x[-4] + x[1] * x[-3] + x[2] * x[-2] + x[3] * x[-1] + x[4] * x[0]) =$$
$$1/5*(9*0+-4*0+-3*0+2*0+4*9) = 1/5*(9*0+-4*0+-3*0+2*0+4*9) = 7.2$$
$$\text{B12[0]} = \text{r12[0]}/\text{Z} = 25.2/25.2 = 1.0$$
$$\text{B12[1]} = \text{r12[1]}/\text{Z} = \text{-4.4}/25.2 = \text{-0.174603174603}$$
$$\text{B12[2]} = \text{r12[2]}/\text{Z} = \text{-9.4}/25.2 = \text{-0.373015873016}$$
$$\text{B12[3]} = \text{r12[3]}/\text{Z} = 0.4/25.2 = 0.015873015873$$
$$\text{B12[4]} = \text{r12[4]}/\text{Z} = 7.2/25.2 = 0.285714285714$$